Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/38-1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [400]. This is test number [38].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (400)	0.00 (0)
Mathematica	98.50 (394)	1.50 (6)
Maple	89.25 (357)	10.75 (43)
Fricas	88.00 (352)	12.00 (48)
Giac	87.75 (351)	12.25 (49)
Maxima	72.75 (291)	27.25 (109)
Mupad	48.75 (195)	51.25 (205)
Sympy	47.25 (189)	52.75 (211)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

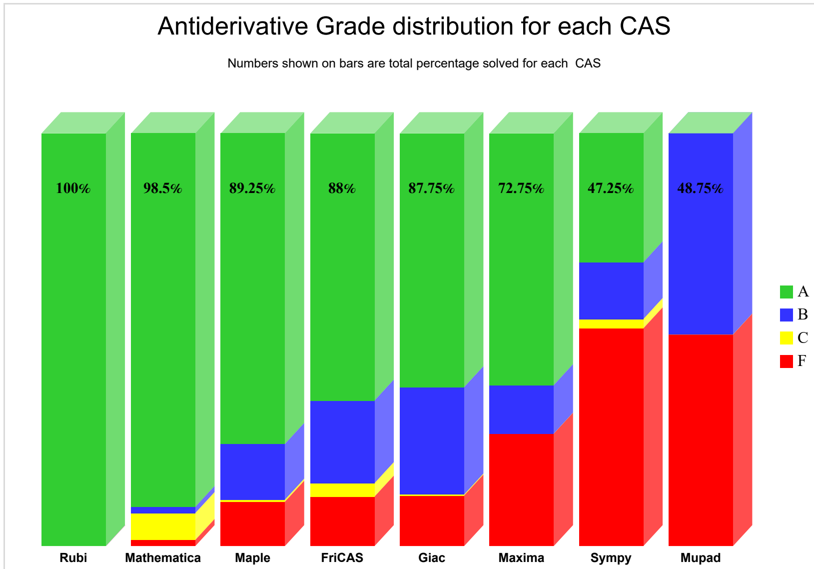
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

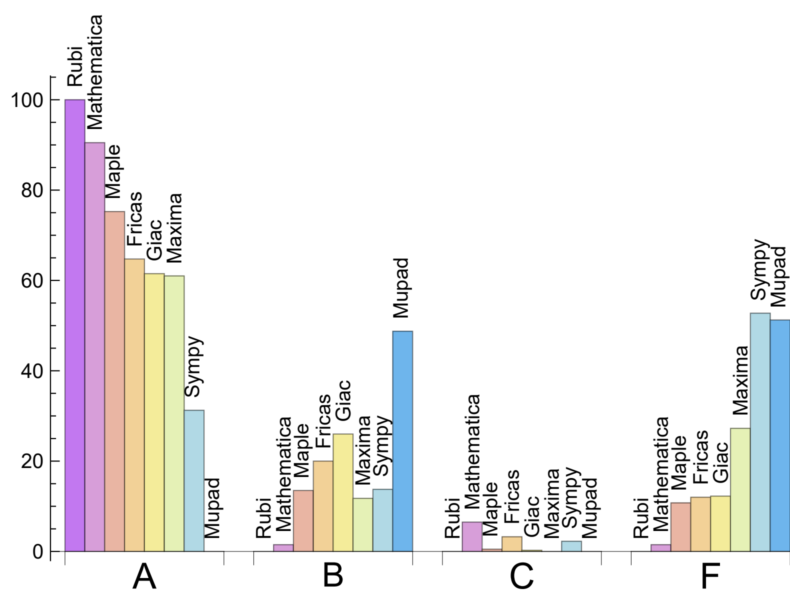
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	90.500	1.500	6.500	1.500
Maple	75.250	13.500	0.500	10.750
Fricas	64.750	20.000	3.250	12.000
Giac	61.500	26.000	0.250	12.250
Maxima	61.000	11.750	0.000	27.250
Sympy	31.250	13.750	2.250	52.750
Mupad	0.000	48.750	0.000	51.250

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	6	100.00	0.00	0.00
Maple	43	27.91	72.09	0.00
Fricas	48	25.00	75.00	0.00
Giac	49	67.35	8.16	24.49
Maxima	109	32.11	0.92	66.97
Mupad	205	0.00	100.00	0.00
Sympy	211	72.99	26.54	0.47

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.27
Giac	0.44
Rubi	0.61
Maple	0.78
Mathematica	2.44
Sympy	3.46
Fricas	3.70
Mupad	9.66

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	243.09	1.04	175.00	1.03
Maxima	419.13	2.05	150.00	1.07
Mathematica	479.92	1.25	136.00	0.95
Fricas	567.76	2.43	190.00	1.39
Maple	770.34	2.12	161.00	0.97
Mupad	773.44	3.16	185.00	1.29
Giac	920.53	2.68	191.00	1.19
Sympy	1766.56	5.34	185.00	1.40

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

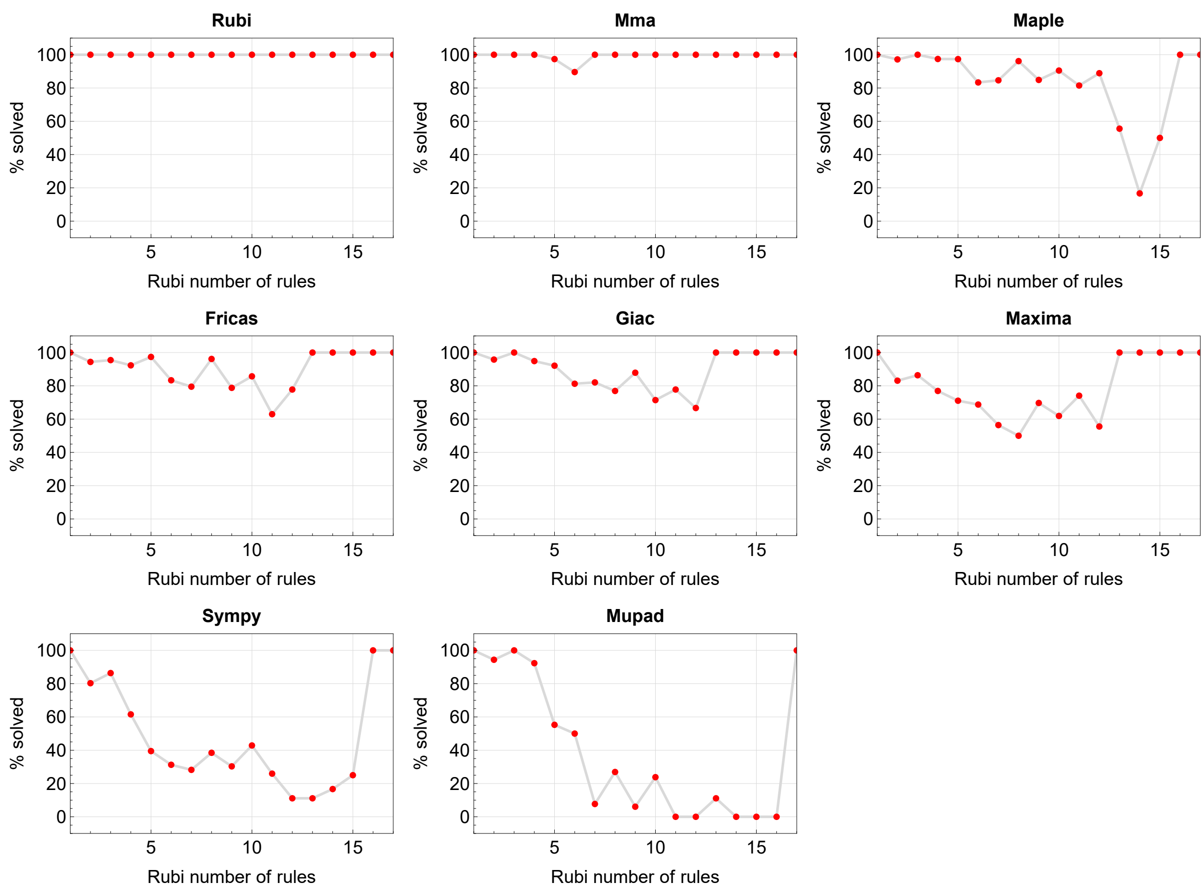


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

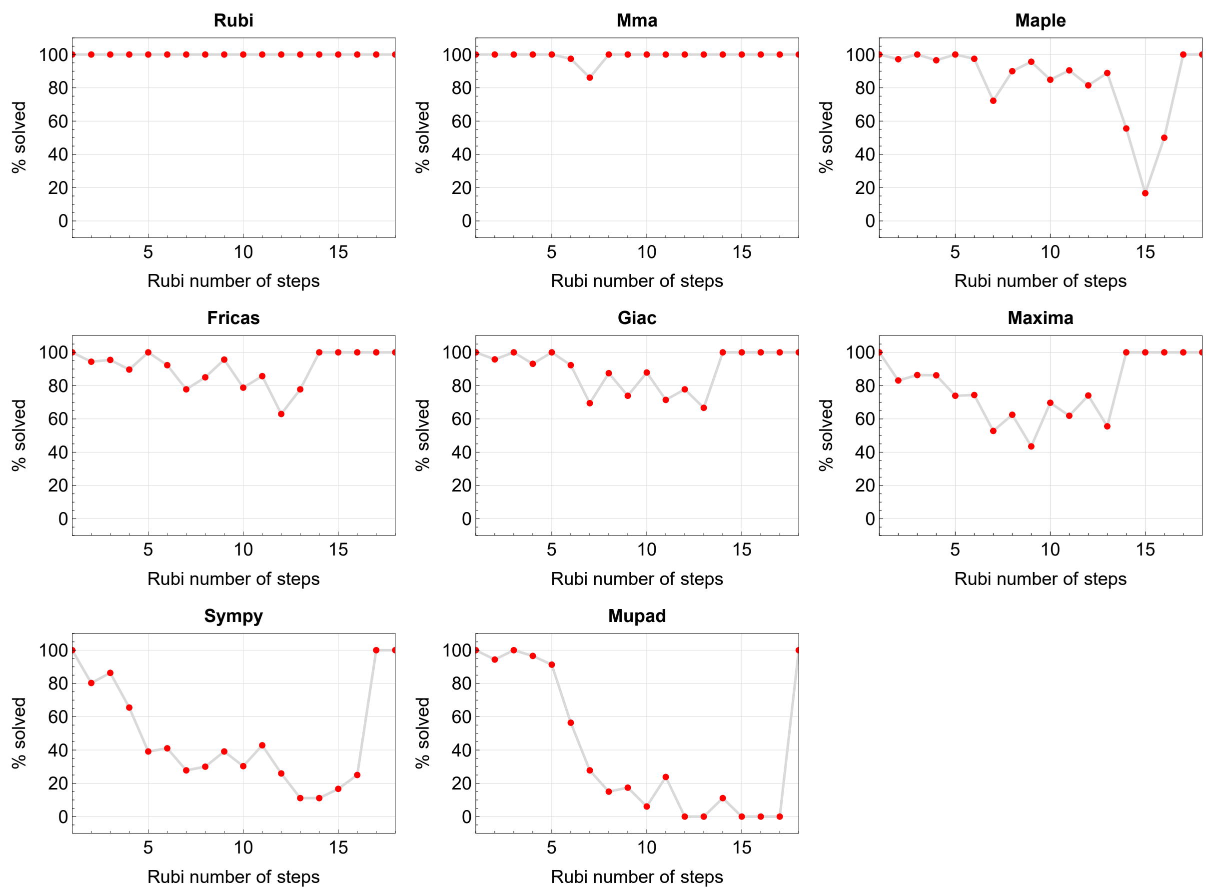


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

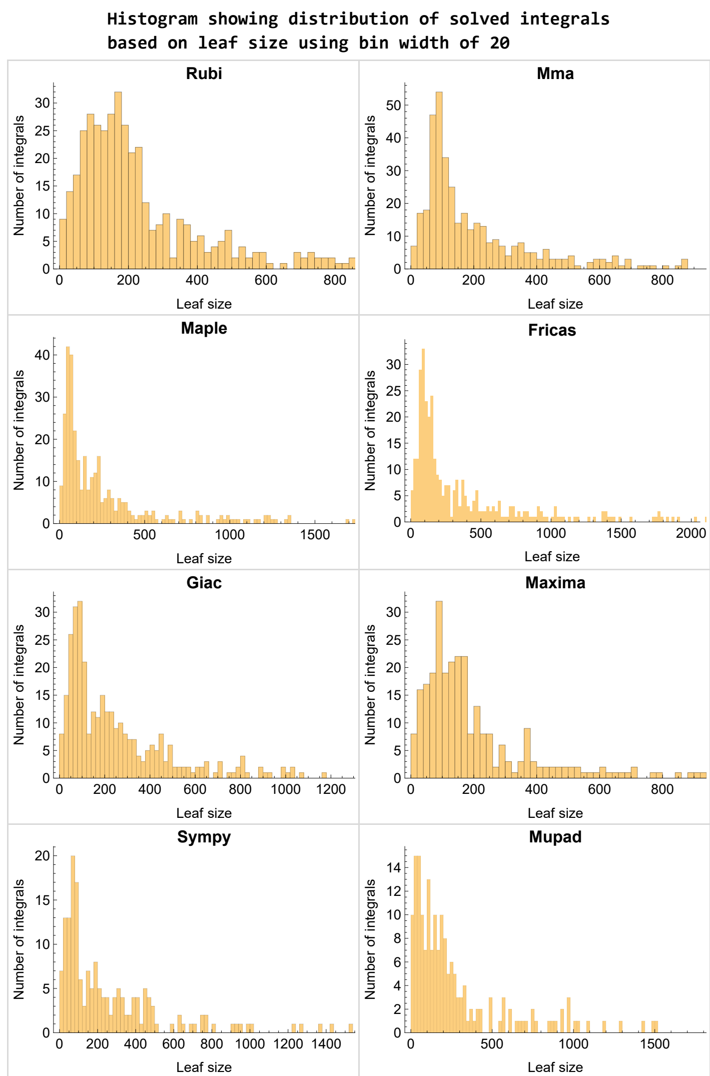


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

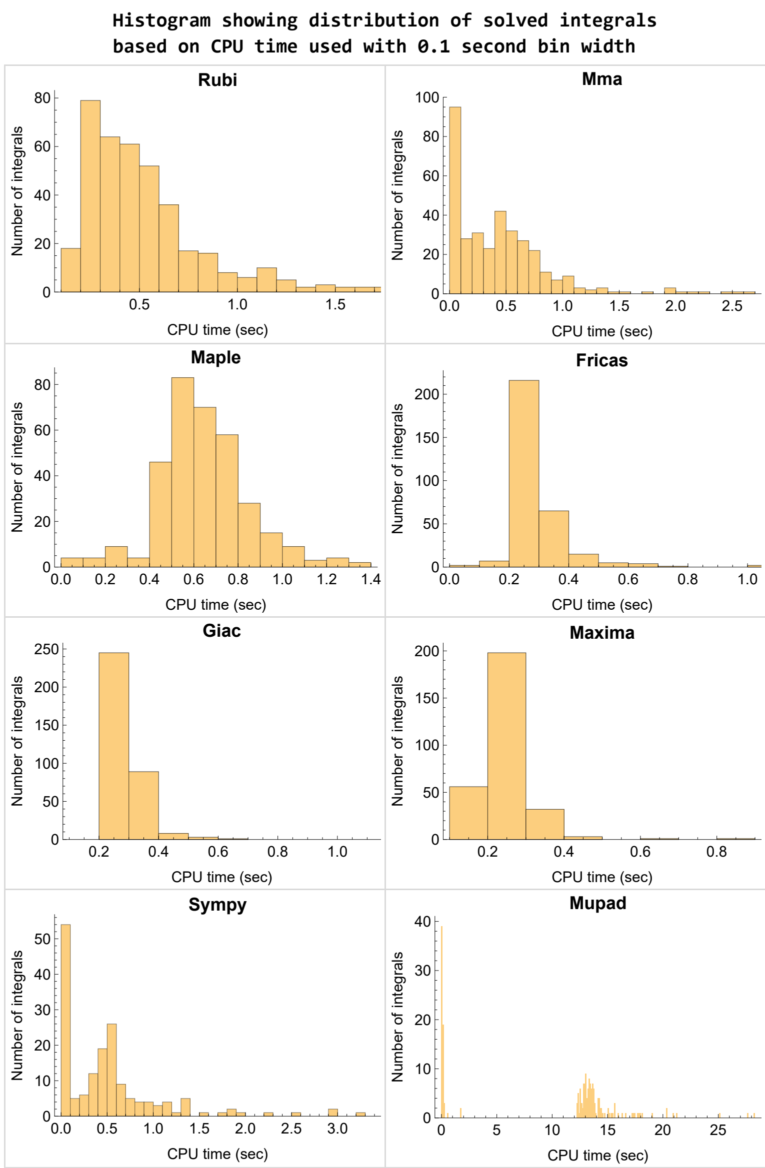


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

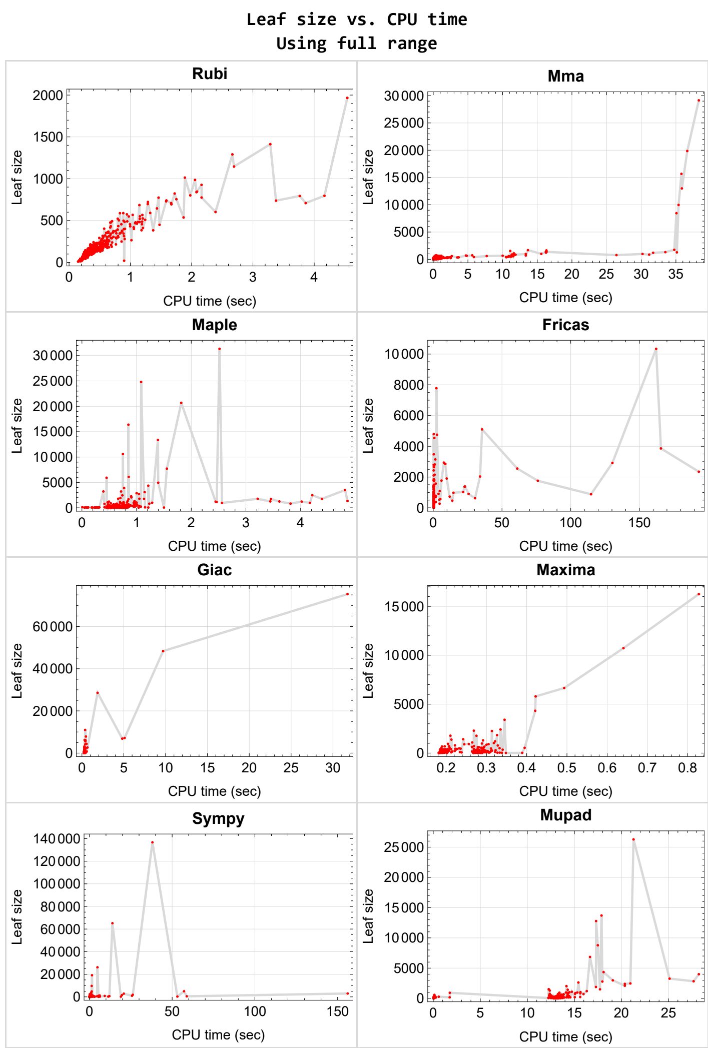


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{264, 265, 398, 399, 400\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

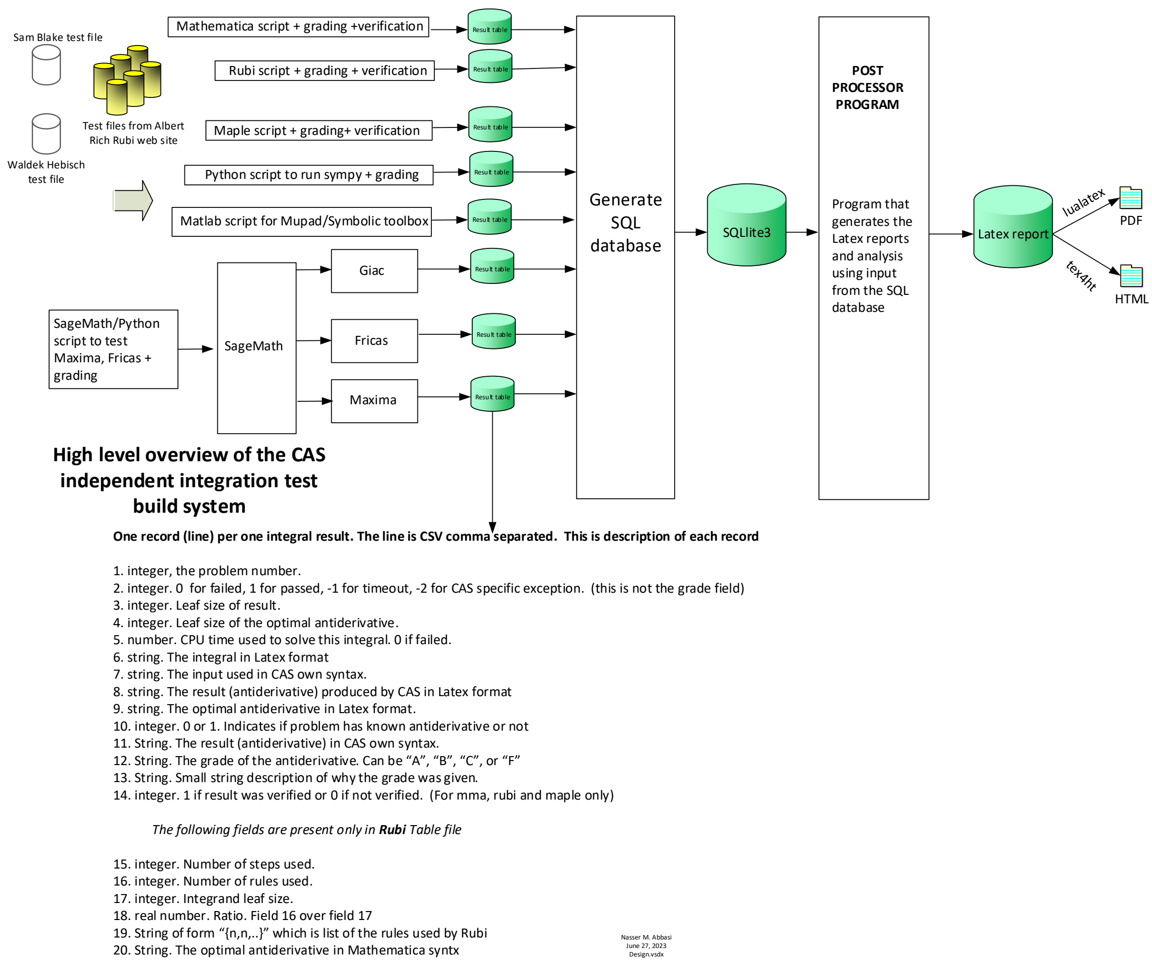
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394, 398, 399, 400 }

B grade { 39, 40, 41, 42, 114, 278 }

C grade { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

F normal fail { 136, 137, 138, 272, 273, 274 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 199, 200, 201, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 263, 264, 265, 266, 267, 268, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 332, 333, 334, 335, 344, 345, 352, 353, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade { 41, 42, 64, 84, 85, 86, 87, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 147, 155, 158, 159, 178, 185, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 231, 232, 233, 234, 237, 238, 239, 259, 262, 269, 270, 271, 278, 366, 367, 368, 369 }

C grade { 133, 255 }

F normal fail { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 327, 328, 329, 330, 331, 336, 337, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade { 31, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 57, 58, 61, 64, 65, 66, 67, 86, 87, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 185, 196, 197, 198, 232, 233, 234, 235, 236, 237, 238, 239, 253, 258, 277, 278, 310, 311, 315, 316, 317, 318, 319, 322, 323, 360, 365, 366, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271 }

F normal fail { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 56, 62, 63, 82, 83, 84, 85, 92, 93, 94, 95, 96, 97, 98, 99, 106, 153, 154, 158, 159, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade { 8, 9, 15, 16, 17, 39, 40, 41, 42, 56, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 99, 106, 107, 112, 113, 114, 131, 177, 252, 253, 278, 317, 323, 343, 358, 359, 360, 367, 368, 369, 377, 383, 389, 395 }

C grade { }

F normal fail { 7, 136, 137, 138, 139, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 366, 370, 371, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

F(-1) timedout fail { 6 }

F(-2) exception fail { 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 372, 373 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 6, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 350, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 386, 387, 388, 389, 392, 393, 394, 395, 396 }

B grade { 5, 7, 9, 16, 17, 39, 40, 41, 42, 56, 61, 62, 63, 64, 65, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 122, 123, 128, 129, 134, 147, 154, 158, 159, 178, 184, 185, 192, 193, 195, 196, 197, 198, 202, 203, 206, 207, 212, 213, 218, 219, 224, 225, 232, 233, 234, 237, 239, 244, 245, 250, 251, 256, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 347, 348, 349, 355, 356, 357, 362, 363, 364, 367, 368, 369, 379, 385, 390, 391, 397 }

C grade { 8 }

F normal fail { 83, 86, 93, 96, 113, 136, 137, 138, 139, 194, 238, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 370, 371, 398, 399, 400 }

F(-1) timedout fail { 191, 201, 204, 205 }

F(-2) exception fail { 15, 82, 92, 105, 106, 190, 200, 230, 231, 283, 378, 384 }

2.1.7 Mupad

A grade { }

B grade { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 182, 183, 184, 185, 186, 187, 188, 189, 208, 209, 210, 236, 254, 258, 275, 276, 277, 278, 279, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 367, 368, 369, 372, 373, 374, 375, 376 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 136, 137, 138, 139, 178, 179, 181, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 91, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 181, 208, 209, 210, 214, 215, 216, 220, 221, 222, 228, 240, 241, 242, 279, 280, 281, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321, 324, 325, 334, 335, 344, 345, 374, 375, 376, 380, 381, 382, 386, 387, 388 }

B grade { 1, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 78, 88, 89, 90, 100, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 155, 156, 157, 177, 178, 179, 180, 186, 187, 188, 189, 196, 197, 198, 199, 226, 227, 229, 275, 276, 277, 278, 282, 367, 368, 369 }

C grade { 304, 305, 306, 311, 312, 313, 318, 319, 320 }

F normal fail { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 105, 106, 107, 108, 109, 112, 121, 122, 123, 124, 125, 127, 128, 130, 131, 137, 182, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 206, 211, 212, 213, 217, 218, 219, 223, 224, 225, 230, 231, 232, 233, 234, 235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 283, 284, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 377, 378, 379, 383, 384, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

F(-1) timedout fail { 47, 48, 49, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 98, 99, 113, 114, 129, 133, 134, 135, 136, 138, 152, 153, 154, 158, 159, 183, 184, 185, 205, 207, 238, 239, 272, 274, 308, 309, 310, 315, 316, 317, 322, 323, 365, 366, 370, 371, 372, 373, 398, 399, 400 }

F(-2) exception fail { 139 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	238	217	218	371	211	466	224	0
N.S.	1	1.01	0.92	0.92	1.57	0.89	1.97	0.95	0.00
time (sec)	N/A	0.733	1.070	0.625	0.277	0.450	0.562	0.296	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	180	181	180	223	173	328	181	0
N.S.	1	0.97	0.97	0.97	1.20	0.93	1.76	0.97	0.00
time (sec)	N/A	0.470	0.874	0.536	0.280	0.379	0.582	0.312	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	120	114	107	137	108	165	96	0
N.S.	1	0.96	0.91	0.86	1.10	0.86	1.32	0.77	0.00
time (sec)	N/A	0.253	0.405	0.493	0.285	0.337	0.348	0.297	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	139	115	111	171	112	0	114	0
N.S.	1	0.94	0.78	0.75	1.16	0.76	0.00	0.77	0.00
time (sec)	N/A	0.359	0.556	0.585	0.311	0.332	0.000	0.294	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	156	121	196	197	190	0	325	0
N.S.	1	0.92	0.71	1.15	1.16	1.12	0.00	1.91	0.00
time (sec)	N/A	0.370	0.594	0.539	0.293	0.309	0.000	0.312	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	143	125	238	0	258	0	274	0
N.S.	1	0.96	0.84	1.60	0.00	1.73	0.00	1.84	0.00
time (sec)	N/A	0.361	0.755	0.587	0.000	0.298	0.000	0.299	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	125	293	0	304	0	441	0
N.S.	1	1.00	0.64	1.49	0.00	1.55	0.00	2.25	0.00
time (sec)	N/A	0.404	0.902	0.578	0.000	0.293	0.000	0.296	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	169	109	116	945	320	0	608	601
N.S.	1	0.94	0.61	0.64	5.25	1.78	0.00	3.38	3.34
time (sec)	N/A	0.383	0.793	0.658	0.204	0.381	0.000	0.309	13.558

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	222	144	152	1378	399	0	715	960
N.S.	1	0.95	0.62	0.65	5.89	1.71	0.00	3.06	4.10
time (sec)	N/A	0.422	1.029	0.796	0.213	0.350	0.000	0.295	14.181

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	265	185	182	423	178	416	187	0
N.S.	1	1.12	0.78	0.77	1.79	0.75	1.76	0.79	0.00
time (sec)	N/A	1.026	0.837	0.793	0.269	0.292	0.579	0.303	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	207	150	146	286	145	280	149	0
N.S.	1	1.08	0.79	0.76	1.50	0.76	1.47	0.78	0.00
time (sec)	N/A	0.701	0.704	0.678	0.274	0.303	0.555	0.318	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	149	114	110	171	109	180	112	270
N.S.	1	1.04	0.80	0.77	1.20	0.76	1.26	0.78	1.89
time (sec)	N/A	0.445	0.497	0.529	0.270	0.311	0.537	0.304	13.763

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	81	72	91	71	124	62	148
N.S.	1	1.02	0.93	0.83	1.05	0.82	1.43	0.71	1.70
time (sec)	N/A	0.231	0.287	0.470	0.272	0.374	0.314	0.292	13.337

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	104	96	127	138	155	0	109	0
N.S.	1	1.01	0.93	1.23	1.34	1.50	0.00	1.06	0.00
time (sec)	N/A	0.320	0.507	0.531	0.273	0.303	0.000	0.291	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	107	195	317	221	0	0	0
N.S.	1	1.00	0.66	1.20	1.94	1.36	0.00	0.00	0.00
time (sec)	N/A	0.376	0.581	0.526	0.281	0.300	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	169	103	110	608	244	0	350	109
N.S.	1	0.94	0.57	0.61	3.38	1.36	0.00	1.94	0.61
time (sec)	N/A	0.375	0.589	0.566	0.279	0.320	0.000	0.297	12.574

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	222	139	146	975	320	0	550	204
N.S.	1	0.95	0.59	0.62	4.17	1.37	0.00	2.35	0.87
time (sec)	N/A	0.417	0.750	0.581	0.289	0.332	0.000	0.293	12.543

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	208	213	202	202	257	248	206
N.S.	1	0.99	1.19	1.22	1.15	1.15	1.47	1.42	1.18
time (sec)	N/A	0.501	0.054	0.521	0.195	0.306	0.038	0.274	0.090

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	173	150	148	141	141	173	171	143
N.S.	1	0.99	0.86	0.85	0.81	0.81	0.99	0.98	0.82
time (sec)	N/A	0.425	0.037	0.549	0.201	0.293	0.029	0.259	12.366

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	80	80	97	94	80
N.S.	1	1.00	1.00	0.92	0.93	0.93	1.13	1.09	0.93
time (sec)	N/A	0.298	0.018	0.116	0.187	0.273	0.021	0.288	12.594

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.204	0.009	0.108	0.182	0.262	0.020	0.267	0.025

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	143	136	161	159	161	148	181	175
N.S.	1	0.99	0.94	1.11	1.10	1.11	1.02	1.25	1.21
time (sec)	N/A	0.423	0.048	0.457	0.187	0.267	0.244	0.286	12.647

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	151	142	172	169	250	185	248	192
N.S.	1	0.99	0.93	1.12	1.10	1.63	1.21	1.62	1.25
time (sec)	N/A	0.404	0.090	0.451	0.198	0.290	0.532	0.298	0.092

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	154	176	169	177	273	206	178	185
N.S.	1	0.99	1.13	1.08	1.13	1.75	1.32	1.14	1.19
time (sec)	N/A	0.387	0.060	0.457	0.184	0.277	1.794	0.272	0.094

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	301	335	371	360	360	445	432	332
N.S.	1	0.99	1.10	1.22	1.18	1.18	1.46	1.42	1.09
time (sec)	N/A	0.732	0.091	0.501	0.191	0.273	0.042	0.308	0.160

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	216	241	263	257	257	311	302	244
N.S.	1	1.00	1.11	1.21	1.18	1.18	1.43	1.39	1.12
time (sec)	N/A	0.580	0.059	0.485	0.188	0.267	0.035	0.270	0.104

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	144	151	154	154	180	172	140
N.S.	1	1.00	1.12	1.18	1.20	1.20	1.41	1.34	1.09
time (sec)	N/A	0.390	0.034	0.480	0.188	0.278	0.027	0.269	12.837

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	69	75	74	74	83	76	74
N.S.	1	1.00	1.03	1.12	1.10	1.10	1.24	1.13	1.10
time (sec)	N/A	0.227	0.019	0.491	0.186	0.322	0.024	0.265	0.038

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	295	285	375	377	379	359	445	422
N.S.	1	0.99	0.96	1.26	1.27	1.28	1.21	1.50	1.42
time (sec)	N/A	0.761	0.106	0.523	0.188	0.303	0.472	0.274	12.505

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	289	272	401	392	553	416	520	575
N.S.	1	0.99	0.93	1.37	1.34	1.89	1.42	1.78	1.97
time (sec)	N/A	0.683	0.181	0.486	0.199	0.285	1.088	0.277	0.123

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	292	274	392	402	608	474	426	495
N.S.	1	0.99	0.93	1.33	1.36	2.06	1.61	1.44	1.68
time (sec)	N/A	0.699	0.079	0.479	0.203	0.291	4.595	0.271	12.316

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	400	459	531	512	512	646	618	490
N.S.	1	0.99	1.14	1.31	1.27	1.27	1.60	1.53	1.21
time (sec)	N/A	0.965	0.134	0.495	0.188	0.257	0.058	0.274	12.549

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	288	329	376	367	367	447	432	343
N.S.	1	1.00	1.14	1.30	1.27	1.27	1.55	1.49	1.19
time (sec)	N/A	0.711	0.084	0.496	0.185	0.266	0.046	0.261	12.382

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	196	223	222	222	265	249	187
N.S.	1	1.00	1.16	1.32	1.31	1.31	1.57	1.47	1.11
time (sec)	N/A	0.466	0.046	0.565	0.190	0.271	0.036	0.270	0.119

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	100	109	108	108	122	111	103
N.S.	1	1.00	1.15	1.25	1.24	1.24	1.40	1.28	1.18
time (sec)	N/A	0.255	0.021	0.539	0.187	0.258	0.030	0.268	12.245

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	487	498	679	672	674	685	817	741
N.S.	1	0.99	1.02	1.39	1.37	1.38	1.40	1.67	1.51
time (sec)	N/A	1.183	0.296	0.578	0.190	0.275	0.735	0.275	12.377

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	483	641	709	691	932	748	882	1511
N.S.	1	0.99	1.32	1.46	1.42	1.92	1.54	1.81	3.11
time (sec)	N/A	1.103	0.227	0.617	0.196	0.288	1.861	0.294	12.272

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	463	438	700	701	1025	816	780	1290
N.S.	1	0.99	0.94	1.50	1.50	2.20	1.75	1.67	2.77
time (sec)	N/A	1.079	0.130	0.476	0.208	0.299	9.233	0.286	12.381

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	27	82	78	73	111	85
N.S.	1	1.00	3.65	1.59	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.199	0.024	0.483	0.189	0.265	0.181	0.266	0.076

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	62	27	82	78	73	111	85
N.S.	1	1.00	3.65	1.59	4.82	4.59	4.29	6.53	5.00
time (sec)	N/A	0.186	0.008	0.453	0.185	0.276	0.171	0.267	12.481

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	38	160	120	153	216	252
N.S.	1	1.00	5.29	2.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.211	0.026	0.469	0.194	0.282	0.285	0.281	12.801

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	90	38	160	120	153	216	252
N.S.	1	1.00	5.29	2.24	9.41	7.06	9.00	12.71	14.82
time (sec)	N/A	0.186	0.013	0.569	0.194	0.272	0.267	0.273	0.050

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	237	223	260	244	592	1008	289	277
N.S.	1	0.99	0.93	1.08	1.02	2.47	4.20	1.20	1.15
time (sec)	N/A	0.530	0.133	0.681	0.273	0.278	5.517	0.269	12.874

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	166	155	169	161	404	638	178	181
N.S.	1	0.99	0.92	1.01	0.96	2.40	3.80	1.06	1.08
time (sec)	N/A	0.406	0.099	0.654	0.264	0.284	1.393	0.269	12.922

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	86	84	86	206	337	86	97
N.S.	1	1.00	0.92	0.90	0.92	2.22	3.62	0.92	1.04
time (sec)	N/A	0.276	0.055	0.663	0.275	0.274	0.714	0.392	12.513

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	56	47	48	125	156	48	56
N.S.	1	1.00	1.02	0.85	0.87	2.27	2.84	0.87	1.02
time (sec)	N/A	0.216	0.026	0.504	0.276	0.276	0.235	0.343	12.243

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	120	112	123	262	0	123	840
N.S.	1	1.00	0.90	0.84	0.92	1.97	0.00	0.92	6.32
time (sec)	N/A	0.343	0.062	0.565	0.273	4.613	0.000	0.342	15.629

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	188	204	255	904	0	277	1199
N.S.	1	1.00	0.88	0.95	1.19	4.22	0.00	1.29	5.60
time (sec)	N/A	0.493	0.184	0.624	0.274	25.697	0.000	0.266	16.313

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	277	317	495	1759	0	516	2980
N.S.	1	1.00	0.91	1.04	1.62	5.77	0.00	1.69	9.77
time (sec)	N/A	0.703	0.185	0.599	0.290	76.070	0.000	0.271	19.082

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	212	233	284	287	931	952	299	303
N.S.	1	0.98	1.08	1.31	1.33	4.31	4.41	1.38	1.40
time (sec)	N/A	0.524	0.126	0.649	0.275	0.310	25.718	0.266	13.005

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	154	175	187	188	631	593	186	195
N.S.	1	1.05	1.20	1.28	1.29	4.32	4.06	1.27	1.34
time (sec)	N/A	0.414	0.086	0.654	0.279	0.301	6.042	0.262	0.228

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	102	108	113	337	318	107	191
N.S.	1	1.06	1.05	1.11	1.16	3.47	3.28	1.10	1.97
time (sec)	N/A	0.265	0.059	0.582	0.272	0.311	2.228	0.275	0.148

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	65	62	195	116	60	60
N.S.	1	1.00	0.99	0.94	0.90	2.83	1.68	0.87	0.87
time (sec)	N/A	0.207	0.038	0.599	0.269	0.288	0.331	0.272	0.098

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	245	195	293	293	1024	0	364	1493
N.S.	1	1.08	0.86	1.30	1.30	4.53	0.00	1.61	6.61
time (sec)	N/A	0.562	0.136	0.612	0.272	21.753	0.000	0.272	17.721

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	384	320	424	604	2916	0	638	2094
N.S.	1	1.03	0.86	1.13	1.61	7.80	0.00	1.71	5.60
time (sec)	N/A	1.173	0.248	0.641	0.280	130.336	0.000	0.280	20.360

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	537	466	643	1030	0	0	1013	2828
N.S.	1	1.02	0.89	1.23	1.97	0.00	0.00	1.93	5.40
time (sec)	N/A	1.832	0.373	0.684	0.318	0.000	0.000	0.273	27.671

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	224	281	333	379	1138	0	358	920
N.S.	1	1.07	1.34	1.59	1.81	5.44	0.00	1.71	4.40
time (sec)	N/A	0.434	0.155	0.573	0.279	0.485	0.000	0.274	1.761

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	195	211	222	253	806	0	256	230
N.S.	1	1.25	1.35	1.42	1.62	5.17	0.00	1.64	1.47
time (sec)	N/A	0.399	0.088	0.589	0.269	0.501	0.000	0.271	12.660

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	141	137	124	160	470	240	146	128
N.S.	1	1.08	1.05	0.95	1.23	3.62	1.85	1.12	0.98
time (sec)	N/A	0.296	0.065	0.576	0.268	0.356	11.400	0.273	0.148

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	90	83	98	314	156	84	88
N.S.	1	1.00	0.92	0.85	1.00	3.20	1.59	0.86	0.90
time (sec)	N/A	0.228	0.045	0.550	0.279	0.439	0.590	0.266	12.901

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	397	321	605	655	2346	0	751	2392
N.S.	1	1.12	0.91	1.71	1.86	6.65	0.00	2.13	6.78
time (sec)	N/A	0.874	0.269	0.811	0.295	193.202	0.000	0.276	20.362

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	603	498	833	1196	0	0	1175	6848
N.S.	1	1.06	0.87	1.46	2.09	0.00	0.00	2.06	11.99
time (sec)	N/A	2.392	0.436	0.736	0.321	0.000	0.000	0.296	16.659

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	794	672	1055	1835	0	0	1614	8774
N.S.	1	1.05	0.89	1.40	2.44	0.00	0.00	2.14	11.65
time (sec)	N/A	3.751	0.582	0.823	0.326	0.000	0.000	0.280	17.487

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	224	437	544	599	1864	0	659	669
N.S.	1	0.96	1.87	2.32	2.56	7.97	0.00	2.82	2.86
time (sec)	N/A	0.419	0.182	0.583	0.282	0.373	0.000	0.270	13.859

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	314	350	383	457	1378	0	487	402
N.S.	1	1.24	1.38	1.51	1.80	5.43	0.00	1.92	1.58
time (sec)	N/A	0.553	0.181	0.596	0.292	0.474	0.000	0.268	13.678

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	224	266	268	323	1062	0	331	287
N.S.	1	1.00	1.18	1.19	1.44	4.72	0.00	1.47	1.28
time (sec)	N/A	0.426	0.100	0.602	0.274	0.537	0.000	0.274	0.228

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	167	171	149	208	636	0	187	164
N.S.	1	1.01	1.04	0.90	1.26	3.85	0.00	1.13	0.99
time (sec)	N/A	0.316	0.084	0.598	0.282	0.397	0.000	0.264	13.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	123	112	100	133	430	196	109	116
N.S.	1	0.98	0.89	0.79	1.06	3.41	1.56	0.87	0.92
time (sec)	N/A	0.247	0.060	0.517	0.267	0.294	0.901	0.256	12.913

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	38	29	30	29	46	29	29	30
N.S.	1	0.88	0.67	0.70	0.67	1.07	0.67	0.67	0.70
time (sec)	N/A	0.220	0.015	0.510	0.269	0.275	0.054	0.258	0.034

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	24	23	40	20	23	23
N.S.	1	1.00	0.90	0.80	0.77	1.33	0.67	0.77	0.77
time (sec)	N/A	0.209	0.008	0.473	0.278	0.407	0.048	0.271	0.035

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	26	23	24	23	33	20	23	25
N.S.	1	0.90	0.79	0.83	0.79	1.14	0.69	0.79	0.86
time (sec)	N/A	0.181	0.007	0.493	0.389	0.583	0.057	0.258	0.032

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	10	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.71	0.86	1.00
time (sec)	N/A	0.157	0.005	0.473	0.348	0.261	0.048	0.267	12.938

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	32	28	26	25	41	24	26	32
N.S.	1	1.03	0.90	0.84	0.81	1.32	0.77	0.84	1.03
time (sec)	N/A	0.216	0.008	0.510	0.338	0.309	0.066	0.282	0.044

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	30	34	49	31	35	38
N.S.	1	1.00	1.00	0.91	1.03	1.48	0.94	1.06	1.15
time (sec)	N/A	0.218	0.012	0.485	0.327	0.350	0.066	0.272	12.851

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	42	39	38	41	61	42	43	47
N.S.	1	0.93	0.87	0.84	0.91	1.36	0.93	0.96	1.04
time (sec)	N/A	0.261	0.012	0.527	0.333	0.444	0.083	0.257	0.040

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	22	12	13	12	18	8	12	12
N.S.	1	1.83	1.00	1.08	1.00	1.50	0.67	1.00	1.00
time (sec)	N/A	0.158	0.010	0.545	0.290	0.289	0.053	0.267	12.769

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	21	25	20	21	21
N.S.	1	1.00	1.00	0.78	0.78	0.93	0.74	0.78	0.78
time (sec)	N/A	0.169	0.009	0.566	0.287	0.273	0.054	0.275	0.035

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	365	358	358	436	855	756	463	0
N.S.	1	0.94	0.92	0.92	1.12	2.19	1.94	1.19	0.00
time (sec)	N/A	0.716	1.005	0.813	0.223	0.332	0.580	0.293	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	267	245	272	305	595	484	313	0
N.S.	1	0.95	0.88	0.97	1.09	2.12	1.73	1.12	0.00
time (sec)	N/A	0.515	0.721	0.694	0.199	0.461	0.568	0.281	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	178	145	153	169	329	258	175	0
N.S.	1	1.02	0.83	0.87	0.97	1.88	1.47	1.00	0.00
time (sec)	N/A	0.367	0.486	0.537	0.202	0.453	0.530	0.273	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	101	87	75	96	190	124	85	0
N.S.	1	0.95	0.82	0.71	0.91	1.79	1.17	0.80	0.00
time (sec)	N/A	0.234	0.214	0.493	0.197	0.274	0.336	0.279	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	217	213	316	362	0	0	0	0
N.S.	1	1.05	1.03	1.53	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	0.691	0.550	0.225	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	323	217	515	478	0	0	0	0
N.S.	1	1.05	0.70	1.67	1.55	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	0.970	0.565	0.232	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	311	280	946	927	0	0	911	0
N.S.	1	1.05	0.95	3.20	3.13	0.00	0.00	3.08	0.00
time (sec)	N/A	0.654	1.735	0.705	0.256	0.000	0.000	0.325	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	362	382	2448	1772	0	0	1701	0
N.S.	1	1.15	1.22	7.80	5.64	0.00	0.00	5.42	0.00
time (sec)	N/A	0.635	10.531	0.740	0.275	0.000	0.000	0.366	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	304	439	3903	3404	2552	0	0	0
N.S.	1	0.97	1.40	12.47	10.88	8.15	0.00	0.00	0.00
time (sec)	N/A	0.550	10.746	0.772	0.345	61.128	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	434	583	6085	5793	3862	0	4170	0
N.S.	1	1.00	1.35	14.05	13.38	8.92	0.00	9.63	0.00
time (sec)	N/A	0.802	10.881	0.856	0.422	165.638	0.000	0.400	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	392	474	406	525	1177	1428	636	0
N.S.	1	0.85	1.03	0.88	1.14	2.55	3.09	1.38	0.00
time (sec)	N/A	0.780	1.331	0.687	0.209	0.332	0.663	0.306	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	293	332	320	380	831	933	441	0
N.S.	1	0.85	0.96	0.92	1.10	2.40	2.70	1.27	0.00
time (sec)	N/A	0.550	1.052	0.611	0.195	0.326	0.640	0.302	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	200	197	194	211	477	507	257	0
N.S.	1	0.94	0.92	0.91	0.99	2.24	2.38	1.21	0.00
time (sec)	N/A	0.392	0.718	0.526	0.189	0.286	0.587	0.304	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	123	117	111	131	262	209	126	0
N.S.	1	0.90	0.85	0.81	0.96	1.91	1.53	0.92	0.00
time (sec)	N/A	0.241	0.366	0.506	0.188	0.281	0.397	0.301	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	345	360	563	632	0	0	0	0
N.S.	1	1.06	1.10	1.73	1.94	0.00	0.00	0.00	0.00
time (sec)	N/A	0.879	1.293	0.629	0.264	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	456	364	736	708	0	0	0	0
N.S.	1	1.06	0.84	1.70	1.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.977	1.486	0.740	0.269	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	490	361	1167	1299	0	0	1021	0
N.S.	1	1.00	0.74	2.39	2.66	0.00	0.00	2.09	0.00
time (sec)	N/A	0.949	2.013	0.696	0.292	0.000	0.000	0.347	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	484	517	1917	2415	0	0	1878	0
N.S.	1	1.02	1.09	4.04	5.08	0.00	0.00	3.95	0.00
time (sec)	N/A	0.898	11.029	0.703	0.335	0.000	0.000	0.385	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	551	575	3214	4326	0	0	0	0
N.S.	1	1.08	1.13	6.29	8.47	0.00	0.00	0.00	0.00
time (sec)	N/A	1.140	11.550	0.716	0.421	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	574	639	10598	6650	0	0	4363	0
N.S.	1	1.13	1.26	20.90	13.12	0.00	0.00	8.61	0.00
time (sec)	N/A	0.919	11.287	0.750	0.493	0.000	0.000	0.547	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	368	696	16383	10724	0	0	6061	0
N.S.	1	0.91	1.72	40.55	26.54	0.00	0.00	15.00	0.00
time (sec)	N/A	0.620	11.408	0.850	0.640	0.000	0.000	0.424	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	498	863	24805	16249	0	0	7857	0
N.S.	1	0.94	1.62	46.63	30.54	0.00	0.00	14.77	0.00
time (sec)	N/A	0.869	11.313	1.083	0.828	0.000	0.000	0.474	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	145	142	148	166	333	328	168	0
N.S.	1	0.86	0.85	0.88	0.99	1.98	1.95	1.00	0.00
time (sec)	N/A	0.264	0.499	0.596	0.195	0.301	0.483	0.279	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	343	244	284	349	559	394	306	0
N.S.	1	1.06	0.75	0.87	1.07	1.72	1.21	0.94	0.00
time (sec)	N/A	0.676	0.737	0.765	0.207	0.319	0.551	0.290	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	241	165	166	230	381	265	201	0
N.S.	1	1.08	0.74	0.74	1.03	1.71	1.19	0.90	0.00
time (sec)	N/A	0.517	0.529	0.598	0.190	0.318	0.511	0.286	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	152	96	87	126	199	150	107	227
N.S.	1	1.12	0.71	0.64	0.93	1.46	1.10	0.79	1.67
time (sec)	N/A	0.352	0.413	0.530	0.196	0.278	0.496	0.296	13.753

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	70	52	61	124	92	57	107
N.S.	1	1.04	0.95	0.70	0.82	1.68	1.24	0.77	1.45
time (sec)	N/A	0.220	0.320	0.482	0.189	0.267	0.305	0.283	13.064

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	135	137	197	218	881	0	0	0
N.S.	1	1.04	1.05	1.52	1.68	6.78	0.00	0.00	0.00
time (sec)	N/A	0.353	0.457	0.547	0.221	114.782	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	186	177	390	419	0	0	0	0
N.S.	1	1.11	1.05	2.32	2.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.853	0.605	0.227	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	237	203	810	896	1088	0	839	0
N.S.	1	1.05	0.90	3.60	3.98	4.84	0.00	3.73	0.00
time (sec)	N/A	0.502	1.094	0.638	0.245	4.715	0.000	0.301	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	249	256	285	346	758	0	331	0
N.S.	1	1.09	1.12	1.24	1.51	3.31	0.00	1.45	0.00
time (sec)	N/A	0.500	0.866	0.848	0.197	0.321	0.000	0.288	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	168	165	193	227	530	0	214	0
N.S.	1	1.13	1.11	1.30	1.52	3.56	0.00	1.44	0.00
time (sec)	N/A	0.404	0.646	0.668	0.206	0.338	0.000	0.280	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	106	104	113	126	278	209	113	151
N.S.	1	1.06	1.04	1.13	1.26	2.78	2.09	1.13	1.51
time (sec)	N/A	0.280	0.463	0.514	0.196	0.298	6.334	0.294	13.482

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	70	61	181	87	62	68
N.S.	1	1.00	1.02	1.15	1.00	2.97	1.43	1.02	1.11
time (sec)	N/A	0.210	0.352	0.458	0.202	0.284	2.996	0.282	12.750

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	147	386	453	721	0	288	0
N.S.	1	1.00	1.07	2.80	3.28	5.22	0.00	2.09	0.00
time (sec)	N/A	0.341	0.663	0.514	0.236	1.055	0.000	0.288	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	240	248	872	1085	1573	0	0	0
N.S.	1	1.00	1.04	3.65	4.54	6.58	0.00	0.00	0.00
time (sec)	N/A	0.581	1.356	0.529	0.265	1.823	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	383	1537	1787	2254	2853	0	1421	0
N.S.	1	1.02	4.11	4.78	6.03	7.63	0.00	3.80	0.00
time (sec)	N/A	1.348	11.111	0.663	0.314	8.792	0.000	0.313	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	47	83	68	194	48	59
N.S.	1	1.00	0.75	0.70	1.24	1.01	2.90	0.72	0.88
time (sec)	N/A	0.216	0.454	0.540	0.199	0.273	5.473	0.279	12.969

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	94	71	72	118	103	638	80	93
N.S.	1	0.97	0.73	0.74	1.22	1.06	6.58	0.82	0.96
time (sec)	N/A	0.228	0.523	0.582	0.204	0.274	12.062	0.274	12.958

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	121	92	96	153	137	1880	112	115
N.S.	1	0.95	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.246	0.636	0.545	0.215	0.274	26.214	0.270	12.990

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	128	66	45	78	60	94	54	45
N.S.	1	1.21	0.62	0.42	0.74	0.57	0.89	0.51	0.42
time (sec)	N/A	0.298	0.209	0.543	0.285	0.265	0.336	0.264	0.051

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	100	58	40	64	54	75	48	40
N.S.	1	1.22	0.71	0.49	0.78	0.66	0.91	0.59	0.49
time (sec)	N/A	0.260	0.164	0.500	0.284	0.295	0.226	0.274	12.852

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	73	56	35	50	49	63	44	35
N.S.	1	1.18	0.90	0.56	0.81	0.79	1.02	0.71	0.56
time (sec)	N/A	0.220	0.126	0.471	0.273	0.255	0.151	0.259	0.034

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	89	55	58	88	0	99	61
N.S.	1	1.00	1.33	0.82	0.87	1.31	0.00	1.48	0.91
time (sec)	N/A	0.240	0.273	0.513	0.270	0.267	0.000	0.291	13.213

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	75	92	62	65	106	0	191	68
N.S.	1	1.06	1.30	0.87	0.92	1.49	0.00	2.69	0.96
time (sec)	N/A	0.253	0.328	0.524	0.270	0.267	0.000	0.392	13.177

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	82	71	65	76	89	0	180	77
N.S.	1	1.06	0.92	0.84	0.99	1.16	0.00	2.34	1.00
time (sec)	N/A	0.246	0.386	0.492	0.275	0.263	0.000	0.292	0.112

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	95	66	45	78	76	0	54	110
N.S.	1	1.09	0.76	0.52	0.90	0.87	0.00	0.62	1.26
time (sec)	N/A	0.338	0.251	0.497	0.277	0.281	0.000	0.273	0.059

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	61	40	64	72	0	49	105
N.S.	1	1.04	0.86	0.56	0.90	1.01	0.00	0.69	1.48
time (sec)	N/A	0.280	0.218	0.514	0.270	0.255	0.000	0.266	13.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	58	56	35	50	67	114	44	100
N.S.	1	1.05	1.02	0.64	0.91	1.22	2.07	0.80	1.82
time (sec)	N/A	0.213	0.181	0.463	0.268	0.255	5.579	0.268	0.038

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	48	58	83	0	82	106
N.S.	1	1.00	0.96	0.91	1.09	1.57	0.00	1.55	2.00
time (sec)	N/A	0.221	0.510	0.472	0.270	0.302	0.000	0.280	0.134

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	60	84	103	0	168	157
N.S.	1	1.00	0.95	0.80	1.12	1.37	0.00	2.24	2.09
time (sec)	N/A	0.248	0.454	0.484	0.273	0.319	0.000	0.288	13.136

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	95	81	65	124	119	0	196	180
N.S.	1	0.98	0.84	0.67	1.28	1.23	0.00	2.02	1.86
time (sec)	N/A	0.328	0.485	0.505	0.278	0.268	0.000	0.293	13.088

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	81	64	45	105	87	0	53	212
N.S.	1	1.11	0.88	0.62	1.44	1.19	0.00	0.73	2.90
time (sec)	N/A	0.300	0.323	0.621	0.268	0.271	0.000	0.264	0.056

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	63	61	40	91	83	0	48	200
N.S.	1	1.05	1.02	0.67	1.52	1.38	0.00	0.80	3.33
time (sec)	N/A	0.259	0.284	0.591	0.269	0.257	0.000	0.264	0.049

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	30	27	50	40	180	25	185
N.S.	1	1.00	0.73	0.66	1.22	0.98	4.39	0.61	4.51
time (sec)	N/A	0.206	0.223	0.523	0.195	0.260	18.992	0.275	0.049

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	78	58	74	81	103	0	91	218
N.S.	1	1.07	0.79	1.01	1.11	1.41	0.00	1.25	2.99
time (sec)	N/A	0.252	0.600	0.563	0.275	0.259	0.000	0.281	0.134

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	101	91	70	107	134	0	233	270
N.S.	1	1.06	0.96	0.74	1.13	1.41	0.00	2.45	2.84
time (sec)	N/A	0.337	0.739	0.512	0.292	0.265	0.000	0.300	13.075

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	128	91	75	147	149	0	183	301
N.S.	1	1.09	0.78	0.64	1.26	1.27	0.00	1.56	2.57
time (sec)	N/A	0.425	0.622	0.487	0.288	0.283	0.000	0.284	12.666

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	420	417	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	403	403	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.568	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	474	476	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.671	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	228	165	0	0	0	0	0	0
N.S.	1	1.03	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.580	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	256	266	263	263	320	308	244
N.S.	1	1.00	1.01	1.05	1.04	1.04	1.26	1.21	0.96
time (sec)	N/A	0.547	0.060	0.701	0.196	0.261	0.042	0.262	0.150

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	163	165	165	165	197	187	149
N.S.	1	1.00	1.01	1.02	1.02	1.02	1.22	1.16	0.93
time (sec)	N/A	0.394	0.030	0.687	0.189	0.250	0.031	0.265	13.679

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	90	87	87	102	99	88
N.S.	1	1.00	1.00	0.94	0.91	0.91	1.06	1.03	0.92
time (sec)	N/A	0.294	0.016	0.652	0.189	0.301	0.024	0.269	13.637

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.85
time (sec)	N/A	0.192	0.006	0.089	0.194	0.296	0.021	0.279	0.024

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	82	0	265	413	78	224
N.S.	1	1.00	1.04	1.01	0.00	3.27	5.10	0.96	2.77
time (sec)	N/A	0.256	0.059	0.661	0.000	0.262	0.598	0.261	0.189

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	115	0	511	376	108	172
N.S.	1	1.00	0.98	1.15	0.00	5.11	3.76	1.08	1.72
time (sec)	N/A	0.247	0.050	0.603	0.000	0.313	0.587	0.261	13.069

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	158	160	269	0	1199	774	217	401
N.S.	1	0.98	0.99	1.67	0.00	7.45	4.81	1.35	2.49
time (sec)	N/A	0.294	0.130	0.631	0.000	0.293	1.174	0.264	13.096

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	203	204	510	0	2103	1224	407	698
N.S.	1	0.99	0.99	2.48	0.00	10.21	5.94	1.98	3.39
time (sec)	N/A	0.353	0.224	0.648	0.000	0.319	1.985	0.264	13.249

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	585	869	0	2150	4972	805	967
N.S.	1	1.00	0.99	1.47	0.00	3.64	8.41	1.36	1.64
time (sec)	N/A	1.319	0.323	0.997	0.000	1.001	57.189	0.268	15.319

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	345	453	0	1273	2839	433	557
N.S.	1	1.00	0.99	1.30	0.00	3.66	8.16	1.24	1.60
time (sec)	N/A	0.769	0.205	0.944	0.000	0.484	20.767	0.277	13.689

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	173	192	0	654	1265	189	273
N.S.	1	1.00	0.98	1.08	0.00	3.69	7.15	1.07	1.54
time (sec)	N/A	0.435	0.099	0.819	0.000	0.315	6.203	0.267	0.583

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	95	93	0	302	488	89	132
N.S.	1	1.00	1.03	1.01	0.00	3.28	5.30	0.97	1.43
time (sec)	N/A	0.281	0.040	0.795	0.000	0.278	1.100	0.258	0.258

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	193	179	0	625	0	199	2467
N.S.	1	1.00	0.98	0.91	0.00	3.19	0.00	1.02	12.59
time (sec)	N/A	0.491	0.115	0.694	0.000	30.410	0.000	0.272	20.951

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	281	346	0	0	0	459	3991
N.S.	1	1.00	0.89	1.09	0.00	0.00	0.00	1.45	12.63
time (sec)	N/A	0.789	0.305	0.721	0.000	0.000	0.000	0.266	28.225

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	504	629	0	0	0	1065	12784
N.S.	1	1.00	0.99	1.24	0.00	0.00	0.00	2.09	25.12
time (sec)	N/A	1.220	0.391	0.993	0.000	0.000	0.000	0.270	17.307

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	301	398	628	0	2771	2966	547	742
N.S.	1	1.05	1.38	2.18	0.00	9.62	10.30	1.90	2.58
time (sec)	N/A	0.772	0.482	0.771	0.000	0.548	156.090	0.267	15.068

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	200	225	308	0	1413	1535	271	376
N.S.	1	1.12	1.26	1.73	0.00	7.94	8.62	1.52	2.11
time (sec)	N/A	0.451	0.225	0.684	0.000	0.370	19.635	0.264	14.451

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	114	133	0	632	459	125	203
N.S.	1	1.00	0.97	1.13	0.00	5.36	3.89	1.06	1.72
time (sec)	N/A	0.265	0.058	0.609	0.000	0.310	0.996	0.259	13.568

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	456	405	809	0	0	0	886	13698
N.S.	1	1.12	1.00	1.99	0.00	0.00	0.00	2.18	33.66
time (sec)	N/A	1.139	0.491	0.960	0.000	0.000	0.000	0.271	17.883

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	709	650	1345	0	0	0	1506	26278
N.S.	1	1.05	0.97	2.00	0.00	0.00	0.00	2.24	39.05
time (sec)	N/A	3.846	1.133	0.837	0.000	0.000	0.000	0.292	21.285

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	65	60	53	51	75	60	51	55
N.S.	1	1.05	0.97	0.85	0.82	1.21	0.97	0.82	0.89
time (sec)	N/A	0.281	0.023	0.745	0.267	0.293	0.068	0.264	0.042

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	60	55	46	46	70	54	46	48
N.S.	1	1.09	1.00	0.84	0.84	1.27	0.98	0.84	0.87
time (sec)	N/A	0.265	0.018	0.776	0.268	0.275	0.067	0.260	0.044

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	55	52	45	43	60	53	43	59
N.S.	1	1.06	1.00	0.87	0.83	1.15	1.02	0.83	1.13
time (sec)	N/A	0.229	0.014	0.737	0.280	0.267	0.071	0.262	13.204

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	32	41	41	32	35
N.S.	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	0.85
time (sec)	N/A	0.183	0.015	0.619	0.267	0.291	0.057	0.256	13.487

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	56	48	47	72	54	48	58
N.S.	1	1.09	1.00	0.86	0.84	1.29	0.96	0.86	1.04
time (sec)	N/A	0.241	0.017	0.621	0.282	0.320	0.079	0.259	0.104

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	64	61	55	54	85	65	55	68
N.S.	1	1.05	1.00	0.90	0.89	1.39	1.07	0.90	1.11
time (sec)	N/A	0.284	0.017	0.671	0.274	0.302	0.086	0.258	13.745

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	66	60	63	98	71	63	75
N.S.	1	1.04	0.97	0.88	0.93	1.44	1.04	0.93	1.10
time (sec)	N/A	0.290	0.020	0.622	0.279	0.299	0.102	0.259	0.102

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	0.80	1.00
time (sec)	N/A	0.143	0.005	0.730	0.182	0.269	0.041	0.263	0.044

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	36	27	29
N.S.	1	1.00	1.00	0.90	0.87	0.87	1.16	0.87	0.94
time (sec)	N/A	0.185	0.007	0.758	0.270	0.302	0.052	0.268	0.032

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.96	0.91	0.91
time (sec)	N/A	0.184	0.007	0.766	0.269	0.285	0.046	0.268	0.044

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	18	17	25	14	18	17
N.S.	1	1.00	0.90	0.86	0.81	1.19	0.67	0.86	0.81
time (sec)	N/A	0.183	0.007	0.579	0.185	0.280	0.036	0.262	13.130

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	14	16	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.78	0.89	0.78
time (sec)	N/A	0.178	0.004	0.512	0.190	0.278	0.051	0.255	0.044

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	1.00	0.86
time (sec)	N/A	0.177	0.004	0.524	0.181	0.291	0.058	0.253	13.162

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	21	21	22	21	17
N.S.	1	1.00	1.15	0.81	0.78	0.78	0.81	0.78	0.63
time (sec)	N/A	0.189	0.006	0.646	0.286	0.262	0.050	0.269	13.248

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	30	36	55	46	45	35
N.S.	1	1.00	1.00	0.62	0.75	1.15	0.96	0.94	0.73
time (sec)	N/A	0.245	0.029	0.562	0.267	0.273	0.056	0.257	0.114

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	19	19	15	19	17
N.S.	1	1.00	1.00	0.81	0.90	0.90	0.71	0.90	0.81
time (sec)	N/A	0.160	0.007	0.536	0.186	0.264	0.051	0.274	13.020

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	44	39	32	30	39	37	30	36
N.S.	1	1.13	1.00	0.82	0.77	1.00	0.95	0.77	0.92
time (sec)	N/A	0.187	0.017	1.037	0.273	0.251	0.063	0.302	12.893

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	33	33	31	11	11
N.S.	1	1.00	1.00	1.09	3.00	3.00	2.82	1.00	1.00
time (sec)	N/A	0.146	0.005	0.763	0.185	0.269	0.050	0.272	12.849

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	241	360	479	0	953	2691	480	0
N.S.	1	0.90	1.35	1.79	0.00	3.57	10.08	1.80	0.00
time (sec)	N/A	0.400	3.664	0.881	0.000	0.361	0.578	0.300	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	198	230	285	0	605	775	295	0
N.S.	1	0.93	1.08	1.34	0.00	2.85	3.66	1.39	0.00
time (sec)	N/A	0.339	1.986	0.764	0.000	0.312	0.506	0.291	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	155	145	151	0	355	308	158	240
N.S.	1	0.99	0.92	0.96	0.00	2.26	1.96	1.01	1.53
time (sec)	N/A	0.303	0.781	0.697	0.000	0.290	0.433	0.294	13.748

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	110	88	76	0	203	189	82	0
N.S.	1	1.06	0.85	0.73	0.00	1.95	1.82	0.79	0.00
time (sec)	N/A	0.267	0.404	0.668	0.000	0.300	0.367	0.293	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	145	0	403	0	108	108
N.S.	1	1.00	0.97	1.48	0.00	4.11	0.00	1.10	1.10
time (sec)	N/A	0.259	0.592	0.711	0.000	0.375	0.000	0.287	13.339

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	128	0	242	0	193	127
N.S.	1	1.00	0.94	1.12	0.00	2.12	0.00	1.69	1.11
time (sec)	N/A	0.253	0.808	0.679	0.000	0.673	0.000	0.288	13.395

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	165	234	294	0	563	0	452	578
N.S.	1	0.99	1.40	1.76	0.00	3.37	0.00	2.71	3.46
time (sec)	N/A	0.293	2.519	0.672	0.000	5.164	0.000	0.287	13.679

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	215	410	524	0	978	0	805	1018
N.S.	1	0.98	1.86	2.38	0.00	4.45	0.00	3.66	4.63
time (sec)	N/A	0.337	5.870	0.756	0.000	14.683	0.000	0.293	14.480

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	930	755	1093	1757	0	2817	4820	1657	3262
N.S.	1	0.81	1.18	1.89	0.00	3.03	5.18	1.78	3.51
time (sec)	N/A	1.719	11.638	1.136	0.000	1.118	1.383	0.294	25.114

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	478	657	1027	0	1791	2440	983	1881
N.S.	1	0.82	1.12	1.76	0.00	3.07	4.18	1.68	3.22
time (sec)	N/A	1.063	10.008	1.000	0.000	0.690	1.187	0.314	17.289

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	280	345	488	0	1009	993	478	877
N.S.	1	0.87	1.07	1.52	0.00	3.13	3.08	1.48	2.72
time (sec)	N/A	0.530	3.465	0.769	0.000	0.465	1.036	0.297	15.104

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	167	174	197	0	465	384	204	320
N.S.	1	0.95	0.99	1.13	0.00	2.66	2.19	1.17	1.83
time (sec)	N/A	0.334	0.071	0.672	0.000	0.298	0.535	0.302	13.542

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	345	324	515	0	0	0	0	0
N.S.	1	1.07	1.01	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	2.212	0.808	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	482	430	704	0	0	0	0	0
N.S.	1	1.05	0.94	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.134	10.507	0.852	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	468	500	1238	0	0	0	2364	0
N.S.	1	1.04	1.12	2.76	0.00	0.00	0.00	5.28	0.00
time (sec)	N/A	1.066	10.865	0.961	0.000	0.000	0.000	0.467	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	645	574	3090	0	0	0	6846	0
N.S.	1	1.07	0.95	5.12	0.00	0.00	0.00	11.35	0.00
time (sec)	N/A	1.400	11.380	1.148	0.000	0.000	0.000	4.818	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	456	693	4940	0	0	0	0	0
N.S.	1	0.92	1.39	9.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	13.347	1.396	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	730	1334	7714	0	0	0	28577	0
N.S.	1	0.89	1.62	9.36	0.00	0.00	0.00	34.68	0.00
time (sec)	N/A	1.517	16.236	1.553	0.000	0.000	0.000	1.870	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1169	801	1683	2878	0	4751	19122	2902	0
N.S.	1	0.69	1.44	2.46	0.00	4.06	16.36	2.48	0.00
time (sec)	N/A	1.976	13.655	1.000	0.000	2.746	1.543	0.334	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	753	525	877	1680	0	3145	9687	1802	0
N.S.	1	0.70	1.16	2.23	0.00	4.18	12.86	2.39	0.00
time (sec)	N/A	1.198	11.445	0.852	0.000	1.496	1.333	0.323	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	323	601	936	0	1833	3990	925	0
N.S.	1	0.77	1.44	2.24	0.00	4.39	9.55	2.21	0.00
time (sec)	N/A	0.581	7.707	0.766	0.000	0.651	1.128	0.309	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	210	293	398	0	839	1360	403	0
N.S.	1	0.89	1.24	1.69	0.00	3.56	5.76	1.71	0.00
time (sec)	N/A	0.380	0.153	0.660	0.000	0.364	0.562	0.287	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	660	694	728	990	0	0	0	0	0
N.S.	1	1.05	1.10	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.652	5.623	0.947	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	754	775	641	1324	0	0	0	0	0
N.S.	1	1.03	0.85	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.136	4.828	0.962	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	824	840	817	1781	0	0	0	2628	0
N.S.	1	1.02	0.99	2.16	0.00	0.00	0.00	3.19	0.00
time (sec)	N/A	1.972	11.634	1.051	0.000	0.000	0.000	0.665	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	833	846	983	2704	0	0	0	7155	0
N.S.	1	1.02	1.18	3.25	0.00	0.00	0.00	8.59	0.00
time (sec)	N/A	2.076	13.344	1.044	0.000	0.000	0.000	5.099	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1097	1146	1005	4359	0	0	0	0	0
N.S.	1	1.04	0.92	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.662	15.586	1.214	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1226	1291	1363	13372	0	0	0	0	0
N.S.	1	1.05	1.11	10.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.642	16.342	1.390	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	657	549	1222	20684	0	0	0	48343	0
N.S.	1	0.84	1.86	31.48	0.00	0.00	0.00	73.58	0.00
time (sec)	N/A	0.965	16.264	1.817	0.000	0.000	0.000	9.703	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1062	823	1636	31330	0	0	0	75375	0
N.S.	1	0.77	1.54	29.50	0.00	0.00	0.00	70.97	0.00
time (sec)	N/A	1.708	16.323	2.517	0.000	0.000	0.000	31.742	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	163	80	60	126	83	76	78	170
N.S.	1	1.14	0.56	0.42	0.88	0.58	0.53	0.55	1.19
time (sec)	N/A	0.383	0.422	0.700	0.270	0.337	0.478	0.283	14.570

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	133	75	55	109	78	70	73	153
N.S.	1	1.13	0.64	0.47	0.92	0.66	0.59	0.62	1.30
time (sec)	N/A	0.337	0.340	0.672	0.272	0.321	0.465	0.293	14.200

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	103	70	50	92	73	63	68	136
N.S.	1	1.11	0.75	0.54	0.99	0.78	0.68	0.73	1.46
time (sec)	N/A	0.270	0.264	0.672	0.276	0.271	0.492	0.287	13.873

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	104	70	96	115	0	126	0
N.S.	1	1.06	1.03	0.69	0.95	1.14	0.00	1.25	0.00
time (sec)	N/A	0.350	0.242	0.689	0.292	0.306	0.000	0.312	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	114	110	87	103	133	0	380	0
N.S.	1	1.06	1.02	0.81	0.95	1.23	0.00	3.52	0.00
time (sec)	N/A	0.340	0.419	0.657	0.271	0.350	0.000	0.486	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	121	111	87	114	149	0	251	0
N.S.	1	1.05	0.97	0.76	0.99	1.30	0.00	2.18	0.00
time (sec)	N/A	0.357	0.451	0.765	0.278	0.280	0.000	0.318	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	183	90	70	155	93	90	88	0
N.S.	1	1.16	0.57	0.44	0.98	0.59	0.57	0.56	0.00
time (sec)	N/A	0.445	0.646	0.789	0.281	0.268	0.580	0.272	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	161	85	65	138	88	82	83	0
N.S.	1	1.14	0.60	0.46	0.98	0.62	0.58	0.59	0.00
time (sec)	N/A	0.382	0.544	0.794	0.265	0.270	0.535	0.288	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	131	80	60	121	83	76	78	0
N.S.	1	1.13	0.69	0.52	1.04	0.72	0.66	0.67	0.00
time (sec)	N/A	0.305	0.402	0.761	0.269	0.272	0.553	0.282	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	135	114	80	125	125	0	136	0
N.S.	1	1.09	0.92	0.65	1.01	1.01	0.00	1.10	0.00
time (sec)	N/A	0.392	0.382	0.663	0.286	0.267	0.000	0.395	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	142	121	97	132	143	0	570	0
N.S.	1	1.08	0.92	0.74	1.01	1.09	0.00	4.35	0.00
time (sec)	N/A	0.388	0.450	0.743	0.280	0.262	0.000	0.502	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	153	121	97	143	159	0	261	0
N.S.	1	1.11	0.88	0.70	1.04	1.15	0.00	1.89	0.00
time (sec)	N/A	0.387	0.589	0.698	0.304	0.267	0.000	0.307	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	219	100	80	184	103	104	98	0
N.S.	1	1.16	0.53	0.42	0.97	0.54	0.55	0.52	0.00
time (sec)	N/A	0.445	0.838	0.681	0.296	0.257	0.750	0.284	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	189	95	75	167	98	97	93	0
N.S.	1	1.15	0.58	0.46	1.02	0.60	0.59	0.57	0.00
time (sec)	N/A	0.400	0.764	0.670	0.279	0.259	0.661	0.280	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	159	90	70	150	93	88	88	0
N.S.	1	1.14	0.65	0.50	1.08	0.67	0.63	0.63	0.00
time (sec)	N/A	0.326	0.638	0.680	0.280	0.248	0.658	0.271	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	163	124	90	154	135	0	146	0
N.S.	1	1.11	0.84	0.61	1.05	0.92	0.00	0.99	0.00
time (sec)	N/A	0.423	0.621	0.694	0.304	0.275	0.000	0.327	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	170	131	107	161	153	0	760	0
N.S.	1	1.10	0.85	0.69	1.05	0.99	0.00	4.94	0.00
time (sec)	N/A	0.429	0.657	0.776	0.285	0.273	0.000	0.571	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	177	131	107	172	169	0	271	0
N.S.	1	1.10	0.81	0.66	1.07	1.05	0.00	1.68	0.00
time (sec)	N/A	0.442	0.784	0.807	0.288	0.267	0.000	0.313	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	693	709	588	810	0	1435	1613	798	0
N.S.	1	1.02	0.85	1.17	0.00	2.07	2.33	1.15	0.00
time (sec)	N/A	1.638	2.670	1.232	0.000	0.495	1.276	0.312	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	432	341	437	0	861	910	442	0
N.S.	1	1.03	0.81	1.04	0.00	2.05	2.17	1.05	0.00
time (sec)	N/A	0.993	1.264	0.896	0.000	0.361	1.100	0.307	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	232	178	192	0	461	437	201	0
N.S.	1	1.04	0.80	0.86	0.00	2.07	1.96	0.90	0.00
time (sec)	N/A	0.463	0.630	0.764	0.000	0.409	0.973	0.362	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	98	86	0	227	226	93	0
N.S.	1	1.04	0.84	0.74	0.00	1.96	1.95	0.80	0.00
time (sec)	N/A	0.286	0.032	0.681	0.000	0.277	0.368	0.298	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	188	187	251	0	0	0	0	0
N.S.	1	1.05	1.04	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	0.685	0.773	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	277	236	485	0	0	0	0	0
N.S.	1	1.15	0.98	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	1.045	0.836	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	356	471	1013	0	2034	0	2279	0
N.S.	1	1.06	1.40	3.01	0.00	6.05	0.00	6.78	0.00
time (sec)	N/A	0.694	11.211	0.993	0.000	34.051	0.000	0.331	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	504	520	694	997	0	2937	0	1028	0
N.S.	1	1.03	1.38	1.98	0.00	5.83	0.00	2.04	0.00
time (sec)	N/A	1.151	4.716	1.285	0.000	7.715	0.000	0.300	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	302	390	548	0	1769	0	565	0
N.S.	1	1.04	1.35	1.90	0.00	6.12	0.00	1.96	0.00
time (sec)	N/A	0.605	2.452	0.959	0.000	5.980	0.000	0.295	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	199	194	311	0	905	0	263	0
N.S.	1	1.07	1.04	1.67	0.00	4.87	0.00	1.41	0.00
time (sec)	N/A	0.443	0.907	0.803	0.000	3.865	0.000	0.360	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	201	0	429	0	118	143
N.S.	1	1.00	0.95	1.81	0.00	3.86	0.00	1.06	1.29
time (sec)	N/A	0.263	0.145	0.708	0.000	0.474	0.000	0.349	13.602

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	235	547	0	1905	0	708	0
N.S.	1	1.00	1.04	2.43	0.00	8.47	0.00	3.15	0.00
time (sec)	N/A	0.470	1.020	0.764	0.000	9.787	0.000	0.306	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	419	518	1115	0	5098	0	0	0
N.S.	1	1.00	1.23	2.65	0.00	12.11	0.00	0.00	0.00
time (sec)	N/A	1.046	10.972	0.760	0.000	35.522	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	713	738	847	2268	0	10340	0	5562	0
N.S.	1	1.04	1.19	3.18	0.00	14.50	0.00	7.80	0.00
time (sec)	N/A	3.286	11.870	0.901	0.000	162.168	0.000	0.408	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	135	70	50	97	73	63	68	0
N.S.	1	1.12	0.58	0.42	0.81	0.61	0.52	0.57	0.00
time (sec)	N/A	0.351	0.341	0.698	0.276	0.280	0.519	0.274	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	105	65	45	80	68	56	63	0
N.S.	1	1.11	0.68	0.47	0.84	0.72	0.59	0.66	0.00
time (sec)	N/A	0.304	0.285	0.695	0.282	0.253	0.494	0.280	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	75	60	40	63	63	49	58	0
N.S.	1	1.07	0.86	0.57	0.90	0.90	0.70	0.83	0.00
time (sec)	N/A	0.247	0.211	0.671	0.277	0.259	0.521	0.295	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	83	93	60	67	105	0	116	0
N.S.	1	1.06	1.19	0.77	0.86	1.35	0.00	1.49	0.00
time (sec)	N/A	0.292	0.233	0.706	0.290	0.270	0.000	0.315	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	86	99	67	74	123	0	191	0
N.S.	1	1.04	1.19	0.81	0.89	1.48	0.00	2.30	0.00
time (sec)	N/A	0.297	0.290	0.674	0.282	0.285	0.000	0.424	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	77	68	82	96	0	204	0
N.S.	1	1.06	0.87	0.76	0.92	1.08	0.00	2.29	0.00
time (sec)	N/A	0.280	0.341	0.707	0.297	0.269	0.000	0.320	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	70	50	97	97	0	67	0
N.S.	1	1.04	0.68	0.49	0.94	0.94	0.00	0.65	0.00
time (sec)	N/A	0.373	0.479	0.694	0.276	0.262	0.000	0.291	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	84	65	45	80	92	0	62	0
N.S.	1	1.02	0.79	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.316	0.438	0.838	0.281	0.256	0.000	0.347	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	60	40	63	87	0	57	0
N.S.	1	1.02	0.95	0.63	1.00	1.38	0.00	0.90	0.00
time (sec)	N/A	0.239	0.357	0.810	0.305	0.278	0.000	0.292	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	51	64	96	0	91	0
N.S.	1	1.00	1.13	0.82	1.03	1.55	0.00	1.47	0.00
time (sec)	N/A	0.252	0.368	0.799	0.278	0.274	0.000	0.310	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	90	90	63	96	106	0	168	0
N.S.	1	1.03	1.03	0.72	1.10	1.22	0.00	1.93	0.00
time (sec)	N/A	0.285	0.407	0.732	0.281	0.270	0.000	0.303	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	87	68	145	126	0	223	0
N.S.	1	1.04	0.78	0.61	1.29	1.12	0.00	1.99	0.00
time (sec)	N/A	0.380	0.493	0.675	0.284	0.259	0.000	0.315	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	90	70	50	202	117	0	67	0
N.S.	1	1.05	0.81	0.58	2.35	1.36	0.00	0.78	0.00
time (sec)	N/A	0.327	0.651	0.672	0.294	0.259	0.000	0.303	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	65	45	185	112	0	62	0
N.S.	1	1.04	0.96	0.66	2.72	1.65	0.00	0.91	0.00
time (sec)	N/A	0.281	0.563	0.691	0.289	0.248	0.000	0.297	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	76	51	0	28	49
N.S.	1	1.00	0.70	0.64	1.62	1.09	0.00	0.60	1.04
time (sec)	N/A	0.213	0.405	0.728	0.212	0.260	0.000	0.280	13.336

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	80	80	93	126	0	101	0
N.S.	1	1.06	0.94	0.94	1.09	1.48	0.00	1.19	0.00
time (sec)	N/A	0.283	0.504	0.660	0.294	0.264	0.000	0.292	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	118	92	73	125	141	0	233	0
N.S.	1	1.07	0.84	0.66	1.14	1.28	0.00	2.12	0.00
time (sec)	N/A	0.363	0.579	0.701	0.282	0.267	0.000	0.318	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	146	105	78	174	156	0	233	0
N.S.	1	1.08	0.78	0.58	1.29	1.16	0.00	1.73	0.00
time (sec)	N/A	0.475	0.649	0.703	0.290	0.274	0.000	0.308	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	217	219	324	0	465	0	0	1089
N.S.	1	1.04	1.05	1.56	0.00	2.24	0.00	0.00	5.24
time (sec)	N/A	0.566	0.291	0.934	0.000	13.842	0.000	0.000	14.730

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	906	927	15669	1736	0	1023	0	0	0
N.S.	1	1.02	17.29	1.92	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	2.194	35.798	3.458	0.000	0.106	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	685	9965	1202	0	764	0	0	0
N.S.	1	1.03	14.92	1.80	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	1.214	35.384	2.446	0.000	0.105	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	749	774	1276	1215	0	814	0	0	0
N.S.	1	1.03	1.70	1.62	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	1.477	35.140	4.020	0.000	0.118	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	722	8456	1340	0	1385	0	0	0
N.S.	1	1.01	11.88	1.88	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	1.302	35.077	4.858	0.000	0.148	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	992	1013	12997	1766	0	2430	0	0	0
N.S.	1	1.02	13.10	1.78	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	1.899	35.861	3.215	0.000	0.308	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1363	1413	19853	2484	0	4543	0	0	0
N.S.	1	1.04	14.57	1.82	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	3.274	36.637	4.214	0.000	0.779	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1904	1966	29140	3498	0	7780	0	0	0
N.S.	1	1.03	15.30	1.84	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	4.553	38.306	4.813	0.000	2.280	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	724	740	1314	1261	0	761	0	0	0
N.S.	1	1.02	1.81	1.74	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	1.561	33.480	3.446	0.000	0.107	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	557	568	862	955	0	571	0	0	0
N.S.	1	1.02	1.55	1.71	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	1.038	31.143	2.562	0.000	0.099	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	476	980	823	0	451	0	0	0
N.S.	1	1.01	2.08	1.75	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.684	30.184	3.816	0.000	0.090	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	522	772	965	0	708	0	0	0
N.S.	1	1.03	1.52	1.90	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.804	26.426	4.171	0.000	0.107	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	684	697	1194	1248	0	1305	0	0	0
N.S.	1	1.02	1.75	1.82	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	1.328	31.720	3.611	0.000	0.154	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	944	986	1746	1757	0	2656	0	0	0
N.S.	1	1.04	1.85	1.86	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	2.018	34.764	4.391	0.000	0.402	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	510	503	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.788	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	496	492	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.701	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	590	589	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	119	36	59	75	211	141	58
N.S.	1	1.00	2.90	0.88	1.44	1.83	5.15	3.44	1.41
time (sec)	N/A	0.207	0.277	0.496	0.307	0.260	3.225	0.293	13.562

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	39	66	83	280	181	78
N.S.	1	1.00	0.74	0.85	1.43	1.80	6.09	3.93	1.70
time (sec)	N/A	0.231	0.433	0.684	0.238	0.266	53.167	0.305	13.489

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	51	98	123	483	296	120
N.S.	1	1.00	0.75	0.89	1.72	2.16	8.47	5.19	2.11
time (sec)	N/A	0.291	0.744	0.658	0.234	0.263	58.819	0.304	13.572

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	167	2052	1779	1779	2281	2467	2026
N.S.	1	1.00	8.35	102.60	88.95	88.95	114.05	123.35	101.30
time (sec)	N/A	0.865	0.300	0.906	0.210	0.272	0.162	0.284	14.160

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	20	20	18
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69
time (sec)	N/A	0.187	0.008	0.518	0.203	0.252	0.045	0.281	0.052

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	370	282	314	0	701	700	321	0
N.S.	1	1.07	0.82	0.91	0.00	2.03	2.02	0.93	0.00
time (sec)	N/A	0.847	1.097	0.837	0.000	0.366	0.873	0.320	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	261	199	210	0	499	479	221	0
N.S.	1	1.07	0.81	0.86	0.00	2.04	1.96	0.90	0.00
time (sec)	N/A	0.569	0.708	0.755	0.000	0.304	0.840	0.329	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	190	139	134	0	341	347	144	0
N.S.	1	1.07	0.79	0.76	0.00	1.93	1.96	0.81	0.00
time (sec)	N/A	0.427	0.522	0.727	0.000	0.303	0.476	0.338	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	168	135	223	0	733	0	0	0
N.S.	1	1.08	0.87	1.44	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.511	0.638	0.717	0.000	2.076	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	152	146	156	0	703	0	170	166
N.S.	1	1.09	1.05	1.12	0.00	5.06	0.00	1.22	1.19
time (sec)	N/A	0.471	0.934	0.775	0.000	1.343	0.000	0.345	13.669

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	175	170	134	0	783	0	346	0
N.S.	1	1.10	1.07	0.84	0.00	4.92	0.00	2.18	0.00
time (sec)	N/A	0.517	1.131	0.746	0.000	1.745	0.000	0.346	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	201	178	150	0	365	0	680	0
N.S.	1	1.08	0.96	0.81	0.00	1.96	0.00	3.66	0.00
time (sec)	N/A	0.524	0.926	0.818	0.000	1.889	0.000	0.306	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	291	237	236	0	525	0	1433	0
N.S.	1	1.08	0.88	0.87	0.00	1.94	0.00	5.31	0.00
time (sec)	N/A	0.676	1.573	0.857	0.000	4.408	0.000	0.328	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	401	328	350	0	727	0	2155	0
N.S.	1	1.08	0.88	0.94	0.00	1.96	0.00	5.81	0.00
time (sec)	N/A	0.907	2.137	0.986	0.000	11.939	0.000	0.313	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	212	200	206	206	230	237	196
N.S.	1	1.00	0.82	0.78	0.80	0.80	0.89	0.92	0.76
time (sec)	N/A	0.572	0.027	0.475	0.200	0.244	0.040	0.275	0.127

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	136	139	145	145	158	160	137
N.S.	1	1.00	0.87	0.89	0.92	0.92	1.01	1.02	0.87
time (sec)	N/A	0.414	0.021	0.541	0.194	0.275	0.029	0.287	13.307

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	78	79	79	87	83	77
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.94	0.89	0.83
time (sec)	N/A	0.327	0.009	0.072	0.191	0.245	0.024	0.265	0.049

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	34	37	34	34
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	0.81
time (sec)	N/A	0.196	0.002	0.047	0.194	0.241	0.023	0.268	0.027

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	179	220	228	230	235	249	260
N.S.	1	1.00	0.79	0.96	1.00	1.01	1.03	1.09	1.14
time (sec)	N/A	0.428	0.038	0.576	0.196	0.241	0.244	0.302	0.065

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	223	232	234	319	238	325	363
N.S.	1	1.00	0.98	1.02	1.03	1.40	1.04	1.43	1.59
time (sec)	N/A	0.447	0.052	0.463	0.204	0.243	0.448	0.430	13.313

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	225	240	360	248	237	297
N.S.	1	1.00	0.88	0.97	1.04	1.56	1.07	1.03	1.29
time (sec)	N/A	0.445	0.039	0.461	0.207	0.248	0.854	0.353	0.095

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	277	254	263	263	298	305	251
N.S.	1	1.00	0.71	0.65	0.67	0.67	0.76	0.78	0.64
time (sec)	N/A	0.792	0.026	0.546	0.195	0.236	0.045	0.271	13.410

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	177	185	185	206	206	175
N.S.	1	1.00	1.00	0.88	0.92	0.92	1.02	1.02	0.87
time (sec)	N/A	0.495	0.022	0.559	0.194	0.251	0.036	0.265	0.132

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	100	105	105	112	107	101
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.93	0.88	0.83
time (sec)	N/A	0.403	0.012	0.526	0.194	0.235	0.030	0.277	0.092

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	45	44	44	56	44	44
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.93	0.73	0.73
time (sec)	N/A	0.216	0.001	0.529	0.197	0.229	0.024	0.267	0.036

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	262	354	366	368	372	416	434
N.S.	1	1.00	0.74	1.01	1.04	1.05	1.06	1.18	1.23
time (sec)	N/A	0.601	0.079	0.556	0.189	0.234	0.360	0.264	0.082

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	342	366	372	490	393	491	939
N.S.	1	1.00	0.97	1.04	1.05	1.39	1.11	1.39	2.66
time (sec)	N/A	0.621	0.088	0.555	0.195	0.241	0.696	0.266	13.408

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	311	359	378	545	394	392	771
N.S.	1	1.00	0.88	1.01	1.07	1.54	1.11	1.11	2.18
time (sec)	N/A	0.637	0.060	0.529	0.197	0.252	1.357	0.271	13.331

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	344	361	390	587	401	383	560
N.S.	1	1.00	0.96	1.00	1.08	1.63	1.11	1.06	1.56
time (sec)	N/A	0.625	0.083	0.546	0.212	0.247	2.541	0.260	13.261

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	178	222	206	206	450	221	397
N.S.	1	1.00	0.81	1.00	0.93	0.93	2.04	1.00	1.80
time (sec)	N/A	0.421	0.077	1.079	0.280	0.255	0.750	0.269	13.402

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	130	147	141	141	303	146	223
N.S.	1	1.00	0.83	0.94	0.90	0.90	1.94	0.94	1.43
time (sec)	N/A	0.360	0.053	0.675	0.288	0.249	0.535	0.258	13.737

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	86	83	84	84	163	82	107
N.S.	1	1.00	0.87	0.84	0.85	0.85	1.65	0.83	1.08
time (sec)	N/A	0.279	0.034	0.654	0.281	0.244	0.339	0.281	0.075

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	43	43	61	43	45
N.S.	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.80
time (sec)	N/A	0.212	0.014	0.629	0.277	0.250	0.062	0.277	0.043

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	146	142	160	171	0	159	713
N.S.	1	1.00	0.87	0.85	0.95	1.02	0.00	0.95	4.24
time (sec)	N/A	0.402	0.076	0.776	0.277	0.281	0.000	0.286	15.969

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	233	213	294	416	0	357	312
N.S.	1	1.00	1.00	0.91	1.26	1.79	0.00	1.53	1.34
time (sec)	N/A	0.476	0.102	0.969	0.289	0.322	0.000	0.280	14.221

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	278	298	498	698	0	438	493
N.S.	1	1.00	0.88	0.94	1.57	2.20	0.00	1.38	1.56
time (sec)	N/A	0.553	0.263	0.843	0.290	0.396	0.000	0.294	13.773

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	209	214	212	350	444	215	333
N.S.	1	1.00	1.11	1.13	1.12	1.85	2.35	1.14	1.76
time (sec)	N/A	0.518	0.099	0.809	0.276	0.263	1.345	0.272	0.155

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	143	150	145	147	245	298	146	211
N.S.	1	1.02	1.07	1.04	1.05	1.75	2.13	1.04	1.51
time (sec)	N/A	0.432	0.074	0.787	0.279	0.257	0.921	0.271	0.116

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	96	87	90	147	165	88	115
N.S.	1	1.01	0.99	0.90	0.93	1.52	1.70	0.91	1.19
time (sec)	N/A	0.365	0.103	0.758	0.275	0.255	0.617	0.281	13.437

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	66	59	50	52	78	65	52	52
N.S.	1	1.05	0.94	0.79	0.83	1.24	1.03	0.83	0.83
time (sec)	N/A	0.247	0.023	0.741	0.279	0.243	0.076	0.270	0.053

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	230	186	214	289	479	0	296	330
N.S.	1	1.03	0.83	0.96	1.29	2.14	0.00	1.32	1.47
time (sec)	N/A	0.604	0.100	0.951	0.283	0.317	0.000	0.280	14.222

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	314	270	303	548	910	0	584	601
N.S.	1	1.00	0.86	0.97	1.75	2.91	0.00	1.87	1.92
time (sec)	N/A	0.878	0.152	0.873	0.287	0.350	0.000	0.272	14.283

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	411	363	400	851	1499	0	648	887
N.S.	1	1.00	0.88	0.97	2.07	3.64	0.00	1.57	2.15
time (sec)	N/A	1.180	0.218	0.862	0.300	0.447	0.000	0.298	14.361

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	181	209	209	222	441	469	210	299
N.S.	1	1.06	1.22	1.22	1.30	2.58	2.74	1.23	1.75
time (sec)	N/A	0.644	0.122	0.844	0.282	0.252	2.949	0.261	0.160

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	142	146	146	155	302	304	145	203
N.S.	1	1.06	1.09	1.09	1.16	2.25	2.27	1.08	1.51
time (sec)	N/A	0.535	0.108	0.852	0.282	0.256	1.882	0.297	13.255

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	116	107	91	101	172	163	91	125
N.S.	1	1.13	1.04	0.88	0.98	1.67	1.58	0.88	1.21
time (sec)	N/A	0.390	0.055	0.762	0.284	0.246	0.880	0.269	13.555

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	72	53	47	56	75	61	46	55
N.S.	1	1.12	0.83	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.238	0.026	0.785	0.281	0.253	0.078	0.265	13.336

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	352	282	359	571	1052	0	495	641
N.S.	1	1.07	0.86	1.09	1.74	3.20	0.00	1.50	1.95
time (sec)	N/A	0.811	0.199	1.051	0.293	0.388	0.000	0.281	13.844

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	449	389	473	916	1734	0	806	965
N.S.	1	1.01	0.88	1.07	2.07	3.91	0.00	1.82	2.18
time (sec)	N/A	1.428	0.287	1.016	0.305	0.519	0.000	0.318	14.105

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	163	80	60	126	83	76	78	170
N.S.	1	1.14	0.56	0.42	0.88	0.58	0.53	0.55	1.19
time (sec)	N/A	0.455	0.892	0.795	0.287	0.247	0.629	0.267	1.745

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	144	75	55	109	78	70	73	153
N.S.	1	1.16	0.60	0.44	0.88	0.63	0.56	0.59	1.23
time (sec)	N/A	0.363	0.372	0.829	0.280	0.246	0.425	0.306	13.963

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	165	103	127	128	125	0	129	0
N.S.	1	1.11	0.69	0.85	0.86	0.84	0.00	0.87	0.00
time (sec)	N/A	0.532	0.418	0.007	0.284	0.258	0.000	0.288	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	163	110	0	132	143	0	531	0
N.S.	1	1.09	0.74	0.00	0.89	0.96	0.00	3.56	0.00
time (sec)	N/A	0.536	0.491	180.000	0.283	0.257	0.000	0.333	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	158	110	0	143	159	0	258	0
N.S.	1	1.05	0.73	0.00	0.95	1.05	0.00	1.71	0.00
time (sec)	N/A	0.520	0.593	180.000	0.291	0.262	0.000	0.304	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	172	110	0	160	173	0	304	0
N.S.	1	1.09	0.70	0.00	1.01	1.09	0.00	1.92	0.00
time (sec)	N/A	0.539	0.679	180.000	0.294	0.270	0.000	0.298	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	183	110	0	181	189	0	327	0
N.S.	1	1.11	0.67	0.00	1.10	1.15	0.00	1.98	0.00
time (sec)	N/A	0.547	0.657	180.000	0.298	0.275	0.000	0.323	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	183	110	0	222	203	0	387	0
N.S.	1	1.11	0.67	0.00	1.35	1.23	0.00	2.35	0.00
time (sec)	N/A	0.537	0.755	180.000	0.289	0.285	0.000	0.299	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	189	86	88	250	156	0	405	0
N.S.	1	1.12	0.51	0.52	1.48	0.92	0.00	2.40	0.00
time (sec)	N/A	0.509	0.777	1.154	0.314	0.259	0.000	0.292	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	219	91	93	301	171	0	456	0
N.S.	1	1.13	0.47	0.48	1.55	0.88	0.00	2.35	0.00
time (sec)	N/A	0.558	1.019	0.412	0.295	0.263	0.000	0.307	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	191	90	70	155	93	90	88	0
N.S.	1	1.15	0.54	0.42	0.93	0.56	0.54	0.53	0.00
time (sec)	N/A	0.480	0.679	0.439	0.272	0.270	0.715	0.290	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	172	85	65	138	88	83	83	0
N.S.	1	1.17	0.58	0.44	0.94	0.60	0.56	0.56	0.00
time (sec)	N/A	0.398	0.567	0.251	0.288	0.253	0.496	0.290	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	191	113	0	157	135	0	139	0
N.S.	1	1.11	0.66	0.00	0.91	0.78	0.00	0.81	0.00
time (sec)	N/A	0.587	0.689	180.000	0.290	0.283	0.000	0.310	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	184	120	0	161	153	0	707	0
N.S.	1	1.07	0.70	0.00	0.94	0.89	0.00	4.11	0.00
time (sec)	N/A	0.605	0.722	180.000	0.284	0.267	0.000	0.356	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	186	120	0	172	169	0	268	0
N.S.	1	1.07	0.69	0.00	0.99	0.97	0.00	1.54	0.00
time (sec)	N/A	0.587	0.780	180.000	0.297	0.271	0.000	0.298	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	196	120	0	189	183	0	314	0
N.S.	1	1.08	0.66	0.00	1.04	1.01	0.00	1.73	0.00
time (sec)	N/A	0.598	0.845	180.000	0.297	0.272	0.000	0.288	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	207	120	0	210	199	0	503	0
N.S.	1	1.10	0.64	0.00	1.12	1.06	0.00	2.68	0.00
time (sec)	N/A	0.610	0.774	180.000	0.303	0.268	0.000	0.361	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	218	120	0	251	213	0	406	0
N.S.	1	1.12	0.62	0.00	1.29	1.09	0.00	2.08	0.00
time (sec)	N/A	0.628	0.892	180.000	0.312	0.284	0.000	0.308	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	218	120	0	297	229	0	452	0
N.S.	1	1.12	0.62	0.00	1.52	1.17	0.00	2.32	0.00
time (sec)	N/A	0.629	0.966	180.000	0.323	0.271	0.000	0.297	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	218	120	0	348	243	0	489	0
N.S.	1	1.12	0.62	0.00	1.78	1.25	0.00	2.51	0.00
time (sec)	N/A	0.624	1.187	180.000	0.340	0.273	0.000	0.312	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	133	70	50	96	73	60	68	0
N.S.	1	1.11	0.58	0.42	0.80	0.61	0.50	0.57	0.00
time (sec)	N/A	0.423	0.326	1.500	0.301	0.252	0.639	0.279	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	116	65	45	80	68	56	63	0
N.S.	1	1.15	0.64	0.45	0.79	0.67	0.55	0.62	0.00
time (sec)	N/A	0.345	0.288	0.235	0.308	0.254	0.433	0.283	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	137	93	0	99	115	0	119	0
N.S.	1	1.09	0.74	0.00	0.79	0.91	0.00	0.94	0.00
time (sec)	N/A	0.503	0.333	180.000	0.301	0.278	0.000	0.292	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	130	100	0	103	133	0	339	0
N.S.	1	1.03	0.79	0.00	0.82	1.06	0.00	2.69	0.00
time (sec)	N/A	0.504	0.509	180.000	0.301	0.260	0.000	0.344	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	134	100	0	114	149	0	248	0
N.S.	1	1.05	0.78	0.00	0.89	1.16	0.00	1.94	0.00
time (sec)	N/A	0.491	0.482	180.000	0.322	0.267	0.000	0.299	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	148	100	0	131	163	0	285	0
N.S.	1	1.10	0.74	0.00	0.97	1.21	0.00	2.11	0.00
time (sec)	N/A	0.505	0.540	180.000	0.293	0.270	0.000	0.284	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	154	76	0	149	125	0	164	0
N.S.	1	1.11	0.55	0.00	1.07	0.90	0.00	1.18	0.00
time (sec)	N/A	0.469	0.590	180.000	0.298	0.252	0.000	0.313	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	139	75	0	114	102	0	72	0
N.S.	1	1.12	0.60	0.00	0.92	0.82	0.00	0.58	0.00
time (sec)	N/A	0.457	0.506	180.000	0.275	0.256	0.000	0.280	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	118	70	50	97	97	0	67	0
N.S.	1	1.15	0.68	0.49	0.94	0.94	0.00	0.65	0.00
time (sec)	N/A	0.369	0.462	1.214	0.286	0.257	0.000	0.291	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	65	45	80	92	0	62	0
N.S.	1	1.10	0.79	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.278	0.414	0.235	0.287	0.254	0.000	0.275	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	102	93	0	99	149	0	118	0
N.S.	1	1.01	0.92	0.00	0.98	1.48	0.00	1.17	0.00
time (sec)	N/A	0.400	0.467	180.000	0.287	0.277	0.000	0.286	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	114	110	0	116	157	0	225	0
N.S.	1	1.06	1.02	0.00	1.07	1.45	0.00	2.08	0.00
time (sec)	N/A	0.410	0.526	180.000	0.293	0.266	0.000	0.347	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	118	76	0	149	126	0	220	0
N.S.	1	1.05	0.68	0.00	1.33	1.12	0.00	1.96	0.00
time (sec)	N/A	0.379	0.487	180.000	0.284	0.278	0.000	0.297	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	143	81	0	217	141	0	271	0
N.S.	1	1.04	0.59	0.00	1.58	1.03	0.00	1.98	0.00
time (sec)	N/A	0.482	0.563	180.000	0.296	0.267	0.000	0.284	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	118	75	0	219	122	0	71	0
N.S.	1	1.12	0.71	0.00	2.09	1.16	0.00	0.68	0.00
time (sec)	N/A	0.401	0.537	180.000	0.293	0.273	0.000	0.310	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	94	70	0	202	117	0	66	0
N.S.	1	1.09	0.81	0.00	2.35	1.36	0.00	0.77	0.00
time (sec)	N/A	0.311	0.639	180.000	0.288	0.261	0.000	0.285	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	71	65	0	185	112	0	62	0
N.S.	1	1.04	0.96	0.00	2.72	1.65	0.00	0.91	0.00
time (sec)	N/A	0.260	0.488	180.000	0.285	0.266	0.000	0.282	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	90	69	0	110	126	0	92	0
N.S.	1	1.06	0.81	0.00	1.29	1.48	0.00	1.08	0.00
time (sec)	N/A	0.350	0.524	180.000	0.284	0.260	0.000	0.292	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	120	81	0	127	141	0	206	0
N.S.	1	1.09	0.74	0.00	1.15	1.28	0.00	1.87	0.00
time (sec)	N/A	0.394	0.653	180.000	0.282	0.265	0.000	0.345	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	146	94	0	178	155	0	228	0
N.S.	1	1.08	0.70	0.00	1.32	1.15	0.00	1.69	0.00
time (sec)	N/A	0.493	0.619	180.000	0.297	0.273	0.000	0.290	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	173	91	0	246	170	0	279	0
N.S.	1	1.08	0.57	0.00	1.54	1.06	0.00	1.74	0.00
time (sec)	N/A	0.595	0.706	180.000	0.298	0.261	0.000	0.310	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	354	377	315	0	0	1373	0	463	0
N.S.	1	1.06	0.89	0.00	0.00	3.88	0.00	1.31	0.00
time (sec)	N/A	0.765	1.969	180.000	0.000	22.785	0.000	0.312	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	374	316	1147	0	1385	0	488	0
N.S.	1	1.06	0.90	3.25	0.00	3.92	0.00	1.38	0.00
time (sec)	N/A	0.715	1.973	2.465	0.000	23.128	0.000	0.296	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	588	588	537	5924	2292	4795	136733	10965	4341
N.S.	1	1.00	0.91	10.07	3.90	8.15	232.54	18.65	7.38
time (sec)	N/A	0.822	0.402	0.451	0.269	0.348	38.078	0.387	18.103

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	432	432	391	3222	1414	2796	65193	6226	2625
N.S.	1	1.00	0.91	7.46	3.27	6.47	150.91	14.41	6.08
time (sec)	N/A	0.651	0.283	0.393	0.242	0.305	13.895	0.325	15.425

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	261	1220	788	1448	26165	3099	1425
N.S.	1	1.00	0.89	4.18	2.70	4.96	89.61	10.61	4.88
time (sec)	N/A	0.480	0.186	0.495	0.222	0.288	4.847	0.288	14.290

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	221	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	377	378	441	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.864	1.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	567	488	769	0	3480	0	646	1027
N.S.	1	1.07	0.92	1.46	0.00	6.59	0.00	1.22	1.95
time (sec)	N/A	1.201	0.607	0.413	0.000	0.371	0.000	0.266	15.649

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	765	765	754	1086	0	2643	0	981	2779
N.S.	1	1.00	0.99	1.42	0.00	3.45	0.00	1.28	3.63
time (sec)	N/A	4.048	0.412	0.470	0.000	0.617	0.000	0.258	17.975

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	249	99	75	177	97	99	92	221
N.S.	1	1.20	0.48	0.36	0.85	0.47	0.48	0.44	1.06
time (sec)	N/A	0.853	0.723	0.263	0.282	0.283	0.482	0.287	15.619

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	197	89	65	143	87	85	82	187
N.S.	1	1.19	0.54	0.39	0.86	0.52	0.51	0.49	1.13
time (sec)	N/A	0.580	0.480	0.279	0.288	0.274	0.426	0.283	15.043

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	145	79	55	109	77	71	72	153
N.S.	1	1.17	0.64	0.44	0.88	0.62	0.57	0.58	1.23
time (sec)	N/A	0.397	0.339	0.179	0.301	0.260	0.404	0.275	14.393

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	214	234	223	500	304	0	144	0
N.S.	1	1.14	1.25	1.19	2.67	1.63	0.00	0.77	0.00
time (sec)	N/A	0.575	0.347	0.618	0.333	0.275	0.000	0.313	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	226	427	235	0	378	0	0	0
N.S.	1	1.14	2.15	1.18	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.528	0.534	0.500	0.000	0.263	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	237	602	231	0	390	0	378	0
N.S.	1	1.11	2.83	1.08	0.00	1.83	0.00	1.77	0.00
time (sec)	N/A	0.509	0.717	0.472	0.000	0.270	0.000	0.299	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	279	109	85	206	107	112	102	0
N.S.	1	1.21	0.47	0.37	0.89	0.46	0.48	0.44	0.00
time (sec)	N/A	0.888	0.864	0.245	0.285	0.247	0.630	0.281	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	227	99	75	172	97	99	92	0
N.S.	1	1.20	0.52	0.40	0.91	0.51	0.52	0.49	0.00
time (sec)	N/A	0.621	0.714	0.319	0.290	0.253	0.512	0.283	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	175	89	65	138	87	85	82	0
N.S.	1	1.19	0.61	0.44	0.94	0.59	0.58	0.56	0.00
time (sec)	N/A	0.428	0.509	0.530	0.284	0.256	0.457	0.276	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	242	254	233	535	326	0	154	0
N.S.	1	1.15	1.21	1.11	2.55	1.55	0.00	0.73	0.00
time (sec)	N/A	0.639	0.558	0.506	0.395	0.290	0.000	0.313	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	254	447	245	0	378	0	0	0
N.S.	1	1.14	2.01	1.10	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.639	0.699	0.513	0.000	0.285	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	266	636	245	0	447	0	411	0
N.S.	1	1.14	2.72	1.05	0.00	1.91	0.00	1.76	0.00
time (sec)	N/A	0.649	0.809	0.529	0.000	0.292	0.000	0.332	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	215	89	65	148	87	85	82	0
N.S.	1	1.16	0.48	0.35	0.80	0.47	0.46	0.44	0.00
time (sec)	N/A	0.791	0.600	0.260	0.288	0.254	0.468	0.282	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	167	79	55	114	77	71	72	0
N.S.	1	1.17	0.55	0.38	0.80	0.54	0.50	0.50	0.00
time (sec)	N/A	0.545	0.430	0.304	0.281	0.251	0.453	0.286	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	115	69	45	80	67	58	62	0
N.S.	1	1.14	0.68	0.45	0.79	0.66	0.57	0.61	0.00
time (sec)	N/A	0.364	0.294	0.198	0.287	0.249	0.385	0.285	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	186	211	204	465	297	0	125	0
N.S.	1	1.13	1.29	1.24	2.84	1.81	0.00	0.76	0.00
time (sec)	N/A	0.419	0.286	0.490	0.311	0.268	0.000	0.330	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	197	352	221	0	330	0	276	0
N.S.	1	1.11	1.98	1.24	0.00	1.85	0.00	1.55	0.00
time (sec)	N/A	0.393	0.438	0.452	0.000	0.255	0.000	0.311	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	237	433	231	0	390	0	378	0
N.S.	1	1.04	1.91	1.02	0.00	1.72	0.00	1.67	0.00
time (sec)	N/A	0.511	0.696	0.493	0.000	0.291	0.000	0.303	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	187	89	65	148	112	0	81	0
N.S.	1	1.13	0.54	0.39	0.89	0.67	0.00	0.49	0.00
time (sec)	N/A	0.677	0.534	0.307	0.278	0.264	0.000	0.280	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	132	79	55	114	102	0	71	0
N.S.	1	1.06	0.64	0.44	0.92	0.82	0.00	0.57	0.00
time (sec)	N/A	0.450	0.637	0.244	0.283	0.255	0.000	0.289	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	91	69	45	80	92	0	62	0
N.S.	1	1.11	0.84	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.295	0.412	0.278	0.286	0.258	0.000	0.297	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	185	199	209	777	333	0	112	0
N.S.	1	1.11	1.20	1.26	4.68	2.01	0.00	0.67	0.00
time (sec)	N/A	0.404	0.401	0.436	0.328	0.273	0.000	0.284	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	225	416	231	0	392	0	295	0
N.S.	1	1.05	1.93	1.07	0.00	1.82	0.00	1.37	0.00
time (sec)	N/A	0.507	0.662	0.475	0.000	0.274	0.000	0.296	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	265	607	241	0	452	0	397	0
N.S.	1	1.06	2.43	0.96	0.00	1.81	0.00	1.59	0.00
time (sec)	N/A	0.616	0.925	0.523	0.000	0.373	0.000	0.296	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	236	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	302	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.400	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [374] had the largest ratio of [.4857139999999999979]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	11	1.01	34	0.324
2	A	10	9	0.97	32	0.281
3	A	7	6	0.96	27	0.222
4	A	7	6	0.94	34	0.176
5	A	7	6	0.92	34	0.176
6	A	8	7	0.96	34	0.206
7	A	2	2	1.00	34	0.059
8	A	5	5	0.94	34	0.147
9	A	6	6	0.95	34	0.176
10	A	13	12	1.12	34	0.353
11	A	11	10	1.08	34	0.294
12	A	9	8	1.04	32	0.250
13	A	6	5	1.02	27	0.185
14	A	7	6	1.01	34	0.176
15	A	2	2	1.00	34	0.059
16	A	5	5	0.94	34	0.147
17	A	6	6	0.95	34	0.176
18	A	2	2	0.99	25	0.080
19	A	2	2	0.99	25	0.080
20	A	2	2	1.00	23	0.087
21	A	2	2	1.00	18	0.111
22	A	2	2	0.99	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	0.99	25	0.080
24	A	2	2	0.99	25	0.080
25	A	3	3	0.99	27	0.111
26	A	3	3	1.00	27	0.111
27	A	3	3	1.00	25	0.120
28	A	3	3	1.00	20	0.150
29	A	2	2	0.99	27	0.074
30	A	2	2	0.99	27	0.074
31	A	2	2	0.99	27	0.074
32	A	3	3	0.99	27	0.111
33	A	3	3	1.00	27	0.111
34	A	3	3	1.00	25	0.120
35	A	3	3	1.00	20	0.150
36	A	2	2	0.99	27	0.074
37	A	2	2	0.99	27	0.074
38	A	2	2	0.99	27	0.074
39	A	1	1	1.00	32	0.031
40	A	1	1	1.00	31	0.032
41	A	1	1	1.00	34	0.029
42	A	1	1	1.00	33	0.030
43	A	2	2	0.99	27	0.074
44	A	2	2	0.99	27	0.074
45	A	2	2	1.00	25	0.080
46	A	2	2	1.00	20	0.100
47	A	2	2	1.00	27	0.074
48	A	2	2	1.00	27	0.074
49	A	2	2	1.00	27	0.074
50	A	4	4	0.98	27	0.148
51	A	4	4	1.05	27	0.148
52	A	5	5	1.06	25	0.200
53	A	4	4	1.00	20	0.200
54	A	5	5	1.08	27	0.185
55	A	4	4	1.03	27	0.148
56	A	4	4	1.02	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	6	6	1.07	27	0.222
58	A	4	4	1.25	27	0.148
59	A	4	4	1.08	25	0.160
60	A	5	5	1.00	20	0.250
61	A	8	8	1.12	27	0.296
62	A	6	6	1.06	27	0.222
63	A	6	6	1.05	27	0.222
64	A	5	5	0.96	27	0.185
65	A	5	5	1.24	27	0.185
66	A	5	5	1.00	27	0.185
67	A	5	5	1.01	25	0.200
68	A	6	6	0.98	20	0.300
69	A	4	4	0.88	17	0.235
70	A	4	4	1.00	17	0.235
71	A	5	5	0.90	15	0.333
72	A	3	3	1.00	14	0.214
73	A	4	4	1.03	17	0.235
74	A	4	4	1.00	17	0.235
75	A	4	4	0.93	17	0.235
76	A	3	3	1.83	16	0.188
77	A	3	3	1.00	18	0.167
78	A	10	9	0.94	29	0.310
79	A	9	8	0.95	29	0.276
80	A	7	6	1.02	27	0.222
81	A	6	5	0.95	22	0.227
82	A	10	9	1.05	29	0.310
83	A	11	10	1.05	29	0.345
84	A	10	9	1.05	29	0.310
85	A	10	9	1.15	29	0.310
86	A	7	6	0.97	29	0.207
87	A	10	9	1.00	29	0.310
88	A	11	10	0.85	29	0.345
89	A	10	9	0.85	29	0.310
90	A	8	7	0.94	27	0.259

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	7	6	0.90	22	0.273
92	A	12	11	1.06	29	0.379
93	A	13	12	1.06	29	0.414
94	A	12	11	1.00	29	0.379
95	A	12	11	1.02	29	0.379
96	A	12	11	1.08	29	0.379
97	A	12	11	1.13	29	0.379
98	A	8	7	0.91	29	0.241
99	A	11	10	0.94	29	0.345
100	A	8	7	0.86	22	0.318
101	A	9	8	1.06	29	0.276
102	A	8	7	1.08	29	0.241
103	A	6	5	1.12	27	0.185
104	A	5	4	1.04	22	0.182
105	A	8	7	1.04	29	0.241
106	A	8	7	1.11	29	0.241
107	A	6	5	1.05	29	0.172
108	A	7	6	1.09	29	0.207
109	A	6	5	1.13	29	0.172
110	A	6	5	1.06	27	0.185
111	A	6	5	1.00	22	0.227
112	A	6	5	1.00	29	0.172
113	A	6	5	1.00	29	0.172
114	A	8	7	1.02	29	0.241
115	A	4	4	1.00	22	0.182
116	A	5	5	0.97	22	0.227
117	A	6	6	0.95	22	0.273
118	A	8	8	1.21	29	0.276
119	A	6	6	1.22	29	0.207
120	A	4	4	1.18	27	0.148
121	A	7	6	1.00	29	0.207
122	A	7	6	1.06	29	0.207
123	A	6	5	1.06	29	0.172
124	A	8	8	1.09	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	6	1.04	29	0.207
126	A	4	4	1.05	27	0.148
127	A	5	4	1.00	29	0.138
128	A	6	5	1.00	29	0.172
129	A	8	7	0.98	29	0.241
130	A	6	6	1.11	29	0.207
131	A	5	5	1.05	29	0.172
132	A	3	3	1.00	27	0.111
133	A	7	6	1.07	29	0.207
134	A	8	7	1.06	29	0.241
135	A	10	9	1.09	29	0.310
136	A	7	6	0.99	27	0.222
137	A	7	6	1.00	29	0.207
138	A	7	6	1.00	31	0.194
139	A	7	7	1.03	69	0.101
140	A	2	2	1.00	20	0.100
141	A	2	2	1.00	20	0.100
142	A	2	2	1.00	20	0.100
143	A	2	2	1.00	18	0.111
144	A	2	2	1.00	20	0.100
145	A	5	4	1.00	20	0.200
146	A	6	5	0.98	20	0.250
147	A	7	6	0.99	20	0.300
148	A	2	2	1.00	30	0.067
149	A	2	2	1.00	30	0.067
150	A	2	2	1.00	28	0.071
151	A	2	2	1.00	23	0.087
152	A	2	2	1.00	30	0.067
153	A	2	2	1.00	30	0.067
154	A	2	2	1.00	30	0.067
155	A	3	3	1.05	30	0.100
156	A	6	5	1.12	28	0.179
157	A	5	4	1.00	23	0.174
158	A	4	4	1.12	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	3	3	1.05	30	0.100
160	A	4	4	1.05	20	0.200
161	A	3	3	1.09	20	0.150
162	A	7	6	1.06	18	0.333
163	A	5	4	1.00	17	0.235
164	A	3	3	1.09	20	0.150
165	A	3	3	1.05	20	0.150
166	A	3	3	1.04	20	0.150
167	A	1	1	1.00	16	0.062
168	A	2	2	1.00	14	0.143
169	A	2	2	1.00	16	0.125
170	A	2	2	1.00	18	0.111
171	A	2	2	1.00	16	0.125
172	A	2	2	1.00	19	0.105
173	A	2	2	1.00	23	0.087
174	A	2	2	1.00	19	0.105
175	A	2	2	1.00	23	0.087
176	A	5	4	1.13	17	0.235
177	A	1	1	1.00	19	0.053
178	A	9	8	0.90	22	0.364
179	A	8	7	0.93	22	0.318
180	A	7	6	0.99	22	0.273
181	A	6	5	1.06	22	0.227
182	A	5	4	1.00	22	0.182
183	A	3	3	1.00	22	0.136
184	A	4	4	0.99	22	0.182
185	A	5	5	0.98	22	0.227
186	A	11	10	0.81	32	0.312
187	A	9	8	0.82	32	0.250
188	A	7	6	0.87	30	0.200
189	A	7	6	0.95	25	0.240
190	A	10	9	1.07	32	0.281
191	A	10	9	1.05	32	0.281
192	A	11	10	1.04	32	0.312

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	10	9	1.07	32	0.281
194	A	7	6	0.92	32	0.188
195	A	9	8	0.89	32	0.250
196	A	12	11	0.69	32	0.344
197	A	10	9	0.70	32	0.281
198	A	8	7	0.77	30	0.233
199	A	8	7	0.89	25	0.280
200	A	12	11	1.05	32	0.344
201	A	12	11	1.03	32	0.344
202	A	13	12	1.02	32	0.375
203	A	12	11	1.02	32	0.344
204	A	12	11	1.04	32	0.344
205	A	12	11	1.05	32	0.344
206	A	8	7	0.84	32	0.219
207	A	10	9	0.77	32	0.281
208	A	11	10	1.14	32	0.312
209	A	9	8	1.13	32	0.250
210	A	7	6	1.11	30	0.200
211	A	10	9	1.06	32	0.281
212	A	10	9	1.06	32	0.281
213	A	10	9	1.05	32	0.281
214	A	12	11	1.16	32	0.344
215	A	10	9	1.14	32	0.281
216	A	8	7	1.13	30	0.233
217	A	12	11	1.09	32	0.344
218	A	12	11	1.08	32	0.344
219	A	12	11	1.11	32	0.344
220	A	12	11	1.16	32	0.344
221	A	11	10	1.15	32	0.312
222	A	9	8	1.14	30	0.267
223	A	14	13	1.11	32	0.406
224	A	14	13	1.10	32	0.406
225	A	14	13	1.10	32	0.406
226	A	10	9	1.02	32	0.281

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	8	7	1.03	32	0.219
228	A	6	5	1.04	30	0.167
229	A	6	5	1.04	25	0.200
230	A	8	7	1.05	32	0.219
231	A	8	7	1.15	32	0.219
232	A	6	5	1.06	32	0.156
233	A	8	7	1.03	32	0.219
234	A	6	5	1.04	32	0.156
235	A	6	5	1.07	30	0.167
236	A	5	4	1.00	25	0.160
237	A	5	4	1.00	32	0.125
238	A	6	5	1.00	32	0.156
239	A	8	7	1.04	32	0.219
240	A	9	8	1.12	32	0.250
241	A	8	7	1.11	32	0.219
242	A	6	5	1.07	30	0.167
243	A	8	7	1.06	32	0.219
244	A	8	7	1.04	32	0.219
245	A	6	5	1.06	32	0.156
246	A	10	9	1.04	32	0.281
247	A	8	7	1.02	32	0.219
248	A	6	5	1.02	30	0.167
249	A	5	4	1.00	32	0.125
250	A	6	5	1.03	32	0.156
251	A	8	7	1.04	32	0.219
252	A	8	7	1.05	32	0.219
253	A	7	6	1.04	32	0.188
254	A	3	3	1.00	30	0.100
255	A	7	6	1.06	32	0.188
256	A	8	7	1.07	32	0.219
257	A	10	9	1.08	32	0.281
258	A	4	4	1.04	47	0.085
259	A	11	10	1.02	34	0.294
260	A	9	8	1.03	34	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	9	8	1.03	34	0.235
262	A	9	8	1.01	34	0.235
263	A	9	8	1.02	34	0.235
264	A	11	10	1.04	34	0.294
265	A	13	12	1.03	34	0.353
266	A	11	10	1.02	34	0.294
267	A	9	8	1.02	34	0.235
268	A	7	6	1.01	34	0.176
269	A	7	6	1.03	34	0.176
270	A	9	8	1.02	34	0.235
271	A	11	10	1.04	34	0.294
272	A	7	6	0.99	30	0.200
273	A	6	5	0.99	32	0.156
274	A	7	6	1.00	34	0.176
275	A	3	3	1.00	42	0.071
276	A	4	4	1.00	46	0.087
277	A	4	4	1.00	69	0.058
278	A	2	2	1.00	75	0.027
279	A	3	3	1.00	16	0.188
280	A	10	9	1.07	33	0.273
281	A	8	7	1.07	31	0.226
282	A	8	7	1.07	30	0.233
283	A	10	9	1.08	33	0.273
284	A	9	8	1.09	33	0.242
285	A	10	9	1.10	33	0.273
286	A	8	7	1.08	33	0.212
287	A	10	9	1.08	33	0.273
288	A	12	11	1.08	33	0.333
289	A	2	2	1.00	36	0.056
290	A	2	2	1.00	36	0.056
291	A	2	2	1.00	34	0.059
292	A	2	2	1.00	29	0.069
293	A	2	2	1.00	36	0.056
294	A	2	2	1.00	36	0.056

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	2	2	1.00	36	0.056
296	A	2	2	1.00	38	0.053
297	A	2	2	1.00	38	0.053
298	A	2	2	1.00	36	0.056
299	A	2	2	1.00	31	0.065
300	A	2	2	1.00	38	0.053
301	A	2	2	1.00	38	0.053
302	A	2	2	1.00	38	0.053
303	A	2	2	1.00	38	0.053
304	A	2	2	1.00	38	0.053
305	A	2	2	1.00	38	0.053
306	A	2	2	1.00	36	0.056
307	A	2	2	1.00	31	0.065
308	A	2	2	1.00	38	0.053
309	A	2	2	1.00	38	0.053
310	A	2	2	1.00	38	0.053
311	A	4	4	1.00	38	0.105
312	A	4	4	1.02	38	0.105
313	A	4	4	1.01	36	0.111
314	A	4	4	1.05	31	0.129
315	A	4	4	1.03	38	0.105
316	A	4	4	1.00	38	0.105
317	A	4	4	1.00	38	0.105
318	A	6	6	1.06	38	0.158
319	A	6	6	1.06	38	0.158
320	A	10	9	1.13	36	0.250
321	A	7	6	1.12	31	0.194
322	A	6	6	1.07	38	0.158
323	A	6	6	1.01	38	0.158
324	A	11	10	1.14	38	0.263
325	A	11	10	1.16	33	0.303
326	A	14	13	1.11	40	0.325
327	A	14	13	1.09	40	0.325
328	A	14	13	1.05	40	0.325

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	12	11	1.09	40	0.275
330	A	14	13	1.11	40	0.325
331	A	13	12	1.11	40	0.300
332	A	10	9	1.12	40	0.225
333	A	13	12	1.13	40	0.300
334	A	12	11	1.15	38	0.289
335	A	12	11	1.17	33	0.333
336	A	15	14	1.11	40	0.350
337	A	16	15	1.07	40	0.375
338	A	15	14	1.07	40	0.350
339	A	14	13	1.08	40	0.325
340	A	15	14	1.10	40	0.350
341	A	15	14	1.12	40	0.350
342	A	15	14	1.12	40	0.350
343	A	16	15	1.12	40	0.375
344	A	10	9	1.11	38	0.237
345	A	10	9	1.15	33	0.273
346	A	12	11	1.09	40	0.275
347	A	12	11	1.03	40	0.275
348	A	12	11	1.05	40	0.275
349	A	11	10	1.10	40	0.250
350	A	9	8	1.11	40	0.200
351	A	11	10	1.12	40	0.250
352	A	9	8	1.15	38	0.211
353	A	8	7	1.10	33	0.212
354	A	10	9	1.01	40	0.225
355	A	10	9	1.06	40	0.225
356	A	8	7	1.05	40	0.175
357	A	10	9	1.04	40	0.225
358	A	10	9	1.12	40	0.225
359	A	8	7	1.09	38	0.184
360	A	7	6	1.04	33	0.182
361	A	7	6	1.06	40	0.150
362	A	8	7	1.09	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	10	9	1.08	40	0.225
364	A	12	11	1.08	40	0.275
365	A	7	6	1.06	35	0.171
366	A	7	6	1.06	36	0.167
367	A	2	2	1.00	38	0.053
368	A	2	2	1.00	38	0.053
369	A	2	2	1.00	36	0.056
370	A	2	2	1.00	38	0.053
371	A	4	4	1.00	38	0.105
372	A	8	7	1.07	38	0.184
373	A	2	2	1.00	53	0.038
374	A	18	17	1.20	35	0.486
375	A	14	13	1.19	35	0.371
376	A	11	10	1.17	33	0.303
377	A	11	10	1.14	35	0.286
378	A	11	10	1.14	35	0.286
379	A	9	8	1.11	35	0.229
380	A	17	16	1.21	35	0.457
381	A	15	14	1.20	35	0.400
382	A	11	10	1.19	33	0.303
383	A	13	12	1.15	35	0.343
384	A	13	12	1.14	35	0.343
385	A	13	12	1.14	35	0.343
386	A	16	15	1.16	35	0.429
387	A	12	11	1.17	35	0.314
388	A	9	8	1.14	33	0.242
389	A	9	8	1.13	35	0.229
390	A	7	6	1.11	35	0.171
391	A	9	8	1.04	35	0.229
392	A	16	15	1.13	35	0.429
393	A	12	11	1.06	35	0.314
394	A	8	7	1.11	33	0.212
395	A	7	6	1.11	35	0.171
396	A	9	8	1.05	35	0.229

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	11	10	1.06	35	0.286
398	A	7	7	1.00	26	0.269
399	A	8	7	1.00	24	0.292
400	A	2	2	1.00	29	0.069

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int (d+ex)^2 (A+Bx+Cx^2) \sqrt{d^2-e^2x^2} \, dx$

3.1.1 Optimal result

Integrand size = 34, antiderivative size = 236

$$\begin{aligned}\int (d+ex)^2 (A+Bx+Cx^2) \sqrt{d^2-e^2x^2} \, dx &= \frac{d^2(3Cd^2+4Bde+10Ae^2)\, x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d(4Cd^2+e(7Bd+10Ae))\,(d^2-e^2x^2)^{3/2}}{15e^3} \\ &\quad - \frac{(3Cd^2+2e(2Bd+Ae))\, x(d^2-e^2x^2)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2(d^2-e^2x^2)^{3/2}}{5e} \\ &\quad - \frac{1}{6}Cx^3(d^2-e^2x^2)^{3/2} + \frac{d^4(3Cd^2+4Bde+10Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^3}\end{aligned}$$

output
```
-1/15*d*(4*C*d^2+e*(10*A*e+7*B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/8*(3*C*d^2+2
*e*(A*e+2*B*d))*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*(B*e+2*C*d)*x^2*(-e^2*x^2+d
^2)^(3/2)/e-1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)+1/16*d^4*(10*A*e^2+4*B*d*e+3*C*
d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^2*(10*A*e^2+4*B*d*e+3*C*d
^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.1.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{\sqrt{d^2-e^2x^2}(C(-64d^5-45d^4ex-32d^3e^2x^2+50d^2e^3x^3+96de^4x^4+40e^5x^5)+2e(5Ae(-16d^3+9d^2ex+}{}$$

input
```
Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(C*(-64*d^5 - 45*d^4*e*x - 32*d^3*e^2*x^2 + 50*d^2*e^
3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-16*d^3 + 9*d^2*e*x + 16*
d*e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 32*d^2*e^2*x^2 + 60*d*e
^3*x^3 + 24*e^4*x^4))) - 30*d^4*(3*C*d^2 + 2*e*(2*B*d + 5*A*e))*ArcTan[(e*
x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(240*e^3)
```

3.1.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2346, 27, 2346, 25, 2346, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \sqrt{d^2-e^2x^2} \left(A+Bx+Cx^2\right) \, dx$$
$$\Big\downarrow \text{2346}$$
$$-\frac{\int -3\sqrt{d^2-e^2x^2}\left(2e^3(2Cd+Be)x^3+e^2\left(3Cd^2+2e(2Bd+Ae)\right)x^2+2de^2(Bd+2Ae)x+2Ad^2e^2\right)dx}{6e^2}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \sqrt{d^2-e^2x^2}\left(2e^3(2Cd+Be)x^3+e^2\left(3Cd^2+2e(2Bd+Ae)\right)x^2+2de^2(Bd+2Ae)x+2Ad^2e^2\right)dx}{2e^2}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 2346$$

$$\frac{-\frac{\int -\sqrt{d^2-e^2x^2}\left(10Ad^2e^4+5\left(3Cd^2+2e(2Bd+Ae)\right)x^2e^4+2d\left(4Cd^2+e(7Bd+10Ae)\right)xe^3\right)dx}{5e^2}-\frac{2}{5}ex^2\left(d^2-e^2x^2\right)^{3/2}(Be+2Cd)}{2e^2}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 25$$

$$\frac{\frac{\int \sqrt{d^2-e^2x^2}\left(10Ad^2e^4+5\left(3Cd^2+2e(2Bd+Ae)\right)x^2e^4+2d\left(4Cd^2+e(7Bd+10Ae)\right)xe^3\right)dx}{5e^2}-\frac{2}{5}ex^2\left(d^2-e^2x^2\right)^{3/2}(Be+2Cd)}{2e^2}-\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 2346$$

$$\frac{\frac{-\frac{\int -de^4\left(5d\left(3Cd^2+4Bed+10Ae^2\right)+8e\left(4Cd^2+e(7Bd+10Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{5e^2}-\frac{2}{5}ex^2\left(d^2-e^2x^2\right)^{3/2}}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 25$$

$$\frac{\frac{\frac{\int de^4\left(5d\left(3Cd^2+4Bed+10Ae^2\right)+8e\left(4Cd^2+e(7Bd+10Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{5e^2}-\frac{2}{5}ex^2\left(d^2-e^2x^2\right)^{3/2}(B}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{1}{4}de^2\int\left(5d\left(3Cd^2+4Bed+10Ae^2\right)+8e\left(4Cd^2+e(7Bd+10Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{5e^2}-\frac{2}{5}ex^2\left(d^2-e^2x^2\right)}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 455$$

$$\frac{\frac{\frac{1}{4}de^2\left(5d\left(10Ae^2+4Bde+3Cd^2\right)\int\sqrt{d^2-e^2x^2}dx-\frac{8\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{3e}\right)-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{5e^2}-\frac{2}{5}ex^2(d}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\downarrow 211$$

$$\frac{\frac{\frac{1}{4}de^2\left(5d(10Ae^2+4Bde+3Cd^2)\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{3e}\right)-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bc\cdots\right.}{5e^2}\cdots}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{1}{4}de^2\left(5d(10Ae^2+4Bde+3Cd^2)\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{8\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{3e}\right)-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(\cdots\right.}{5e^2}\cdots}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{1}{4}de^2\left(5d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)(10Ae^2+4Bde+3Cd^2)-\frac{8\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{3e}\right)-\frac{5}{4}e^2x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+\cdots\right.}{5e^2}\cdots}{2e^2}$$

$$\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2}$$

input
```
Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/6*(C*x^3*(d^2 - e^2*x^2)^(3/2)) + ((-2*e*(2*C*d + B*e)*x^2*(d^2 - e^2*x
^2)^(3/2))/5 + ((-5*e^2*(3*C*d^2 + 2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3
/2))/4 + (d*e^2*((-8*(4*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))
/(3*e) + 5*d*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*((x*Sqrt[d^2 - e^2*x^2])/2 + (
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/4)/(5*e^2))/(2*e^2)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\left(-40 e^{5} C x^{5}-48 B x^{4} e^{5}-96 C d e^{4} x^{4}-60 A e^{5} x^{3}-120 x^{3} d B e^{4}-50 C d^{2} e^{3} x^{3}-160 A d e^{4} x^{2}-64 x^{2} d^{2} B e^{3}+32 C d^{3} e^{2} x^{2}-90 A d^{2} e^{3} x+6\cdots\right.}{240 e^{3}}$
default	$A d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)+e^{2}C\left(-\frac{x^{3}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{6e^{2}}+\frac{d^{2}\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4e^{2}}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+\cdots}}{2}\cdots\right.}{\cdots}\right.}{2e^{2}}\cdots\right.$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240/e^3*(-40*C*e^5*x^5-48*B*e^5*x^4-96*C*d*e^4*x^4-60*A*e^5*x^3-120*B*d
*e^4*x^3-50*C*d^2*e^3*x^3-160*A*d*e^4*x^2-64*B*d^2*e^3*x^2+32*C*d^3*e^2*x^
2-90*A*d^2*e^3*x+60*B*d^3*e^2*x+45*C*d^4*e*x+160*A*d^3*e^2+112*B*d^4*e+64*
C*d^5)*(-e^2*x^2+d^2)^(1/2)+1/16*d^4/e^2*(10*A*e^2+4*B*d*e+3*C*d^2)/(e^2)^
(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx =$$

$$-\frac{30\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,Ce^5x^5-64\,Cd^5-112\,Bd^4e-160\,Ad^3e^2\right.}{}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-e^2*x^2
 + d^2))/(e*x)) - (40*C*e^5*x^5 - 64*C*d^5 - 112*B*d^4*e - 160*A*d^3*e^2 +
 48*(2*C*d*e^4 + B*e^5)*x^4 + 10*(5*C*d^2*e^3 + 12*B*d*e^4 + 6*A*e^5)*x^3
- 32*(C*d^3*e^2 - 2*B*d^2*e^3 - 5*A*d*e^4)*x^2 - 15*(3*C*d^4*e + 4*B*d^3*e
^2 - 6*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(219) = 438.

Time = 0.56 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.97

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$

$$=\begin{cases}\sqrt{d^2-e^2x^2}\left(\frac{Ce^2x^5}{6}-\frac{x^4\left(-Be^4-2Cde^3\right)}{5e^2}-\frac{x^3\left(-Ae^4-2Bde^3-\frac{5Cd^2e^2}{6}\right)}{4e^2}-\frac{x^2\left(-2Ade^3+2Cd^3e+\frac{4d^2\left(-Be^4-2Cde^3\right)}{5e^2}\right)}{3e^2}-\frac{x\left(2B\right.}{} \right. \\ \left(Ad^2x+\frac{Ce^2x^5}{5}+\frac{x^4\left(Be^2+2Cde\right)}{4}+\frac{x^3\left(Ae^2+2Bde+Cd^2\right)}{3}+\frac{x^2\cdot\left(2Ade+Bd^2\right)}{2}\right)\sqrt{d^2}\end{cases}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(C*e**2*x**5/6 - x**4*(-B*e**4 - 2*C*d*e
**3)/(5*e**2) - x**3*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2*e**2/6)/(4*e**2) - x
**2*(-2*A*d*e**3 + 2*C*d**3*e + 4*d**2*(-B*e**4 - 2*C*d*e**3)/(5*e**2))/(3
*e**2) - x*(2*B*d**3*e + C*d**4 + 3*d**2*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2*
e**2/6)/(4*e**2))/(2*e**2) - (2*A*d**3*e + B*d**4 + 2*d**2*(-2*A*d*e**3 +
2*C*d**3*e + 4*d**2*(-B*e**4 - 2*C*d*e**3)/(5*e**2))/(3*e**2))/e**2) + (A*
d**4 + d**2*(2*B*d**3*e + C*d**4 + 3*d**2*(-A*e**4 - 2*B*d*e**3 - 5*C*d**2
*e**2/6)/(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt
(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2),
 True)), Ne(e**2, 0)), ((A*d**2*x + C*e**2*x**5/5 + x**4*(B*e**2 + 2*C*d*e
)/4 + x**3*(A*e**2 + 2*B*d*e + C*d**2)/3 + x**2*(2*A*d*e + B*d**2)/2)*sqrt
(d**2), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.57

$$\begin{aligned}\int (d+ex)^2\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx =& -\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^3+\frac{Ad^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} \\ &+\frac{Cd^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ad^2x \\ &+\frac{\sqrt{-e^2x^2+d^2}Cd^4x}{16\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cd^2x}{8\,e^2} \\ &+\frac{(Cd^2+2\,Bde+Ae^2)d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} \\ &-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Bd^2}{3\,e^2}-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Ad}{3\,e} \\ &+\frac{\sqrt{-e^2x^2+d^2}(Cd^2+2\,Bde+Ae^2)d^2x}{8\,e^2} \\ &-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(2\,Cde+Be^2)x^2}{5\,e^2} \\ &-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(Cd^2+2\,Bde+Ae^2)x}{4\,e^2} \\ &-\frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(2\,Cde+Be^2)d^2}{15\,e^4}\end{aligned}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/6*(-e^2*x^2 + d^2)^(3/2)*C*x^3 + 1/2*A*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/
sqrt(e^2) + 1/16*C*d^6*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/2*s
qrt(-e^2*x^2 + d^2)*A*d^2*x + 1/16*sqrt(-e^2*x^2 + d^2)*C*d^4*x/e^2 - 1/8*
(-e^2*x^2 + d^2)^(3/2)*C*d^2*x/e^2 + 1/8*(C*d^2 + 2*B*d*e + A*e^2)*d^4*arc
sin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d^
2/e^2 - 2/3*(-e^2*x^2 + d^2)^(3/2)*A*d/e + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d^2
 + 2*B*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^
2)*x^2/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 -
2/15*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*d^2/e^4
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.95

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{1}{240}\sqrt{-e^2x^2+d^2}\Bigg(\Bigg(\Bigg(2\left(\left(4\left(5\,Ce^2x+\frac{6\,(2\,Cde^9+Be^{10})}{e^8}\right)x+\frac{5\,(5\,Cd^2e^8+12\,Bde^9+6\,Ae^{10})}{e^8}\right)x-\frac{16\,(\cdots}{\cdots}$$
$$+\frac{(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2)\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{16\,e^2|e|}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
1/240*sqrt(-e^2*x^2 + d^2)*((2*((4*(5*C*e^2*x + 6*(2*C*d*e^9 + B*e^10)/e^8
)*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)/e^8)*x - 16*(C*d^3*e^7 - 2*B
*d^2*e^8 - 5*A*d*e^9)/e^8)*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*e^8
)/e^8)*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)/e^8) + 1/16*(3*C*
d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx = \int \sqrt{d^2-e^2\,x^2}\, (d+e\,x)^2 \left(C\,x^2+B\,x+A\right)\, dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2), x)
```

3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

3.2.1 Optimal result

Integrand size = 32, antiderivative size = 186

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = \frac{d(Cd^2+e(Bd+4Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd^2+5e(Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{(Cd+Be)x(d^2-e^2x^2)^{3/2}}{4e^2} - \frac{Cx^2(d^2-e^2x^2)^{3/2}}{5e} + \frac{d^3(Cd^2+e(Bd+4Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
-1/15*(2*C*d^2+5*e*(A*e+B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*(B*e+C*d)*x*(-e
^2*x^2+d^2)^(3/2)/e^2-1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e+1/8*d^3*(C*d^2+e*(4
*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d*(C*d^2+e*(4*A*e+B*d)
)*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.2.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$
$$=\frac{\sqrt{d^2-e^2x^2}(C(-16d^4-15d^3ex-8d^2e^2x^2+30de^3x^3+24e^4x^4)-5e(-4Ae(-2d^2+3dex+2e^2x^2)+B(8\ldots}{120e^3}$$

input
```
Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*d^2*e^2*x^2 + 30*d*e^3*x
^3 + 24*e^4*x^4) - 5*e*(-4*A*e*(-2*d^2 + 3*d*e*x + 2*e^2*x^2) + B*(8*d^3 +
 3*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))) - 30*d^3*(C*d^2 + e*(B*d + 4*A*e))
*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(120*e^3)
```

3.2.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2346, 25, 2346, 25, 27, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt{d^2-e^2x^2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\sqrt{d^2-e^2x^2}\left(5(Cd+Be)x^2e^2+5Ade^2+\left(2Cd^2+5e(Bd+Ae)\right)xe\right)dx}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sqrt{d^2-e^2x^2}\left(5(Cd+Be)x^2e^2+5Ade^2+\left(2Cd^2+5e(Bd+Ae)\right)xe\right)dx}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{-\frac{\int -e^2\left(5d\left(Cd^2+e(Bd+4Ae)\right)+4e\left(2Cd^2+5e(Bd+Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{5}{4}x\left(d^2-e^2x^2\right)^{3/2}(Be+Cd)}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int e^2\left(5d(Cd^2+e(Bd+4Ae))+4e\left(2Cd^2+5e(Bd+Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{5}{4}x\left(d^2-e^2x^2\right)^{3/2}(Be+Cd)}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{4}\int\left(5d(Cd^2+e(Bd+4Ae))+4e\left(2Cd^2+5e(Bd+Ae)\right)x\right)\sqrt{d^2-e^2x^2}dx-\frac{5}{4}x\left(d^2-e^2x^2\right)^{3/2}(Be+Cd)}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 455$$

$$\frac{\frac{1}{4}\left(5d\left(e(4Ae+Bd)+Cd^2\right)\int\sqrt{d^2-e^2x^2}dx-\frac{4\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{3e}\right)-\frac{5}{4}x\left(d^2-e^2x^2\right)^{3/2}(Be+Cd)}{5e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 211$$

$$\frac{\frac{1}{4}\left(5d\left(e(4Ae+Bd)+Cd^2\right)\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{4\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{3e}\right)-\frac{5}{4}x\left(d^2-e^{2}\right.}{5e^2}$$

$$\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 224$$

$$\frac{\frac{1}{4}\left(5d\left(e(4Ae+Bd)+Cd^2\right)\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{4\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{3e}\right)-\frac{5}{4}x}{5e^2}$$

$$\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

$$\Big\downarrow 216$$

$$\frac{\frac{1}{4}\left(5d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\left(e(4Ae+Bd)+Cd^2\right)-\frac{4\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{3e}\right)-\frac{5}{4}x\left(d^2-\right.}{5e^2}$$

$$\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}$$

3.2. $\int(d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

input
```
Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/5*(C*x^2*(d^2 - e^2*x^2)^(3/2))/e + ((-5*(C*d + B*e)*x*(d^2 - e^2*x^2)^
(3/2))/4 + ((-4*(2*C*d^2 + 5*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(3*e) +
 5*d*(C*d^2 + e*(B*d + 4*A*e))*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e
*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)/(5*e^2)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-24e^4Cx^4-30x^3Be^4-30Cde^3x^3-40Ae^4x^2-40x^2dBe^3+8Cd^2e^2x^2-60Ade^3x+15xBd^2e^2+15Cd^3xe+40Ad^2e^2+40Bd^3e+}{120e^3}$
default	$dA\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)+eC\left(-\frac{x^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{5e^2}-\frac{2d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{15e^4}\right)+(Be+Cd)$

input
```
int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/e^3*(-24*C*e^4*x^4-30*B*e^4*x^3-30*C*d*e^3*x^3-40*A*e^4*x^2-40*B*d*
e^3*x^2+8*C*d^2*e^2*x^2-60*A*d*e^3*x+15*B*d^2*e^2*x+15*C*d^3*e*x+40*A*d^2*
e^2+40*B*d^3*e+16*C*d^4)*(-e^2*x^2+d^2)^(1/2)+1/8*d^3/e^2*(4*A*e^2+B*d*e+C
*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx =$$
$$-\frac{30\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,Ce^4x^4-16\,Cd^4-40\,Bd^3e-40\,Ad^2e^2+30\right.}{120\,e^3}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^
2))/(e*x)) - (24*C*e^4*x^4 - 16*C*d^4 - 40*B*d^3*e - 40*A*d^2*e^2 + 30*(C*
d*e^3 + B*e^4)*x^3 - 8*(C*d^2*e^2 - 5*B*d*e^3 - 5*A*e^4)*x^2 - 15*(C*d^3*e
 + B*d^2*e^2 - 4*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.76

$$\int (d+ex) \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx$$
$$= \begin{cases} \sqrt{d^2-e^2x^2}\left(\frac{Cex^4}{5} - \frac{x^3(-Be^3-Cde^2)}{4e^2} - \frac{x^2\left(-Ae^3-Bde^2+\frac{Cd^2e}{5}\right)}{3e^2} - \frac{x\left(-Ade^2+Bd^2e+Cd^3+\frac{3d^2\left(-Be^3-Cde^2\right)}{4e^2}\right)}{2e^2} - \frac{Ad^2e+Bd \cdots}{} \right. \\ \left(Adx+\frac{Cex^4}{4}+\frac{x^3(Be+Cd)}{3}+\frac{x^2(Ae+Bd)}{2}\right)\sqrt{d^2} \end{cases}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(C*e*x**4/5 - x**3*(-B*e**3 - C*d*e**2)/
(4*e**2) - x**2*(-A*e**3 - B*d*e**2 + C*d**2*e/5)/(3*e**2) - x*(-A*d*e**2
+ B*d**2*e + C*d**3 + 3*d**2*(-B*e**3 - C*d*e**2)/(4*e**2))/(2*e**2) - (A*
d**2*e + B*d**3 + 2*d**2*(-A*e**3 - B*d*e**2 + C*d**2*e/5)/(3*e**2))/e**2)
 + (A*d**3 + d**2*(-A*d*e**2 + B*d**2*e + C*d**3 + 3*d**2*(-B*e**3 - C*d*e
**2)/(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**
2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tru
e)), Ne(e**2, 0)), ((A*d*x + C*e*x**4/4 + x**3*(B*e + C*d)/3 + x**2*(A*e +
 B*d)/2)*sqrt(d**2), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.20

$$\int (d+ex) \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx$$
$$= \frac{Ad^3 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{1}{2}\sqrt{-e^2x^2+d^2}Adx - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^2}{5\,e}$$
$$+ \frac{(Cd+Be)d^4 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}(Cd+Be)d^2x}{8\,e^2} - \frac{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cd^2}{15\,e^3}$$
$$- \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Bd}{3\,e^2} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}A}{3\,e} - \frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}(Cd+Be)x}{4\,e^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
1/2*A*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*sqrt(-e^2*x^2 + d^2)
*A*d*x - 1/5*(-e^2*x^2 + d^2)^(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*arcsin(e
^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)
*d^2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*C*d^2/e^3 - 1/3*(-e^2*x^2 + d^2)^
(3/2)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*A/e - 1/4*(-e^2*x^2 + d^2)^(3/2
)*(C*d + B*e)*x/e^2
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$
$$= \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(\left(2\left(3\left(4\,Cex+\frac{5\,(Cde^6+Be^7)}{e^6}\right)x-\frac{4\,(Cd^2e^5-5\,Bde^6-5\,Ae^7)}{e^6}\right)x-\frac{15\,(Cd^3e^4+\right.\right.\right.$$
$$+\frac{(Cd^5+Bd^4e+4\,Ad^3e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(-e^2*x^2 + d^2)*((2*(3*(4*C*e*x + 5*(C*d*e^6 + B*e^7)/e^6)*x -
4*(C*d^2*e^5 - 5*B*d*e^6 - 5*A*e^7)/e^6)*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4
*A*d*e^6)/e^6)*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)/e^6) + 1/8*
(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\int \sqrt{d^2-e^2\,x^2}\,(d+e\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)
```

3.3 $\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx$

3.3.1 Optimal result

Integrand size = 27, antiderivative size = 125

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx=\frac{1}{8}\left(4A+\frac{Cd^2}{e^2}\right)x\sqrt{d^2-e^2x^2}-\frac{B(d^2-e^2x^2)^{3/2}}{3e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}+\frac{d^2(Cd^2+4Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
-1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2-1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^2*(
4*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*(4*A+C*d^2/e^2)*x*
(-e^2*x^2+d^2)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx=\frac{e\sqrt{d^2-e^2x^2}(-8Bd^2-3Cd^2x+12Ae^2x+8Be^2x^2+6Ce^2x^3)-6d^2(Cd^2+4Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e^3}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
(e*Sqrt[d^2 - e^2*x^2]*(-8*B*d^2 - 3*C*d^2*x + 12*A*e^2*x + 8*B*e^2*x^2 +
6*C*e^2*x^3) - 6*d^2*(C*d^2 + 4*A*e^2)*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2
- e^2*x^2])])/(24*e^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2346, 25, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d^2-e^2x^2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2346}$$

$$-\frac{\int -\left(\left(Cd^2+4Ae^2+4Be^2x\right)\sqrt{d^2-e^2x^2}\right)dx}{4e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \left(Cd^2+4Ae^2+4Be^2x\right)\sqrt{d^2-e^2x^2}dx}{4e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{\left(4Ae^2+Cd^2\right)\int \sqrt{d^2-e^2x^2}dx-\frac{4}{3}B\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\left(4Ae^2+Cd^2\right)\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{4}{3}B\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\left(4Ae^2+Cd^2\right)\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{4}{3}B\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{(4Ae^2+Cd^2)\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{4}{3}B(d^2-e^2x^2)^{3/2}}{4e^2}-\frac{Cx(d^2-e^2x^2)^{3/2}}{4e^2}$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/4*(C*x*(d^2 - e^2*x^2)^(3/2))/e^2 + ((-4*B*(d^2 - e^2*x^2)^(3/2))/3 + (
C*d^2 + 4*A*e^2)*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]])/(2*e)))/(4*e^2)
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\left(6C\,e^{2}x^{3}+8B\,e^{2}x^{2}+12A\,e^{2}x-3C\,d^{2}x-8B\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e^{2}}+\frac{d^{2}\left(4A\,e^{2}+C\,d^{2}\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8e^{2}\sqrt{e^{2}}}$
default	$A\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)+C\left(-\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4e^{2}}+\frac{d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4e^{2}}\right.$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*e^2*x^3+8*B*e^2*x^2+12*A*e^2*x-3*C*d^2*x-8*B*d^2)/e^2*(-e^2*x^2+
d^2)^(1/2)+1/8*d^2*(4*A*e^2+C*d^2)/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-
e^2*x^2+d^2)^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int\left(A+Bx+Cx^{2}\right)\sqrt{d^{2}-e^{2}x^{2}}\,dx=$$
$$-\frac{6\left(Cd^{4}+4\,Ad^{2}e^{2}\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)-\left(6\,Ce^{3}x^{3}+8\,Be^{3}x^{2}-8\,Bd^{2}e-3\left(Cd^{2}e-4\,Ae^{3}\right)x\right)\sqrt{-e^{2}x^{2}}}{24\,e^{3}}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
 (6*C*e^3*x^3 + 8*B*e^3*x^2 - 8*B*d^2*e - 3*(C*d^2*e - 4*A*e^3)*x)*sqrt(-e
^2*x^2 + d^2))/e^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$$
$$=\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{Bd^2}{3e^2}+\frac{Bx^2}{3}+\frac{Cx^3}{4}-\frac{x\left(-Ae^2+\frac{Cd^2}{4}\right)}{2e^2}\right)+\left(Ad^2+\frac{d^2\left(-Ae^2+\frac{Cd^2}{4}\right)}{2e^2}\right)\left(\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} \\ \frac{x\log(x)}{\sqrt{-e^2x^2}}\end{cases}\right.\right. \\ \left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}\right)\sqrt{d^2}\end{cases}$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(-B*d**2/(3*e**2) + B*x**2/3 + C*x**3/4
- x*(-A*e**2 + C*d**2/4)/(2*e**2)) + (A*d**2 + d**2*(-A*e**2 + C*d**2/4)/(
2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/
sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)), Ne(e**2, 0)
), ((A*x + B*x**2/2 + C*x**3/3)*sqrt(d**2), True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx=\frac{Ad^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}}+\frac{Cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2}+\frac{1}{2}\sqrt{-e^2x^2+d^2}Ax+\frac{\sqrt{-e^2x^2+d^2}Cd^2x}{8e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}B}{3e^2}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*A*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/8*C*d^4*arcsin(e^2*x/(
d*sqrt(e^2)))/(sqrt(e^2)*e^2) + 1/2*sqrt(-e^2*x^2 + d^2)*A*x + 1/8*sqrt(-e
^2*x^2 + d^2)*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*C*x/e^2 - 1/3*(-e^2
*x^2 + d^2)^(3/2)*B/e^2
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx$$
$$=\frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\,(3\,Cx+4\,B)x-\frac{3\,(Cd^2e^2-4\,Ae^4)}{e^4}\right)x-\frac{8\,Bd^2}{e^2}\right)$$
$$+\frac{(Cd^4+4\,Ad^2e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(-e^2*x^2 + d^2)*((2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)/e^
4)*x - 8*B*d^2/e^2) + 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(e*x/d)*sgn(d)*sgn(e
)/(e^2*abs(e))
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\sqrt{d^2-e^2x^2}\,dx=\int \sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

3.4 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

3.4.1 Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{(Cd^2-e(Bd-2Ae))\sqrt{d^2-e^2x^2}}{2e^3} - \frac{C(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{(Cd-Be)(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} + \frac{d(Cd^2-e(Bd-2Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
-1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*(-B*e+C*d)*(-e^2*x^2+d^2)^(3/2)/e^3/(e
*x+d)+1/2*d*(C*d^2-e*(-2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/
2*(C*d^2-e*(-2*A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(3e(-2Bd+2Ae+Bex)+C(4d^2-3dex+2e^2x^2)) - 6d(Cd^2+e(-Bd+2Ae))\arctan\left(\frac{\ }{\sqrt{d^2 \cdots}}\right.}{6e^3}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x +
2*e^2*x^2)) - 6*d*(C*d^2 + e*(-(B*d) + 2*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - S
qrt[d^2 - e^2*x^2])])/(6*e^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2170, 27, 667, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}\left(A+Bx+Cx^2\right)}{d+ex}dx$$

$$\Big\downarrow \text{2170}$$

$$-\frac{\int -\frac{3e^3(Ae-(Cd-Be)x)\sqrt{d^2-e^2x^2}}{d+ex}dx}{3e^4}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(Ae-(Cd-Be)x)\sqrt{d^2-e^2x^2}}{d+ex}dx}{e}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

$$\Big\downarrow \text{667}$$

$$\frac{\int \frac{(d-ex)(Ae+(Be-Cd)x)}{\sqrt{d^2-e^2x^2}}dx}{e}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{d(Cd^2-e(Bd-2Ae))\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e}+\sqrt{d^2-e^2x^2}\left(A+\frac{d(Cd-Be)}{e^2}\right)-\frac{x\sqrt{d^2-e^2x^2}(Cd-Be)}{2e}}{e}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{d(Cd^2-e(Bd-2Ae))\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e}+\sqrt{d^2-e^2x^2}\left(A+\frac{d(Cd-Be)}{e^2}\right)-\frac{x\sqrt{d^2-e^2x^2}(Cd-Be)}{2e}}{e}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd^2-e(Bd-2Ae))}{2e^2}+\sqrt{d^2-e^2x^2}\left(A+\frac{d(Cd-Be)}{e^2}\right)-\frac{x\sqrt{d^2-e^2x^2}(Cd-Be)}{2e}}{e}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

output
```
-1/3*(C*(d^2 - e^2*x^2)^(3/2))/e^3 + ((A + (d*(C*d - B*e))/e^2)*Sqrt[d^2 -
 e^2*x^2] - ((C*d - B*e)*x*Sqrt[d^2 - e^2*x^2])/(2*e) + (d*(C*d^2 - e*(B*d
 - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^2))/e
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 667
```
Int[(((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(
x_)), x_Symbol] :> Int[(a/d + c*(x/e))*(f + g*x)^n*(a + c*x^2)^(p - 1), x]
/; FreeQ[{a, c, d, e, f, g, n, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\left(2C\,e^{2}x^{2}+3xB\,e^{2}-3Cdex+6A\,e^{2}-6Bde+4C\,d^{2}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{6e^{3}}+\frac{d\left(2A\,e^{2}-Bde+C\,d^{2}\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}$
default	$\frac{Be\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)-\frac{C\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{3e}-Cd\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{e^{2}}+\frac{\left(A\,e^{2}-Bde+C\right.}{}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*e^2*x^2+3*B*e^2*x-3*C*d*e*x+6*A*e^2-6*B*d*e+4*C*d^2)/e^3*(-e^2*x^
2+d^2)^(1/2)+1/2*d/e^2*(2*A*e^2-B*d*e+C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{6\,(Cd^3-Bd^2e+2\,Ade^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-(2\,Ce^2x^2+4\,Cd^2-6\,Bde+6\,Ae^2-3\,(Cde-Be^2}{6\,e^3}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas"
)
```

output
```
-1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(
e*x)) - (2*C*e^2*x^2 + 4*C*d^2 - 6*B*d*e + 6*A*e^2 - 3*(C*d*e - B*e^2)*x)*
sqrt(-e^2*x^2 + d^2))/e^3
```

3.4.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{d+ex}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{Cd^3\arcsin\left(\frac{ex}{d}\right)}{2\,e^3} - \frac{Bd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^2} + \frac{Ad\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2+d^2}Cdx}{2\,e^2} + \frac{\sqrt{-e^2x^2+d^2}Bx}{2\,e} + \frac{\sqrt{-e^2x^2+d^2}Cd^2}{e^3} - \frac{\sqrt{-e^2x^2+d^2}Bd}{e^2} + \frac{\sqrt{-e^2x^2+d^2}A}{e} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}C}{3\,e^3}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima"
)
```

output
```
1/2*C*d^3*arcsin(e*x/d)/e^3 - 1/2*B*d^2*arcsin(e*x/d)/e^2 + A*d*arcsin(e*x
/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*C*d*x/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*B*x/
e + sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 + sqrt(-
e^2*x^2 + d^2)*A/e - 1/3*(-e^2*x^2 + d^2)^(3/2)*C/e^3
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \frac{1}{6}\sqrt{-e^2x^2+d^2}\left(\left(\frac{2\,Cx}{e}-\frac{3\,(Cde^3-Be^4)}{e^5}\right)x+\frac{2\,(2\,Cd^2e^2-3\,Bde^3+3\,Ae^4)}{e^5}\right) + \frac{(Cd^3-Bd^2e+2\,Ade^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
1/6*sqrt(-e^2*x^2 + d^2)*((2*C*x/e - 3*(C*d*e^3 - B*e^4)/e^5)*x + 2*(2*C*d
^2*e^2 - 3*B*d*e^3 + 3*A*e^4)/e^5) + 1/2*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arc
sin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,(C\,x^2+B\,x+A)}{d+e\,x}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x),x)
```

output
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x), x)
```

3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx$

3.5.1 Optimal result

Integrand size = 34, antiderivative size = 170

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{(5Cd^2-2e(2Bd-Ae))\sqrt{d^2-e^2x^2}}{2de^3} - \frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{de^3(d+ex)^2} - \frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)} - \frac{(5Cd^2-2e(2Bd-Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^2-1/2*C*(-e^2*x^2+
d^2)^(3/2)/e^3/(e*x+d)-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*arctan(e*x/(-e^2*x^2
+d^2)^(1/2))/e^3-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*(-e^2*x^2+d^2)^(1/2)/d/e^3
```

3.5.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(2e(3Bd-2Ae+Bex)+C(-8d^2-3dex+e^2x^2))}{d+ex}+2(5Cd^2+2e(-2Bd+Ae))\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*x +
 e^2*x^2)))/(d + e*x) + 2*(5*C*d^2 + 2*e*(-2*B*d + A*e))*ArcTan[(e*x)/(Sqr
t[d^2] - Sqrt[d^2 - e^2*x^2])])/(2*e^3)
```

3.5.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2170, 27, 671, 466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}(A+Bx+Cx^2)}{(d+ex)^2}dx$$

$$\Big\downarrow \ 2170$$

$$-\frac{\int \frac{e^2(Cd^2-2Ae^2+e(3Cd-2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^2}dx}{2e^4}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(Cd^2-2Ae^2+e(3Cd-2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^2}dx}{2e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

$$\Big\downarrow \ 671$$

$$-\frac{\frac{(5Cd^2-2e(2Bd-Ae))\int \frac{\sqrt{d^2-e^2x^2}}{d+ex}dx}{d}+\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de(d+ex)^2}}{2e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

$$\Big\downarrow \; 466$$

$$-\frac{\frac{(5Cd^2-2e(2Bd-Ae))\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{d}+\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de(d+ex)^2}}{2e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{(5Cd^2-2e(2Bd-Ae))\left(d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)}{d}+\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de(d+ex)^2}}{2e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)(5Cd^2-2e(2Bd-Ae))}{d}+\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de(d+ex)^2}}{2e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{2e^3(d+ex)}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

output
```
-1/2*(C*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)) - ((2*(C*d^2 - B*d*e + A*e^
2)*(d^2 - e^2*x^2)^(3/2))/(d*e*(d + e*x)^2) + ((5*C*d^2 - 2*e*(2*B*d - A*e
))*(Sqrt[d^2 - e^2*x^2]/e + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/d)/(
2*e^2)
```

3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

method	result
risch	$\frac{(Cxe+2Be-4Cd)\sqrt{-e^2x^2+d^2}}{2e^3} - \frac{\frac{2Ae^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{5Cd^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{4Bde\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}} + \frac{4\left(Ae^2-Bc\right.}{}}{2e^2}$
default	$\frac{C\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{e^2} + \frac{(Be-2Cd)\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^3}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*e*x+2*B*e-4*C*d)/e^3*(-e^2*x^2+d^2)^(1/2)-1/2/e^2*(2*A*e^2/(e^2)^(1
/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+5*C*d^2/(e^2)^(1/2)*arctan(
(e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-4*B*d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)
*x/(-e^2*x^2+d^2)^(1/2))+4*(A*e^2-B*d*e+C*d^2)/e^2/(x+d/e)*(-(x+d/e)^2*e^2
+2*d*e*(x+d/e))^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx =$$
$$-\frac{8\,Cd^3-6\,Bd^2e+4\,Ade^2+2\,(4\,Cd^2e-3\,Bde^2+2\,Ae^3)x-2\,(5\,Cd^3-4\,Bd^2e+2\,Ade^2+(5\,Cd^2e-4}{2\,(}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="frica
s")
```

output
```
-1/2*(8*C*d^3 - 6*B*d^2*e + 4*A*d*e^2 + 2*(4*C*d^2*e - 3*B*d*e^2 + 2*A*e^3
)*x - 2*(5*C*d^3 - 4*B*d^2*e + 2*A*d*e^2 + (5*C*d^2*e - 4*B*d*e^2 + 2*A*e^
3)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (C*e^2*x^2 - 8*C*d^2 + 6
*B*d*e - 4*A*e^2 - (3*C*d*e - 2*B*e^2)*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d
*e^3)
```

3.5.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{e^4x+de^3}+\frac{2\sqrt{-e^2x^2+d^2}Bd}{e^3x+de^2} - \frac{5\,Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{2\,Bd\arcsin\left(\frac{ex}{d}\right)}{e^2}-\frac{A\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{2\sqrt{-e^2x^2+d^2}A}{e^2x+de}+\frac{\sqrt{-e^2x^2+d^2}Cx}{2\,e^2} - \frac{2\sqrt{-e^2x^2+d^2}Cd}{e^3}+\frac{\sqrt{-e^2x^2+d^2}B}{e^2}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxim
a")
```

output
```
-2*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^4*x + d*e^3) + 2*sqrt(-e^2*x^2 + d^2)*B*d
/(e^3*x + d*e^2) - 5/2*C*d^2*arcsin(e*x/d)/e^3 + 2*B*d*arcsin(e*x/d)/e^2 -
 A*arcsin(e*x/d)/e - 2*sqrt(-e^2*x^2 + d^2)*A/(e^2*x + d*e) + 1/2*sqrt(-e^
2*x^2 + d^2)*C*x/e^2 - 2*sqrt(-e^2*x^2 + d^2)*C*d/e^3 + sqrt(-e^2*x^2 + d^
2)*B/e^2
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(156) = 312.

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{\left(8\,Cd^3e^3\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)-8\,Bd^2e^4\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+8\,Ade^5\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac"
)
```

output
```
-1/4*(8*C*d^3*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) - 8*B*d^
2*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) + 8*A*d*e^5*sqrt(2*d
/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) - 4*(5*C*d^3*e^3*sgn(1/(e*x + d))*
sgn(e) - 4*B*d^2*e^4*sgn(1/(e*x + d))*sgn(e) + 2*A*d*e^5*sgn(1/(e*x + d))*
sgn(e))*arctan(sqrt(2*d/(e*x + d) - 1)) + (5*C*d^3*e^3*(2*d/(e*x + d) - 1)
^(3/2)*sgn(1/(e*x + d))*sgn(e) - 2*B*d^2*e^4*(2*d/(e*x + d) - 1)^(3/2)*sgn
(1/(e*x + d))*sgn(e) + 3*C*d^3*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d)
)*sgn(e) - 2*B*d^2*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))*(e
*x + d)^2/d^2)*abs(e)/(d*e^7)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,(C\,x^2+B\,x+A)}{(d+e\,x)^2}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

output
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2, x)
```

3.6 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

3.6.1 Optimal result

Integrand size = 34, antiderivative size = 149

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2} + \frac{(3Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-C*(-e^2*x^2+
d^2)^(3/2)/e^3/(e*x+d)^2+(-B*e+3*C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3
+2*(-B*e+3*C*d)*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

3.6.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(Cd(14d^2+19dex+3e^2x^2)+e(Ae(-d+ex)-Bd(5d+7ex)))}{d(d+ex)^2} + 6(-3Cd+Be)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d +
e*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) + 6*(-3*C*d + B*e)*ArcTan[(e*x
)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(3*e^3)
```

3.6.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2170, 27, 671, 463, 25, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}\left(A+Bx+Cx^2\right)}{(d+ex)^3}dx$$

$$\Big\downarrow \text{2170}$$

$$-\frac{\int \frac{e^2\left(2Cd^2-Ae^2+e(3Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{e^4}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\left(2Cd^2-Ae^2+e(3Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{e^2}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{671}$$

$$-\frac{(3Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}dx-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{3(d+ex)^3}}{e^2}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{463}$$

$$-\frac{(3Cd-Be)\left(\int -\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{3(d+ex)^3}}{e^2}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{(3Cd-Be)\left(-\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{3(d+ex)^3}}{e^2}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \ 224$$

$$-\frac{(3Cd-Be)\left(-\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{(d^2-e^2x^2)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{3(d+ex)^3}}{e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \ 216$$

$$-\frac{\left(-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)(3Cd-Be)-\frac{(d^2-e^2x^2)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{3(d+ex)^3}}{e^2}-\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^2}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

output
```
-((C*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^2)) - (-1/3*((B - (C*d)/e - (A*
e)/d)*(d^2 - e^2*x^2)^(3/2))/(d + e*x)^3 + (3*C*d - B*e)*((-2*Sqrt[d^2 - e
^2*x^2])/(e*(d + e*x)) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e))/e^2
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.60

method	result
risch	$\frac{C\sqrt{-e^2x^2+d^2}}{e^3}-\frac{\frac{(Be-3Cd)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e\sqrt{e^2}}-\frac{\left(A\,e^2-3Bde+5C\,d^2\right)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^3d\left(x+\frac{d}{e}\right)}-\frac{2d\left(A\,e^2-Bde+C\,d^2\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right.}}{3de}\right.}{}}{e}$
default	$\frac{C\left(\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{de\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^2}}\right)}{e^3}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{de\left(x+\frac{d}{e}\right)^2}-\frac{e\left(\sqrt{-\left(x\right.}\right.}{}\right.}{}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
C*(-e^2*x^2+d^2)^(1/2)/e^3-1/e*((B*e-3*C*d)/e/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^3*(A*e^2-3*B*d*e+5*C*d^2)/d/(x+d/e)*(-(x+d/
e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2*d*(A*e^2-B*d*e+C*d^2)/e^3*(-1/3/d/e/(x+d/e
)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/e)*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2)))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$$
$$=\frac{14\,Cd^4-5\,Bd^3e-Ad^2e^2+(14\,Cd^2e^2-5\,Bde^3-Ae^4)x^2+2\,(14\,Cd^3e-5\,Bd^2e^2-Ade^3)x-6\,(3\,Cd^4-}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="frica
s")
```

output
```
1/3*(14*C*d^4 - 5*B*d^3*e - A*d^2*e^2 + (14*C*d^2*e^2 - 5*B*d*e^3 - A*e^4)
*x^2 + 2*(14*C*d^3*e - 5*B*d^2*e^2 - A*d*e^3)*x - 6*(3*C*d^4 - B*d^3*e + (
3*C*d^2*e^2 - B*d*e^3)*x^2 + 2*(3*C*d^3*e - B*d^2*e^2)*x)*arctan(-(d - sqr
t(-e^2*x^2 + d^2))/(e*x)) + (3*C*d*e^2*x^2 + 14*C*d^3 - 5*B*d^2*e - A*d*e^
2 + (19*C*d^2*e - 7*B*d*e^2 + A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^2 +
 2*d^2*e^4*x + d^3*e^3)
```

3.6.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

3.6.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxim
a")
```

output
```
Timed out
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.84

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \frac{(3\,Cd-Be)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|}+\frac{\sqrt{-e^2x^2+d^2}C}{e^3}$$
$$-\frac{2\left(11\,Cd^2-5\,Bde-Ae^2+\frac{24\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}-\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2Cd^2}{e^4x^2}-3\right.}{3\,de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^3|e|}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac"
)
```

output
```
(3*C*d - B*e)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + sqrt(-e^2*x^2 + d
^2)*C/e^3 - 2/3*(11*C*d^2 - 5*B*d*e - A*e^2 + 24*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))*C*d^2/(e^2*x) - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x
) + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*C*d^2/(e^4*x^2) - 3*(d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*x^2) - 3*(d*e + sqrt(-e^2*x^2 + d^2)*
abs(e))^2*A/(e^2*x^2))/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x)
 + 1)^3*abs(e))
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,(C\,x^2+B\,x+A)}{(d+e\,x)^3}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

output
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)
```

3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

3.7.1 Optimal result

Integrand size = 34, antiderivative size = 196

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e^2x^2)^{3/2}}{3de^3(d+ex)^3} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{15d^2e^3(d+ex)^3} - \frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^4+1/3*(-B*e+2*
C*d)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-1/15*(A*e^2-B*d*e+C*d^2)*(-e^2*x
^2+d^2)^(3/2)/d^2/e^3/(e*x+d)^3-C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*C
*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

3.7.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \frac{-\frac{\sqrt{d^2-e^2x^2}(3Cd^2(8d^2+19dex+13e^2x^2)+e(d-ex)(Ae(4d+ex)+Bd(d+4ex)))}{d^2(d+ex)^3}+30C\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{15e^3}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(-((Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d -
e*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*(d + e*x)^3)) + 30*C*ArcTa
n[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(15*e^3)
```

3.7.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}(A+Bx+Cx^2)}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{2168}$$

$$\int \left(\frac{\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{e^2(d+ex)^4}+\frac{\sqrt{d^2-e^2x^2}(Be-2Cd)}{e^2(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^2(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{15d^2e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^4}-\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{(d^2-e^2x^2)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

output
```
(-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
 - e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^
(3/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3
/2))/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2168
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + b*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, b,
d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.49

method	result
default	$\frac{C\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{d e\left(x+\frac{d}{e}\right)^{2}}-\frac{e\left(\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right)}+\frac{d e \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-\left(x+\frac{d}{e}\right)^{2} e^{2}+2 d e\left(x+\frac{d}{e}\right)}}\right)}{\sqrt{e^{2}}}\right)}{d}\right)}{e^{4}}-\frac{(B e-2 C d)\left(-\left(x+\frac{d}{e}\right.\right.}{3 e^{6} d\left(\right.}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
C/e^4*(-1/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-e/d*((-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2)+d*e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e
)^2*e^2+2*d*e*(x+d/e))^(1/2))))-1/3*(B*e-2*C*d)/e^6/d/(x+d/e)^3*(-(x+d/e)^
2*e^2+2*d*e*(x+d/e))^(3/2)+(A*e^2-B*d*e+C*d^2)/e^6*(-1/5/d/e/(x+d/e)^4*(-(
x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-1/15/d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e
*(x+d/e))^(3/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx =$$
$$-\frac{24\,Cd^5+Bd^4e+4\,Ad^3e^2+(24\,Cd^2e^3+Bde^4+4\,Ae^5)x^3+3\,(24\,Cd^3e^2+Bd^2e^3+4\,Ade^4)x^2+3\,(24\,C}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="frica
s")
```

output
```
-1/15*(24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^
5)*x^3 + 3*(24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4)*x^2 + 3*(24*C*d^4*e + B*
d^3*e^2 + 4*A*d^2*e^3)*x - 30*(C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e
*x + C*d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*C*d^4 + B*d^3*
e + 4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4)*x^2 + 3*(19*C*d^3*e +
 B*d^2*e^2 - A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^3 + 3*d^3*e^5*x^
2 + 3*d^4*e^4*x + d^5*e^3)
```

3.7.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^4}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

3.7.7 Maxima [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{-e^2x^2+d^2}(Cx^2+Bx+A)}{(ex+d)^4}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxim
a")
```

output
```
integrate(sqrt(-e^2*x^2 + d^2)*(C*x^2 + B*x + A)/(e*x + d)^4, x)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(180) = 360$.

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = -\frac{C\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} + \frac{2\left(24\,Cd^2+Bde+4\,Ae^2+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{105\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{165\,(\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac"
)
```

output
```
-C*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) + 2/15*(24*C*d^2 + B*d*e + 4*A
*e^2 + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*A/x + 105*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))*C*d^2/(e^2*x) + 5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*B*d
/(e*x) + 165*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*C*d^2/(e^4*x^2) - 5*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*x^2) + 25*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^2*A/(e^2*x^2) + 75*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*C
*d^2/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) +
15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*A/(e^4*x^3) + 15*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^4*C*d^2/(e^8*x^4) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^4*A/(e^6*x^4))/(d^2*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x)
+ 1)^5*abs(e))
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{d^2-e^2\,x^2}\,(C\,x^2+B\,x+A)}{(d+e\,x)^4}\,dx$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4,x)
```

output
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)
```

3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

3.8.1 Optimal result

Integrand size = 34, antiderivative size = 180

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{(Cd^2-Bde+Ae^2)\,(d^2-e^2x^2)^{3/2}}{7de^3(d+ex)^5}+\frac{C(d^2-e^2x^2)^{3/2}}{e^3(d+ex)^4} - \frac{(23Cd^2+e(5Bd+2Ae))\,(d^2-e^2x^2)^{3/2}}{35d^2e^3(d+ex)^4} - \frac{(23Cd^2+e(5Bd+2Ae))\,(d^2-e^2x^2)^{3/2}}{105d^3e^3(d+ex)^3}$$

output
```
-1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^5+C*(-e^2*x^2+
d^2)^(3/2)/e^3/(e*x+d)^4-1/35*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3
/2)/d^2/e^3/(e*x+d)^4-1/105*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2
)/d^3/e^3/(e*x+d)^3
```

3.8.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{(d-ex)\sqrt{d^2-e^2x^2}(Cd^2(2d^2+10dex+23e^2x^2)+e(5Bd(d^2+5dex+e^2x^2)+Ae(23d^2+10dex+2e^2}{105d^3e^3(d+ex)^4}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

output
```
-1/105*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^
2) + e*(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x
^2))))/(d^3*e^3*(d + e*x)^4)
```

3.8.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2170, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}\left(A+Bx+Cx^2\right)}{(d+ex)^5}dx$$

$$\Big\downarrow \ 2170$$

$$\frac{\int \frac{e^2\left(4Cd^2+Ae^2+e(3Cd+Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}dx}{e^4}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(4Cd^2+Ae^2+e(3Cd+Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}dx}{e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4}$$

$$\Big\downarrow \ 671$$

$$\frac{\frac{\left(e(2Ae+5Bd)+23Cd^2\right)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx}{7d}+\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{7(d+ex)^5}}{e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4}$$

$$\Big\downarrow \ 461$$

$$\frac{\frac{\left(e(2Ae+5Bd)+23Cd^2\right)\left(\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{5d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}\right)}{7d}+\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{7(d+ex)^5}}{e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4}$$

$$\Big\downarrow \ 460$$

$$\frac{\frac{\left(d^2-e^2x^2\right)^{3/2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{7(d+ex)^5}+\frac{\left(-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}\right)(e(2Ae+5Bd)+23Cd^2)}{7d}}{e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

output
```
(C*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) + (((B - (C*d)/e - (A*e)/d)*(d
^2 - e^2*x^2)^(3/2))/(7*(d + e*x)^5) + ((23*C*d^2 + e*(5*B*d + 2*A*e))*(-1
/5*(d^2 - e^2*x^2)^(3/2)/(d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2
*e*(d + e*x)^3)))/(7*d))/e^2
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

method	result
gosper	$-\frac{(-ex+d)\left(2A\,e^{4}x^{2}+5x^{2}dB\,e^{3}+23C\,d^{2}e^{2}x^{2}+10Ad\,e^{3}x+25xB\,d^{2}e^{2}+10C\,d^{3}xe+23A\,d^{2}e^{2}+5B\,d^{3}e+2C\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{105(ex+d)^{4}d^{3}e^{3}}$
trager	$-\frac{(-2A\,e^{5}x^{3}-5x^{3}dB\,e^{4}-23C\,d^{2}e^{3}x^{3}-8Ad\,e^{4}x^{2}-20x^{2}d^{2}B\,e^{3}+13C\,d^{3}e^{2}x^{2}-13A\,d^{2}e^{3}x+20x\,d^{3}B\,e^{2}+8C\,d^{4}ex+23A\,d^{3}e^{2}+5B\,d^{4}e-}{105d^{3}(ex+d)^{4}e^{3}}$
default	$-\frac{C\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{3e^{6}d\left(x+\frac{d}{e}\right)^{3}}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^{4}}-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^{2}\left(x+\frac{d}{e}\right)^{3}}\right)}{e^{6}}+\frac{(A\,e^{2}-Bde}{}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-e*x+d)*(2*A*e^4*x^2+5*B*d*e^3*x^2+23*C*d^2*e^2*x^2+10*A*d*e^3*x+2
5*B*d^2*e^2*x+10*C*d^3*e*x+23*A*d^2*e^2+5*B*d^3*e+2*C*d^4)*(-e^2*x^2+d^2)^
(1/2)/(e*x+d)^4/d^3/e^3
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.78

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx =$$
$$-\frac{2\,Cd^6+5\,Bd^5e+23\,Ad^4e^2+(2\,Cd^2e^4+5\,Bde^5+23\,Ae^6)x^4+4\,(2\,Cd^3e^3+5\,Bd^2e^4+23\,Ade^5)x^3+6}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="frica
s")
```

output
```
-1/105*(2*C*d^6 + 5*B*d^5*e + 23*A*d^4*e^2 + (2*C*d^2*e^4 + 5*B*d*e^5 + 23
*A*e^6)*x^4 + 4*(2*C*d^3*e^3 + 5*B*d^2*e^4 + 23*A*d*e^5)*x^3 + 6*(2*C*d^4*
e^2 + 5*B*d^3*e^3 + 23*A*d^2*e^4)*x^2 + 4*(2*C*d^5*e + 5*B*d^4*e^2 + 23*A*
d^3*e^3)*x + (2*C*d^5 + 5*B*d^4*e + 23*A*d^3*e^2 - (23*C*d^2*e^3 + 5*B*d*e
^4 + 2*A*e^5)*x^3 + (13*C*d^3*e^2 - 20*B*d^2*e^3 - 8*A*d*e^4)*x^2 + (8*C*d
^4*e + 20*B*d^3*e^2 - 13*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^4
+ 4*d^4*e^6*x^3 + 6*d^5*e^5*x^2 + 4*d^6*e^4*x + d^7*e^3)
```

3.8.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^5}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(166) = 332$.

Time = 0.20 (sec) , antiderivative size = 945, normalized size of antiderivative = 5.25

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = -\frac{2\sqrt{-e^2x^2+d^2}Cd^2}{7\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)} + \frac{\sqrt{-e^2x^2+d^2}Cd^2}{35\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)} + \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{105\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)} + \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{105\,(d^3e^4x+d^4e^3)} + \frac{2\sqrt{-e^2x^2+d^2}Bd}{7\,(e^6x^4+4\,de^5x^3+6\,d^2e^4x^2+4\,d^3e^3x+d^4e^2)} - \frac{\sqrt{-e^2x^2+d^2}Bd}{35\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)} - \frac{2\sqrt{-e^2x^2+d^2}Bd}{105\,(d^2e^4x^2+2\,d^3e^3x+d^4e^2)} - \frac{2\sqrt{-e^2x^2+d^2}Bd}{105\,(d^3e^3x+d^4e^2)} + \frac{4\sqrt{-e^2x^2+d^2}Cd}{5\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)} - \frac{2\sqrt{-e^2x^2+d^2}Cd}{15\,(de^5x^2+2\,d^2e^4x+d^3e^3)} - \frac{2\sqrt{-e^2x^2+d^2}Cd}{15\,(d^2e^4x+d^3e^3)} - \frac{2\sqrt{-e^2x^2+d^2}A}{7\,(e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e)} + \frac{\sqrt{-e^2x^2+d^2}A}{35\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)} + \frac{2\sqrt{-e^2x^2+d^2}A}{105\,(d^2e^3x^2+2\,d^3e^2x+d^4e)} + \frac{2\sqrt{-e^2x^2+d^2}A}{105\,(d^3e^2x+d^4e)} - \frac{2\sqrt{-e^2x^2+d^2}B}{5\,(e^5x^3+3\,de^4x^2+3\,d^2e^3x+d^3e^2)} + \frac{\sqrt{-e^2x^2+d^2}B}{15\,(de^4x^2+2\,d^2e^3x+d^3e^2)} + \frac{\sqrt{-e^2x^2+d^2}B}{15\,(d^2e^3x+d^3e^2)} - \frac{2\sqrt{-e^2x^2+d^2}C}{3\,(e^5x^2+2\,de^4x+d^2e^3)} + \frac{\sqrt{-e^2x^2+d^2}C}{3\,(de^4x+d^2e^3)}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxim
a")
```

output
```
-2/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4
*d^3*e^4*x + d^4*e^3) + 1/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2
*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*
e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e
^4*x + d^4*e^3) + 2/7*sqrt(-e^2*x^2 + d^2)*B*d/(e^6*x^4 + 4*d*e^5*x^3 + 6*
d^2*e^4*x^2 + 4*d^3*e^3*x + d^4*e^2) - 1/35*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^
5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2
)*B*d/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B
*d/(d^3*e^3*x + d^4*e^2) + 4/5*sqrt(-e^2*x^2 + d^2)*C*d/(e^6*x^3 + 3*d*e^5
*x^2 + 3*d^2*e^4*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^5*x^2 +
 2*d^2*e^4*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4*x + d^3*e
^3) - 2/7*sqrt(-e^2*x^2 + d^2)*A/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 +
4*d^3*e^2*x + d^4*e) + 1/35*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*
x^2 + 3*d^3*e^2*x + d^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2
*d^3*e^2*x + d^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 2
/5*sqrt(-e^2*x^2 + d^2)*B/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2)
+ 1/15*sqrt(-e^2*x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/15*s
qrt(-e^2*x^2 + d^2)*B/(d^2*e^3*x + d^3*e^2) - 2/3*sqrt(-e^2*x^2 + d^2)*C/(
e^5*x^2 + 2*d*e^4*x + d^2*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2
*e^3)
```

3.8.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.38

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx =$$

$$-\frac{1}{420}\left(\frac{3\left(5\left(\frac{2d}{ex+d}-1\right)^{\frac{7}{2}}+21\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}+35\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2d}{ex+d}-1}\right)C\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)-35\left(\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac"
)
```

output
```
-1/420*((3*(5*(2*d/(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 3
5*(2*d/(e*x + d) - 1)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d
))*sgn(e) - 35*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2)
 + 15*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d))*sgn(e) + 280*((2*d/(e*x
+ d) - 1)^(3/2) + 3*sqrt(2*d/(e*x + d) - 1))*C*sgn(1/(e*x + d))*sgn(e) - 3
*(5*(2*d/(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 35*(2*d/(e*
x + d) - 1)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e
)/d + 21*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2) + 15*
sqrt(2*d/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e)/d - 70*((2*d/(e*x + d
) - 1)^(3/2) + 3*sqrt(2*d/(e*x + d) - 1))*B*e*sgn(1/(e*x + d))*sgn(e)/d +
3*(5*(2*d/(e*x + d) - 1)^(7/2) + 21*(2*d/(e*x + d) - 1)^(5/2) + 35*(2*d/(e
*x + d) - 1)^(3/2) + 35*sqrt(2*d/(e*x + d) - 1))*A*e^2*sgn(1/(e*x + d))*sg
n(e)/d^2 - 7*(3*(2*d/(e*x + d) - 1)^(5/2) + 10*(2*d/(e*x + d) - 1)^(3/2) +
 15*sqrt(2*d/(e*x + d) - 1))*A*e^2*sgn(1/(e*x + d))*sgn(e)/d^2 - 420*C*sqr
t(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e))/(d*e^4) + 4*(23*I*C*d^2 + 5*
I*B*d*e + 2*I*A*e^2)*sgn(1/(e*x + d))*sgn(e)/(d^3*e^4))*abs(e)
```

3.8. $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

3.8.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.34

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{B\sqrt{d^2-e^2x^2}}{21\,(d^3e^2+x\,d^2e^3)} - \frac{3\,B\sqrt{d^2-e^2x^2}}{7\,(d^3e^2+3\,d^2e^3x+3\,d\,e^4x^2+e^5x^3)} + \frac{2\,A\sqrt{d^2-e^2x^2}}{105\,(d^4e+2\,d^3e^2x+d^2e^3x^2)} + \frac{B\sqrt{d^2-e^2x^2}}{21\,(d^3e^2+2\,d^2e^3x+d\,e^4x^2)} - \frac{82\,C\sqrt{d^2-e^2x^2}}{105\,(d^2e^3+2\,d\,e^4x+e^5x^2)} + \frac{2\,A\sqrt{d^2-e^2x^2}}{105\,(d^4e+x\,d^3e^2)} + \frac{23\,C\sqrt{d^2-e^2x^2}}{105\,(d^2e^3+x\,d\,e^4)} - \frac{2\,A\sqrt{d^2-e^2x^2}}{7\,(d^4e+4\,d^3e^2x+6\,d^2e^3x^2+4\,d\,e^4x^3+e^5x^4)} + \frac{A\sqrt{d^2-e^2x^2}}{35\,(d^4e+3\,d^3e^2x+3\,d^2e^3x^2+d\,e^4x^3)} - \frac{2\,C\,d^2\sqrt{d^2-e^2x^2}}{7\,(d^4e^3+4\,d^3e^4x+6\,d^2e^5x^2+4\,d\,e^6x^3+e^7x^4)} + \frac{2\,B\,d\sqrt{d^2-e^2x^2}}{7\,(d^4e^2+4\,d^3e^3x+6\,d^2e^4x^2+4\,d\,e^5x^3+e^6x^4)} + \frac{29\,C\,d\sqrt{d^2-e^2x^2}}{35\,(d^3e^3+3\,d^2e^4x+3\,d\,e^5x^2+e^6x^3)}$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)
```

output
```
(B*(d^2 - e^2*x^2)^(1/2))/(21*(d^3*e^2 + d^2*e^3*x)) - (3*B*(d^2 - e^2*x^2
)^(1/2))/(7*(d^3*e^2 + e^5*x^3 + 3*d^2*e^3*x + 3*d*e^4*x^2)) + (2*A*(d^2 -
 e^2*x^2)^(1/2))/(105*(d^4*e + 2*d^3*e^2*x + d^2*e^3*x^2)) + (B*(d^2 - e^2
*x^2)^(1/2))/(21*(d^3*e^2 + 2*d^2*e^3*x + d*e^4*x^2)) - (82*C*(d^2 - e^2*x
^2)^(1/2))/(105*(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)) + (2*A*(d^2 - e^2*x^2)^(1
/2))/(105*(d^4*e + d^3*e^2*x)) + (23*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^2*e^
3 + d*e^4*x)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e + e^5*x^4 + 4*d^3*e^
2*x + 4*d*e^4*x^3 + 6*d^2*e^3*x^2)) + (A*(d^2 - e^2*x^2)^(1/2))/(35*(d^4*e
 + 3*d^3*e^2*x + d*e^4*x^3 + 3*d^2*e^3*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^(1
/2))/(7*(d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) +
 (2*B*d*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e
^5*x^3 + 6*d^2*e^4*x^2)) + (29*C*d*(d^2 - e^2*x^2)^(1/2))/(35*(d^3*e^3 + e
^6*x^3 + 3*d^2*e^4*x + 3*d*e^5*x^2))
```

3.8. $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

3.9 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

3.9.1 Optimal result

Integrand size = 34, antiderivative size = 234

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = -\frac{(Cd^2-Bde+Ae^2)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{(11Cd^2+2e(2Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{42d^2e^3(d+ex)^5} - \frac{(11Cd^2+2e(2Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{105d^3e^3(d+ex)^4} - \frac{(11Cd^2+2e(2Bd+Ae))\left(d^2-e^2x^2\right)^{3/2}}{315d^4e^3(d+ex)^3}$$

output
```
-1/9*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^6+1/2*C*(-e^2*
x^2+d^2)^(3/2)/e^3/(e*x+d)^5-1/42*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2
)^(3/2)/d^2/e^3/(e*x+d)^5-1/105*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^
(3/2)/d^3/e^3/(e*x+d)^4-1/315*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3
/2)/d^4/e^3/(e*x+d)^3
```

3.9.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx =$$
$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}(Cd^2(4d^3+24d^2ex+66de^2x^2+11e^3x^3)+e(Ae(58d^3+33d^2ex+12de^2x^2+2e^3x^3}{315d^4e^3(d+ex)^5}$$

input
```
Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

output
```
-1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^
2*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x
^3) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d
+ e*x)^5)
```

3.9.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2170, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d^2-e^2x^2}(A+Bx+Cx^2)}{(d+ex)^6}dx$$

$$\Big\downarrow \text{2170}$$

$$\frac{\int \frac{e^2(5Cd^2+2Ae^2+e(3Cd+2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^6}dx}{2e^4}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(5Cd^2+2Ae^2+e(3Cd+2Be)x)\sqrt{d^2-e^2x^2}}{(d+ex)^6}dx}{2e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

$$\Big\downarrow \text{671}$$

$$\frac{\frac{(2e(Ae+2Bd)+11Cd^2)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^5}dx}{3d}-\frac{2\left(d^2-e^2x^2\right)^{3/2}(Ae^2-Bde+Cd^2)}{9de(d+ex)^6}}{2e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{(2e(Ae+2Bd)+11Cd^2)\left(\frac{2\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^4}dx}{7d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7de(d+ex)^5}\right)}{3d}-\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{9de(d+ex)^6}}{2e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{(2e(Ae+2Bd)+11Cd^2)\left(\frac{2\left(\frac{\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}dx}{5d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7de(d+ex)^5}\right)}{3d}-\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{9de(d+ex)^6}}{2e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{\left(\frac{2\left(-\frac{\left(d^2-e^2x^2\right)^{3/2}}{15d^2e(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{5de(d+ex)^4}\right)}{7d}-\frac{\left(d^2-e^2x^2\right)^{3/2}}{7de(d+ex)^5}\right)(2e(Ae+2Bd)+11Cd^2)}{3d}-\frac{2(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{9de(d+ex)^6}}{2e^2}+\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5}$$

input
```
Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

output
```
(C*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) + ((-2*(C*d^2 - B*d*e + A*e^
2)*(d^2 - e^2*x^2)^(3/2))/(9*d*e*(d + e*x)^6) + ((11*C*d^2 + 2*e*(2*B*d +
A*e))*(-1/7*(d^2 - e^2*x^2)^(3/2)/(d*e*(d + e*x)^5) + (2*(-1/5*(d^2 - e^2*
x^2)^(3/2)/(d*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(3/2)/(15*d^2*e*(d + e*x)^3
)))/(7*d)))/(3*d))/(2*e^2)
```

3.9. $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

method	result
gosper	$-\frac{(-ex+d)\left(2A\,e^{5}x^{3}+4x^{3}dB\,e^{4}+11C\,d^{2}e^{3}x^{3}+12Ad\,e^{4}x^{2}+24x^{2}d^{2}B\,e^{3}+66C\,d^{3}e^{2}x^{2}+33A\,d^{2}e^{3}x+66x\,d^{3}B\,e^{2}+24C\,d^{4}ex+58A\,d^{3}e^{2}\right.}{315(ex+d)^{5}d^{4}e^{3}}$
trager	$-\frac{\left(-2A\,e^{6}x^{4}-4Bd\,e^{5}x^{4}-11C\,d^{2}e^{4}x^{4}-10Ad\,e^{5}x^{3}-20B\,d^{2}e^{4}x^{3}-55C\,d^{3}e^{3}x^{3}-21A\,d^{2}e^{4}x^{2}-42B\,d^{3}e^{3}x^{2}+42C\,d^{4}e^{2}x^{2}-25A\,d^{3}e^{3}x+\right.}{315d^{4}(ex+d)^{5}e^{3}}$
default	$\frac{C\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{5de\left(x+\frac{d}{e}\right)^{4}}-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{15d^{2}\left(x+\frac{d}{e}\right)^{3}}\right)}{e^{6}}+\frac{(Be-2Cd)\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}{7de\left(x+\frac{d}{e}\right)^{5}}+\frac{2e\left(-\frac{\left(-\left(x+\frac{d}{e}\right)^{2}\right.}{5d}\right.}{}\right.}{e^{7}}$

input
```
int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(-e*x+d)*(2*A*e^5*x^3+4*B*d*e^4*x^3+11*C*d^2*e^3*x^3+12*A*d*e^4*x^2
+24*B*d^2*e^3*x^2+66*C*d^3*e^2*x^2+33*A*d^2*e^3*x+66*B*d^3*e^2*x+24*C*d^4*
e*x+58*A*d^3*e^2+11*B*d^4*e+4*C*d^5)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5/d^4/e^
3
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = -\frac{4\,Cd^7+11\,Bd^6e+58\,Ad^5e^2+(4\,Cd^2e^5+11\,Bde^6+58\,Ae^7)x^5+5\,(4\,Cd^3e^4+11\,Bd^2e^5+58\,Ade^6)x^4}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="frica
s")
```

output
```
-1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 +
58*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*
d^4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^
3 + 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4
*C*d^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*
x^4 - 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2
*B*d^3*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x
)*sqrt(-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10
*d^7*e^5*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

3.9.6 Sympy [F]

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \int \frac{\sqrt{-(-d+ex)(d+ex)}(A+Bx+Cx^2)}{(d+ex)^6}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

output
```
Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**6, x)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. 2(214) = 428.

Time = 0.21 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.89

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxim
a")
```

output
```
-2/9*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 +
10*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3) + 1/63*sqrt(-e^2*x^2 + d^2)*C*d^2/
(d*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2 + 4*d^4*e^4*x + d^5*e^3) + 1/10
5*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*d^4*e^4*x +
d^5*e^3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^5*x^2 + 2*d^4*e^4*x + d
^5*e^3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e^4*x + d^5*e^3) + 2/9*sqr
t(-e^2*x^2 + d^2)*B*d/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*d^3*e^4
*x^2 + 5*d^4*e^3*x + d^5*e^2) - 1/63*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 +
 4*d^2*e^5*x^3 + 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) - 1/105*sqrt(-e^2*
x^2 + d^2)*B*d/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) - 2/3
15*sqrt(-e^2*x^2 + d^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) - 2/315*
sqrt(-e^2*x^2 + d^2)*B*d/(d^4*e^3*x + d^5*e^2) + 4/7*sqrt(-e^2*x^2 + d^2)*
C*d/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3) - 2/35
*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e
^3) - 4/105*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
 - 4/105*sqrt(-e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^4*e^3) - 2/9*sqrt(-e^2*x^
2 + d^2)*A/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^
4*e^2*x + d^5*e) + 1/63*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3
+ 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d^5*e) + 1/105*sqrt(-e^2*x^2 + d^2)*A/(d^2
*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 +...
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(214) = 428$.

Time = 0.30 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.06

$$\int \frac{(A + Bx + Cx^2)\sqrt{d^2 - e^2x^2}}{(d + ex)^6}\,dx$$

$$= \frac{2\left(4\,Cd^2 + 11\,Bde + 58\,Ae^2 + \frac{207\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x} + \frac{36\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x} + \frac{99\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex} + \frac{14}{}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac"
)
```

output
```
2/315*(4*C*d^2 + 11*B*d*e + 58*A*e^2 + 207*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))*A/x + 36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 99*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 144*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^2*C*d^2/(e^4*x^2) + 81*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/
(e^3*x^2) + 1143*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) - 84*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*C*d^2/(e^6*x^3) + 609*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) + 2247*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*A/(e^4*x^3) + 504*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*C*d^2/(e^8*
x^4) + 441*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*B*d/(e^7*x^4) + 3843*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^4*A/(e^6*x^4) + 945*(d*e + sqrt(-e^2*x^2 +
 d^2)*abs(e))^5*B*d/(e^9*x^5) + 3465*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5
*A/(e^8*x^5) + 420*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*C*d^2/(e^12*x^6)
+ 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*B*d/(e^11*x^6) + 2625*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^6*A/(e^10*x^6) + 315*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))^7*B*d/(e^13*x^7) + 945*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*A/
(e^12*x^7) + 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*A/(e^14*x^8))/(d^4*
e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^9*abs(e))
```

3.9. $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

3.9.9 Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.10

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$$
$$= \frac{B\sqrt{d^2-e^2x^2}}{63\,(d^4e^2+x\,d^3e^3)}+\frac{C\sqrt{d^2-e^2x^2}}{135\,(d^3e^3+x\,d^2e^4)}$$
$$-\frac{19\,B\sqrt{d^2-e^2x^2}}{63\,(d^4e^2+4\,d^3e^3x+6\,d^2e^4x^2+4\,d\,e^5x^3+e^6x^4)}$$
$$+\frac{A\sqrt{d^2-e^2x^2}}{105\,(d^5e+3\,d^4e^2x+3\,d^3e^3x^2+d^2e^4x^3)}$$
$$+\frac{2\,B\sqrt{d^2-e^2x^2}}{105\,(d^4e^2+3\,d^3e^3x+3\,d^2e^4x^2+d\,e^5x^3)}-\frac{47\,C\sqrt{d^2-e^2x^2}}{105\,(d^3e^3+3\,d^2e^4x+3\,d\,e^5x^2+e^6x^3)}$$
$$+\frac{2\,A\sqrt{d^2-e^2x^2}}{315\,(d^5e+2\,d^4e^2x+d^3e^3x^2)}+\frac{11\,C\sqrt{d^2-e^2x^2}}{315\,(d^3e^3+2\,d^2e^4x+d\,e^5x^2)}$$
$$-\frac{2\,A\sqrt{d^2-e^2x^2}}{9\,(d^5e+5\,d^4e^2x+10\,d^3e^3x^2+10\,d^2e^4x^3+5\,d\,e^5x^4+e^6x^5)}$$
$$+\frac{A\sqrt{d^2-e^2x^2}}{63\,(d^5e+4\,d^4e^2x+6\,d^3e^3x^2+4\,d^2e^4x^3+d\,e^5x^4)}$$
$$-\frac{2\,A\sqrt{d^2-e^2x^2}}{945\,(d^5e+x\,d^4e^2)}+\frac{4\,B\sqrt{d^2-e^2x^2}}{315\,(d^4e^2+2\,d^3e^3x+d^2e^4x^2)}$$
$$+\frac{2\,B\,d\sqrt{d^2-e^2x^2}}{9\,(d^5e^2+5\,d^4e^3x+10\,d^3e^4x^2+10\,d^2e^5x^3+5\,d\,e^6x^4+e^7x^5)}$$
$$+\frac{37\,C\,d\sqrt{d^2-e^2x^2}}{63\,(d^4e^3+4\,d^3e^4x+6\,d^2e^5x^2+4\,d\,e^6x^3+e^7x^4)}$$
$$-\frac{2\,C\,d^2\sqrt{d^2-e^2x^2}}{9\,(d^5e^3+5\,d^4e^4x+10\,d^3e^5x^2+10\,d^2e^6x^3+5\,d\,e^7x^4+e^8x^5)}$$
$$+\frac{8\,A\,e^2\sqrt{d^2-e^2x^2}}{945\,(d^5e^3+x\,d^4e^4)}+\frac{26\,C\,d^2\sqrt{d^2-e^2x^2}}{945\,(d^5e^3+x\,d^4e^4)}-\frac{B\,d\,e\sqrt{d^2-e^2x^2}}{315\,(d^5e^3+x\,d^4e^4)}$$

input
```
int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^6,x)
```

output
```
(B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^
(1/2))/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4
*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e
^2*x^2)^(1/2))/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) +
 (2*B*(d^2 - e^2*x^2)^(1/2))/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d
^2*e^4*x^2)) - (47*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^3*e^3 + e^6*x^3 + 3*d^
2*e^4*x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e + 2*d^4*
e^2*x + d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^(1/2))/(315*(d^3*e^3 + 2*d^2
*e^4*x + d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e + e^6*x^5 + 5
*d^4*e^2*x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e
^2*x^2)^(1/2))/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^
2*e^4*x^3)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e + d^4*e^2*x)) + (4*B
*(d^2 - e^2*x^2)^(1/2))/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B
*d*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^
4 + 10*d^3*e^4*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^(1/2))/(63
*(d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d
^2*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^
4 + 10*d^3*e^5*x^2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^(1/2))/(9
45*(d^5*e^3 + d^4*e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3
 + d^4*e^4*x)) - (B*d*e*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e^3 + d^4*e^4*...
```

3.9. $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

3.10 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

3.10.1 Optimal result

Integrand size = 34, antiderivative size = 236

$$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{d^2(38Cd^2+45Bde+55Ae^2)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d(13Cd^2+15Bde+12Ae^2)\,x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(19Cd^2+5e(3Bd+Ae))\,x^2\sqrt{d^2-e^2x^2}}{15e} - \frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} + \frac{d^3(13Cd^2+15Bde+20Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

output
```
1/8*d^3*(20*A*e^2+15*B*d*e+13*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-
1/15*d^2*(55*A*e^2+45*B*d*e+38*C*d^2)*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*d*(12*A
*e^2+15*B*d*e+13*C*d^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/15*(19*C*d^2+5*e*(A*e
+3*B*d))*x^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(B*e+3*C*d)*x^3*(-e^2*x^2+d^2)^(1/
2)-1/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{\sqrt{d^2-e^2x^2}(C(304d^4+195d^3ex+152d^2e^2x^2+90de^3x^3+24e^4x^4)+5e(4Ae(22d^2+9dex+2e^2x^2)+3}{120e^3}$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/120*(Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 +
90*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3
*B*(24*d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3))) + 30*d^3*(13*C*d^2 +
5*e*(3*B*d + 4*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.10.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2346, 25, 2346, 25, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \ 2346$$

$$-\frac{\int -\frac{5e^4(3Cd+Be)x^4+e^3\left(19Cd^2+5e(3Bd+Ae)\right)x^3+5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2+5d^2e^2(Bd+3Ae)x+5Ad^3e^2}{\sqrt{d^2-e^2x^2}}dx}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5e^4(3Cd+Be)x^4+e^3\left(19Cd^2+5e(3Bd+Ae)\right)x^3+5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2+5d^2e^2(Bd+3Ae)x+5Ad^3e^2}{\sqrt{d^2-e^2x^2}}dx}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 2346$$

$$-\frac{\frac{\int -\frac{4\left(19Cd^2+5e(3Bd+Ae)\right)x^3e^5+20Ad^3e^4+5d\left(13Cd^2+15Bed+12Ae^2\right)x^2e^4+20d^2(Bd+3Ae)xe^4}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{5}{4}e^2x^3\sqrt{d^2-e^2x^2}(Be+3Cd)}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{4\left(19Cd^2+5e(3Bd+Ae)\right)x^3e^5+20Ad^3e^4+5d\left(13Cd^2+15Bed+12Ae^2\right)x^2e^4+20d^2(Bd+3Ae)xe^4}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{5}{4}e^2x^3\sqrt{d^2-e^2x^2}(Be+3Cd)}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 2346

$$\frac{\frac{-\frac{\int -\frac{60Ad^3e^6+15d\left(13Cd^2+15Bed+12Ae^2\right)x^2e^6+4d^2\left(38Cd^2+45Bed+55Ae^2\right)xe^5}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{4e^2}-\frac{5}{4}e^2x^3\sqrt{d^2-e^2\ldots}}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{60Ad^3e^6+15d\left(13Cd^2+15Bed+12Ae^2\right)x^2e^6+4d^2\left(38Cd^2+45Bed+55Ae^2\right)xe^5}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd^2\right)}{4e^2}-\frac{5}{4}e^2x^3\sqrt{d^2-e^2x^2}(\ldots}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 2346

$$\frac{\frac{\frac{-\frac{\int -\frac{d^2e^6\left(15d\left(13Cd^2+15Bed+20Ae^2\right)+8e\left(38Cd^2+45Bed+55Ae^2\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+\ldots\right.}{4e^2}\ldots}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\frac{\int \frac{d^2e^6\left(15d\left(13Cd^2+5e(3Bd+4Ae)\right)+8e\left(38Cd^2+45Bed+55Ae^2\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19C\ldots\right.}{4e^2}\ldots}{5e^2}-\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

3.10. $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{1}{2}d^2e^4\int \frac{15d\left(13Cd^2+5e(3Bd+4Ae)\right)+8e\left(38Cd^2+45Bed+55Ae^2\right)x}{\sqrt{d^2-e^2x^2}}dx-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x^2}\left(5e(Ae+3Bd)+19Cd\right.}{4e^2}}{5e^2}$$

$$\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\downarrow$ 455

$$\frac{\frac{\frac{\frac{1}{2}d^2e^4\left(15d\left(20Ae^2+15Bde+13Cd^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+38Cd^2\right)}{e}\right)-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2-e^2x}}{4e^2}}{5e^2}$$

$$\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\frac{1}{2}d^2e^4\left(15d\left(20Ae^2+15Bde+13Cd^2\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+38Cd^2\right)}{e}\right)-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x}{4e^2}}{5e^2}$$

$$\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

$\downarrow$ 216

$$\frac{\frac{\frac{\frac{1}{2}d^2e^4\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(20Ae^2+15Bde+13Cd^2\right)}{e}-\frac{8\sqrt{d^2-e^2x^2}\left(55Ae^2+45Bde+38Cd^2\right)}{e}\right)-\frac{15}{2}de^4x\sqrt{d^2-e^2x^2}\left(12Ae^2+15Bde+13Cd^2\right)}{3e^2}-\frac{4}{3}e^3x^2\sqrt{d^2}}{4e^2}}{5e^2}$$

$$\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2}$$

input

```
Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output

```
-1/5*(C*e*x^4*Sqrt[d^2 - e^2*x^2]) + ((-5*e^2*(3*C*d + B*e)*x^3*Sqrt[d^2 -
 e^2*x^2])/4 + ((-4*e^3*(19*C*d^2 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*
x^2])/3 + ((-15*d*e^4*(13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^
2])/2 + (d^2*e^4*((-8*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2]
)/e + (15*d*(13*C*d^2 + 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/e))/2)/(3*e^2))/(4*e^2))/(5*e^2)
```

3.10. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(24e^4C\,x^4+30x^3B\,e^4+90Cd\,e^3x^3+40A\,e^4x^2+120x^2dB\,e^3+152C\,d^2e^2x^2+180Ad\,e^3x+225xB\,d^2e^2+195C\,d^3xe+440A\,d^2e^2+360}{120e^3}$
default	$\frac{A\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^3C\left(-\frac{x^4\sqrt{-e^2x^2+d^2}}{5e^2}+\frac{4d^2\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)}{5e^2}\right)+(B\,e^3+3d\,e^2C)$

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/e^3*(24*C*e^4*x^4+30*B*e^4*x^3+90*C*d*e^3*x^3+40*A*e^4*x^2+120*B*d*
e^3*x^2+152*C*d^2*e^2*x^2+180*A*d*e^3*x+225*B*d^2*e^2*x+195*C*d^3*e*x+440*
A*d^2*e^2+360*B*d^3*e+304*C*d^4)*(-e^2*x^2+d^2)^(1/2)+1/8*d^3/e^2*(20*A*e^
2+15*B*d*e+13*C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{30\,(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(24\,Ce^4x^4+304\,Cd^4+360\,Bd^3e+440\,A\ldots}{}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) + (24*C*e^4*x^4 + 304*C*d^4 + 360*B*d^3*e + 440*A*d^2*e^
2 + 30*(3*C*d*e^3 + B*e^4)*x^3 + 8*(19*C*d^2*e^2 + 15*B*d*e^3 + 5*A*e^4)*x
^2 + 15*(13*C*d^3*e + 15*B*d^2*e^2 + 12*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/
e^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$$
$$=\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{Cex^4}{5}-\frac{x^3(Be^3+3Cde^2)}{4e^2}-\frac{x^2\left(Ae^3+3Bde^2+\frac{19Cd^2e}{5}\right)}{3e^2}-\frac{x\left(3Ade^2+3Bd^2e+Cd^3+\frac{3d^2\left(Be^3+3Cde^2\right)}{4e^2}\right)}{2e^2}-\frac{3Ad^2e\ldots}{}\right. \\ \frac{Ad^3x+\frac{Ce^3x^6}{6}+\frac{x^5\left(Be^3+3Cde^2\right)}{5}+\frac{x^4\left(Ae^3+3Bde^2+3Cd^2e\right)}{4}+\frac{x^3\cdot\left(3Ade^2+3Bd^2e+Cd^3\right)}{3}+\frac{x^2\cdot\left(3Ad^2e+Bd^3\right)}{2}}{\sqrt{d^2}}\end{cases}$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(-C*e*x**4/5 - x**3*(B*e**3 + 3*C*d*e**2
)/(4*e**2) - x**2*(A*e**3 + 3*B*d*e**2 + 19*C*d**2*e/5)/(3*e**2) - x*(3*A*
d*e**2 + 3*B*d**2*e + C*d**3 + 3*d**2*(B*e**3 + 3*C*d*e**2)/(4*e**2))/(2*e
**2) - (3*A*d**2*e + B*d**3 + 2*d**2*(A*e**3 + 3*B*d*e**2 + 19*C*d**2*e/5)
/(3*e**2))/e**2) + (A*d**3 + d**2*(3*A*d*e**2 + 3*B*d**2*e + C*d**3 + 3*d*
*2*(B*e**3 + 3*C*d*e**2)/(4*e**2))/(2*e**2))*Piecewise((log(-2*e**2*x + 2*
sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/s
qrt(-e**2*x**2), True)), Ne(e**2, 0)), ((A*d**3*x + C*e**3*x**6/6 + x**5*(
B*e**3 + 3*C*d*e**2)/5 + x**4*(A*e**3 + 3*B*d*e**2 + 3*C*d**2*e)/4 + x**3*
(3*A*d*e**2 + 3*B*d**2*e + C*d**3)/3 + x**2*(3*A*d**2*e + B*d**3)/2)/sqrt(
d**2), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{(d+ex)^3(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =& -\frac{1}{5}\sqrt{-e^2x^2+d^2}Cex^4 - \frac{4\sqrt{-e^2x^2+d^2}Cd^2x^2}{15\,e} \\ &+ \frac{Ad^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{8\sqrt{-e^2x^2+d^2}Cd^4}{15\,e^3} \\ &- \frac{\sqrt{-e^2x^2+d^2}Bd^3}{e^2} - \frac{3\sqrt{-e^2x^2+d^2}Ad^2}{e} \\ &- \frac{(3\,Cde^2+Be^3)\sqrt{-e^2x^2+d^2}x^3}{4\,e^2} \\ &- \frac{(3\,Cd^2e+3\,Bde^2+Ae^3)\sqrt{-e^2x^2+d^2}x^2}{3\,e^2} \\ &+ \frac{3\,(3\,Cde^2+Be^3)d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^4} \\ &+ \frac{(Cd^3+3\,Bd^2e+3\,Ade^2)d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} \\ &- \frac{3\,(3\,Cde^2+Be^3)\sqrt{-e^2x^2+d^2}d^2x}{8\,e^4} \\ &- \frac{(Cd^3+3\,Bd^2e+3\,Ade^2)\sqrt{-e^2x^2+d^2}x}{2\,e^2} \\ &- \frac{2\,(3\,Cd^2e+3\,Bde^2+Ae^3)\sqrt{-e^2x^2+d^2}d^2}{3\,e^4}\end{aligned}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*C*e*x^4 - 4/15*sqrt(-e^2*x^2 + d^2)*C*d^2*x^2/e
+ A*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 8/15*sqrt(-e^2*x^2 + d^2)*
C*d^4/e^3 - sqrt(-e^2*x^2 + d^2)*B*d^3/e^2 - 3*sqrt(-e^2*x^2 + d^2)*A*d^2/
e - 1/4*(3*C*d*e^2 + B*e^3)*sqrt(-e^2*x^2 + d^2)*x^3/e^2 - 1/3*(3*C*d^2*e
+ 3*B*d*e^2 + A*e^3)*sqrt(-e^2*x^2 + d^2)*x^2/e^2 + 3/8*(3*C*d*e^2 + B*e^3
)*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 1/2*(C*d^3 + 3*B*d^2*e
 + 3*A*d*e^2)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*(3*C*d
*e^2 + B*e^3)*sqrt(-e^2*x^2 + d^2)*d^2*x/e^4 - 1/2*(C*d^3 + 3*B*d^2*e + 3*
A*d*e^2)*sqrt(-e^2*x^2 + d^2)*x/e^2 - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*
sqrt(-e^2*x^2 + d^2)*d^2/e^4
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\left(4\,Cex+\frac{5\,(3\,Cde^6+Be^7)}{e^6}\right)x+\frac{4\,(19\,Cd^2e^5+15\,Bde^6+5\,Ae^7)}{e^6}\right)x+\frac{15\,(1\right.\right.$$
$$+\frac{(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,e^2|e|}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
-1/120*sqrt(-e^2*x^2 + d^2)*((2*(3*(4*C*e*x + 5*(3*C*d*e^6 + B*e^7)/e^6)*x
 + 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)/e^6)*x + 15*(13*C*d^3*e^4 + 15*
B*d^2*e^5 + 12*A*d*e^6)/e^6)*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*d^2
*e^5)/e^6) + 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(e*x/d)*sgn(
d)*sgn(e)/(e^2*abs(e))
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^3\,(C\,x^2+B\,x+A)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.11 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

3.11.1 Optimal result

Integrand size = 34, antiderivative size = 191

$$\begin{aligned}\int \frac{(d+ex)^2(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =& -\frac{d(4Cd^2+e(5Bd+6Ae))\sqrt{d^2-e^2x^2}}{3e^3} \\ &-\frac{(7Cd^2+4e(2Bd+Ae))\,x\sqrt{d^2-e^2x^2}}{8e^2} \\ &-\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} \\ &+\frac{d^2(7Cd^2+8Bde+12Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}\end{aligned}$$

output
```
1/8*d^2*(12*A*e^2+8*B*d*e+7*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/
3*d*(4*C*d^2+e*(6*A*e+5*B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*(7*C*d^2+4*e*(A
*e+2*B*d))*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/3*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2)^(
1/2)/e-1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$

$$-\frac{\sqrt{d^2-e^2x^2}(C(32d^3+21d^2ex+16de^2x^2+6e^3x^3)+4e(3Ae(4d+ex)+2B(5d^2+3dex+e^2x^2)))+6d}{24e^3}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/24*(Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*
x^3) + 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2))) + 6*d^2*
(7*C*d^2 + 4*e*(2*B*d + 3*A*e))*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x
^2])])/e^3
```

3.11.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2346, 25, 2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{4e^3(2Cd+Be)x^3+e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2+4de^2(Bd+2Ae)x+4Ad^2e^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4e^3(2Cd+Be)x^3+e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2+4de^2(Bd+2Ae)x+4Ad^2e^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\frac{\int -\frac{12Ad^2e^4+3\left(7Cd^2+4e(2Bd+Ae)\right)x^2e^4+4d\left(4Cd^2+e(5Bd+6Ae)\right)xe^3}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{12Ad^2e^4+3\left(7Cd^2+4e(2Bd+Ae)\right)x^2e^4+4d\left(4Cd^2+e(5Bd+6Ae)\right)xe^3}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 2346$$

$$\frac{-\frac{\frac{\int -\frac{de^4\left(3d\left(7Cd^2+8Bed+12Ae^2\right)+8e\left(4Cd^2+e(5Bd+6Ae)\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{de^4\left(3d\left(7Cd^2+8Bed+12Ae^2\right)+8e\left(4Cd^2+e(5Bd+6Ae)\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{1}{2}de^2\int \frac{3d\left(7Cd^2+8Bed+12Ae^2\right)+8e\left(4Cd^2+e(5Bd+6Ae)\right)x}{\sqrt{d^2-e^2x^2}}dx-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2x^2}(Be+2Cd)}{4e^2}-\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 455$$

$$\frac{\frac{\frac{1}{2}de^2\left(3d\left(12Ae^2+8Bde+7Cd^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{e}\right)-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-e^2}}{4e^2}$$

$$\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \ 224$$

3.11. $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

$$\frac{\frac{\frac{1}{2}de^2\left(3d(12Ae^2+8Bde+7Cd^2)\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{e}\right)-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\ldots}{4e^2}$$

$$\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

$\big\downarrow$ 216

$$\frac{\frac{\frac{1}{2}de^2\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(12Ae^2+8Bde+7Cd^2)}{e}-\frac{8\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{e}\right)-\frac{3}{2}e^2x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{3e^2}-\frac{4}{3}ex^2\sqrt{d^2-\ldots}}{4e^2}$$

$$\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/4*(C*x^3*Sqrt[d^2 - e^2*x^2]) + ((-4*e*(2*C*d + B*e)*x^2*Sqrt[d^2 - e^2
*x^2])/3 + ((-3*e^2*(7*C*d^2 + 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/2
 + (d*e^2*((-8*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/e + (3*d
*(7*C*d^2 + 8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2)/
(3*e^2))/(4*e^2)
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(6C e^3 x^3+8B e^3 x^2+16C d e^2 x^2+12A e^3 x+24B d e^2 x+21C d^2 e x+48A d e^2+40B d^2 e+32d^3 C)\sqrt{-e^2x^2+d^2}}{24e^3}+\frac{d^2(12A e^2+8Bde+$
default	$\frac{A d^2 \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^2 C\left(-\frac{x^3\sqrt{-e^2x^2+d^2}}{4e^2}+\frac{3d^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)}{4e^2}\right)+(B e^2+2$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/e^3*(6*C*e^3*x^3+8*B*e^3*x^2+16*C*d*e^2*x^2+12*A*e^3*x+24*B*d*e^2*x+
21*C*d^2*e*x+48*A*d*e^2+40*B*d^2*e+32*C*d^3)*(-e^2*x^2+d^2)^(1/2)+1/8*d^2/
e^2*(12*A*e^2+8*B*d*e+7*C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+
d^2)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{6\,(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(6\,Ce^3x^3+32\,Cd^3+40\,Bd^2e+48\,Ade^2+8\,(}{24\,e^3}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (6*C*e^3*x^3 + 32*C*d^3 + 40*B*d^2*e + 48*A*d*e^2 + 8*(2*C
*d*e^2 + B*e^3)*x^2 + 3*(7*C*d^2*e + 8*B*d*e^2 + 4*A*e^3)*x)*sqrt(-e^2*x^2
 + d^2))/e^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$$
$$=\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{Cx^3}{4}-\frac{x^2(Be^2+2Cde)}{3e^2}-\frac{x\left(Ae^2+2Bde+\frac{7Cd^2}{4}\right)}{2e^2}-\frac{2Ade+Bd^2+\frac{2d^2\left(Be^2+2Cde\right)}{3e^2}}{e^2}\right)+\left(Ad^2+\frac{d^2\left(Ae^2+2Bde\right.}{2e^2}\right. \\ \frac{Ad^2x+\frac{Ce^2x^5}{5}+\frac{x^4\left(Be^2+2Cde\right)}{4}+\frac{x^3\left(Ae^2+2Bde+Cd^2\right)}{3}+\frac{x^2\cdot\left(2Ade+Bd^2\right)}{2}}{\sqrt{d^2}}\end{cases}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(-C*x**3/4 - x**2*(B*e**2 + 2*C*d*e)/(3*
e**2) - x*(A*e**2 + 2*B*d*e + 7*C*d**2/4)/(2*e**2) - (2*A*d*e + B*d**2 + 2
*d**2*(B*e**2 + 2*C*d*e)/(3*e**2))/e**2) + (A*d**2 + d**2*(A*e**2 + 2*B*d*
e + 7*C*d**2/4)/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d*
*2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), Tr
ue)), Ne(e**2, 0)), ((A*d**2*x + C*e**2*x**5/5 + x**4*(B*e**2 + 2*C*d*e)/4
 + x**3*(A*e**2 + 2*B*d*e + C*d**2)/3 + x**2*(2*A*d*e + B*d**2)/2)/sqrt(d*
*2), True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}\sqrt{-e^2x^2+d^2}Cx^3 + \frac{Ad^2 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{3\,Cd^4 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}e^2} - \frac{3\sqrt{-e^2x^2+d^2}Cd^2x}{8\,e^2} - \frac{\sqrt{-e^2x^2+d^2}Bd^2}{e^2} - \frac{2\sqrt{-e^2x^2+d^2}Ad}{e} - \frac{\sqrt{-e^2x^2+d^2}(2\,Cde+Be^2)x^2}{3\,e^2} + \frac{(Cd^2+2\,Bde+Ae^2)d^2 \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}(Cd^2+2\,Bde+Ae^2)x}{2\,e^2} - \frac{2\sqrt{-e^2x^2+d^2}(2\,Cde+Be^2)d^2}{3\,e^4}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/4*sqrt(-e^2*x^2 + d^2)*C*x^3 + A*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e
^2) + 3/8*C*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^
2*x^2 + d^2)*C*d^2*x/e^2 - sqrt(-e^2*x^2 + d^2)*B*d^2/e^2 - 2*sqrt(-e^2*x^
2 + d^2)*A*d/e - 1/3*sqrt(-e^2*x^2 + d^2)*(2*C*d*e + B*e^2)*x^2/e^2 + 1/2*
(C*d^2 + 2*B*d*e + A*e^2)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2)
- 1/2*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/3*sqrt(-e^2
*x^2 + d^2)*(2*C*d*e + B*e^2)*d^2/e^4
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\left(2\left(3\,Cx+\frac{4\,(2\,Cde^4+Be^5)}{e^5}\right)x+\frac{3\,(7\,Cd^2e^3+8\,Bde^4+4\,Ae^5)}{e^5}\right)x+\frac{8\,(4\,Cd^3e^2+\right. + \frac{(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2)\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,e^2|e|}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
-1/24*sqrt(-e^2*x^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)/e^5)*x + 3*(
7*C*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)/e^5)*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 +
 6*A*d*e^4)/e^5) + 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(e*x/d)*
sgn(d)*sgn(e)/(e^2*abs(e))
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^2\,(C\,x^2+B\,x+A)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

3.12 $\int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$

3.12.1 Optimal result

Integrand size = 32, antiderivative size = 143

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(2Cd^2+3e(Bd+Ae))\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{d(Cd^2+e(Bd+2Ae))\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
1/2*d*(C*d^2+e*(2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(2*C*
d^2+3*e*(A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*(B*e+C*d)*x*(-e^2*x^2+d^2)
^(1/2)/e^2-1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e
```

3.12.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(3e(2Bd+2Ae+Bex)+C(4d^2+3dex+2e^2x^2))+6d(Cd^2+e(Bd+2Ae))\arctan\left(\frac{\cdots}{\sqrt{d^2-\cdots}}\right.}{6e^3}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/6*(Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*
x + 2*e^2*x^2)) + 6*d*(C*d^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/(Sqrt[d^2] -
Sqrt[d^2 - e^2*x^2])])/e^3
```

3.12.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2346, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{3(Cd+Be)x^2e^2+3Ade^2+\left(2Cd^2+3e(Bd+Ae)\right)xe}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3(Cd+Be)x^2e^2+3Ade^2+\left(2Cd^2+3e(Bd+Ae)\right)xe}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{2346}$$

$$\frac{-\frac{\int -\frac{e^2\left(3d\left(Cd^2+e(Bd+2Ae)\right)+2e\left(2Cd^2+3e(Bd+Ae)\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{e^2\left(3d\left(Cd^2+e(Bd+2Ae)\right)+2e\left(2Cd^2+3e(Bd+Ae)\right)x\right)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3d(Cd^2+e(Bd+2Ae))+2e(2Cd^2+3e(Bd+Ae))x}{\sqrt{d^2-e^2x^2}}dx-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{\frac{1}{2}\left(3d\left(e(2Ae+Bd)+Cd^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{e}\right)-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}\left(3d\left(e(2Ae+Bd)+Cd^2\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{e}\right)-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

$\big\downarrow$ 216

$$\frac{\frac{1}{2}\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{e}-\frac{2\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{e}\right)-\frac{3}{2}x\sqrt{d^2-e^2x^2}(Be+Cd)}{3e^2}-\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/3*(C*x^2*Sqrt[d^2 - e^2*x^2])/e + ((-3*(C*d + B*e)*x*Sqrt[d^2 - e^2*x^2
])/2 + ((-2*(2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/e + (3*d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)/2)/(3*e^2)
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(2C e^2 x^2+3xB e^2+3Cdex+6A e^2+6Bde+4C d^2)\sqrt{-e^2x^2+d^2}}{6e^3}+\frac{d(2A e^2+Bde+C d^2)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$
default	$\frac{dA\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+eC\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+(Be+Cd)\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\right.}{2e}\right.$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*C*e^2*x^2+3*B*e^2*x+3*C*d*e*x+6*A*e^2+6*B*d*e+4*C*d^2)/e^3*(-e^2*x
^2+d^2)^(1/2)+1/2*d/e^2*(2*A*e^2+B*d*e+C*d^2)/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,(Cd^3+Bd^2e+2\,Ade^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,Ce^2x^2+4\,Cd^2+6\,Bde+6\,Ae^2+3\,(Cde+Be^2}{6\,e^3}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(
e*x)) + (2*C*e^2*x^2 + 4*C*d^2 + 6*B*d*e + 6*A*e^2 + 3*(C*d*e + B*e^2)*x)*
sqrt(-e^2*x^2 + d^2))/e^3
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \sqrt{d^2-e^2x^2}\left(-\frac{Cx^2}{3e}-\frac{x(Be+Cd)}{2e^2}-\frac{Ae+Bd+\frac{2Cd^2}{3e}}{e^2}\right)+\left(Ad+\frac{d^2(Be+Cd)}{2e^2}\right)\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{othe}\end{cases}\right. \\ \frac{Adx+\frac{Cex^4}{4}+\frac{x^3(Be+Cd)}{3}+\frac{x^2(Ae+Bd)}{2}}{\sqrt{d^2}} \end{cases}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((sqrt(d**2 - e**2*x**2)*(-C*x**2/(3*e) - x*(B*e + C*d)/(2*e**2)
- (A*e + B*d + 2*C*d**2/(3*e))/e**2) + (A*d + d**2*(B*e + C*d)/(2*e**2))*P
iecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2
), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)), Ne(e**2, 0)), ((A*d*x
 + C*e*x**4/4 + x**3*(B*e + C*d)/3 + x**2*(A*e + B*d)/2)/sqrt(d**2), True)
)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cx^2}{3\,e} + \frac{Ad\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{(Cd+Be)d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{3\,e^3} - \frac{\sqrt{-e^2x^2+d^2}Bd}{e^2} - \frac{\sqrt{-e^2x^2+d^2}A}{e} - \frac{\sqrt{-e^2x^2+d^2}(Cd+Be)x}{2\,e^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*C*x^2/e + A*d*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e
^2) + 1/2*(C*d + B*e)*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 2/
3*sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 - sqrt(-e^
2*x^2 + d^2)*A/e - 1/2*sqrt(-e^2*x^2 + d^2)*(C*d + B*e)*x/e^2
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{6}\sqrt{-e^2x^2+d^2}\left(\left(\frac{2\,Cx}{e}+\frac{3\,(Cde^3+Be^4)}{e^5}\right)x+\frac{2\,(2\,Cd^2e^2+3\,Bde^3+3\,Ae^4)}{e^5}\right) + \frac{(Cd^3+Bd^2e+2\,Ade^2)\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,e^2|e|}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(-e^2*x^2 + d^2)*((2*C*x/e + 3*(C*d*e^3 + B*e^4)/e^5)*x + 2*(2*C*
d^2*e^2 + 3*B*d*e^3 + 3*A*e^4)/e^5) + 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*ar
csin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{d^2}} \\ \frac{A\,d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{A\,\sqrt{d^2-e^2\,x^2}}{e} - \frac{B\,d\,\sqrt{d^2-e^2\,x^2}}{e^2} - \frac{B\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e} - \frac{C\,\sqrt{d^2-e^2\,x^2}\,(2\,d^2+e^2\,x^2)}{3\,e^3} - \frac{C\,d^3\,\ln\left(2\,x\,\sqrt{-e\right.}}{2\,(-} \end{cases}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
piecewise(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^(1/2)), e ~=
0, - (A*(d^2 - e^2*x^2)^(1/2))/e + (A*d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^
2)^(1/2)))/(-e^2)^(1/2) - (B*d*(d^2 - e^2*x^2)^(1/2))/e^2 - (B*x*(d^2 - e^
2*x^2)^(1/2))/(2*e) - (C*(d^2 - e^2*x^2)^(1/2)*(2*d^2 + e^2*x^2))/(3*e^3)
- (C*d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2))
 - (B*d^2*e*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/
2)) - (C*d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.13.1 Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(Cd^2+2Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

output
```
1/2*(2*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-B*(-e^2*x^2+d^2)^
(1/2)/e^2-1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx=\frac{(-2B-Cx)\sqrt{d^2-e^2x^2}}{2e^2}+\frac{(-Cd^2-2Ae^2)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
((-2*B - C*x)*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((-(C*d^2) - 2*A*e^2)*ArcTan[
(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.13.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2346, 25, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{Cd^2+2Ae^2+2Be^2x}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Cd^2+2Ae^2+2Be^2x}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{(2Ae^2+Cd^2)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-2B\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2Ae^2+Cd^2)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-2B\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(2Ae^2+Cd^2)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-2B\sqrt{d^2-e^2x^2}}{2e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/2*(C*x*Sqrt[d^2 - e^2*x^2])/e^2 + (-2*B*Sqrt[d^2 - e^2*x^2] + ((C*d^2 +
 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)/(2*e^2)
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{(Cx+2B)\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{(2A\,e^2+C\,d^2)\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}$	72
default	$\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+C\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{B\sqrt{-e^2x^2+d^2}}{e^2}$	109

input
```
int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(C*x+2*B)/e^2*(-e^2*x^2+d^2)^(1/2)+1/2*(2*A*e^2+C*d^2)/e^2/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,(Cd^2+2\,Ae^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(Cex+2\,Be)}{2\,e^3}$$

input
```
integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt
(-e^2*x^2 + d^2)*(C*e*x + 2*B*e))/e^3
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \left(A+\frac{Cd^2}{2e^2}\right)\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)+\sqrt{d^2-e^2x^2}\left(-\frac{B}{e^2}-\frac{Cx}{2e^2}\right) & \text{for } e^2\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise(((A + C*d**2/(2*e**2))*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*
sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x*
*2), True)) + sqrt(d**2 - e**2*x**2)*(-B/e**2 - C*x/(2*e**2)), Ne(e**2, 0)
), ((A*x + B*x**2/2 + C*x**3/3)/sqrt(d**2), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{A\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{Cd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}Cx}{2\,e^2} - \frac{\sqrt{-e^2x^2+d^2}B}{e^2}$$

input
```
integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
A*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*C*d^2*arcsin(e^2*x/(d*sqrt(e
^2)))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*C*x/e^2 - sqrt(-e^2*x^2 +
 d^2)*B/e^2
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{2}\sqrt{-e^2x^2+d^2}\left(\frac{Cx}{e^2}+\frac{2\,B}{e^2}\right) + \frac{(Cd^2+2\,Ae^2)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,e^2|e|}$$

input
```
integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-e^2*x^2 + d^2)*(C*x/e^2 + 2*B/e^2) + 1/2*(C*d^2 + 2*A*e^2)*arcs
in(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{d^2}} & \text{if } e=0 \\ \frac{A\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{B\,\sqrt{d^2-e^2\,x^2}}{e^2} - \frac{C\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2} - \frac{C\,d^2\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} & \text{if } e\neq 0 \end{cases}$$

input
```
int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*
log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*
x^2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e
^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))
```

3.13. $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

3.14.1 Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
-(-B*e+C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-C*(-e^2*x^2+d^2)^(1/2)/e^
3-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

3.14.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{-\frac{\sqrt{d^2-e^2x^2}(e(-Bd+Ae)+Cd(2d+ex))}{d(d+ex)}+2(Cd-Be)\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(-((Sqrt[d^2 - e^2*x^2]*(e*(-(B*d) + A*e) + C*d*(2*d + e*x)))/(d*(d + e*x)
)) + 2*(C*d - B*e)*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e^3
```

3.14.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2170, 25, 27, 671, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \; 2170$$

$$-\frac{\int -\frac{e^3(Ae-(Cd-Be)x)}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{e^3(Ae-(Cd-Be)x)}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{e^4}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{Ae-(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{e}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

$$\Big\downarrow \; 671$$

$$\frac{-\frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{e}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{A}{d}+\frac{Cd-Be}{e^2}\right)}{d+ex}}{e}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

$$\Big\downarrow \; 224$$

$$\frac{-\frac{(Cd-Be)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{e}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{A}{d}+\frac{Cd-Be}{e^2}\right)}{d+ex}}{e}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

$$\Big\downarrow \; 216$$

$$\frac{-\frac{\sqrt{d^2-e^2x^2}\left(\frac{A}{d}+\frac{Cd-Be}{e^2}\right)}{d+ex}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(Cd-Be)}{e^2}}{e}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-((C*Sqrt[d^2 - e^2*x^2])/e^3) + (-(((A/d + (C*d - B*e)/e^2)*Sqrt[d^2 - e^
2*x^2])/(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^2)/
e
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{C\sqrt{-e^{2}x^{2}+d^{2}}}{e^{3}}+\frac{\frac{(Be-Cd)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{e\sqrt{e^{2}}}-\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{3}d\left(x+\frac{d}{e}\right)}}{e}$	127
default	$\frac{\frac{Be\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{C\sqrt{-e^{2}x^{2}+d^{2}}}{e}-\frac{Cd\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}}{e^{2}}-\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{4}d\left(x+\frac{d}{e}\right)}$	149

input
```
int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-C*(-e^2*x^2+d^2)^(1/2)/e^3+1/e*((B*e-C*d)/e/(e^2)^(1/2)*arctan((e^2)^(1/2
)*x/(-e^2*x^2+d^2)^(1/2))-(A*e^2-B*d*e+C*d^2)/e^3/d/(x+d/e)*(-(x+d/e)^2*e^
2+2*d*e*(x+d/e))^(1/2))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{2\,Cd^3-Bd^2e+Ade^2+(2\,Cd^2e-Bde^2+Ae^3)x-2\,(Cd^3-Bd^2e+(Cd^2e-Bde^2)x)\arctan\left(-\frac{d-\sqrt{-}}{}\right.}{de^4x+d^2e^3}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
-(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3
 - B*d^2*e + (C*d^2*e - B*d*e^2)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*
x)) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*sqrt(-e^2*x^2 + d^2))/(d*e^4*x +
 d^2*e^3)
```

3.14.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cd}{e^4x+de^3} - \frac{\sqrt{-e^2x^2+d^2}A}{de^2x+d^2e} + \frac{\sqrt{-e^2x^2+d^2}B}{e^3x+de^2} - \frac{Cd\arcsin\left(\frac{ex}{d}\right)}{e^3} + \frac{B\arcsin\left(\frac{ex}{d}\right)}{e^2} - \frac{\sqrt{-e^2x^2+d^2}C}{e^3}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
-sqrt(-e^2*x^2 + d^2)*C*d/(e^4*x + d*e^3) - sqrt(-e^2*x^2 + d^2)*A/(d*e^2*
x + d^2*e) + sqrt(-e^2*x^2 + d^2)*B/(e^3*x + d*e^2) - C*d*arcsin(e*x/d)/e^
3 + B*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2)*C/e^3
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd-Be)\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{e^2|e|} - \frac{\sqrt{-e^2x^2+d^2}C}{e^3} + \frac{2\,(Cd^2-Bde+Ae^2)}{de^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
-(C*d - B*e)*arcsin(e*x/d)*sgn(d)*sgn(e)/(e^2*abs(e)) - sqrt(-e^2*x^2 + d^
2)*C/e^3 + 2*(C*d^2 - B*d*e + A*e^2)/(d*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))/(e^2*x) + 1)*abs(e))
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

3.15.1 Optimal result

Integrand size = 34, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{3d^2e^3(d+ex)}+\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

output
```
C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d
^2)^(1/2)/d/e^3/(e*x+d)^2+(-B*e+2*C*d)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)-
1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d^2/e^3/(e*x+d)
```

3.15.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(Cd^2(4d+5ex)-e(Ae(2d+ex)+Bd(d+2ex)))}{d^2(d+ex)^2}-6C\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
 2*e*x))))/(d^2*(d + e*x)^2) - 6*C*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^
2*x^2])])/(3*e^3)
```

3.15.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2168}$$

$$\int \left(\frac{Ae^2-Bde+Cd^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{Be-2Cd}{e^2(d+ex)\sqrt{d^2-e^2x^2}}+\frac{C}{e^2\sqrt{d^2-e^2x^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^2}+\frac{C\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}+\frac{\sqrt{d^2-e^2x^2}(2Cd-Be)}{de^3(d+ex)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/3*((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)^2) + (
(2*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^
2 - e^2*x^2]])/e^3
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2168
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + b*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, b,
d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

3.15.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

method	result
default	$\frac{C\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}-\frac{(Be-2Cd)\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{e^4d\left(x+\frac{d}{e}\right)}+\frac{(Ae^2-Bde+Cd^2)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-(x+\cdots}}{3\cdots}\right.}{e^4}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/e^4*(B*e-2*
C*d)/d/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/e^4*(A*e^2-B*d*e+C*d
^2)*(-1/3/d/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/3/d^2/(x+d/
e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$$
$$=\frac{4\,Cd^4-Bd^3e-2\,Ad^2e^2+(4\,Cd^2e^2-Bde^3-2\,Ae^4)x^2+2\,(4\,Cd^3e-Bd^2e^2-2\,Ade^3)x-6\,(Cd^2e^2x^2+\cdots}{3\,(d^2e^5x^2+\cdots}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
1/3*(4*C*d^4 - B*d^3*e - 2*A*d^2*e^2 + (4*C*d^2*e^2 - B*d*e^3 - 2*A*e^4)*x
^2 + 2*(4*C*d^3*e - B*d^2*e^2 - 2*A*d*e^3)*x - 6*(C*d^2*e^2*x^2 + 2*C*d^3*
e*x + C*d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*C*d^3 - B*d^2*
e - 2*A*d*e^2 + (5*C*d^2*e - 2*B*d*e^2 - A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(
d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
```

3.15.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(151) = 302$.

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}Cd^2}{3\,(de^5x^2+2\,d^2e^4x+d^3e^3)} - \frac{\sqrt{-e^2x^2+d^2}Cd^2}{3\,(d^2e^4x+d^3e^3)}\\ &+\frac{\sqrt{-e^2x^2+d^2}Bd}{3\,(de^4x^2+2\,d^2e^3x+d^3e^2)} + \frac{\sqrt{-e^2x^2+d^2}Bd}{3\,(d^2e^3x+d^3e^2)}\\ &-\frac{\sqrt{-e^2x^2+d^2}A}{3\,(de^3x^2+2\,d^2e^2x+d^3e)} - \frac{\sqrt{-e^2x^2+d^2}A}{3\,(d^2e^2x+d^3e)}\\ &-\frac{\sqrt{-e^2x^2+d^2}B}{de^3x+d^2e^2} + \frac{2\sqrt{-e^2x^2+d^2}C}{e^4x+de^3} + \frac{C\arcsin\left(\frac{ex}{d}\right)}{e^3}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*
sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^4*x + d^3*e^3) + 1/3*sqrt(-e^2*x^2 + d^2
)*B*d/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/3*sqrt(-e^2*x^2 + d^2)*B*d/(
d^2*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d*e^3*x^2 + 2*d^2*e^2*x
 + d^3*e) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^2*x + d^3*e) - sqrt(-e^2*x^2
 + d^2)*B/(d*e^3*x + d^2*e^2) + 2*sqrt(-e^2*x^2 + d^2)*C/(e^4*x + d*e^3) +
 C*arcsin(e*x/d)/e^3
```

3.15.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 \sqrt{d^2 - e^2x^2}} \, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 1/ab
s(sageVARe)*(-((-i)*sageVARA*sageVARe^2+(-2*i)*sageVARB*sageVARd*sageVARe-
6*sageVARC*sageVARd^2*atan(i)+5*i*sageVARC*sageVARd^2)/3/sageVARd^2/sageVA
Re^2*sign((sageVARe
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 \sqrt{d^2 - e^2x^2}} \, dx = \int \frac{C\,x^2 + B\,x + A}{\sqrt{d^2 - e^2\,x^2}\,{\left(d + e\,x\right)}^2} \, dx$$

input
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2), x)
```

3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.16.1 Optimal result

Integrand size = 34, antiderivative size = 180

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} - \frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^2e^3(d+ex)^2} - \frac{(7Cd^2+e(3Bd+2Ae))\sqrt{d^2-e^2x^2}}{15d^3e^3(d+ex)}$$

output
```
-1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^3+C*(-e^2*x^2+
d^2)^(1/2)/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/
2)/d^2/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)/d
^3/e^3/(e*x+d)
```

3.16.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(Cd^2(2d^2+6dex+7e^2x^2)+e(3Bd(d^2+3dex+e^2x^2)+Ae(7d^2+6dex+2e^2x^2)))}{15d^3e^3(d+ex)^3}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d
*(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3
*(d + e*x)^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2170, 27, 671, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 2170}$$

$$\frac{\int \frac{e^2(2Cd^2+Ae^2+e(Cd+Be)x)}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{e^4}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2Cd^2+Ae^2+e(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{e^2}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{ 671}$$

$$\frac{\frac{(e(2Ae+3Bd)+7Cd^2)\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}+\frac{\sqrt{d^2-e^2x^2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{5(d+ex)^3}}{e^2}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{\frac{(e(2Ae+3Bd)+7Cd^2)\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{5(d+ex)^3}}{e^2}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(-\frac{Ae}{d}+B-\frac{Cd}{e}\right)}{5(d+ex)^3}+\frac{\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)(e(2Ae+3Bd)+7Cd^2)}{5d}}{e^2}+\frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) + (((B - (C*d)/e - (A*e)/d)*Sqrt
[d^2 - e^2*x^2])/(5*(d + e*x)^3) + ((7*C*d^2 + e*(3*B*d + 2*A*e))*(-1/3*Sq
rt[d^2 - e^2*x^2]/(d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d + e*
x))))/(5*d))/e^2
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

method	result
trager	$-\frac{\left(2A\,e^{4}x^{2}+3x^{2}dB\,e^{3}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9xB\,d^{2}e^{2}+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{3}e^{3}\left(ex+d\right)^{3}}$
gosper	$-\frac{\left(-ex+d\right)\left(2A\,e^{4}x^{2}+3x^{2}dB\,e^{3}+7C\,d^{2}e^{2}x^{2}+6Ad\,e^{3}x+9xB\,d^{2}e^{2}+6C\,d^{3}xe+7A\,d^{2}e^{2}+3B\,d^{3}e+2C\,d^{4}\right)}{15\left(ex+d\right)^{2}d^{3}e^{3}\sqrt{-e^{2}x^{2}+d^{2}}}$
default	$-\frac{C\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{4}d\left(x+\frac{d}{e}\right)}+\frac{\left(Be-2Cd\right)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^{2}e^{2}+2de\left(x+\frac{d}{e}\right)}}{3d^{2}\left(x+\frac{d}{e}\right)}\right)}{e^{4}}+\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\left(\cdots\right.}{\cdots}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d^2*e^2*x
+6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/d^3/e^3/(e*x+d)^3*(-e^2*x^2+d^
2)^(1/2)
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{2\,Cd^5+3\,Bd^4e+7\,Ad^3e^2+(2\,Cd^2e^3+3\,Bde^4+7\,Ae^5)x^3+3\,(2\,Cd^3e^2+3\,Bd^2e^3+7\,Ade^4)x^2+3\,(2\,C\cdots}{1\cdots}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/15*(2*C*d^5 + 3*B*d^4*e + 7*A*d^3*e^2 + (2*C*d^2*e^3 + 3*B*d*e^4 + 7*A*
e^5)*x^3 + 3*(2*C*d^3*e^2 + 3*B*d^2*e^3 + 7*A*d*e^4)*x^2 + 3*(2*C*d^4*e +
3*B*d^3*e^2 + 7*A*d^2*e^3)*x + (2*C*d^4 + 3*B*d^3*e + 7*A*d^2*e^2 + (7*C*d
^2*e^2 + 3*B*d*e^3 + 2*A*e^4)*x^2 + 3*(2*C*d^3*e + 3*B*d^2*e^2 + 2*A*d*e^3
)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^
6*e^3)
```

3.16.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^3}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(166) = 332$.

Time = 0.28 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.38

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cd^2}{5\,(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3)} - \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{15\,(d^2e^5x^2+2\,d^3e^4x+d^4e^3)} - \frac{2\sqrt{-e^2x^2+d^2}Cd^2}{15\,(d^3e^4x+d^4e^3)} + \frac{\sqrt{-e^2x^2+d^2}Bd}{5\,(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2)} + \frac{2\sqrt{-e^2x^2+d^2}Bd}{15\,(d^2e^4x^2+2\,d^3e^3x+d^4e^2)} + \frac{2\sqrt{-e^2x^2+d^2}Bd}{15\,(d^3e^3x+d^4e^2)} + \frac{2\sqrt{-e^2x^2+d^2}Cd}{3\,(de^5x^2+2\,d^2e^4x+d^3e^3)} + \frac{2\sqrt{-e^2x^2+d^2}Cd}{3\,(d^2e^4x+d^3e^3)} - \frac{\sqrt{-e^2x^2+d^2}A}{5\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)} - \frac{2\sqrt{-e^2x^2+d^2}A}{15\,(d^2e^3x^2+2\,d^3e^2x+d^4e)} - \frac{2\sqrt{-e^2x^2+d^2}A}{15\,(d^3e^2x+d^4e)} - \frac{\sqrt{-e^2x^2+d^2}B}{3\,(de^4x^2+2\,d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}B}{3\,(d^2e^3x+d^3e^2)} - \frac{\sqrt{-e^2x^2+d^2}C}{de^4x+d^2e^3}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x +
 d^4*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5*x^2 + 2*d^3*e^4*x + d
^4*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x + d^4*e^3) + 1/5*sqrt
(-e^2*x^2 + d^2)*B*d/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) +
 2/15*sqrt(-e^2*x^2 + d^2)*B*d/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) + 2/1
5*sqrt(-e^2*x^2 + d^2)*B*d/(d^3*e^3*x + d^4*e^2) + 2/3*sqrt(-e^2*x^2 + d^2
)*C*d/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/(
d^2*e^4*x + d^3*e^3) - 1/5*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x
^2 + 3*d^3*e^2*x + d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d
^3*e^2*x + d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 1/3*
sqrt(-e^2*x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2
*x^2 + d^2)*B/(d^2*e^3*x + d^3*e^2) - sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^
2*e^3)
```

3.16. $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{2\left(2\,Cd^2+3\,Bde+7\,Ae^2+\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x}+\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex}+\frac{20\left(de\right.}{}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
2/15*(2*C*d^2 + 3*B*d*e + 7*A*e^2 + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
*A/x + 10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 15*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^2*C*d^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d/(e^3*
x^2) + 40*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) + 15*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) + 30*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^3*A/(e^4*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*A/(e^6*
x^4))/(d^3*e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

3.16.9 Mupad [B] (verification not implemented)

Time = 12.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,C\,d^4+6\,C\,d^3\,e\,x+3\,B\,d^3\,e+7\,C\,d^2\,e^2\,x^2+9\,B\,d^2\,e^2\,x+7\,A\,d^2\,e^2+3\,B\,d\,e^3\,x^2+6\,A\,d\,e\right.}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3}$$

input
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(2*C*d^4 + 7*A*d^2*e^2 + 2*A*e^4*x^2 + 3*B*d^3*e +
 7*C*d^2*e^2*x^2 + 6*A*d*e^3*x + 6*C*d^3*e*x + 9*B*d^2*e^2*x + 3*B*d*e^3*x
^2))/(15*d^3*e^3*(d + e*x)^3)
```

3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

3.17.1 Optimal result

Integrand size = 34, antiderivative size = 234

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{70d^2e^3(d+ex)^3} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^3e^3(d+ex)^2} - \frac{(13Cd^2+8Bde+6Ae^2)\sqrt{d^2-e^2x^2}}{105d^4e^3(d+ex)}$$

output
```
-1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^4+1/2*C*(-e^2*
x^2+d^2)^(1/2)/e^3/(e*x+d)^3-1/70*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2
)^(1/2)/d^2/e^3/(e*x+d)^3-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^
(1/2)/d^3/e^3/(e*x+d)^2-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1
/2)/d^4/e^3/(e*x+d)
```

3.17.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{\sqrt{d^2-e^2x^2}(Cd^2(8d^3+32d^2ex+52de^2x^2+13e^3x^3)+e(3Ae(12d^3+13d^2ex+8de^2x^2+2e^3x^3)+Bd(}{105d^4e^3(d+ex)^4}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/105*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13
*e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d
*(13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)
```

3.17.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2170, 27, 671, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$$
$$\Big\downarrow \ 2170$$
$$\frac{\int \frac{e^2(3Cd^2+2Ae^2+e(Cd+2Be)x)}{(d+ex)^4\sqrt{d^2-e^2x^2}}dx}{2e^4}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{3Cd^2+2Ae^2+e(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$
$$\Big\downarrow \ 671$$
$$\frac{\frac{(6Ae^2+8Bde+13Cd^2)\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{7d}-\frac{2\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{7de(d+ex)^4}}{2e^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{(6Ae^2+8Bde+13Cd^2)\left(\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)}{7d}-\frac{2\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{7de(d+ex)^4}}{2e^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{(6Ae^2+8Bde+13Cd^2)\left(\frac{2\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)}{7d}-\frac{2\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{7de(d+ex)^4}}{2e^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{\left(\frac{2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)(6Ae^2+8Bde+13Cd^2)}{7d}-\frac{2\sqrt{d^2-e^2x^2}(Ae^2-Bde+Cd^2)}{7de(d+ex)^4}}{2e^2}+\frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(C*Sqrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) + ((-2*(C*d^2 - B*d*e + A*e^2)
*Sqrt[d^2 - e^2*x^2])/(7*d*e*(d + e*x)^4) + ((13*C*d^2 + 8*B*d*e + 6*A*e^2
)*(-1/5*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^3) + (2*(-1/3*Sqrt[d^2 - e^2*x^
2]/(d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d + e*x))))/(5*d)))/(
7*d))/(2*e^2)
```

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 671
```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Simp[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
 + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
 + 1, 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.62

method	result
trager	$-\frac{(6A\,e^5x^3+8x^3dB\,e^4+13C\,d^2e^3x^3+24Ad\,e^4x^2+32x^2d^2B\,e^3+52C\,d^3e^2x^2+39A\,d^2e^3x+52x\,d^3B\,e^2+32C\,d^4ex+36A\,d^3e^2+13B\,d^4}{105d^4(ex+d)^4e^3}$
gosper	$-\frac{(-ex+d)(6A\,e^5x^3+8x^3dB\,e^4+13C\,d^2e^3x^3+24Ad\,e^4x^2+32x^2d^2B\,e^3+52C\,d^3e^2x^2+39A\,d^2e^3x+52x\,d^3B\,e^2+32C\,d^4ex+36A\,d^3e^2}{105(ex+d)^3d^4e^3\sqrt{-e^2x^2+d^2}}$
default	$\frac{C\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^2}-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{3d^2\left(x+\frac{d}{e}\right)}\right)}{e^4}+\frac{(Be-2Cd)\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}{5de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(-\frac{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de}}{3de\left(x+\frac{d}{e}\right)^2}\right.}{}\right.}{e^5}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(6*A*e^5*x^3+8*B*d*e^4*x^3+13*C*d^2*e^3*x^3+24*A*d*e^4*x^2+32*B*d^2
*e^3*x^2+52*C*d^3*e^2*x^2+39*A*d^2*e^3*x+52*B*d^3*e^2*x+32*C*d^4*e*x+36*A*
d^3*e^2+13*B*d^4*e+8*C*d^5)/d^4/(e*x+d)^4/e^3*(-e^2*x^2+d^2)^(1/2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx =$$
$$-\frac{8\,Cd^6+13\,Bd^5e+36\,Ad^4e^2+(8\,Cd^2e^4+13\,Bde^5+36\,Ae^6)x^4+4\,(8\,Cd^3e^3+13\,Bd^2e^4+36\,Ade^5)x^3}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 +
36*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d
^4*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 +
36*A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8
*B*d*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 +
 (32*C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*
e^7*x^4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)
```

3.17.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(d+ex\right)^4}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(214) = 428$.

Time = 0.29 (sec) , antiderivative size = 975, normalized size of antiderivative = 4.17

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}Cd^2}{7\left(de^7x^4+4\,d^2e^6x^3+6\,d^3e^5x^2+4\,d^4e^4x+d^5e^3\right)} - \frac{3\,\sqrt{-e^2x^2+d^2}Cd^2}{35\left(d^2e^6x^3+3\,d^3e^5x^2+3\,d^4e^4x+d^5e^3\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}Cd^2}{35\left(d^3e^5x^2+2\,d^4e^4x+d^5e^3\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}Cd^2}{35\left(d^4e^4x+d^5e^3\right)} + \frac{\sqrt{-e^2x^2+d^2}Bd}{7\left(de^6x^4+4\,d^2e^5x^3+6\,d^3e^4x^2+4\,d^4e^3x+d^5e^2\right)} + \frac{3\,\sqrt{-e^2x^2+d^2}Bd}{35\left(d^2e^5x^3+3\,d^3e^4x^2+3\,d^4e^3x+d^5e^2\right)} + \frac{2\,\sqrt{-e^2x^2+d^2}Bd}{35\left(d^3e^4x^2+2\,d^4e^3x+d^5e^2\right)} + \frac{2\,\sqrt{-e^2x^2+d^2}Bd}{35\left(d^4e^3x+d^5e^2\right)} + \frac{2\,\sqrt{-e^2x^2+d^2}Cd}{5\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)} + \frac{4\,\sqrt{-e^2x^2+d^2}Cd}{15\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)} + \frac{4\,\sqrt{-e^2x^2+d^2}Cd}{15\left(d^3e^4x+d^4e^3\right)} - \frac{\sqrt{-e^2x^2+d^2}A}{7\left(de^5x^4+4\,d^2e^4x^3+6\,d^3e^3x^2+4\,d^4e^2x+d^5e\right)} - \frac{3\,\sqrt{-e^2x^2+d^2}A}{35\left(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}A}{35\left(d^3e^3x^2+2\,d^4e^2x+d^5e\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}A}{35\left(d^4e^2x+d^5e\right)} - \frac{\sqrt{-e^2x^2+d^2}B}{5\left(de^5x^3+3\,d^2e^4x^2+3\,d^3e^3x+d^4e^2\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}B}{15\left(d^2e^4x^2+2\,d^3e^3x+d^4e^2\right)} - \frac{2\,\sqrt{-e^2x^2+d^2}B}{15\left(d^3e^3x+d^4e^2\right)} - \frac{\sqrt{-e^2x^2+d^2}C}{3\left(de^5x^2+2\,d^2e^4x+d^3e^3\right)} - \frac{\sqrt{-e^2x^2+d^2}C}{3\left(d^2e^4x+d^3e^3\right)}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2
 + 4*d^4*e^4*x + d^5*e^3) - 3/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 +
 3*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/
(d^3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d
^4*e^4*x + d^5*e^3) + 1/7*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e^5*
x^3 + 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) + 3/35*sqrt(-e^2*x^2 + d^2)*B
*d/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*
x^2 + d^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2
+ d^2)*B*d/(d^4*e^3*x + d^5*e^2) + 2/5*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^6*x^3
 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/
(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^3
*e^4*x + d^4*e^3) - 1/7*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3
+ 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d^5*e) - 3/35*sqrt(-e^2*x^2 + d^2)*A/(d^2*
e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)
*A/(d^3*e^3*x^2 + 2*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^4*
e^2*x + d^5*e) - 1/5*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3
*d^3*e^3*x + d^4*e^2) - 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e
^3*x + d^4*e^2) - 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 1/3*
sqrt(-e^2*x^2 + d^2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*sqrt(-e^2
*x^2 + d^2)*C/(d^2*e^4*x + d^3*e^3)
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(214) = 428$.

Time = 0.29 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.35

$$\int \frac{A + Bx + Cx^2}{(d + ex)^4\sqrt{d^2 - e^2x^2}}\, dx$$
$$= \frac{2\left(8\,Cd^2 + 13\,Bde + 36\,Ae^2 + \frac{147\left(de+\sqrt{-e^2x^2+d^2}|e|\right)A}{x} + \frac{56\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Cd^2}{e^2x} + \frac{91\left(de+\sqrt{-e^2x^2+d^2}|e|\right)Bd}{ex} + \frac{16}{}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac"
)
```

output
```
2/105*(8*C*d^2 + 13*B*d*e + 36*A*e^2 + 147*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))*A/x + 56*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*C*d^2/(e^2*x) + 91*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))*B*d/(e*x) + 168*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^2*C*d^2/(e^4*x^2) + 168*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*B*d
/(e^3*x^2) + 441*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*A/(e^2*x^2) + 140*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*C*d^2/(e^6*x^3) + 280*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^3*B*d/(e^5*x^3) + 630*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*A/(e^4*x^3) + 140*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*C*d^2/(e^8*
x^4) + 175*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*B*d/(e^7*x^4) + 630*(d*e
+ sqrt(-e^2*x^2 + d^2)*abs(e))^4*A/(e^6*x^4) + 105*(d*e + sqrt(-e^2*x^2 +
d^2)*abs(e))^5*B*d/(e^9*x^5) + 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*A
/(e^8*x^5) + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*A/(e^10*x^6))/(d^4*
e^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^7*abs(e))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{C}{5\,e^3}-\frac{-4\,C\,d^2+4\,B\,d\,e+3\,A\,e^2}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{A}{7\,d\,e}+\frac{d\,\left(\frac{C}{7\,e^2}-\frac{B}{7\,d\,e}\right)}{e}\right)}{{\left(d+e\,x\right)}^4} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^3\,e^3\,{\left(d+e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e^2\right)}{105\,d^4\,e^3\,\left(d+e\,x\right)}$$

input
```
int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(C/(5*e^3) - (3*A*e^2 - 4*C*d^2 + 4*B*d*e)/(35*d^2*
e^3)))/(d + e*x)^3 - ((d^2 - e^2*x^2)^(1/2)*(A/(7*d*e) + (d*(C/(7*e^2) - B
/(7*d*e)))/e))/(d + e*x)^4 - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d^2 +
8*B*d*e))/(105*d^3*e^3*(d + e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13
*C*d^2 + 8*B*d*e))/(105*d^4*e^3*(d + e*x))
```

3.18 $\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2) \, dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2) \, dx = \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^4}{4e^5} - \frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))(d+ex)^5}{5e^5} + \frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))(d+ex)^6}{6e^5} - \frac{c(4Cd-Be)(d+ex)^7}{7e^5} + \frac{cC(d+ex)^8}{8e^5}$$

output
```
1/4*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^4/e^5-1/5*(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^5/e^5+1/6*(a*C*e^2+c*(6*C*d^2-e*
(-A*e+3*B*d)))*(e*x+d)^6/e^5-1/7*c*(-B*e+4*C*d)*(e*x+d)^7/e^5+1/8*c*C*(e*x
+d)^8/e^5
```

3.18.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx$$
$$= aAd^3x + \frac{1}{2}ad^2(Bd+3Ae)x^2 + \frac{1}{3}d\left(ad(Cd+3Be)+A\left(cd^2+3ae^2\right)\right)x^3 + \frac{1}{4}\left(Bcd^3+3Acd^2e+3aCd^2e+3aBde^2+aAe^3\right)x^4 + \frac{1}{5}\left(cCd^3+3cde(Bd+Ae)+ae^2(3Cd+Be)\right)x^5 + \frac{1}{6}e\left(3cCd^2+aCe^2+ce(3Bd+Ae)\right)x^6 + \frac{1}{7}ce^2(3Cd+Be)x^7 + \frac{1}{8}cCe^3x^8$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
 + 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2
+ a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))
*x^5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*
C*d + B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

3.18.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{(d+ex)^4\left(-ae^2(2Cd-Be)+cde(3Bd-2Ae)-4cCd^3\right)}{e^4}+\frac{(d+ex)^5\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{e^4}+\frac{(\right.$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{(d+ex)^5\left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{5e^5}+\frac{(d+ex)^6\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{6e^5}+\frac{(d+ex)^4\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{4e^5}-\frac{c(d+ex)^7(4Cd-Be)}{7e^5}+\frac{cC(d+ex)^8}{8e^5}
$$

input
```
Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^
3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((
6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d
- B*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.18.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.22

method	result
norman	$\frac{c e^3 C x^8}{8}+\left(\frac{1}{7}Bce^3+\frac{3}{7}cde^2C\right)x^7+\left(\frac{1}{6}Ace^3+\frac{1}{2}Bcde^2+\frac{1}{6}Cae^3+\frac{1}{2}Ccd^2e\right)x^6+\big(\frac{3}{5}Acde^2+$
default	$\frac{c e^3 C x^8}{8}+\frac{(Bce^3+3cde^2C)x^7}{7}+\frac{((ae^3+3cd^2e)C+3Bcde^2+Ace^3)x^6}{6}+\frac{((3ade^2+cd^3)C+(ae^3+3cd^2e)B+3Acde^2)x}{5}$
gosper	$\frac{1}{8}ce^3Cx^8+\frac{1}{7}Be^3cx^7+\frac{3}{7}x^7cde^2C+\frac{1}{6}x^6Ace^3+\frac{1}{2}x^6Bcde^2+\frac{1}{6}x^6Cae^3+\frac{1}{2}x^6Ccd^2e+\frac{3}{5}x^5$
risch	$\frac{1}{8}ce^3Cx^8+\frac{1}{7}Be^3cx^7+\frac{3}{7}x^7cde^2C+\frac{1}{6}x^6Ace^3+\frac{1}{2}x^6Bcde^2+\frac{1}{6}x^6Cae^3+\frac{1}{2}x^6Ccd^2e+\frac{3}{5}x^5$
parallelrisch	$\frac{1}{8}ce^3Cx^8+\frac{1}{7}Be^3cx^7+\frac{3}{7}x^7cde^2C+\frac{1}{6}x^6Ace^3+\frac{1}{2}x^6Bcde^2+\frac{1}{6}x^6Cae^3+\frac{1}{2}x^6Ccd^2e+\frac{3}{5}x^5$

input
```
int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*e^3*C*x^8+(1/7*B*c*e^3+3/7*c*d*e^2*C)*x^7+(1/6*A*c*e^3+1/2*B*c*d*e^2
+1/6*C*a*e^3+1/2*C*c*d^2*e)*x^6+(3/5*A*c*d*e^2+1/5*B*e^3*a+3/5*B*c*d^2*e+3
/5*C*a*d*e^2+1/5*C*c*d^3)*x^5+(1/4*A*a*e^3+3/4*A*c*d^2*e+3/4*B*a*d*e^2+1/4
*B*c*d^3+3/4*a*d^2*e*C)*x^4+(A*a*d*e^2+1/3*A*c*d^3+B*a*d^2*e+1/3*a*d^3*C)*
x^3+(3/2*a*A*d^2*e+1/2*B*a*d^3)*x^2+A*d^3*a*x
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx \\
&= \frac{1}{8} Cce^3x^8 + \frac{1}{7} \left(3\,Ccde^2 + Bce^3\right)x^7 + \frac{1}{6} \left(3\,Ccd^2e + 3\,Bcde^2 + (Ca+Ac)e^3\right)x^6 \\
&\quad + Aad^3x + \frac{1}{5} \left(Ccd^3 + 3\,Bcd^2e + Bae^3 + 3\,(Ca+Ac)de^2\right)x^5 \\
&\quad + \frac{1}{4} \left(Bcd^3 + 3\,Bade^2 + Aae^3 + 3\,(Ca+Ac)d^2e\right)x^4 \\
&\quad + \frac{1}{3} \left(3\,Bad^2e + 3\,Aade^2 + (Ca+Ac)d^3\right)x^3 + \frac{1}{2} \left(Bad^3 + 3\,Aad^2e\right)x^2
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B
*c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e +
 B*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3
 + 3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)
*d^3)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.47

$$\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\, dx = Aad^3x + \frac{Cce^3x^8}{8} + x^7\left(\frac{Bce^3}{7} + \frac{3Ccde^2}{7}\right) + x^6\left(\frac{Ace^3}{6} + \frac{Bcde^2}{2} + \frac{Cae^3}{6} + \frac{Ccd^2e}{2}\right) + x^5 \cdot \left(\frac{3Acde^2}{5} + \frac{Bae^3}{5} + \frac{3Bcd^2e}{5} + \frac{3Cade^2}{5} + \frac{Ccd^3}{5}\right) + x^4\left(\frac{Aae^3}{4} + \frac{3Acd^2e}{4} + \frac{3Bade^2}{4} + \frac{Bcd^3}{4} + \frac{3Cad^2e}{4}\right) + x^3\left(Aade^2 + \frac{Acd^3}{3} + Bad^2e + \frac{Cad^3}{3}\right) + x^2 \cdot \left(\frac{3Aad^2e}{2} + \frac{Bad^3}{2}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

output
```
A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(
A*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**
2/5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A
*a*e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4)
 + x**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*
d**2*e/2 + B*a*d**3/2)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2)\, dx = \frac{1}{8}Cce^3x^8 + \frac{1}{7}(3Ccde^2 + Bce^3)x^7 + \frac{1}{6}(3Ccd^2e + 3Bcde^2 + (Ca+Ac)e^3)x^6 + Aad^3x + \frac{1}{5}(Ccd^3 + 3Bcd^2e + Bae^3 + 3(Ca+Ac)de^2)x^5 + \frac{1}{4}(Bcd^3 + 3Bade^2 + Aae^3 + 3(Ca+Ac)d^2e)x^4 + \frac{1}{3}(3Bad^2e + 3Aade^2 + (Ca+Ac)d^3)x^3 + \frac{1}{2}(Bad^3 + 3Aad^2e)x^2$$

input
```
integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B
*c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e +
 B*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3
 + 3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)
*d^3)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (d+ex)^3 (a+cx^2) (A+Bx+Cx^2) \, dx = {} & \frac{1}{8}Cce^3x^8 + \frac{3}{7}Ccde^2x^7 + \frac{1}{7}Bce^3x^7 + \frac{1}{2}Ccd^2ex^6 \\ & + \frac{1}{2}Bcde^2x^6 + \frac{1}{6}Cae^3x^6 + \frac{1}{6}Ace^3x^6 \\ & + \frac{1}{5}Ccd^3x^5 + \frac{3}{5}Bcd^2ex^5 + \frac{3}{5}Cade^2x^5 \\ & + \frac{3}{5}Acde^2x^5 + \frac{1}{5}Bae^3x^5 + \frac{1}{4}Bcd^3x^4 \\ & + \frac{3}{4}Cad^2ex^4 + \frac{3}{4}Acd^2ex^4 + \frac{3}{4}Bade^2x^4 \\ & + \frac{1}{4}Aae^3x^4 + \frac{1}{3}Cad^3x^3 + \frac{1}{3}Acd^3x^3 + Bad^2ex^3 \\ & + Aade^2x^3 + \frac{1}{2}Bad^3x^2 + \frac{3}{2}Aad^2ex^2 + Aad^3x\end{aligned}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*C*c*e^3*x^8 + 3/7*C*c*d*e^2*x^7 + 1/7*B*c*e^3*x^7 + 1/2*C*c*d^2*e*x^6
+ 1/2*B*c*d*e^2*x^6 + 1/6*C*a*e^3*x^6 + 1/6*A*c*e^3*x^6 + 1/5*C*c*d^3*x^5
+ 3/5*B*c*d^2*e*x^5 + 3/5*C*a*d*e^2*x^5 + 3/5*A*c*d*e^2*x^5 + 1/5*B*a*e^3*
x^5 + 1/4*B*c*d^3*x^4 + 3/4*C*a*d^2*e*x^4 + 3/4*A*c*d^2*e*x^4 + 3/4*B*a*d*
e^2*x^4 + 1/4*A*a*e^3*x^4 + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^3 + B*a*d^2*e*
x^3 + A*a*d*e^2*x^3 + 1/2*B*a*d^3*x^2 + 3/2*A*a*d^2*e*x^2 + A*a*d^3*x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx = x^3 \left(\frac{A\,c\,d^3}{3}+\frac{C\,a\,d^3}{3}+A\,a\,d\,e^2+B\,a\,d^2\,e\right)+x^6 \left(\frac{A\,c\,e^3}{6}+\frac{C\,a\,e^3}{6}+\frac{B\,c\,d\,e^2}{2}+\frac{C\,c\,d^2\,e}{2}\right)+x^4 \left(\frac{A\,a\,e^3}{4}+\frac{B\,c\,d^3}{4}+\frac{3\,B\,a\,d\,e^2}{4}+\frac{3\,A\,c\,d^2\,e}{4}+\frac{3\,C\,a\,d^2\,e}{4}\right)+x^5 \left(\frac{B\,a\,e^3}{5}+\frac{C\,c\,d^3}{5}+\frac{3\,A\,c\,d\,e^2}{5}+\frac{3\,C\,a\,d\,e^2}{5}+\frac{3\,B\,c\,d^2\,e}{5}\right)+A\,a\,d^3\,x+\frac{C\,c\,e^3\,x^8}{8}+\frac{a\,d^2\,x^2\,(3\,A\,e+B\,d)}{2}+\frac{c\,e^2\,x^7\,(B\,e+3\,C\,d)}{7}$$

input
```
int((a + c*x^2)*(d + e*x)^3*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6
 + (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*
d^3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*
e^3)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5
) + A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7
*(B*e + 3*C*d))/7
```

3.19 $\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) \, dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) \, dx &= \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)(d+ex)^3}{3e^5} \\ &\quad - \frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))(d+ex)^4}{4e^5} \\ &\quad + \frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))(d+ex)^5}{5e^5} - \frac{c(4Cd-Be)(d+ex)^6}{6e^5} + \frac{cC(d+ex)^7}{7e^5}\end{aligned}$$

output
```
1/3*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^3/e^5-1/4*(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^4/e^5+1/5*(a*C*e^2+c*(6*C*d^2-e*
(-A*e+3*B*d)))*(e*x+d)^5/e^5-1/6*c*(-B*e+4*C*d)*(e*x+d)^6/e^5+1/7*c*C*(e*x
+d)^7/e^5
```

3.19.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86

$$\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx = aAd^2x+\frac{1}{2}ad(Bd+2Ae)x^2 + \frac{1}{3}\left(Acd^2+aCd^2+2aBde+aAe^2\right)x^3 + \frac{1}{4}\left(Bcd^2+2Acde+2aCde+aBe^2\right)x^4 + \frac{1}{5}\left(cCd^2+2Bcde+Ace^2+aCe^2\right)x^5 + \frac{1}{6}ce(2Cd+Be)x^6+\frac{1}{7}cCe^2x^7$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e +
a*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((
c*C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/
6 + (c*C*e^2*x^7)/7
```

3.19.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)(d+ex)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{(d+ex)^3\left(-ae^2(2Cd-Be)+cde(3Bd-2Ae)-4cCd^3\right)}{e^4}+\frac{(d+ex)^4\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{e^4}+\frac{(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d+ex)^4\left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{4e^5}+\frac{(d+ex)^5\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{5e^5}+\frac{(d+ex)^3\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{3e^5}-\frac{c(d+ex)^6(4Cd-Be)}{6e^5}+\frac{cC(d+ex)^7}{7e^5}$$

input
```
Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^
3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((
6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d
- B*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.19.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

method	result
default	$\frac{c e^2 C x^7}{7}+\frac{(c e^2 B+2cdeC)x^6}{6}+\frac{((e^2a+c d^2)C+2Bcde+Ac e^2)x^5}{5}+\frac{(2adeC+(e^2a+c d^2)B+2Acde)x^4}{4}+\frac{(a d^2C+2Bc$
norman	$\frac{c e^2 C x^7}{7}+\left(\frac{1}{6}c e^2 B+\frac{1}{3}cdeC\right)x^6+\left(\frac{1}{5}Ac e^2+\frac{2}{5}Bcde+\frac{1}{5}aC e^2+\frac{1}{5}Cc d^2\right)x^5+\left(\frac{1}{2}Acde+\frac{1}{4}Bc\right.$
gosper	$\frac{1}{7}c e^2 C x^7+\frac{1}{6}Bc e^2 x^6+\frac{1}{3}x^6 cdeC+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{5}x^5 aC e^2+\frac{1}{5}x^5 Cc d^2+\frac{1}{2}x^4 Acd$
risch	$\frac{1}{7}c e^2 C x^7+\frac{1}{6}Bc e^2 x^6+\frac{1}{3}x^6 cdeC+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{5}x^5 aC e^2+\frac{1}{5}x^5 Cc d^2+\frac{1}{2}x^4 Acd$
parallelrisch	$\frac{1}{7}c e^2 C x^7+\frac{1}{6}Bc e^2 x^6+\frac{1}{3}x^6 cdeC+\frac{1}{5}x^5 Ac e^2+\frac{2}{5}x^5 Bcde+\frac{1}{5}x^5 aC e^2+\frac{1}{5}x^5 Cc d^2+\frac{1}{2}x^4 Acd$

input
```
int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*B*c*d*e
+A*c*e^2)*x^5+1/4*(2*a*d*e*C+(a*e^2+c*d^2)*B+2*A*c*d*e)*x^4+1/3*(a*d^2*C+2
*B*a*d*e+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+A*d^2*a*x
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\frac{1}{7}Cce^2x^7+\frac{1}{6}\left(2\,Ccde+Bce^2\right)x^6+\frac{1}{5}\left(Ccd^2+2\,Bcde+(Ca+Ac)e^2\right)x^5+Aad^2x+\frac{1}{4}\left(Bcd^2+Bae^2+2\,(Ca+Ac)de\right)x^4+\frac{1}{3}\left(2\,Bade+Aae^2+(Ca+Ac)d^2\right)x^3+\frac{1}{2}\left(Bad^2+2\,Aade\right)x^2$$

input
```
integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
 + (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*
c)*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d
^2 + 2*A*a*d*e)*x^2
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\int (d+ex)^2\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=Aad^2x+\frac{Cce^2x^7}{7}+x^6\left(\frac{Bce^2}{6}+\frac{Ccde}{3}\right)+x^5\left(\frac{Ace^2}{5}+\frac{2Bcde}{5}+\frac{Cae^2}{5}+\frac{Ccd^2}{5}\right)+x^4\left(\frac{Acde}{2}+\frac{Bae^2}{4}+\frac{Bcd^2}{4}+\frac{Cade}{2}\right)+x^3\left(\frac{Aae^2}{3}+\frac{Acd^2}{3}+\frac{2Bade}{3}+\frac{Cad^2}{3}\right)+x^2\left(Aade+\frac{Bad^2}{2}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

output
```
A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e
**2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**
2/4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/
3 + C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \,dx = \frac{1}{7} Cce^2x^7 + \frac{1}{6} \left(2\,Ccde + Bce^2\right)x^6 + \frac{1}{5} \left(Ccd^2 + 2\,Bcde + (Ca+Ac)e^2\right)x^5 + Aad^2x + \frac{1}{4} \left(Bcd^2 + Bae^2 + 2\,(Ca+Ac)de\right)x^4 + \frac{1}{3} \left(2\,Bade + Aae^2 + (Ca+Ac)d^2\right)x^3 + \frac{1}{2} \left(Bad^2 + 2\,Aade\right)x^2$$

input
```
integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
 + (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*
c)*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d
^2 + 2*A*a*d*e)*x^2
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2)\, dx = \frac{1}{7}Cce^2x^7 + \frac{1}{3}Ccdex^6 + \frac{1}{6}Bce^2x^6 + \frac{1}{5}Ccd^2x^5 + \frac{2}{5}Bcdex^5 + \frac{1}{5}Cae^2x^5 + \frac{1}{5}Ace^2x^5 + \frac{1}{4}Bcd^2x^4 + \frac{1}{2}Cadex^4 + \frac{1}{2}Acdex^4 + \frac{1}{4}Bae^2x^4 + \frac{1}{3}Cad^2x^3 + \frac{1}{3}Acd^2x^3 + \frac{2}{3}Badex^3 + \frac{1}{3}Aae^2x^3 + \frac{1}{2}Bad^2x^2 + Aadex^2 + Aad^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*C*c*e^2*x^7 + 1/3*C*c*d*e*x^6 + 1/6*B*c*e^2*x^6 + 1/5*C*c*d^2*x^5 + 2/
5*B*c*d*e*x^5 + 1/5*C*a*e^2*x^5 + 1/5*A*c*e^2*x^5 + 1/4*B*c*d^2*x^4 + 1/2*
C*a*d*e*x^4 + 1/2*A*c*d*e*x^4 + 1/4*B*a*e^2*x^4 + 1/3*C*a*d^2*x^3 + 1/3*A*
c*d^2*x^3 + 2/3*B*a*d*e*x^3 + 1/3*A*a*e^2*x^3 + 1/2*B*a*d^2*x^2 + A*a*d*e*
x^2 + A*a*d^2*x
```

3.19.9 Mupad [B] (verification not implemented)

Time = 12.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2)\, dx = x^3 \left(\frac{A\,a\,e^2}{3} + \frac{A\,c\,d^2}{3} + \frac{C\,a\,d^2}{3} + \frac{2\,B\,a\,d\,e}{3}\right) + x^5 \left(\frac{A\,c\,e^2}{5} + \frac{C\,a\,e^2}{5} + \frac{C\,c\,d^2}{5} + \frac{2\,B\,c\,d\,e}{5}\right) + x^4 \left(\frac{B\,a\,e^2}{4} + \frac{B\,c\,d^2}{4} + \frac{A\,c\,d\,e}{2} + \frac{C\,a\,d\,e}{2}\right) + A\,a\,d^2\,x + \frac{a\,d\,x^2\,(2\,A\,e + B\,d)}{2} + \frac{c\,e\,x^6\,(B\,e + 2\,C\,d)}{6} + \frac{C\,c\,e^2\,x^7}{7}$$

input
```
int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*
e^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (
B*c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*
d))/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7
```

3.19. $\int (d+ex)^2 (a+cx^2) (A+Bx+Cx^2) \, dx$

3.20 $\int (d+ex)(a+cx^2)(A+Bx+Cx^2)\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+aCe)x^4+\frac{1}{5}c(Cd+Be)x^5+\frac{1}{6}cCex^6$$

output
```
a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e+C*a*d)*x^3+1/4*(A*c*e+B*c*d+C
*a*e)*x^4+1/5*c*(B*e+C*d)*x^5+1/6*c*C*e*x^6
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+aCe)x^4+\frac{1}{5}c(Cd+Be)x^5+\frac{1}{6}cCex^6$$

input
```
Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*
d + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

3.20.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)(d+ex)(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(x^3(aCe+Ace+Bcd)+x^2(aBe+aCd+Acd)+ax(Ae+Bd)+aAd+cx^4(Be+Cd)+cCex^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

input
```
Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*
d + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.20.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result
default	$\frac{cCe\,x^6}{6}+\frac{(Bce+cdC)x^5}{5}+\frac{(Ace+Bcd+Cae)x^4}{4}+\frac{(Acd+Bae+Cad)x^3}{3}+\frac{(aAe+Bad)x^2}{2}+aAdx$
norman	$\frac{cCe\,x^6}{6}+\left(\frac{1}{5}Bce+\frac{1}{5}cdC\right)x^5+\left(\frac{1}{4}Ace+\frac{1}{4}Bcd+\frac{1}{4}Cae\right)x^4+\left(\frac{1}{3}Acd+\frac{1}{3}Bae+\frac{1}{3}Cad\right)x^3+\big($
gosper	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3C$
risch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3C$
parallelrisch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}Bce\,x^5+\frac{1}{5}x^5cdC+\frac{1}{4}x^4Ace+\frac{1}{4}x^4Bcd+\frac{1}{4}x^4Cae+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bae+\frac{1}{3}x^3C$

input
```
int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d
+B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\frac{1}{6}Ccex^6+\frac{1}{5}(Ccd+Bce)x^5+\frac{1}{4}(Bcd+(Ca+Ac)e)x^4+Aadx+\frac{1}{3}(Bae+(Ca+Ac)d)x^3+\frac{1}{2}(Bad+Aae)x^2$$

input
```
integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4
+ A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = Aadx + \frac{Ccex^6}{6} + x^5\left(\frac{Bce}{5}+\frac{Ccd}{5}\right) + x^4\left(\frac{Ace}{4}+\frac{Bcd}{4}+\frac{Cae}{4}\right) + x^3\left(\frac{Acd}{3}+\frac{Bae}{3}+\frac{Cad}{3}\right) + x^2\left(\frac{Aae}{2}+\frac{Bad}{2}\right)$$

input
```
integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

output
```
A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/
4 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/
2)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx = \frac{1}{6}Ccex^6 + \frac{1}{5}(Ccd+Bce)x^5 + \frac{1}{4}(Bcd+(Ca+Ac)e)x^4 + Aadx + \frac{1}{3}(Bae+(Ca+Ac)d)x^3 + \frac{1}{2}(Bad+Aae)x^2$$

input
```
integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4
+ A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int (d+ex) \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx = \frac{1}{6} Ccex^6 + \frac{1}{5} Ccdx^5 + \frac{1}{5} Bcex^5 + \frac{1}{4} Bcdx^4 + \frac{1}{4} Caex^4 + \frac{1}{4} Acex^4 + \frac{1}{3} Cadx^3 + \frac{1}{3} Acdx^3 + \frac{1}{3} Baex^3 + \frac{1}{2} Badx^2 + \frac{1}{2} Aaex^2 + Aadx$$

input
```
integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*C*c*e*x^6 + 1/5*C*c*d*x^5 + 1/5*B*c*e*x^5 + 1/4*B*c*d*x^4 + 1/4*C*a*e*
x^4 + 1/4*A*c*e*x^4 + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*e*x^3 + 1/2*
B*a*d*x^2 + 1/2*A*a*e*x^2 + A*a*d*x
```

3.20.9 Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int (d+ex) \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx = \frac{C\,c\,e\,x^6}{6} + \frac{c\,(B\,e+C\,d)\,x^5}{5} + \left(\frac{A\,c\,e}{4} + \frac{B\,c\,d}{4} + \frac{C\,a\,e}{4}\right) x^4 + \left(\frac{A\,c\,d}{3} + \frac{B\,a\,e}{3} + \frac{C\,a\,d}{3}\right) x^3 + \frac{a\,(A\,e+B\,d)\,x^2}{2} + A\,a\,d\,x$$

input
```
int((a + c*x^2)*(d + e*x)*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*c*d)/3 + (B*a*e)/3 + (C*a*d)/3) + x^4*((A*c*e)/4 + (B*c*d)/4 + (C*
a*e)/4) + (a*x^2*(A*e + B*d))/2 + (c*x^5*(B*e + C*d))/5 + (C*c*e*x^6)/6 +
A*a*d*x
```

3.21 $\int (a+cx^2)(A+Bx+Cx^2)\,dx$

3.21.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

input
```
Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

3.21.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^2(aC+Ac)+aA+aBx+Bcx^3+cCx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

input
```
Int[(a + c*x^2)*(A + B*x + C*x^2),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.21.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^2}{2}+\frac{(Ac+Ca)x^3}{3}+\frac{Bc\,x^4}{4}+\frac{cC\,x^5}{5}$	39
norman	$\frac{cC\,x^5}{5}+\frac{Bc\,x^4}{4}+\left(\frac{Ac}{3}+\frac{Ca}{3}\right)x^3+\frac{Ba\,x^2}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41
risch	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41
parallelrisch	$\frac{1}{5}cC\,x^5+\frac{1}{4}Bc\,x^4+\frac{1}{3}Ac\,x^3+\frac{1}{3}x^3Ca+\frac{1}{2}Ba\,x^2+aAx$	41

input
```
int((c*x^2+a)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx=\frac{1}{5}Ccx^5+\frac{1}{4}Bcx^4+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ac)x^3+Aax$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx=Aax+\frac{Bax^2}{2}+\frac{Bcx^4}{4}+\frac{Ccx^5}{5}+x^3\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx=\frac{1}{5}Ccx^5+\frac{1}{4}Bcx^4+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ac)x^3+Aax$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx=\frac{1}{5}Ccx^5+\frac{1}{4}Bcx^4+\frac{1}{3}Cax^3+\frac{1}{3}Acx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*
x
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int (a+cx^2)(A+Bx+Cx^2)\,dx=\frac{C\,c\,x^5}{5}+\frac{B\,c\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)x^3+\frac{B\,a\,x^2}{2}+A\,a\,x$$

input
```
int((a + c*x^2)*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*c)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5
```

3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = -\frac{(ae^2(Cd-Be)+cd(Cd^2-e(Bd-Ae)))\,x}{e^4} + \frac{(aCe^2+c(Cd^2-e(Bd-Ae)))\,x^2}{2e^3} - \frac{c(Cd-Be)x^3}{3e^2} + \frac{cCx^4}{4e} + \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)\log(d+ex)}{e^5}$$

output
```
-(a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))*x/e^4+1/2*(a*C*e^2+c*(C*d^2-e
*(-A*e+B*d)))*x^2/e^3-1/3*c*(-B*e+C*d)*x^3/e^2+1/4*c*C*x^4/e+(a*e^2+c*d^2)
*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^5
```

3.22.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{ex(6ae^2(-2Cd+2Be+Cex)+cC(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2ce(3Ae(-2d+ex)+B(6d^2-}{12e^5}$$

input
```
Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
(e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^
2*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^
2*x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12
*e^5)
```

3.22.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-ae^2(Cd-Be)-c(Cd^3-de(Bd-Ae))}{e^4}+\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^4(d+ex)}+\frac{x(aCe^2-ce(Bd-Ae)+cC\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3)}{e^4}+\frac{(ae^2+cd^2)\log(d+ex)(Ae^2-Bde+Cd^2)}{e^5}+\frac{x^2(aCe^2-ce(Bd-Ae)+cCd^2)}{2e^3}-\frac{cx^3(Cd-Be)}{3e^2}+\frac{cCx^4}{4e}$$

input
```
Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
-(((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
 a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (
c*C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^
5
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.22.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

method	result
norman	$\frac{(Ac\,e^2-Bcde+aC\,e^2+Cc\,d^2)x^2}{2e^3}-\frac{(Acd\,e^2-B\,e^3a-Bc\,d^2e+Cad\,e^2+Cc\,d^3)x}{e^4}+\frac{cC\,x^4}{4e}+\frac{c(Be-Cd)x^3}{3e^2}+\frac{(Aa\,e^4+Ac\,d^2 \cdots}{}$
default	$-\frac{-\frac{1}{4}cC\,x^4e^3-\frac{1}{3}Bc\,x^3e^3+\frac{1}{3}Ccd\,e^2x^3-\frac{1}{2}Ac\,e^3x^2+\frac{1}{2}B\,x^2cd\,e^2-\frac{1}{2}Ca\,e^3x^2-\frac{1}{2}Cc\,d^2e\,x^2+Acd\,e^2x-Bxa\,e^3-Bc\,d^2ex+Cad\,e^2 \cdots}{e^4}$
risch	$\frac{cC\,x^4}{4e}+\frac{Bc\,x^3}{3e}-\frac{Ccd\,x^3}{3e^2}+\frac{Ac\,x^2}{2e}-\frac{B\,x^2cd}{2e^2}+\frac{Ca\,x^2}{2e}+\frac{Cc\,d^2x^2}{2e^3}-\frac{Acdx}{e^2}+\frac{Bxa}{e}+\frac{Bc\,d^2x}{e^3}-\frac{Cadx}{e^2}-\frac{Cc\,d^3x}{e^4} \cdots$
parallelrisch	$\frac{3cC\,x^4e^4+4B\,x^3c\,e^4-4C\,x^3cd\,e^3+6A\,x^2c\,e^4-6B\,x^2cd\,e^3+6C\,x^2a\,e^4+6C\,x^2c\,d^2e^2+12A\ln(ex+d)a\,e^4+12A\ln(ex+d)c\,d^2e^2- \cdots}{}$

input
```
int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^3*(A*c*e^2-B*c*d*e+C*a*e^2+C*c*d^2)*x^2-(A*c*d*e^2-B*a*e^3-B*c*d^2*e
+C*a*d*e^2+C*c*d^3)/e^4*x+1/4*c*C*x^4/e+1/3*c/e^2*(B*e-C*d)*x^3+(A*a*e^4+A
*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)/e^5*ln(e*x+d)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$$
$$=\frac{3\,Cce^4x^4-4\,(Ccde^3-Bce^4)x^3+6\,(Ccd^2e^2-Bcde^3+(Ca+Ac)e^4)x^2-12\,(Ccd^3e-Bcd^2e^2-Bae^4+\cdots}{12\,e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*C*c*e^4*x^4 - 4*(C*c*d*e^3 - B*c*e^4)*x^3 + 6*(C*c*d^2*e^2 - B*c*d
*e^3 + (C*a + A*c)*e^4)*x^2 - 12*(C*c*d^3*e - B*c*d^2*e^2 - B*a*e^4 + (C*a
 + A*c)*d*e^3)*x + 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a +
A*c)*d^2*e^2)*log(e*x + d))/e^5
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{Ccx^4}{4e} + x^3\left(\frac{Bc}{3e} - \frac{Ccd}{3e^2}\right) + x^2\left(\frac{Ac}{2e} - \frac{Bcd}{2e^2} + \frac{Ca}{2e} + \frac{Ccd^2}{2e^3}\right) + x\left(-\frac{Acd}{e^2} + \frac{Ba}{e} + \frac{Bcd^2}{e^3} - \frac{Cad}{e^2} - \frac{Ccd^3}{e^4}\right) + \frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)\log(d+ex)}{e^5}$$

input
```
integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

output
```
C*c*x**4/(4*e) + x**3*(B*c/(3*e) - C*c*d/(3*e**2)) + x**2*(A*c/(2*e) - B*c
*d/(2*e**2) + C*a/(2*e) + C*c*d**2/(2*e**3)) + x*(-A*c*d/e**2 + B*a/e + B*
c*d**2/e**3 - C*a*d/e**2 - C*c*d**3/e**4) + (a*e**2 + c*d**2)*(A*e**2 - B*
d*e + C*d**2)*log(d + e*x)/e**5
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{3\,Cce^3x^4 - 4\,(Ccde^2 - Bce^3)x^3 + 6\,(Ccd^2e - Bcde^2 + (Ca+Ac)e^3)x^2 - 12\,(Ccd^3 - Bcd^2e - Bae^3 + (C}{12\,e^4} + \frac{(Ccd^4 - Bcd^3e - Bade^3 + Aae^4 + (Ca+Ac)d^2e^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*C*c*e^3*x^4 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c*d^2*e - B*c*d*e
^2 + (C*a + A*c)*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a*e^3 + (C*a + A*c
)*d*e^2)*x)/e^4 + (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)
*d^2*e^2)*log(e*x + d)/e^5
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = \frac{3\,Cce^3x^4 - 4\,Ccde^2x^3 + 4\,Bce^3x^3 + 6\,Ccd^2ex^2 - 6\,Bcde^2x^2 + 6\,Cae^3x^2 + 6\,Ace^3x^2 - 12\,Ccd^3x + 12\,Bcd}{12\,e^4} + \frac{(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4)\log(|ex+d|)}{e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*C*c*e^3*x^4 - 4*C*c*d*e^2*x^3 + 4*B*c*e^3*x^3 + 6*C*c*d^2*e*x^2 -
6*B*c*d*e^2*x^2 + 6*C*a*e^3*x^2 + 6*A*c*e^3*x^2 - 12*C*c*d^3*x + 12*B*c*d^
2*e*x - 12*C*a*d*e^2*x - 12*A*c*d*e^2*x + 12*B*a*e^3*x)/e^4 + (C*c*d^4 - B
*c*d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*log(abs(e*x +
d))/e^5
```

3.22.9 Mupad [B] (verification not implemented)

Time = 12.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx = x^3\left(\frac{B\,c}{3\,e} - \frac{C\,c\,d}{3\,e^2}\right) - x\left(\frac{d\left(\frac{A\,c+C\,a}{e} - \frac{d\left(\frac{B\,c}{e} - \frac{C\,c\,d}{e^2}\right)}{e}\right)}{e} - \frac{B\,a}{e}\right) + x^2\left(\frac{A\,c+C\,a}{2\,e} - \frac{d\left(\frac{B\,c}{e} - \frac{C\,c\,d}{e^2}\right)}{2\,e}\right) + \frac{\ln(d+e\,x)\,\left(A\,a\,e^4 + C\,c\,d^4 - B\,a\,d\,e^3 - B\,c\,d^3\,e + A\,c\,d^2\,e^2 + C\,a\,d^2\,e^2\right)}{e^5} + \frac{C\,c\,x^4}{4\,e}$$

input
```
int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)
```

output
```
x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e -
 (C*c*d)/e^2))/e))/e - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (
C*c*d)/e^2))/(2*e)) + (log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d
^3*e + A*c*d^2*e^2 + C*a*d^2*e^2))/e^5 + (C*c*x^4)/(4*e)
```

3.22. $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex} \, dx$

3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{(aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}\\&\quad+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{e^5(d+ex)}\\&\quad-\frac{(ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\log(d+ex)}{e^5}\end{aligned}$$

output
```
(a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^4-1/2*c*(-B*e+2*C*d)*x^2/e^3+1/3*
c*C*x^3/e^2-(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)/e^5/(e*x+d)-(a*e^2*(-B*e+2*C
*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*ln(e*x+d)/e^5
```

3.23.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{6e(3cCd^2+aCe^2+ce(-2Bd+Ae))\,x+3ce^2(-2Cd+Be)x^2+2cCe^3x^3-\frac{6(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+6}{6e^5}$$

input
```
Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
(6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)
*x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d +
 e*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[
d + e*x])/(6*e^5)
```

3.23.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-ae^2(2Cd-Be)+cde(3Bd-2Ae)-4cCd^3}{e^4(d+ex)}+\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^4(d+ex)^2}+\frac{aCe^2-ce(2Bd-Ae)+\dots}{e^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(d+ex)(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{e^5}-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}+\frac{x(aCe^2-ce(2Bd-Ae)+3cCd^2)}{e^4}-\frac{cx^2(2Cd-Be)}{2e^3}+\frac{cCx^3}{3e^2}$$

input
```
Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/
(2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e
^5*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))
*Log[d + e*x])/e^5
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.23.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

method	result
default	$\frac{\frac{1}{3}cC\,x^{3}e^{2}+\frac{1}{2}Bc\,e^{2}x^{2}-Ccde\,x^{2}+Ac\,e^{2}x-2Bcdex+aC\,e^{2}x+3Cc\,d^{2}x}{e^{4}}-\frac{Aa\,e^{4}+Ac\,d^{2}e^{2}-Bad\,e^{3}-Bc\,d^{3}e+Ca\,d^{2}e^{2}+Cc\,d^{4}}{e^{5}(ex+d)}+$
norman	$\frac{\frac{\left(Aa\,e^{4}+2Ac\,d^{2}e^{2}-Bad\,e^{3}-3Bc\,d^{3}e+2Ca\,d^{2}e^{2}+4Cc\,d^{4}\right)x}{e^{4}d}+\frac{\left(2Ac\,e^{2}-3Bcde+2aC\,e^{2}+4Cc\,d^{2}\right)x^{2}}{2e^{3}}+\frac{cC\,x^{4}}{3e}+\frac{c(3Be-4Cd)x^{3}}{6e^{2}}}{ex+d}-\frac{(2Acd\,e$
risch	$\frac{cC\,x^{3}}{3e^{2}}+\frac{Bc\,x^{2}}{2e^{2}}-\frac{Ccd\,x^{2}}{e^{3}}+\frac{Acx}{e^{2}}-\frac{2Bcdx}{e^{3}}+\frac{aCx}{e^{2}}+\frac{3Cc\,d^{2}x}{e^{4}}-\frac{Aa}{e(ex+d)}-\frac{Ac\,d^{2}}{e^{3}(ex+d)}+\frac{Bad}{e^{2}(ex+d)}+\frac{Bc\,d^{3}}{e^{4}(ex+d)}$
parallelrisch	$-\frac{6Aa\,e^{4}+24Cc\,d^{4}+12A\ln(ex+d)xcd\,e^{3}+24C\ln(ex+d)xc\,d^{3}e+24C\ln(ex+d)c\,d^{4}-3B\,x^{3}c\,e^{4}-6A\,x^{2}c\,e^{4}-6C\,x^{2}a\,e^{4}-2cC\,x^{4}}{}$

input
```
int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2-C*c*d*e*x^2+A*c*e^2*x-2*B*c*d*e*x+a
*C*e^2*x+3*C*c*d^2*x)-(A*a*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2
+C*c*d^4)/e^5/(e*x+d)+(-2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e-2*C*a*d*e^2-4*C*c*
d^3)/e^5*ln(e*x+d)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.63

$$\int \frac{(a+cx^2)\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$=\frac{2\,Cce^4x^4-6\,Ccd^4+6\,Bcd^3e+6\,Bade^3-6\,Aae^4-6\,(Ca+Ac)d^2e^2-(4\,Ccde^3-3\,Bce^4)x^3+3\,(4\,Ccd^2}{}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/6*(2*C*c*e^4*x^4 - 6*C*c*d^4 + 6*B*c*d^3*e + 6*B*a*d*e^3 - 6*A*a*e^4 - 6
*(C*a + A*c)*d^2*e^2 - (4*C*c*d*e^3 - 3*B*c*e^4)*x^3 + 3*(4*C*c*d^2*e^2 -
3*B*c*d*e^3 + 2*(C*a + A*c)*e^4)*x^2 + 6*(3*C*c*d^3*e - 2*B*c*d^2*e^2 + (C
*a + A*c)*d*e^3)*x - 6*(4*C*c*d^4 - 3*B*c*d^3*e - B*a*d*e^3 + 2*(C*a + A*c
)*d^2*e^2 + (4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*
x)*log(e*x + d))/(e^6*x + d*e^5)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{Ccx^3}{3e^2}+x^2\left(\frac{Bc}{2e^2}-\frac{Ccd}{e^3}\right)+x\left(\frac{Ac}{e^2}-\frac{2Bcd}{e^3}+\frac{Ca}{e^2}+\frac{3Ccd^2}{e^4}\right)+\frac{-Aae^4-Acd^2e^2+Bade^3+Bcd^3e-Cad^2e^2-Ccd^4}{de^5+e^6x}-\frac{(2Acde^2-Bae^3-3Bcd^2e+2Cade^2+4Ccd^3)\log(d+ex)}{e^5}$$

input
```
integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

output
```
C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c
*d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d
*e**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*
d*e**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)
/e**5
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx = -\frac{Ccd^4-Bcd^3e-Bade^3+Aae^4+(Ca+Ac)d^2e^2}{e^6x+de^5}+\frac{2\,Cce^2x^3-3\,(2\,Ccde-Bce^2)x^2+6\,(3\,Ccd^2-2\,Bcde+(Ca+Ac)e^2)x}{6\,e^4}-\frac{(4\,Ccd^3-3\,Bcd^2e-Bae^3+2\,(Ca+Ac)de^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x
+ d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2
 - 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^
3 + 2*(C*a + A*c)*d*e^2)*log(e*x + d)/e^5
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{\left(2\,Cc-\frac{3\,(4\,Ccde-Bce^2)}{(ex+d)e}+\frac{6\,(6\,Ccd^2e^2-3\,Bcde^3+Cae^4+Ace^4)}{(ex+d)^2e^2}\right)(ex+d)^3}{6\,e^5} + \frac{(4\,Ccd^3-3\,Bcd^2e+2\,Cade^2+2\,Acde^2-Bae^3)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} - \frac{\frac{Ccd^4e^3}{ex+d}-\frac{Bcd^3e^4}{ex+d}+\frac{Cad^2e^5}{ex+d}+\frac{Acd^2e^5}{ex+d}-\frac{Bade^6}{ex+d}+\frac{Aae^7}{ex+d}}{e^8}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)/((e*x + d)*e) + 6*(6*C*c*d^2*e^2 - 3*
B*c*d*e^3 + C*a*e^4 + A*c*e^4)/((e*x + d)^2*e^2))*(e*x + d)^3/e^5 + (4*C*c
*d^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*log(abs(e*x + d)
/((e*x + d)^2*abs(e)))/e^5 - (C*c*d^4*e^3/(e*x + d) - B*c*d^3*e^4/(e*x + d
) + C*a*d^2*e^5/(e*x + d) + A*c*d^2*e^5/(e*x + d) - B*a*d*e^6/(e*x + d) +
A*a*e^7/(e*x + d))/e^8
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= x^2\left(\frac{B\,c}{2\,e^2}-\frac{C\,c\,d}{e^3}\right)-x\left(\frac{2\,d\left(\frac{B\,c}{e^2}-\frac{2\,C\,c\,d}{e^3}\right)}{e}-\frac{A\,c+C\,a}{e^2}+\frac{C\,c\,d^2}{e^4}\right)$$
$$-\frac{\ln(d+e\,x)\,\left(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\,e^2-3\,B\,c\,d^2\,e\right)}{e^5}$$
$$-\frac{A\,a\,e^4+C\,c\,d^4-B\,a\,d\,e^3-B\,c\,d^3\,e+A\,c\,d^2\,e^2+C\,a\,d^2\,e^2}{e\,(x\,e^5+d\,e^4)}+\frac{C\,c\,x^3}{3\,e^2}$$

input
```
int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

output
```
x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e
 - (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (log(d + e*x)*(4*C*c*d^3 - B*a*e^3 +
 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*
d*e^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*
x^3)/(3*e^2)
```

3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

3.24.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = -\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3} - \frac{(cd^2+ae^2)(Cd^2-Bde+Ae^2)}{2e^5(d+ex)^2} + \frac{ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae))}{e^5(d+ex)} + \frac{(aCe^2+c(6Cd^2-e(3Bd-Ae)))\log(d+ex)}{e^5}$$

output
```
-c*(-B*e+3*C*d)*x/e^4+1/2*c*C*x^2/e^3-1/2*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2
)/e^5/(e*x+d)^2+(a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))/e^5/(e
*x+d)+(a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*ln(e*x+d)/e^5
```

3.24.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{c(-3Cd+Be)x}{e^4}+\frac{cCx^2}{2e^3} + \frac{-cCd^4+Bcd^3e-Acd^2e^2-aCd^2e^2+aBde^3-aAe^4}{2e^5(d+ex)^2} + \frac{4cCd^3-3Bcd^2e+2Acde^2+2aCde^2-aBe^3}{e^5(d+ex)} + \frac{(6cCd^2-3Bcde+Ace^2+aCe^2)\log(d+ex)}{e^5}$$

input
```
Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
(c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A
*c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c
*C*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)
) + ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

3.24.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-ae^2(2Cd-Be)+cde(3Bd-2Ae)-4cCd^3}{e^4(d+ex)^2}+\frac{aCe^2-ce(3Bd-Ae)+6cCd^2}{e^4(d+ex)}+\frac{(ae^2+cd^2)(Ae^2-Bde}{e^4(d+ex)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)}-\frac{(ae^2+cd^2)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{e^5}-\frac{cx(3Cd-Be)}{e^4}+\frac{cCx^2}{2e^3}$$

input
```
Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
-((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
 B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e)
+ a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*
d - A*e))*Log[d + e*x])/e^5
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.24.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

method	result
default	$\frac{c\left(\frac{1}{2}C\,x^2e+Bex-3Cdx\right)}{e^4}-\frac{-2Acd\,e^2+B\,e^3a+3Bc\,d^2e-2Cad\,e^2-4Cc\,d^3}{e^5(ex+d)}+\frac{\left(Ac\,e^2-3Bcde+aC\,e^2+6Cc\,d^2\right)\ln(ex+d)}{e^5}-$
norman	$\frac{\frac{\left(2Acd\,e^2-B\,e^3a-6Bc\,d^2e+2Cad\,e^2+12Cc\,d^3\right)x}{e^4}+\frac{c(Be-2Cd)x^3}{e^2}-\frac{Aa\,e^4-3Ac\,d^2e^2+Bad\,e^3+9Bc\,d^3e-3Ca\,d^2e^2-18Cc\,d^4}{2e^5}+\frac{cC\,x^4}{2e}}{(ex+d)^2}+$
risch	$\frac{cC\,x^2}{2e^3}+\frac{Bcx}{e^3}-\frac{3cCdx}{e^4}+\frac{(2Acd\,e^2-B\,e^3a-3Bc\,d^2e+2Cad\,e^2+4Cc\,d^3)x-\frac{Aa\,e^4-3Ac\,d^2e^2+Bad\,e^3+5Bc\,d^3e-3Ca\,d^2e^2-7C}{2e}}{e^4(ex+d)^2}$
parallelrisch	$\frac{-Aa\,e^4+18Cc\,d^4+4A\ln(ex+d)xcd\,e^3-6B\ln(ex+d)x^2cd\,e^3+12C\ln(ex+d)x^2c\,d^2e^2+24C\ln(ex+d)xc\,d^3e+12C\ln(ex+d)c\,d^4}{}$

input
```
int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/2*C*x^2*e+B*e*x-3*C*d*x)-(-2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e-2*C*a*
d*e^2-4*C*c*d^3)/e^5/(e*x+d)+1/e^5*(A*c*e^2-3*B*c*d*e+C*a*e^2+6*C*c*d^2)*l
n(e*x+d)-1/2*(A*a*e^4+A*c*d^2*e^2-B*a*d*e^3-B*c*d^3*e+C*a*d^2*e^2+C*c*d^4)
/e^5/(e*x+d)^2
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.75

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{Cce^4x^4+7\,Ccd^4-5\,Bcd^3e-Bade^3-Aae^4+3\,(Ca+Ac)d^2e^2-2\,(2\,Ccde^3-Bce^4)x^3-(11\,Ccd^2e^2-$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(C*c*e^4*x^4 + 7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a
+ A*c)*d^2*e^2 - 2*(2*C*c*d*e^3 - B*c*e^4)*x^3 - (11*C*c*d^2*e^2 - 4*B*c*d
*e^3)*x^2 + 2*(C*c*d^3*e - 2*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*
x + 2*(6*C*c*d^4 - 3*B*c*d^3*e + (C*a + A*c)*d^2*e^2 + (6*C*c*d^2*e^2 - 3*
B*c*d*e^3 + (C*a + A*c)*e^4)*x^2 + 2*(6*C*c*d^3*e - 3*B*c*d^2*e^2 + (C*a +
 A*c)*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{Ccx^2}{2e^3}+x\left(\frac{Bc}{e^3}-\frac{3Ccd}{e^4}\right) + \frac{-Aae^4+3Acd^2e^2-Bade^3-5Bcd^3e+3Cad^2e^2+7Ccd^4+x(4Acde^3-2Bae^4-6Bcd^2e^2+4Cade^3}{2d^2e^5+4de^6x+2e^7x^2} + \frac{(Ace^2-3Bcde+Cae^2+6Ccd^2)\log{(d+ex)}}{e^5}$$

input
```
integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

output
```
C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*
e**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c
*d*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*
d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 +
 6*C*c*d**2)*log(d + e*x)/e**5
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{7\,Ccd^4 - 5\,Bcd^3e - Bade^3 - Aae^4 + 3\,(Ca+Ac)d^2e^2 + 2\,(4\,Ccd^3e - 3\,Bcd^2e^2 - Bae^4 + 2\,(Ca+Ac)de^3}{2\,(e^7x^2 + 2\,de^6x + d^2e^5)}$$
$$+ \frac{Ccex^2 - 2\,(3\,Ccd - Bce)x}{2\,e^4} + \frac{(6\,Ccd^2 - 3\,Bcde + (Ca+Ac)e^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a + A*c)*d^2*e^2
 + 2*(4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)/(e^7
*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(C*c*e*x^2 - 2*(3*C*c*d - B*c*e)*x)/e^4
+ (6*C*c*d^2 - 3*B*c*d*e + (C*a + A*c)*e^2)*log(e*x + d)/e^5
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{(6\,Ccd^2 - 3\,Bcde + Cae^2 + Ace^2)\log(|ex+d|)}{e^5} + \frac{Cce^3x^2 - 6\,Ccde^2x + 2\,Bce^3x}{2\,e^6}$$
$$+ \frac{7\,Ccd^4 - 5\,Bcd^3e + 3\,Cad^2e^2 + 3\,Acd^2e^2 - Bade^3 - Aae^4 + 2\,(4\,Ccd^3e - 3\,Bcd^2e^2 + 2\,Cade^3 + 2\,Acde}{2\,(ex+d)^2e^5}$$

input
```
integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

output
```
(6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*log(abs(e*x + d))/e^5 + 1/2*(C
*c*e^3*x^2 - 6*C*c*d*e^2*x + 2*B*c*e^3*x)/e^6 + 1/2*(7*C*c*d^4 - 5*B*c*d^3
*e + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3*e
- 3*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)/((e*x + d)^2*e^5
)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3}\,dx = \frac{x\,(4\,C\,c\,d^3-B\,a\,e^3+2\,A\,c\,d\,e^2+2\,C\,a\,d\,e^2-3\,B\,c\,d^2\,e)-\frac{A\,a\,e^4-7\,C\,c\,d^4+B\,a\,d\,e^3+5\,B\,c\,d^3\,e-3\,A\,c\,d^2\,e^2-3\,C\,a\,d^2\,e^2}{2\,e}}{d^2\,e^4+2\,d\,e^5\,x+e^6\,x^2} + x\left(\frac{B\,c}{e^3}-\frac{3\,C\,c\,d}{e^4}\right)+\frac{\ln(d+e\,x)\,(A\,c\,e^2+C\,a\,e^2+6\,C\,c\,d^2-3\,B\,c\,d\,e)}{e^5}+\frac{C\,c\,x^2}{2\,e^3}$$

input
```
int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

output
```
(x*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e) - (A*a*
e^4 - 7*C*c*d^4 + B*a*d*e^3 + 5*B*c*d^3*e - 3*A*c*d^2*e^2 - 3*C*a*d^2*e^2)
/(2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + x*((B*c)/e^3 - (3*C*c*d)/e^4) +
(log(d + e*x)*(A*c*e^2 + C*a*e^2 + 6*C*c*d^2 - 3*B*c*d*e))/e^5 + (C*c*x^2)
/(2*e^3)
```

3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

3.25.1 Optimal result

Integrand size = 27, antiderivative size = 304

$$\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx \\
&= a^2Ad^3x+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3+\frac{1}{4}a^2e\left(3Cd^2+e(3Bd+Ae)\right)x^4 \\
&\quad+\frac{1}{5}\left(Acd\left(cd^2+6ae^2\right)+a\left(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\right)\right)x^5 \\
&\quad+\frac{1}{6}ae\left(aCe^2+2c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^6 \\
&\quad+\frac{1}{7}c\left(2ae^2(3Cd+Be)+cd\left(Cd^2+3e(Bd+Ae)\right)\right)x^7 \\
&\quad+\frac{1}{8}ce\left(2aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^8 \\
&\quad+\frac{1}{9}c^2e^2(3Cd+Be)x^9+\frac{1}{10}c^2Ce^3x^{10}+\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}
\end{aligned}$$

output
```
a^2*A*d^3*x+1/3*a*d*(a*d*(3*B*e+C*d)+A*(3*a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(3
*C*d^2+e*(A*e+3*B*d))*x^4+1/5*(A*c*d*(6*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+
2*c*d^2*(3*B*e+C*d)))*x^5+1/6*a*e*(a*C*e^2+2*c*(3*C*d^2+e*(A*e+3*B*d)))*x^
6+1/7*c*(2*a*e^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^7+1/8*c*e*(2*a*C
*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*e^2*(B*e+3*C*d)*x^9+1/10*c^2*C
*e^3*x^10+1/6*d^2*(3*A*e+B*d)*(c*x^2+a)^3/c
```

3.25.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.10

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = a^2Ad^3x+\frac{1}{2}a^2d^2(Bd+3Ae)x^2 +\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3 +\frac{1}{4}a\left(2Bcd^3+6Acd^2e+3aCd^2e+3aBde^2 +aAe^3\right)x^4+\frac{1}{5}\left(Acd\left(cd^2+6ae^2\right) +a\left(ae^2(3Cd+Be)+2cd^2(Cd+3Be)\right)\right)x^5 +\frac{1}{6}\left(aCe\left(6cd^2+ae^2\right)+Ace\left(3cd^2+2ae^2\right) +Bcd\left(cd^2+6ae^2\right)\right)x^6+\frac{1}{7}c\left(cCd^3 +3cde(Bd+Ae)+2ae^2(3Cd+Be)\right)x^7 +\frac{1}{8}ce\left(3cCd^2+2aCe^2+ce(3Bd+Ae)\right)x^8 +\frac{1}{9}c^2e^2(3Cd+Be)x^9+\frac{1}{10}c^2Ce^3x^{10}$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*
(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*
d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*
c*e*(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 +
3*c*d*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*
a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2
*C*e^3*x^10)/10
```

3.25.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (d+ex)^3 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)^2 \left((d+ex)^3 \left(Cx^2+Bx+A\right)-\left(Bd^3+3Aed^2\right) x\right) dx+\frac{d^2 \left(a+cx^2\right)^3 (3Ae+Bd)}{6c}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(c^2Ce^3x^9+c^2e^2(3Cd+Be)x^8+ce\left(3cCd^2+2aCe^2+ce(3Bd+Ae)\right)x^7+c\left(cCd^3+3ce(Bd+Ae)d+2ae^2(3\right.\right.$$
$$\frac{d^2 \left(a+cx^2\right)^3 (3Ae+Bd)}{6c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{6}aex^6\left(aCe^2+2ce(Ae+3Bd)+6cCd^2\right)+\frac{1}{5}x^5\left(Acd\left(6ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+2cd^2(3Be+Cd)\right)\right)+\frac{1}{3}adx^3\left(A\left(3ae^2+2cd^2\right)+ad(3Be+Cd)\right)+\frac{d^2\left(a+cx^2\right)^3(3Ae+Bd)}{6c}+\frac{1}{9}c^2e^2x^9(Be+3Cd)+\frac{1}{10}c^2Ce^3x^{10}$$

input
```
Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a
^2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a
*e^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*
C*e^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) +
2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d +
 A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2
*(B*d + 3*A*e)*(a + c*x^2)^3)/(6*c)
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.25.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.22

method	result
norman	$\frac{c^2 C e^3 x^{10}}{10}+\left(\frac{1}{9}e^3 c^2 B+\frac{1}{3}c^2 d e^2 C\right)x^9+\left(\frac{1}{8}e^3 c^2 A+\frac{3}{8}c^2 d e^2 B+\frac{1}{4}C a c e^3+\frac{3}{8}C c^2 d^2 e\right)x^8+\left(\frac{3}{7}c^2\right.$
default	$\frac{c^2 C e^3 x^{10}}{10}+\frac{(e^3 c^2 B+3 c^2 d e^2 C)x^9}{9}+\frac{((2c e^3 a+3c^2 d^2 e)C+3c^2 d e^2 B+e^3 c^2 A)x^8}{8}+\frac{((6acde^2+c^2d^3)C+(2ce^3a+3c^2d^2e)}{7}$
gosper	$\frac{6}{7}x^7 C a c d e^2+x^6 B a c d e^2+x^6 C a c d^2 e+\frac{6}{5}x^5 A a c d e^2+\frac{6}{5}x^5 B a c d^2 e+\frac{1}{9}B c^2 e^3 x^9+\frac{1}{8}x^8 A c^2 e^3$
risch	$\frac{6}{7}x^7 C a c d e^2+x^6 B a c d e^2+x^6 C a c d^2 e+\frac{6}{5}x^5 A a c d e^2+\frac{6}{5}x^5 B a c d^2 e+\frac{1}{9}B c^2 e^3 x^9+\frac{1}{8}x^8 A c^2 e^3$
parallelrisch	$\frac{6}{7}x^7 C a c d e^2+x^6 B a c d e^2+x^6 C a c d^2 e+\frac{6}{5}x^5 A a c d e^2+\frac{6}{5}x^5 B a c d^2 e+\frac{1}{9}B c^2 e^3 x^9+\frac{1}{8}x^8 A c^2 e^3$

input
```
int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*C*e^3*x^10+(1/9*e^3*c^2*B+1/3*c^2*d*e^2*C)*x^9+(1/8*e^3*c^2*A+3/8
*c^2*d*e^2*B+1/4*C*a*c*e^3+3/8*C*c^2*d^2*e)*x^8+(3/7*c^2*d*e^2*A+2/7*B*e^3
*a*c+3/7*B*c^2*d^2*e+6/7*C*a*c*d*e^2+1/7*C*c^2*d^3)*x^7+(1/3*A*a*c*e^3+1/2
*A*c^2*d^2*e+B*a*c*d*e^2+1/6*B*c^2*d^3+1/6*C*a^2*e^3+C*a*c*d^2*e)*x^6+(6/5
*A*a*c*d*e^2+1/5*A*d^3*c^2+1/5*a^2*B*e^3+6/5*B*a*c*d^2*e+3/5*C*a^2*d*e^2+2
/5*C*a*c*d^3)*x^5+(1/4*A*a^2*e^3+3/2*A*a*c*d^2*e+3/4*B*a^2*d*e^2+1/2*B*a*c
*d^3+3/4*d^2*e*a^2*C)*x^4+(A*a^2*d*e^2+2/3*A*d^3*a*c+B*a^2*d^2*e+1/3*d^3*a
^2*C)*x^3+(3/2*A*a^2*d^2*e+1/2*B*a^2*d^3)*x^2+A*d^3*a^2*x
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$
$$= \frac{1}{10} Cc^2e^3x^{10} + \frac{1}{9} \left(3 Cc^2de^2 + Bc^2e^3\right)x^9 + \frac{1}{8} \left(3 Cc^2d^2e + 3 Bc^2de^2 + \left(2 Cac + Ac^2\right)e^3\right)x^8 + \frac{1}{7} \left(Cc^2d^3 + 3 Bc^2d^2e + 2 Bace^3 + 3 \left(2 Cac + Ac^2\right)de^2\right)x^7 + Aa^2d^3x + \frac{1}{6} \left(Bc^2d^3 + 6 Bacde^2 + 3 \left(2 Cac + Ac^2\right)d^2e + \left(Ca^2 + 2 Aac\right)e^3\right)x^6 + \frac{1}{5} \left(6 Bacd^2e + Ba^2e^3 + \left(2 Cac + Ac^2\right)d^3 + 3 \left(Ca^2 + 2 Aac\right)de^2\right)x^5 + \frac{1}{4} \left(2 Bacd^3 + 3 Ba^2de^2 + Aa^2e^3 + 3 \left(Ca^2 + 2 Aac\right)d^2e\right)x^4 + \frac{1}{3} \left(3 Ba^2d^2e + 3 Aa^2de^2 + \left(Ca^2 + 2 Aac\right)d^3\right)x^3 + \frac{1}{2} \left(Ba^2d^3 + 3 Aa^2d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d
^2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c
^2*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/
6*(B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*
c)*e^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(
C*a^2 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3
 + 3*(C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (
C*a^2 + 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.46

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = Aa^2d^3x + \frac{Cc^2e^3x^{10}}{10} + x^9\left(\frac{Bc^2e^3}{9} + \frac{Cc^2de^2}{3}\right) + x^8\left(\frac{Ac^2e^3}{8} + \frac{3Bc^2de^2}{8} + \frac{Cace^3}{4} + \frac{3Cc^2d^2e}{8}\right) + x^7 \cdot \left(\frac{3Ac^2de^2}{7} + \frac{2Bace^3}{7} + \frac{3Bc^2d^2e}{7} + \frac{6Cacde^2}{7} + \frac{Cc^2d^3}{7}\right) + x^6\left(\frac{Aace^3}{3} + \frac{Ac^2d^2e}{2} + Bacde^2 + \frac{Bc^2d^3}{6} + \frac{Ca^2e^3}{6} + Cacd^2e\right) + x^5 \cdot \left(\frac{6Aacde^2}{5} + \frac{Ac^2d^3}{5} + \frac{Ba^2e^3}{5} + \frac{6Bacd^2e}{5} + \frac{3Ca^2de^2}{5} + \frac{2Cacd^3}{5}\right) + x^4\left(\frac{Aa^2e^3}{4} + \frac{3Aacd^2e}{2} + \frac{3Ba^2de^2}{4} + \frac{Bacd^3}{2} + \frac{3Ca^2d^2e}{4}\right) + x^3\left(Aa^2de^2 + \frac{2Aacd^3}{3} + Ba^2d^2e + \frac{Ca^2d^3}{3}\right) + x^2 \cdot \left(\frac{3Aa^2d^2e}{2} + \frac{Ba^2d^3}{2}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

output
```
A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2
/3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d*
*2*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6
*C*a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 +
B*a*c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a
*c*d*e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*
d*e**2/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*
a**2*d*e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 +
2*A*a*c*d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2
+ B*a**2*d**3/2)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$
$$= \frac{1}{10} Cc^2e^3x^{10} + \frac{1}{9} \left(3 Cc^2de^2 + Bc^2e^3\right)x^9 + \frac{1}{8} \left(3 Cc^2d^2e + 3 Bc^2de^2 + \left(2 Cac + Ac^2\right)e^3\right)x^8 + \frac{1}{7} \left(Cc^2d^3 + 3 Bc^2d^2e + 2 Bace^3 + 3 \left(2 Cac + Ac^2\right)de^2\right)x^7 + Aa^2d^3x + \frac{1}{6} \left(Bc^2d^3 + 6 Bacde^2 + 3 \left(2 Cac + Ac^2\right)d^2e + \left(Ca^2 + 2 Aac\right)e^3\right)x^6 + \frac{1}{5} \left(6 Bacd^2e + Ba^2e^3 + \left(2 Cac + Ac^2\right)d^3 + 3 \left(Ca^2 + 2 Aac\right)de^2\right)x^5 + \frac{1}{4} \left(2 Bacd^3 + 3 Ba^2de^2 + Aa^2e^3 + 3 \left(Ca^2 + 2 Aac\right)d^2e\right)x^4 + \frac{1}{3} \left(3 Ba^2d^2e + 3 Aa^2de^2 + \left(Ca^2 + 2 Aac\right)d^3\right)x^3 + \frac{1}{2} \left(Ba^2d^3 + 3 Aa^2d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d
^2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c
^2*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/
6*(B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*
c)*e^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(
C*a^2 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3
 + 3*(C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (
C*a^2 + 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.42

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{10}Cc^2e^3x^{10} + \frac{1}{3}Cc^2de^2x^9 + \frac{1}{9}Bc^2e^3x^9 + \frac{3}{8}Cc^2d^2ex^8 + \frac{3}{8}Bc^2de^2x^8 + \frac{1}{4}Cace^3x^8 + \frac{1}{8}Ac^2e^3x^8 + \frac{1}{7}Cc^2d^3x^7 + \frac{3}{7}Bc^2d^2ex^7 + \frac{6}{7}Cacde^2x^7 + \frac{3}{7}Ac^2de^2x^7 + \frac{2}{7}Bace^3x^7 + \frac{1}{6}Bc^2d^3x^6 + Cacd^2ex^6 + \frac{1}{2}Ac^2d^2ex^6 + Bacde^2x^6 + \frac{1}{6}Ca^2e^3x^6 + \frac{1}{3}Aace^3x^6 + \frac{2}{5}Cacd^3x^5 + \frac{1}{5}Ac^2d^3x^5 + \frac{6}{5}Bacd^2ex^5 + \frac{3}{5}Ca^2de^2x^5 + \frac{6}{5}Aacde^2x^5 + \frac{1}{5}Ba^2e^3x^5 + \frac{1}{2}Bacd^3x^4 + \frac{3}{4}Ca^2d^2ex^4 + \frac{3}{2}Aacd^2ex^4 + \frac{3}{4}Ba^2de^2x^4 + \frac{1}{4}Aa^2e^3x^4 + \frac{1}{3}Ca^2d^3x^3 + \frac{2}{3}Aacd^3x^3 + Ba^2d^2ex^3 + Aa^2de^2x^3 + \frac{1}{2}Ba^2d^3x^2 + \frac{3}{2}Aa^2d^2ex^2 + Aa^2d^3x$$

input

```
integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

output

```
1/10*C*c^2*e^3*x^10 + 1/3*C*c^2*d*e^2*x^9 + 1/9*B*c^2*e^3*x^9 + 3/8*C*c^2*
d^2*e*x^8 + 3/8*B*c^2*d*e^2*x^8 + 1/4*C*a*c*e^3*x^8 + 1/8*A*c^2*e^3*x^8 +
1/7*C*c^2*d^3*x^7 + 3/7*B*c^2*d^2*e*x^7 + 6/7*C*a*c*d*e^2*x^7 + 3/7*A*c^2*
d*e^2*x^7 + 2/7*B*a*c*e^3*x^7 + 1/6*B*c^2*d^3*x^6 + C*a*c*d^2*e*x^6 + 1/2*
A*c^2*d^2*e*x^6 + B*a*c*d*e^2*x^6 + 1/6*C*a^2*e^3*x^6 + 1/3*A*a*c*e^3*x^6
+ 2/5*C*a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 6/5*B*a*c*d^2*e*x^5 + 3/5*C*a^2*
d*e^2*x^5 + 6/5*A*a*c*d*e^2*x^5 + 1/5*B*a^2*e^3*x^5 + 1/2*B*a*c*d^3*x^4 +
3/4*C*a^2*d^2*e*x^4 + 3/2*A*a*c*d^2*e*x^4 + 3/4*B*a^2*d*e^2*x^4 + 1/4*A*a^
2*e^3*x^4 + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + B*a^2*d^2*e*x^3 + A*a^
2*d*e^2*x^3 + 1/2*B*a^2*d^3*x^2 + 3/2*A*a^2*d^2*e*x^2 + A*a^2*d^3*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx = x^5 \left(\frac{3\,C\,a^2\,d\,e^2}{5}+\frac{B\,a^2\,e^3}{5}+\frac{2\,C\,a\,c\,d^3}{5}+\frac{6\,B\,a\,c\,d^2\,e}{5}+\frac{6\,A\,a\,c\,d\,e^2}{5}+\frac{A\,c^2\,d^3}{5}\right) + x^6 \left(\frac{C\,a^2\,e^3}{6}+C\,a\,c\,d^2\,e+B\,a\,c\,d\,e^2+\frac{A\,a\,c\,e^3}{3}+\frac{B\,c^2\,d^3}{6}+\frac{A\,c^2\,d^2\,e}{2}\right) + \frac{a\,x^4\,(A\,a\,e^3+2\,B\,c\,d^3+3\,B\,a\,d\,e^2+6\,A\,c\,d^2\,e+3\,C\,a\,d^2\,e)}{4} + \frac{c\,x^7\,(2\,B\,a\,e^3+C\,c\,d^3+3\,A\,c\,d\,e^2+6\,C\,a\,d\,e^2+3\,B\,c\,d^2\,e)}{7} + \frac{C\,c^2\,e^3\,x^{10}}{10} + \frac{a^2\,d^2\,x^2\,(3\,A\,e+B\,d)}{2} + \frac{c^2\,e^2\,x^9\,(B\,e+3\,C\,d)}{9} + \frac{a\,d\,x^3\,(3\,A\,a\,e^2+2\,A\,c\,d^2+C\,a\,d^2+3\,B\,a\,d\,e)}{3} + \frac{c\,e\,x^8\,(A\,c\,e^2+2\,C\,a\,e^2+3\,C\,c\,d^2+3\,B\,c\,d\,e)}{8} + A\,a^2\,d^3\,x$$

input
```
int((a + c*x^2)^2*(d + e*x)^3*(A + B*x + C*x^2),x)
```

output
```
x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (2*C*a*c*d^3)/5 + (3*C*a^2*d*e^2)/5 +
 (6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5) + x^6*((B*c^2*d^3)/6 + (C*a^2*e^3)
/6 + (A*a*c*e^3)/3 + (A*c^2*d^2*e)/2 + B*a*c*d*e^2 + C*a*c*d^2*e) + (a*x^4
*(A*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c*x
^7*(2*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 6*C*a*d*e^2 + 3*B*c*d^2*e))/7 + (C
*c^2*e^3*x^10)/10 + (a^2*d^2*x^2*(3*A*e + B*d))/2 + (c^2*e^2*x^9*(B*e + 3*
C*d))/9 + (a*d*x^3*(3*A*a*e^2 + 2*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (c*e
*x^8*(A*c*e^2 + 2*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/8 + A*a^2*d^3*x
```

3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$

3.26.1 Optimal result

Integrand size = 27, antiderivative size = 217

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx = a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^2e(2Cd+Be)x^4+\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right) + a\left(aCe^2+2cd(Cd+2Be)\right)\right)x^5 + \frac{1}{3}ace(2Cd+Be)x^6 + \frac{1}{7}c\left(2aCe^2+c\left(Cd^2+e(2Bd+Ae)\right)\right)x^7 + \frac{1}{8}c^2e(2Cd+Be)x^8+\frac{1}{9}c^2Ce^2x^9 + \frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}$$

output

```
a^2*A*d^2*x+1/3*a*(a*d*(2*B*e+C*d)+A*(a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(B*e+2
*C*d)*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+a*(a*C*e^2+2*c*d*(2*B*e+C*d)))*x^5+1/3*
a*c*e*(B*e+2*C*d)*x^6+1/7*c*(2*a*C*e^2+c*(C*d^2+e*(A*e+2*B*d)))*x^7+1/8*c^
2*e*(B*e+2*C*d)*x^8+1/9*c^2*C*e^2*x^9+1/6*d*(2*A*e+B*d)*(c*x^2+a)^3/c
```

3.26.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx = a^2Ad^2x+\frac{1}{2}a^2d(Bd+2Ae)x^2 +\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3 +\frac{1}{4}a\left(2Bcd^2+4Acde+2aCde+aBe^2\right)x^4 +\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right) +a\left(aCe^2+2cd(Cd+2Be)\right)\right)x^5 +\frac{1}{6}c\left(Bcd^2+2Acde+4aCde+2aBe^2\right)x^6 +\frac{1}{7}c\left(cCd^2+2aCe^2+ce(2Bd+Ae)\right)x^7 +\frac{1}{8}c^2e(2Cd+Be)x^8+\frac{1}{9}c^2Ce^2x^9$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c
*d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x
^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/
5 + (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2
+ 2*a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c
^2*C*e^2*x^9)/9
```

3.26.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2 (d+ex)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)^2 \left(\left(d+ex\right)^2 \left(Cx^2+Bx+A\right)-\left(Bd^2+2Aed\right)x\right)dx+\frac{d\left(a+cx^2\right)^3(2Ae+Bd)}{6c}$$

$\big\downarrow$ 2341

$$\int \left(c^2Ce^2x^8+c^2e(2Cd+Be)x^7+c\left(cCd^2+2aCe^2+ce(2Bd+Ae)\right)x^6+2ace(2Cd+Be)x^5+\left(Ac\left(cd^2+2ae^2\right)\right.\right.$$
$$\frac{d\left(a+cx^2\right)^3(2Ae+Bd)}{6c}$$

$\big\downarrow$ 2009

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+$$
$$\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd)\right)\right)+\frac{1}{3}ax^3\left(A\left(ae^2+2cd^2\right)+ad(2Be+Cd)\right)+$$
$$\frac{d\left(a+cx^2\right)^3(2Ae+Bd)}{6c}+\frac{1}{3}acex^6(Be+2Cd)+\frac{1}{8}c^2ex^8(Be+2Cd)+\frac{1}{9}c^2Ce^2x^9$$

input
```
Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e
*(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d
+ 2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2
+ c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9
)/9 + (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.26.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

method	result
norman	$\frac{c^2 C e^2 x^9}{9}+\left(\frac{1}{8} B c^2 e^2+\frac{1}{4} c^2 d e C\right) x^8+\left(\frac{1}{7} A c^2 e^2+\frac{2}{7} B c^2 d e+\frac{2}{7} C a c e^2+\frac{1}{7} C c^2 d^2\right) x^7+\left(\frac{1}{3} A c^2 d\right.$
default	$\frac{c^2 C e^2 x^9}{9}+\frac{\left(B c^2 e^2+2 c^2 d e C\right) x^8}{8}+\frac{\left(\left(2 a c e^2+c^2 d^2\right) C+2 B c^2 d e+A c^2 e^2\right) x^7}{7}+\frac{\left(4 a c d e C+\left(2 a c e^2+c^2 d^2\right) B+2 A c^2 d e\right) x^6}{6}-$
gosper	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x$
risch	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x$
parallelrisch	$\frac{2}{3} x^6 a c d e C+\frac{4}{5} x^5 B a c d e+x^4 A a c d e+\frac{1}{2} x^4 B a c d^2+\frac{1}{8} B c^2 e^2 x^8+\frac{2}{3} x^3 A d^2 a c+\frac{2}{3} x^3 B a^2 d e+x$

input
```
int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*c^2*C*e^2*x^9+(1/8*B*c^2*e^2+1/4*c^2*d*e*C)*x^8+(1/7*A*c^2*e^2+2/7*B*c
^2*d*e+2/7*C*a*c*e^2+1/7*C*c^2*d^2)*x^7+(1/3*A*c^2*d*e+1/3*B*e^2*a*c+1/6*B
*c^2*d^2+2/3*a*c*d*e*C)*x^6+(2/5*A*a*c*e^2+1/5*A*c^2*d^2+4/5*B*a*c*d*e+1/5
*a^2*C*e^2+2/5*C*a*c*d^2)*x^5+(A*a*c*d*e+1/4*a^2*B*e^2+1/2*B*a*c*d^2+1/2*d
*e*a^2*C)*x^4+(1/3*A*a^2*e^2+2/3*A*d^2*a*c+2/3*B*a^2*d*e+1/3*a^2*d^2*C)*x^
3+(d*e*a^2*A+1/2*a^2*d^2*B)*x^2+A*d^2*a^2*x
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18

$$\begin{aligned}
&\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx\\
&=\frac{1}{9}Cc^2e^2x^9+\frac{1}{8}\left(2Cc^2de+Bc^2e^2\right)x^8+\frac{1}{7}\left(Cc^2d^2+2Bc^2de+\left(2Cac+Ac^2\right)e^2\right)x^7\\
&\quad+\frac{1}{6}\left(Bc^2d^2+2Bace^2+2\left(2Cac+Ac^2\right)de\right)x^6+Aa^2d^2x\\
&\quad+\frac{1}{5}\left(4Bacde+\left(2Cac+Ac^2\right)d^2+\left(Ca^2+2Aac\right)e^2\right)x^5\\
&\quad+\frac{1}{4}\left(2Bacd^2+Ba^2e^2+2\left(Ca^2+2Aac\right)de\right)x^4\\
&\quad+\frac{1}{3}\left(2Ba^2de+Aa^2e^2+\left(Ca^2+2Aac\right)d^2\right)x^3+\frac{1}{2}\left(Ba^2d^2+2Aa^2de\right)x^2
\end{aligned}$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2
*B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2
*(2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c +
A*c^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2
*(C*a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*
a*c)*d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.43

$$\int (d+ex)^2 (a+cx^2)^2 (A+Bx+Cx^2)\,dx = Aa^2d^2x + \frac{Cc^2e^2x^9}{9} + x^8\left(\frac{Bc^2e^2}{8} + \frac{Cc^2de}{4}\right) + x^7\left(\frac{Ac^2e^2}{7} + \frac{2Bc^2de}{7} + \frac{2Cace^2}{7} + \frac{Cc^2d^2}{7}\right) + x^6\left(\frac{Ac^2de}{3} + \frac{Bace^2}{3} + \frac{Bc^2d^2}{6} + \frac{2Cacde}{3}\right) + x^5\cdot\left(\frac{2Aace^2}{5} + \frac{Ac^2d^2}{5} + \frac{4Bacde}{5} + \frac{Ca^2e^2}{5} + \frac{2Cacd^2}{5}\right) + x^4\left(Aacde + \frac{Ba^2e^2}{4} + \frac{Bacd^2}{2} + \frac{Ca^2de}{2}\right) + x^3\left(\frac{Aa^2e^2}{3} + \frac{2Aacd^2}{3} + \frac{2Ba^2de}{3} + \frac{Ca^2d^2}{3}\right) + x^2\left(Aa^2de + \frac{Ba^2d^2}{2}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

output
```
A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
 x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) +
x**6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*
(2*A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*
d**2/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) +
 x**3*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) +
x**2*(A*a**2*d*e + B*a**2*d**2/2)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{9}\,Cc^2e^2x^9+\frac{1}{8}\left(2\,Cc^2de+Bc^2e^2\right)x^8+\frac{1}{7}\left(Cc^2d^2+2\,Bc^2de+\left(2\,Cac+Ac^2\right)e^2\right)x^7 \\
&\quad+\frac{1}{6}\left(Bc^2d^2+2\,Bace^2+2\left(2\,Cac+Ac^2\right)de\right)x^6+Aa^2d^2x \\
&\quad+\frac{1}{5}\left(4\,Bacde+\left(2\,Cac+Ac^2\right)d^2+\left(Ca^2+2\,Aac\right)e^2\right)x^5 \\
&\quad+\frac{1}{4}\left(2\,Bacd^2+Ba^2e^2+2\left(Ca^2+2\,Aac\right)de\right)x^4 \\
&\quad+\frac{1}{3}\left(2\,Ba^2de+Aa^2e^2+\left(Ca^2+2\,Aac\right)d^2\right)x^3+\frac{1}{2}\left(Ba^2d^2+2\,Aa^2de\right)x^2
\end{aligned}
$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2
*B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2
*(2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c +
A*c^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2
*(C*a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*
a*c)*d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.39

$$\int (d+ex)^2 (a+cx^2)^2 (A+Bx+Cx^2)\, dx = \frac{1}{9}Cc^2e^2x^9 + \frac{1}{4}Cc^2dex^8 + \frac{1}{8}Bc^2e^2x^8 + \frac{1}{7}Cc^2d^2x^7 + \frac{2}{7}Bc^2dex^7 + \frac{2}{7}Cace^2x^7 + \frac{1}{7}Ac^2e^2x^7 + \frac{1}{6}Bc^2d^2x^6 + \frac{2}{3}Cacdex^6 + \frac{1}{3}Ac^2dex^6 + \frac{1}{3}Bace^2x^6 + \frac{2}{5}Cacd^2x^5 + \frac{1}{5}Ac^2d^2x^5 + \frac{4}{5}Bacdex^5 + \frac{1}{5}Ca^2e^2x^5 + \frac{2}{5}Aace^2x^5 + \frac{1}{2}Bacd^2x^4 + \frac{1}{2}Ca^2dex^4 + Aacdex^4 + \frac{1}{4}Ba^2e^2x^4 + \frac{1}{3}Ca^2d^2x^3 + \frac{2}{3}Aacd^2x^3 + \frac{2}{3}Ba^2dex^3 + \frac{1}{3}Aa^2e^2x^3 + \frac{1}{2}Ba^2d^2x^2 + Aa^2dex^2 + Aa^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/4*C*c^2*d*e*x^8 + 1/8*B*c^2*e^2*x^8 + 1/7*C*c^2*d^2*
x^7 + 2/7*B*c^2*d*e*x^7 + 2/7*C*a*c*e^2*x^7 + 1/7*A*c^2*e^2*x^7 + 1/6*B*c^
2*d^2*x^6 + 2/3*C*a*c*d*e*x^6 + 1/3*A*c^2*d*e*x^6 + 1/3*B*a*c*e^2*x^6 + 2/
5*C*a*c*d^2*x^5 + 1/5*A*c^2*d^2*x^5 + 4/5*B*a*c*d*e*x^5 + 1/5*C*a^2*e^2*x^
5 + 2/5*A*a*c*e^2*x^5 + 1/2*B*a*c*d^2*x^4 + 1/2*C*a^2*d*e*x^4 + A*a*c*d*e*
x^4 + 1/4*B*a^2*e^2*x^4 + 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 2/3*B*a^
2*d*e*x^3 + 1/3*A*a^2*e^2*x^3 + 1/2*B*a^2*d^2*x^2 + A*a^2*d*e*x^2 + A*a^2*
d^2*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

$$\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$
$$= x^3 \left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right) + x^7 \left(\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right) + x^5 \left(\frac{C\,a^2\,e^2}{5}+\frac{2\,C\,a\,c\,d^2}{5}+\frac{4\,B\,a\,c\,d\,e}{5}+\frac{2\,A\,a\,c\,e^2}{5}+\frac{A\,c^2\,d^2}{5}\right) + \frac{a\,x^4\,(B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e)}{4} + \frac{c\,x^6\,(2\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+4\,C\,a\,d\,e)}{6} + \frac{C\,c^2\,e^2\,x^9}{9} + A\,a^2\,d^2\,x + \frac{a^2\,d\,x^2\,(2\,A\,e+B\,d)}{2} + \frac{c^2\,e\,x^8\,(B\,e+2\,C\,d)}{8}$$

input
```
int((a + c*x^2)^2*(d + e*x)^2*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) +
x^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) +
x^5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (
4*B*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4
+ (c*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9
)/9 + A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d)
)/8
```

3.27 $\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx$

3.27.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= a^2Adx+\frac{1}{3}a(2Acd+aCd+aBe)x^3\\&\quad+\frac{1}{4}a^2Cex^4+\frac{1}{5}c(Acd+2a(Cd+Be))x^5\\&\quad+\frac{1}{3}acCex^6+\frac{1}{7}c^2(Cd+Be)x^7\\&\quad+\frac{1}{8}c^2Cex^8+\frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c}\end{aligned}$$

output
```
a^2*A*d*x+1/3*a*(2*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^2*C*e*x^4+1/5*c*(A*c*d+2*a
*(B*e+C*d))*x^5+1/3*a*c*C*e*x^6+1/7*c^2*(B*e+C*d)*x^7+1/8*c^2*C*e*x^8+1/6*
(A*e+B*d)*(c*x^2+a)^3/c
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = a^2Adx+\frac{1}{2}a^2(Bd+Ae)x^2+\frac{1}{3}a(2Acd+aCd+aBe)x^3+\frac{1}{4}a(2Bcd+2Ace+aCe)x^4+\frac{1}{5}c(Acd+2aCd+2aBe)x^5+\frac{1}{6}c(Bcd+Ace+2aCe)x^6+\frac{1}{7}c^2(Cd+Be)x^7+\frac{1}{8}c^2Cex^8$$

input
```
Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3
+ (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x
^5)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c
^2*C*e*x^8)/8
```

3.27.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^2(d+ex)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)^2\left((d+ex)\left(Cx^2+Bx+A\right)-(Bd+Ae)x\right)dx+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(c^2Cex^7 + c^2(Cd+Be)x^6 + 2acCex^5 + c(Acd+2a(Cd+Be))x^4 + a^2Cex^3 + a(2Acd+aCd+aBe)x^2 + a^2A\right.$$

$$\frac{(a+cx^2)^3(Ae+Bd)}{6c}$$

$$\Big\downarrow \text{2009}$$

$$a^2Adx + \frac{1}{4}a^2Cex^4 + \frac{1}{5}cx^5(2a(Be+Cd)+Acd) + \frac{1}{3}ax^3(aBe+aCd+2Acd) + \frac{(a+cx^2)^3(Ae+Bd)}{6c} + \frac{1}{3}acCex^6 + \frac{1}{7}c^2x^7(Be+Cd) + \frac{1}{8}c^2Cex^8$$

input
```
Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*
c*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7
+ (c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.27.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

method	result
default	$\frac{c^2Ce\,x^8}{8}+\frac{(c^2eB+c^2dC)x^7}{7}+\frac{(c^2eA+c^2dB+2aceC)x^6}{6}+\frac{(A\,c^2d+2Bace+2acdC)x^5}{5}+\frac{(2aAce+2Bacd+a^2eC)x^4}{4}+$
norman	$\frac{c^2Ce\,x^8}{8}+\left(\frac{1}{7}c^2eB+\frac{1}{7}c^2dC\right)x^7+\left(\frac{1}{6}c^2eA+\frac{1}{6}c^2dB+\frac{1}{3}aceC\right)x^6+\left(\frac{1}{5}A\,c^2d+\frac{2}{5}Bace+\frac{2}{5}acdC\right.$
gosper	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$
risch	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$
parallelrisch	$\frac{1}{8}c^2Ce\,x^8+\frac{1}{7}B\,c^2e\,x^7+\frac{1}{7}x^7c^2dC+\frac{1}{6}x^6A\,c^2e+\frac{1}{6}x^6B\,c^2d+\frac{1}{3}acCe\,x^6+\frac{1}{5}x^5A\,c^2d+\frac{2}{5}x^5Bac$

input
```
int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*
x^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e
)*x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*
d*x
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\begin{aligned}\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx &= \frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\left(Cc^2d+Bc^2e\right)x^7\\&+\frac{1}{6}\left(Bc^2d+\left(2\,Cac+Ac^2\right)e\right)x^6\\&+\frac{1}{5}\left(2\,Bace+\left(2\,Cac+Ac^2\right)d\right)x^5+Aa^2dx\\&+\frac{1}{4}\left(2\,Bacd+\left(Ca^2+2\,Aac\right)e\right)x^4\\&+\frac{1}{3}\left(Ba^2e+\left(Ca^2+2\,Aac\right)d\right)x^3\\&+\frac{1}{2}\left(Ba^2d+Aa^2e\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c +
A*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/
4*(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)
*d)*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.41

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = Aa^2dx + \frac{Cc^2ex^8}{8} + x^7\left(\frac{Bc^2e}{7}+\frac{Cc^2d}{7}\right) + x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}+\frac{Cace}{3}\right) + x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}+\frac{2Cacd}{5}\right) + x^4\left(\frac{Aace}{2}+\frac{Bacd}{2}+\frac{Ca^2e}{4}\right) + x^3\cdot\left(\frac{2Aacd}{3}+\frac{Ba^2e}{3}+\frac{Ca^2d}{3}\right) + x^2\left(\frac{Aa^2e}{2}+\frac{Ba^2d}{2}\right)$$

input
```
integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

output
```
A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c*
*2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*
c*d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B
*a**2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx=\frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\,(Cc^2d+Bc^2e)x^7+\frac{1}{6}\,(Bc^2d+(2\,Cac+Ac^2)e)x^6+\frac{1}{5}\,(2\,Bace+(2\,Cac+Ac^2)d)x^5+Aa^2dx+\frac{1}{4}\,(2\,Bacd+(Ca^2+2\,Aac)e)x^4+\frac{1}{3}\,(Ba^2e+(Ca^2+2\,Aac)d)x^3+\frac{1}{2}\,(Ba^2d+Aa^2e)x^2$$

input
```
integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c +
A*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/
4*(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)
*d)*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.34

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx=\frac{1}{8}\,Cc^2ex^8+\frac{1}{7}\,Cc^2dx^7+\frac{1}{7}\,Bc^2ex^7+\frac{1}{6}\,Bc^2dx^6+\frac{1}{3}\,Cacex^6+\frac{1}{6}\,Ac^2ex^6+\frac{2}{5}\,Cacdx^5+\frac{1}{5}\,Ac^2dx^5+\frac{2}{5}\,Bacex^5+\frac{1}{2}\,Bacdx^4+\frac{1}{4}\,Ca^2ex^4+\frac{1}{2}\,Aacex^4+\frac{1}{3}\,Ca^2dx^3+\frac{2}{3}\,Aacdx^3+\frac{1}{3}\,Ba^2ex^3+\frac{1}{2}\,Ba^2dx^2+\frac{1}{2}\,Aa^2ex^2+Aa^2dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*C*c^2*e*x^8 + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*e*x^7 + 1/6*B*c^2*d*x^6 + 1/
3*C*a*c*e*x^6 + 1/6*A*c^2*e*x^6 + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*
B*a*c*e*x^5 + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*e*x^4 + 1/2*A*a*c*e*x^4 + 1/3*C*
a^2*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*e*x^3 + 1/2*B*a^2*d*x^2 + 1/2*A*a^
2*e*x^2 + A*a^2*d*x
```

3.27.9 Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx = x^3\left(\frac{B\,a^2\,e}{3}+\frac{C\,a^2\,d}{3}+\frac{2\,A\,a\,c\,d}{3}\right) + x^6\left(\frac{A\,c^2\,e}{6}+\frac{B\,c^2\,d}{6}+\frac{C\,a\,c\,e}{3}\right) + \frac{c\,x^5\,(A\,c\,d+2\,B\,a\,e+2\,C\,a\,d)}{5} + \frac{a\,x^4\,(2\,A\,c\,e+2\,B\,c\,d+C\,a\,e)}{4} + \frac{a^2\,x^2\,(A\,e+B\,d)}{2} + \frac{c^2\,x^7\,(B\,e+C\,d)}{7} + A\,a^2\,d\,x + \frac{C\,c^2\,e\,x^8}{8}$$

input
```
int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2),x)
```

output
```
x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^
2*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*
A*c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*
d))/7 + A*a^2*d*x + (C*c^2*e*x^8)/8
```

3.28 $\int (a+cx^2)^2 (A+Bx+Cx^2) \, dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int (a+cx^2)^2 (A+Bx+Cx^2) \, dx = a^2Ax + \frac{1}{3}a(2Ac+aC)x^3 + \frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B(a+cx^2)^3}{6c}$$

output
```
a^2*A*x+1/3*a*(2*A*c+C*a)*x^3+1/5*c*(A*c+2*C*a)*x^5+1/7*c^2*C*x^7+1/6*B*(c
*x^2+a)^3/c
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int (a+cx^2)^2 (A+Bx+Cx^2) \, dx = \frac{1}{210}x\left(35a^2(6A+x(3B+2Cx)) + 7acx^2(20A+3x(5B+4Cx)) + c^2x^4(42A+5x(7B+6Cx))\right)$$

input
```
Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
(x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x))
+ c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

3.28.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (cx^2+a)^2 (Cx^2+A)\, dx+\frac{B(a+cx^2)^3}{6c}$$

$$\Big\downarrow \text{290}$$

$$\int (c^2Cx^6+c(Ac+2aC)x^4+a(2Ac+aC)x^2+a^2A)\, dx+\frac{B(a+cx^2)^3}{6c}$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B(a+cx^2)^3}{6c}+\frac{1}{7}c^2Cx^7$$

input
```
Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/
7 + (B*(a + c*x^2)^3)/(6*c)
```

3.28.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.28.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result
default	$\frac{c^2 C x^7}{7}+\frac{B c^2 x^6}{6}+\frac{(A c^2+2acC)x^5}{5}+\frac{Bac x^4}{2}+\frac{(2Aac+C a^2)x^3}{3}+\frac{B a^2 x^2}{2}+a^2Ax$
norman	$\frac{c^2 C x^7}{7}+\frac{B c^2 x^6}{6}+\left(\frac{1}{5}A c^2+\frac{2}{5}acC\right)x^5+\frac{Bac x^4}{2}+\left(\frac{2}{3}Aac+\frac{1}{3}C a^2\right)x^3+\frac{B a^2 x^2}{2}+a^2Ax$
gosper	$\frac{1}{7}c^2C x^7+\frac{1}{6}B c^2x^6+\frac{1}{5}A c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac x^4+\frac{2}{3}aAc x^3+\frac{1}{3}x^3C a^2+\frac{1}{2}B a^2x^2+a^2A$
risch	$\frac{1}{7}c^2C x^7+\frac{1}{6}B c^2x^6+\frac{1}{5}A c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac x^4+\frac{2}{3}aAc x^3+\frac{1}{3}x^3C a^2+\frac{1}{2}B a^2x^2+a^2A$
parallelrisch	$\frac{1}{7}c^2C x^7+\frac{1}{6}B c^2x^6+\frac{1}{5}A c^2x^5+\frac{2}{5}x^5acC+\frac{1}{2}Bac x^4+\frac{2}{3}aAc x^3+\frac{1}{3}x^3C a^2+\frac{1}{2}B a^2x^2+a^2A$

input
```
int((c*x^2+a)^2*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*B*a*c*x^4+1/3*(2*A
*a*c+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\,dx = \frac{1}{7}Cc^2x^7+\frac{1}{6}Bc^2x^6+\frac{1}{2}Bacx^4+\frac{1}{5}(2Cac+Ac^2)x^5 + \frac{1}{2}Ba^2x^2+Aa^2x+\frac{1}{3}(Ca^2+2Aac)x^3$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5
+ 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\,dx = Aa^2x + \frac{Ba^2x^2}{2} + \frac{Bacx^4}{2} + \frac{Bc^2x^6}{6} + \frac{Cc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Cac}{5}\right) + x^3\cdot\left(\frac{2Aac}{3} + \frac{Ca^2}{3}\right)$$

input
```
integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 +
x**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\,dx = \frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{1}{2}Bacx^4 + \frac{1}{5}(2Cac + Ac^2)x^5 + \frac{1}{2}Ba^2x^2 + Aa^2x + \frac{1}{3}(Ca^2 + 2Aac)x^3$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5
+ 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\,dx = \frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bacx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aacx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*
x^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int (a+cx^2)^2 (A+Bx+Cx^2)\ dx = x^3 \left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}\right)+x^5 \left(\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right) + \frac{B\,a^2\,x^2}{2}+\frac{B\,c^2\,x^6}{6}+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{B\,a\,c\,x^4}{2}$$

input
```
int((a + c*x^2)^2*(A + B*x + C*x^2),x)
```

output
```
x^3*((C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (2*C*a*c)/5) + (B*a^2*x^2
)/2 + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + A*a^2*x + (B*a*c*x^4)/2
```

3.29 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$

3.29.1 Optimal result

Integrand size = 27, antiderivative size = 297

$$\begin{aligned}\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx &= -\frac{(a^2e^4(Cd-Be)+c^2d^3(Cd^2-e(Bd-Ae))+2acde^2(Cd^2-e(Bd-Ae)))\,x}{e^6} \\ &\quad+\frac{(a^2Ce^4+c^2d^2(Cd^2-e(Bd-Ae))+2ace^2(Cd^2-e(Bd-Ae)))\,x^2}{2e^5} \\ &\quad-\frac{c(2ae^2(Cd-Be)+cd(Cd^2-e(Bd-Ae)))\,x^3}{3e^4}+\frac{c(2aCe^2+c(Cd^2-e(Bd-Ae)))\,x^4}{4e^3} \\ &\quad-\frac{c^2(Cd-Be)x^5}{5e^2}+\frac{c^2Cx^6}{6e}+\frac{(cd^2+ae^2)^2(Cd^2-Bde+Ae^2)\log(d+ex)}{e^7}\end{aligned}$$

output
```
-(a^2*e^4*(-B*e+C*d)+c^2*d^3*(C*d^2-e*(-A*e+B*d))+2*a*c*d*e^2*(C*d^2-e*(-A
*e+B*d)))*x/e^6+1/2*(a^2*C*e^4+c^2*d^2*(C*d^2-e*(-A*e+B*d))+2*a*c*e^2*(C*d
^2-e*(-A*e+B*d)))*x^2/e^5-1/3*c*(2*a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d
)))*x^3/e^4+1/4*c*(2*a*C*e^2+c*(C*d^2-e*(-A*e+B*d)))*x^4/e^3-1/5*c^2*(-B*e
+C*d)*x^5/e^2+1/6*c^2*C*x^6/e+(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*ln(e*x+d
)/e^7
```

3.29.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$$
$$= \frac{ex(30a^2e^4(-2Cd+2Be+Cex)+10ace^2(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+$$

input
```
Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
(e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d
^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x -
4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*
e^3*x^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))
*Log[d + e*x])/(60*e^7)
```

3.29.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{e^5}+\frac{-a^2e^4(Cd-Be)-2acde^2\left(Cd^2-e(B\right.}{e^6}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^5-d^3e(Bd-Ae)\right)\right)}{e^6}-\frac{cx^3\left(2ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3\right)}{3e^4}+\frac{\left(ae^2+cd^2\right)^2\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e^7}+\frac{cx^4\left(2aCe^2-ce(Bd-Ae)+cCd^2\right)}{4e^3}-\frac{c^2x^5(Cd-Be)}{5e^2}+\frac{c^2Cx^6}{6e}$$

input
```
Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
-(((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
 - d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d -
 A*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d
*e*(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e
^2 - c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*
C*x^6)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^
7
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.29.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.26

method	result
norman	$\frac{\left(2Aac\,e^4+A\,c^2d^2e^2-2Bacd\,e^3-B\,c^2d^3e+a^2C\,e^4+2Cac\,d^2e^2+C\,c^2d^4\right)x^2}{2e^5}-\frac{(2Aacd\,e^4+A\,c^2d^3e^2-B\,e^5a^2-2Bac\,d^2e^3-B\,c^{}}{e^6}$
default	$-\frac{2Cac\,d^3e^2x-2Bxac\,d^2e^3+2Axacd\,e^4+B\,x^2acd\,e^4-\frac{1}{2}C\,a^2e^5x^2-\frac{1}{6}c^2C\,x^6e^5-Bx\,a^2e^5-\frac{1}{4}A\,x^4c^2e^5-\frac{1}{5}B\,x^5c^2e^5+C\,c^2d^5x-}{}$
risch	$-\frac{Ax\,c^2d^3}{e^4}+\frac{B\,c^2x^5}{5e}+\frac{\ln(ex+d)A\,a^2}{e}+\frac{Bx\,a^2}{e}+\frac{A\,x^4c^2}{4e}+\frac{C\,a^2x^2}{2e}-\frac{C\,c^2d^5x}{e^6}-\frac{C\,c^2d\,x^5}{5e^2}+\frac{2\ln(ex+d)Aac\,d^2}{e^3}-$
parallelrisch	$\frac{-30B\,x^2c^2d^3e^3-12C\,x^5c^2d\,e^5-15B\,x^4c^2d\,e^5+60C\ln(ex+d)a^2d^2e^4+60A\ln(ex+d)c^2d^4e^2-60B\ln(ex+d)a^2d\,e^5-60B\ln(ex+}{}$

input
```
int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^5*(2*A*a*c*e^4+A*c^2*d^2*e^2-2*B*a*c*d*e^3-B*c^2*d^3*e+C*a^2*e^4+2*C
*a*c*d^2*e^2+C*c^2*d^4)*x^2-(2*A*a*c*d*e^4+A*c^2*d^3*e^2-B*a^2*e^5-2*B*a*c
*d^2*e^3-B*c^2*d^4*e+C*a^2*d*e^4+2*C*a*c*d^3*e^2+C*c^2*d^5)/e^6*x-1/3*c/e^
4*(A*c*d*e^2-2*B*a*e^3-B*c*d^2*e+2*C*a*d*e^2+C*c*d^3)*x^3+1/4*c/e^3*(A*c*e
^2-B*c*d*e+2*C*a*e^2+C*c*d^2)*x^4+1/6*c^2*C*x^6/e+1/5*c^2/e^2*(B*e-C*d)*x^
5+(A*a^2*e^6+2*A*a*c*d^2*e^4+A*c^2*d^4*e^2-B*a^2*d*e^5-2*B*a*c*d^3*e^3-B*c
^2*d^5*e+C*a^2*d^2*e^4+2*C*a*c*d^4*e^2+C*c^2*d^6)/e^7*ln(e*x+d)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.28

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{10\,Cc^2e^6x^6 - 12\,(Cc^2de^5 - Bc^2e^6)x^5 + 15\,(Cc^2d^2e^4 - Bc^2de^5 + (2\,Cac + Ac^2)e^6)x^4 - 20\,(Cc^2d^3e^3 - Bc^2}{}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

output
```
1/60*(10*C*c^2*e^6*x^6 - 12*(C*c^2*d*e^5 - B*c^2*e^6)*x^5 + 15*(C*c^2*d^2*
e^4 - B*c^2*d*e^5 + (2*C*a*c + A*c^2)*e^6)*x^4 - 20*(C*c^2*d^3*e^3 - B*c^2
*d^2*e^4 - 2*B*a*c*e^6 + (2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(C*c^2*d^4*e^2
- B*c^2*d^3*e^3 - 2*B*a*c*d*e^5 + (2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A
*a*c)*e^6)*x^2 - 60*(C*c^2*d^5*e - B*c^2*d^4*e^2 - 2*B*a*c*d^2*e^4 - B*a^2
*e^6 + (2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x + 60*(C*c^2*
d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c +
 A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d))/e^7
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.21

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{Cc^2x^6}{6e} + x^5\left(\frac{Bc^2}{5e} - \frac{Cc^2d}{5e^2}\right) + x^4\left(\frac{Ac^2}{4e} - \frac{Bc^2d}{4e^2} + \frac{Cac}{2e} + \frac{Cc^2d^2}{4e^3}\right) + x^3\left(-\frac{Ac^2d}{3e^2} + \frac{2Bac}{3e} + \frac{Bc^2d^2}{3e^3} - \frac{2Cacd}{3e^2} - \frac{Cc^2d^3}{3e^4}\right) + x^2\left(\frac{Aac}{e} + \frac{Ac^2d^2}{2e^3} - \frac{Bacd}{e^2} - \frac{Bc^2d^3}{2e^4} + \frac{Ca^2}{2e} + \frac{Cacd^2}{e^3} + \frac{Cc^2d^4}{2e^5}\right) + x\left(-\frac{2Aacd}{e^2} - \frac{Ac^2d^3}{e^4} + \frac{Ba^2}{e} + \frac{2Bacd^2}{e^3} + \frac{Bc^2d^4}{e^5} - \frac{Ca^2d}{e^2} - \frac{2Cacd^3}{e^4} - \frac{Cc^2d^5}{e^6}\right) + \frac{(ae^2+cd^2)^2(Ae^2-Bde+Cd^2)\log(d+ex)}{e^7}$$

input
```
integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

output
```
C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2
/(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-
A*c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e*
*2) - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c
*d/e**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d
**4/(2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c
*d**2/e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2
*d**5/e**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/
e**7
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{10\,Cc^2e^5x^6 - 12\,(Cc^2de^4 - Bc^2e^5)x^5 + 15\,(Cc^2d^2e^3 - Bc^2de^4 + (2\,Cac + Ac^2)e^5)x^4 - 20\,(Cc^2d^3e^2 - Bc^2}{} + \frac{(Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + (2\,Cac + Ac^2)d^4e^2 + (Ca^2 + 2\,Aac)d^2e^4)\log(ex +}{e^7}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

output
```
1/60*(10*C*c^2*e^5*x^6 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2*d^2*
e^3 - B*c^2*d*e^4 + (2*C*a*c + A*c^2)*e^5)*x^4 - 20*(C*c^2*d^3*e^2 - B*c^2
*d^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c + A*c^2)*d*e^4)*x^3 + 30*(C*c^2*d^4*e -
B*c^2*d^3*e^2 - 2*B*a*c*d*e^4 + (2*C*a*c + A*c^2)*d^2*e^3 + (C*a^2 + 2*A*a
*c)*e^5)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3 - B*a^2*e^5 +
 (2*C*a*c + A*c^2)*d^3*e^2 + (C*a^2 + 2*A*a*c)*d*e^4)*x)/e^6 + (C*c^2*d^6
- B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c
^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d)/e^7
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{d+ex}\,dx$$
$$= \frac{10\,Cc^2e^5x^6 - 12\,Cc^2de^4x^5 + 12\,Bc^2e^5x^5 + 15\,Cc^2d^2e^3x^4 - 15\,Bc^2de^4x^4 + 30\,Cace^5x^4 + 15\,Ac^2e^5x^4 - 20\,\cdots}{\cdots}$$
$$+ \frac{(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6)\log(\cdots}{e^7}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

output
```
1/60*(10*C*c^2*e^5*x^6 - 12*C*c^2*d*e^4*x^5 + 12*B*c^2*e^5*x^5 + 15*C*c^2*
d^2*e^3*x^4 - 15*B*c^2*d*e^4*x^4 + 30*C*a*c*e^5*x^4 + 15*A*c^2*e^5*x^4 - 2
0*C*c^2*d^3*e^2*x^3 + 20*B*c^2*d^2*e^3*x^3 - 40*C*a*c*d*e^4*x^3 - 20*A*c^2
*d*e^4*x^3 + 40*B*a*c*e^5*x^3 + 30*C*c^2*d^4*e*x^2 - 30*B*c^2*d^3*e^2*x^2
+ 60*C*a*c*d^2*e^3*x^2 + 30*A*c^2*d^2*e^3*x^2 - 60*B*a*c*d*e^4*x^2 + 30*C*
a^2*e^5*x^2 + 60*A*a*c*e^5*x^2 - 60*C*c^2*d^5*x + 60*B*c^2*d^4*e*x - 120*C
*a*c*d^3*e^2*x - 60*A*c^2*d^3*e^2*x + 120*B*a*c*d^2*e^3*x - 60*C*a^2*d*e^4
*x - 120*A*a*c*d*e^4*x + 60*B*a^2*e^5*x)/e^6 + (C*c^2*d^6 - B*c^2*d^5*e +
2*C*a*c*d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3 + C*a^2*d^2*e^4 + 2*A*a*
c*d^2*e^4 - B*a^2*d*e^5 + A*a^2*e^6)*log(abs(e*x + d))/e^7
```

3.29.9 Mupad [B] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{d+ex}\,dx = x^5\left(\frac{B\,c^2}{5\,e}-\frac{C\,c^2\,d}{5\,e^2}\right) - x\left(\frac{d\left(\frac{C\,a^2+2\,A\,c\,a}{e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}\right)}{e}-\frac{B\,a^2}{e}\right) + x^4\left(\frac{A\,c^2+2\,C\,a\,c}{4\,e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{4\,e}\right) - x^3\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{3\,e}-\frac{2\,B\,a\,c}{3\,e}\right) + x^2\left(\frac{C\,a^2+2\,A\,c\,a}{2\,e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{2\,e}\right) + \frac{\ln(d+e\,x)\,\left(C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5+A\,a^2\,e^6+2\,C\,a\,c\,d^4\,e^2-2\,B\,a\,c\,d^3\,e^3+2\,A\,a\,c\,d^2\,e^4+C\,c^2\,d^6-B\,c\right.}{e^7} + \frac{C\,c^2\,x^6}{6\,e}$$

input
```
int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x)
```

output
```
x^5*((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x*((d*((C*a^2 + 2*A*a*c)/e + (d*
((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*
c)/e))/e))/e - (B*a^2)/e) + x^4*((A*c^2 + 2*C*a*c)/(4*e) - (d*((B*c^2)/e -
 (C*c^2*d)/e^2))/(4*e)) - x^3*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (
C*c^2*d)/e^2))/e))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((C*a^2 + 2*A*a*c)/(2*e)
 + (d*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (
2*B*a*c)/e))/(2*e)) + (log(d + e*x)*(A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 -
 B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d
^3*e^3 + 2*C*a*c*d^4*e^2))/e^7 + (C*c^2*x^6)/(6*e)
```

3.29. $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx$

3.30 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.30.1 Optimal result

Integrand size = 27, antiderivative size = 292

$$\begin{aligned}\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{(a^2Ce^4+c^2d^2(5Cd^2-e(4Bd-3Ae))+2ace^2(3Cd^2-e(2Bd-Ae)))\,x}{e^6} \\ &- \frac{c(2ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\,x^2}{2e^5} \\ &+ \frac{c(2aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x^3}{3e^4} - \frac{c^2(2Cd-Be)x^4}{4e^3} \\ &+ \frac{c^2Cx^5}{5e^2} - \frac{(cd^2+ae^2)^2(Cd^2-Bde+Ae^2)}{e^7(d+ex)} \\ &- \frac{(cd^2+ae^2)(ae^2(2Cd-Be)+cd(6Cd^2-e(5Bd-4Ae)))\log(d+ex)}{e^7}\end{aligned}$$

output
```
(a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+2*a*c*e^2*(3*C*d^2-e*(-A*e+2
*B*d)))*x/e^6-1/2*c*(2*a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*
x^2/e^5+1/3*c*(2*a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^4-1/4*c^2*(-B*e
+2*C*d)*x^4/e^3+1/5*c^2*C*x^5/e^2-(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/
(e*x+d)-(a*e^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))*
ln(e*x+d)/e^7
```

3.30.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2} dx$$
$$= \frac{60e(a^2Ce^4+2ace^2(3Cd^2+e(-2Bd+Ae))+c^2(5Cd^4+d^2e(-4Bd+3Ae)))\,x-30ce^2(4cCd^3+cde(-3}{}$$

input
```
Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
(60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
 d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e)
 - 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2
*B*d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*
(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^
2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x]
)/(60*e^7)
```

3.30.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2} dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)}{e^6}+\frac{cx\left(-2ae^2(2Cd-Be)+cde(3Bd-\right.}{e^5}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{e^6}-\frac{cx^2\left(2ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{2e^5}-\frac{\left(ae^2+cd^2\right)^2\left(Ae^2-Bde+Cd^2\right)}{e^7(d+ex)}+\frac{cx^3\left(2aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{3e^4}-\frac{\left(ae^2+cd^2\right)\log(d+ex)\left(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3\right)}{e^7}-\frac{c^2x^4(2Cd-Be)}{4e^3}+\frac{c^2Cx^5}{5e^2}$$

input
```
Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
 e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^
2*(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A
*e))*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2)
- ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 +
a*e^2)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e
*x])/e^7
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.30.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.37

method	result
norman	$\frac{\left(A a^2 e^6+4 A a c d^2 e^4+4 A c^2 d^4 e^2-B a^2 d e^5-6 B a c d^3 e^3-5 B c^2 d^5 e+2 C a^2 d^2 e^4+8 C a c d^4 e^2+6 C c^2 d^6\right) x}{e^6 d}+\frac{\left(4 A a c e^4+4 A c^2 d^2 e^2-6 B a c d e^3-\right.}{}$
default	$\frac{\frac{1}{5} c^2 C x^5 e^4+\frac{1}{4} B c^2 e^4 x^4-\frac{1}{2} C c^2 d e^3 x^4+\frac{1}{3} A c^2 e^4 x^3-\frac{2}{3} B c^2 d e^3 x^3+\frac{2}{3} C a c e^4 x^3+C c^2 d^2 e^2 x^3-A c^2 d e^3 x^2+B a c e^4 x^2+\frac{3}{2} B c^2 d^2 e}{e^6}$
risch	$-\frac{C c^2 d x^4}{2 e^3}-\frac{2 B c^2 d x^3}{3 e^3}+\frac{B c^2 d^5}{e^6 (e x+d)}-\frac{C a^2 d^2}{e^3 (e x+d)}-\frac{C c^2 d^6}{e^7 (e x+d)}+\frac{6 C a c d^2 x}{e^4}+\frac{A c^2 x^3}{3 e^2}+\frac{a^2 C x}{e^2}+\frac{B c^2 x^4}{4 e^2}+\frac{\ln(ex}{}$
parallelrisch	$-\frac{150 B x^2 c^2 d^3 e^3+18 C x^5 c^2 d e^5+25 B x^4 c^2 d e^5+120 C \ln(e x+d) a^2 d^2 e^4+240 A \ln(e x+d) c^2 d^4 e^2-60 B \ln(e x+d) a^2 d e^5-300 B \ln}{}$

input
```
int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^2*e^6+4*A*a*c*d^2*e^4+4*A*c^2*d^4*e^2-B*a^2*d*e^5-6*B*a*c*d^3*e^3-5*
B*c^2*d^5*e+2*C*a^2*d^2*e^4+8*C*a*c*d^4*e^2+6*C*c^2*d^6)/e^6/d*x+1/2*(4*A*
a*c*e^4+4*A*c^2*d^2*e^2-6*B*a*c*d*e^3-5*B*c^2*d^3*e+2*C*a^2*e^4+8*C*a*c*d^
2*e^2+6*C*c^2*d^4)/e^5*x^2+1/12*c*(4*A*c*e^2-5*B*c*d*e+8*C*a*e^2+6*C*c*d^2
)/e^3*x^4-1/6*c*(4*A*c*d*e^2-6*B*a*e^3-5*B*c*d^2*e+8*C*a*d*e^2+6*C*c*d^3)/
e^4*x^3+1/5*c^2*C*x^6/e+1/20*c^2*(5*B*e-6*C*d)/e^2*x^5)/(e*x+d)-(4*A*a*c*d
*e^4+4*A*c^2*d^3*e^2-B*a^2*e^5-6*B*a*c*d^2*e^3-5*B*c^2*d^4*e+2*C*a^2*d*e^4
+8*C*a*c*d^3*e^2+6*C*c^2*d^5)/e^7*ln(e*x+d)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.89

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2} \, dx$$
$$= \frac{12 \, C c^2 e^6 x^6 - 60 \, C c^2 d^6 + 60 \, B c^2 d^5 e + 120 \, B a c d^3 e^3 + 60 \, B a^2 d e^5 - 60 \, A a^2 e^6 - 60 \, (2 \, C a c + A c^2) d^4 e^2 - 60}{}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/60*(12*C*c^2*e^6*x^6 - 60*C*c^2*d^6 + 60*B*c^2*d^5*e + 120*B*a*c*d^3*e^3
 + 60*B*a^2*d*e^5 - 60*A*a^2*e^6 - 60*(2*C*a*c + A*c^2)*d^4*e^2 - 60*(C*a^
2 + 2*A*a*c)*d^2*e^4 - 3*(6*C*c^2*d*e^5 - 5*B*c^2*e^6)*x^5 + 5*(6*C*c^2*d^
2*e^4 - 5*B*c^2*d*e^5 + 4*(2*C*a*c + A*c^2)*e^6)*x^4 - 10*(6*C*c^2*d^3*e^3
 - 5*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 4*(2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(6*
C*c^2*d^4*e^2 - 5*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 4*(2*C*a*c + A*c^2)*d^2*
e^4 + 2*(C*a^2 + 2*A*a*c)*e^6)*x^2 + 60*(5*C*c^2*d^5*e - 4*B*c^2*d^4*e^2 -
 4*B*a*c*d^2*e^4 + 3*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*
x - 60*(6*C*c^2*d^6 - 5*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 - B*a^2*d*e^5 + 4*(2
*C*a*c + A*c^2)*d^4*e^2 + 2*(C*a^2 + 2*A*a*c)*d^2*e^4 + (6*C*c^2*d^5*e - 5
*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^2)*d^3*e^3
 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx &= \frac{Cc^2x^5}{5e^2} + x^4\left(\frac{Bc^2}{4e^2} - \frac{Cc^2d}{2e^3}\right)\\ &+ x^3\left(\frac{Ac^2}{3e^2} - \frac{2Bc^2d}{3e^3} + \frac{2Cac}{3e^2} + \frac{Cc^2d^2}{e^4}\right) + x^2\left(-\frac{Ac^2d}{e^3} + \frac{Bac}{e^2} + \frac{3Bc^2d^2}{2e^4} - \frac{2Cacd}{e^3} - \frac{2Cc^2d^3}{e^5}\right)\\ &+ x\left(\frac{2Aac}{e^2} + \frac{3Ac^2d^2}{e^4} - \frac{4Bacd}{e^3} - \frac{4Bc^2d^3}{e^5} + \frac{Ca^2}{e^2} + \frac{6Cacd^2}{e^4} + \frac{5Cc^2d^4}{e^6}\right)\\ &+ \frac{-Aa^2e^6 - 2Aacd^2e^4 - Ac^2d^4e^2 + Ba^2de^5 + 2Bacd^3e^3 + Bc^2d^5e - Ca^2d^2e^4 - 2Cacd^4e^2 - Cc^2d^6}{de^7 + e^8x}\\ &- \frac{(ae^2 + cd^2)\,(4Acde^2 - Bae^3 - 5Bcd^2e + 2Cade^2 + 6Ccd^3)\log(d + ex)}{e^7}\end{aligned}$$

input
```
integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

output
```
C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(
A*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**
4) + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*
d/e**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*
a*c*d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c*
*2*d**4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a
**2*d*e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*
c*d**4*e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*
e**2 - B*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e
**7
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2}\,dx = -\frac{Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + (2\,Cac + Ac^2)d^4e^2 + (Ca^2 + 2\,Aac)d^2e^4}{e^8x + de^7} + \frac{12\,Cc^2e^4x^5 - 15\,(2\,Cc^2de^3 - Bc^2e^4)x^4 + 20\,(3\,Cc^2d^2e^2 - 2\,Bc^2de^3 + (2\,Cac + Ac^2)e^4)x^3 - 30\,(4\,Cc^2d^3}{} - \frac{(6\,Cc^2d^5 - 5\,Bc^2d^4e - 6\,Bacd^2e^3 - Ba^2e^5 + 4\,(2\,Cac + Ac^2)d^3e^2 + 2\,(Ca^2 + 2\,Aac)de^4)\log(ex + d)}{e^7}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2
*C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)/(e^8*x + d*e^7) + 1/6
0*(12*C*c^2*e^4*x^5 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^2
*e^2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*
B*c^2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2
*d^4 - 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^
2 + 2*A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3
- B*a^2*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log
(e*x + d)/e^7
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.78

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2}\,dx = \frac{\left(12\,Cc^2 - \frac{15\,(6\,Cc^2de - Bc^2e^2)}{(ex+d)e} + \frac{20\,(15\,Cc^2d^2e^2 - 5\,Bc^2de^3 + 2\,Cace^4 + Ac^2e^4)}{(ex+d)^2e^2} - \frac{60\,(10\,Cc^2d^3e^3 - 5\,Bc^2d^2e^4 + 4\,Cacde^5 + 2\,Ac^2de^5 - Ba}{(ex+d)^3e^3}\right.}{60\,e^7} + \frac{(6\,Cc^2d^5 - 5\,Bc^2d^4e + 8\,Cacd^3e^2 + 4\,Ac^2d^3e^2 - 6\,Bacd^2e^3 + 2\,Ca^2de^4 + 4\,Aacde^4 - Ba^2e^5)\log\left(\frac{|ex+}{(ex+d)}\right.}{e^7} - \frac{\frac{Cc^2d^6e^5}{ex+d} - \frac{Bc^2d^5e^6}{ex+d} + \frac{2\,Cacd^4e^7}{ex+d} + \frac{Ac^2d^4e^7}{ex+d} - \frac{2\,Bacd^3e^8}{ex+d} + \frac{Ca^2d^2e^9}{ex+d} + \frac{2\,Aacd^2e^9}{ex+d} - \frac{Ba^2de^{10}}{ex+d} + \frac{Aa^2e^{11}}{ex+d}}{e^{12}}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)/((e*x + d)*e) + 20*(15*C*c^2
*d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)/((e*x + d)^2*e^2) - 60
*(10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*a
*c*e^6)/((e*x + d)^3*e^3) + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*C
*a*c*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)/
((e*x + d)^4*e^4))*(e*x + d)^5/e^7 + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C*a*
c*d^3*e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*d*
e^4 - B*a^2*e^5)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^7 - (C*c^2*d^6*e
^5/(e*x + d) - B*c^2*d^5*e^6/(e*x + d) + 2*C*a*c*d^4*e^7/(e*x + d) + A*c^2
*d^4*e^7/(e*x + d) - 2*B*a*c*d^3*e^8/(e*x + d) + C*a^2*d^2*e^9/(e*x + d) +
 2*A*a*c*d^2*e^9/(e*x + d) - B*a^2*d*e^10/(e*x + d) + A*a^2*e^11/(e*x + d)
)/e^12
```

3.30. $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.30.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.97

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

$$= x^4\left(\frac{B\,c^2}{4\,e^2}-\frac{C\,c^2\,d}{2\,e^3}\right)+x\left(\frac{C\,a^2+2\,A\,c\,a}{e^2}+\frac{d^2\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e^2}-\frac{2\,d\left(\frac{2\,d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e^2}+\frac{2\,B\,a\,c}{e^2}\right)}{e}\right)$$

$$-x^3\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{3\,e}-\frac{A\,c^2+2\,C\,a\,c}{3\,e^2}+\frac{C\,c^2\,d^2}{3\,e^4}\right)$$

$$+x^2\left(\frac{d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{2\,e^2}+\frac{B\,a\,c}{e^2}\right)$$

$$-\frac{C\,a^2\,d^2\,e^4-B\,a^2\,d\,e^5+A\,a^2\,e^6+2\,C\,a\,c\,d^4\,e^2-2\,B\,a\,c\,d^3\,e^3+2\,A\,a\,c\,d^2\,e^4+C\,c^2\,d^6-B\,c^2\,d^5\,e+A\,c^2}{e\,(x\,e^7+d\,e^6)}$$

$$-\frac{\ln(d+e\,x)\,(2\,C\,a^2\,d\,e^4-B\,a^2\,e^5+8\,C\,a\,c\,d^3\,e^2-6\,B\,a\,c\,d^2\,e^3+4\,A\,a\,c\,d\,e^4+6\,C\,c^2\,d^5-5\,B\,c^2\,d^4\,e+}{e^7}$$

$$+\frac{C\,c^2\,x^5}{5\,e^2}$$

input
```
int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

output
```
x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^
2*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^
2*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (
A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*
d)/e^3))/e^2 + (2*B*a*c)/e^2))/e) - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e
^3))/(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((
2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^
2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2)
- (A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a
^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^
6 + e^7*x)) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B
*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d
^3*e^2))/e^7 + (C*c^2*x^5)/(5*e^2)
```

3.30. $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3} dx$

3.31.1 Optimal result

Integrand size = 27, antiderivative size = 295

$$\begin{aligned}\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3} dx &= -\frac{c(2ae^2(3Cd-Be)+cd(10Cd^2-3e(2Bd-Ae)))x}{e^6} \\ &+ \frac{c(2aCe^2+c(6Cd^2-e(3Bd-Ae)))x^2}{2e^5} - \frac{c^2(3Cd-Be)x^3}{3e^4} \\ &+ \frac{c^2Cx^4}{4e^3} - \frac{(cd^2+ae^2)^2(Cd^2-Bde+Ae^2)}{2e^7(d+ex)^2} \\ &+ \frac{(cd^2+ae^2)(ae^2(2Cd-Be)+cd(6Cd^2-e(5Bd-4Ae)))}{e^7(d+ex)} \\ &+ \frac{(a^2Ce^4+c^2d^2(15Cd^2-2e(5Bd-3Ae))+2ace^2(6Cd^2-e(3Bd-Ae)))\log(d+ex)}{e^7}\end{aligned}$$

output
```
-c*(2*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^6+1/2*c*(2*a
*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^5-1/3*c^2*(-B*e+3*C*d)*x^3/e^4+1/
4*c^2*C*x^4/e^3-1/2*(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x+d)^2+(a*e
^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))/e^7/(e*x+d)+
(a^2*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+2*a*c*e^2*(6*C*d^2-e*(-A*
e+3*B*d)))*ln(e*x+d)/e^7
```

3.31.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.93

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{-12ce(10cCd^3+3cde(-2Bd+Ae)-2ae^2(-3Cd+Be))x+6ce^2(6cCd^2+2aCe^2+ce(-3Bd+Ae))x}{}$$

input
```
Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
(-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x
+ 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3
*C*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d
) + A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d +
4*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*
d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d
 + e*x])/(12*e^7)
```

3.31.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{a^2Ce^4+2ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae))}{e^6(d+ex)}+\frac{c(-2ae^2(3Cd-Be)+3cde(2Bd}{e^6}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{e^7}-\frac{cx\left(2ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3\right)}{e^6}-\frac{\left(ae^2+cd^2\right)^2\left(Ae^2-Bde+Cd^2\right)}{2e^7(d+ex)^2}+\frac{cx^2\left(2aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{2e^5}+\frac{\left(ae^2+cd^2\right)\left(ae^2(2Cd-Be)-cde(5Bd-4Ae)+6cCd^3\right)}{e^7(d+ex)}-\frac{c^2x^3(3Cd-Be)}{3e^4}+\frac{c^2Cx^4}{4e^3}$$

input
```
Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
-((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6)
+ (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*
d - B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 -
B*d*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*
(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^
2*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A
*e)))*Log[d + e*x])/e^7
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.31.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.33

method	result
norman	$\frac{\left(4Aacd\,e^4+12A\,c^2d^3e^2-B\,e^5a^2-12Bac\,d^2e^3-20B\,c^2d^4e+2C\,a^2d\,e^4+24Cac\,d^3e^2+30C\,c^2d^5\right)x}{e^6}-\frac{A\,a^2e^6-6Aac\,d^2e^4-18A\,c^2d^4e^2+B\,a^2d}{}$
default	$-\frac{c\left(-\frac{1}{4}cC\,x^4e^3-\frac{1}{3}Bc\,x^3e^3+Ccd\,e^2x^3-\frac{1}{2}Ac\,e^3x^2+\frac{3}{2}B\,x^2cd\,e^2-Ca\,e^3x^2-3Cc\,d^2e\,x^2+3Acd\,e^2x-2Bxa\,e^3-6Bc\,d^2ex+6Cad\right.}{e^6}$
risch	$\frac{c^2C\,x^4}{4e^3}+\frac{c^2B\,x^3}{3e^3}-\frac{c^2Cd\,x^3}{e^4}+\frac{c^2A\,x^2}{2e^3}-\frac{3c^2B\,x^2d}{2e^4}+\frac{cCa\,x^2}{e^3}+\frac{3c^2C\,d^2x^2}{e^5}-\frac{3c^2Adx}{e^4}+\frac{2cBxa}{e^3}+\frac{6c^2B\,d^2x}{e^5}-$
parallelrisch	$12C\ln(ex+d)x^2a^2e^6-6C\,x^5c^2d\,e^5-10B\,x^4c^2d\,e^5+12C\ln(ex+d)a^2d^2e^4+72A\ln(ex+d)c^2d^4e^2-120B\ln(ex+d)c^2d^5e+12C\,x^4$

input
```
int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((4*A*a*c*d*e^4+12*A*c^2*d^3*e^2-B*a^2*e^5-12*B*a*c*d^2*e^3-20*B*c^2*d^4*e
+2*C*a^2*d*e^4+24*C*a*c*d^3*e^2+30*C*c^2*d^5)/e^6*x-1/2*(A*a^2*e^6-6*A*a*c
*d^2*e^4-18*A*c^2*d^4*e^2+B*a^2*d*e^5+18*B*a*c*d^3*e^3+30*B*c^2*d^5*e-3*C*
a^2*d^2*e^4-36*C*a*c*d^4*e^2-45*C*c^2*d^6)/e^7+1/12*c*(6*A*c*e^2-10*B*c*d*
e+12*C*a*e^2+15*C*c*d^2)/e^3*x^4-1/3*c*(6*A*c*d*e^2-6*B*a*e^3-10*B*c*d^2*e
+12*C*a*d*e^2+15*C*c*d^3)/e^4*x^3+1/4*c^2*C*x^6/e+1/6*c^2*(2*B*e-3*C*d)/e^
2*x^5)/(e*x+d)^2+1/e^7*(2*A*a*c*e^4+6*A*c^2*d^2*e^2-6*B*a*c*d*e^3-10*B*c^2
*d^3*e+C*a^2*e^4+12*C*a*c*d^2*e^2+15*C*c^2*d^4)*ln(e*x+d)
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(287) = 574$.

Time = 0.29 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.06

$$\int \frac{(a+cx^2)^2\,(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$=\frac{3\,Cc^2e^6x^6+66\,Cc^2d^6-54\,Bc^2d^5e-60\,Bacd^3e^3-6\,Ba^2de^5-6\,Aa^2e^6+42\,(2\,Cac+Ac^2)d^4e^2+18\,(Ca^2}{}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/12*(3*C*c^2*e^6*x^6 + 66*C*c^2*d^6 - 54*B*c^2*d^5*e - 60*B*a*c*d^3*e^3 -
 6*B*a^2*d*e^5 - 6*A*a^2*e^6 + 42*(2*C*a*c + A*c^2)*d^4*e^2 + 18*(C*a^2 +
2*A*a*c)*d^2*e^4 - 2*(3*C*c^2*d*e^5 - 2*B*c^2*e^6)*x^5 + (15*C*c^2*d^2*e^4
 - 10*B*c^2*d*e^5 + 6*(2*C*a*c + A*c^2)*e^6)*x^4 - 4*(15*C*c^2*d^3*e^3 - 1
0*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 6*(2*C*a*c + A*c^2)*d*e^5)*x^3 - 6*(34*C*c
^2*d^4*e^2 - 21*B*c^2*d^3*e^3 - 8*B*a*c*d*e^5 + 11*(2*C*a*c + A*c^2)*d^2*e
^4)*x^2 - 12*(4*C*c^2*d^5*e - B*c^2*d^4*e^2 + 4*B*a*c*d^2*e^4 + B*a^2*e^6
- (2*C*a*c + A*c^2)*d^3*e^3 - 2*(C*a^2 + 2*A*a*c)*d*e^5)*x + 12*(15*C*c^2*
d^6 - 10*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 + 6*(2*C*a*c + A*c^2)*d^4*e^2 + (C*
a^2 + 2*A*a*c)*d^2*e^4 + (15*C*c^2*d^4*e^2 - 10*B*c^2*d^3*e^3 - 6*B*a*c*d*
e^5 + 6*(2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c)*e^6)*x^2 + 2*(15*C*c
^2*d^5*e - 10*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 + 6*(2*C*a*c + A*c^2)*d^3*e^
3 + (C*a^2 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e
^7)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.61

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{Cc^2x^4}{4e^3} + x^3\left(\frac{Bc^2}{3e^3} - \frac{Cc^2d}{e^4}\right) + x^2\left(\frac{Ac^2}{2e^3} - \frac{3Bc^2d}{2e^4} + \frac{Cac}{e^3} + \frac{3Cc^2d^2}{e^5}\right)$$
$$+ x\left(-\frac{3Ac^2d}{e^4} + \frac{2Bac}{e^3} + \frac{6Bc^2d^2}{e^5} - \frac{6Cacd}{e^4} - \frac{10Cc^2d^3}{e^6}\right)$$
$$+ \frac{-Aa^2e^6 + 6Aacd^2e^4 + 7Ac^2d^4e^2 - Ba^2de^5 - 10Bacd^3e^3 - 9Bc^2d^5e + 3Ca^2d^2e^4 + 14Cacd^4e^2 + 11Cc^2}{2d^2e^7 + 4d}$$
$$+ \frac{(2Aace^4 + 6Ac^2d^2e^2 - 6Bacde^3 - 10Bc^2d^3e + Ca^2e^4 + 12Cacd^2e^2 + 15Cc^2d^4)\log(d+ex)}{e^7}$$

input
```
integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

output
```
C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c*
*2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(
-3*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10
*C*c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**
2 - B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e
**4 + 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d
**3*e**3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*
a**2*d*e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e
**8*x + 2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3
 - 10*B*c**2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*l
og(d + e*x)/e**7
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{11\,Cc^2d^6 - 9\,Bc^2d^5e - 10\,Bacd^3e^3 - Ba^2de^5 - Aa^2e^6 + 7\,(2\,Cac + Ac^2)d^4e^2 + 3\,(Ca^2 + 2\,Aac)d^2e^4 + 2\,(\cdots}{2\,(e^9x^2 + 2\,de^8x + \cdots}$$
$$+ \frac{3\,Cc^2e^3x^4 - 4\,(3\,Cc^2de^2 - Bc^2e^3)x^3 + 6\,(6\,Cc^2d^2e - 3\,Bc^2de^2 + (2\,Cac + Ac^2)e^3)x^2 - 12\,(10\,Cc^2d^3 - \cdots}{12\,e^6}$$
$$+ \frac{(15\,Cc^2d^4 - 10\,Bc^2d^3e - 6\,Bacde^3 + 6\,(2\,Cac + Ac^2)d^2e^2 + (Ca^2 + 2\,Aac)e^4)\log{(ex + d)}}{e^7}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e - 10*B*a*c*d^3*e^3 - B*a^2*d*e^5 - A*a^2
*e^6 + 7*(2*C*a*c + A*c^2)*d^4*e^2 + 3*(C*a^2 + 2*A*a*c)*d^2*e^4 + 2*(6*C*
c^2*d^5*e - 5*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A
*c^2)*d^3*e^3 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e
^7) + 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c
^2*d^2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 -
 6*B*c^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c
^2*d^4 - 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C
*a^2 + 2*A*a*c)*e^4)*log(e*x + d)/e^7
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.44

$$\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4)\log(|ex+d|)}{e^7}$$
$$+ \frac{11\,Cc^2d^6 - 9\,Bc^2d^5e + 14\,Cacd^4e^2 + 7\,Ac^2d^4e^2 - 10\,Bacd^3e^3 + 3\,Ca^2d^2e^4 + 6\,Aacd^2e^4 - Ba^2de^5 - Aa}{2\,(ex -}$$
$$+ \frac{3\,Cc^2e^9x^4 - 12\,Cc^2de^8x^3 + 4\,Bc^2e^9x^3 + 36\,Cc^2d^2e^7x^2 - 18\,Bc^2de^8x^2 + 12\,Cace^9x^2 + 6\,Ac^2e^9x^2 - 120}{12\,e^{12}}$$

input
```
integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

output
```
(15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*
a*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*log(abs(e*x + d))/e^7 + 1/2*(11*C*c^2
*d^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^2*d^4*e^2 - 10*B*a*c*d^3*e
^3 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*e^5 - A*a^2*e^6 + 2*(6*C*
c^2*d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*A*c^2*d^3*e^3 - 6*B*a*c*
d^2*e^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6)*x)/((e*x + d)^2*e^7)
+ 1/12*(3*C*c^2*e^9*x^4 - 12*C*c^2*d*e^8*x^3 + 4*B*c^2*e^9*x^3 + 36*C*c^2*
d^2*e^7*x^2 - 18*B*c^2*d*e^8*x^2 + 12*C*a*c*e^9*x^2 + 6*A*c^2*e^9*x^2 - 12
0*C*c^2*d^3*e^6*x + 72*B*c^2*d^2*e^7*x - 72*C*a*c*d*e^8*x - 36*A*c^2*d*e^8
*x + 24*B*a*c*e^9*x)/e^12
```

3.31.9 Mupad [B] (verification not implemented)

Time = 12.32 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.68

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= x\left(\frac{3d\left(\frac{3d\left(\frac{Bc^2}{e^3}-\frac{3Cc^2d}{e^4}\right)}{e}-\frac{Ac^2+2Cac}{e^3}+\frac{3Cc^2d^2}{e^5}\right)}{e}-\frac{3d^2\left(\frac{Bc^2}{e^3}-\frac{3Cc^2d}{e^4}\right)}{e^2}+\frac{2Bac}{e^3}-\frac{Cc^2d^3}{e^6}\right)+x^3\left(\frac{Bc^2}{3e^3}-\frac{Cc^2d}{e^4}\right)-x^2\left(\frac{3d\left(\frac{Bc^2}{e^3}-\frac{3Cc^2d}{e^4}\right)}{2e}-\frac{Ac^2+2Cac}{2e^3}+\frac{3Cc^2d^2}{2e^5}\right)$$
$$+\frac{\frac{3Ca^2d^2e^4-Ba^2de^5-Aa^2e^6+14Cacd^4e^2-10Bacd^3e^3+6Aacd^2e^4+11Cc^2d^6-9Bc^2d^5e+7Ac^2d^4e^2}{2e}+x\,(2Ca^2de^4-B}{d^2e^6+2de^7x+e^8}$$
$$+\frac{\ln(d+ex)\,(Ca^2e^4+12Cacd^2e^2-6Bacde^3+2Aace^4+15Cc^2d^4-10Bc^2d^3e+6Ac^2d^2e^2)}{e^7}$$
$$+\frac{Cc^2x^4}{4e^3}$$

input
```
int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

output
```
x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 +
 (3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*
B*a*c)/e^3 - (C*c^2*d^3)/e^6) + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^
2*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3)
 + (3*C*c^2*d^2)/(2*e^5)) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B
*c^2*d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*
c*d^3*e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2
*d*e^4 - 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3
 + 8*C*a*c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^
2*e^4 + 15*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*
B*a*c*d*e^3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3)
```

3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

3.32.1 Optimal result

Integrand size = 27, antiderivative size = 404

$$\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx \\
&= a^3Ad^3x+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e\left(3Cd^2+e(3Bd+Ae)\right)x^4 \\
&\quad +\frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right)+a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5 \\
&\quad +\frac{1}{6}a^2e\left(aCe^2+3c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^6 \\
&\quad +\frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right)+3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7 \\
&\quad +\frac{3}{8}ace\left(aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^8 \\
&\quad +\frac{1}{9}c^2\left(3ae^2(3Cd+Be)+cd\left(Cd^2+3e(Bd+Ae)\right)\right)x^9 \\
&\quad +\frac{1}{10}c^2e\left(3aCe^2+c\left(3Cd^2+e(3Bd+Ae)\right)\right)x^{10} \\
&\quad +\frac{1}{11}c^3e^2(3Cd+Be)x^{11}+\frac{1}{12}c^3Ce^3x^{12}+\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}
\end{aligned}$$

output
```
a^3*A*d^3*x+1/3*a^2*d*(a*d*(3*B*e+C*d)+3*A*(a*e^2+c*d^2))*x^3+1/4*a^3*e*(3
*C*d^2+e*(A*e+3*B*d))*x^4+1/5*a*(3*A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C
*d)+3*c*d^2*(3*B*e+C*d)))*x^5+1/6*a^2*e*(a*C*e^2+3*c*(3*C*d^2+e*(A*e+3*B*d
)))*x^6+1/7*c*(A*c*d*(9*a*e^2+c*d^2)+3*a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C
*d)))*x^7+3/8*a*c*e*(a*C*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*(3*a*e
^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^9+1/10*c^2*e*(3*a*C*e^2+c*(3*C
*d^2+e*(A*e+3*B*d)))*x^10+1/11*c^3*e^2*(B*e+3*C*d)*x^11+1/12*c^3*C*e^3*x^1
2+1/8*d^2*(3*A*e+B*d)*(c*x^2+a)^4/c
```

3.32.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.14

$$\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = a^3Ad^3x+\frac{1}{2}a^3d^2(Bd+3Ae)x^2 + \frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^2\left(3Bcd^3+9Acd^2e+3aCd^2e+3aBde^2 + aAe^3\right)x^4+\frac{1}{5}a\left(3Acd\left(cd^2+3ae^2\right) + a\left(ae^2(3Cd+Be)+3cd^2(Cd+3Be)\right)\right)x^5 + \frac{1}{6}a\left(3Ace\left(3cd^2+ae^2\right)+aCe\left(9cd^2+ae^2\right) +3Bcd\left(cd^2+3ae^2\right)\right)x^6+\frac{1}{7}c\left(Acd\left(cd^2+9ae^2\right) + 3a\left(ae^2(3Cd+Be)+cd^2(Cd+3Be)\right)\right)x^7 + \frac{1}{8}c\left(Bcd\left(cd^2+9ae^2\right) + 3e\left(Ac\left(cd^2+ae^2\right)+aC\left(3cd^2+ae^2\right)\right)\right)x^8 + \frac{1}{9}c^2\left(cCd^3+3cde(Bd+Ae) + 3ae^2(3Cd+Be)\right)x^9 + \frac{1}{10}c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^{10} + \frac{1}{11}c^3e^2(3Cd+Be)x^{11}+\frac{1}{12}c^3Ce^3x^{12}$$

input
```
Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) +
3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e
+ 3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2
*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a
*e^2) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(
A*c*d*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))
*x^7)/7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*
c*d^2 + a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3
*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x
^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

3.32.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^3 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)^3 \left((d+ex)^3 \left(Cx^2+Bx+A\right)-\left(Bd^3+3Aed^2\right)x\right)dx+\frac{d^2\left(a+cx^2\right)^4(3Ae+Bd)}{8c}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(c^3Ce^3x^{11}+c^3e^2(3Cd+Be)x^{10}+c^2e\left(3cCd^2+3aCe^2+ce(3Bd+Ae)\right)x^9+c^2\left(cCd^3+3ce(Bd+Ae)d+3ae\right.\right.$$

$$\frac{d^2\left(a+cx^2\right)^4(3Ae+Bd)}{8c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^3Ad^3x+\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{9}c^2x^9\left(3ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{10}c^2ex^{10}\left(3aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{3}{8}acex^8\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(Acd\left(9ae^2+cd^2\right)+3a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)+\frac{1}{5}ax^5\left(3Acd\left(3ae^2+cd^2\right)+a\left(ae^2(Be+3Cd)+3cd^2(3Be+Cd)\right)\right)+\frac{d^2\left(a+cx^2\right)^4(3Ae+Bd)}{8c}+\frac{1}{11}c^3e^2x^{11}(Be+3Cd)+\frac{1}{12}c^3Ce^3x^{12}$$

input
```
Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a
^3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) +
a*(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^
2 + a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) +
3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*
d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d +
 A*e) + 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*
e*(3*B*d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x
^12)/12 + (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.32.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.31

method	result
norman	$\frac{c^3 C e^3 x^{12}}{12} + \left(\frac{1}{11} B c^3 e^3 + \frac{3}{11} c^3 d e^2 C\right) x^{11} + \left(\frac{1}{10} c^3 e^3 A + \frac{3}{10} c^3 d e^2 B + \frac{3}{10} C a c^2 e^3 + \frac{3}{10} C c^3 d^2 e\right) x^1$
default	$\frac{c^3 C e^3 x^{12}}{12} + \frac{(B c^3 e^3 + 3c^3 d e^2 C) x^{11}}{11} + \frac{((3a c^2 e^3 + 3c^3 d^2 e) C + 3c^3 d e^2 B + c^3 e^3 A) x^{10}}{10} + \frac{((9a c^2 d e^2 + c^3 d^3) C + (3a c^2 e^3 + 3}{9}$
gosper	$\frac{1}{11} B c^3 e^3 x^{11} + \frac{3}{8} x^8 C a^2 c e^3 + \frac{3}{7} x^7 B e^3 c a^2 + \frac{3}{7} x^7 C a c^2 d^3 + \frac{3}{8} x^8 A a c^2 e^3 + \frac{3}{8} x^8 A c^3 d^2 e + \frac{1}{3} x^9 B$
risch	$\frac{1}{11} B c^3 e^3 x^{11} + \frac{3}{8} x^8 C a^2 c e^3 + \frac{3}{7} x^7 B e^3 c a^2 + \frac{3}{7} x^7 C a c^2 d^3 + \frac{3}{8} x^8 A a c^2 e^3 + \frac{3}{8} x^8 A c^3 d^2 e + \frac{1}{3} x^9 B$
parallelrisch	$\frac{1}{11} B c^3 e^3 x^{11} + \frac{3}{8} x^8 C a^2 c e^3 + \frac{3}{7} x^7 B e^3 c a^2 + \frac{3}{7} x^7 C a c^2 d^3 + \frac{3}{8} x^8 A a c^2 e^3 + \frac{3}{8} x^8 A c^3 d^2 e + \frac{1}{3} x^9 B$

input
```
int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*c^3*C*e^3*x^12+(1/11*B*c^3*e^3+3/11*c^3*d*e^2*C)*x^11+(1/10*c^3*e^3*A
+3/10*c^3*d*e^2*B+3/10*C*a*c^2*e^3+3/10*C*c^3*d^2*e)*x^10+(1/3*A*c^3*d*e^2
+1/3*B*e^3*a*c^2+1/3*B*c^3*d^2*e+C*a*c^2*d*e^2+1/9*C*c^3*d^3)*x^9+(3/8*A*a
*c^2*e^3+3/8*A*c^3*d^2*e+9/8*B*a*c^2*d*e^2+1/8*B*c^3*d^3+3/8*C*a^2*c*e^3+9
/8*C*a*c^2*d^2*e)*x^8+(9/7*A*a*c^2*d*e^2+1/7*A*c^3*d^3+3/7*B*e^3*c*a^2+9/7
*B*a*c^2*d^2*e+9/7*C*a^2*c*d*e^2+3/7*C*a*c^2*d^3)*x^7+(1/2*A*a^2*c*e^3+3/2
*A*a*c^2*d^2*e+3/2*B*a^2*c*d*e^2+1/2*B*a*c^2*d^3+1/6*a^3*C*e^3+3/2*C*a^2*c
*d^2*e)*x^6+(9/5*A*a^2*c*d*e^2+3/5*A*d^3*a*c^2+1/5*B*e^3*a^3+9/5*B*a^2*c*d
^2*e+3/5*C*a^3*d*e^2+3/5*C*a^2*c*d^3)*x^5+(1/4*A*a^3*e^3+9/4*A*a^2*c*d^2*e
+3/4*B*a^3*d*e^2+3/4*B*a^2*c*d^3+3/4*a^3*d^2*e*C)*x^4+(A*a^3*d*e^2+A*d^3*c
*a^2+a^3*d^2*e*B+1/3*a^3*d^3*C)*x^3+(3/2*A*a^3*d^2*e+1/2*B*a^3*d^3)*x^2+A*
d^3*a^3*x
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{12}Cc^3e^3x^{12} + \frac{1}{11}\left(3\,Cc^3de^2 + Bc^3e^3\right)x^{11} \\
&\quad + \frac{1}{10}\left(3\,Cc^3d^2e + 3\,Bc^3de^2 + \left(3\,Cac^2 + Ac^3\right)e^3\right)x^{10} \\
&\quad + \frac{1}{9}\left(Cc^3d^3 + 3\,Bc^3d^2e + 3\,Bac^2e^3 + 3\left(3\,Cac^2 + Ac^3\right)de^2\right)x^9 \\
&\quad + \frac{1}{8}\left(Bc^3d^3 + 9\,Bac^2de^2 + 3\left(3\,Cac^2 + Ac^3\right)d^2e + 3\left(Ca^2c + Aac^2\right)e^3\right)x^8 + Aa^3d^3x \\
&\quad + \frac{1}{7}\left(9\,Bac^2d^2e + 3\,Ba^2ce^3 + \left(3\,Cac^2 + Ac^3\right)d^3 + 9\left(Ca^2c + Aac^2\right)de^2\right)x^7 \\
&\quad + \frac{1}{6}\left(3\,Bac^2d^3 + 9\,Ba^2cde^2 + 9\left(Ca^2c + Aac^2\right)d^2e + \left(Ca^3 + 3\,Aa^2c\right)e^3\right)x^6 \\
&\quad + \frac{1}{5}\left(9\,Ba^2cd^2e + Ba^3e^3 + 3\left(Ca^2c + Aac^2\right)d^3 + 3\left(Ca^3 + 3\,Aa^2c\right)de^2\right)x^5 \\
&\quad + \frac{1}{4}\left(3\,Ba^2cd^3 + 3\,Ba^3de^2 + Aa^3e^3 + 3\left(Ca^3 + 3\,Aa^2c\right)d^2e\right)x^4 \\
&\quad + \frac{1}{3}\left(3\,Ba^3d^2e + 3\,Aa^3de^2 + \left(Ca^3 + 3\,Aa^2c\right)d^3\right)x^3 + \frac{1}{2}\left(Ba^3d^3 + 3\,Aa^3d^2e\right)x^2
\end{aligned}$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^
3*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 +
 3*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B
*c^3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*
a*c^2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*
C*a*c^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d
^3 + 9*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e
^3)*x^6 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3
*(C*a^3 + 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a
^3*e^3 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d
*e^2 + (C*a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.60

$$\int (d+ex)^3 (a+cx^2)^3 (A+Bx+Cx^2)\,dx$$
$$= Aa^3d^3x + \frac{Cc^3e^3x^{12}}{12} + x^{11}\left(\frac{Bc^3e^3}{11} + \frac{3Cc^3de^2}{11}\right)$$
$$+ x^{10}\left(\frac{Ac^3e^3}{10} + \frac{3Bc^3de^2}{10} + \frac{3Cac^2e^3}{10} + \frac{3Cc^3d^2e}{10}\right)$$
$$+ x^9\left(\frac{Ac^3de^2}{3} + \frac{Bac^2e^3}{3} + \frac{Bc^3d^2e}{3} + Cac^2de^2 + \frac{Cc^3d^3}{9}\right) + x^8$$
$$\cdot\left(\frac{3Aac^2e^3}{8} + \frac{3Ac^3d^2e}{8} + \frac{9Bac^2de^2}{8} + \frac{Bc^3d^3}{8} + \frac{3Ca^2ce^3}{8} + \frac{9Cac^2d^2e}{8}\right) + x^7$$
$$\cdot\left(\frac{9Aac^2de^2}{7} + \frac{Ac^3d^3}{7} + \frac{3Ba^2ce^3}{7} + \frac{9Bac^2d^2e}{7} + \frac{9Ca^2cde^2}{7} + \frac{3Cac^2d^3}{7}\right)$$
$$+ x^6\left(\frac{Aa^2ce^3}{2} + \frac{3Aac^2d^2e}{2} + \frac{3Ba^2cde^2}{2} + \frac{Bac^2d^3}{2} + \frac{Ca^3e^3}{6} + \frac{3Ca^2cd^2e}{2}\right)$$
$$+ x^5\cdot\left(\frac{9Aa^2cde^2}{5} + \frac{3Aac^2d^3}{5} + \frac{Ba^3e^3}{5} + \frac{9Ba^2cd^2e}{5} + \frac{3Ca^3de^2}{5} + \frac{3Ca^2cd^3}{5}\right)$$
$$+ x^4\left(\frac{Aa^3e^3}{4} + \frac{9Aa^2cd^2e}{4} + \frac{3Ba^3de^2}{4} + \frac{3Ba^2cd^3}{4} + \frac{3Ca^3d^2e}{4}\right)$$
$$+ x^3\left(Aa^3de^2 + Aa^2cd^3 + Ba^3d^2e + \frac{Ca^3d^3}{3}\right) + x^2\cdot\left(\frac{3Aa^3d^2e}{2} + \frac{Ba^3d^3}{2}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

output
```
A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*
e**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10
 + 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*
d**2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*
A*c**3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8
+ 9*C*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a
**2*c*e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3
/7) + x**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 +
B*a*c**2*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*
d*e**2/5 + 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a
**3*d*e**2/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*
e/4 + 3*B*a**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A
*a**3*d*e**2 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*
a**3*d**2*e/2 + B*a**3*d**3/2)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{12}Cc^3e^3x^{12} + \frac{1}{11}\left(3\,Cc^3de^2 + Bc^3e^3\right)x^{11} \\
&\quad + \frac{1}{10}\left(3\,Cc^3d^2e + 3\,Bc^3de^2 + \left(3\,Cac^2 + Ac^3\right)e^3\right)x^{10} \\
&\quad + \frac{1}{9}\left(Cc^3d^3 + 3\,Bc^3d^2e + 3\,Bac^2e^3 + 3\left(3\,Cac^2 + Ac^3\right)de^2\right)x^9 \\
&\quad + \frac{1}{8}\left(Bc^3d^3 + 9\,Bac^2de^2 + 3\left(3\,Cac^2 + Ac^3\right)d^2e + 3\left(Ca^2c + Aac^2\right)e^3\right)x^8 + Aa^3d^3x \\
&\quad + \frac{1}{7}\left(9\,Bac^2d^2e + 3\,Ba^2ce^3 + \left(3\,Cac^2 + Ac^3\right)d^3 + 9\left(Ca^2c + Aac^2\right)de^2\right)x^7 \\
&\quad + \frac{1}{6}\left(3\,Bac^2d^3 + 9\,Ba^2cde^2 + 9\left(Ca^2c + Aac^2\right)d^2e + \left(Ca^3 + 3\,Aa^2c\right)e^3\right)x^6 \\
&\quad + \frac{1}{5}\left(9\,Ba^2cd^2e + Ba^3e^3 + 3\left(Ca^2c + Aac^2\right)d^3 + 3\left(Ca^3 + 3\,Aa^2c\right)de^2\right)x^5 \\
&\quad + \frac{1}{4}\left(3\,Ba^2cd^3 + 3\,Ba^3de^2 + Aa^3e^3 + 3\left(Ca^3 + 3\,Aa^2c\right)d^2e\right)x^4 \\
&\quad + \frac{1}{3}\left(3\,Ba^3d^2e + 3\,Aa^3de^2 + \left(Ca^3 + 3\,Aa^2c\right)d^3\right)x^3 + \frac{1}{2}\left(Ba^3d^3 + 3\,Aa^3d^2e\right)x^2
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^
3*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 +
 3*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B
*c^3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*
a*c^2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*
C*a*c^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d
^3 + 9*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e
^3)*x^6 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3
*(C*a^3 + 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a
^3*e^3 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d
*e^2 + (C*a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.53

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx = \frac{1}{12} Cc^3e^3x^{12} + \frac{3}{11} Cc^3de^2x^{11} + \frac{1}{11} Bc^3e^3x^{11} + \frac{3}{10} Cc^3d^2ex^{10} + \frac{3}{10} Bc^3de^2x^{10} + \frac{3}{10} Cac^2e^3x^{10} + \frac{1}{10} Ac^3e^3x^{10} + \frac{1}{9} Cc^3d^3x^9 + \frac{1}{3} Bc^3d^2ex^9 + Cac^2de^2x^9 + \frac{1}{3} Ac^3de^2x^9 + \frac{1}{3} Bac^2e^3x^9 + \frac{1}{8} Bc^3d^3x^8 + \frac{9}{8} Cac^2d^2ex^8 + \frac{3}{8} Ac^3d^2ex^8 + \frac{9}{8} Bac^2de^2x^8 + \frac{3}{8} Ca^2ce^3x^8 + \frac{3}{8} Aac^2e^3x^8 + \frac{3}{7} Cac^2d^3x^7 + \frac{1}{7} Ac^3d^3x^7 + \frac{9}{7} Bac^2d^2ex^7 + \frac{9}{7} Ca^2cde^2x^7 + \frac{9}{7} Aac^2de^2x^7 + \frac{3}{7} Ba^2ce^3x^7 + \frac{1}{2} Bac^2d^3x^6 + \frac{3}{2} Ca^2cd^2ex^6 + \frac{3}{2} Aac^2d^2ex^6 + \frac{3}{2} Ba^2cde^2x^6 + \frac{1}{6} Ca^3e^3x^6 + \frac{1}{2} Aa^2ce^3x^6 + \frac{3}{5} Ca^2cd^3x^5 + \frac{3}{5} Aac^2d^3x^5 + \frac{9}{5} Ba^2cd^2ex^5 + \frac{3}{5} Ca^3de^2x^5 + \frac{9}{5} Aa^2cde^2x^5 + \frac{1}{5} Ba^3e^3x^5 + \frac{3}{4} Ba^2cd^3x^4 + \frac{3}{4} Ca^3d^2ex^4 + \frac{9}{4} Aa^2cd^2ex^4 + \frac{3}{4} Ba^3de^2x^4 + \frac{1}{4} Aa^3e^3x^4 + \frac{1}{3} Ca^3d^3x^3 + Aa^2cd^3x^3 + Ba^3d^2ex^3 + Aa^3de^2x^3 + \frac{1}{2} Ba^3d^3x^2 + \frac{3}{2} Aa^3d^2ex^2 + Aa^3d^3x$$

input
```
integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/12*C*c^3*e^3*x^12 + 3/11*C*c^3*d*e^2*x^11 + 1/11*B*c^3*e^3*x^11 + 3/10*C
*c^3*d^2*e*x^10 + 3/10*B*c^3*d*e^2*x^10 + 3/10*C*a*c^2*e^3*x^10 + 1/10*A*c
^3*e^3*x^10 + 1/9*C*c^3*d^3*x^9 + 1/3*B*c^3*d^2*e*x^9 + C*a*c^2*d*e^2*x^9
+ 1/3*A*c^3*d*e^2*x^9 + 1/3*B*a*c^2*e^3*x^9 + 1/8*B*c^3*d^3*x^8 + 9/8*C*a*
c^2*d^2*e*x^8 + 3/8*A*c^3*d^2*e*x^8 + 9/8*B*a*c^2*d*e^2*x^8 + 3/8*C*a^2*c*
e^3*x^8 + 3/8*A*a*c^2*e^3*x^8 + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 +
9/7*B*a*c^2*d^2*e*x^7 + 9/7*C*a^2*c*d*e^2*x^7 + 9/7*A*a*c^2*d*e^2*x^7 + 3/
7*B*a^2*c*e^3*x^7 + 1/2*B*a*c^2*d^3*x^6 + 3/2*C*a^2*c*d^2*e*x^6 + 3/2*A*a*
c^2*d^2*e*x^6 + 3/2*B*a^2*c*d*e^2*x^6 + 1/6*C*a^3*e^3*x^6 + 1/2*A*a^2*c*e^
3*x^6 + 3/5*C*a^2*c*d^3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 9/5*B*a^2*c*d^2*e*x^5
+ 3/5*C*a^3*d*e^2*x^5 + 9/5*A*a^2*c*d*e^2*x^5 + 1/5*B*a^3*e^3*x^5 + 3/4*B*
a^2*c*d^3*x^4 + 3/4*C*a^3*d^2*e*x^4 + 9/4*A*a^2*c*d^2*e*x^4 + 3/4*B*a^3*d*
e^2*x^4 + 1/4*A*a^3*e^3*x^4 + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + B*a^3*
d^2*e*x^3 + A*a^3*d*e^2*x^3 + 1/2*B*a^3*d^3*x^2 + 3/2*A*a^3*d^2*e*x^2 + A*
a^3*d^3*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.21

$$\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$$
$$= x^5 \left(\frac{3\,C\,a^3\,d\,e^2}{5} + \frac{B\,a^3\,e^3}{5} + \frac{3\,C\,a^2\,c\,d^3}{5} + \frac{9\,B\,a^2\,c\,d^2\,e}{5} + \frac{9\,A\,a^2\,c\,d\,e^2}{5} + \frac{3\,A\,a\,c^2\,d^3}{5}\right)$$
$$+ x^8 \left(\frac{3\,C\,a^2\,c\,e^3}{8} + \frac{9\,C\,a\,c^2\,d^2\,e}{8} + \frac{9\,B\,a\,c^2\,d\,e^2}{8} + \frac{3\,A\,a\,c^2\,e^3}{8} + \frac{B\,c^3\,d^3}{8} + \frac{3\,A\,c^3\,d^2\,e}{8}\right)$$
$$+ x^6 \left(\frac{C\,a^3\,e^3}{6} + \frac{3\,C\,a^2\,c\,d^2\,e}{2} + \frac{3\,B\,a^2\,c\,d\,e^2}{2} + \frac{A\,a^2\,c\,e^3}{2} + \frac{B\,a\,c^2\,d^3}{2} + \frac{3\,A\,a\,c^2\,d^2\,e}{2}\right)$$
$$+ x^7 \left(\frac{9\,C\,a^2\,c\,d\,e^2}{7} + \frac{3\,B\,a^2\,c\,e^3}{7} + \frac{3\,C\,a\,c^2\,d^3}{7} + \frac{9\,B\,a\,c^2\,d^2\,e}{7} + \frac{9\,A\,a\,c^2\,d\,e^2}{7} + \frac{A\,c^3\,d^3}{7}\right)$$
$$+ \frac{a^2\,x^4\,\left(A\,a\,e^3 + 3\,B\,c\,d^3 + 3\,B\,a\,d\,e^2 + 9\,A\,c\,d^2\,e + 3\,C\,a\,d^2\,e\right)}{4}$$
$$+ \frac{c^2\,x^9\,\left(3\,B\,a\,e^3 + C\,c\,d^3 + 3\,A\,c\,d\,e^2 + 9\,C\,a\,d\,e^2 + 3\,B\,c\,d^2\,e\right)}{9}$$
$$+ \frac{C\,c^3\,e^3\,x^{12}}{12} + \frac{a^3\,d^2\,x^2\,\left(3\,A\,e + B\,d\right)}{2} + \frac{c^3\,e^2\,x^{11}\,\left(B\,e + 3\,C\,d\right)}{11}$$
$$+ A\,a^3\,d^3\,x + \frac{a^2\,d\,x^3\,\left(3\,A\,a\,e^2 + 3\,A\,c\,d^2 + C\,a\,d^2 + 3\,B\,a\,d\,e\right)}{3}$$
$$+ \frac{c^2\,e\,x^{10}\,\left(A\,c\,e^2 + 3\,C\,a\,e^2 + 3\,C\,c\,d^2 + 3\,B\,c\,d\,e\right)}{10}$$

input
```
int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

output
```
x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^
2)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (
3*A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^
2)/8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*
c^2*d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)
/2) + x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*
c^2*d*e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*
e^3 + 3*B*c*d^3 + 3*B*a*d*e^2 + 9*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c^2*x^9*(
3*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 9*C*a*d*e^2 + 3*B*c*d^2*e))/9 + (C*c^3
*e^3*x^12)/12 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^11*(B*e + 3*C*d
))/11 + A*a^3*d^3*x + (a^2*d*x^3*(3*A*a*e^2 + 3*A*c*d^2 + C*a*d^2 + 3*B*a*
d*e))/3 + (c^2*e*x^10*(A*c*e^2 + 3*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/10
```

3.32. $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

3.33 $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$

3.33.1 Optimal result

Integrand size = 27, antiderivative size = 289

$$\begin{aligned}\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx = {} & a^3Ad^2x+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 \\ & +\frac{1}{4}a^3e(2Cd+Be)x^4+\frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)\right. \\ & \left.+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5 \\ & +\frac{1}{2}a^2ce(2Cd+Be)x^6+\frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)\right. \\ & \left.+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7 \\ & +\frac{3}{8}ac^2e(2Cd+Be)x^8 \\ & +\frac{1}{9}c^2\left(3aCe^2+c\left(Cd^2+e(2Bd+Ae)\right)\right)x^9 \\ & +\frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11} \\ & +\frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c}\end{aligned}$$

output
```
a^3*A*d^2*x+1/3*a^2*(a*d*(2*B*e+C*d)+A*(a*e^2+3*c*d^2))*x^3+1/4*a^3*e*(B*e
+2*C*d)*x^4+1/5*a*(3*A*c*(a*e^2+c*d^2)+a*(a*C*e^2+3*c*d*(2*B*e+C*d)))*x^5+
1/2*a^2*c*e*(B*e+2*C*d)*x^6+1/7*c*(A*c*(3*a*e^2+c*d^2)+3*a*(a*C*e^2+c*d*(2
*B*e+C*d)))*x^7+3/8*a*c^2*e*(B*e+2*C*d)*x^8+1/9*c^2*(3*a*C*e^2+c*(C*d^2+e*
(A*e+2*B*d)))*x^9+1/10*c^3*e*(B*e+2*C*d)*x^10+1/11*c^3*C*e^2*x^11+1/8*d*(2
*A*e+B*d)*(c*x^2+a)^4/c
```

3.33.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.14

$$
\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx \\
&= a^3Ad^2x+\frac{1}{2}a^3d(Bd+2Ae)x^2+\frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 \\
&\quad+\frac{1}{4}a^2\left(3Bcd^2+6Acde+2aCde+aBe^2\right)x^4 \\
&\quad+\frac{1}{5}a\left(3Ac\left(cd^2+ae^2\right)+a\left(aCe^2+3cd(Cd+2Be)\right)\right)x^5 \\
&\quad+\frac{1}{2}ac\left(2(Ac+aC)de+B\left(cd^2+ae^2\right)\right)x^6 \\
&\quad+\frac{1}{7}c\left(Ac\left(cd^2+3ae^2\right)+3a\left(aCe^2+cd(Cd+2Be)\right)\right)x^7 \\
&\quad+\frac{1}{8}c^2\left(Bcd^2+2Acde+6aCde+3aBe^2\right)x^8 \\
&\quad+\frac{1}{9}c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right)x^9+\frac{1}{10}c^3e(2Cd+Be)x^{10}+\frac{1}{11}c^3Ce^2x^{11}
\end{aligned}
$$

input
```
Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3
*c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^
2)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))
*x^5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c
*d^2 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^
2 + 2*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2
+ c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x
^11)/11
```

3.33.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3 (d+ex)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)^3 \left((d+ex)^2 \left(Cx^2+Bx+A\right)-\left(Bd^2+2Aed\right) x\right) dx+\frac{d\left(a+cx^2\right)^4 (2Ae+Bd)}{8c}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(c^3Ce^2x^{10}+c^3e(2Cd+Be)x^9+c^2\left(cCd^2+3aCe^2+ce(2Bd+Ae)\right) x^8+3ac^2e(2Cd+Be)x^7+c\big(Ac\big(cd^2+3a$$
$$\frac{d\left(a+cx^2\right)^4 (2Ae+Bd)}{8c}$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ad^2x+\frac{1}{4}a^3ex^4(Be+2Cd)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{2}a^2cex^6(Be+2Cd)+\frac{1}{9}c^2x^9\left(3aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{7}cx^7\left(Ac\left(3ae^2+cd^2\right)+3a\left(aCe^2+cd(2Be+Cd)\right)\right)+\frac{1}{5}ax^5\left(3Ac\left(ae^2+cd^2\right)+a\left(aCe^2+3cd(2Be+Cd)\right)\right)+\frac{d\left(a+cx^2\right)^4 (2Ae+Bd)}{8c}+\frac{3}{8}ac^2ex^8(Be+2Cd)+\frac{1}{10}c^3ex^{10}(Be+2Cd)+\frac{1}{11}c^3Ce^2x^{11}$$

input
```
Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3
*e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(
C*d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3
*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d +
B*e)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3
*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*
x^2)^4)/(8*c)
```

3.33. $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) \, dx$

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.33.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.30

method	result
norman	$\frac{c^3 C e^2 x^{11}}{11}+\left(\frac{1}{10}c^3e^2B+\frac{1}{5}c^3deC\right)x^{10}+\left(\frac{1}{9}c^3e^2A+\frac{2}{9}c^3deB+\frac{1}{3}Ca\,c^2e^2+\frac{1}{9}C\,c^3d^2\right)x^9+\left(\frac{1}{4}c^3d\right.$
default	$\frac{c^3Ce^2x^{11}}{11}+\frac{(c^3e^2B+2c^3deC)x^{10}}{10}+\frac{((3a\,c^2e^2+c^3d^2)C+2c^3deB+c^3e^2A)x^9}{9}+\frac{(6a\,c^2deC+(3a\,c^2e^2+c^3d^2)B+2c^3deA)}{8}$
gosper	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$
risch	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$
parallelrisch	$\frac{1}{10}B\,c^3e^2x^{10}+\frac{1}{3}x^3d^2a^3C+\frac{1}{2}x^2B\,a^3d^2+\frac{1}{3}x^3A\,a^3e^2+\frac{1}{4}x^4B\,e^2a^3+\frac{1}{8}x^8B\,c^3d^2+\frac{1}{7}x^7A\,c^3d^2+$

input
```
int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*c^3*C*e^2*x^11+(1/10*c^3*e^2*B+1/5*c^3*d*e*C)*x^10+(1/9*c^3*e^2*A+2/9
*c^3*d*e*B+1/3*C*a*c^2*e^2+1/9*C*c^3*d^2)*x^9+(1/4*c^3*d*e*A+3/8*B*e^2*a*c
^2+1/8*B*c^3*d^2+3/4*a*c^2*d*e*C)*x^8+(3/7*A*a*c^2*e^2+1/7*A*c^3*d^2+6/7*B
*a*c^2*d*e+3/7*C*a^2*c*e^2+3/7*C*a*c^2*d^2)*x^7+(A*a*c^2*d*e+1/2*B*e^2*c*a
^2+1/2*B*a*c^2*d^2+a^2*c*d*e*C)*x^6+(3/5*A*a^2*c*e^2+3/5*A*d^2*a*c^2+6/5*a
^2*c*d*e*B+1/5*C*a^3*e^2+3/5*C*a^2*c*d^2)*x^5+(3/2*A*a^2*c*d*e+1/4*B*e^2*a
^3+3/4*B*a^2*c*d^2+1/2*d*e*a^3*C)*x^4+(1/3*A*a^3*e^2+A*d^2*c*a^2+2/3*d*e*a
^3*B+1/3*d^2*a^3*C)*x^3+(A*a^3*d*e+1/2*B*a^3*d^2)*x^2+A*d^2*a^3*x
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\, dx \\
&= \frac{1}{11} Cc^3e^2x^{11} + \frac{1}{10} \left(2\, Cc^3de + Bc^3e^2\right)x^{10} + \frac{1}{9} \left(Cc^3d^2 + 2\, Bc^3de + \left(3\, Cac^2 + Ac^3\right)e^2\right)x^9 \\
&\quad + \frac{1}{8} \left(Bc^3d^2 + 3\, Bac^2e^2 + 2\left(3\, Cac^2 + Ac^3\right)de\right)x^8 \\
&\quad + \frac{1}{7} \left(6\, Bac^2de + \left(3\, Cac^2 + Ac^3\right)d^2 + 3\left(Ca^2c + Aac^2\right)e^2\right)x^7 \\
&\quad + Aa^3d^2x + \frac{1}{2} \left(Bac^2d^2 + Ba^2ce^2 + 2\left(Ca^2c + Aac^2\right)de\right)x^6 \\
&\quad + \frac{1}{5} \left(6\, Ba^2cde + 3\left(Ca^2c + Aac^2\right)d^2 + \left(Ca^3 + 3\, Aa^2c\right)e^2\right)x^5 \\
&\quad + \frac{1}{4} \left(3\, Ba^2cd^2 + Ba^3e^2 + 2\left(Ca^3 + 3\, Aa^2c\right)de\right)x^4 \\
&\quad + \frac{1}{3} \left(2\, Ba^3de + Aa^3e^2 + \left(Ca^3 + 3\, Aa^2c\right)d^2\right)x^3 + \frac{1}{2} \left(Ba^3d^2 + 2\, Aa^3de\right)x^2
\end{aligned}$$

input

```
integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

output

```
1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
 + 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2
*e^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 +
A*c^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d
^2 + B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3
*(C*a^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d
^2 + B*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3
*e^2 + (C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.55

$$\int (d+ex)^2 (a+cx^2)^3 (A+Bx+Cx^2)\, dx = Aa^3d^2x + \frac{Cc^3e^2x^{11}}{11} + x^{10}\left(\frac{Bc^3e^2}{10} + \frac{Cc^3de}{5}\right) + x^9\left(\frac{Ac^3e^2}{9} + \frac{2Bc^3de}{9} + \frac{Cac^2e^2}{3} + \frac{Cc^3d^2}{9}\right) + x^8\left(\frac{Ac^3de}{4} + \frac{3Bac^2e^2}{8} + \frac{Bc^3d^2}{8} + \frac{3Cac^2de}{4}\right) + x^7\cdot\left(\frac{3Aac^2e^2}{7} + \frac{Ac^3d^2}{7} + \frac{6Bac^2de}{7} + \frac{3Ca^2ce^2}{7} + \frac{3Cac^2d^2}{7}\right) + x^6\left(Aac^2de + \frac{Ba^2ce^2}{2} + \frac{Bac^2d^2}{2} + Ca^2cde\right) + x^5\cdot\left(\frac{3Aa^2ce^2}{5} + \frac{3Aac^2d^2}{5} + \frac{6Ba^2cde}{5} + \frac{Ca^3e^2}{5} + \frac{3Ca^2cd^2}{5}\right) + x^4 \cdot\left(\frac{3Aa^2cde}{2} + \frac{Ba^3e^2}{4} + \frac{3Ba^2cd^2}{4} + \frac{Ca^3de}{2}\right) + x^3\left(\frac{Aa^3e^2}{3} + Aa^2cd^2 + \frac{2Ba^3de}{3} + \frac{Ca^3d^2}{3}\right) + x^2\left(Aa^3de + \frac{Ba^3d^2}{2}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

output
```
A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/
5) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/
9) + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d
*e/4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a
**2*c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 +
 B*a*c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d*
*2/5 + 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a
**2*c*d*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*
a**3*e**2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a*
*3*d*e + B*a**3*d**2/2)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.27

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{1}{11}Cc^3e^2x^{11} + \frac{1}{10}\left(2Cc^3de + Bc^3e^2\right)x^{10} + \frac{1}{9}\left(Cc^3d^2 + 2Bc^3de + \left(3Cac^2 + Ac^3\right)e^2\right)x^9$$
$$+ \frac{1}{8}\left(Bc^3d^2 + 3Bac^2e^2 + 2\left(3Cac^2 + Ac^3\right)de\right)x^8$$
$$+ \frac{1}{7}\left(6Bac^2de + \left(3Cac^2 + Ac^3\right)d^2 + 3\left(Ca^2c + Aac^2\right)e^2\right)x^7$$
$$+ Aa^3d^2x + \frac{1}{2}\left(Bac^2d^2 + Ba^2ce^2 + 2\left(Ca^2c + Aac^2\right)de\right)x^6$$
$$+ \frac{1}{5}\left(6Ba^2cde + 3\left(Ca^2c + Aac^2\right)d^2 + \left(Ca^3 + 3Aa^2c\right)e^2\right)x^5$$
$$+ \frac{1}{4}\left(3Ba^2cd^2 + Ba^3e^2 + 2\left(Ca^3 + 3Aa^2c\right)de\right)x^4$$
$$+ \frac{1}{3}\left(2Ba^3de + Aa^3e^2 + \left(Ca^3 + 3Aa^2c\right)d^2\right)x^3 + \frac{1}{2}\left(Ba^3d^2 + 2Aa^3de\right)x^2$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
 + 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2
*e^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 +
A*c^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d
^2 + B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3
*(C*a^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d
^2 + B*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3
*e^2 + (C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.49

$$\int (d+ex)^2 (a+cx^2)^3 (A+Bx+Cx^2)\,dx = \frac{1}{11}Cc^3e^2x^{11} + \frac{1}{5}Cc^3dex^{10} + \frac{1}{10}Bc^3e^2x^{10} + \frac{1}{9}Cc^3d^2x^9 + \frac{2}{9}Bc^3dex^9 + \frac{1}{3}Cac^2e^2x^9 + \frac{1}{9}Ac^3e^2x^9 + \frac{1}{8}Bc^3d^2x^8 + \frac{3}{4}Cac^2dex^8 + \frac{1}{4}Ac^3dex^8 + \frac{3}{8}Bac^2e^2x^8 + \frac{3}{7}Cac^2d^2x^7 + \frac{1}{7}Ac^3d^2x^7 + \frac{6}{7}Bac^2dex^7 + \frac{3}{7}Ca^2ce^2x^7 + \frac{3}{7}Aac^2e^2x^7 + \frac{1}{2}Bac^2d^2x^6 + Ca^2cdex^6 + Aac^2dex^6 + \frac{1}{2}Ba^2ce^2x^6 + \frac{3}{5}Ca^2cd^2x^5 + \frac{3}{5}Aac^2d^2x^5 + \frac{6}{5}Ba^2cdex^5 + \frac{1}{5}Ca^3e^2x^5 + \frac{3}{5}Aa^2ce^2x^5 + \frac{3}{4}Ba^2cd^2x^4 + \frac{1}{2}Ca^3dex^4 + \frac{3}{2}Aa^2cdex^4 + \frac{1}{4}Ba^3e^2x^4 + \frac{1}{3}Ca^3d^2x^3 + Aa^2cd^2x^3 + \frac{2}{3}Ba^3dex^3 + \frac{1}{3}Aa^3e^2x^3 + \frac{1}{2}Ba^3d^2x^2 + Aa^3dex^2 + Aa^3d^2x$$

input
```
integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*C*c^3*e^2*x^11 + 1/5*C*c^3*d*e*x^10 + 1/10*B*c^3*e^2*x^10 + 1/9*C*c^3
*d^2*x^9 + 2/9*B*c^3*d*e*x^9 + 1/3*C*a*c^2*e^2*x^9 + 1/9*A*c^3*e^2*x^9 + 1
/8*B*c^3*d^2*x^8 + 3/4*C*a*c^2*d*e*x^8 + 1/4*A*c^3*d*e*x^8 + 3/8*B*a*c^2*e
^2*x^8 + 3/7*C*a*c^2*d^2*x^7 + 1/7*A*c^3*d^2*x^7 + 6/7*B*a*c^2*d*e*x^7 + 3
/7*C*a^2*c*e^2*x^7 + 3/7*A*a*c^2*e^2*x^7 + 1/2*B*a*c^2*d^2*x^6 + C*a^2*c*d
*e*x^6 + A*a*c^2*d*e*x^6 + 1/2*B*a^2*c*e^2*x^6 + 3/5*C*a^2*c*d^2*x^5 + 3/5
*A*a*c^2*d^2*x^5 + 6/5*B*a^2*c*d*e*x^5 + 1/5*C*a^3*e^2*x^5 + 3/5*A*a^2*c*e
^2*x^5 + 3/4*B*a^2*c*d^2*x^4 + 1/2*C*a^3*d*e*x^4 + 3/2*A*a^2*c*d*e*x^4 + 1
/4*B*a^3*e^2*x^4 + 1/3*C*a^3*d^2*x^3 + A*a^2*c*d^2*x^3 + 2/3*B*a^3*d*e*x^3
 + 1/3*A*a^3*e^2*x^3 + 1/2*B*a^3*d^2*x^2 + A*a^3*d*e*x^2 + A*a^3*d^2*x
```

3.33.9 Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.19

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx$$
$$= x^3 \left(\frac{C\,a^3\,d^2}{3}+\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right) + x^9 \left(\frac{C\,c^3\,d^2}{9}+\frac{2\,B\,c^3\,d\,e}{9}+\frac{A\,c^3\,e^2}{9}+\frac{C\,a\,c^2\,e^2}{3}\right) + x^5 \left(\frac{C\,a^3\,e^2}{5}+\frac{3\,C\,a^2\,c\,d^2}{5}+\frac{6\,B\,a^2\,c\,d\,e}{5}+\frac{3\,A\,a^2\,c\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^2}{5}\right) + x^7 \left(\frac{3\,C\,a^2\,c\,e^2}{7}+\frac{3\,C\,a\,c^2\,d^2}{7}+\frac{6\,B\,a\,c^2\,d\,e}{7}+\frac{3\,A\,a\,c^2\,e^2}{7}+\frac{A\,c^3\,d^2}{7}\right) + \frac{a^2\,x^4\,(B\,a\,e^2+3\,B\,c\,d^2+6\,A\,c\,d\,e+2\,C\,a\,d\,e)}{4} + \frac{c^2\,x^8\,(3\,B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+6\,C\,a\,d\,e)}{8} + \frac{C\,c^3\,e^2\,x^{11}}{11} + \frac{a\,c\,x^6\,(B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e)}{2} + A\,a^3\,d^2\,x + \frac{a^3\,d\,x^2\,(2\,A\,e+B\,d)}{2} + \frac{c^3\,e\,x^{10}\,(B\,e+2\,C\,d)}{10}$$

input
```
int((a + c*x^2)^3*(d + e*x)^2*(A + B*x + C*x^2),x)
```

output
```
x^3*((A*a^3*e^2)/3 + (C*a^3*d^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^9*
((A*c^3*e^2)/9 + (C*c^3*d^2)/9 + (2*B*c^3*d*e)/9 + (C*a*c^2*e^2)/3) + x^5*
((C*a^3*e^2)/5 + (3*A*a*c^2*d^2)/5 + (3*A*a^2*c*e^2)/5 + (3*C*a^2*c*d^2)/5
 + (6*B*a^2*c*d*e)/5) + x^7*((A*c^3*d^2)/7 + (3*A*a*c^2*e^2)/7 + (3*C*a*c^
2*d^2)/7 + (3*C*a^2*c*e^2)/7 + (6*B*a*c^2*d*e)/7) + (a^2*x^4*(B*a*e^2 + 3*
B*c*d^2 + 6*A*c*d*e + 2*C*a*d*e))/4 + (c^2*x^8*(3*B*a*e^2 + B*c*d^2 + 2*A*
c*d*e + 6*C*a*d*e))/8 + (C*c^3*e^2*x^11)/11 + (a*c*x^6*(B*a*e^2 + B*c*d^2
+ 2*A*c*d*e + 2*C*a*d*e))/2 + A*a^3*d^2*x + (a^3*d*x^2*(2*A*e + B*d))/2 +
(c^3*e*x^10*(B*e + 2*C*d))/10
```

3.34 $\int (d+ex)(a+cx^2)^3(A+Bx+Cx^2)\,dx$

3.34.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\begin{aligned}\int (d+ex)(a+cx^2)^3(A+Bx+Cx^2)\,dx &= a^3Adx+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3\\&+\frac{1}{4}a^3Cex^4+\frac{3}{5}ac(Acd+aCd+aBe)x^5\\&+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2(Acd+3a(Cd+Be))x^7\\&+\frac{3}{8}ac^2Cex^8+\frac{1}{9}c^3(Cd+Be)x^9\\&+\frac{1}{10}c^3Cex^{10}+\frac{(Bd+Ae)(a+cx^2)^4}{8c}\end{aligned}$$

output
```
a^3*A*d*x+1/3*a^2*(3*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^3*C*e*x^4+3/5*a*c*(A*c*d
+B*a*e+C*a*d)*x^5+1/2*a^2*c*C*e*x^6+1/7*c^2*(A*c*d+3*a*(B*e+C*d))*x^7+3/8*
a*c^2*C*e*x^8+1/9*c^3*(B*e+C*d)*x^9+1/10*c^3*C*e*x^10+1/8*(A*e+B*d)*(c*x^2
+a)^4/c
```

3.34.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.16

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx=a^3Adx+\frac{1}{2}a^3(Bd+Ae)x^2+\frac{1}{3}a^2(3Acd+aCd+aBe)x^3+\frac{1}{4}a^2(3Bcd+3Ace+aCe)x^4+\frac{3}{5}ac(Acd+aCd+aBe)x^5+\frac{1}{2}ac(Bcd+Ace+aCe)x^6+\frac{1}{7}c^2(Acd+3aCd+3aBe)x^7+\frac{1}{8}c^2(Bcd+Ace+3aCe)x^8+\frac{1}{9}c^3(Cd+Be)x^9+\frac{1}{10}c^3Cex^{10}$$

input
```
Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/
3 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*
e)*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d +
3*a*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)
*x^9)/9 + (c^3*C*e*x^10)/10
```

3.34.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^3(d+ex)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(cx^2+a\right)^3 \left((d+ex)\left(Cx^2+Bx+A\right)-(Bd+Ae)x\right)dx+\frac{\left(a+cx^2\right)^4 (Ae+Bd)}{8c}$$

$\big\downarrow$ 2341

$$\int \left(c^3Cex^9+c^3(Cd+Be)x^8+3ac^2Cex^7+c^2(Acd+3a(Cd+Be))x^6+3a^2cCex^5+3ac(Acd+aCd+aBe)x^4 +\right.$$
$$\frac{\left(a+cx^2\right)^4 (Ae+Bd)}{8c}$$

$\big\downarrow$ 2009

$$a^3Adx+\frac{1}{4}a^3Cex^4+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{\left(a+cx^2\right)^4 (Ae+Bd)}{8c}+\frac{3}{8}ac^2Cex^8+\frac{1}{9}c^3x^9(Be+Cd)+\frac{1}{10}c^3Cex^{10}$$

input
```
Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a
*c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*
(C*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3
*C*e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.34.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.32

method	result
default	$\frac{c^3 C e x^{10}}{10}+\frac{(e c^3 B+c^3 d C) x^9}{9}+\frac{(e c^3 A+c^3 d B+3 a c^2 e C) x^8}{8}+\frac{(A d c^3+3 a c^2 e B+3 a c^2 d C) x^7}{7}+\frac{(3 A a c^2 e+3 B a c^2 d+3 a^2}{6}$
norman	$\frac{c^3 C e x^{10}}{10}+\left(\frac{1}{9} e c^3 B+\frac{1}{9} c^3 d C\right) x^9+\left(\frac{1}{8} e c^3 A+\frac{1}{8} c^3 d B+\frac{3}{8} a c^2 e C\right) x^8+\left(\frac{1}{7} A d c^3+\frac{3}{7} a c^2 e B+\frac{3}{7}\right.$
gosper	$\frac{1}{10} c^3 C e x^{10}+\frac{1}{9} B c^3 e x^9+\frac{1}{9} x^9 c^3 d C+\frac{1}{8} x^8 A c^3 e+\frac{1}{8} x^8 B c^3 d+\frac{3}{8} a c^2 C e x^8+\frac{1}{7} x^7 A d c^3+\frac{3}{7} x^7 B$
risch	$\frac{1}{10} c^3 C e x^{10}+\frac{1}{9} B c^3 e x^9+\frac{1}{9} x^9 c^3 d C+\frac{1}{8} x^8 A c^3 e+\frac{1}{8} x^8 B c^3 d+\frac{3}{8} a c^2 C e x^8+\frac{1}{7} x^7 A d c^3+\frac{3}{7} x^7 B$
parallelrisch	$\frac{1}{10} c^3 C e x^{10}+\frac{1}{9} B c^3 e x^9+\frac{1}{9} x^9 c^3 d C+\frac{1}{8} x^8 A c^3 e+\frac{1}{8} x^8 B c^3 d+\frac{3}{8} a c^2 C e x^8+\frac{1}{7} x^7 A d c^3+\frac{3}{7} x^7 B$

input
```
int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2
*e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c
^2*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3
*A*a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+
1/2*(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{10}Cc^3ex^{10}+\frac{1}{9}\left(Cc^3d+Bc^3e\right)x^9+\frac{1}{8}\left(Bc^3d+\left(3\,Cac^2+Ac^3\right)e\right)x^8 \\
&\quad+\frac{1}{7}\left(3\,Bac^2e+\left(3\,Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\left(Bac^2d+\left(Ca^2c+Aac^2\right)e\right)x^6+Aa^3dx \\
&\quad+\frac{3}{5}\left(Ba^2ce+\left(Ca^2c+Aac^2\right)d\right)x^5+\frac{1}{4}\left(3\,Ba^2cd+\left(Ca^3+3\,Aa^2c\right)e\right)x^4 \\
&\quad+\frac{1}{3}\left(Ba^3e+\left(Ca^3+3\,Aa^2c\right)d\right)x^3+\frac{1}{2}\left(Ba^3d+Aa^3e\right)x^2
\end{aligned}$$

input
```
integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^
2 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B
*a*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^
2*c + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/
3*(B*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.57

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = Aa^3dx+\frac{Cc^3ex^{10}}{10}+x^9\left(\frac{Bc^3e}{9}+\frac{Cc^3d}{9}\right) + x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}+\frac{3Cac^2e}{8}\right) + x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}+\frac{3Cac^2d}{7}\right) + x^6\left(\frac{Aac^2e}{2}+\frac{Bac^2d}{2}+\frac{Ca^2ce}{2}\right)+x^5 \cdot\left(\frac{3Aac^2d}{5}+\frac{3Ba^2ce}{5}+\frac{3Ca^2cd}{5}\right) + x^4\cdot\left(\frac{3Aa^2ce}{4}+\frac{3Ba^2cd}{4}+\frac{Ca^3e}{4}\right) + x^3\left(Aa^2cd+\frac{Ba^3e}{3}+\frac{Ca^3d}{3}\right) + x^2\left(\frac{Aa^3e}{2}+\frac{Ba^3d}{2}\right)$$

input
```
integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

output
```
A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*
c**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/
7 + 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) +
x**5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c
*e/4 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a*
*3*d/3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = \frac{1}{10}\,Cc^3ex^{10}+\frac{1}{9}\,(Cc^3d+Bc^3e)x^9+\frac{1}{8}\,\left(Bc^3d+\left(3\,Cac^2+Ac^3\right)e\right)x^8 + \frac{1}{7}\,\left(3\,Bac^2e+\left(3\,Cac^2+Ac^3\right)d\right)x^7+\frac{1}{2}\,\left(Bac^2d+\left(Ca^2c+Aac^2\right)e\right)x^6+Aa^3dx + \frac{3}{5}\,\left(Ba^2ce+\left(Ca^2c+Aac^2\right)d\right)x^5+\frac{1}{4}\,\left(3\,Ba^2cd+\left(Ca^3+3\,Aa^2c\right)e\right)x^4 + \frac{1}{3}\,\left(Ba^3e+\left(Ca^3+3\,Aa^2c\right)d\right)x^3+\frac{1}{2}\,(Ba^3d+Aa^3e)x^2$$

input
```
integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^
2 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B
*a*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^
2*c + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/
3*(B*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.47

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = \frac{1}{10}\,Cc^3ex^{10}+\frac{1}{9}\,Cc^3dx^9+\frac{1}{9}\,Bc^3ex^9+\frac{1}{8}\,Bc^3dx^8 + \frac{3}{8}\,Cac^2ex^8+\frac{1}{8}\,Ac^3ex^8+\frac{3}{7}\,Cac^2dx^7 + \frac{1}{7}\,Ac^3dx^7+\frac{3}{7}\,Bac^2ex^7+\frac{1}{2}\,Bac^2dx^6 + \frac{1}{2}\,Ca^2cex^6+\frac{1}{2}\,Aac^2ex^6+\frac{3}{5}\,Ca^2cdx^5 + \frac{3}{5}\,Aac^2dx^5+\frac{3}{5}\,Ba^2cex^5+\frac{3}{4}\,Ba^2cdx^4 + \frac{1}{4}\,Ca^3ex^4+\frac{3}{4}\,Aa^2cex^4+\frac{1}{3}\,Ca^3dx^3+Aa^2cdx^3 + \frac{1}{3}\,Ba^3ex^3+\frac{1}{2}\,Ba^3dx^2+\frac{1}{2}\,Aa^3ex^2+Aa^3dx$$

input
```
integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*C*c^3*e*x^10 + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*e*x^9 + 1/8*B*c^3*d*x^8 +
3/8*C*a*c^2*e*x^8 + 1/8*A*c^3*e*x^8 + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7
+ 3/7*B*a*c^2*e*x^7 + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*e*x^6 + 1/2*A*a*c^2*
e*x^6 + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*e*x^5 + 3/4*B*
a^2*c*d*x^4 + 1/4*C*a^3*e*x^4 + 3/4*A*a^2*c*e*x^4 + 1/3*C*a^3*d*x^3 + A*a^
2*c*d*x^3 + 1/3*B*a^3*e*x^3 + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*e*x^2 + A*a^3*d*
x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = x^3\left(\frac{B\,a^3\,e}{3}+\frac{C\,a^3\,d}{3}+A\,a^2\,c\,d\right)+x^8\left(\frac{A\,c^3\,e}{8}+\frac{B\,c^3\,d}{8}+\frac{3\,C\,a\,c^2\,e}{8}\right)+\frac{a^3\,x^2\,(A\,e+B\,d)}{2}+\frac{c^3\,x^9\,(B\,e+C\,d)}{9}+\frac{c^2\,x^7\,(A\,c\,d+3\,B\,a\,e+3\,C\,a\,d)}{7}+\frac{a^2\,x^4\,(3\,A\,c\,e+3\,B\,c\,d+C\,a\,e)}{4}+A\,a^3\,d\,x+\frac{3\,a\,c\,x^5\,(A\,c\,d+B\,a\,e+C\,a\,d)}{5}+\frac{a\,c\,x^6\,(A\,c\,e+B\,c\,d+C\,a\,e)}{2}+\frac{C\,c^3\,e\,x^{10}}{10}$$

input
```
int((a + c*x^2)^3*(d + e*x)*(A + B*x + C*x^2),x)
```

output
```
x^3*((B*a^3*e)/3 + (C*a^3*d)/3 + A*a^2*c*d) + x^8*((A*c^3*e)/8 + (B*c^3*d)
/8 + (3*C*a*c^2*e)/8) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^9*(B*e + C*d))/9
+ (c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d))/7 + (a^2*x^4*(3*A*c*e + 3*B*c*d +
C*a*e))/4 + A*a^3*d*x + (3*a*c*x^5*(A*c*d + B*a*e + C*a*d))/5 + (a*c*x^6*(
A*c*e + B*c*d + C*a*e))/2 + (C*c^3*e*x^10)/10
```

3.35 $\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 87

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx = a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}c^3Cx^9 + \frac{B(a+cx^2)^4}{8c}$$

output
```
a^3*A*x+1/3*a^2*(3*A*c+C*a)*x^3+3/5*a*c*(A*c+C*a)*x^5+1/7*c^2*(A*c+3*C*a)*
x^7+1/9*c^3*C*x^9+1/8*B*(c*x^2+a)^4/c
```

3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx = \frac{1}{6}a^3x(6A+x(3B+2Cx)) + \frac{1}{20}a^2cx^3(20A+3x(5B+4Cx)) + \frac{1}{70}ac^2x^5(42A+5x(7B+6Cx)) + \frac{1}{504}c^3x^7(72A+7x(9B+8Cx))$$

input
```
Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
(a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))
/20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*
B + 8*C*x)))/504
```

3.35.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (cx^2+a)^3 (Cx^2+A)\, dx+\frac{B(a+cx^2)^4}{8c}$$

$$\Big\downarrow \text{290}$$

$$\int \left(c^3Cx^8+c^2(Ac+3aC)x^6+3ac(Ac+aC)x^4+a^2(3Ac+aC)x^2+a^3A\right) dx+\frac{B(a+cx^2)^4}{8c}$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{3}a^2x^3(aC+3Ac)+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B(a+cx^2)^4}{8c}+\frac{1}{9}c^3Cx^9$$

input
```
Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

output
```
a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*
c + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

3.35.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

3.35.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
norman	$\frac{c^3 C x^9}{9}+\frac{B c^3 x^8}{8}+\left(\frac{1}{7}A c^3+\frac{3}{7}a c^2 C\right)x^7+\frac{B a c^2 x^6}{2}+\left(\frac{3}{5}A a c^2+\frac{3}{5}c a^2 C\right)x^5+\frac{3B a^2 c x^4}{4}+\left(A a^2 c+\right.$
default	$\frac{c^3 C x^9}{9}+\frac{B c^3 x^8}{8}+\frac{(A c^3+3a c^2 C)x^7}{7}+\frac{B a c^2 x^6}{2}+\frac{(3A a c^2+3c a^2 C)x^5}{5}+\frac{3B a^2 c x^4}{4}+\frac{(3A a^2 c+a^3 C)x^3}{3}+\frac{B a^3 x}{2}$
gosper	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4+$
risch	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4+$
parallelrisch	$\frac{1}{9}c^3 C x^9+\frac{1}{8}B c^3 x^8+\frac{1}{7}A c^3 x^7+\frac{3}{7}x^7 a c^2 C+\frac{1}{2}B a c^2 x^6+\frac{3}{5}a A c^2 x^5+\frac{3}{5}x^5 c a^2 C+\frac{3}{4}B a^2 c x^4+$

input
```
int((c*x^2+a)^3*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*C*x^9+1/8*B*c^3*x^8+(1/7*A*c^3+3/7*a*c^2*C)*x^7+1/2*B*a*c^2*x^6+(3
/5*A*a*c^2+3/5*c*a^2*C)*x^5+3/4*B*a^2*c*x^4+(A*a^2*c+1/3*a^3*C)*x^3+1/2*B*
a^3*x^2+a^3*A*x
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cc^3x^9+\frac{1}{8}\,Bc^3x^8+\frac{1}{2}\,Bac^2x^6+\frac{3}{4}\,Ba^2cx^4 + \frac{1}{7}\,(3\,Cac^2+Ac^3)x^7+\frac{1}{2}\,Ba^3x^2 + \frac{3}{5}\,(Ca^2c+Aac^2)x^5+Aa^3x+\frac{1}{3}\,(Ca^3+3\,Aa^2c)x^3$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3
*C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^
3*x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = Aa^3x+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}\right)+x^5\cdot\left(\frac{3Aac^2}{5}+\frac{3Ca^2c}{5}\right)+x^3\left(Aa^2c+\frac{Ca^3}{3}\right)$$

input
```
integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x*
*8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5
 + 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right)\,dx = \frac{1}{9}\,Cc^3x^9+\frac{1}{8}\,Bc^3x^8+\frac{1}{2}\,Bac^2x^6+\frac{3}{4}\,Ba^2cx^4+\frac{1}{7}\,(3\,Cac^2+Ac^3)x^7+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2c+Aac^2)x^5+Aa^3x+\frac{1}{3}\,(Ca^3+3\,Aa^2c)x^3$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3
*C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^
3*x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\,dx = \frac{1}{9}Cc^3x^9+\frac{1}{8}Bc^3x^8+\frac{3}{7}Cac^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bac^2x^6+\frac{3}{5}Ca^2cx^5+\frac{3}{5}Aac^2x^5+\frac{3}{4}Ba^2cx^4+\frac{1}{3}Ca^3x^3+Aa^2cx^3+\frac{1}{2}Ba^3x^2+Aa^3x$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*
c^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*
x^3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

3.35.9 Mupad [B] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

$$\int (a+cx^2)^3 (A+Bx+Cx^2)\,dx = x^3\left(\frac{C\,a^3}{3}+A\,c\,a^2\right)+x^7\left(\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right)+\frac{B\,a^3\,x^2}{2}+\frac{B\,c^3\,x^8}{8}+\frac{C\,c^3\,x^9}{9}+A\,a^3\,x+\frac{3\,a\,c\,x^5\,(A\,c+C\,a)}{5}+\frac{3\,B\,a^2\,c\,x^4}{4}+\frac{B\,a\,c^2\,x^6}{2}$$

input
```
int((a + c*x^2)^3*(A + B*x + C*x^2),x)
```

output
```
x^3*((C*a^3)/3 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7) + (B*a^3*x^2)/
2 + (B*c^3*x^8)/8 + (C*c^3*x^9)/9 + A*a^3*x + (3*a*c*x^5*(A*c + C*a))/5 +
(3*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

3.36 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

3.36.1 Optimal result

Integrand size = 27, antiderivative size = 490

$$\begin{aligned}
&\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx \\
&= -\frac{(cd^2+ae^2)^2(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))\,x}{e^8} \\
&\quad+\frac{(cd^2+ae^2)(a^2Ce^4+c^2d^2(28Cd^2-3e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))(d+ex)^2}{2e^9} \\
&\quad-\frac{c(3a^2e^4(4Cd-Be)+c^2d^3(56Cd^2-5e(7Bd-4Ae))+6acde^2(10Cd^2-e(5Bd-2Ae)))(d+ex)^3}{3e^9} \\
&\quad+\frac{c(3a^2Ce^4+5c^2d^2(14Cd^2-e(7Bd-3Ae))+3ace^2(15Cd^2-e(5Bd-Ae)))(d+ex)^4}{4e^9} \\
&\quad-\frac{c^2(3ae^2(6Cd-Be)+cd(56Cd^2-3e(7Bd-2Ae)))(d+ex)^5}{5e^9} \\
&\quad+\frac{c^2(3aCe^2+c(28Cd^2-e(7Bd-Ae)))(d+ex)^6}{6e^9}-\frac{c^3(8Cd-Be)(d+ex)^7}{7e^9} \\
&\quad+\frac{c^3C(d+ex)^8}{8e^9}+\frac{(cd^2+ae^2)^3(Cd^2-Bde+Ae^2)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
-(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*x/e^8
+1/2*(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^
2*(17*C*d^2-3*e*(-A*e+3*B*d)))*(e*x+d)^2/e^9-1/3*c*(3*a^2*e^4*(-B*e+4*C*d)
+c^2*d^3*(56*C*d^2-5*e*(-4*A*e+7*B*d))+6*a*c*d*e^2*(10*C*d^2-e*(-2*A*e+5*B
*d)))*(e*x+d)^3/e^9+1/4*c*(3*a^2*C*e^4+5*c^2*d^2*(14*C*d^2-e*(-3*A*e+7*B*d
))+3*a*c*e^2*(15*C*d^2-e*(-A*e+5*B*d)))*(e*x+d)^4/e^9-1/5*c^2*(3*a*e^2*(-B
*e+6*C*d)+c*d*(56*C*d^2-3*e*(-2*A*e+7*B*d)))*(e*x+d)^5/e^9+1/6*c^2*(3*a*C*
e^2+c*(28*C*d^2-e*(-A*e+7*B*d)))*(e*x+d)^6/e^9-1/7*c^3*(-B*e+8*C*d)*(e*x+d
)^7/e^9+1/8*c^3*C*(e*x+d)^8/e^9+(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)*ln(e*x
+d)/e^9
```

3.36.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.02

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx = \frac{x(420a^3e^6(-2Cd+2Be+Cex)+210a^2ce^4(C(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+2e(3Ae(-2d+ex)+\cdots}{\cdots} + \frac{(cd^2+ae^2)^3\,(Cd^2+e(-Bd+Ae))\log(d+ex)}{e^9}$$

input
```
Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
(x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d
^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3
*d*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2
*x^2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6
*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*
x^2 - 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*
d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*
e^6*x^6 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2
 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x
+ 140*d^4*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e
^6*x^6)))))/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
d + e*x])/e^9
```

3.36.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{c(d+ex)^3 \left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2\left(14Cd^4-d^2e(7Bd-3Ae)\right)\right)}{e^8}+\frac{(d+ex)\left(ae^2+cd\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^4 \left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2\left(14Cd^4-d^2e(7Bd-3Ae)\right)\right)}{4e^9}+\frac{(d+ex)^2 \left(ae^2+cd^2\right) \left(a^2Ce^4+ace^2\left(17Cd^2-3e(3Bd-Ae)\right)+c^2\left(28Cd^4-3d^2e(7Bd-5Ae)\right)\right)}{2e^9}-\frac{c(d+ex)^3 \left(3a^2e^4(4Cd-Be)+6acde^2\left(10Cd^2-e(5Bd-2Ae)\right)+c^2\left(56Cd^5-5d^3e(7Bd-4Ae)\right)\right)}{3e^9}-\frac{c^2(d+ex)^5 \left(3ae^2(6Cd-Be)-3cde(7Bd-2Ae)+56cCd^3\right)}{5e^9}+\frac{c^2(d+ex)^6 \left(3aCe^2-ce(7Bd-Ae)+28cCd^2\right)}{6e^9}+\frac{\left(ae^2+cd^2\right)^3 \log(d+ex) \left(Ae^2-Bde+Cd^2\right)}{e^9}-\frac{x\left(ae^2+cd^2\right)^2 \left(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3\right)}{e^8}-\frac{c^3(d+ex)^7(8Cd-Be)}{7e^9}+\frac{c^3C(d+ex)^8}{8e^9}$$

input
```
Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

output
```
-(((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B
*e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*
d - 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9)
 - (c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e))
+ 6*a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3
*a^2*C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^
2 - e*(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7
*B*d - 2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C
*d^2 + 3*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d -
 B*e)*(d + e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2
)^3*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.36.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.39

method	result
norman	$\frac{\left(3 A \,a^{2} c \,e^{6}+3 A a \,c^{2} d^{2} e^{4}+A \,c^{3} d^{4} e^{2}-3 B \,a^{2} c d \,e^{5}-3 B a \,c^{2} d^{3} e^{3}-B \,c^{3} d^{5} e+a^{3} C \,e^{6}+3 C \,a^{2} c \,d^{2} e^{4}+3 C a \,c^{2} d^{4} e^{2}+C \,c^{3} d^{6}\right) x^{2}}{2 e^{7}}-\frac{(3A}{}$
default	$-\frac{C \,a^{3} d \,e^{6} x+C \,a^{2} c d \,e^{6} x^{3}+C a \,c^{2} d^{3} e^{4} x^{3}-\frac{3}{2} C a \,c^{2} d^{4} e^{3} x^{2}-\frac{3}{2} A \,x^{2} a \,c^{2} d^{2} e^{5}+\frac{3}{2} B \,x^{2} a^{2} c d \,e^{6}+\frac{3}{2} B \,x^{2} a \,c^{2} d^{3} e^{4}+3 A x \,a^{2} c d \,e^{6}+3 A x}{}$
risch	$-\frac{C \,c^{3} d \,x^{7}}{7 e^{2}}+\frac{C a \,c^{2} x^{6}}{2 e}-\frac{3 C a \,c^{2} d^{5} x}{e^{6}}-\frac{3 C \,a^{2} c \,d^{3} x}{e^{4}}-\frac{C \,a^{2} c d \,x^{3}}{e^{2}}+\frac{3 A \,x^{2} a \,c^{2} d^{2}}{2 e^{3}}-\frac{3 B \,x^{2} a^{2} c d}{2 e^{2}}-\frac{3 B \,x^{2} a \,c^{2} d^{3}}{2 e^{4}}-3A$
parallelrisch	$\frac{-280 C \,x^{3} c^{3} d^{5} e^{3}+1260 A \,x^{2} a^{2} c \,e^{8}+420 A \,x^{2} c^{3} d^{4} e^{4}-420 B \,x^{2} c^{3} d^{5} e^{3}+420 C \,x^{2} c^{3} d^{6} e^{2}-840 A x \,c^{3} d^{5} e^{3}+840 B x \,c^{3} d^{6} e^{2}-840 C x}{}$

input
```
int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/e^7*(3*A*a^2*c*e^6+3*A*a*c^2*d^2*e^4+A*c^3*d^4*e^2-3*B*a^2*c*d*e^5-3*B
*a*c^2*d^3*e^3-B*c^3*d^5*e+C*a^3*e^6+3*C*a^2*c*d^2*e^4+3*C*a*c^2*d^4*e^2+C
*c^3*d^6)*x^2-(3*A*a^2*c*d*e^6+3*A*a*c^2*d^3*e^4+A*c^3*d^5*e^2-B*a^3*e^7-3
*B*a^2*c*d^2*e^5-3*B*a*c^2*d^4*e^3-B*c^3*d^6*e+C*a^3*d*e^6+3*C*a^2*c*d^3*e
^4+3*C*a*c^2*d^5*e^2+C*c^3*d^7)/e^8*x+1/8*C*c^3/e*x^8-1/3*c/e^6*(3*A*a*c*d
*e^4+A*c^2*d^3*e^2-3*B*a^2*e^5-3*B*a*c*d^2*e^3-B*c^2*d^4*e+3*C*a^2*d*e^4+3
*C*a*c*d^3*e^2+C*c^2*d^5)*x^3+1/4*c/e^5*(3*A*a*c*e^4+A*c^2*d^2*e^2-3*B*a*c
*d*e^3-B*c^2*d^3*e+3*C*a^2*e^4+3*C*a*c*d^2*e^2+C*c^2*d^4)*x^4-1/5*c^2/e^4*
(A*c*d*e^2-3*B*a*e^3-B*c*d^2*e+3*C*a*d*e^2+C*c*d^3)*x^5+1/6*c^2/e^3*(A*c*e
^2-B*c*d*e+3*C*a*e^2+C*c*d^2)*x^6+1/7*c^3/e^2*(B*e-C*d)*x^7+(A*a^3*e^8+3*A
*a^2*c*d^2*e^6+3*A*a*c^2*d^4*e^4+A*c^3*d^6*e^2-B*a^3*d*e^7-3*B*a^2*c*d^3*e
^5-3*B*a*c^2*d^5*e^3-B*c^3*d^7*e+C*a^3*d^2*e^6+3*C*a^2*c*d^4*e^4+3*C*a*c^2
*d^6*e^2+C*c^3*d^8)/e^9*ln(e*x+d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.38

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} dx = \frac{105 Cc^3e^8x^8 - 120 (Cc^3de^7 - Bc^3e^8)x^7 + 140 (Cc^3d^2e^6 - Bc^3de^7 + (3 Cac^2 + Ac^3)e^8)x^6 - 168 (Cc^3d^3e^5}{}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

output
```
1/840*(105*C*c^3*e^8*x^8 - 120*(C*c^3*d*e^7 - B*c^3*e^8)*x^7 + 140*(C*c^3*
d^2*e^6 - B*c^3*d*e^7 + (3*C*a*c^2 + A*c^3)*e^8)*x^6 - 168*(C*c^3*d^3*e^5
- B*c^3*d^2*e^6 - 3*B*a*c^2*e^8 + (3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 210*(C*
c^3*d^4*e^4 - B*c^3*d^3*e^5 - 3*B*a*c^2*d*e^7 + (3*C*a*c^2 + A*c^3)*d^2*e^
6 + 3*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 280*(C*c^3*d^5*e^3 - B*c^3*d^4*e^4 -
3*B*a*c^2*d^2*e^6 - 3*B*a^2*c*e^8 + (3*C*a*c^2 + A*c^3)*d^3*e^5 + 3*(C*a^2
*c + A*a*c^2)*d*e^7)*x^3 + 420*(C*c^3*d^6*e^2 - B*c^3*d^5*e^3 - 3*B*a*c^2*
d^3*e^5 - 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a
*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^
6*e^2 - 3*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A
*c^3)*d^5*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)
*x + 840*(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5
- B*a^3*d*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a
*c^2)*d^4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d))/e^9
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.40

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx = \frac{Cc^3x^8}{8e} + x^7\left(\frac{Bc^3}{7e} - \frac{Cc^3d}{7e^2}\right) + x^6\left(\frac{Ac^3}{6e} - \frac{Bc^3d}{6e^2} + \frac{Cac^2}{2e} + \frac{Cc^3d^2}{6e^3}\right) + x^5\left(-\frac{Ac^3d}{5e^2} + \frac{3Bac^2}{5e} + \frac{Bc^3d^2}{5e^3} - \frac{3Cac^2d}{5e^2} - \frac{Cc^3d^3}{5e^4}\right) + x^4\cdot\left(\frac{3Aac^2}{4e} + \frac{Ac^3d^2}{4e^3} - \frac{3Bac^2d}{4e^2} - \frac{Bc^3d^3}{4e^4} + \frac{3Ca^2c}{4e} + \frac{3Cac^2d^2}{4e^3} + \frac{Cc^3d^4}{4e^5}\right) + x^3\left(-\frac{Aac^2d}{e^2} - \frac{Ac^3d^3}{3e^4} + \frac{Ba^2c}{e} + \frac{Bac^2d^2}{e^3} + \frac{Bc^3d^4}{3e^5} - \frac{Ca^2cd}{e^2} - \frac{Cac^2d^3}{e^4} - \frac{Cc^3d^5}{3e^6}\right) + x^2\cdot\left(\frac{3Aa^2c}{2e} + \frac{3Aac^2d^2}{2e^3} + \frac{Ac^3d^4}{2e^5} - \frac{3Ba^2cd}{2e^2} - \frac{3Bac^2d^3}{2e^4} - \frac{Bc^3d^5}{2e^6} + \frac{Ca^3}{2e} + \frac{3Ca^2cd^2}{2e^3} + \frac{3Cac^2d^4}{2e^5} + \frac{Cc^3d^6}{2e^7}\right) + x\left(-\frac{3Aa^2cd}{e^2} - \frac{3Aac^2d^3}{e^4} - \frac{Ac^3d^5}{e^6} + \frac{Ba^3}{e} + \frac{3Ba^2cd^2}{e^3} + \frac{3Bac^2d^4}{e^5} + \frac{Bc^3d^6}{e^7} - \frac{Ca^3d}{e^2} - \frac{3Ca^2cd^3}{e^4} - \frac{3Cac^2d^5}{e^6} - \frac{Cc^3d^7}{e^8}\right) + \frac{(ae^2+cd^2)^3 (Ae^2 - Bde + Cd^2)\log(d+ex)}{e^9}$$

input
```
integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)
```

output
```
C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3
/(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5
*(-A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**
2*d/(5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**
2/(4*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*
e) + 3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/
e**2 - A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**
4/(3*e**5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6))
+ x**2*(3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5)
 - 3*B*a**2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6)
 + C*a**3/(2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*
c**3*d**6/(2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**
3*d**5/e**6 + B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c
**3*d**6/e**7 - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**
6 - C*c**3*d**7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log
(d + e*x)/e**9
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.37

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx$$
$$= \frac{105\,Cc^3e^7x^8 - 120\,(Cc^3de^6 - Bc^3e^7)x^7 + 140\,(Cc^3d^2e^5 - Bc^3de^6 + (3\,Cac^2 + Ac^3)e^7)x^6 - 168\,(Cc^3d^3e^4}{}$$
$$+ \frac{(Cc^3d^8 - Bc^3d^7e - 3\,Bac^2d^5e^3 - 3\,Ba^2cd^3e^5 - Ba^3de^7 + Aa^3e^8 + (3\,Cac^2 + Ac^3)d^6e^2 + 3\,(Ca^2c + Aa}{e^9}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

output
```
1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*
d^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4
- B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*
c^3*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^
5 + 3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 -
3*B*a*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2
*c + A*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^
3*e^4 - 3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c
^2)*d^2*e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e
- 3*B*a*c^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*
d^5*e^2 + 3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^
8 + (C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a
^3*d*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)
*d^4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.67

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex} dx$$
$$= \frac{105\, Cc^3e^7x^8 - 120\, Cc^3de^6x^7 + 120\, Bc^3e^7x^7 + 140\, Cc^3d^2e^5x^6 - 140\, Bc^3de^6x^6 + 420\, Cac^2e^7x^6 + 140\, Ac^3e}{}$$
$$+ \frac{(Cc^3d^8 - Bc^3d^7e + 3\, Cac^2d^6e^2 + Ac^3d^6e^2 - 3\, Bac^2d^5e^3 + 3\, Ca^2cd^4e^4 + 3\, Aac^2d^4e^4 - 3\, Ba^2cd^3e^5 + Ca}{e^9}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

output
```
1/840*(105*C*c^3*e^7*x^8 - 120*C*c^3*d*e^6*x^7 + 120*B*c^3*e^7*x^7 + 140*C
*c^3*d^2*e^5*x^6 - 140*B*c^3*d*e^6*x^6 + 420*C*a*c^2*e^7*x^6 + 140*A*c^3*e
^7*x^6 - 168*C*c^3*d^3*e^4*x^5 + 168*B*c^3*d^2*e^5*x^5 - 504*C*a*c^2*d*e^6
*x^5 - 168*A*c^3*d*e^6*x^5 + 504*B*a*c^2*e^7*x^5 + 210*C*c^3*d^4*e^3*x^4 -
 210*B*c^3*d^3*e^4*x^4 + 630*C*a*c^2*d^2*e^5*x^4 + 210*A*c^3*d^2*e^5*x^4 -
 630*B*a*c^2*d*e^6*x^4 + 630*C*a^2*c*e^7*x^4 + 630*A*a*c^2*e^7*x^4 - 280*C
*c^3*d^5*e^2*x^3 + 280*B*c^3*d^4*e^3*x^3 - 840*C*a*c^2*d^3*e^4*x^3 - 280*A
*c^3*d^3*e^4*x^3 + 840*B*a*c^2*d^2*e^5*x^3 - 840*C*a^2*c*d*e^6*x^3 - 840*A
*a*c^2*d*e^6*x^3 + 840*B*a^2*c*e^7*x^3 + 420*C*c^3*d^6*e*x^2 - 420*B*c^3*d
^5*e^2*x^2 + 1260*C*a*c^2*d^4*e^3*x^2 + 420*A*c^3*d^4*e^3*x^2 - 1260*B*a*c
^2*d^3*e^4*x^2 + 1260*C*a^2*c*d^2*e^5*x^2 + 1260*A*a*c^2*d^2*e^5*x^2 - 126
0*B*a^2*c*d*e^6*x^2 + 420*C*a^3*e^7*x^2 + 1260*A*a^2*c*e^7*x^2 - 840*C*c^3
*d^7*x + 840*B*c^3*d^6*e*x - 2520*C*a*c^2*d^5*e^2*x - 840*A*c^3*d^5*e^2*x
+ 2520*B*a*c^2*d^4*e^3*x - 2520*C*a^2*c*d^3*e^4*x - 2520*A*a*c^2*d^3*e^4*x
 + 2520*B*a^2*c*d^2*e^5*x - 840*C*a^3*d*e^6*x - 2520*A*a^2*c*d*e^6*x + 840
*B*a^3*e^7*x)/e^8 + (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d
^6*e^2 - 3*B*a*c^2*d^5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a
^2*c*d^3*e^5 + C*a^3*d^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8
)*log(abs(e*x + d))/e^9
```

3.36. $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

3.36.9 Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.51

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{d+ex}\,dx = x\left(\frac{B\,a^3}{e}\right.$$

input
```
int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x)
```

output
```
x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*
c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*
c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^
3*d)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)
/e^2))/e))/(5*e) - (3*B*a*c^2)/(5*e)) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e
 - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*
(A*c + C*a))/(4*e)) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*
c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e
))/e + (3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e)) + x^6*((A*c^3 +
3*C*a*c^2)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((
d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*
c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (log(d + e*x)*
(A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3
*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B
*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)
/(8*e)
```

3.36. $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

3.37 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.37.1 Optimal result

Integrand size = 27, antiderivative size = 486

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{(a^3Ce^6+c^3d^4(7Cd^2-e(6Bd-5Ae))+3ac^2d^2e^2(5Cd^2-e(4Bd-3Ae))+3a^2ce^4(3Cd^2-e(2Bd-Ae}{e^8}$$
$$-\frac{c(3a^2e^4(2Cd-Be)+c^2d^3(6Cd^2-e(5Bd-4Ae))+3acde^2(4Cd^2-e(3Bd-2Ae)))\,x^2}{2e^7}$$
$$+\frac{c(3a^2Ce^4+c^2d^2(5Cd^2-e(4Bd-3Ae))+3ace^2(3Cd^2-e(2Bd-Ae)))\,x^3}{3e^6}$$
$$-\frac{c^2(3ae^2(2Cd-Be)+cd(4Cd^2-e(3Bd-2Ae)))\,x^4}{4e^5}$$
$$+\frac{c^2(3aCe^2+c(3Cd^2-e(2Bd-Ae)))\,x^5}{5e^4}-\frac{c^3(2Cd-Be)x^6}{6e^3}$$
$$+\frac{c^3Cx^7}{7e^2}-\frac{(cd^2+ae^2)^3\,(Cd^2-Bde+Ae^2)}{e^9(d+ex)}$$
$$-\frac{(cd^2+ae^2)^2\,(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))\log(d+ex)}{e^9}$$

output
```
(a^3*C*e^6+c^3*d^4*(7*C*d^2-e*(-5*A*e+6*B*d))+3*a*c^2*d^2*e^2*(5*C*d^2-e*(
-3*A*e+4*B*d))+3*a^2*c*e^4*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^8-1/2*c*(3*a^2*e^
4*(-B*e+2*C*d)+c^2*d^3*(6*C*d^2-e*(-4*A*e+5*B*d))+3*a*c*d*e^2*(4*C*d^2-e*(
-2*A*e+3*B*d)))*x^2/e^7+1/3*c*(3*a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*
d))+3*a*c*e^2*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^6-1/4*c^2*(3*a*e^2*(-B*e+2*C
*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^4/e^5+1/5*c^2*(3*a*C*e^2+c*(3*C*d^2-
e*(-A*e+2*B*d)))*x^5/e^4-1/6*c^3*(-B*e+2*C*d)*x^6/e^3+1/7*c^3*C*x^7/e^2-(a
*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)-(a*e^2+c*d^2)^2*(a*e^2*(-B*e
+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*ln(e*x+d)/e^9
```

3.37.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.32

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{420a^3e^6(e(Bd-Ae)+C(-d^2+dex+e^2x^2))+210a^2ce^4(2C(-3d^4+9d^3ex+6d^2e^2x^2-2de^3x^3+e^4x^4)}{}$$

input
```
Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
(420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*
(2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2
*A*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x
^3))) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*
e^3*x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 +
9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*
x - 30*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4
*C*(105*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4
*x^4 + 42*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*
(6*A*e*(-10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4
*x^4 - 3*d*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^
2 + 70*d^4*e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e
^7*x^7))) - 420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*
e^2*(2*C*d - B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

3.37.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(\frac{cx^2 (3a^2Ce^4+3ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae)))}{e^6}+\frac{cx(-3a^2e^4(2Cd-Be)-3ac\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{cx^3 (3a^2Ce^4+3ace^2(3Cd^2-e(2Bd-Ae))+c^2(5Cd^4-d^2e(4Bd-3Ae)))}{3e^6}-\frac{cx^2 (3a^2e^4(2Cd-Be)+3acde^2(4Cd^2-e(3Bd-2Ae))+c^2(6Cd^5-d^3e(5Bd-4Ae)))}{2e^7}+\frac{x (a^3Ce^6+3a^2ce^4(3Cd^2-e(2Bd-Ae))+3ac^2d^2e^2(5Cd^2-e(4Bd-3Ae))+c^3(7Cd^6-d^4e(6Bd-5Ae)))}{e^8}-\frac{c^2x^4 (3ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3)}{4e^5}+\frac{c^2x^5 (3aCe^2-ce(2Bd-Ae)+3cCd^2)}{5e^4}-\frac{(ae^2+cd^2)^3 (Ae^2-Bde+Cd^2)}{e^9(d+ex)}-\frac{(ae^2+cd^2)^2 \log(d+ex) (ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3)}{e^9}-\frac{c^3x^6(2Cd-Be)}{6e^3}+\frac{c^3Cx^7}{7e^2}$$

input
```
Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

output
```
((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C
*d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^
8 - (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) +
3*a*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4
+ c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d -
A*e)))*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2
*C*d - B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e
))*x^5)/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) -
((c*d^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*
e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e
*x])/e^9
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.37.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.46

method	result
norman	$\frac{\left(A a^3 e^8+6A a^2 c d^2 e^6+12Aa c^2 d^4 e^4+6A c^3 d^6 e^2-B a^3 d e^7-9B a^2 c d^3 e^5-15Ba c^2 d^5 e^3-7B c^3 d^7 e+2C a^3 d^2 e^6+12C a^2 c d^4 e^4+18Ca c^2 d^6 e}{e^8 d}$
default	$3Ca c^2 d^2 e^4 x^3-3C c^3 d^5 e x^2+3A a^2 c e^6 x+a^3 C e^6 x+7C c^3 d^6 x+\frac{1}{6}B c^3 e^6 x^6+\frac{1}{5}A c^3 e^6 x^5+\frac{1}{7}c^3 C x^7 e^6+Aa c^2 e^6 x^3+A c^3 d^2 e^4 x^3+$
risch	$-\frac{A c^3 d^6}{e^7(ex+d)}+\frac{B a^3 d}{e^2(ex+d)}+\frac{B c^3 d^7}{e^8(ex+d)}-\frac{C a^3 d^2}{e^3(ex+d)}-\frac{C c^3 d^8}{e^9(ex+d)}-\frac{6\ln(ex+d)A c^3 d^5}{e^7}+\frac{7\ln(ex+d)B c^3 d^6}{e^8}-\frac{2\ln(ex$
parallelrisch	Expression too large to display

input
```
int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^3*e^8+6*A*a^2*c*d^2*e^6+12*A*a*c^2*d^4*e^4+6*A*c^3*d^6*e^2-B*a^3*d*e
^7-9*B*a^2*c*d^3*e^5-15*B*a*c^2*d^5*e^3-7*B*c^3*d^7*e+2*C*a^3*d^2*e^6+12*C
*a^2*c*d^4*e^4+18*C*a*c^2*d^6*e^2+8*C*c^3*d^8)/e^8/d*x+1/2*(6*A*a^2*c*e^6+
12*A*a*c^2*d^2*e^4+6*A*c^3*d^4*e^2-9*B*a^2*c*d*e^5-15*B*a*c^2*d^3*e^3-7*B*
c^3*d^5*e+2*C*a^3*e^6+12*C*a^2*c*d^2*e^4+18*C*a*c^2*d^4*e^2+8*C*c^3*d^6)/e
^7*x^2+1/7*C*c^3/e*x^8+1/12*c*(12*A*a*c*e^4+6*A*c^2*d^2*e^2-15*B*a*c*d*e^3
-7*B*c^2*d^3*e+12*C*a^2*e^4+18*C*a*c*d^2*e^2+8*C*c^2*d^4)/e^5*x^4-1/6*c*(1
2*A*a*c*d*e^4+6*A*c^2*d^3*e^2-9*B*a^2*e^5-15*B*a*c*d^2*e^3-7*B*c^2*d^4*e+1
2*C*a^2*d*e^4+18*C*a*c*d^3*e^2+8*C*c^2*d^5)/e^6*x^3+1/30*c^2*(6*A*c*e^2-7*
B*c*d*e+18*C*a*e^2+8*C*c*d^2)/e^3*x^6-1/20*c^2*(6*A*c*d*e^2-15*B*a*e^3-7*B
*c*d^2*e+18*C*a*d*e^2+8*C*c*d^3)/e^4*x^5+1/42*c^3*(7*B*e-8*C*d)/e^2*x^7)/(
e*x+d)-(6*A*a^2*c*d*e^6+12*A*a*c^2*d^3*e^4+6*A*c^3*d^5*e^2-B*a^3*e^7-9*B*a
^2*c*d^2*e^5-15*B*a*c^2*d^4*e^3-7*B*c^3*d^6*e+2*C*a^3*d*e^6+12*C*a^2*c*d^3
*e^4+18*C*a*c^2*d^5*e^2+8*C*c^3*d^7)/e^9*ln(e*x+d)
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 932, normalized size of antiderivative = 1.92

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} \, dx$$
$$= \frac{60\, Cc^3e^8x^8 - 420\, Cc^3d^8 + 420\, Bc^3d^7e + 1260\, Bac^2d^5e^3 + 1260\, Ba^2cd^3e^5 + 420\, Ba^3de^7 - 420\, Aa^3e^8 - 42}{}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

output

```
1/420*(60*C*c^3*e^8*x^8 - 420*C*c^3*d^8 + 420*B*c^3*d^7*e + 1260*B*a*c^2*d
^5*e^3 + 1260*B*a^2*c*d^3*e^5 + 420*B*a^3*d*e^7 - 420*A*a^3*e^8 - 420*(3*C
*a*c^2 + A*c^3)*d^6*e^2 - 1260*(C*a^2*c + A*a*c^2)*d^4*e^4 - 420*(C*a^3 +
3*A*a^2*c)*d^2*e^6 - 10*(8*C*c^3*d*e^7 - 7*B*c^3*e^8)*x^7 + 14*(8*C*c^3*d^
2*e^6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e
^5 - 7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 +
 35*(8*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 +
 A*c^3)*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 -
7*B*c^3*d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^
3)*d^3*e^5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*
B*c^3*d^5*e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^
3)*d^4*e^4 + 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x
^2 + 420*(7*C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c
*d^2*e^6 + 5*(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 +
 (C*a^3 + 3*A*a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*
c^2*d^5*e^3 - 9*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*
e^2 + 12*(C*a^2*c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*
C*c^3*d^7*e - 7*B*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B
*a^3*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5
+ 2*(C*a^3 + 3*A*a^2*c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

3.37. $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.37.6 Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.54

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$
$$= \frac{Cc^3x^7}{7e^2} + x^6\left(\frac{Bc^3}{6e^2} - \frac{Cc^3d}{3e^3}\right) + x^5\left(\frac{Ac^3}{5e^2} - \frac{2Bc^3d}{5e^3} + \frac{3Cac^2}{5e^2} + \frac{3Cc^3d^2}{5e^4}\right)$$
$$+ x^4\left(-\frac{Ac^3d}{2e^3} + \frac{3Bac^2}{4e^2} + \frac{3Bc^3d^2}{4e^4} - \frac{3Cac^2d}{2e^3} - \frac{Cc^3d^3}{e^5}\right)$$
$$+ x^3\left(\frac{Aac^2}{e^2} + \frac{Ac^3d^2}{e^4} - \frac{2Bac^2d}{e^3} - \frac{4Bc^3d^3}{3e^5} + \frac{Ca^2c}{e^2} + \frac{3Cac^2d^2}{e^4} + \frac{5Cc^3d^4}{3e^6}\right)$$
$$+ x^2\left(-\frac{3Aac^2d}{e^3} - \frac{2Ac^3d^3}{e^5} + \frac{3Ba^2c}{2e^2} + \frac{9Bac^2d^2}{2e^4} + \frac{5Bc^3d^4}{2e^6} - \frac{3Ca^2cd}{e^3} - \frac{6Cac^2d^3}{e^5} - \frac{3Cc^3d^5}{e^7}\right)$$
$$+ x\left(\frac{3Aa^2c}{e^2} + \frac{9Aac^2d^2}{e^4} + \frac{5Ac^3d^4}{e^6} - \frac{6Ba^2cd}{e^3} - \frac{12Bac^2d^3}{e^5} - \frac{6Bc^3d^5}{e^7} + \frac{Ca^3}{e^2} + \frac{9Ca^2cd^2}{e^4} + \frac{15Cac^2d^4}{e^6} + \frac{7Cc^3d^6}{e^8}\right)$$
$$+ \frac{-Aa^3e^8 - 3Aa^2cd^2e^6 - 3Aac^2d^4e^4 - Ac^3d^6e^2 + Ba^3de^7 + 3Ba^2cd^3e^5 + 3Bac^2d^5e^3 + Bc^3d^7e - Ca^3d^2e}{de^9 + e^{10}x}$$
$$- \frac{(ae^2+cd^2)^2 \cdot (6Acde^2 - Bae^3 - 7Bcd^2e + 2Cade^2 + 8Ccd^3)\log(d+ex)}{e^9}$$

input
```
integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

output
```
C*c**3*x**7/(7*e**2) + x**6*(B*c**3/(6*e**2) - C*c**3*d/(3*e**3)) + x**5*(
A*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3) + 3*C*a*c**2/(5*e**2) + 3*C*c**3*d**
2/(5*e**4)) + x**4*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d*
*2/(4*e**4) - 3*C*a*c**2*d/(2*e**3) - C*c**3*d**3/e**5) + x**3*(A*a*c**2/e
**2 + A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3*e**5) + C*a*
*2*c/e**2 + 3*C*a*c**2*d**2/e**4 + 5*C*c**3*d**4/(3*e**6)) + x**2*(-3*A*a*
c**2*d/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e**2) + 9*B*a*c**2*d**2/(
2*e**4) + 5*B*c**3*d**4/(2*e**6) - 3*C*a**2*c*d/e**3 - 6*C*a*c**2*d**3/e**
5 - 3*C*c**3*d**5/e**7) + x*(3*A*a**2*c/e**2 + 9*A*a*c**2*d**2/e**4 + 5*A*
c**3*d**4/e**6 - 6*B*a**2*c*d/e**3 - 12*B*a*c**2*d**3/e**5 - 6*B*c**3*d**5
/e**7 + C*a**3/e**2 + 9*C*a**2*c*d**2/e**4 + 15*C*a*c**2*d**4/e**6 + 7*C*c
**3*d**6/e**8) + (-A*a**3*e**8 - 3*A*a**2*c*d**2*e**6 - 3*A*a*c**2*d**4*e*
*4 - A*c**3*d**6*e**2 + B*a**3*d*e**7 + 3*B*a**2*c*d**3*e**5 + 3*B*a*c**2*
d**5*e**3 + B*c**3*d**7*e - C*a**3*d**2*e**6 - 3*C*a**2*c*d**4*e**4 - 3*C*
a*c**2*d**6*e**2 - C*c**3*d**8)/(d*e**9 + e**10*x) - (a*e**2 + c*d**2)**2*
(6*A*c*d*e**2 - B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c*d**3)*log(d
 + e*x)/e**9
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} dx =$$
$$-\frac{Cc^3d^8 - Bc^3d^7e - 3\,Bac^2d^5e^3 - 3\,Ba^2cd^3e^5 - Ba^3de^7 + Aa^3e^8 + (3\,Cac^2 + Ac^3)d^6e^2 + 3\,(Ca^2c + Aac}{e^{10}x + de^9}$$
$$+\frac{60\,Cc^3e^6x^7 - 70\,(2\,Cc^3de^5 - Bc^3e^6)x^6 + 84\,(3\,Cc^3d^2e^4 - 2\,Bc^3de^5 + (3\,Cac^2 + Ac^3)e^6)x^5 - 105\,(4\,Cc^3}{}$$
$$-\frac{(8\,Cc^3d^7 - 7\,Bc^3d^6e - 15\,Bac^2d^4e^3 - 9\,Ba^2cd^2e^5 - Ba^3e^7 + 6\,(3\,Cac^2 + Ac^3)d^5e^2 + 12\,(Ca^2c + Aac^2)}{e^9}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*
d*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^
4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)/(e^10*x + d*e^9) + 1/420*(60*C*c^3*e^
6*x^7 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*B*c^3
*d*e^5 + (3*C*a*c^2 + A*c^3)*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c^3*d^2
*e^4 - 3*B*a*c^2*e^6 + 2*(3*C*a*c^2 + A*c^3)*d*e^5)*x^4 + 140*(5*C*c^3*d^4
*e^2 - 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*(3*C*a*c^2 + A*c^3)*d^2*e^4 +
 3*(C*a^2*c + A*a*c^2)*e^6)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*c^3*d^4*e^2 - 9
*B*a*c^2*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2 + A*c^3)*d^3*e^3 + 6*(C*a^
2*c + A*a*c^2)*d*e^5)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^3*d^5*e - 12*B*a*c^2*
d^3*e^3 - 6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 9*(C*a^2*c + A
*a*c^2)*d^2*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*x)/e^8 - (8*C*c^3*d^7 - 7*B*c^3
*d^6*e - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2
 + A*c^3)*d^5*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)
*d*e^6)*log(e*x + d)/e^9
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.81

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2}\,dx$$

$$= \frac{\left(60\,Cc^3 - \frac{70\,(8\,Cc^3de - Bc^3e^2)}{(ex+d)e} + \frac{84\,(28\,Cc^3d^2e^2 - 7\,Bc^3de^3 + 3\,Cac^2e^4 + Ac^3e^4)}{(ex+d)^2e^2} - \frac{105\,(56\,Cc^3d^3e^3 - 21\,Bc^3d^2e^4 + 18\,Cac^2de^5 + 6\,Ac^3de}{(ex+d)^3e^3}}{}$$

$$+ \frac{(8\,Cc^3d^7 - 7\,Bc^3d^6e + 18\,Cac^2d^5e^2 + 6\,Ac^3d^5e^2 - 15\,Bac^2d^4e^3 + 12\,Ca^2cd^3e^4 + 12\,Aac^2d^3e^4 - 9\,Ba^2c}{e^9}$$

$$- \frac{\frac{Cc^3d^8e^7}{ex+d} - \frac{Bc^3d^7e^8}{ex+d} + \frac{3\,Cac^2d^6e^9}{ex+d} + \frac{Ac^3d^6e^9}{ex+d} - \frac{3\,Bac^2d^5e^{10}}{ex+d} + \frac{3\,Ca^2cd^4e^{11}}{ex+d} + \frac{3\,Aac^2d^4e^{11}}{ex+d} - \frac{3\,Ba^2cd^3e^{12}}{ex+d} + \frac{Ca^3d^2e^{13}}{ex+d} + 3}{e^{16}}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)/((e*x + d)*e) + 84*(28*C*c^
3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)/((e*x + d)^2*e^2) -
 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*e
^5 - 3*B*a*c^2*e^6)/((e*x + d)^3*e^3) + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*d
^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^
2*c*e^8 + 3*A*a*c^2*e^8)/((e*x + d)^4*e^4) - 210*(56*C*c^3*d^5*e^5 - 35*B*
c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 +
 12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)/((e*x + d)^5*e^5) +
 420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*
d^4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 -
 9*B*a^2*c*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)/((e*x + d)^6*e^6))*(e*x +
 d)^7/e^9 + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^
5*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B
*a^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*log(abs(e*x
+ d)/((e*x + d)^2*abs(e)))/e^9 - (C*c^3*d^8*e^7/(e*x + d) - B*c^3*d^7*e^8/
(e*x + d) + 3*C*a*c^2*d^6*e^9/(e*x + d) + A*c^3*d^6*e^9/(e*x + d) - 3*B*a*
c^2*d^5*e^10/(e*x + d) + 3*C*a^2*c*d^4*e^11/(e*x + d) + 3*A*a*c^2*d^4*e^11
/(e*x + d) - 3*B*a^2*c*d^3*e^12/(e*x + d) + C*a^3*d^2*e^13/(e*x + d) + 3*A
*a^2*c*d^2*e^13/(e*x + d) - B*a^3*d*e^14/(e*x + d) + A*a^3*e^15/(e*x + d))
/e^16
```

3.37.9 Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 1511, normalized size of antiderivative = 3.11

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^2} \, dx = \text{Too large to display}$$

input
```
int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

output

```
x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^
3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*
d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*
c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)
/e^2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (
2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*
((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)
/e^2))/e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 +
3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (
2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*
((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c +
 C*a))/e^2))/e^2) + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*
c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*
c^3*d)/e^3))/(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)
/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e
^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*
c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2
 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((
B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e
^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/...
```

3.37. $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

3.38 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

3.38.1 Optimal result

Integrand size = 27, antiderivative size = 466

$$\begin{aligned}\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx &= \\ &-\frac{c(3a^2e^4(3Cd-Be)+c^2d^3(21Cd^2-5e(3Bd-2Ae))+3acde^2(10Cd^2-3e(2Bd-Ae)))\,x}{e^8} \\ &+\frac{c(3a^2Ce^4+c^2d^2(15Cd^2-2e(5Bd-3Ae))+3ace^2(6Cd^2-e(3Bd-Ae)))\,x^2}{2e^7} \\ &-\frac{c^2(3ae^2(3Cd-Be)+cd(10Cd^2-3e(2Bd-Ae)))\,x^3}{3e^6} \\ &+\frac{c^2(3aCe^2+c(6Cd^2-e(3Bd-Ae)))\,x^4}{4e^5}-\frac{c^3(3Cd-Be)x^5}{5e^4} \\ &+\frac{c^3Cx^6}{6e^3}-\frac{(cd^2+ae^2)^3(Cd^2-Bde+Ae^2)}{2e^9(d+ex)^2} \\ &+\frac{(cd^2+ae^2)^2(ae^2(2Cd-Be)+cd(8Cd^2-e(7Bd-6Ae)))}{e^9(d+ex)} \\ &+\frac{(cd^2+ae^2)(a^2Ce^4+c^2d^2(28Cd^2-3e(7Bd-5Ae))+ace^2(17Cd^2-3e(3Bd-Ae)))\log(d+ex)}{e^9}\end{aligned}$$

output
```
-c*(3*a^2*e^4*(-B*e+3*C*d)+c^2*d^3*(21*C*d^2-5*e*(-2*A*e+3*B*d))+3*a*c*d*e
^2*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^8+1/2*c*(3*a^2*C*e^4+c^2*d^2*(15*C*d^2
-2*e*(-3*A*e+5*B*d))+3*a*c*e^2*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^7-1/3*c^2*(
3*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x^3/e^6+1/4*c^2*(3*a
*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^4/e^5-1/5*c^3*(-B*e+3*C*d)*x^5/e^4+1/
6*c^3*C*x^6/e^3-1/2*(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)^2+(a*e
^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))/e^9/(e*x+d
)+(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*(
17*C*d^2-3*e*(-A*e+3*B*d)))*ln(e*x+d)/e^9
```

3.38.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} \, dx$$
$$= \frac{-60ce(-3a^2e^4(-3Cd+Be)+3acde^2(10Cd^2+3e(-2Bd+Ae))+c^2(21Cd^5+5d^3e(-3Bd+2Ae)))\,x+}{}$$

input
```
Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
(-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d
+ A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C
*e^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-
5*B*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3
*a*e^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B
*d + A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c
*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e
^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x
) + 60*(c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)
) + c^2*(28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

3.38.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3} dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{(ae^2+cd^2)\left(a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae))\right)}{e^8(d+ex)}+\frac{cx(3a^2Ce^4+3ace\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ae^2+cd^2)\log(d+ex)\left(a^2Ce^4+ace^2(17Cd^2-3e(3Bd-Ae))+c^2(28Cd^4-3d^2e(7Bd-5Ae))\right)}{e^9}+\frac{cx^2\left(3a^2Ce^4+3ace^2(6Cd^2-e(3Bd-Ae))+c^2(15Cd^4-2d^2e(5Bd-3Ae))\right)}{2e^7}-\frac{cx\left(3a^2e^4(3Cd-Be)+3acde^2(10Cd^2-3e(2Bd-Ae))+c^2(21Cd^5-5d^3e(3Bd-2Ae))\right)}{e^8}-\frac{c^2x^3\left(3ae^2(3Cd-Be)-3cde(2Bd-Ae)+10cCd^3\right)}{3e^6}+\frac{c^2x^4\left(3aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{4e^5}-\frac{(ae^2+cd^2)^3(Ae^2-Bde+Cd^2)}{2e^9(d+ex)^2}+\frac{(ae^2+cd^2)^2\left(ae^2(2Cd-Be)-cde(7Bd-6Ae)+8cCd^3\right)}{e^9(d+ex)}-\frac{c^3x^5(3Cd-Be)}{5e^4}+\frac{c^3Cx^6}{6e^3}$$

input
```
Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

output
```
-((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) +
 3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c
^2*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d -
A*e)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(
3*C*d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*
e))*x^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) -
 ((c*d^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2
 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^
9*(d + e*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*
d - 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.38.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.50

method	result
norman	$\frac{\left(6A a^2 c d e^6+36A a c^2 d^3 e^4+30A c^3 d^5 e^2-B a^3 e^7-18B a^2 c d^2 e^5-60B a c^2 d^4 e^3-42B c^3 d^6 e+2C a^3 d e^6+36C a^2 c d^3 e^4+90C a c^2 d^5 e^2+56C c^3\right.}{e^8}$
default	$-\frac{c\left(30Cac d^3 e^2 x-18Bxac d^2 e^3+9Axacd e^4+\frac{9}{2}B x^2 acd e^4-\frac{3}{2}C a^2 e^5 x^2-\frac{1}{6}c^2 C x^6 e^5-3Bx a^2 e^5-\frac{1}{4}A x^4 c^2 e^5-\frac{1}{5}B x^5 c^2 e^5+21C\right.}{}$
risch	$\frac{3cC a^2 x^2}{2e^3}+\frac{3cBx a^2}{e^3}-\frac{10c^3 Ax d^3}{e^6}+\frac{15c^3 Bx d^4}{e^7}-\frac{3c^3 B x^4 d}{4e^4}-\frac{10c^3 C d^3 x^3}{3e^6}+\frac{15c^3 C d^4 x^2}{2e^7}-\frac{3c^2 Cad x^3}{e^4}-\frac{9cC a^2}{e^4}$
parallelrisch	Expression too large to display

input
```
int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
((6*A*a^2*c*d*e^6+36*A*a*c^2*d^3*e^4+30*A*c^3*d^5*e^2-B*a^3*e^7-18*B*a^2*c
*d^2*e^5-60*B*a*c^2*d^4*e^3-42*B*c^3*d^6*e+2*C*a^3*d*e^6+36*C*a^2*c*d^3*e^
4+90*C*a*c^2*d^5*e^2+56*C*c^3*d^7)/e^8*x-1/2*(A*a^3*e^8-9*A*a^2*c*d^2*e^6-
54*A*a*c^2*d^4*e^4-45*A*c^3*d^6*e^2+B*a^3*d*e^7+27*B*a^2*c*d^3*e^5+90*B*a*
c^2*d^5*e^3+63*B*c^3*d^7*e-3*C*a^3*d^2*e^6-54*C*a^2*c*d^4*e^4-135*C*a*c^2*
d^6*e^2-84*C*c^3*d^8)/e^9+1/6*C*c^3/e*x^8+1/12*c*(18*A*a*c*e^4+15*A*c^2*d^
2*e^2-30*B*a*c*d*e^3-21*B*c^2*d^3*e+18*C*a^2*e^4+45*C*a*c*d^2*e^2+28*C*c^2
*d^4)/e^5*x^4-1/3*c*(18*A*a*c*d*e^4+15*A*c^2*d^3*e^2-9*B*a^2*e^5-30*B*a*c*
d^2*e^3-21*B*c^2*d^4*e+18*C*a^2*d*e^4+45*C*a*c*d^3*e^2+28*C*c^2*d^5)/e^6*x
^3+1/60*c^2*(15*A*c*e^2-21*B*c*d*e+45*C*a*e^2+28*C*c*d^2)/e^3*x^6-1/30*c^2
*(15*A*c*d*e^2-30*B*a*e^3-21*B*c*d^2*e+45*C*a*d*e^2+28*C*c*d^3)/e^4*x^5+1/
15*c^3*(3*B*e-4*C*d)/e^2*x^7)/(e*x+d)^2+1/e^9*(3*A*a^2*c*e^6+18*A*a*c^2*d^
2*e^4+15*A*c^3*d^4*e^2-9*B*a^2*c*d*e^5-30*B*a*c^2*d^3*e^3-21*B*c^3*d^5*e+C
*a^3*e^6+18*C*a^2*c*d^2*e^4+45*C*a*c^2*d^4*e^2+28*C*c^3*d^6)*ln(e*x+d)
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(454) = 908$.

Time = 0.30 (sec) , antiderivative size = 1025, normalized size of antiderivative = 2.20

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{10\,Cc^3e^8x^8 + 450\,Cc^3d^8 - 390\,Bc^3d^7e - 810\,Bac^2d^5e^3 - 450\,Ba^2cd^3e^5 - 30\,Ba^3de^7 - 30\,Aa^3e^8 + 330\,(3\,\cdots}{\cdots}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5
*e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^
2 + A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2
*c)*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21
*B*c^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*
B*c^3*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28
*C*c^3*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c
^3)*d^2*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*
B*c^3*d^4*e^4 - 30*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 15*(3*C*a*c^2 + A*c^3
)*d^3*e^5 + 18*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 - 30*(69*C*c^3*d^6*e^2 - 50*
B*c^3*d^5*e^3 - 63*B*a*c^2*d^3*e^5 - 12*B*a^2*c*d*e^7 + 34*(3*C*a*c^2 + A*
c^3)*d^4*e^4 + 33*(C*a^2*c + A*a*c^2)*d^2*e^6)*x^2 - 60*(13*C*c^3*d^7*e -
8*B*c^3*d^6*e^2 - 3*B*a*c^2*d^4*e^4 + 6*B*a^2*c*d^2*e^6 + B*a^3*e^8 + 4*(3
*C*a*c^2 + A*c^3)*d^5*e^3 - 3*(C*a^2*c + A*a*c^2)*d^3*e^5 - 2*(C*a^3 + 3*A
*a^2*c)*d*e^7)*x + 60*(28*C*c^3*d^8 - 21*B*c^3*d^7*e - 30*B*a*c^2*d^5*e^3
- 9*B*a^2*c*d^3*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 18*(C*a^2*c + A*a*c
^2)*d^4*e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6 + (28*C*c^3*d^6*e^2 - 21*B*c^3*d
^5*e^3 - 30*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^4
*e^4 + 18*(C*a^2*c + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 2*(
28*C*c^3*d^7*e - 21*B*c^3*d^6*e^2 - 30*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*...
```

3.38.6 Sympy [A] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.75

$$\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{Cc^3x^6}{6e^3} + x^5\left(\frac{Bc^3}{5e^3} - \frac{3Cc^3d}{5e^4}\right) + x^4\left(\frac{Ac^3}{4e^3} - \frac{3Bc^3d}{4e^4} + \frac{3Cac^2}{4e^3} + \frac{3Cc^3d^2}{2e^5}\right)$$
$$+ x^3\left(-\frac{Ac^3d}{e^4} + \frac{Bac^2}{e^3} + \frac{2Bc^3d^2}{e^5} - \frac{3Cac^2d}{e^4} - \frac{10Cc^3d^3}{3e^6}\right) + x^2$$
$$\cdot\left(\frac{3Aac^2}{2e^3} + \frac{3Ac^3d^2}{e^5} - \frac{9Bac^2d}{2e^4} - \frac{5Bc^3d^3}{e^6} + \frac{3Ca^2c}{2e^3} + \frac{9Cac^2d^2}{e^5} + \frac{15Cc^3d^4}{2e^7}\right) + x\left(-\frac{9Aac^2d}{e^4}\right.$$
$$\left. - \frac{10Ac^3d^3}{e^6} + \frac{3Ba^2c}{e^3} + \frac{18Bac^2d^2}{e^5} + \frac{15Bc^3d^4}{e^7} - \frac{9Ca^2cd}{e^4} - \frac{30Cac^2d^3}{e^6} - \frac{21Cc^3d^5}{e^8}\right)$$
$$+ \frac{-Aa^3e^8 + 9Aa^2cd^2e^6 + 21Aac^2d^4e^4 + 11Ac^3d^6e^2 - Ba^3de^7 - 15Ba^2cd^3e^5 - 27Bac^2d^5e^3 - 13Bc^3d^7e +}{}$$
$$+ \frac{(ae^2+cd^2)(3Aace^4 + 15Ac^2d^2e^2 - 9Bacde^3 - 21Bc^2d^3e + Ca^2e^4 + 17Cacd^2e^2 + 28Cc^2d^4)\log(d+ex}{e^9}$$

input
```
integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

output
```
C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4
*(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d
**2/(2*e**5)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e**5
- 3*C*a*c**2*d/e**4 - 10*C*c**3*d**3/(3*e**6)) + x**2*(3*A*a*c**2/(2*e**3)
 + 3*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**4) - 5*B*c**3*d**3/e**6 + 3*C*a
**2*c/(2*e**3) + 9*C*a*c**2*d**2/e**5 + 15*C*c**3*d**4/(2*e**7)) + x*(-9*A
*a*c**2*d/e**4 - 10*A*c**3*d**3/e**6 + 3*B*a**2*c/e**3 + 18*B*a*c**2*d**2/
e**5 + 15*B*c**3*d**4/e**7 - 9*C*a**2*c*d/e**4 - 30*C*a*c**2*d**3/e**6 - 2
1*C*c**3*d**5/e**8) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d
**4*e**4 + 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 2
7*B*a*c**2*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c
*d**4*e**4 + 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**
7 + 24*A*a*c**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**
2*c*d**2*e**6 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e
**7 + 24*C*a**2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(
2*d**2*e**9 + 4*d*e**10*x + 2*e**11*x**2) + (a*e**2 + c*d**2)*(3*A*a*c*e**
4 + 15*A*c**2*d**2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d**3*e + C*a**2*e**4
+ 17*C*a*c*d**2*e**2 + 28*C*c**2*d**4)*log(d + e*x)/e**9
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.50

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{15\,Cc^3d^8 - 13\,Bc^3d^7e - 27\,Bac^2d^5e^3 - 15\,Ba^2cd^3e^5 - Ba^3de^7 - Aa^3e^8 + 11\,(3\,Cac^2 + Ac^3)d^6e^2 + 21\,(C\cdots}{\cdots}$$
$$+ \frac{10\,Cc^3e^5x^6 - 12\,(3\,Cc^3de^4 - Bc^3e^5)x^5 + 15\,(6\,Cc^3d^2e^3 - 3\,Bc^3de^4 + (3\,Cac^2 + Ac^3)e^5)x^4 - 20\,(10\,Cc^3\cdots}{\cdots}$$
$$+ \frac{(28\,Cc^3d^6 - 21\,Bc^3d^5e - 30\,Bac^2d^3e^3 - 9\,Ba^2cde^5 + 15\,(3\,Cac^2 + Ac^3)d^4e^2 + 18\,(Ca^2c + Aac^2)d^2e^4 + \cdots}{e^9}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e
^5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*
c + A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e -
7*B*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(
3*C*a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3
*A*a^2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5
*x^6 - 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*
d*e^4 + (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*
e^3 - 3*B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*
e - 10*B*c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3
*(C*a^2*c + A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a
*c^2*d^2*e^3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c
 + A*a*c^2)*d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^
3*e^3 - 9*B*a^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A
*a*c^2)*d^2*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.67

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3}\,dx$$
$$= \frac{(28\,Cc^3d^6 - 21\,Bc^3d^5e + 45\,Cac^2d^4e^2 + 15\,Ac^3d^4e^2 - 30\,Bac^2d^3e^3 + 18\,Ca^2cd^2e^4 + 18\,Aac^2d^2e^4 - 9\,Ba^2c}{e^9}$$
$$+ \frac{15\,Cc^3d^8 - 13\,Bc^3d^7e + 33\,Cac^2d^6e^2 + 11\,Ac^3d^6e^2 - 27\,Bac^2d^5e^3 + 21\,Ca^2cd^4e^4 + 21\,Aac^2d^4e^4 - 15\,B}{}$$
$$+ \frac{10\,Cc^3e^{15}x^6 - 36\,Cc^3de^{14}x^5 + 12\,Bc^3e^{15}x^5 + 90\,Cc^3d^2e^{13}x^4 - 45\,Bc^3de^{14}x^4 + 45\,Cac^2e^{15}x^4 + 15\,Ac^3e^{1}}{}$$

input
```
integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

output
```
(28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 3
0*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*
e^5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*log(abs(e*x + d))/e^9 + 1/2*(15*C*c^3*d^8
 - 13*B*c^3*d^7*e + 33*C*a*c^2*d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5
*e^3 + 21*C*a^2*c*d^4*e^4 + 21*A*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*
a^3*d^2*e^6 + 9*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7
*e - 7*B*c^3*d^6*e^2 + 18*C*a*c^2*d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d
^4*e^4 + 12*C*a^2*c*d^3*e^5 + 12*A*a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C
*a^3*d*e^7 + 6*A*a^2*c*d*e^7 - B*a^3*e^8)*x)/((e*x + d)^2*e^9) + 1/60*(10*
C*c^3*e^15*x^6 - 36*C*c^3*d*e^14*x^5 + 12*B*c^3*e^15*x^5 + 90*C*c^3*d^2*e^
13*x^4 - 45*B*c^3*d*e^14*x^4 + 45*C*a*c^2*e^15*x^4 + 15*A*c^3*e^15*x^4 - 2
00*C*c^3*d^3*e^12*x^3 + 120*B*c^3*d^2*e^13*x^3 - 180*C*a*c^2*d*e^14*x^3 -
60*A*c^3*d*e^14*x^3 + 60*B*a*c^2*e^15*x^3 + 450*C*c^3*d^4*e^11*x^2 - 300*B
*c^3*d^3*e^12*x^2 + 540*C*a*c^2*d^2*e^13*x^2 + 180*A*c^3*d^2*e^13*x^2 - 27
0*B*a*c^2*d*e^14*x^2 + 90*C*a^2*c*e^15*x^2 + 90*A*a*c^2*e^15*x^2 - 1260*C*
c^3*d^5*e^10*x + 900*B*c^3*d^4*e^11*x - 1800*C*a*c^2*d^3*e^12*x - 600*A*c^
3*d^3*e^12*x + 1080*B*a*c^2*d^2*e^13*x - 540*C*a^2*c*d*e^14*x - 540*A*a*c^
2*d*e^14*x + 180*B*a^2*c*e^15*x)/e^18
```

3.38.9 Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 1290, normalized size of antiderivative = 2.77

$$\int \frac{(a + cx^2)^3 (A + Bx + Cx^2)}{(d + ex)^3} \, dx = \text{Too large to display}$$

input
```
int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

output
```
x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3
 + (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*
a*c^2)/e^3 - (C*c^3*d^3)/(3*e^6)) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3
 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e -
 (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^
3)/e^6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*
C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e
^4))/e^3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^
3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*
((3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 +
 (3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*
B*a*c^2)/e^3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3) + x^5*((B*c^3)/(5*
e^3) - (3*C*c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(
4*e) - (A*c^3 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((
3*d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (
3*C*c^3*d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*
a*c^2)/e^3 - (C*c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^
3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^
3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3))
+ ((15*C*c^3*d^8 - A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*...
```

3.38. $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx$

3.39.1 Optimal result

Integrand size = 32, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{(a+bx^2)^2}{c+dx}$$

output

```
(b*x^2+a)^2/(d*x+c)
```

3.39.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

input

```
Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

output

```
(a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^
4))/(d^4*(c + d*x))
```

3.39.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 2023$$
$$\frac{(a+bx^2)^2}{c+dx}$$

input
```
Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

output
```
(a + b*x^2)^2/(c + d*x)
```

3.39.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.39.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{dx+c}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{d\,a^2x}{c}}{dx+c}$	34
parallelrisch	$\frac{b^2c\,x^4+2abc\,x^2-a^2dx}{c(dx+c)}$	36
default	$\frac{b(b\,x^3d^2-bcd\,x^2+2a\,d^2x+b\,c^2x)}{d^3}-\frac{-a^2d^4-2ab\,c^2d^2-b^2c^4}{d^4(dx+c)}$	76
risch	$\frac{b^2x^3}{d}-\frac{b^2c\,x^2}{d^2}+\frac{2bax}{d}+\frac{b^2c^2x}{d^3}+\frac{a^2}{dx+c}+\frac{2ab\,c^2}{d^2(dx+c)}+\frac{b^2c^4}{d^4(dx+c)}$	88

input
```
int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2*x^4+2*a*b*x^2+a^2)/(d*x+c)
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

input
```
integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^
3*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = -\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right)+\frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

input
```
integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

output
```
-b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**
4 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.82

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{b^2c^4+2\,abc^2d^2+a^2d^4}{d^5x+cd^4}+\frac{b^2d^2x^3-b^2cdx^2+(b^2c^2+2\,abd^2)x}{d^3}$$

input
```
integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c
*d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = \frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

input
```
integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*
(d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d
^7/(d*x + c))/d^7
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2}\,dx = x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,(x\,d^4 + c\,d^3)} - \frac{b^2\,c\,x^2}{d^2}$$

input
```
int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)
```

output
```
x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c
^2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$

3.40.1 Optimal result

Integrand size = 31, antiderivative size = 17

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

output
```
(b*x^2+a)^2/(d*x+c)
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{a^2d^4+2abd^2(c^2+cdx+d^2x^2)+b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

input
```
Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

output
```
(a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^
4))/(d^4*(c + d*x))
```

3.40.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\,(bx(4c+3dx)-ad)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx^2)^2}{c+dx}$$

input
```
Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

output
```
(a + b*x^2)^2/(c + d*x)
```

3.40.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.40.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{b^2x^4+2ab\,x^2+a^2}{dx+c}$	27
norman	$\frac{b^2x^4+2ab\,x^2-\frac{d\,a^2x}{c}}{dx+c}$	34
parallelrisch	$\frac{b^2c\,x^4+2abc\,x^2-a^2dx}{c(dx+c)}$	36
default	$\frac{b\left(b\,x^3d^2-bcd\,x^2+2a\,d^2x+b\,c^2x\right)}{d^3}-\frac{-a^2d^4-2ab\,c^2d^2-b^2c^4}{d^4(dx+c)}$	76
risch	$\frac{b^2x^3}{d}-\frac{b^2c\,x^2}{d^2}+\frac{2bax}{d}+\frac{b^2c^2x}{d^3}+\frac{a^2}{dx+c}+\frac{2ab\,c^2}{d^2(dx+c)}+\frac{b^2c^4}{d^4(dx+c)}$	88

input
```
int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2*x^4+2*a*b*x^2+a^2)/(d*x+c)
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.59

$$\int \frac{(a+bx^2)\left(-ad+bx(4c+3dx)\right)}{(c+dx)^2}\,dx = \frac{b^2d^4x^4+2\,abd^4x^2+b^2c^4+2\,abc^2d^2+a^2d^4+(b^2c^3d+2\,abcd^3)x}{d^5x+cd^4}$$

input
```
integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas"
)
```

output
```
(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^
3*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{(a+bx^2)\,(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = -\frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d} + x\left(\frac{2ab}{d} + \frac{b^2c^2}{d^3}\right) + \frac{a^2d^4 + 2abc^2d^2 + b^2c^4}{cd^4 + d^5x}$$

input
```
integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

output
```
-b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**
4 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.82

$$\int \frac{(a+bx^2)\,(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + (b^2c^2 + 2\,abd^2)x}{d^3}$$

input
```
integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima"
)
```

output
```
(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c
*d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.53

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = \frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

input
```
integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

output
```
(b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*
(d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d
^7/(d*x + c))/d^7
```

3.40.9 Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2}\,dx = x\left(\frac{b^2\,c^2}{d^3} + \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^3}{d} + \frac{a^2\,d^4 + 2\,a\,b\,c^2\,d^2 + b^2\,c^4}{d\,(x\,d^4 + c\,d^3)} - \frac{b^2\,c\,x^2}{d^2}$$

input
```
int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)
```

output
```
x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c
^2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$

3.41.1 Optimal result

Integrand size = 34, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

output
```
(b*x^2+a)^3/(d*x+c)
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} dx = \frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

input
```
Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

output
```
(a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*
x + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

3.41.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (-ad+6bcx+5bdx^2)}{(c+dx)^2} dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx^2)^3}{c+dx}$$

input
```
Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

output
```
(a + b*x^2)^3/(c + d*x)
```

3.41.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^{3}x^{6}+3b^{2}a\,x^{4}+3a^{2}b\,x^{2}+a^{3}}{dx+c}$
norman	$\frac{b^{3}x^{6}+3a^{2}b\,x^{2}+3b^{2}a\,x^{4}-\frac{d\,a^{3}x}{c}}{dx+c}$
parallelrisch	$\frac{b^{3}d\,x^{6}+3a\,b^{2}d\,x^{4}+3a^{2}bd\,x^{2}+d\,a^{3}}{(dx+c)d}$
default	$\frac{b(b^{2}x^{5}d^{4}-x^{4}b^{2}d^{3}c+3ab\,d^{4}x^{3}+b^{2}c^{2}d^{2}x^{3}-3abc\,d^{3}x^{2}-b^{2}c^{3}d\,x^{2}+3a^{2}d^{4}x+3ab\,c^{2}d^{2}x+b^{2}c^{4}x)}{d^{5}}-\frac{-a^{3}d^{6}-3a^{2}b\,c^{2}d^{4}-3a\,b^{2}c^{4}d^{2}}{d^{6}(dx+c)}$
risch	$\frac{b^{3}x^{5}}{d}-\frac{b^{3}x^{4}c}{d^{2}}+\frac{3b^{2}a\,x^{3}}{d}+\frac{b^{3}c^{2}x^{3}}{d^{3}}-\frac{3b^{2}ac\,x^{2}}{d^{2}}-\frac{b^{3}c^{3}x^{2}}{d^{4}}+\frac{3b\,a^{2}x}{d}+\frac{3b^{2}a\,c^{2}x}{d^{3}}+\frac{b^{3}c^{4}x}{d^{5}}+\frac{a^{3}}{dx+c}+\frac{3a^{2}b\,c^{2}}{d^{2}(dx+c)}+$

input
```
int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(d*x+c)
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx$$
$$=\frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5)x}{d^7x+cd^6}$$

input
```
integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fric
as")
```

output
```
(b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d
^2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*
d^5)*x)/(d^7*x + c*d^6)
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.00

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = -\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

input
```
integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

output
```
-b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**
2*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3
 + b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2
+ b**3*c**6)/(c*d**6 + d**7*x)
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 9.41

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = \frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6}+\frac{b^3d^4x^5-b^3cd^3x^4+(b^3c^2d^2+3\,ab^2d^4)x^3-(b^3c^3d+3\,ab^2cd^3)x^2+(b^3c^4+3\,ab^2c^2d^2+3\,a^2bd^4)x}{d^5}$$

input
```
integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxi
ma")
```

output
```
(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) +
(b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*
d + 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.71

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx$$
$$= \frac{\left(b^3 - \frac{6\,b^3c}{dx+c} + \frac{15\,b^3c^2}{(dx+c)^2} - \frac{20\,b^3c^3}{(dx+c)^3} + \frac{15\,b^3c^4}{(dx+c)^4} + \frac{3\,ab^2d^2}{(dx+c)^2} - \frac{12\,ab^2cd^2}{(dx+c)^3} + \frac{18\,ab^2c^2d^2}{(dx+c)^4} + \frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c} + \frac{3\,ab^2c^4d^7}{dx+c} + \frac{3\,a^2bc^2d^9}{dx+c} + \frac{a^3d^{11}}{dx+c}}{d^{11}}$$

input
```
integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac
")
```

output
```
(b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
 + 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x
+ c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)
^5/d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*
d^9/(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

3.41.9 Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.82

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2}\,dx = x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right) - x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d} + \frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right) + x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{6\,a\,b^2\,c}{d^2}\right) + \frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,(x\,d^6+c\,d^5)} + \frac{b^3\,x^5}{d}-\frac{b^3\,c\,x^4}{d^2}$$

input
```
int(((a + b*x^2)^2*(6*b*c*x - a*d + 5*b*d*x^2))/(c + d*x)^2,x)
```

output
```
x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*
b^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d +
(3*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2
)/d + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^
2*c^4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^
4)/d^2
```

3.42 $\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$

3.42.1 Optimal result

Integrand size = 33, antiderivative size = 17

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^3}{c+dx}$$

output
```
(b*x^2+a)^3/(d*x+c)
```

3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(17) = 34.

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.29

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{a^3d^6+3a^2bd^4(c^2+cdx+d^2x^2)+3ab^2d^2(c^4+c^3dx+d^4x^4)+b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

input
```
Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

output
```
(a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*
x + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

3.42.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (bx(6c+5dx)-ad)}{(c+dx)^2} dx$$

$$\Big\downarrow \text{2023}$$

$$\frac{(a+bx^2)^3}{c+dx}$$

input
```
Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

output
```
(a + b*x^2)^3/(c + d*x)
```

3.42.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.24

method	result
gosper	$\frac{b^3x^6+3b^2a\,x^4+3a^2b\,x^2+a^3}{dx+c}$
norman	$\frac{b^3x^6+3a^2b\,x^2+3b^2a\,x^4-\frac{d\,a^3x}{c}}{dx+c}$
parallelrisch	$\frac{b^3d\,x^6+3a\,b^2d\,x^4+3a^2bd\,x^2+d\,a^3}{(dx+c)d}$
default	$\frac{b\left(b^2x^5d^4-x^4b^2d^3c+3ab\,d^4x^3+b^2c^2d^2x^3-3abc\,d^3x^2-b^2c^3d\,x^2+3a^2d^4x+3ab\,c^2d^2x+b^2c^4x\right)}{d^5}-\frac{-a^3d^6-3a^2b\,c^2d^4-3a\,b^2c^4d^2}{d^6(dx+c)}$
risch	$\frac{b^3x^5}{d}-\frac{b^3x^4c}{d^2}+\frac{3b^2a\,x^3}{d}+\frac{b^3c^2x^3}{d^3}-\frac{3b^2ac\,x^2}{d^2}-\frac{b^3c^3x^2}{d^4}+\frac{3b\,a^2x}{d}+\frac{3b^2a\,c^2x}{d^3}+\frac{b^3c^4x}{d^5}+\frac{a^3}{dx+c}+\frac{3a^2b\,c^2}{d^2(dx+c)}+$

input
```
int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3*x^6+3*a*b^2*x^4+3*a^2*b*x^2+a^3)/(d*x+c)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx^2)^2\left(-ad+bx(6c+5dx)\right)}{(c+dx)^2}\,dx$$
$$= \frac{b^3d^6x^6+3\,ab^2d^6x^4+3\,a^2bd^6x^2+b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6+(b^3c^5d+3\,ab^2c^3d^3+3\,a^2bcd^5)x}{d^7x+cd^6}$$

input
```
integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="frica
s")
```

output
```
(b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d
^2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*
d^5)*x)/(d^7*x + c*d^6)
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.00

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = -\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\cdot\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d^2+b^3c^6}{cd^6+d^7x}$$

input
```
integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

output
```
-b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**
2*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3
 + b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2
+ b**3*c**6)/(c*d**6 + d**7*x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 9.41

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = \frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6}+\frac{b^3d^4x^5-b^3cd^3x^4+(b^3c^2d^2+3\,ab^2d^4)x^3-(b^3c^3d+3\,ab^2cd^3)x^2+(b^3c^4+3\,ab^2c^2d^2+3\,a^2bd^4)x}{d^5}$$

input
```
integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxim
a")
```

output
```
(b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) +
(b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*
d + 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.71

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx$$
$$= \frac{\left(b^3-\frac{6\,b^3c}{dx+c}+\frac{15\,b^3c^2}{(dx+c)^2}-\frac{20\,b^3c^3}{(dx+c)^3}+\frac{15\,b^3c^4}{(dx+c)^4}+\frac{3\,ab^2d^2}{(dx+c)^2}-\frac{12\,ab^2cd^2}{(dx+c)^3}+\frac{18\,ab^2c^2d^2}{(dx+c)^4}+\frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6}$$
$$+\frac{\frac{b^3c^6d^5}{dx+c}+\frac{3\,ab^2c^4d^7}{dx+c}+\frac{3\,a^2bc^2d^9}{dx+c}+\frac{a^3d^{11}}{dx+c}}{d^{11}}$$

input
```
integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac"
)
```

output
```
(b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
 + 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x
+ c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)
^5/d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*
d^9/(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

3.42.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 252, normalized size of antiderivative = 14.82

$$\int \frac{(a+bx^2)^2(-ad+bx(6c+5dx))}{(c+dx)^2}\,dx = x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right) - x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d} + \frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right) + x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{6\,a\,b^2\,c}{d^2}\right) + \frac{a^3\,d^6+3\,a^2\,b\,c^2\,d^4+3\,a\,b^2\,c^4\,d^2+b^3\,c^6}{d\,(x\,d^6+c\,d^5)} + \frac{b^3\,x^5}{d}-\frac{b^3\,c\,x^4}{d^2}$$

input
```
int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)
```

output
```
x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*
b^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d +
(3*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2
)/d + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^
2*c^4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^
4)/d^2
```

3.43 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2}\,dx$

3.43.1 Optimal result

Integrand size = 27, antiderivative size = 240

$$\begin{aligned}\int \frac{(d+ex)^3(A+Bx+Cx^2)}{a+cx^2}\,dx &= -\frac{(ae^2(3Cd+Be)-cd(Cd^2+3e(Bd+Ae)))\,x}{c^2} \\ &\quad -\frac{e(aCe^2-c(3Cd^2+e(3Bd+Ae)))\,x^2}{2c^2}+\frac{e^2(3Cd+Be)x^3}{3c}+\frac{Ce^3x^4}{4c} \\ &\quad +\frac{(Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}} \\ &\quad +\frac{(Bcd(cd^2-3ae^2)+(Ac-aC)e(3cd^2-ae^2))\log(a+cx^2)}{2c^3}\end{aligned}$$

output
```
-(a*e^2*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))*x/c^2-1/2*e*(a*C*e^2-c*(3*C
*d^2+e*(A*e+3*B*d)))*x^2/c^2+1/3*e^2*(B*e+3*C*d)*x^3/c+1/4*C*e^3*x^4/c+1/2
*(B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/c^3+(A*
c*d*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(
1/2)/a^(1/2))/c^(5/2)/a^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$= \frac{(Acd(cd^2-3ae^2)+a(ae^2(3Cd+Be)-cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}$$
$$+ \frac{cx(-6ae^2(6Cd+2Be+Cex)+3cC(4d^3+6d^2ex+4de^2x^2+e^3x^3)+2ce(3Ae(6d+ex)+B(18d^2+9}{12c^3}$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))
*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d +
2*B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e
*(3*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2
 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$\Big\downarrow \text{ 2160}$$
$$\int \left(\frac{-ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3}{c^2}+\frac{x\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)+Acd\left(cd^2-\right.}{c^2\left(a+cx^2\right)}\right.$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)+a\left(ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{\sqrt{a}c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)}{2c^3}+\frac{x\left(-ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)}{c^2}+\frac{ex^2\left(-aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{2c^2}+\frac{e^2x^3(Be+3Cd)}{3c}+\frac{Ce^3x^4}{4c}$$

input
```
Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*
d^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/
(3*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d +
B*e) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)
) + ((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c
*x^2])/(2*c^3)
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.43.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.08

method	result
default	$\frac{\frac{1}{4}cC\,x^4e^3+\frac{1}{3}Bc\,x^3e^3+Ccd\,e^2x^3+\frac{1}{2}Ac\,e^3x^2+\frac{3}{2}B\,x^2cd\,e^2-\frac{1}{2}Ca\,e^3x^2+\frac{3}{2}Cc\,d^2e\,x^2+3Acd\,e^2x-Bxa\,e^3+3Bc\,d^2ex-3Cad\,e^2x+Cc\,d^3x}{c^2}$
risch	Expression too large to display

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/4*c*C*x^4*e^3+1/3*B*c*x^3*e^3+C*c*d*e^2*x^3+1/2*A*c*e^3*x^2+3/2*B
*x^2*c*d*e^2-1/2*C*a*e^3*x^2+3/2*C*c*d^2*e*x^2+3*A*c*d*e^2*x-B*x*a*e^3+3*B
*c*d^2*e*x-3*C*a*d*e^2*x+C*c*d^3*x)+1/c^2*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e-3
*B*a*c*d*e^2+B*c^2*d^3+C*a^2*e^3-3*C*a*c*d^2*e)/c*ln(c*x^2+a)+(-3*A*a*c*d*
e^2+A*c^2*d^3+B*a^2*e^3-3*B*a*c*d^2*e+3*C*a^2*d*e^2-C*a*c*d^3)/(a*c)^(1/2)
*arctan(c*x/(a*c)^(1/2)))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.47

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2}\,dx$$

$$= \left[\frac{3\,Cac^2e^3x^4 + 4\,(3\,Cac^2de^2 + Bac^2e^3)x^3 + 6\,(3\,Cac^2d^2e + 3\,Bac^2de^2 - (Ca^2c - Aac^2)e^3)x^2 + 6\,(3\,Bacd}{}\right.$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^3 + 6*(3*C*
a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*x^2 + 6*(3*B*a*c*
d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a
*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(C*a*c^2*d^3 + 3*B*
a*c^2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^
3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)
*log(c*x^2 + a))/(a*c^3), 1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B
*a*c^2*e^3)*x^3 + 6*(3*C*a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^
2)*e^3)*x^2 - 12*(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a
^2 - A*a*c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 12*(C*a*c^2*d^3 + 3*B
*a*c^2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d
^3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3
)*log(c*x^2 + a))/(a*c^3)]
```

3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(224) = 448.

Time = 5.52 (sec) , antiderivative size = 1008, normalized size of antiderivative = 4.20

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{Ce^3x^4}{4c} + x^3\left(\frac{Be^3}{3c}+\frac{Cde^2}{c}\right) + x^2\left(\frac{Ae^3}{2c}+\frac{3Bde^2}{2c}-\frac{Cae^3}{2c^2}+\frac{3Cd^2e}{2c}\right) + x\left(\frac{3Ade^2}{c}-\frac{Bae^3}{c^2}+\frac{3Bd^2e}{c}-\frac{3Cade^2}{c^2}+\frac{Cd^3}{c}\right) + \left(\frac{-Aace^3+3Ac^2d^2e-3Bacde^2+Bc^2d^3+Ca^2e^3-3Cacd^2e}{2c^3} - \frac{\sqrt{-ac^7}(-3Aacde^2+Ac^2d^3+Ba^2e^3-3Bacd^2e+3Ca^2de^2-Cacd^3)}{2ac^6}\right)\log\left(x+\frac{Aa^2ce^3-3Aac^2d^2e \cdots}{\cdots}\right) + \left(\frac{-Aace^3+3Ac^2d^2e-3Bacde^2+Bc^2d^3+Ca^2e^3-3Cacd^2e}{2c^3} + \frac{\sqrt{-ac^7}(-3Aacde^2+Ac^2d^3+Ba^2e^3-3Bacd^2e+3Ca^2de^2-Cacd^3)}{2ac^6}\right)\log\left(x+\frac{Aa^2ce^3-3Aac^2d^2e \cdots}{\cdots}\right)$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

output
```
C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c)
+ 3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2
/c - B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a
*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3
*C*a*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B
*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6))*lo
g(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d*
*3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c**2*d
**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c*
*3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c
*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2 +
 A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a
*c**2*d**3)) + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d
**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e
**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c
*d**3)/(2*a*c**6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c
*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*
a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 -
3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 +
B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)...
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$= -\frac{\left(3\,Bacd^2e - Ba^2e^3 + (Cac - Ac^2)d^3 - 3\,(Ca^2 - Aac)de^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2}$$
$$+ \frac{3\,Cce^3x^4 + 4\,(3\,Ccde^2 + Bce^3)x^3 + 6\,(3\,Ccd^2e + 3\,Bcde^2 - (Ca - Ac)e^3)x^2 + 12\,(Ccd^3 + 3\,Bcd^2e - B}{12\,c^2}$$
$$+ \frac{\left(Bc^2d^3 - 3\,Bacde^2 - 3\,(Cac - Ac^2)d^2e + (Ca^2 - Aac)e^3\right)\log\left(cx^2 + a\right)}{2\,c^3}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

output
```
-(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^
2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*C*c*e^3*x^4 + 4*(3*C*c*
d*e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - (C*a - A*c)*e^3)*x^2
 + 12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a - A*c)*d*e^2)*x)/c^2 + 1/2
*(B*c^2*d^3 - 3*B*a*c*d*e^2 - 3*(C*a*c - A*c^2)*d^2*e + (C*a^2 - A*a*c)*e^
3)*log(c*x^2 + a)/c^3
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2} \, dx$$
$$= -\frac{(Cacd^3 - Ac^2d^3 + 3\,Bacd^2e - 3\,Ca^2de^2 + 3\,Aacde^2 - Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2}$$
$$+ \frac{(Bc^2d^3 - 3\,Cacd^2e + 3\,Ac^2d^2e - 3\,Bacde^2 + Ca^2e^3 - Aace^3)\log(cx^2 + a)}{2\,c^3}$$
$$+ \frac{3\,Cc^3e^3x^4 + 12\,Cc^3de^2x^3 + 4\,Bc^3e^3x^3 + 18\,Cc^3d^2ex^2 + 18\,Bc^3de^2x^2 - 6\,Cac^2e^3x^2 + 6\,Ac^3e^3x^2 + 12\,Cc}{12\,c^4}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

output
```
-(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 -
B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*
c*d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2
 + a)/c^3 + 1/12*(3*C*c^3*e^3*x^4 + 12*C*c^3*d*e^2*x^3 + 4*B*c^3*e^3*x^3 +
 18*C*c^3*d^2*e*x^2 + 18*B*c^3*d*e^2*x^2 - 6*C*a*c^2*e^3*x^2 + 6*A*c^3*e^3
*x^2 + 12*C*c^3*d^3*x + 36*B*c^3*d^2*e*x - 36*C*a*c^2*d*e^2*x + 36*A*c^3*d
*e^2*x - 12*B*a*c^2*e^3*x)/c^4
```

3.43.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{a+cx^2}\,dx = x^2\left(\frac{3\,C\,d^2\,e+3\,B\,d\,e^2+A\,e^3}{2\,c}-\frac{C\,a\,e^3}{2\,c^2}\right) + x\left(\frac{C\,d^3+3\,B\,d^2\,e+3\,A\,d\,e^2}{c}-\frac{a\,(B\,e^3+3\,C\,d\,e^2)}{c^2}\right)+\frac{x^3\,(B\,e^3+3\,C\,d\,e^2)}{3\,c}+\frac{C\,e^3\,x^4}{4\,c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{\sqrt{a}\,c^{5/2}} + \frac{\ln(c\,x^2+a)\,(4\,C\,a^3\,c^3\,e^3-12\,C\,a^2\,c^4\,d^2\,e-12\,B\,a^2\,c^4\,d\,e^2-4\,A\,a^2\,c^4\,e^3+4\,B\,a\,c^5\,d^3+12\,A\,a\,c^5\,d^2\,e)}{8\,a\,c^6}$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x)
```

output
```
x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^
3 + 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3
+ 3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^
2*d^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^
2*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3
 + 4*C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*
d^2*e))/(8*a*c^6)
```

3.44 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx$

3.44.1 Optimal result

Integrand size = 27, antiderivative size = 168

$$\begin{aligned}\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx &= -\frac{(aCe^2-c(Cd^2+e(2Bd+Ae)))\,x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}\\ &\quad+\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}\\ &\quad+\frac{(Bcd^2+2Acde-2aCde-aBe^2)\log(a+cx^2)}{2c^2}\end{aligned}$$

output
```
-(a*C*e^2-c*(C*d^2+e*(A*e+2*B*d)))*x/c^2+1/2*e*(B*e+2*C*d)*x^2/c+1/3*C*e^2
*x^3/c+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*ln(c*x^2+a)/c^2+(A*c*(-a*
e^2+c*d^2)+a*(a*C*e^2-c*d*(2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/c^(5/2)/
a^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$=\frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}$$
$$+\frac{x(-6aCe^2+3ce(4Bd+2Ae+Bex)+2cC(3d^2+3dex+e^2x^2))+3(Bcd^2+2Acde-2aCde-aBe^2)}{6c^2}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*
e*x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C
*d*e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

3.44.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$\Big\downarrow \ 2160$$
$$\int \left(\frac{cx\left(-aBe^2-2aCde+2Acde+Bcd^2\right)+Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(2Be+Cd)\right)}{c^2\left(a+cx^2\right)}+\frac{-aCe^2+ce(Ae+2B}{c^2}\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(cd^2-ae^2)+a(aCe^2-cd(2Be+Cd))\right)}{\sqrt{a}c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}+\frac{x\left(-aCe^2+ce(Ae+2Bd)+cCd^2\right)}{c^2}+\frac{ex^2(Be+2Cd)}{2c}+\frac{Ce^2x^3}{3c}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2
*c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*
c*d*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.44.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{1}{3}cC\,x^3e^2+\frac{1}{2}Bc\,e^2x^2+Ccde\,x^2+Ac\,e^2x+2Bcdex-aC\,e^2x+Cc\,d^2x}{c^2}+\frac{\frac{\left(2A\,c^2de-B\,e^2ac+B\,c^2d^2-2acdeC\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-Aac\,e^2+A\,c^2\right.}{}}{c^2}$
risch	Expression too large to display

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/3*c*C*x^3*e^2+1/2*B*c*e^2*x^2+C*c*d*e*x^2+A*c*e^2*x+2*B*c*d*e*x-a
*C*e^2*x+C*c*d^2*x)+1/c^2*(1/2*(2*A*c^2*d*e-B*a*c*e^2+B*c^2*d^2-2*C*a*c*d*
e)/c*ln(c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2-2*B*a*c*d*e+C*a^2*e^2-C*a*c*d^2)/(a
*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{a+cx^2}\,dx$$

$$= \left[\frac{2\,Cac^2e^2x^3 + 3\,(2\,Cac^2de + Bac^2e^2)x^2 - 3\,(2\,Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)\sqrt{-ac}\log\left(\frac{cx^2 \cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a*c^2*e^2)*x^2 - 3*(2*B*a*c
*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(-a*c)*log((c*x^2 +
2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2
*c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2
)*d*e)*log(c*x^2 + a))/(a*c^3), 1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e
+ B*a*c^2*e^2)*x^2 - 6*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c
)*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e -
(C*a^2*c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A
*a*c^2)*d*e)*log(c*x^2 + a))/(a*c^3)]
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(156) = 312$.

Time = 1.39 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.80

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{Ce^2x^3}{3c} + x^2\left(\frac{Be^2}{2c}+\frac{Cde}{c}\right) + x\left(\frac{Ae^2}{c}+\frac{2Bde}{c}-\frac{Cae^2}{c^2}+\frac{Cd^2}{c}\right) + \left(-\frac{-2Acde+Bae^2-Bcd^2+2Cade}{2c^2} - \frac{\sqrt{-ac^5}(-Aace^2+Ac^2d^2-2Bacde+Ca^2e^2-Cacd^2)}{2ac^5}\right)\log\left(x+\frac{-2Aacde+Ba^2e^2-Bacd^2+2Ca}{-}\right) + \left(-\frac{-2Acde+Bae^2-Bcd^2+2Cade}{2c^2} + \frac{\sqrt{-ac^5}(-Aace^2+Ac^2d^2-2Bacde+Ca^2e^2-Cacd^2)}{2ac^5}\right)\log\left(x+\frac{-2Aacde+Ba^2e^2-Bacd^2+2Ca}{-}\right)$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

output
```
C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/
c - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*
a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e +
 C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2
 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**
2 + 2*C*a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a
*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2
 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B
*c*d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 -
 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e
 + B*a**2*e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*
e**2 - B*c*d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**
2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2
 + A*c**2*d**2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2} dx = \frac{(Bcd^2 - Bae^2 - 2(Ca - Ac)de) \log(cx^2+a)}{2c^2} - \frac{(2Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cce^2x^3 + 3(2Ccde + Bce^2)x^2 + 6(Ccd^2 + 2Bcde - (Ca - Ac)e^2)x}{6c^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/c^2 - (2*B*a*c*
d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*c^2) + 1/6*(2*C*c*e^2*x^3 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*d
^2 + 2*B*c*d*e - (C*a - A*c)*e^2)*x)/c^2
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2} dx = \frac{(Bcd^2 - 2Cade + 2Acde - Bae^2) \log(cx^2+a)}{2c^2} - \frac{(Cacd^2 - Ac^2d^2 + 2Bacde - Ca^2e^2 + Aace^2) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c^2} + \frac{2Cc^2e^2x^3 + 6Cc^2dex^2 + 3Bc^2e^2x^2 + 6Cc^2d^2x + 12Bc^2dex - 6Cace^2x + 6Ac^2e^2x}{6c^3}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*
c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a
*c))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*e^2*x^3 + 6*C*c^2*d*e*x^2 + 3*B*c^2*e^
2*x^2 + 6*C*c^2*d^2*x + 12*B*c^2*d*e*x - 6*C*a*c*e^2*x + 6*A*c^2*e^2*x)/c^
3
```

3.44.9 Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{a+cx^2} \, dx$$
$$= x\left(\frac{C\,d^2+2\,B\,d\,e+A\,e^2}{c}-\frac{C\,a\,e^2}{c^2}\right)+\frac{x^2\,\left(B\,e^2+2\,C\,d\,e\right)}{2\,c}+\frac{C\,e^2\,x^3}{3\,c}$$
$$-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2\right)}{\sqrt{a}\,c^{5/2}}$$
$$+\frac{\ln\left(c\,x^2+a\right)\,\left(-8\,C\,a^2\,c^3\,d\,e-4\,B\,a^2\,c^3\,e^2+4\,B\,a\,c^4\,d^2+8\,A\,a\,c^4\,d\,e\right)}{8\,a\,c^5}$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x)
```

output
```
x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/
(2*c) + (C*e^2*x^3)/(3*c) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*c*e^2 - C*a^2*
e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^(1/2)*c^(5/2)) + (log(a + c
*x^2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))
/(8*a*c^5)
```

3.45 $\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2}\,dx$

3.45.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2}\,dx=\frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{(Bcd+Ace-aCe)\log(a+cx^2)}{2c^2}$$

output
```
(B*e+C*d)*x/c+1/2*C*e*x^2/c+1/2*(A*c*e+B*c*d-C*a*e)*ln(c*x^2+a)/c^2+(A*c*d
-a*(B*e+C*d))*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2}\,dx=\frac{cx(2Cd+2Be+Cex)-\frac{2\sqrt{c}(-Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}}+(Bcd+Ace-aCe)\log(a+cx^2)}{2c^2}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
(c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/
(2*c^2)
```

3.45.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$

$$\Big\downarrow \text{ 2160}$$

$$\int \left(\frac{x(-aCe+Ace+Bcd)-a(Be+Cd)+Acd}{c\left(a+cx^2\right)}+\frac{Be+Cd}{c}+\frac{Cex}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}c^{3/2}}+\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

output
```
((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x
^2])/(2*c^2)
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.45.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{1}{2}C\,x^2e+Bex+Cdx}{c}+\frac{\frac{(Ace+Bcd-Cae)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd-Bae-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Ce\,x^2}{2c}+\frac{Bex}{c}+\frac{Cdx}{c}+\frac{\ln\left(dAac-Be\,a^2-a^2dC-\sqrt{-ac(Acd-Bae-Cad)^2}\,x\right)Ae}{2c}+\frac{\ln\left(dAac-Be\,a^2-a^2dC-\sqrt{-ac(Acd-Be\right.}}{2c}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*C*x^2*e+B*e*x+C*d*x)+1/c*(1/2*(A*c*e+B*c*d-C*a*e)/c*ln(c*x^2+a)+(
A*c*d-B*a*e-C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.22

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$$
$$=\left[\frac{Cacex^2-(Bae+(Ca-Ac)d)\sqrt{-ac}\log\left(\frac{cx^2+2\sqrt{-ac}x-a}{cx^2+a}\right)+2\,(Cacd+Bace)x+(Bacd-(Ca^2-Aac)}{2\,ac^2}\right.$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(C*a*c*e*x^2 - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt
(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d - (C*a^2 -
 A*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(C*a*c*e*x^2 - 2*(B*a*e + (C*a - A
*c)*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*
d - (C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

3.45.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(88) = 176$.

Time = 0.71 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.62

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \frac{Cex^2}{2c} + x\left(\frac{Be}{c}+\frac{Cd}{c}\right) + \left(-\frac{-Ace-Bcd+Cae}{2c^2} - \frac{\sqrt{-ac^5}(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x + \frac{Aace+Bacd-Ca^2e-2ac^2\left(-\frac{-Ace-Bcd+Cae}{2c^2} - \frac{\sqrt{-ac^5}(-Acd+\ldots}{2ac^4}\ldots\right.}{-Ac^2d+Bace+Cacd}\ldots\right.$$
$$+\left(-\frac{-Ace-Bcd+Cae}{2c^2} + \frac{\sqrt{-ac^5}(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x + \frac{Aace+Bacd-Ca^2e-2ac^2\left(-\frac{-Ace-Bcd+Cae}{2c^2} + \frac{\sqrt{-ac^5}(-Acd+\ldots}{2ac^4}\ldots\right.}{-Ac^2d+Bace+Cacd}\ldots\right.$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

output
```
C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) -
 sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a
*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c
**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)
) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e +
C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*
c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*
a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = -\frac{(Bae+(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{Cex^2+2(Cd+Be)x}{2c} + \frac{(Bcd-(Ca-Ac)e)\log\left(cx^2+a\right)}{2c^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

output
```
-(B*a*e + (C*a - A*c)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(C*e*x^
2 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - (C*a - A*c)*e)*log(c*x^2 + a)/c^2
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2}\,dx = -\frac{(Cad-Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c} + \frac{(Bcd-Cae+Ace)\log(cx^2+a)}{2c^2} + \frac{Ccex^2+2Ccdx+2Bcex}{2c^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

output
```
-(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d
- C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*e*x^2 + 2*C*c*d*x + 2*B*c*e
*x)/c^2
```

3.45.9 Mupad [B] (verification not implemented)

Time = 12.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{a+cx^2}\,dx = \frac{x(B\,e+C\,d)}{c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(B\,a\,e-A\,c\,d+C\,a\,d)}{\sqrt{a}\,c^{3/2}} + \frac{C\,e\,x^2}{2\,c} + \frac{\ln(c\,x^2+a)\,(4\,A\,a\,c^3\,e+4\,B\,a\,c^3\,d-4\,C\,a^2\,c^2\,e)}{8\,a\,c^4}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x)
```

output
```
(x*(B*e + C*d))/c - (atan((c^(1/2)*x)/a^(1/2))*(B*a*e - A*c*d + C*a*d))/(a
^(1/2)*c^(3/2)) + (C*e*x^2)/(2*c) + (log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c
^3*d - 4*C*a^2*c^2*e))/(8*a*c^4)
```

3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2} dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{A+Bx+Cx^2}{a+cx^2} dx = \frac{Cx}{c} + \frac{(Ac-aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{B\log(a+cx^2)}{2c}$$

output
```
C*x/c+1/2*B*ln(c*x^2+a)/c+(A*c-C*a)*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1
/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+cx^2} dx = \frac{Cx}{c} - \frac{(-Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}} + \frac{B\log(a+cx^2)}{2c}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
```

output
```
(C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
 (B*Log[a + c*x^2])/(2*c)
```

3.46.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{a+cx^2} \, dx$$

$$\Big\downarrow \ 2341$$

$$\int \left(\frac{-aC+Ac+Bcx}{c\left(a+cx^2\right)}+\frac{C}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(Ac-aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2),x]
```

output
```
(C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B
*Log[a + c*x^2])/(2*c)
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.46.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

method	result
default	$\frac{Cx}{c}+\frac{\frac{B\ln\left(c\,x^{2}+a\right)}{2}+\frac{(Ac-Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{c}$
risch	$\frac{Cx}{c}+\frac{\ln\left(Aac-C\,a^{2}-\sqrt{-ac(Ac-Ca)^{2}}\,x\right)B}{2c}+\frac{\ln\left(Aac-C\,a^{2}-\sqrt{-ac(Ac-Ca)^{2}}\,x\right)\sqrt{-ac(Ac-Ca)^{2}}}{2c^{2}a}+\frac{\ln\left(Aac-C\,a^{2}+\sqrt{-ac\right.}{2c}$

input
```
int((C*x^2+B*x+A)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/c*(1/2*B*ln(c*x^2+a)+(A*c-C*a)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx$$
$$=\left[\frac{2\,Cacx+Bac\log\left(cx^2+a\right)+(Ca-Ac)\sqrt{-ac}\log\left(\frac{cx^2-2\,\sqrt{-ac}x-a}{cx^2+a}\right)}{2\,ac^2},\frac{2\,Cacx+Bac\log\left(cx^2+a\right)-2\,(Ca}{2\,ac^2}\right.$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c
*x^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(48) = 96.

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \left(\frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} - \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right) + \left(\frac{B}{2c} + \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)\log\left(x + \frac{Ba - 2ac\left(\frac{B}{2c} + \frac{\sqrt{-ac^3}(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right)$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

output
```
C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2
*a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B
/(2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2
*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

output
```
C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a
*c)*c)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{Cx}{c} + \frac{B\log\left(cx^2+a\right)}{2\,c} - \frac{(Ca-Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

output
```
C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a
*c)*c)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx = \frac{B\,\ln\left(c\,x^2+a\right)}{2\,c} + \frac{C\,x}{c} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2),x)
```

output
```
(B*log(a + c*x^2))/(2*c) + (C*x)/c + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2
)*c^(1/2)) - (C*a^(1/2)*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2)
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx$

3.47.1 Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)}\,dx = \frac{(Acd-aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)} + \frac{(Cd^2-Bde+Ae^2)\log(d+ex)}{e\,(cd^2+ae^2)} + \frac{(Bcd-Ace+aCe)\log\left(a+cx^2\right)}{2c\,(cd^2+ae^2)}$$

output
```
(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e)*ln(
c*x^2+a)/c/(a*e^2+c*d^2)+(A*c*d+B*a*e-C*a*d)*arctan(x*c^(1/2)/a^(1/2))/(a*
e^2+c*d^2)/a^(1/2)/c^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)}\,dx = \frac{2\sqrt{c}e(Acd-aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)+\sqrt{a}(2c(Cd^2-Bde+Ae^2)\log(d+ex)+e(Bcd-Ace+aCe)\,\mathrm{l}}{2\sqrt{a}ce\,(cd^2+ae^2)}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

output
```
(2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]
*(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log
[a + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

3.47.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)(d+ex)} dx$$

$$\Big\downarrow \text{ 2160}$$

$$\int \left(\frac{x(aCe-Ace+Bcd)+aBe-aCd+Acd}{(a+cx^2)(ae^2+cd^2)}+\frac{Ae^2-Bde+Cd^2}{(d+ex)(ae^2+cd^2)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}+\frac{\log(a+cx^2)(aCe-Ace+Bcd)}{2c(ae^2+cd^2)}+\frac{\log(d+ex)(Ae^2-Bde+Cd^2)}{e(ae^2+cd^2)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

output
```
((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*
d^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2))
 + ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.47.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{(-Ace+Bcd+Cae)\ln\left(c\,x^2+a\right)}{2c}+\frac{(Acd+Bae-Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{e^2a+c\,d^2}+\frac{\left(A\,e^2-Bde+C\,d^2\right)\ln(ex+d)}{e\left(e^2a+c\,d^2\right)}$
risch	$\frac{\ln(ex+d)eA}{e^2a+c\,d^2}-\frac{\ln(ex+d)Bd}{e^2a+c\,d^2}+\frac{\ln(ex+d)C\,d^2}{e\left(e^2a+c\,d^2\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2c^2e^2+a\,c^3d^2\right)\_Z^2+\left(2Aa\,c^2e-2Ba\,c^2d-2a^2ceC\right)\_Z+A^2c^2-2ACa\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)*(1/2*(-A*c*e+B*c*d+C*a*e)/c*ln(c*x^2+a)+(A*c*d+B*a*e-C*a*d
)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))+(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*
e^2+c*d^2)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)}\,dx$$
$$=\left[-\frac{(Bae^2-(Ca-Ac)de)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-(Bacde+(Ca^2-Aac)e^2)\log\left(cx^2+a\right)-2\left(Cacd\right.}{2\left(ac^2d^2e+a^2ce^3\right)}\right.$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x
- a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(
C*a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)
, 1/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*
a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e +
 A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

3.47.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

output
```
Timed out
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \frac{(Bcd+(Ca-Ac)e)\log(cx^2+a)}{2(c^2d^2+ace^2)} + \frac{(Cd^2-Bde+Ae^2)\log(ex+d)}{cd^2e+ae^3} + \frac{(Bae-(Ca-Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c*d + (C*a - A*c)*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 -
B*d*e + A*e^2)*log(e*x + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*ar
ctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \frac{(Bcd+Cae-Ace)\log(cx^2+a)}{2(c^2d^2+ace^2)} + \frac{(Cd^2-Bde+Ae^2)\log(|ex+d|)}{cd^2e+ae^3} - \frac{(Cad-Acd-Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(cd^2+ae^2)\sqrt{ac}}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 -
B*d*e + A*e^2)*log(abs(e*x + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*
e)*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

3.47.9 Mupad [B] (verification not implemented)

Time = 15.63 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.32

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx = \frac{\ln(d+e\,x)\,\left(C\,d^2-B\,d\,e+A\,e^2\right)}{c\,d^2\,e+a\,e^3}$$

$$-\frac{\ln\left(x\,(c\,e\,B^2-c\,d\,B\,C+a\,e\,C^2-A\,c\,e\,C)+C^2\,a\,d+\frac{\left(c^2\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\left(\frac{C\,a^2\,e}{2}-\frac{A\,d\,\sqrt{-a\,c^3}}{2}\right)+\frac{B\,a\,e\,\sqrt{-a\,c^3}}{2}-\frac{C\,a\,d}{}\right.}{}\right.}{}$$

$$-\frac{\ln\left(x\,(c\,e\,B^2-c\,d\,B\,C+a\,e\,C^2-A\,c\,e\,C)+C^2\,a\,d+\frac{\left(c^2\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\left(\frac{C\,a^2\,e}{2}+\frac{A\,d\,\sqrt{-a\,c^3}}{2}\right)-\frac{B\,a\,e\,\sqrt{-a\,c^3}}{2}+\frac{C\,a\,d}{}\right.}{}\right.}{}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)),x)
```

output
```
(log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (log(x*(C^2*a*e
 + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2)
- c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e*(-a*c^3)^(1/2))/2 - (C
*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*
(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) +
(B*a*e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2
*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^
2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*
d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2)
 + (B*a*e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*
c^2*e^2) - (log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^
2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(1/2))/2) - (B*
a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^
3*d^2*e) + 8*a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 +
(A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2)
)/2))/(a*c^3*d^2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e
^2 - B*c^2*d*e) + B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c
^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2
 + (A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1
/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2)
```

3.47. $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\,dx$

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$

3.48.1 Optimal result

Integrand size = 27, antiderivative size = 214

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)}\,dx = -\frac{Cd^2-Bde+Ae^2}{e\,(cd^2+ae^2)\,(d+ex)} + \frac{(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\,(cd^2+ae^2)^2} - \frac{(Bcd^2-2Acde+2aCde-aBe^2)\log(d+ex)}{(cd^2+ae^2)^2} + \frac{(Bcd^2-2Acde+2aCde-aBe^2)\log\left(a+cx^2\right)}{2\,(cd^2+ae^2)^2}$$

output
```
(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)-(-2*A*c*d*e-B*a*e^2+B*c*d^2+2
*C*a*d*e)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*C*a*
d*e)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*
e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{-\frac{2(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{e(d+ex)} + \frac{2(Ac(cd^2-ae^2)+a(aCe^2+cd(-Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + (-2Bcd^2+4Acde-4aCde+}{2(cd^2+ae^2)^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(
c*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sq
rt[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^
2)*Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^
2])/(2*(c*d^2 + a*e^2)^2)
```

3.48.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)(d+ex)^2}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(\frac{cx(-aBe^2+2aCde-2Acde+Bcd^2)+Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be))}{(a+cx^2)(ae^2+cd^2)^2} + \frac{Ae^2-Bde+Cd^2}{(d+ex)^2(ae^2+cd^2)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(cd^2-ae^2)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}+\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{\left(ae^2+cd^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

output
```
-((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 -
a*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sq
rt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B
*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e
- a*B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.48.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(-2A\,c^2de-B\,e^2ac+B\,c^2d^2+2acdeC\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(-Aac\,e^2+A\,c^2d^2+2Bacde+a^2C\,e^2-Cac\,d^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{\left(e^2a+c\,d^2\right)^2}-\frac{A\,e^2-Bde+C\,d^2}{\left(e^2a+c\,d^2\right)e(ex+d)}+$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)^2*(1/2*(-2*A*c^2*d*e-B*a*c*e^2+B*c^2*d^2+2*C*a*c*d*e)/c*ln
(c*x^2+a)+(-A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2-C*a*c*d^2)/(a*c)^(1/
2)*arctan(c*x/(a*c)^(1/2)))-(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)+(2
*A*c*d*e+B*a*e^2-B*c*d^2-2*C*a*d*e)/(a*e^2+c*d^2)^2*ln(e*x+d)
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(204) = 408$.

Time = 25.70 (sec) , antiderivative size = 904, normalized size of antiderivative = 4.22

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)} dx$$

$$= \left[-\frac{2\,Cac^2d^4 - 2\,Bac^2d^3e - 2\,Ba^2cde^3 + 2\,Aa^2ce^4 + 2\,(Ca^2c + Aac^2)d^2e^2 - (2\,Bacd^2e^2 - (Cac - Ac^2)d^3}{} \right.$$

$$-\frac{2\,Cac^2d^4 - 2\,Bac^2d^3e - 2\,Ba^2cde^3 + 2\,Aa^2ce^4 + 2\,(Ca^2c + Aac^2)d^2e^2 - 2\,(2\,Bacd^2e^2 - (Cac - Ac^2)d^3}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 +
 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - (2*B*a*c*d^2*e^2 - (C*a*c - A*c^2)*d^3*e
+ (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2)*d^2*e^2 + (C*a^
2 - A*a*c)*e^4)*x)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)
) - (B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*
c^2*d^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a)
 + 2*(B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a
*c^2*d^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))
/(a*c^3*d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c
^2*d^2*e^4 + a^3*c*e^6)*x), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^
2*c*d*e^3 + 2*A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - 2*(2*B*a*c*d^2
*e^2 - (C*a*c - A*c^2)*d^3*e + (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C
*a*c - A*c^2)*d^2*e^2 + (C*a^2 - A*a*c)*e^4)*x)*sqrt(a*c)*arctan(sqrt(a*c)
*x/a) - (B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (
B*a*c^2*d^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2
+ a) + 2*(B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 +
(B*a*c^2*d^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x +
 d))/(a*c^3*d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a
^2*c^2*d^2*e^4 + a^3*c*e^6)*x)]
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx = \frac{(Bcd^2-Bae^2+2(Ca-Ac)de)\log(cx^2+a)}{2(c^2d^4+2acd^2e^2+a^2e^4)} - \frac{(Bcd^2-Bae^2+2(Ca-Ac)de)\log(ex+d)}{c^2d^4+2acd^2e^2+a^2e^4} + \frac{(2Bacde-(Cac-Ac^2)d^2+(Ca^2-Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^2d^4+2acd^2e^2+a^2e^4)\sqrt{ac}} - \frac{Cd^2-Bde+Ae^2}{cd^3e+ade^3+(cd^2e^2+ae^4)x}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/(c^2*d^4 + 2*a*
c*d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(e*x + d
)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e - (C*a*c - A*c^2)*d^2
 + (C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(a*c)) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2*
e^2 + a*e^4)*x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{(Bcd^2+2\,Cade-2\,Acde-Bae^2)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} - \frac{\frac{Cd^2e}{ex+d}-\frac{Bde^2}{ex+d}+\frac{Ae^3}{ex+d}}{cd^2e^2+ae^4} - \frac{(Cacd^2e^2-Ac^2d^2e^2-2\,Bacde^3-Ca^2e^4+Aace^4)\arctan\left(\frac{cd-\frac{cd^2}{ex+d}-\frac{ae^2}{ex+d}}{\sqrt{ac}e}\right)}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}e^2}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c*d^2 + 2*C*a*d*e - 2*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(e*x + d) +
c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
 - (C*d^2*e/(e*x + d) - B*d*e^2/(e*x + d) + A*e^3/(e*x + d))/(c*d^2*e^2 +
a*e^4) - (C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*
c*e^4)*arctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)*e^2)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.60

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$$
$$= \frac{\ln\left(C\,c\,d^4\,(-a\,c)^{3/2}-A\,a\,e^4\,(-a\,c)^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x+A\,c^3\,d^4\,\sqrt{-a\,c}-C\,a^3\,e^4\,\sqrt{-\right.}{}$$
$$-\frac{\ln(d+e\,x)\,\left(c\,(B\,d^2-2\,A\,d\,e)-a\,(B\,e^2-2\,C\,d\,e)\right)}{a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4}$$
$$-\frac{\ln\left(A\,a\,e^4\,(-a\,c)^{3/2}-C\,c\,d^4\,(-a\,c)^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x-A\,c^3\,d^4\,\sqrt{-a\,c}+C\,a^3\,e^4\,\sqrt{\right.}{}$$
$$-\frac{C\,d^2-B\,d\,e+A\,e^2}{e\,(c\,d^2+a\,e^2)\,(d+e\,x)}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^2),x)
```

output
```
(log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3
*c*e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C
*a*c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^
2*(-a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*
c^2*d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 + 8*B*a*d*e^3*(-a*c)^(3
/2) - 8*B*c*d^3*e*(-a*c)^(3/2) + 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*
e - 8*C*a^3*c*d*e^3 + 14*C*a^2*c^2*d^2*e^2*x + 8*A*c*d*e^3*x*(-a*c)^(3/2)
- 8*C*a*d*e^3*x*(-a*c)^(3/2) + 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*
e*x + 8*A*c^3*d^3*e*x*(-a*c)^(1/2) - 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*
a*c^3*d^2*e^2*x - 8*B*a^2*c^2*d*e^3*x)*(c^2*(a*((B*d^2)/2 - A*d*e) + (A*d^
2*(-a*c)^(1/2))/2) - c*(a^2*((B*e^2)/2 - C*d*e) + a*((A*e^2*(-a*c)^(1/2))/
2 + (C*d^2*(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) + (C*a^2*e^2*(-a*c)^(1/2
))/2))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (log(d + e*x)*(c*(B*d
^2 - 2*A*d*e) - a*(B*e^2 - 2*C*d*e)))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)
- (log(A*a*e^4*(-a*c)^(3/2) - C*c*d^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a
^3*c*e^4 + A*c^4*d^4*x - A*c^3*d^4*(-a*c)^(1/2) + C*a^3*e^4*(-a*c)^(1/2) -
 C*a*c^3*d^4*x - C*a^3*c*e^4*x - 14*A*c*d^2*e^2*(-a*c)^(3/2) + 14*C*a*d^2*
e^2*(-a*c)^(3/2) + 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^
2*c^2*d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 - 8*B*a*d*e^3*(-a*c)^
(3/2) + 8*B*c*d^3*e*(-a*c)^(3/2) - 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3...
```

3.48. $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)}\,dx$

3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)} dx$

3.49.1 Optimal result

Integrand size = 27, antiderivative size = 305

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx &= -\frac{Cd^2-Bde+Ae^2}{2e(cd^2+ae^2)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{(cd^2+ae^2)^2(d+ex)} \\ &+\frac{\sqrt{c}(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}(cd^2+ae^2)^3} \\ &-\frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(d+ex)}{(cd^2+ae^2)^3} \\ &+\frac{(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))\log(a+cx^2)}{2(cd^2+ae^2)^3}\end{aligned}$$

output
```
1/2*(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c
*d^2+2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(B*c*d*(-3*a*e^2+c*d^2)-(A*c-C*a)*
e*(-a*e^2+3*c*d^2))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*(B*c*d*(-3*a*e^2+c*d^2)-
(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+(A*c*d*(-3*a*e^2
+c*d^2)-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2
))*c^(1/2)/(a*e^2+c*d^2)^3/a^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx$$
$$= \frac{-\frac{(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{e(d+ex)^2}+\frac{2(cd^2+ae^2)(Bcd^2-2Acde+2aCde-aBe^2)}{d+ex}+\frac{2\sqrt{c}(Acd(cd^2-3ae^2)+a(ae^2(3Cd-Be)+cd^2(-Cd+3Be}{\sqrt{a}}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
(-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c
*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*
Sqrt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d)
+ 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^
2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e
^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^
3)
```

3.49.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2160}$$
$$\int \left(\frac{c\left(x\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)+Acd(cd^2-3ae^2)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right.}{(a+cx^2)\left(ae^2+cd^2\right)^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(cd^2-3ae^2)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{\sqrt{a}\left(ae^2+cd^2\right)^3}+\frac{\log\left(a+cx^2\right)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+Bcd^2}{(d+ex)\left(ae^2+cd^2\right)^2}-\frac{\log(d+ex)\left(Bcd(cd^2-3ae^2)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{\left(ae^2+cd^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

output
```
-1/2*(C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 -
2*A*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*
(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2
- 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2
)^3 + ((B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a +
 c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.49.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.04

method	result
default	$-\frac{c\left(\frac{\left(-Aac\,e^3+3A\,c^2d^2e+3Bacd\,e^2-B\,c^2d^3+C\,a^2e^3-3Cac\,d^2e\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(3Aacd\,e^2-A\,d^3c^2+a^2B\,e^3-3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)\text{arct}}{\sqrt{ac}}\right.}{\left(e^2a+c\,d^2\right)^3}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)^3*(1/2*(-A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3+
C*a^2*e^3-3*C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2-A*c^2*d^3+B*a^2*e^3-
3*B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
)-1/2*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)/e/(e*x+d)^2-(2*A*c*d*e+B*a*e^2-B*c
*d^2-2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(A*a*c*e^3-3*A*c^2*d^2*e-3*B*a*c*d
*e^2+B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/(a*e^2+c*d^2)^3*ln(e*x+d)
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(291) = 582$.

Time = 76.07 (sec) , antiderivative size = 1759, normalized size of antiderivative = 5.77

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(C*c^2*d^6 - 3*B*c^2*d^5*e - 2*B*a*c*d^3*e^3 + B*a^2*d*e^5 + A*a^2*e
^6 - (2*C*a*c - 5*A*c^2)*d^4*e^2 - 3*(C*a^2 - 2*A*a*c)*d^2*e^4 + (3*B*a*c*
d^4*e^2 - B*a^2*d^2*e^4 - (C*a*c - A*c^2)*d^5*e + 3*(C*a^2 - A*a*c)*d^3*e^
3 + (3*B*a*c*d^2*e^4 - B*a^2*e^6 - (C*a*c - A*c^2)*d^3*e^3 + 3*(C*a^2 - A*
a*c)*d*e^5)*x^2 + 2*(3*B*a*c*d^3*e^3 - B*a^2*d*e^5 - (C*a*c - A*c^2)*d^4*e
^2 + 3*(C*a^2 - A*a*c)*d^2*e^4)*x)*sqrt(-c/a)*log((c*x^2 - 2*a*x*sqrt(-c/a
) - a)/(c*x^2 + a)) - 2*(B*c^2*d^4*e^2 - B*a^2*e^6 + 2*(C*a*c - A*c^2)*d^3
*e^3 + 2*(C*a^2 - A*a*c)*d*e^5)*x - (B*c^2*d^5*e - 3*B*a*c*d^3*e^3 + 3*(C*
a*c - A*c^2)*d^4*e^2 - (C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*a*c*
d*e^5 + 3*(C*a*c - A*c^2)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2*d^
4*e^2 - 3*B*a*c*d^2*e^4 + 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*e^
5)*x)*log(c*x^2 + a) + 2*(B*c^2*d^5*e - 3*B*a*c*d^3*e^3 + 3*(C*a*c - A*c^2
)*d^4*e^2 - (C*a^2 - A*a*c)*d^2*e^4 + (B*c^2*d^3*e^3 - 3*B*a*c*d*e^5 + 3*(
C*a*c - A*c^2)*d^2*e^4 - (C*a^2 - A*a*c)*e^6)*x^2 + 2*(B*c^2*d^4*e^2 - 3*B
*a*c*d^2*e^4 + 3*(C*a*c - A*c^2)*d^3*e^3 - (C*a^2 - A*a*c)*d*e^5)*x)*log(e
*x + d))/(c^3*d^8*e + 3*a*c^2*d^6*e^3 + 3*a^2*c*d^4*e^5 + a^3*d^2*e^7 + (c
^3*d^6*e^3 + 3*a*c^2*d^4*e^5 + 3*a^2*c*d^2*e^7 + a^3*e^9)*x^2 + 2*(c^3*d^7
*e^2 + 3*a*c^2*d^5*e^4 + 3*a^2*c*d^3*e^6 + a^3*d*e^8)*x), -1/2*(C*c^2*d^6
- 3*B*c^2*d^5*e - 2*B*a*c*d^3*e^3 + B*a^2*d*e^5 + A*a^2*e^6 - (2*C*a*c - 5
*A*c^2)*d^4*e^2 - 3*(C*a^2 - 2*A*a*c)*d^2*e^4 - 2*(3*B*a*c*d^4*e^2 - B*...
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \frac{(Bc^2d^3-3\,Bacde^2+3\,(Cac-Ac^2)d^2e-(Ca^2-Aac)e^3)\log{(cx^2+a)}}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)} - \frac{(Bc^2d^3-3\,Bacde^2+3\,(Cac-Ac^2)d^2e-(Ca^2-Aac)e^3)\log{(ex+d)}}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6} + \frac{(3\,Bac^2d^2e-Ba^2ce^3-(Cac^2-Ac^3)d^3+3\,(Ca^2c-Aac^2)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}} - \frac{Ccd^4-3\,Bcd^3e+Bade^3+Aae^4-(3\,Ca-5\,Ac)d^2e^2-2\,(Bcd^2e^2-Bae^4+2\,(Ca-Ac)de^3)x}{2\,(c^2d^6e+2\,acd^4e^3+a^2d^2e^5+(c^2d^4e^3+2\,acd^2e^5+a^2e^7)x^2+2\,(c^2d^5e^2+2\,acd^3e^4+a^2de^6)x)}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)
*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^
6) - (B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c
)*e^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6
) + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 - A*c^3)*d^3 + 3*(C*a^2*c -
A*a*c^2)*d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*
c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c*d^4 - 3*B*c*d^3*e + B*a*d*e^3 +
 A*a*e^4 - (3*C*a - 5*A*c)*d^2*e^2 - 2*(B*c*d^2*e^2 - B*a*e^4 + 2*(C*a - A
*c)*d*e^3)*x)/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 + 2*
a*c*d^2*e^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 + 2*a*c*d^3*e^4 + a^2*d*e^6)*x
)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx$$
$$= \frac{(Bc^2d^3+3\,Cacd^2e-3\,Ac^2d^2e-3\,Bacde^2-Ca^2e^3+Aace^3)\log\left(cx^2+a\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)}$$
$$-\frac{(Bc^2d^3e+3\,Cacd^2e^2-3\,Ac^2d^2e^2-3\,Bacde^3-Ca^2e^4+Aace^4)\log\left(|ex+d|\right)}{c^3d^6e+3\,ac^2d^4e^3+3\,a^2cd^2e^5+a^3e^7}$$
$$-\frac{(Cac^2d^3-Ac^3d^3-3\,Bac^2d^2e-3\,Ca^2cde^2+3\,Aac^2de^2+Ba^2ce^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)\sqrt{ac}}$$
$$-\frac{Cc^2d^6-3\,Bc^2d^5e-2\,Cacd^4e^2+5\,Ac^2d^4e^2-2\,Bacd^3e^3-3\,Ca^2d^2e^4+6\,Aacd^2e^4+Ba^2de^5+Aa^2e^6-}{2\,(cd^2+ae^2)^3(ex+d)^2e}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
 + A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4
+ a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*
e^3 - C*a^2*e^4 + A*a*c*e^4)*log(abs(e*x + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^
3 + 3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*
e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))
/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2
*(C*c^2*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*
d^3*e^3 - 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*
(B*c^2*d^4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a
*c*d*e^5 - B*a^2*e^6)*x)/((c*d^2 + a*e^2)^3*(e*x + d)^2*e)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 2980, normalized size of antiderivative = 9.77

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^3),x)
```

output
```
(log(d + e*x)*(e^3*(C*a^2 - A*a*c) - B*c^2*d^3 + d^2*e*(3*A*c^2 - 3*C*a*c)
 + 3*B*a*c*d*e^2))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
 - (log(9*A^2*a^5*e^10*(-a*c)^(5/2) + A^2*c^5*d^10*(-a*c)^(5/2) - B^2*a^7*
e^10*(-a*c)^(3/2) - 9*B^2*c^3*d^10*(-a*c)^(7/2) + 9*C^2*a^9*e^10*(-a*c)^(1
/2) + C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x - 6*A^2*a*d^4*e^6*(-a*c
)^(9/2) - 6*B^2*a*d^6*e^4*(-a*c)^(9/2) + 106*A^2*c*d^6*e^4*(-a*c)^(9/2) +
77*C^2*a*d^8*e^2*(-a*c)^(9/2) - 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^
8*d^10*x + 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*
x + C^2*a^4*c^6*d^10*x + 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) - 106*B^2*a^3*d^4
*e^6*(-a*c)^(7/2) + 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) - 77*A^2*c^3*d^8*e^2*(
-a*c)^(7/2) - 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) - 6*C^2*a^5*d^4*e^6*(-a*c)^
(5/2) + 27*C^2*a^7*d^2*e^8*(-a*c)^(3/2) + 18*A*C*a^7*e^10*(-a*c)^(3/2) + 2
*A*C*c^3*d^10*(-a*c)^(7/2) + 224*A*B*a*d^5*e^5*(-a*c)^(9/2) - 48*A*B*a^5*d
*e^9*(-a*c)^(5/2) - 212*A*C*a*d^6*e^4*(-a*c)^(9/2) + 64*A*B*c*d^7*e^3*(-a*
c)^(9/2) + 48*A*B*c^3*d^9*e*(-a*c)^(7/2) - 64*B*C*a*d^7*e^3*(-a*c)^(9/2) -
 48*B*C*a^7*d*e^9*(-a*c)^(3/2) - 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a
^3*c^7*d^8*e^2*x + 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 2
7*A^2*a^6*c^4*d^2*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4
*x + 106*B^2*a^6*c^4*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5
*d^8*e^2*x + 106*C^2*a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C...
```

3.49. $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)}\,dx$

3.50 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

3.50.1 Optimal result

Integrand size = 27, antiderivative size = 216

$$\begin{aligned}\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx &= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2}-\frac{(Ac-2aC)e^3x^2}{2ac^2}-\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)}\\&+\frac{\left(Acd\left(cd^2+3ae^2\right)-a\left(3ae^2(3Cd+Be)-cd^2(Cd+3Be)\right)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}}\\&-\frac{e\left(2aCe^2-c\left(3Cd^2+e(3Bd+Ae)\right)\right)\log\left(a+cx^2\right)}{2c^3}\end{aligned}$$

output
```
-3/2*e^2*(A*c*d-a*(B*e+3*C*d))*x/a/c^2-1/2*(A*c-2*C*a)*e^3*x^2/a/c^2-1/2*(
a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)+1/2*(A*c*d*(3*a*e^2+c*d^2)-a*(3*a
*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(
5/2)-1/2*e*(2*a*C*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))*ln(c*x^2+a)/c^3
```

3.50.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2} \, dx$$
$$= \frac{2ce^2(3Cd+Be)x + cCe^3x^2 + \frac{-a^3Ce^3+Ac^3d^3x-ac^2d(Cd^2x+3Ae(d+ex)+Bd(d+3ex))+a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a(a+cx^2)} +}{2}$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
(2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c
^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e
*x) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 +
 3*a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*L
og[a + c*x^2])/(2*c^3)
```

3.50.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2176, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2} \, dx$$
$$\Big\downarrow \ 2176$$
$$-\frac{\int -\frac{(d+ex)^2(Acd+aCd+3aBe-2(Ac-2aC)ex)}{cx^2+a}dx}{2ac} - \frac{(d+ex)^3(aB-x(Ac-aC))}{2ac\,(a+cx^2)}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{(d+ex)^2(Acd+aCd+3aBe-2(Ac-2aC)ex)}{cx^2+a}dx}{2ac} - \frac{(d+ex)^3(aB-x(Ac-aC))}{2ac\,(a+cx^2)}$$
$$\Big\downarrow \ 657$$

$$\frac{\int \left(-\frac{2(Ac-2aC)xe^3}{c}-\frac{3(Acd-3aCd-aBe)e^2}{c}+\frac{Acd(cd^2+3ae^2)-a(3ae^2(3Cd+Be)-cd^2(Cd+3Be))-2ae(2aCe^2-c(3Cd^2+e(3Bd+Ae)))x}{c(cx^2+a)}\right.}{2ac} - \frac{(d+ex)^3(aB-x(Ac-aC))}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(Acd(3ae^2+cd^2)-a(3ae^2(Be+3Cd)-cd^2(3Be+Cd)))}{\sqrt{a}c^{3/2}}-\frac{ae\log(a+cx^2)(2aCe^2-c(e(Ae+3Bd)+3Cd^2))}{c^2}-\frac{3e^2x(-aBe-3aCd+}{c}}{2ac} - \frac{(d+ex)^3(aB-x(Ac-aC))}{2ac\,(a+cx^2)}$$

input
```
Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)) + ((-3*e^2*(A*c
*d - 3*a*C*d - a*B*e)*x)/c - ((A*c - 2*a*C)*e^3*x^2)/c + ((A*c*d*(c*d^2 +
3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) - (a*e*(2*a*C*e^2 - c*(3*C*d^2 + e*(3*B*d
 + A*e)))*Log[a + c*x^2])/c^2)/(2*a*c)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.50.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.31

method	result
default	$\frac{e^2\left(\frac{1}{2}C x^2 e+Bex+3Cdx\right)}{c^2}+\frac{-\frac{\left(3Aacd e^2-A d^3 c^2-a^2 B e^3+3Bac d^2 e-3C a^2 d e^2+Cac d^3\right)x}{2a}+\frac{Aac e^3-3A c^2 d^2 e+3Bacd e^2-B c^2 d^3-C a^2 e^3\cdots}{2c}}{c x^2+a}$
risch	Expression too large to display

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2/c^2*(1/2*C*x^2*e+B*e*x+3*C*d*x)+1/c^2*((-1/2*(3*A*a*c*d*e^2-A*c^2*d^3-
B*a^2*e^3+3*B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a*x+1/2*(A*a*c*e^3-3*A*c^
2*d^2*e+3*B*a*c*d*e^2-B*c^2*d^3-C*a^2*e^3+3*C*a*c*d^2*e)/c)/(c*x^2+a)+1/2/
a*(1/2*(2*A*a*c*e^3+6*B*a*c*d*e^2-4*C*a^2*e^3+6*C*a*c*d^2*e)/c*ln(c*x^2+a)
+(3*A*a*c*d*e^2+A*c^2*d^3-3*B*a^2*e^3+3*B*a*c*d^2*e-9*C*a^2*d*e^2+C*a*c*d^
3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(196) = 392$.

Time = 0.31 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$
$$=\left[\frac{2\,Ca^2c^2e^3x^4+2\,Ca^3ce^3x^2-2\,Ba^2c^2d^3+6\,Ba^3cde^2+6\,(Ca^3c-Aa^2c^2)d^2e-2\,(Ca^4-Aa^3c)e^3+4\,(3\,C\cdots}{\cdots}\right.$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*
c*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C
*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*
a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*
B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*
sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*
d^2*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c
^2)*d*e^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*
e^3 + (3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3
)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C
*a^3*c*e^3*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)
*d^2*e - (C*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3
 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 -
 A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3
 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (
3*B*a^2*c^2*d^2*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3
*c - A*a^2*c^2)*d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 -
 A*a^3*c)*e^3 + (3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^
2*c^2)*e^3)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(197) = 394$.

Time = 25.72 (sec) , antiderivative size = 952, normalized size of antiderivative = 4.41

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2} \, dx$$
$$= \frac{Ce^3x^2}{2c^2} + x\left(\frac{Be^3}{c^2} + \frac{3Cde^2}{c^2}\right) + \left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}\right.$$
$$\left.-\frac{\sqrt{-a^3c^7}(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)\log\left(x+\frac{2Aa^2ce^3+6Ba^2cd\ldots}{\ldots}\right.$$
$$+\left(-\frac{e(-Ace^2-3Bcde+2Cae^2-3Ccd^2)}{2c^3}\right.$$
$$\left.+\frac{\sqrt{-a^3c^7}(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3Bacd^2e+9Ca^2de^2-Cacd^3)}{4a^3c^6}\right)\log\left(x+\frac{2Aa^2ce^3+6Ba^2cd\ldots}{\ldots}\right.$$
$$+\frac{Aa^2ce^3-3Aac^2d^2e+3Ba^2cde^2-Bac^2d^3-Ca^3e^3+3Ca^2cd^2e+x(-3Aac^2de^2+Ac^3d^3+Ba^2ce^3-3\ldots}{2a^2c^3+2ac^4x^2}$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

output
```
C*e**3*x**2/(2*c**2) + x*(B*e**3/c**2 + 3*C*d*e**2/c**2) + (-e*(-A*c*e**2
- 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a
*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2
 - C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2
 - 4*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*
d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**
2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c
*d**3)/(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3
 - 3*B*a*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (-e*(-A*c*e**
2 - 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A
*a*c*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e*
*2 - C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e*
*2 - 4*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*
c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e
**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a
*c*d**3)/(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e*
*3 - 3*B*a*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (A*a**2*c*e
**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3
+ 3*C*a**2*c*d**2*e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3
- 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))/(2*a**2*c**3 ...
```

3.50. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^2} \, dx$

3.50.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = -\frac{Bac^2d^3 - 3\,Ba^2cde^2 - 3\,(Ca^2c - Aac^2)d^2e + (Ca^3 - Aa^2c)e^3 + (3\,Bac^2d^2e - Ba^2ce^3 + (Cac^2 - Ac^3)d}{2\,(ac^4x^2 + a^2c^3)} + \frac{Ce^3x^2 + 2\,(3\,Cde^2 + Be^3)x}{2\,c^2} + \frac{(3\,Ccd^2e + 3\,Bcde^2 - (2\,Ca - Ac)e^3)\log(cx^2 + a)}{2\,c^3} + \frac{(3\,Bacd^2e - 3\,Ba^2e^3 + (Cac + Ac^2)d^3 - 3\,(3\,Ca^2 - Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a*c^2*d^3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3
 - A*a^2*c)*e^3 + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 + (C*a*c^2 - A*c^3)*d^3 -
 3*(C*a^2*c - A*a*c^2)*d*e^2)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(C*e^3*x^2 +
2*(3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - (2*C*a - A
*c)*e^3)*log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e - 3*B*a^2*e^3 + (C*a*c +
A*c^2)*d^3 - 3*(3*C*a^2 - A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a
*c^2)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{(3\,Ccd^2e + 3\,Bcde^2 - 2\,Cae^3 + Ace^3)\log(cx^2 + a)}{2\,c^3} + \frac{(Cacd^3 + Ac^2d^3 + 3\,Bacd^2e - 9\,Ca^2de^2 + 3\,Aacde^2 - 3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} + \frac{Cc^2e^3x^2 + 6\,Cc^2de^2x + 2\,Bc^2e^3x}{2\,c^4} - \frac{Bac^2d^3 - 3\,Ca^2cd^2e + 3\,Aac^2d^2e - 3\,Ba^2cde^2 + Ca^3e^3 - Aa^2ce^3 + (Cac^2d^3 - Ac^3d^3 + 3\,Bac^2d^2e - 3\,C}{2\,(cx^2 + a)ac^3}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
 1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^
2 - 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*e^3*
x^2 + 6*C*c^2*d*e^2*x + 2*B*c^2*e^3*x)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*
d^2*e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a
*c^2*d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2
 - B*a^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{x\,(B\,e^3+3\,C\,d\,e^2)}{c^2} - \frac{\frac{C\,a^2\,e^3-3\,C\,a\,c\,d^2\,e-3\,B\,a\,c\,d\,e^2-A\,a\,c\,e^3+B\,c^2\,d^3+3\,A\,c^2\,d^2\,e}{2\,c}-\frac{x\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{2\,a}}{c^3\,x^2+a\,c^2} + \frac{C\,e^3\,x^2}{2\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(-9\,C\,a^2\,d\,e^2-3\,B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+A\,c^2\,d^3)}{2\,a^{3/2}\,c^{5/2}} + \frac{\ln(c\,x^2+a)\,(-32\,C\,a^4\,c^3\,e^3+48\,C\,a^3\,c^4\,d^2\,e+48\,B\,a^3\,c^4\,d\,e^2+16\,A\,a^3\,c^4\,e^3)}{32\,a^3\,c^6}$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

output
```
(x*(B*e^3 + 3*C*d*e^2))/c^2 - ((B*c^2*d^3 + C*a^2*e^3 - A*a*c*e^3 + 3*A*c^
2*d^2*e - 3*B*a*c*d*e^2 - 3*C*a*c*d^2*e)/(2*c) - (x*(A*c^2*d^3 + B*a^2*e^3
 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a))/(a*c
^2 + c^3*x^2) + (C*e^3*x^2)/(2*c^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^
3 - 3*B*a^2*e^3 + C*a*c*d^3 - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*
e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*A*a^3*c^4*e^3 - 32*C*a^4*c^3
*e^3 + 48*B*a^3*c^4*d*e^2 + 48*C*a^3*c^4*d^2*e))/(32*a^3*c^6)
```

3.51 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^2} dx$

3.51.1 Optimal result

Integrand size = 27, antiderivative size = 146

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^2} dx = -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac(a+cx^2)} + \frac{(a(Ac-3aC)e^2+cd(Acd+aCd+2aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{5/2}} + \frac{e(2Cd+Be)\log(a+cx^2)}{2c^2}$$

output
```
-1/2*(A*c-3*C*a)*e^2*x/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)
+1/2*(a*(A*c-3*C*a)*e^2+c*d*(A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2
))/a^(3/2)/c^(5/2)+1/2*e*(B*e+2*C*d)*ln(c*x^2+a)/c^2
```

3.51.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^2} dx = \frac{2\sqrt{c}Ce^2x + \frac{\sqrt{c}(Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex)))}{a(a+cx^2)} + \frac{(Ac(cd^2+ae^2)+a(-3aCe^2+cd(Cd+2Be)))\,\text{arcta}}{a^{3/2}}}{2c^{5/2}}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
(2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
 a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A
*c*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

3.51.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2176, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(d+ex)(Acd+aCd+2aBe-(Ac-3aC)ex)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)(Acd+aCd+2aBe-(Ac-3aC)ex)}{cx^2+a}dx}{2ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(\frac{Ac(cd^2+ae^2)-a(3aCe^2-cd(Cd+2Be))+2ace(2Cd+Be)x}{c(cx^2+a)}-\frac{(Ac-3aC)e^2}{c}\right)dx}{2ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(ae^2+cd^2)-a(3aCe^2-cd(2Be+Cd))\right)}{\sqrt{a}c^{3/2}}-\frac{e^2x(Ac-3aC)}{c}+\frac{ae\log(a+cx^2)(Be+2Cd)}{c}}{2ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)) + (-(((A*c - 3*
a*C)*e^2*x)/c) + ((A*c*(c*d^2 + a*e^2) - a*(3*a*C*e^2 - c*d*(C*d + 2*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (a*e*(2*C*d + B*e)*Log[
a + c*x^2])/c)/(2*a*c)
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.51.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.28

method	result
default	$\frac{e^2Cx}{c^2}+\frac{\frac{-\frac{\left(Aac\,e^2-A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2\right)x}{2a}-Acde+\frac{Ba\,e^2}{2}-\frac{Bc\,d^2}{2}+adeC}{c\,x^2+a}+\frac{\frac{\left(2B\,e^2ac+4acdeC\right)\ln\left(c\,x^2+a\right)}{2c}+\frac{\left(Aac\,e^2+A\,c^2d^2+2\right.}{}}{2a}}{c^2}$
risch	$\frac{e^2Cx}{c^2}+\frac{-\frac{\left(Aac\,e^2-A\,c^2d^2+2Bacde-a^2C\,e^2+Cac\,d^2\right)x}{2a}-Acde+\frac{Ba\,e^2}{2}-\frac{Bc\,d^2}{2}+adeC}{c^2\left(c\,x^2+a\right)}+\frac{\ln\left(A\,a^2c\,e^2+A\,d^2a\,c^2+2a^2cdeB-3C\,a^3e^2+\right.}{}$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2*C/c^2*x+1/c^2*((-1/2*(A*a*c*e^2-A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*
d^2)/a*x-A*c*d*e+1/2*B*a*e^2-1/2*B*c*d^2+a*d*e*C)/(c*x^2+a)+1/2/a*(1/2*(2*
B*a*c*e^2+4*C*a*c*d*e)/c*ln(c*x^2+a)+(A*a*c*e^2+A*c^2*d^2+2*B*a*c*d*e-3*C*
a^2*e^2+C*a*c*d^2)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(132) = 264.

Time = 0.30 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.32

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$
$$=\left[\frac{4\,Ca^2c^2e^2x^3-2\,Ba^2c^2d^2+2\,Ba^3ce^2+4\,(Ca^3c-Aa^2c^2)de-(2\,Ba^2cde+(Ca^2c+Aac^2)d^2-(3\,Ca^3-\right.$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c -
 A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*
a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2
)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(
2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)
*x + 2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^
2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a
^2*c^2*d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e +
(C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*
c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a
*c)*x/a) - (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a
^2*c^2)*e^2)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c
^2*e^2)*x^2)*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(138) = 276$.

Time = 6.04 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.06

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce^2x}{c^2} + \left(\frac{e(Be+2Cd)}{2c^2} - \frac{\sqrt{-a^3c^5}(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+2}{2c^2}\right.}{-Aace^2 - Ac^2d^2}\right. + \left(\frac{e(Be+2Cd)}{2c^2} + \frac{\sqrt{-a^3c^5}(-Aace^2 - Ac^2d^2 - 2Bacde + 3Ca^2e^2 - Cacd^2)}{4a^3c^5}\right)\log\left(x + \frac{2Ba^2e^2 + 4Ca^2de - 4a^2c^2\left(\frac{e(Be+2}{2c^2}\right.}{-Aace^2 - Ac^2d^2}\right. + \frac{-2Aacde + Ba^2e^2 - Bacd^2 + 2Ca^2de + x(-Aace^2 + Ac^2d^2 - 2Bacde + Ca^2e^2 - Cacd^2)}{2a^2c^2 + 2ac^3x^2}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

output
```
C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2
- A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*l
og(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**
2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*
e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*
e + 3*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*
c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**
2)/(4*a**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(
B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*
a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**
2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a*
*2*e**2 - B*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a
*c*d*e + C*a**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$$
$$= \frac{Ce^2x}{c^2} - \frac{Bacd^2 - Ba^2e^2 - 2\,(Ca^2 - Aac)de + (2\,Bacde + (Cac - Ac^2)d^2 - (Ca^2 - Aac)e^2)x}{2\,(ac^3x^2 + a^2c^2)} + \frac{(2\,Cde + Be^2)\log(cx^2 + a)}{2\,c^2} + \frac{(2\,Bacde + (Cac + Ac^2)d^2 - (3\,Ca^2 - Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
C*e^2*x/c^2 - 1/2*(B*a*c*d^2 - B*a^2*e^2 - 2*(C*a^2 - A*a*c)*d*e + (2*B*a*
c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*x)/(a*c^3*x^2 + a^2*c^2
) + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(2*B*a*c*d*e + (C*a*c +
 A*c^2)*d^2 - (3*C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^
2)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce^2x}{c^2} + \frac{(2\,Cde+Be^2)\log\left(cx^2+a\right)}{2\,c^2} + \frac{(Cacd^2+Ac^2d^2+2\,Bacde-3\,Ca^2e^2+Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac^2} - \frac{Bacd^2-2\,Ca^2de+2\,Aacde-Ba^2e^2+(Cacd^2-Ac^2d^2+2\,Bacde-Ca^2e^2+Aace^2)x}{2\,(cx^2+a)ac^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
C*e^2*x/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 +
A*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(
sqrt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2
+ (C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2
 + a)*a*c^2)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{C\,e^2\,x}{c^2} - \frac{\frac{x\,\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2\right)}{2\,a} - \frac{B\,a\,e^2}{2} + \frac{B\,c\,d^2}{2} + A\,c\,d\,e - C\,a\,d\,e}{c^3\,x^2+a\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+A\,c^2\,d^2\right)}{2\,a^{3/2}\,c^{5/2}} + \frac{\ln\left(c\,x^2+a\right)\,\left(16\,B\,a^3\,c^3\,e^2+32\,C\,d\,a^3\,c^3\,e\right)}{32\,a^3\,c^5}$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

output
```
(C*e^2*x)/c^2 - ((x*(A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a
*c*d*e))/(2*a) - (B*a*e^2)/2 + (B*c*d^2)/2 + A*c*d*e - C*a*d*e)/(a*c^2 + c
^3*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^2 - 3*C*a^2*e^2 + A*a*c*e^2
+ C*a*c*d^2 + 2*B*a*c*d*e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*B*a^
3*c^3*e^2 + 32*C*a^3*c^3*d*e))/(32*a^3*c^5)
```

3.52 $\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

3.52.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac(a+cx^2)} + \frac{(Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Ce\log(a+cx^2)}{2c^2}$$

output
```
-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e+C*a*d)*arcta
n(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(3/2)+1/2*C*e*ln(c*x^2+a)/c^2
```

3.52.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{\frac{a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))}{a(a+cx^2)} + \frac{\sqrt{c}(Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}} + Ce\log(a+cx^2)}{2c^2}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
 (Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C
*e*Log[a + c*x^2])/(2*c^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2176, 25, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{Acd+a(Cd+Be)+2aCex}{cx^2+a}dx}{2ac}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Acd+aCd+aBe+2aCex}{cx^2+a}dx}{2ac}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(aBe+aCd+Acd)\int \frac{1}{cx^2+a}dx+2aCe\int \frac{x}{cx^2+a}dx}{2ac}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aCe\int \frac{x}{cx^2+a}dx+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{\sqrt{a}\sqrt{c}}+\frac{aCe\log\left(a+cx^2\right)}{c}}{2ac}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

output
```
-1/2*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)) + (((A*c*d + a*C*
d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + (a*C*e*Log[a +
 c*x^2])/c)/(2*a*c)
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.52.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-Cae}{2c^2}}{c\,x^2+a}+\frac{\frac{Cae\ln\left(c\,x^2+a\right)}{c}+\frac{(Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2ac}$
risch	$\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-Cae}{2c^2}}{c\,x^2+a}+\frac{\ln\left(dAac+Be\,a^2+a^2dC-\sqrt{-ac(Acd+Bae+Cad)^2}\,x\right)eC}{2c^2}+\frac{\ln\left(dAac+Be\,a^2+a^2dC-\sqrt{-a\right.}}{}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2/
a/c*(C*a*e/c*ln(c*x^2+a)+(A*c*d+B*a*e+C*a*d)/(a*c)^(1/2)*arctan(c*x/(a*c)^
(1/2)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.47

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$$

$$= \left[-\frac{2\,Ba^2cd+(Ba^2e+(Bace+(Cac+Ac^2)d)x^2+(Ca^2+Aac)d)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)-2\,(Ca^3-}{4\,(a^2c^3x^2+a^3c^2)}\right.$$

$$-\frac{Ba^2cd-(Ba^2e+(Bace+(Cac+Ac^2)d)x^2+(Ca^2+Aac)d)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)-(Ca^3-Aa^2c)e+}{2\,(a^2c^3x^2+a^3c^2)}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c*d + (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2
 + A*a*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*
(C*a^3 - A*a^2*c)*e + 2*(B*a^2*c*e + (C*a^2*c - A*a*c^2)*d)*x - 2*(C*a^2*c
*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d
 - (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2 + A*a*c)*d)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) - (C*a^3 - A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c -
 A*a*c^2)*d)*x - (C*a^2*c*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 +
a^3*c^2)]
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(90) = 180.

Time = 2.23 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.28

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \left(\frac{Ce}{2c^2} - \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)\log\left(x+\frac{-2Ca^2e+4a^2c^2\left(\frac{Ce}{2c^2}-\frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d+Bace+Cacd}\right) + \left(\frac{Ce}{2c^2} + \frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)\log\left(x+\frac{-2Ca^2e+4a^2c^2\left(\frac{Ce}{2c^2}+\frac{\sqrt{-a^3c^5}(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d+Bace+Cacd}\right) + \frac{-Aace-Bacd+Ca^2e+x(Ac^2d-Bace-Cacd)}{2a^2c^2+2ac^3x^2}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

output
```
(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*lo
g(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d +
 B*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*
c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-
2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e +
 C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (-A*a*c*e - B*a*
c*d + C*a**2*e + x*(A*c**2*d - B*a*c*e - C*a*c*d))/(2*a**2*c**2 + 2*a*c**3
*x**2)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce\log(cx^2+a)}{2\,c^2} - \frac{Bacd-(Ca^2-Aac)e+(Bace+(Cac-Ac^2)d)x}{2\,(ac^3x^2+a^2c^2)} + \frac{(Bae+(Ca+Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*C*e*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d - (C*a^2 - A*a*c)*e + (B*a*c*e +
 (C*a*c - A*c^2)*d)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(B*a*e + (C*a + A*c)*d)
*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{Ce\log(cx^2+a)}{2\,c^2} + \frac{(Cad+Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}ac} - \frac{(Cad-Acd+Bae)x+\frac{Bacd-Ca^2e+Aace}{c}}{2\,(cx^2+a)ac}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a
*c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e
 + A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx = \frac{C\,e\,\ln(c\,x^2+a)}{2\,c^2} - \frac{B\,d}{2\,(c^2\,x^2+a\,c)} - \frac{B\,e\,x}{2\,(c^2\,x^2+a\,c)} - \frac{C\,d\,x}{2\,(c^2\,x^2+a\,c)} - \frac{A\,e}{2\,(c^2\,x^2+a\,c)} + \frac{C\,a\,e}{2\,(c^3\,x^2+a\,c^2)} + \frac{A\,d\,x}{2\,(a^2+c\,a\,x^2)} + \frac{A\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{c}} + \frac{B\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}} + \frac{C\,d\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,c^{3/2}}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

output
```
(C*e*log(a + c*x^2))/(2*c^2) - (B*d)/(2*(a*c + c^2*x^2)) - (B*e*x)/(2*(a*c
 + c^2*x^2)) - (C*d*x)/(2*(a*c + c^2*x^2)) - (A*e)/(2*(a*c + c^2*x^2)) + (
C*a*e)/(2*(a*c^2 + c^3*x^2)) + (A*d*x)/(2*(a^2 + a*c*x^2)) + (A*d*atan((c^
(1/2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) + (B*e*atan((c^(1/2)*x)/a^(1/2)))/(
2*a^(1/2)*c^(3/2)) + (C*d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

3.52. $\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^2}\,dx$

3.53 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx$

3.53.1 Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = -\frac{aB-(Ac-aC)x}{2ac\,(a+cx^2)} + \frac{(Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

output
```
1/2*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)+1/2*(A*c+C*a)*arctan(x*c^(1/2)/a^(1/2
))/a^(3/2)/c^(3/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \frac{-aB+Acx-aCx}{2ac\,(a+cx^2)} + \frac{(Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

output
```
(-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

3.53.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{Ac+aC}{c(cx^2+a)}dx}{2a}-\frac{aB-x(Ac-aC)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{Ac+aC}{c(cx^2+a)}dx}{2a}-\frac{aB-x(Ac-aC)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+Ac)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{aB-x(Ac-aC)}{2ac\,(a+cx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(aC+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\,(a+cx^2)}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

output
```
-1/2*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{(Ac-Ca)x}{2ac}-\frac{B}{2c}}{c\,x^2+a}+\frac{(Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2ac\sqrt{ac}}$	65
risch	$\frac{\frac{(Ac-Ca)x}{2ac}-\frac{B}{2c}}{c\,x^2+a}-\frac{A\ln(cx+\sqrt{-ac})}{4\sqrt{-ac}\,a}-\frac{\ln(cx+\sqrt{-ac})C}{4\sqrt{-ac}\,c}+\frac{A\ln(-cx+\sqrt{-ac})}{4\sqrt{-ac}\,a}+\frac{\ln(-cx+\sqrt{-ac})C}{4\sqrt{-ac}\,c}$	130

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2*(A*c+C*a)/a/c/(a*c)^(1/2)*arct
an(c*x/(a*c)^(1/2))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \left[-\frac{2\,Ba^2c+(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{-ac}\log\left(\frac{cx^2-2\sqrt{-ac}x-a}{cx^2+a}\right)+2\,(Ca^2c-Aac^2)x}{4\,(a^2c^3x^2+a^3c^2)}, -\frac{Ba^2c-(Ca^2+Aac+(Cac+Ac^2)x^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}x}{a}\right)+(Ca^2c-Aac^2)x}{2\,(a^2c^3x^2+a^3c^2)}\right]$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c
*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^
3*x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sq
rt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*
c^2)]
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3c^3}}(Ac+Ca)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c^3}}(Ac+Ca)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4} + \frac{-Ba+x(Ac-Ca)}{2a^2c+2ac^2x^2}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4
+ sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4
+ (-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = -\frac{Ba+(Ca-Ac)x}{2\,(ac^2x^2+a^2c)} + \frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*arctan(c*
x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}ac} - \frac{Cax-Acx+Ba}{2\,(cx^2+a)ac}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
 + B*a)/((c*x^2 + a)*a*c)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(A\,c+C\,a)}{2\,a^{3/2}\,c^{3/2}} - \frac{\frac{B}{2\,c}-\frac{x\,(A\,c-C\,a)}{2\,a\,c}}{c\,x^2+a}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^2,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(A*c + C*a))/(2*a^(3/2)*c^(3/2)) - (B/(2*c) - (
x*(A*c - C*a))/(2*a*c))/(a + c*x^2)
```

3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx$

3.54.1 Optimal result

Integrand size = 27, antiderivative size = 226

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = &-\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} \\ &+\frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd(cd^2+3ae^2)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}\left(cd^2+ae^2\right)^2} \\ &+\frac{e(Cd^2-Bde+Ae^2)\log(d+ex)}{\left(cd^2+ae^2\right)^2} \\ &-\frac{e(Cd^2-Bde+Ae^2)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^2}\end{aligned}$$

output
```
1/2*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c
*x^2+a)+e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2-1/2*e*(A*e^2-B*d*e
+C*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+1/2*(a*(-B*e+C*d)*(-a*e^2+c*d^2)+A*c*d
*(3*a*e^2+c*d^2))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2/c^(1/2
)
```

3.54.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^2}\,dx = \frac{\frac{(cd^2+ae^2)(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex))}{ac(a+cx^2)}+\frac{(a(Cd-Be)(cd^2-ae^2)+Acd(cd^2+3ae^2))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{c}}+2e(Cd^2+e(-Bd$$
$$\overline{\qquad 2(cd^2+ae^2)^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
(((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*
e*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2
+ 3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 +
e*(-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^
2])/(2*(c*d^2 + a*e^2)^2)
```

3.54.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2178, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2(d+ex)}\,dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{c(ad(Cd-Be)+A(cd^2+2ae^2)+e(Acd-aCd+aBe)x)}{(cd^2+ae^2)(d+ex)(cx^2+a)}dx}{2ac}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c(ad(Cd-Be)+A(cd^2+2ae^2)+e(Acd-aCd+aBe)x)}{(cd^2+ae^2)(d+ex)(cx^2+a)}dx}{2ac}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac(a+cx^2)(ae^2+cd^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ad(Cd-Be)+A(cd^2+2ae^2)+e(Acd-aCd+aBe)x}{(d+ex)(cx^2+a)}dx}{2a\left(ae^2+cd^2\right)}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 657}$$

$$\frac{\int \left(\frac{2a(Cd^2-Bed+Ae^2)e^2}{(cd^2+ae^2)(d+ex)}+\frac{a(Cd-Be)(cd^2-ae^2)+Acd(cd^2+3ae^2)-2ace(Cd^2-Bed+Ae^2)x}{(cd^2+ae^2)(cx^2+a)}\right)dx}{2a\left(ae^2+cd^2\right)}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd(3ae^2+cd^2)+a(cd^2-ae^2)(Cd-Be)\right)}{\sqrt{a}\sqrt{c}(ae^2+cd^2)}-\frac{ae\log(a+cx^2)\left(Ae^2-Bde+Cd^2\right)}{ae^2+cd^2}+\frac{2ae\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{ae^2+cd^2}}{2a\left(ae^2+cd^2\right)}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

output
```
-1/2*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
 + a*e^2)*(a + c*x^2)) + (((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
 3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2))
+ (2*a*e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2) - (a*e*(C*d
^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(c*d^2 + a*e^2))/(2*a*(c*d^2 + a*e^2))
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

method	result
default	$\frac{\frac{\frac{\left(Aacd\,e^{2}+A\,d^{3}c^{2}+a^{2}B\,e^{3}+Bac\,d^{2}e-C\,a^{2}d\,e^{2}-Cac\,d^{3}\right)x}{2a}+\frac{Aac\,e^{3}+A\,c^{2}d^{2}e-Bacd\,e^{2}-B\,c^{2}d^{3}-C\,a^{2}e^{3}-Cac\,d^{2}e}{2c}}{c\,x^{2}+a}+\frac{\left(-2Aac\,e^{3}+2Bacd\,e^{2}-2Ca\right.}{2c}}{\left(e^{2}a+c\,d^{2}\right)^{2}}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)^2*((1/2*(A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3+B*a*c*d^2*e-C*a^2
*d*e^2-C*a*c*d^3)/a*x+1/2*(A*a*c*e^3+A*c^2*d^2*e-B*a*c*d*e^2-B*c^2*d^3-C*a
^2*e^3-C*a*c*d^2*e)/c)/(c*x^2+a)+1/2/a*(1/2*(-2*A*a*c*e^3+2*B*a*c*d*e^2-2*
C*a*c*d^2*e)/c*ln(c*x^2+a)+(3*A*a*c*d*e^2+A*c^2*d^3+B*a^2*e^3-B*a*c*d^2*e-
C*a^2*d*e^2+C*a*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))+e*(A*e^2-B*d*
e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(212) = 424$.

Time = 21.75 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.53

$$\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^2}\,dx$$
$$= \left[-\frac{2\,Ba^2c^2d^3+2\,Ba^3cde^2+2\,(Ca^3c-Aa^2c^2)d^2e+2\,(Ca^4-Aa^3c)e^3-(Ba^2cd^2e-Ba^3e^3-(Ca^2c+A\cdots}{\cdots}\right.$$
$$-\frac{Ba^2c^2d^3+Ba^3cde^2+(Ca^3c-Aa^2c^2)d^2e+(Ca^4-Aa^3c)e^3+(Ba^2cd^2e-Ba^3e^3-(Ca^2c+Aac^2)d^3\cdots}{\cdots}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c^2*d^3 + 2*B*a^3*c*d*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d^2*e +
 2*(C*a^4 - A*a^3*c)*e^3 - (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*c + A*a*c^2
)*d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^
2 + A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 -
 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*
a^2*c^2 - A*a*c^3)*d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + 2*(C*a^3*c*d^2*e
 - B*a^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^
2*c^2*e^3)*x^2)*log(c*x^2 + a) - 4*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*
c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x +
 d))/(a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c
^3*d^2*e^2 + a^4*c^2*e^4)*x^2), -1/2*(B*a^2*c^2*d^3 + B*a^3*c*d*e^2 + (C*a
^3*c - A*a^2*c^2)*d^2*e + (C*a^4 - A*a^3*c)*e^3 + (B*a^2*c*d^2*e - B*a^3*e
^3 - (C*a^2*c + A*a*c^2)*d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e
- B*a^2*c*e^3 - (C*a*c^2 + A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*
sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*
c^2 - A*a*c^3)*d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + (C*a^3*c*d^2*e - B*a
^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*
e^3)*x^2)*log(c*x^2 + a) - 2*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3
+ (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(
a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*...
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx &= -\frac{(Cd^2e-Bde^2+Ae^3)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{(Cd^2e-Bde^2+Ae^3)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}\\ &\quad-\frac{(Bacd^2e-Ba^2e^3-(Cac+Ac^2)d^3+(Ca^2-3\,Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}\\ &\quad-\frac{Bacd+(Ca^2-Aac)e-(Bace-(Cac-Ac^2)d)x}{2\left(a^2c^2d^2+a^3ce^2+(ac^3d^2+a^2c^2e^2)x^2\right)}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*
e^2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - B*a^2*e^3 - (C*a*c + A*c^2)*d^3 + (C*a
^2 - 3*A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 +
 a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c*d + (C*a^2 - A*a*c)*e - (B*a*c*e - (C*a*
c - A*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx$$
$$= -\frac{(Cd^2e-Bde^2+Ae^3)\log(cx^2+a)}{2\,(c^2d^4+2\,acd^2e^2+a^2e^4)} + \frac{(Cd^2e^2-Bde^3+Ae^4)\log(|ex+d|)}{c^2d^4e+2\,acd^2e^3+a^2e^5}$$
$$+ \frac{(Cacd^3+Ac^2d^3-Bacd^2e-Ca^2de^2+3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,(ac^2d^4+2\,a^2cd^2e^2+a^3e^4)\sqrt{ac}}$$
$$- \frac{Bac^2d^3+Ca^2cd^2e-Aac^2d^2e+Ba^2cde^2+Ca^3e^3-Aa^2ce^3+(Cac^2d^3-Ac^3d^3-Bac^2d^2e+Ca^2cde^2}{2\,(cd^2+ae^2)^2(cx^2+a)ac}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
 a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(e*x + d))/(c^2*d^4*e + 2
*a*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2
*d*e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*
a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A
*a*c^2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*
c^3*d^3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/
((c*d^2 + a*e^2)^2*(c*x^2 + a)*a*c)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 1493, normalized size of antiderivative = 6.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)),x)
```

output
```
(log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4*c*e^5 -
 B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3
 + C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/
2) - 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^
4*d^5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^
3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^3*(-a^3*c)^(
1/2) - 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*
d*e^4*x - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^
4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^
3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*(-a^3*c)
^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^3*c
)^(1/2) - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(a^2*((B*e^3*(-a^3*c)^(1/2))
/4 - (C*d*e^2*(-a^3*c)^(1/2))/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^
2*e)/2) - a*((C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*
d^2*e*(-a^3*c)^(1/2))/4)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^
3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5
*(-a^3*c)^(1/2) - 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*
a^3*c^2*d^4*e - 8*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)
^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/2) + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d
^3*e^2 + 8*B*a^4*c*d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B...
```

3.54. $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^2}\,dx$

3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2}\,dx$

3.55.1 Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2}\,dx = -\frac{e(Cd^2-Bde+Ae^2)}{(cd^2+ae^2)^2(d+ex)} - \frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))x}{2a(cd^2+ae^2)^2(a+cx^2)} + \frac{(Ac(c^2d^4+6acd^2e^2-3a^2e^4)+a(a^2Ce^4+c^2d^3(Cd-2Be)-6acde^2(Cd-Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}(cd^2+ae^2)^3} - \frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))\log(d+ex)}{(cd^2+ae^2)^3} + \frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))\log(a+cx^2)}{2(cd^2+ae^2)^3}$$

output
```
-e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)+1/2*(-a*(-2*A*c*d*e-B*a*e^2
+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a
/(a*e^2+c*d^2)^2/(c*x^2+a)-e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*A*e+3*
B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*
(-4*A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+1/2*(A*c*(-3*a^2*e^4+6*a*c*d^
2*e^2+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*e+C*d)-6*a*c*d*e^2*(-B*e+C*d)))*
arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3/c^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$$
$$=\frac{-\frac{2e(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{(cd^2+ae^2)(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)}+\frac{(Ac(c^2d^4+6a}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

output
```
((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a
*e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d
- 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^
2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-
(C*d) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C
*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*
c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(
2*(c*d^2 + a*e^2)^3)
```

3.55.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2(d+ex)^2}\,dx$$
$$\Big\downarrow \; 2178$$
$$-\frac{\int -\frac{\frac{ce^2\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^2}+\frac{2ce(Acd-aCd+aBe)x}{cd^2+ae^2}+\frac{c\left(A\left(c^2d^4+5ace^2d^2+2a^2e^4\right)-ad^2\left(aCe^2-cd(Cd-2Be)\right)\right)}{\left(cd^2+ae^2\right)^2}}{(d+ex)^2(cx^2+a)}dx}{2ac}-$$
$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{ce^2\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^2}+\frac{2ce(Acd-aCd+aBe)x}{cd^2+ae^2}+\frac{c\left(A\left(c^2d^4+5ace^2d^2+2a^2e^4\right)-ad^2\left(aCe^2-cd(Cd-2Be)\right)\right)}{\left(cd^2+ae^2\right)^2}}{(d+ex)^2(cx^2+a)}dx}{2ac}-\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\int \left(\frac{2ac(2cCd^3-ce(3Bd-4Ae)d-ae^2(2Cd-Be))e^2}{(cd^2+ae^2)^3(d+ex)}+\frac{2ac(Cd^2-Bed+Ae^2)e^2}{(cd^2+ae^2)^2(d+ex)^2}+\frac{c(Ac(c^2d^4+6ace^2d^2-3a^2e^4)+a(a^2Ce^4-6acd(Cd-Be)e^2+c}{(cd^2+ac}\right.}{2ac}$$

$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac(-3a^2e^4+6acd^2e^2+c^2d^4)+a(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be))\right)}{\sqrt{a}(ae^2+cd^2)^3}-\frac{2ace(Ae^2-Bde+Cd^2)}{(d+ex)(ae^2+cd^2)^2}-\frac{ace\log(a+cx^2)(-c}{}}{2ac}$$

$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

output
```
-1/2*(a*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2)
 + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*x)/(a*(c*d^2 + a*e^2)^2*(a + c*x^2)) +
 ((-2*a*c*e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt
[c]*(A*c*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C
*d - 2*B*e) - 6*a*c*d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*(c*d^2 + a*e^2)^3) + (2*a*c*e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e
^2*(2*C*d - B*e))*Log[d + e*x])/(c*d^2 + a*e^2)^3 - (a*c*e*(2*c*C*d^3 - c*
d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(c*d^2 + a*e^2)
^3)/(2*a*c)
```

3.55. $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2}\,dx$

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\frac{\left(A\,a^{2} c\,e^{4}-A\,d^{4} c^{3}-2 B\,a^{2} c d\,e^{3}-2 B a\,c^{2} d^{3} e-C\,a^{3} e^{4}+C a\,c^{2} d^{4}\right) x}{2 a}-A a c d\,e^{3}-A\,c^{2} d^{3} e-\frac{B\,e^{4} a^{2}}{2}+\frac{B\,c^{2} d^{4}}{2}+C\,a^{2} d\,e^{3}+C a c\,d^{3} e}{c\,x^{2}+a}+\frac{\left(8 A a\,c^{2} d\,e^{3}+2 B \right.}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2+c*d^2)^3*((1/2*(A*a^2*c*e^4-A*c^3*d^4-2*B*a^2*c*d*e^3-2*B*a*c^2*
d^3*e-C*a^3*e^4+C*a*c^2*d^4)/a*x-A*a*c*d*e^3-A*c^2*d^3*e-1/2*B*e^4*a^2+1/2
*B*c^2*d^4+C*a^2*d*e^3+C*a*c*d^3*e)/(c*x^2+a)+1/2/a*(1/2*(8*A*a*c^2*d*e^3+
2*B*a^2*c*e^4-6*B*a*c^2*d^2*e^2-4*C*a^2*c*d*e^3+4*C*a*c^2*d^3*e)/c*ln(c*x^
2+a)+(3*A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-A*c^3*d^4-6*B*a^2*c*d*e^3+2*B*a*c^2*
d^3*e-C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/(a*c)^(1/2)*arctan(c*x/(a*c
)^(1/2))))+e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d^2*e-2*C*a*d*e^2+2*C*c*d^3)/(a*e^
2+c*d^2)^3*ln(e*x+d)-e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(360) = 720$.

Time = 130.34 (sec) , antiderivative size = 2916, normalized size of antiderivative = 7.80

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*B*a^2*c^3*d^5 - 4*B*a^3*c^2*d^3*e^2 + 8*C*a^4*c*d^2*e^3 - 6*B*a^4
*c*d*e^4 + 4*A*a^4*c*e^5 + 4*(2*C*a^3*c^2 - A*a^2*c^3)*d^4*e - 2*(4*B*a^2*
c^3*d^3*e^2 + 4*B*a^3*c^2*d*e^4 - (3*C*a^2*c^3 - A*a*c^4)*d^4*e - 2*(C*a^3
*c^2 + A*a^2*c^3)*d^2*e^3 + (C*a^4*c - 3*A*a^3*c^2)*e^5)*x^2 + (2*B*a^2*c^
2*d^4*e - 6*B*a^3*c*d^2*e^3 - (C*a^2*c^2 + A*a*c^3)*d^5 + 6*(C*a^3*c - A*a
^2*c^2)*d^3*e^2 - (C*a^4 - 3*A*a^3*c)*d*e^4 + (2*B*a*c^3*d^3*e^2 - 6*B*a^2
*c^2*d*e^4 - (C*a*c^3 + A*c^4)*d^4*e + 6*(C*a^2*c^2 - A*a*c^3)*d^2*e^3 - (
C*a^3*c - 3*A*a^2*c^2)*e^5)*x^3 + (2*B*a*c^3*d^4*e - 6*B*a^2*c^2*d^2*e^3 -
 (C*a*c^3 + A*c^4)*d^5 + 6*(C*a^2*c^2 - A*a*c^3)*d^3*e^2 - (C*a^3*c - 3*A*
a^2*c^2)*d*e^4)*x^2 + (2*B*a^2*c^2*d^3*e^2 - 6*B*a^3*c*d*e^4 - (C*a^2*c^2
+ A*a*c^3)*d^4*e + 6*(C*a^3*c - A*a^2*c^2)*d^2*e^3 - (C*a^4 - 3*A*a^3*c)*e
^5)*x)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2
*c^3*d^4*e + 2*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - (C*a^2*c^3 - A*a*c^4)*d^5
 - 2*(C*a^3*c^2 - A*a^2*c^3)*d^3*e^2 - (C*a^4*c - A*a^3*c^2)*d*e^4)*x + 2*
(2*C*a^3*c^2*d^4*e - 3*B*a^3*c^2*d^3*e^2 + B*a^4*c*d*e^4 - 2*(C*a^4*c - 2*
A*a^3*c^2)*d^2*e^3 + (2*C*a^2*c^3*d^3*e^2 - 3*B*a^2*c^3*d^2*e^3 + B*a^3*c^
2*e^5 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d*e^4)*x^3 + (2*C*a^2*c^3*d^4*e - 3*B*
a^2*c^3*d^3*e^2 + B*a^3*c^2*d*e^4 - 2*(C*a^3*c^2 - 2*A*a^2*c^3)*d^2*e^3)*x
^2 + (2*C*a^3*c^2*d^3*e^2 - 3*B*a^3*c^2*d^2*e^3 + B*a^4*c*e^5 - 2*(C*a^4*c
 - 2*A*a^3*c^2)*d*e^4)*x)*log(c*x^2 + a) - 4*(2*C*a^3*c^2*d^4*e - 3*B*a...
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$$
$$= -\frac{\left(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3\right)\log\left(cx^2+a\right)}{2\,\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)}$$
$$+\frac{\left(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3\right)\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6}$$
$$-\frac{\left(2\,Bac^2d^3e-6\,Ba^2cde^3-(Cac^2+Ac^3)d^4+6\,(Ca^2c-Aac^2)d^2e^2-(Ca^3-3\,Aa^2c)e^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\left(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6\right)\sqrt{ac}}$$
$$-\frac{Bacd^3-3\,Ba^2de^2+2\,Aa^2e^3+2\,(2\,Ca^2-Aac)d^2e-(4\,Bacde^2-(3\,Cac-Ac^2)d^2e+(Ca^2-3\,Aac)e}{2\,(a^2c^2d^5+2\,a^3cd^3e^2+a^4de^4+(ac^3d^4e+2\,a^2c^2d^2e^3+a^3ce^5)x^3+(ac^3d^5+2\,a^2c^2d^3e^2}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(c
*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C*c
*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(e*x + d)/(c^
3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*d^3*
e - 6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 + 6*(C*a^2*c - A*a*c^2)*d^2*e^
2 - (C*a^3 - 3*A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2
*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 3*B*a^
2*d*e^2 + 2*A*a^2*e^3 + 2*(2*C*a^2 - A*a*c)*d^2*e - (4*B*a*c*d*e^2 - (3*C*
a*c - A*c^2)*d^2*e + (C*a^2 - 3*A*a*c)*e^3)*x^2 - (B*a*c*d^2*e + B*a^2*e^3
 - (C*a*c - A*c^2)*d^3 - (C*a^2 - A*a*c)*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*
d^3*e^2 + a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 +
(a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3
*c*d^2*e^3 + a^4*e^5)*x)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$$
$$= -\frac{(2\,Ccd^3e-3\,Bcd^2e^2-2\,Cade^3+4\,Acde^3+Bae^4)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6)}$$
$$-\frac{\frac{Cd^2e^5}{ex+d}-\frac{Bde^6}{ex+d}+\frac{Ae^7}{ex+d}}{c^2d^4e^4+2\,acd^2e^6+a^2e^8}$$
$$+\frac{(Cac^2d^4e^2+Ac^3d^4e^2-2\,Bac^2d^3e^3-6\,Ca^2cd^2e^4+6\,Aac^2d^2e^4+6\,Ba^2cde^5+Ca^3e^6-3\,Aa^2ce^6)\,\mathrm{arcta}}{2\,(ac^3d^6+3\,a^2c^2d^4e^2+3\,a^3cd^2e^4+a^4e^6)\sqrt{ac}e^2}$$
$$-\frac{\frac{Cac^2d^3e-Ac^3d^3e-3\,Bac^2d^2e^2-3\,Ca^2cde^3+3\,Aac^2de^3+Ba^2ce^4}{cd^2+ae^2}-\frac{Cac^2d^4e^2-Ac^3d^4e^2-4\,Bac^2d^3e^3-6\,Ca^2cd^2e^4+6\,Aac^2d^2e^4+4\,Ba^2cde^5}{(cd^2+ae^2)(ex+d)e}}{2\,(cd^2+ae^2)^2a\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*l
og(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 +
 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(e*x + d) - B*d
*e^6/(e*x + d) + A*e^7/(e*x + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8)
+ 1/2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2
*e^4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*ar
ctan((c*d - c*d^2/(e*x + d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((a*c^3*d^6
+ 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)*e^2) - 1/2*((C*
a*c^2*d^3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^
2*d*e^3 + B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2
- 4*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*
e^5 + C*a^3*e^6 - A*a^2*c*e^6)/((c*d^2 + a*e^2)*(e*x + d)*e))/((c*d^2 + a*
e^2)^2*a*(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2))
```

3.55.9 Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 2094, normalized size of antiderivative = 5.60

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2),x)
```

output
```
((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2
*e))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*
B*a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d +
 B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3)
- (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3
*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3
*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*
c)^(3/2) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3
*A*a^4*c^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3
*c^3*d^3*e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*
d^3*e^3 - 14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*
a^3*c^3*d^2*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C
*a^4*c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e
^4*(-a^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4
*e^2*(-a^3*c)^(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c
^3*d^5*e*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^
2*e^4*x*(-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^
3*d^5*e*x*(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*
c^2*d*e^5*x*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*
((C*d^4*(-a^3*c)^(1/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(...
```

3.55. $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^2}\,dx$

3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^2}\,dx$

3.56.1 Optimal result

Integrand size = 27, antiderivative size = 524

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx \\
&= -\frac{e(Cd^2-Bde+Ae^2)}{2\,(cd^2+ae^2)^2\,(d+ex)^2}+\frac{e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))}{(cd^2+ae^2)^3\,(d+ex)} \\
&\quad -\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{2a\,(cd^2+ae^2)^3\,(a+cx^2)} \\
&\quad +\frac{\sqrt{c}(Acd(c^2d^4+10acd^2e^2-15a^2e^4)-a(2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)-3a^2e^4(3Cd-Be)))}{2a^{3/2}\,(cd^2+ae^2)^4} \\
&\quad +\frac{e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(d+ex)}{(cd^2+ae^2)^4} \\
&\quad -\frac{e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))\log(a+cx^2)}{2\,(cd^2+ae^2)^4}
\end{aligned}$$

output
```
-1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2+e*(a*e^2*(-B*e+2*C*d)
-c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))/(a*e^2+c*d^2)^3/(e*x+d)+1/2*(-a*(B*c*d*(-
3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*(
c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)+e*(
a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+
3*B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(
-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^
2)^4+1/2*(A*c*d*(-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)-a*(2*a*c*d^2*e^2*(-9*
B*e+7*C*d)-c^2*d^4*(-3*B*e+C*d)-3*a^2*e^4*(-B*e+3*C*d)))*arctan(x*c^(1/2)/
a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^4
```

3.56.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx$$
$$= \frac{-\frac{e(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{(d+ex)^2} - \frac{2e(cd^2+ae^2)(2cCd^3+cde(-3Bd+4Ae)+ae^2(-2Cd+Be))}{d+ex} + \frac{(cd^2+ae^2)(a^3Ce^3+Ac^3d^3x-ac^2d(Cd^2}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

output
```
(-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c
*d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))
/(d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x
+ B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B
*d + A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d
^2*e^2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d -
3*B*e) - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) +
 2*(a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^
2 + 2*e*(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2
+ e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a +
c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

3.56.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^2 (d+ex)^3} \, dx$$

$$\Big\downarrow \; 2178$$

$$-\frac{\int -\frac{\frac{c^2e^3\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x^3}{\left(cd^2+ae^2\right)^3}+\frac{ce^2\left(Ac\left(3c^2d^4-3ace^2d^2-2a^2e^4\right)+a\left(2a^2Ce^4+3acd(Cd+Be)e^2-c^2d^3(3Cd-7Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^3}+\cdots}{(d+\cdots}}{\cdots}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{c^2e^3\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x^3}{\left(cd^2+ae^2\right)^3}+\frac{ce^2\left(Ac\left(3c^2d^4-3ace^2d^2-2a^2e^4\right)+a\left(2a^2Ce^4+3acd(Cd+Be)e^2-c^2d^3(3Cd-7Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^3}+\frac{ce\left(\cdots\right.}{\cdots}}{(d+ex)^3(\cdots}}{2a\cdots}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow \; 2160$$

$$\int \left(\frac{\left(Acd\left(c^2d^4+10ace^2d^2-15a^2e^4\right)-a\left(-c^2(Cd-3Be)d^4+2ace^2(7Cd-9Be)d^2-3a^2e^4(3Cd-Be)\right)-2ae\left(a^2Ce^4-2ac\left(4Cd^2-e(3Bd-Ae)\right)e^2+c^2\left(\cdots\right.\right.\right.}{\left(cd^2+ae^2\right)^4\left(cx^2+a\right)}\cdots\right.$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(-15a^2e^4+10acd^2e^2+c^2d^4\right)-a\left(-3a^2e^4(3Cd-Be)+2acd^2e^2(7Cd-9Be)-c^2d^4(Cd-3Be)\right)\right)}{\sqrt{a}\left(ae^2+cd^2\right)^4}-\frac{ace\log\left(a+cx^2\right)\left(a^2Ce^4-2\cdots\right.}{\cdots}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{2a\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

output
```
-1/2*(a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c*(A
*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*x)
/(a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (-((a*c*e*(C*d^2 - B*d*e + A*e^2))/((
c*d^2 + a*e^2)^2*(d + e*x)^2)) - (2*a*c*e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*
e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d + e*x)) + (c^(3/2)*(A*c*d
*(c^2*d^4 + 10*a*c*d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e
) - c^2*d^4*(C*d - 3*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^4) + (2*a*c*e*(a^2*C*e^4 + c^2*(3*C*d^4
- 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d
+ e*x])/(c*d^2 + a*e^2)^4 - (a*c*e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*
B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(c*
d^2 + a*e^2)^4)/(2*a*c)
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.23

method	result
default	$-\frac{c\left(\frac{\frac{\left(3A\,a^2cd\,e^4+2Aa\,c^2d^3e^2-d^5A\,c^3+B\,e^5a^3-2B\,a^2c\,d^2e^3-3Ba\,c^2d^4e-3C\,a^3d\,e^4-2C\,a^2c\,d^3e^2+C\,c^2d^5a\right)x}{2a}+\frac{A\,a^2c\,e^5-2Aa\,c^2d^2e^3-3A\,c^3d}{}}{c\,x^2+a}\right.}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)^4*((1/2*(3*A*a^2*c*d*e^4+2*A*a*c^2*d^3*e^2-A*c^3*d^5+B*a^
3*e^5-2*B*a^2*c*d^2*e^3-3*B*a*c^2*d^4*e-3*C*a^3*d*e^4-2*C*a^2*c*d^3*e^2+C*
a*c^2*d^5)/a*x+1/2*(A*a^2*c*e^5-2*A*a*c^2*d^2*e^3-3*A*c^3*d^4*e-3*B*a^2*c*
d*e^4-2*B*a*c^2*d^3*e^2+B*c^3*d^5-C*a^3*e^5+2*C*a^2*c*d^2*e^3+3*C*a*c^2*d^
4*e)/c)/(c*x^2+a)+1/2/a*(1/2*(-4*A*a^2*c*e^5+20*A*a*c^2*d^2*e^3+12*B*a^2*c
*d*e^4-12*B*a*c^2*d^3*e^2+2*C*a^3*e^5-16*C*a^2*c*d^2*e^3+6*C*a*c^2*d^4*e)/
c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4-10*A*a*c^2*d^3*e^2-A*c^3*d^5+3*B*a^3*e^5-1
8*B*a^2*c*d^2*e^3+3*B*a*c^2*d^4*e-9*C*a^3*d*e^4+14*C*a^2*c*d^3*e^2-C*a*c^2
*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))-e*(4*A*c*d*e^2+B*a*e^3-3*B*c*d
^2*e-2*C*a*d*e^2+2*C*c*d^3)/(a*e^2+c*d^2)^3/(e*x+d)-1/2*e*(A*e^2-B*d*e+C*d
^2)/(a*e^2+c*d^2)^2/(e*x+d)^2-e*(2*A*a*c*e^4-10*A*c^2*d^2*e^2-6*B*a*c*d*e^
3+6*B*c^2*d^3*e-C*a^2*e^4+8*C*a*c*d^2*e^2-3*C*c^2*d^4)/(a*e^2+c*d^2)^4*ln(
e*x+d)
```

3.56.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

output
```
Timed out
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. 2(505) = 1010.

Time = 0.32 (sec) , antiderivative size = 1030, normalized size of antiderivative = 1.97

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx=$$
$$-\frac{(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\,(4\,Cac-5\,Ac^2)d^2e^3+(Ca^2-2\,Aac)e^5)\log\left(cx^2+a\right)}{2\,(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8)}$$
$$+\frac{(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\,(4\,Cac-5\,Ac^2)d^2e^3+(Ca^2-2\,Aac)e^5)\log\left(ex+d\right)}{c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8}$$
$$-\frac{(3\,Bac^3d^4e-18\,Ba^2c^2d^2e^3+3\,Ba^3ce^5-(Cac^3+Ac^4)d^5+2\,(7\,Ca^2c^2-5\,Aac^3)d^3e^2-3\,(3\,Ca^3c-5\,A}{2\,(ac^4d^8+4\,a^2c^3d^6e^2+6\,a^3c^2d^4e^4+4\,a^4cd^2e^6+a^5e^8)\sqrt{ac}}$$
$$-\frac{Bac^2d^5-10\,Ba^2cd^3e^2+Ba^3de^4+Aa^3e^5+(8\,Ca^2c-3\,Aac^2)d^4e-2\,(2\,Ca^3-5\,Aa^2c)d^2e^3-(9\,Bac^2}{2\,(a^2c^3d^8+3\,a^3c^2d^6e^2+3\,a^4cd^4e^4+a^5d^2e^6}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c
^2)*d^2*e^3 + (C*a^2 - 2*A*a*c)*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6
*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e - 6
*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^2)*d^2*e^3 + (C*a^2 -
2*A*a*c)*e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4
+ 4*a^3*c*d^2*e^6 + a^4*e^8) - 1/2*(3*B*a*c^3*d^4*e - 18*B*a^2*c^2*d^2*e^3
 + 3*B*a^3*c*e^5 - (C*a*c^3 + A*c^4)*d^5 + 2*(7*C*a^2*c^2 - 5*A*a*c^3)*d^3
*e^2 - 3*(3*C*a^3*c - 5*A*a^2*c^2)*d*e^4)*arctan(c*x/sqrt(a*c))/((a*c^4*d^
8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqr
t(a*c)) - 1/2*(B*a*c^2*d^5 - 10*B*a^2*c*d^3*e^2 + B*a^3*d*e^4 + A*a^3*e^5
+ (8*C*a^2*c - 3*A*a*c^2)*d^4*e - 2*(2*C*a^3 - 5*A*a^2*c)*d^2*e^3 - (9*B*a
*c^2*d^2*e^3 - 3*B*a^2*c*e^5 - (5*C*a*c^2 - A*c^3)*d^3*e^2 + (7*C*a^2*c -
11*A*a*c^2)*d*e^4)*x^3 - (12*B*a*c^2*d^3*e^2 - (7*C*a*c^2 - 2*A*c^3)*d^4*e
 + 6*(C*a^2*c - 2*A*a*c^2)*d^2*e^3 + (C*a^3 - 2*A*a^2*c)*e^5)*x^2 - (B*a*c
^2*d^4*e + 11*B*a^2*c*d^2*e^3 - 2*B*a^3*e^5 - (C*a*c^2 - A*c^3)*d^5 - (7*C
*a^2*c - 3*A*a*c^2)*d^3*e^2 + 2*(3*C*a^3 - 5*A*a^2*c)*d*e^4)*x)/(a^2*c^3*d
^8 + 3*a^3*c^2*d^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 +
3*a^2*c^3*d^4*e^4 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e +
3*a^2*c^3*d^5*e^3 + 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*
a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + ...
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(505) = 1010$.

Time = 0.27 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx =$$
$$-\frac{(3\,Cc^2d^4e-6\,Bc^2d^3e^2-8\,Cacd^2e^3+10\,Ac^2d^2e^3+6\,Bacde^4+Ca^2e^5-2\,Aace^5)\log\left(cx^2+a\right)}{2\,(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8)}$$
$$+\frac{(3\,Cc^2d^4e^2-6\,Bc^2d^3e^3-8\,Cacd^2e^4+10\,Ac^2d^2e^4+6\,Bacde^5+Ca^2e^6-2\,Aace^6)\log\left(|ex+d|\right)}{c^4d^8e+4\,ac^3d^6e^3+6\,a^2c^2d^4e^5+4\,a^3cd^2e^7+a^4e^9}$$
$$+\frac{(Cac^3d^5+Ac^4d^5-3\,Bac^3d^4e-14\,Ca^2c^2d^3e^2+10\,Aac^3d^3e^2+18\,Ba^2c^2d^2e^3+9\,Ca^3cde^4-15\,Aa^2c^2d\ldots}{2\,(ac^4d^8+4\,a^2c^3d^6e^2+6\,a^3c^2d^4e^4+4\,a^4cd^2e^6+a^5e^8)\sqrt{ac}}$$
$$-\frac{Bac^3d^7+8\,Ca^2c^2d^6e-3\,Aac^3d^6e-9\,Ba^2c^2d^5e^2+4\,Ca^3cd^4e^3+7\,Aa^2c^2d^4e^3-9\,Ba^3cd^3e^4-4\,Ca^4d^2e\ldots}{\ldots}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
 + 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*
c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^
4*e^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e
^5 + C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(e*x + d))/(c^4*d^8*e + 4*a*c^3*d^6*e
^3 + 6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A
*c^4*d^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 1
8*B*a^2*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5
)*arctan(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^
4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2
*d^6*e - 3*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a
^2*c^2*d^4*e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5
+ B*a^4*d*e^6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3
*d^4*e^3 - 2*C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5
- 7*C*a^3*c*d*e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d
^6*e - 2*A*c^4*d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3
*d^4*e^3 - 12*B*a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5
 - C*a^4*e^7 + 2*A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6
*e + 8*C*a^2*c^2*d^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^
3*c*d^3*e^4 + 7*A*a^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 +...
```

3.56.9 Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 2828, normalized size of antiderivative = 5.40

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^3),x)
```

output
```
(log(C*c^2*d^7*(-a^3*c)^(3/2) - 3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 +
 12*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e
 + 72*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x + 39*A*a^2*d*e^6
*(-a^3*c)^(3/2) + 21*C*a^6*d*e^6*(-a^3*c)^(1/2) - 3*B*c^2*d^6*e*(-a^3*c)^(
3/2) + 12*A*a^2*e^7*x*(-a^3*c)^(3/2) + 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A
*a^5*c^3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^
6*c^2*d^3*e^4 - 94*C*a^6*c^2*d^4*e^3 - A*a^2*c^4*d^7*(-a^3*c)^(1/2) - 93*B
*a^2*d^2*e^5*(-a^3*c)^(3/2) + 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) + 119*C*a^2*d
^3*e^4*(-a^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a
^5*c^3*d^3*e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*
a^5*c^3*d^5*e^2*x - 119*C*a^6*c^2*d^3*e^4*x + 80*A*c^2*d^4*e^3*x*(-a^3*c)^
(3/2) + 72*C*a^2*d^2*e^5*x*(-a^3*c)^(3/2) - 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3
/2) + 21*C*a^7*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x - 14
5*A*a*c*d^3*e^4*(-a^3*c)^(3/2) + 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) - 51*C*a*
c*d^5*e^2*(-a^3*c)^(3/2) - 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) + 20*C*c^2*d^6*
e*x*(-a^3*c)^(3/2) - 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) + 108*B*a*c*d^3*e^
4*x*(-a^3*c)^(3/2) - 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^6*e
*x*(-a^3*c)^(1/2))*(e^2*(3*B*a^3*c^2*d^3 + (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/
2 - (7*C*a^2*c*d^3*(-a^3*c)^(1/2))/2) + e^3*(4*C*a^4*c*d^2 - 5*A*a^3*c^2*d
^2 + (9*B*a^2*c*d^2*(-a^3*c)^(1/2))/2) - e^4*(3*B*a^4*c*d - (9*C*a^3*d*...
```

3.56. $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^2}\,dx$

3.57 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.57.1 Optimal result

Integrand size = 27, antiderivative size = 209

$$\begin{aligned}\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac(a+cx^2)^2}\\ &\quad-\frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2-a(4aCe^2-cd(Cd+3Be))\right)x\right)}{8a^2c^2(a+cx^2)}\\ &\quad+\frac{\left(3ae^2(Acd+3aCd+aBe)+cd^2(3Acd+aCd+3aBe)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}\\ &\quad+\frac{Ce^3\log(a+cx^2)}{2c^3}\end{aligned}$$

output
```
-1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*e*(3*A*c*d
+3*B*a*e+5*C*a*d)-(3*A*c^2*d^2-a*(4*a*C*e^2-c*d*(3*B*e+C*d)))*x)/a^2/c^2/(
c*x^2+a)+1/8*(3*A*c*d*(a*e^2+c*d^2)+a*(3*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*
d)))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)+1/2*C*e^3*ln(c*x^2+a)/c^3
```

3.57.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$$
$$= \frac{\frac{-2a^3Ce^3+2Ac^3d^3x-2ac^2d\left(Cd^2x+3Ae(d+ex)+Bd(d+3ex)\right)+2a^2ce(3Cd(d+ex)+e(3Bd+Ae+Bex))}{a(a+cx^2)^2}+\frac{8a^3Ce^3+3Ac^3d^3x+ac^2d\left(Cd^2+3e(B\right.}{}}{}$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x)) + B*
d*(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a
*(a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d
 + A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))
/(a^2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d
 + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C
*e^3*Log[a + c*x^2])/(8*c^3)
```

3.57.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 25, 684, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$$
$$\Big\downarrow \text{2176}$$
$$-\frac{\int -\frac{(d+ex)^2(3Acd+aCd+3aBe+4aCex)}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{(d+ex)^2(3Acd+aCd+3aBe+4aCex)}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \; 684$$

$$\frac{\frac{\int \frac{8a^2Cxe^3+3a(Acd+3aCd+aBe)e^2+cd^2(3Acd+aCd+3aBe)}{cx^2+a}dx}{2ac}-\frac{(d+ex)(x(4a^2Ce^2-cd(3aBe+aCd+3Acd))+ae(3aBe+5aCd+3Acd))}{2ac(a+cx^2)}}{4ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \; 452$$

$$\frac{\frac{8a^2Ce^3\int \frac{x}{cx^2+a}dx+(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd))\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(x(4a^2Ce^2-cd(3aBe+aCd+3Acd))+ae(3aBe+5}{2ac(a+cx^2)}}{4ac}\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{8a^2Ce^3\int \frac{x}{cx^2+a}dx+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd))}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)(x(4a^2Ce^2-cd(3aBe+aCd+3Acd))+ae(3aBe+5aCd}{2ac(a+cx^2)}}{4ac}\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{\frac{4a^2Ce^3\log(a+cx^2)}{c}+\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd))}{\sqrt{a}\sqrt{c}}}{2ac}-\frac{(d+ex)(x(4a^2Ce^2-cd(3aBe+aCd+3Acd))+ae(3aBe+5aCd}{2ac(a+cx^2)}}{4ac}\frac{(d+ex)^3(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

input
```
Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^2) + (-1/2*((d +
 e*x)*(a*e*(3*A*c*d + 5*a*C*d + 3*a*B*e) + (4*a^2*C*e^2 - c*d*(3*A*c*d + a
*C*d + 3*a*B*e))*x))/(a*c*(a + c*x^2)) + (((3*a*e^2*(A*c*d + 3*a*C*d + a*B
*e) + c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqr
t[a]*Sqrt[c]) + (4*a^2*C*e^3*Log[a + c*x^2])/c)/(2*a*c))/(4*a*c)
```

3.57. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.57. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.57.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.59

method	result
default	$\frac{\frac{\left(3Aacd\,e^2+3A\,d^3c^2-5a^2B\,e^3+3Bac\,d^2e-15C\,a^2d\,e^2+Cac\,d^3\right)x^3}{8c\,a^2}-\frac{e\left(Ac\,e^2+3Bcde-2aC\,e^2+3Cc\,d^2\right)x^2}{2c^2}-\frac{\left(3Aacd\,e^2-5A\,d^3c^2+3a^2B\,e^3+3Bac}{8a\,c^2}}{\left(c\,x^2+a\right)^2}$
risch	$\frac{\frac{\left(3Aacd\,e^2+3A\,d^3c^2-5a^2B\,e^3+3Bac\,d^2e-15C\,a^2d\,e^2+Cac\,d^3\right)x^3}{8c\,a^2}-\frac{e\left(Ac\,e^2+3Bcde-2aC\,e^2+3Cc\,d^2\right)x^2}{2c^2}-\frac{\left(3Aacd\,e^2-5A\,d^3c^2+3a^2B\,e^3+3Bac}{8a\,c^2}}{\left(c\,x^2+a\right)^2}$

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C
*a*c*d^3)/c/a^2*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-
1/8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a
*c*d^3)/a/c^2*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a
^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/8/a^2/c^2*(4*C*a^2*e^3/c*ln(c*x^2
+a)+(3*A*a*c*d*e^2+3*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a
*c*d^3)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2)))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(198) = 396$.

Time = 0.48 (sec) , antiderivative size = 1138, normalized size of antiderivative = 5.44

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 12*(C*a^4*c + A*a^3*c^2)*d^2*
e - 4*(3*C*a^5 - A*a^4*c)*e^3 - 2*(3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (
C*a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 8*(3
*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 +
(3*B*a^3*c*d^2*e + 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a
*c^3 + 3*A*c^4)*d^3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*
A*a^2*c^2)*d^3 + 3*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*
a^3*c*e^3 + (C*a^2*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)
*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(3*B*a^
3*c^2*d^2*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3*C*a^4*c
 + A*a^3*c^2)*d*e^2)*x - 8*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 + C*a^5*
e^3)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^
3*c^2*d^3 + 6*B*a^4*c*d*e^2 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e - 2*(3*C*a^5 -
 A*a^4*c)*e^3 - (3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*a^2*c^3 + 3*A*a*
c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 4*(3*C*a^3*c^2*d^2*e +
 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 - (3*B*a^3*c*d^2*e +
 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^
3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3
*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2
*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sqrt(a*c)...
```

3.57.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3} - \frac{2\,Ba^2c^2d^3 + 6\,Ba^3cde^2 + 6\,(Ca^3c + Aa^2c^2)d^2e - 2\,(3\,Ca^4 - Aa^3c)e^3 - (3\,Bac^3d^2e - 5\,Ba^2c^2e^3 + (Cac}{} + \frac{(3\,Bacd^2e + 3\,Ba^2e^3 + (Cac + 3\,Ac^2)d^3 + 3\,(3\,Ca^2 + Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*C*e^3*log(c*x^2 + a)/c^3 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 6*
(C*a^3*c + A*a^2*c^2)*d^2*e - 2*(3*C*a^4 - A*a^3*c)*e^3 - (3*B*a*c^3*d^2*e
 - 5*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 - 3*(5*C*a^2*c^2 - A*a*c^3)*d
*e^2)*x^3 + 4*(3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*
c^2)*e^3)*x^2 + (3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 - 5*A*a*c^
3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2
+ a^4*c^3) + 1/8*(3*B*a*c*d^2*e + 3*B*a^2*e^3 + (C*a*c + 3*A*c^2)*d^3 + 3*
(3*C*a^2 + A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3} + \frac{(Cacd^3 + 3\,Ac^2d^3 + 3\,Bacd^2e + 9\,Ca^2de^2 + 3\,Aacde^2 + 3\,Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2} + \frac{(Cac^2d^3 + 3\,Ac^3d^3 + 3\,Bac^2d^2e - 15\,Ca^2cde^2 + 3\,Aac^2de^2 - 5\,Ba^2ce^3)x^3 - 4\,(3\,Ca^2cd^2e + 3\,Ba^2cde^2}{}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*
e + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*
C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e
+ 3*B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*
c^2*d^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)
*x - 2*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*
e^2 - 3*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

3.57. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.57.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 920, normalized size of antiderivative = 4.40

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{5\,A\,d^3\,x}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{B\,d^3}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,C\,a^2\,e^3}{4\,\left(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4\right)}-\frac{3\,A\,d^2\,e}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,d^3\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{C\,d^3\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{A\,a\,e^3}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{A\,e^3\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{5\,B\,e^3\,x^3}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,e^3\,\ln\left(c\,x^2+a\right)}{2\,c^3}-\frac{3\,B\,a\,d\,e^2}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{3\,C\,a\,d^2\,e}{4\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}+\frac{3\,A\,c\,d^3\,x^3}{8\,\left(a^4+2\,a^3\,c\,x^2+a^2\,c^2\,x^4\right)}-\frac{3\,B\,a\,e^3\,x}{8\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}-\frac{3\,B\,d\,e^2\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{3\,C\,d^2\,e\,x^2}{2\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{15\,C\,d\,e^2\,x^3}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{C\,a\,e^3\,x^2}{a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4}+\frac{3\,A\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{c}}+\frac{3\,B\,e^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}+\frac{C\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{3\,A\,d\,e^2\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}+\frac{3\,B\,d^2\,e\,x^3}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{3\,A\,d\,e^2\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}-\frac{3\,B\,d^2\,e\,x}{8\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,A\,d\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{3\,B\,d^2\,e\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,c^{3/2}}+\frac{9\,C\,d\,e^2\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,c^{5/2}}-\frac{9\,C\,a\,d\,e^2\,x}{8\,\left(a^2\,c^2+2\,a\,c^3\,x^2+c^4\,x^4\right)}$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

output
```
(5*A*d^3*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^3)/(4*(a^2*c + c^3*
x^4 + 2*a*c^2*x^2)) + (3*C*a^2*e^3)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2))
- (3*A*d^2*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*d^3*x^3)/(8*(a^3 +
2*a^2*c*x^2 + a*c^2*x^4)) - (C*d^3*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2))
- (A*a*e^3)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (A*e^3*x^2)/(2*(a^2*c
+ c^3*x^4 + 2*a*c^2*x^2)) - (5*B*e^3*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^
2)) + (C*e^3*log(a + c*x^2))/(2*c^3) - (3*B*a*d*e^2)/(4*(a^2*c^2 + c^4*x^4
 + 2*a*c^3*x^2)) - (3*C*a*d^2*e)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (
3*A*c*d^3*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) - (3*B*a*e^3*x)/(8*(a
^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (3*B*d*e^2*x^2)/(2*(a^2*c + c^3*x^4 + 2
*a*c^2*x^2)) - (3*C*d^2*e*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (15*C
*d*e^2*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*a*e^3*x^2)/(a^2*c^2 +
 c^4*x^4 + 2*a*c^3*x^2) + (3*A*d^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c
^(1/2)) + (3*B*e^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (C*d^3
*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (3*A*d*e^2*x^3)/(8*(a^3
+ 2*a^2*c*x^2 + a*c^2*x^4)) + (3*B*d^2*e*x^3)/(8*(a^3 + 2*a^2*c*x^2 + a*c^
2*x^4)) - (3*A*d*e^2*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*d^2*e*x
)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (3*A*d*e^2*atan((c^(1/2)*x)/a^(1/2
)))/(8*a^(3/2)*c^(3/2)) + (3*B*d^2*e*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)
*c^(3/2)) + (9*C*d*e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) -...
```

3.57. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.58 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3} dx$

3.58.1 Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3} dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac(a+cx^2)^2} - \frac{(d+ex)(a(Ac+3aC)e-c(3Acd+aCd+2aBe)x)}{8a^2c^2(a+cx^2)} + \frac{(a(Ac+3aC)e^2+cd(3Acd+aCd+2aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

output
```
-1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*(A*c+3*C*a
)*e-c*(3*A*c*d+2*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(a*(A*c+3*C*a)*e^2+
c*d*(3*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} \, dx = \frac{3Ac^2d^2x+ac(Cd^2+e(2Bd+Ae))\,x-a^2e(8Cd+4Be+5Cex)}{8a^2c^2\left(a+cx^2\right)} + \frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex))}{4ac^2\left(a+cx^2\right)^2} + \frac{(Ac(3cd^2+ae^2)+a(3aCe^2+cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{5/2}}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
(3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e +
5*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*
e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*
x^2)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*Arc
Tan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

3.58.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2176, 25, 675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} \, dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(d+ex)(3Acd+aCd+2aBe+(Ac+3aC)ex)}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)(3Acd+aCd+2aBe+(Ac+3aC)ex)}{(cx^2+a)^2}dx}{4ac} - \frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{675}$$

$$\frac{\frac{(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac))\int \frac{1}{cx^2+a}dx}{2ac} - \frac{x(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd))}{2ac(a+cx^2)} - \frac{e(aBe+2aCd+2Acd)}{c(a+cx^2)}}{4ac} - \frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac))}{2a^{3/2}c^{3/2}} - \frac{x(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd))}{2ac(a+cx^2)} - \frac{e(aBe+2aCd+2Acd)}{c(a+cx^2)}}{4ac} - \frac{(d+ex)^2(aB-x(Ac-aC))}{4ac\,(a+cx^2)^2}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^2) + (-((e*(2*A*
c*d + 2*a*C*d + a*B*e))/(c*(a + c*x^2))) - ((a*(A*c + 3*a*C)*e^2 - c*d*(3*
A*c*d + a*C*d + 2*a*B*e))*x)/(2*a*c*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 +
 c*d*(3*A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*
c^(3/2)))/(4*a*c)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.58.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.42

method	result
default	$\frac{\frac{\left(A a c\,e^{2}+3 A\,c^{2} d^{2}+2 B a c d e-5 a^{2} C\,e^{2}+C a c\,d^{2}\right) x^{3}}{8 c\,a^{2}}-\frac{e \left(B e+2 C d\right) x^{2}}{2 c}-\frac{\left(A a c\,e^{2}-5 A\,c^{2} d^{2}+2 B a c d e+3 a^{2} C\,e^{2}+C a c\,d^{2}\right) x}{8 a\,c^{2}}-\frac{2 A c d e+B a\,e^{2}+B c\,d^{2}+2 a d}{4 c^{2}}}{\left(c\,x^{2}+a \right)^{2}}$
risch	$\frac{\frac{\left(A a c\,e^{2}+3 A\,c^{2} d^{2}+2 B a c d e-5 a^{2} C\,e^{2}+C a c\,d^{2}\right) x^{3}}{8 c\,a^{2}}-\frac{e \left(B e+2 C d\right) x^{2}}{2 c}-\frac{\left(A a c\,e^{2}-5 A\,c^{2} d^{2}+2 B a c d e+3 a^{2} C\,e^{2}+C a c\,d^{2}\right) x}{8 a\,c^{2}}-\frac{2 A c d e+B a\,e^{2}+B c\,d^{2}+2 a d}{4 c^{2}}}{\left(c\,x^{2}+a \right)^{2}}$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/c/a^2*x^3-1
/2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+
C*a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a
)^2+1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*a*c*d^2)/a^2/c^2/
(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(141) = 282$.

Time = 0.50 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.17

$$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{4\,Ba^3c^2d^2 + 4\,Ba^4ce^2 - 2\,(2\,Ba^2c^3de + (Ca^2c^3 + 3\,Aac^4)d^2 - (5\,Ca^3c^2 - Aa^2c^3)e^2)x^3 + 8\,(Ca^4c + Ac\right.$$
$$-\frac{2\,Ba^3c^2d^2 + 2\,Ba^4ce^2 - (2\,Ba^2c^3de + (Ca^2c^3 + 3\,Aac^4)d^2 - (5\,Ca^3c^2 - Aa^2c^3)e^2)x^3 + 4\,(Ca^4c + Aa^3c}{}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3
+ 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3
*c^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*
B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 +
 (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*
e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*sqrt(-
a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e +
(C*a^3*c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x
^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*
B*a^2*c^3*d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^
2)*x^3 + 4*(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)
*x^2 - (2*B*a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^
2*c^2 + A*a*c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3
*c)*e^2 + 2*(2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c +
A*a^2*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*B*a^3*c^2*d*e +
(C*a^3*c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x
^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = -\frac{2\,Ba^2cd^2 + 2\,Ba^3e^2 - (2\,Bac^2de + (Cac^2 + 3\,Ac^3)d^2 - (5\,Ca^2c - Aac^2)e^2)x^3 + 4\,(Ca^3 + Aa^2c)de + 4}{8\,(a^2c^4x^4 + 2\,a^3c^3x^2 + a^4}$$
$$+ \frac{(2\,Bacde + (Cac + 3\,Ac^2)d^2 + (3\,Ca^2 + Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e + (C*a*c^2 + 3*A*c^3)*d
^2 - (5*C*a^2*c - A*a*c^2)*e^2)*x^3 + 4*(C*a^3 + A*a^2*c)*d*e + 4*(2*C*a^2
*c*d*e + B*a^2*c*e^2)*x^2 + (2*B*a^2*c*d*e + (C*a^2*c - 5*A*a*c^2)*d^2 + (
3*C*a^3 + A*a^2*c)*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(
2*B*a*c*d*e + (C*a*c + 3*A*c^2)*d^2 + (3*C*a^2 + A*a*c)*e^2)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^2*c^2)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3} dx$$
$$= \frac{(Cacd^2 + 3Ac^2d^2 + 2Bacde + 3Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}a^2c^2} + \frac{Cac^2d^2x^3 + 3Ac^3d^2x^3 + 2Bac^2dex^3 - 5Ca^2ce^2x^3 + Aac^2e^2x^3 - 8Ca^2cdex^2 - 4Ba^2ce^2x^2 - Ca^2cd^2x}{8(cx^2+a)^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arct
an(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2
*x^3 + 2*B*a*c^2*d*e*x^3 - 5*C*a^2*c*e^2*x^3 + A*a*c^2*e^2*x^3 - 8*C*a^2*c
*d*e*x^2 - 4*B*a^2*c*e^2*x^2 - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 2*B*a^2*c
*d*e*x - 3*C*a^3*e^2*x - A*a^2*c*e^2*x - 2*B*a^2*c*d^2 - 4*C*a^3*d*e - 4*A
*a^2*c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^3} dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(3\,C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e + A\,a\,c\,e^2 + 3\,A\,c^2\,d^2)}{8\,a^{5/2}\,c^{5/2}} - \frac{\frac{B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e}{4\,c^2} + \frac{x^2\,(B\,e^2+2\,C\,d\,e)}{2\,c} + \frac{x\,(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-5\,A\,c^2\,d^2)}{8\,a\,c^2} - \frac{x^3\,(-5\,C\,a^2\,e^2+C\,a\,c}{}}{a^2 + 2\,a\,c\,x^2 + c^2\,x^4}$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*
d^2 + 2*B*a*c*d*e))/(8*a^(5/2)*c^(5/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e
+ 2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5
*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c
^2*d^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a
^2 + c^2*x^4 + 2*a*c*x^2)
```

3.59 $\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

3.59.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac(a+cx^2)^2} - \frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2(a+cx^2)} + \frac{(3Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
-1/4*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^2+1/8*(-2*a*(A*c+C*a)*e+c*(3*
A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*c*d+B*a*e+C*a*d)*arctan(x
*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx = \frac{\frac{\sqrt{a}(-4a^2Ce+3Ac^2dx+ac(Cd+Be)x)}{a+cx^2} + \frac{2a^{3/2}(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex)))}{(a+cx^2)^2} + \sqrt{c}(3Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^2}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2
*a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x
^2)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*
a^(5/2)*c^2)
```

3.59.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2176, 25, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{3Acd+a(Cd+Be)+2(Ac+aC)ex}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3Acd+aCd+aBe+2(Ac+aC)ex}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{(aBe+aCd+3Acd)\int \frac{1}{cx^2+a}dx}{2a}-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{2ac\left(a+cx^2\right)}}{4ac}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{2a^{3/2}\sqrt{c}}-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{2ac\left(a+cx^2\right)}}{4ac}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

output
```
-1/4*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^2) + (-1/2*(2*a*(A
*c + a*C)*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(a*c*(a + c*x^2)) + ((3*A*c*d
 + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]))/(4*a*c
)
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.59.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{(3Acd+Bae+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+Cae}{4c^2}}{(c\,x^2+a)^2}+\frac{(3Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$
risch	$\frac{\frac{(3Acd+Bae+Cad)x^3}{8a^2}-\frac{Ce\,x^2}{2c}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+Cae}{4c^2}}{(c\,x^2+a)^2}-\frac{3\ln(cx+\sqrt{-ac})Ad}{16\sqrt{-ac}\,a^2}-\frac{\ln(cx+\sqrt{-ac})Be}{16\sqrt{-ac}\,ca}-\frac{\ln(cx+\sqrt{-ac})C}{16\sqrt{-ac}\,ca}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C*e*x^2/c+1/8*(5*A*c*d-B*a*e-C*a*d)
/a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+1/8*(3*A*c*d+B*a*e+C*a*d)/
a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.62

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{8\,Ca^3cex^2+4\,Ba^3cd-2\,(Ba^2c^2e+(Ca^2c^2+3\,Aac^3)d)x^3+(Ba^3e+(Bac^2e+(Cac^2+3\,Ac^3)d)x^4+\cdots}{\cdots}\right.$$
$$-\frac{4\,Ca^3cex^2+2\,Ba^3cd-(Ba^2c^2e+(Ca^2c^2+3\,Aac^3)d)x^3-(Ba^3e+(Bac^2e+(Cac^2+3\,Ac^3)d)x^4+2\cdots}{8\,(\cdots}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*C*a^3*c*e*x^2 + 4*B*a^3*c*d - 2*(B*a^2*c^2*e + (C*a^2*c^2 + 3*A*
a*c^3)*d)*x^3 + (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*
a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 4*(C*a^4 + A*a^3*c)*e + 2
*(B*a^3*c*e + (C*a^3*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 +
 a^5*c^2), -1/8*(4*C*a^3*c*e*x^2 + 2*B*a^3*c*d - (B*a^2*c^2*e + (C*a^2*c^2
 + 3*A*a*c^3)*d)*x^3 - (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4
+ 2*(B*a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqr
t(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a^4 + A*a^3*c)*e + (B*a^3*c*e + (C*a^3
*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

3.59.6 Sympy [A] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}$$
$$+\frac{-2Aa^2ce-2Ba^2cd-2Ca^3e-4Ca^2cex^2+x^3\cdot\left(3Ac^3d+Bac^2e+Cac^2d\right)+x(5Aac^2d-Ba^2ce-Ca}{8a^4c^2+16a^3c^3x^2+8a^2c^4x^4}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*
c**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c
*sqrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e
 - 4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*
A*a*c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 +
8*a**2*c**4*x**4)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx =$$
$$-\frac{4\,Ca^2cex^2+2\,Ba^2cd-\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^3+2\,\left(Ca^3+Aa^2c\right)e+\left(Ba^2ce+\left(Ca^2c-5\,Aac^2\right)\right.}{8\,\left(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2\right)}$$
$$+\frac{(Bae+(Ca+3\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*C*a^2*c*e*x^2 + 2*B*a^2*c*d - (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*
x^3 + 2*(C*a^3 + A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^
2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = \frac{(Cad+3\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{Cac^2dx^3+3\,Ac^3dx^3+Bac^2ex^3-4\,Ca^2cex^2-Ca^2cdx+5\,Aac^2dx-Ba^2cex-2\,Ba^2cd-2\,Ca^3e-2}{8\left(cx^2+a\right)^2a^2c^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/
8*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*e*x^3 - 4*C*a^2*c*e*x^2 - C*a^2
*c*d*x + 5*A*a*c^2*d*x - B*a^2*c*e*x - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c
*e)/((c*x^2 + a)^2*a^2*c^2)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^{5/2}\,c^{3/2}} - \frac{\frac{A\,c\,e+B\,c\,d+C\,a\,e}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^2}+\frac{C\,e\,x^2}{2\,c}+\frac{x\,\left(B\,a\,e-5\,A\,c\,d+C\,a\,d\right)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(3*A*c*d + B*a*e + C*a*d))/(8*a^(5/2)*c^(3/2))
- ((A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^
2) + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c))/(a^2 + c^2*x
^4 + 2*a*c*x^2)
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = -\frac{aB-(Ac-aC)x}{4ac\,(a+cx^2)^2}+\frac{(3Ac+aC)x}{8a^2c\,(a+cx^2)}+\frac{(3Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

output
```
1/4*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^2+1/8*(3*A*c+C*a)*x/a^2/c/(c*x^2+a)+1
/8*(3*A*c+C*a)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{3Ac^2x^3-a^2(2B+Cx)+acx(5A+Cx^2)}{8a^2c\,(a+cx^2)^2}+\frac{(3Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

output
```
(3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)
^2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

3.60.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2345, 25, 27, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{3Ac+aC}{c(cx^2+a)^2}dx}{4a}-\frac{aB-x(Ac-aC)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3Ac+aC}{c(cx^2+a)^2}dx}{4a}-\frac{aB-x(Ac-aC)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+3Ac)\int \frac{1}{(cx^2+a)^2}dx}{4ac}-\frac{aB-x(Ac-aC)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aC+3Ac)\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4ac}-\frac{aB-x(Ac-aC)}{4ac(a+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aC+3Ac)\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)}{4ac}-\frac{aB-x(Ac-aC)}{4ac(a+cx^2)^2}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

output
```
-1/4*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*(x/(2*a*(a
 + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a*c)
```

3.60.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{(3Ac+Ca)x^3}{8a^2}+\frac{(5Ac-Ca)x}{8ac}-\frac{B}{4c}}{(c\,x^2+a)^2}+\frac{(3Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8a^2c\sqrt{ac}}$	83
risch	$\frac{\frac{(3Ac+Ca)x^3}{8a^2}+\frac{(5Ac-Ca)x}{8ac}-\frac{B}{4c}}{(c\,x^2+a)^2}-\frac{3A\ln(cx+\sqrt{-ac})}{16\sqrt{-ac}\,a^2}-\frac{\ln(cx+\sqrt{-ac})C}{16\sqrt{-ac}\,ca}+\frac{3A\ln(-cx+\sqrt{-ac})}{16\sqrt{-ac}\,a^2}+\frac{\ln(-cx+\sqrt{-ac})C}{16\sqrt{-ac}\,ca}$	153

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+1/8*(3
*A*c+C*a)/a^2/c/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.20

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx$$
$$= \left[-\frac{4\,Ba^3c-2\,(Ca^2c^2+3\,Aac^3)x^3+((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{-ac}\log}{16\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}\right.$$
$$-\frac{2\,Ba^3c-(Ca^2c^2+3\,Aac^3)x^3-((Cac^2+3\,Ac^3)x^4+Ca^3+3\,Aa^2c+2\,(Ca^2c+3\,Aac^2)x^2)\sqrt{ac}\arctan}{8\,(a^3c^4x^4+2\,a^4c^3x^2+a^5c^2)}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x
^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^
2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c
^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^
3)*x^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a
*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a
^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}\cdot(3Ac+Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16} + \frac{-2Ba^2+x^3\cdot(3Ac^2+Cac)+x(5Aac-Ca^2)}{8a^4c+16a^3c^2x^2+8a^2c^3x^4}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/
16 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) +
x)/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a*
*4*c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{(Cac+3\,Ac^2)x^3-2\,Ba^2-(Ca^2-5\,Aac)x}{8\,(a^2c^3x^4+2\,a^3c^2x^2+a^4c)} + \frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((C*a*c + 3*A*c^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*c)*x)/(a^2*c^3*x^4 +
 2*a^3*c^2*x^2 + a^4*c) + 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*a^2*c)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{(Ca+3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2c} + \frac{Cacx^3+3\,Ac^2x^3-Ca^2x+5\,Aacx-2\,Ba^2}{8\,(cx^2+a)^2a^2c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
 + 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3}\,dx = \frac{\frac{x^3\,(3\,A\,c+C\,a)}{8\,a^2}-\frac{B}{4\,c}+\frac{x\,(5\,A\,c-C\,a)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,A\,c+C\,a)}{8\,a^{5/2}\,c^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^3,x)
```

output
```
((x^3*(3*A*c + C*a))/(8*a^2) - B/(4*c) + (x*(5*A*c - C*a))/(8*a*c))/(a^2 +
 c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*c + C*a))/(8*a^(5/
2)*c^(3/2))
```

3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^3}\,dx$

3.61.1 Optimal result

Integrand size = 27, antiderivative size = 353

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e(Cd^2-Bde+Ae^2)+\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd(3cd^2+7ae^2)\right)x}{8a^2\left(cd^2+ae^2\right)^2\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(c^2d^4+6acd^2e^2-3a^2e^4\right)+Acd(3c^2d^4+10acd^2e^2+15a^2e^4)\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}\left(cd^2+ae^2\right)^3} + \frac{e^3(Cd^2-Bde+Ae^2)\log(d+ex)}{\left(cd^2+ae^2\right)^3} - \frac{e^3(Cd^2-Bde+Ae^2)\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^3}$$

output
```
1/4*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c
*x^2+a)^2+1/8*(4*a^2*e*(A*e^2-B*d*e+C*d^2)+(a*(-B*e+C*d)*(-3*a*e^2+c*d^2)+
A*c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+e^3*(A*e^2-B*d*e
+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^3*(A*e^2-B*d*e+C*d^2)*ln(c*x^2+a)/
(a*e^2+c*d^2)^3+1/8*(a*(-B*e+C*d)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+A*c*d
*(15*a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/
(a*e^2+c*d^2)^3/c^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$$
$$= \frac{\frac{2\left(cd^2+ae^2\right)^2\left(-a^2Ce+Ac^2dx+ac(-Bd+Ae-Cdx+Bex)\right)}{ac\left(a+cx^2\right)^2}+\frac{\left(cd^2+ae^2\right)\left(3Ac^2d^3x+acd\left(Cd^2+e(-Bd+7Ae)\right)x+a^2e(Cd(4d-3ex)+e(-4Bd+4}{a^2\left(a+cx^2\right)}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x
+ B*e*x)))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(
C*d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A
*e + 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*
e^2 - 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e)
)*Log[d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^
2 + a*e^2)^3)
```

3.61.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2178, 25, 27, 686, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3(d+ex)}\,dx$$
$$\Big\downarrow \ 2178$$
$$-\frac{\int -\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)+3e(Acd-aCd+aBe)x\right)}{\left(cd^2+ae^2\right)(d+ex)\left(cx^2+a\right)^2}dx}{4ac}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)+3e(Acd-aCd+aBe)x\right)}{(cd^2+ae^2)(d+ex)(cx^2+a)^2}dx}{4ac}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{ad(Cd-Be)+A\left(3cd^2+4ae^2\right)+3e(Acd-aCd+aBe)x}{(d+ex)(cx^2+a)^2}dx}{4a\left(ae^2+cd^2\right)}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 686$$

$$\frac{\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3cd^2\right)+a\left(cd^2-3ae^2\right)(Cd-Be)\right)}{2a(a+cx^2)(ae^2+cd^2)}-\frac{\int -\frac{c\left(ad(Cd-Be)\left(cd^2+5ae^2\right)+A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)+e\left(a(Cd-Be)\left(cd\right.\right.\right.}{(d+ex)\left(cx^2+a\right)}}{2ac(ae^2+cd^2)}}{4a\left(ae^2+cd^2\right)}$$

$$\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{c\left(ad(Cd-Be)\left(cd^2+5ae^2\right)+A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)+e\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd\left(3cd^2+7ae^2\right)\right)x\right)}{(d+ex)\left(cx^2+a\right)}dx}{2ac(ae^2+cd^2)}+\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3\right.\right.}{2a(a+cx^2)(ae^2}}{4a\left(ae^2+cd^2\right)}$$

$$\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{ad(Cd-Be)\left(cd^2+5ae^2\right)+A\left(3c^2d^4+7ace^2d^2+8a^2e^4\right)+e\left(a(Cd-Be)\left(cd^2-3ae^2\right)+Acd\left(3cd^2+7ae^2\right)\right)x}{(d+ex)\left(cx^2+a\right)}dx}{2a(ae^2+cd^2)}+\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3cd^2\right.\right.}{2a(a+cx^2)(ae^2+c}}{4a\left(ae^2+cd^2\right)}$$

$$\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 657$$

$$\frac{\frac{\int \left(\frac{8a^2\left(Cd^2-Bed+Ae^2\right)e^4}{\left(cd^2+ae^2\right)(d+ex)}+\frac{-8a^2c\left(Cd^2-Bed+Ae^2\right)xe^3+a(Cd-Be)\left(c^2d^4+6ace^2d^2-3a^2e^4\right)+Acd\left(3c^2d^4+10ace^2d^2+15a^2e^4\right)}{\left(cd^2+ae^2\right)\left(cx^2+a\right)}\right)dx}{2a(ae^2+cd^2)}+\frac{4a^2e\left(Ae^2-Bde+\right.}{}}{4a\left(ae^2+cd^2\right)}$$

$$\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

$$\Big\downarrow \; 2009$$

3.61. $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)^3}\,dx$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)}-\frac{4a^2e^3\log\left(a+cx^2\right)\left(Ae^2-Bde+Cd^2\right)}{ae^2+cd^2}+\frac{8a^2e^3\log(d+ex)\left(Ae^2-Bde+C\right.}{ae^2+cd^2}}{2a\left(ae^2+cd^2\right)}}{4a\left(ae^2+cd^2\right)}$$

$$\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{4ac\left(a+cx^2\right)^2\left(ae^2+cd^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

output
```
-1/4*(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(a*c*(c*d^2
 + a*e^2)*(a + c*x^2)^2) + ((4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B
*e)*(c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(2*a*(c*d^2 + a*e^2)
*(a + c*x^2)) + (((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A
*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]
])/(Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)) + (8*a^2*e^3*(C*d^2 - B*d*e + A*e^2)*
Log[d + e*x])/(c*d^2 + a*e^2) - (4*a^2*e^3*(C*d^2 - B*d*e + A*e^2)*Log[a +
 c*x^2])/(c*d^2 + a*e^2))/(2*a*(c*d^2 + a*e^2)))/(4*a*(c*d^2 + a*e^2))
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.71

method	result
default	$\frac{\frac{c\left(7A\,a^{2}cd\,e^{4}+10Aa\,c^{2}d^{3}e^{2}+3d^{5}A\,c^{3}+3B\,e^{5}a^{3}+2B\,a^{2}c\,d^{2}e^{3}-Ba\,c^{2}d^{4}e-3C\,a^{3}d\,e^{4}-2C\,a^{2}c\,d^{3}e^{2}+C\,c^{2}d^{5}a\right)x^{3}}{8a^{2}}+\left(\frac{1}{2}Aac\,e^{5}+\frac{1}{2}A\,c^{2}d^{2}e^{3}-\frac{1}{2}Bacd\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2+c*d^2)^3*((1/8*c*(7*A*a^2*c*d*e^4+10*A*a*c^2*d^3*e^2+3*A*c^3*d^5+
3*B*a^3*e^5+2*B*a^2*c*d^2*e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4-2*C*a^2*c*d^3*e^
2+C*a*c^2*d^5)/a^2*x^3+(1/2*A*a*c*e^5+1/2*A*c^2*d^2*e^3-1/2*B*a*c*d*e^4-1/
2*B*c^2*d^3*e^2+1/2*C*a*c*d^2*e^3+1/2*C*c^2*d^4*e)*x^2+1/8*(9*A*a^2*c*d*e^
4+14*A*a*c^2*d^3*e^2+5*A*c^3*d^5+5*B*a^3*e^5+6*B*a^2*c*d^2*e^3+B*a*c^2*d^4
*e-5*C*a^3*d*e^4-6*C*a^2*c*d^3*e^2-C*a*c^2*d^5)/a*x+1/4*(3*A*a^2*c*e^5+4*A
*a*c^2*d^2*e^3+A*c^3*d^4*e-3*B*a^2*c*d*e^4-4*B*a*c^2*d^3*e^2-B*c^3*d^5-C*a
^3*e^5+C*a*c^2*d^4*e)/c)/(c*x^2+a)^2+1/8/a^2*(1/2*(-8*A*a^2*c*e^5+8*B*a^2*
c*d*e^4-8*C*a^2*c*d^2*e^3)/c*ln(c*x^2+a)+(15*A*a^2*c*d*e^4+10*A*a*c^2*d^3*
e^2+3*A*c^3*d^5+3*B*a^3*e^5-6*B*a^2*c*d^2*e^3-B*a*c^2*d^4*e-3*C*a^3*d*e^4+
6*C*a^2*c*d^3*e^2+C*a*c^2*d^5)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))+e^3*(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(337) = 674$.

Time = 193.20 (sec) , antiderivative size = 2346, normalized size of antiderivative = 6.65

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*B*a^3*c^3*d^5 + 16*B*a^4*c^2*d^3*e^2 - 16*A*a^4*c^2*d^2*e^3 + 12
*B*a^5*c*d*e^4 - 4*(C*a^4*c^2 + A*a^3*c^3)*d^4*e + 4*(C*a^6 - 3*A*a^5*c)*e
^5 + 2*(B*a^2*c^4*d^4*e - 2*B*a^3*c^3*d^2*e^3 - 3*B*a^4*c^2*e^5 - (C*a^2*c
^4 + 3*A*a*c^5)*d^5 + 2*(C*a^3*c^3 - 5*A*a^2*c^4)*d^3*e^2 + (3*C*a^4*c^2 -
 7*A*a^3*c^3)*d*e^4)*x^3 - 8*(C*a^3*c^3*d^4*e - B*a^3*c^3*d^3*e^2 - B*a^4*
c^2*d*e^4 + A*a^4*c^2*e^5 + (C*a^4*c^2 + A*a^3*c^3)*d^2*e^3)*x^2 - (B*a^3*
c^2*d^4*e + 6*B*a^4*c*d^2*e^3 - 3*B*a^5*e^5 - (C*a^3*c^2 + 3*A*a^2*c^3)*d^
5 - 2*(3*C*a^4*c + 5*A*a^3*c^2)*d^3*e^2 + 3*(C*a^5 - 5*A*a^4*c)*d*e^4 + (B
*a*c^4*d^4*e + 6*B*a^2*c^3*d^2*e^3 - 3*B*a^3*c^2*e^5 - (C*a*c^4 + 3*A*c^5)
*d^5 - 2*(3*C*a^2*c^3 + 5*A*a*c^4)*d^3*e^2 + 3*(C*a^3*c^2 - 5*A*a^2*c^3)*d
*e^4)*x^4 + 2*(B*a^2*c^3*d^4*e + 6*B*a^3*c^2*d^2*e^3 - 3*B*a^4*c*e^5 - (C*
a^2*c^3 + 3*A*a*c^4)*d^5 - 2*(3*C*a^3*c^2 + 5*A*a^2*c^3)*d^3*e^2 + 3*(C*a^
4*c - 5*A*a^3*c^2)*d*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)
/(c*x^2 + a)) - 2*(B*a^3*c^3*d^4*e + 6*B*a^4*c^2*d^2*e^3 + 5*B*a^5*c*e^5 -
 (C*a^3*c^3 - 5*A*a^2*c^4)*d^5 - 2*(3*C*a^4*c^2 - 7*A*a^3*c^3)*d^3*e^2 - (
5*C*a^5*c - 9*A*a^4*c^2)*d*e^4)*x + 8*(C*a^5*c*d^2*e^3 - B*a^5*c*d*e^4 + A
*a^5*c*e^5 + (C*a^3*c^3*d^2*e^3 - B*a^3*c^3*d*e^4 + A*a^3*c^3*e^5)*x^4 + 2
*(C*a^4*c^2*d^2*e^3 - B*a^4*c^2*d*e^4 + A*a^4*c^2*e^5)*x^2)*log(c*x^2 + a)
 - 16*(C*a^5*c*d^2*e^3 - B*a^5*c*d*e^4 + A*a^5*c*e^5 + (C*a^3*c^3*d^2*e^3
- B*a^3*c^3*d*e^4 + A*a^3*c^3*e^5)*x^4 + 2*(C*a^4*c^2*d^2*e^3 - B*a^4*c...
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

output
```
Timed out
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$$
$$= -\frac{(Cd^2e^3-Bde^4+Ae^5)\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)}+\frac{(Cd^2e^3-Bde^4+Ae^5)\log\left(ex+d\right)}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6}$$
$$-\frac{(Bac^2d^4e+6\,Ba^2cd^2e^3-3\,Ba^3e^5-(Cac^2+3\,Ac^3)d^5-2\,(3\,Ca^2c+5\,Aac^2)d^3e^2+3\,(Ca^3-5\,Aa^2c)de}{8\left(a^2c^3d^6+3\,a^3c^2d^4e^2+3\,a^4cd^2e^4+a^5e^6\right)\sqrt{ac}}$$
$$-\frac{2\,Ba^2c^2d^3+6\,Ba^3cde^2-2\,(Ca^3c+Aa^2c^2)d^2e+2\,(Ca^4-3\,Aa^3c)e^3+(Bac^3d^2e-3\,Ba^2c^2e^3-(Cac^3}{8\left(a^4c^3d^4+2\,a^5c^2d^2e^2+a\right.}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(e*x +
d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*
d^4*e + 6*B*a^2*c*d^2*e^3 - 3*B*a^3*e^5 - (C*a*c^2 + 3*A*c^3)*d^5 - 2*(3*C
*a^2*c + 5*A*a*c^2)*d^3*e^2 + 3*(C*a^3 - 5*A*a^2*c)*d*e^4)*arctan(c*x/sqrt
(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sqrt
(a*c)) - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 2*(C*a^3*c + A*a^2*c^2)*
d^2*e + 2*(C*a^4 - 3*A*a^3*c)*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*
a*c^3 + 3*A*c^4)*d^3 + (3*C*a^2*c^2 - 7*A*a*c^3)*d*e^2)*x^3 - 4*(C*a^2*c^2
*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5*B*a^3
*c*e^3 - (C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c - 9*A*a^2*c^2)*d*e^2)*x)
/(a^4*c^3*d^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a^3*c^4*d
^2*e^2 + a^4*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^5*c^2*e
^4)*x^2)
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(337) = 674$.

Time = 0.28 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx+Cx^2}{(d+ex)\,(a+cx^2)^3}\,dx$$
$$= -\frac{(Cd^2e^3 - Bde^4 + Ae^5)\log(cx^2+a)}{2\,(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6)} + \frac{(Cd^2e^4 - Bde^5 + Ae^6)\log(|ex+d|)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7}$$
$$+ \frac{(Cac^2d^5 + 3\,Ac^3d^5 - Bac^2d^4e + 6\,Ca^2cd^3e^2 + 10\,Aac^2d^3e^2 - 6\,Ba^2cd^2e^3 - 3\,Ca^3de^4 + 15\,Aa^2cde^4 + 3}{8\,(a^2c^3d^6 + 3\,a^3c^2d^4e^2 + 3\,a^4cd^2e^4 + a^5e^6)\sqrt{ac}}$$
$$- \frac{2\,Ba^2c^3d^5 - 2\,Ca^3c^2d^4e - 2\,Aa^2c^3d^4e + 8\,Ba^3c^2d^3e^2 - 8\,Aa^3c^2d^2e^3 + 6\,Ba^4cde^4 + 2\,Ca^5e^5 - 6\,Aa^4ce^5}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(e*
x + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C
*a*c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*
d^3*e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e
^5)*arctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*
e^4 + a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A
*a^2*c^3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e
^4 + 2*C*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^
4*e - 2*C*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C
*a^3*c^2*d*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d
^4*e - B*a^2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c
^2*d*e^4 + A*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3
*d^4*e + 6*C*a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3
+ 5*C*a^4*c*d*e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)
^3*(c*x^2 + a)^2*a^2*c)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 2392, normalized size of antiderivative = 6.78

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
```

output
```
((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2
*e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^
2 - C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d
^3 + 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e
))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2
*c*e^3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))
/(8*a^2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2)
- (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*
c*e^7 + 3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6
*c*d^2*e^5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1
/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d
^2*e^5 - 2*B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3
 + 30*B*a^6*c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*
a^4*c^3*d^4*e^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*
a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*
(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5
*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5
*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e
*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A
*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2...
```

3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3}\,dx$

3.62.1 Optimal result

Integrand size = 27, antiderivative size = 571

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = -\frac{e^3(Cd^2-Bde+Ae^2)}{\left(cd^2+ae^2\right)^3(d+ex)}$$
$$-\frac{a(Bcd^2-2Acde+2aCde-aBe^2)-(Ac(cd^2-ae^2)+a(aCe^2-cd(Cd-2Be)))\,x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$
$$-\frac{4a^2e(ae^2(2Cd-Be)-cd(2Cd^2-e(3Bd-4Ae)))-(Ac(3c^2d^4+12acd^2e^2-7a^2e^4)+a(3a^2Ce^4-2a}{8a^2\left(cd^2+ae^2\right)^3(a+cx^2)}$$
$$+\frac{(3Ac(c^3d^6+5ac^2d^4e^2+15a^2cd^2e^4-5a^3e^6)+a(3a^3Ce^6+ac^2d^3e^2(13Cd-20Be)-3a^2cde^4(11Cd-1}{8a^{5/2}\sqrt{c}\left(cd^2+ae^2\right)^4}$$
$$-\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))\log(d+ex)}{\left(cd^2+ae^2\right)^4}$$
$$+\frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))\log\left(a+cx^2\right)}{2\left(cd^2+ae^2\right)^4}$$

output
```
-e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)+1/4*(-a*(-2*A*c*d*e-B*a*e
^2+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)
/a/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/8*(-4*a^2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*
d^2-e*(-4*A*e+3*B*d)))+(A*c*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)+a*(3*a^2
*C*e^4-2*a*c*d*e^2*(-7*B*e+6*C*d)+c^2*d^3*(-2*B*e+C*d)))*x)/a^2/(a*e^2+c*d
^2)^3/(c*x^2+a)-e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))*ln
(e*x+d)/(a*e^2+c*d^2)^4+1/2*e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e
+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/8*(3*A*c*(-5*a^3*e^6+15*a^2*c*d^2*
e^4+5*a*c^2*d^4*e^2+c^3*d^6)+a*(3*a^3*C*e^6+a*c^2*d^3*e^2*(-20*B*e+13*C*d)
-3*a^2*c*d*e^4*(-10*B*e+11*C*d)+c^3*d^5*(-2*B*e+C*d)))*arctan(x*c^(1/2)/a^
(1/2))/a^(5/2)/(a*e^2+c*d^2)^4/c^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx$$
$$= \frac{-\frac{8e^3(cd^2+ae^2)(Cd^2+e(-Bd+Ae))}{d+ex}+\frac{2(cd^2+ae^2)^2(Ac^2d^2x+a^2e(-2Cd+Be+Cex)-ac(Cd^2x+Bd(d-2ex)+Ae(-2d+ex)))}{a(a+cx^2)^2}+\frac{(cd^2+ae^2)}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
 + a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B
*d*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*
(3*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*
C*d + 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d -
 7*e*x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*
a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3
*e^2*(13*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d +
10*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^
3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4
*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/
(8*(c*d^2 + a*e^2)^4)
```

3.62.3 Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2178, 25, 2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3(d+ex)^2}\,dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{\frac{3ce^2\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^2}+\frac{2ce\left(Acd\left(3cd^2+ae^2\right)-a\left(c(3Cd-4Be)d^2+ae^2(Cd-2Be)\right)\right)x}{\left(cd^2+ae^2\right)^2}+\frac{c\left(A\left(3c^2d^4+9ace^2d^2+4a^2e^4\right)-ad^2\left(a\right.\right.}{\left(cd^2+ae^2\right)^2}}{(d+ex)^2(cx^2+a)^2}}{4ac}$$
$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{3ce^2\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^2}+\frac{2ce\left(Acd\left(3cd^2+ae^2\right)-a\left(c(3Cd-4Be)d^2+ae^2(Cd-2Be)\right)\right)x}{\left(cd^2+ae^2\right)^2}+\frac{c\left(A\left(3c^2d^4+9ace^2d^2+4a^2e^4\right)-ad^2\left(aCe^2-\right.\right.}{\left(cd^2+ae^2\right)^2}}{(d+ex)^2(cx^2+a)^2}}{4ac}$$
$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{2178}$$

$$\frac{\frac{c(x(Ac(-7a^2e^4+12acd^2e^2+3c^2d^4)+a(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)))+4a^2e(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3))}{2a(a+cx^2)(ae^2+cd^2)^3}-\frac{\int}{}}{}$$
$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{e^2\left(Ac\left(3c^2d^4+12ace^2d^2-7a^2e^4\right)+a\left(3a^2Ce^4-2acd(6Cd-7Be)e^2+c^2d^3(Cd-2Be)\right)\right)x^2c^2}{\left(cd^2+ae^2\right)^3}+\frac{\left(A\left(3c^3d^6+12ac^2e^2d^4+33a^2ce^4d^2+8a^3e^6\right)-ad^2\left(5a^2Ce^4-6acd(\cdots\right.\right.}{\left(cd^2+ae^2\right)^3}}{(d+ex)^2\left(cx^2+a\right)}}{2ac}$$

$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

$\downarrow$ 2160

$$\frac{\int\left(\frac{8a^2c^2\left(4cCd^3-ce(5Bd-6Ae)d-ae^2(2Cd-Be)\right)e^4}{\left(cd^2+ae^2\right)^4(d+ex)}+\frac{8a^2c^2\left(Cd^2-Bed+Ae^2\right)e^4}{\left(cd^2+ae^2\right)^3(d+ex)^2}+\frac{c^2\left(-8a^2c\left(4cCd^3-ce(5Bd-6Ae)d-ae^2(2Cd-Be)\right)xe^3+3Ac\left(c^3d^6+5ac^2e^2\cdots\right.\right.}{\cdots}\right)}{2ac}$$

$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

$\downarrow$ 2009

$$\frac{\frac{c(x(Ac(-7a^2e^4+12acd^2e^2+3c^2d^4)+a(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)))+4a^2e(-ae^2(2Cd-Be)-cde(3Bd-4Ae)+2cCd^3))}{2a(a+cx^2)(ae^2+cd^2)^3}+\cdots}{\cdots}$$

$$\frac{a\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-x\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

output
```
-1/4*(a*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2)
 + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*x)/(a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2)
 + ((c*(4*a^2*e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))
+ (A*c*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d
*e^2*(6*C*d - 7*B*e) + c^2*d^3*(C*d - 2*B*e)))*x))/(2*a*(c*d^2 + a*e^2)^3*
(a + c*x^2)) + ((-8*a^2*c^2*e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^
3*(d + e*x)) + (c^(3/2)*(3*A*c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e
^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2
*c*d*e^4*(11*C*d - 10*B*e) + c^3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sq
rt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^4) + (8*a^2*c^2*e^3*(4*c*C*d^3 - c*d*e*(5
*B*d - 6*A*e) - a*e^2*(2*C*d - B*e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (4*
a^2*c^2*e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e))*Log[
a + c*x^2])/(c*d^2 + a*e^2)^4)/(2*a*c))/(4*a*c)
```

3.62. $\int \frac{A+Bx+Cx^2}{(d+ex)^2(a+cx^2)^3}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 833, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\frac{c\left(7A\,a^{3}c\,e^{6}-5A\,a^{2}c^{2}d^{2}e^{4}-15Aa\,c^{3}d^{4}e^{2}-3A\,d^{6}c^{4}-14B\,a^{3}cd\,e^{5}-12B\,a^{2}c^{2}d^{3}e^{3}+2Ba\,c^{3}d^{5}e-3C\,a^{4}e^{6}+9C\,a^{3}c\,d^{2}e^{4}+11C\,a^{2}c^{2}d^{4}e^{2}-Ca\,c^{3}d^{6}\right)}{8a^{2}}}{}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2+c*d^2)^4*((1/8*c*(7*A*a^3*c*e^6-5*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d
^4*e^2-3*A*c^4*d^6-14*B*a^3*c*d*e^5-12*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3
*C*a^4*e^6+9*C*a^3*c*d^2*e^4+11*C*a^2*c^2*d^4*e^2-C*a*c^3*d^6)/a^2*x^3+(-2
*A*a*c^2*d*e^5-2*A*c^3*d^3*e^3-1/2*B*a^2*c*e^6+B*a*c^2*d^2*e^4+3/2*B*c^3*d
^4*e^2+C*a^2*c*d*e^5-C*c^3*d^5*e)*x^2+1/8*(9*A*a^3*c*e^6-3*A*a^2*c^2*d^2*e
^4-17*A*a*c^3*d^4*e^2-5*A*c^4*d^6-18*B*a^3*c*d*e^5-20*B*a^2*c^2*d^3*e^3-2*
B*a*c^3*d^5*e-5*C*a^4*e^6+7*C*a^3*c*d^2*e^4+13*C*a^2*c^2*d^4*e^2+C*a*c^3*d
^6)/a*x-5/2*A*a^2*c*d*e^5-3*A*a*c^2*d^3*e^3-1/2*A*c^3*d^5*e-3/4*B*a^3*e^6+
3/4*B*a^2*c*d^2*e^4+7/4*B*a*c^2*d^4*e^2+1/4*B*c^3*d^6+3/2*C*a^3*d*e^5+C*a^
2*c*d^3*e^3-1/2*C*a*c^2*d^5*e)/(c*x^2+a)^2+1/8/a^2*(1/2*(48*A*a^2*c^2*d*e^
5+8*B*a^3*c*e^6-40*B*a^2*c^2*d^2*e^4-16*C*a^3*c*d*e^5+32*C*a^2*c^2*d^3*e^3
)/c*ln(c*x^2+a)+(15*A*a^3*c*e^6-45*A*a^2*c^2*d^2*e^4-15*A*a*c^3*d^4*e^2-3*
A*c^4*d^6-30*B*a^3*c*d*e^5+20*B*a^2*c^2*d^3*e^3+2*B*a*c^3*d^5*e-3*C*a^4*e^
6+33*C*a^3*c*d^2*e^4-13*C*a^2*c^2*d^4*e^2-C*a*c^3*d^6)/(a*c)^(1/2)*arctan(
c*x/(a*c)^(1/2))))+e^3*(6*A*c*d*e^2+B*a*e^3-5*B*c*d^2*e-2*C*a*d*e^2+4*C*c*
d^3)/(a*e^2+c*d^2)^4*ln(e*x+d)-e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*
x+d)
```

3.62.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
```

output
```
Timed out
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(555) = 1110$.

Time = 0.32 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.09

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*e^5)*log
(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e
^6 + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)
*d*e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^
3*c*d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 - 30*
B*a^3*c*d*e^5 - (C*a*c^3 + 3*A*c^4)*d^6 - (13*C*a^2*c^2 + 15*A*a*c^3)*d^4*
e^2 + 3*(11*C*a^3*c - 15*A*a^2*c^2)*d^2*e^4 - 3*(C*a^4 - 5*A*a^3*c)*e^6)*a
rctan(c*x/sqrt(a*c))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4
 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5 + 12*B*a^3
*c*d^3*e^2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c + A*a^2*c^2)*d^4*e
+ 20*(C*a^4 - A*a^3*c)*d^2*e^3 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 -
 (C*a*c^3 + 3*A*c^4)*d^4*e + 4*(5*C*a^2*c^2 - 3*A*a*c^3)*d^2*e^3 - 3*(C*a^
3*c - 5*A*a^2*c^2)*e^5)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - 4*B
*a^3*c*e^5 - (C*a*c^3 + 3*A*c^4)*d^5 + 4*(C*a^2*c^2 - 3*A*a*c^3)*d^3*e^2 +
 (5*C*a^3*c - 9*A*a^2*c^2)*d*e^4)*x^3 + (10*B*a^2*c^2*d^3*e^2 - 38*B*a^3*c
*d*e^4 - (7*C*a^2*c^2 + 5*A*a*c^3)*d^4*e + 4*(9*C*a^3*c - 7*A*a^2*c^2)*d^2
*e^3 - 5*(C*a^4 - 5*A*a^3*c)*e^5)*x^2 - (6*B*a^3*c*d^2*e^3 + 6*B*a^4*e^5 -
 (C*a^2*c^2 - 5*A*a*c^3)*d^5 - 8*(C*a^3*c - 2*A*a^2*c^2)*d^3*e^2 - (7*C*a^
4 - 11*A*a^3*c)*d*e^4)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e
^4 + a^7*d*e^6 + (a^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5...
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(555) = 1110$.

Time = 0.30 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.06

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2 (a + cx^2)^3}\, dx$$
$$= -\frac{(4\,Ccd^3e^3 - 5\,Bcd^2e^4 - 2\,Cade^5 + 6\,Acde^5 + Bae^6)\log\left(c - \frac{2\,cd}{ex+d} + \frac{cd^2}{(ex+d)^2} + \frac{ae^2}{(ex+d)^2}\right)}{2\,(c^4d^8 + 4\,ac^3d^6e^2 + 6\,a^2c^2d^4e^4 + 4\,a^3cd^2e^6 + a^4e^8)}$$
$$- \frac{\frac{Cd^2e^9}{ex+d} - \frac{Bde^{10}}{ex+d} + \frac{Ae^{11}}{ex+d}}{c^3d^6e^6 + 3\,ac^2d^4e^8 + 3\,a^2cd^2e^{10} + a^3e^{12}}$$
$$+ \frac{(Cac^3d^6e^2 + 3\,Ac^4d^6e^2 - 2\,Bac^3d^5e^3 + 13\,Ca^2c^2d^4e^4 + 15\,Aac^3d^4e^4 - 20\,Ba^2c^2d^3e^5 - 33\,Ca^3cd^2e^6 + 4}{8\,(a^2c^4d^8 + 4\,a^3c^3d^6e^2 + 6\,a^4c^2d^4e^4 + 4\,a^5cd^2e}$$
$$+ \frac{Cac^4d^5e + 3\,Ac^5d^5e - 2\,Bac^4d^4e^2 - 22\,Ca^2c^3d^3e^3 + 14\,Aac^4d^3e^3 + 32\,Ba^2c^3d^2e^4 + 17\,Ca^3c^2de^5 - 29\,A}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*e^5 + B*a*e^6)
*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^4*d^8
 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) - (C*d
^2*e^9/(e*x + d) - B*d*e^10/(e*x + d) + A*e^11/(e*x + d))/(c^3*d^6*e^6 + 3
*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^3*d^6*e^2 + 3*A
*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d^4*e^4 + 15*A*a*c^3*d^4*e
^4 - 20*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 45*A*a^2*c^2*d^2*e^6 + 30
*B*a^3*c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arctan((c*d - c*d^2/(e*x +
d) - a*e^2/(e*x + d))/(sqrt(a*c)*e))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6
*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)*e^2) + 1/8*(C*a*c^
4*d^5*e + 3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*
a*c^4*d^3*e^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d
*e^5 - 6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*
d^5*e^3 - 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^
5 + 77*C*a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*
C*a^4*c*e^8 + 7*A*a^3*c^2*e^8)/((e*x + d)*e) + (3*C*a*c^4*d^7*e^3 + 9*A*c^
5*d^7*e^3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5
+ 140*B*a^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 2
2*B*a^3*c^2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^
10)/((e*x + d)^2*e^2) - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*...
```

3.62.9 Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 6848, normalized size of antiderivative = 11.99

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\,(a+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^2),x)
```

output
```
symsum(log(root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 +
14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^1
2*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a
^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*
c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 3
28*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^
4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*z + 168*A*
C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*c^3*d^3*e
^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180*A*B*a^2
*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z + 4239
*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4*d^8*
e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820*B^2*
a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z
+ 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4*c^
4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90*
A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2*
a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a
^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4
 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c
*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2...
```

3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3 (a+cx^2)^3} dx$

3.63.1 Optimal result

Integrand size = 27, antiderivative size = 753

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3 (a+cx^2)^3} dx$$
$$= -\frac{e^3(Cd^2-Bde+Ae^2)}{2(cd^2+ae^2)^3(d+ex)^2} + \frac{e^3(ae^2(2Cd-Be)-cd(4Cd^2-e(5Bd-6Ae)))}{(cd^2+ae^2)^4(d+ex)}$$
$$-\frac{a(Bcd(cd^2-3ae^2)-(Ac-aC)e(3cd^2-ae^2))-c(Acd(cd^2-3ae^2)-a(cd^2(Cd-3Be)-ae^2(3Cd-}{4a(cd^2+ae^2)^3(a+cx^2)^2}$$
$$+\frac{4a^2e(a^2Ce^4+c^2d^2(3Cd^2-2e(3Bd-5Ae))-2ace^2(4Cd^2-e(3Bd-Ae)))+c(3Acd(c^2d^4+6acd^2e^2}{8a^2(cd^2+ae^2)^4(a+cx}$$
$$+\frac{\sqrt{c}(3Acd(c^3d^6+7ac^2d^4e^2+35a^2cd^2e^4-35a^3e^6)+a(ac^2d^4e^2(23Cd-45Be)-5a^2cd^2e^4(25Cd-27Be}{8a^{5/2}(cd^2+ae^2)^5}$$
$$+\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2d^2(10Cd^2-3e(5Bd-7Ae)))\log(d+ex)}{(cd^2+ae^2)^5}$$
$$-\frac{e^3(a^2Ce^4-ace^2(13Cd^2-9Bde+3Ae^2)+c^2d^2(10Cd^2-3e(5Bd-7Ae)))\log(a+cx^2)}{2(cd^2+ae^2)^5}$$

output
```
-1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2+e^3*(a*e^2*(-B*e+2*
C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))/(a*e^2+c*d^2)^4/(e*x+d)+1/4*(-a*(B*c*
d*(-3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)
-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)
^2+1/8*(4*a^2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*
(4*C*d^2-e*(-A*e+3*B*d)))+c*(3*A*c*d*(-11*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-a
*(2*a*c*d^2*e^2*(-19*B*e+13*C*d)-c^2*d^4*(-3*B*e+C*d)-7*a^2*e^4*(-B*e+3*C*
d)))*x)/a^2/(a*e^2+c*d^2)^4/(c*x^2+a)+e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*
d*e+13*C*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7*A*e+5*B*d)))*ln(e*x+d)/(a*e^2+c*d^
2)^5-1/2*e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C*d^2)+c^2*d^2*(10*C*d
^2-3*e*(-7*A*e+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^5+1/8*(3*A*c*d*(-35*a^3*
e^6+35*a^2*c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)+a*(a*c^2*d^4*e^2*(-45*B*e+23
*C*d)-5*a^2*c*d^2*e^4*(-27*B*e+25*C*d)+c^3*d^6*(-3*B*e+C*d)+15*a^3*e^6*(-B
*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^2+c*d^2)^5
```

3.63.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 672, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\,(a+cx^2)^3}\,dx$$

$$= \frac{-\frac{4e^3(cd^2+ae^2)^2(Cd^2+e(-Bd+Ae))}{(d+ex)^2} - \frac{8e^3(cd^2+ae^2)(4cCd^3+cde(-5Bd+6Ae)+ae^2(-2Cd+Be))}{d+ex} + \frac{2(cd^2+ae^2)^2(a^3Ce^3+Ac^3d^3x-ac^2d\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

output
```
((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^
3*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*
e)))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(
C*d^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) +
 e*(-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C
*e^5 + 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*
e^3*(C*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2
*C*d^2*(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x)
)))/(a^2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*
a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c
*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d +
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*
C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e))
)*Log[d + e*x] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) +
c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^5)
```

3.63.3 Rubi [A] (verified)

Time = 3.75 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2178, 25, 2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^3(d+ex)^3}\,dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{\frac{3c^2e^3\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x^3}{\left(cd^2+ae^2\right)^3}+\frac{ce^2\left(Ac\left(9c^2d^4-15ace^2d^2-4a^2e^4\right)+a\left(4a^2Ce^4+3acd(5Cd+Be)e^2-c^2d^3(9Cd-23Be)\right)\right)\cdots}{\left(cd^2+ae^2\right)^3}\cdots}{\cdots}}{\cdots}\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\frac{3c^2e^3\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x^3}{\left(cd^2+ae^2\right)^3}+\frac{ce^2\left(Ac\left(9c^2d^4-15ace^2d^2-4a^2e^4\right)+a\left(4a^2Ce^4+3acd(5Cd+Be)e^2-c^2d^3(9Cd-23Be)\right)\right)x^2}{\left(cd^2+ae^2\right)^3}+\cdots}{(d+e\cdots}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow 2178$$

$$\frac{c(4e(a^2Ce^4-2ac(4Cd^2-e(3Bd-Ae))e^2+c^2(3Cd^4-2d^2e(3Bd-5Ae)))a^2+c(3Acd(c^2d^4+6ace^2d^2-11a^2e^4)-a(-c^2(Cd-3Be)d^4+2ace^2(13Cd-\cdots}{2a(cd^2+ae^2)^4(cx^2+a)}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^3\left(cx^2+a\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{c(4e(a^2Ce^4-2ac(4Cd^2-e(3Bd-Ae))e^2+c^2(3Cd^4-2d^2e(3Bd-5Ae)))a^2+c(3Acd(c^2d^4+6ace^2d^2-11a^2e^4)-a(-c^2(Cd-3Be)d^4+2ace^2(13Cd-\cdots}{2a(cd^2+ae^2)^4(cx^2+a)}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^3\left(cx^2+a\right)^2}$$

$$\Big\downarrow 2160$$

$$\frac{\int\left(\frac{8a^2c^2\left(a^2Ce^4-ac\left(13Cd^2-9Bed+3Ae^2\right)e^2+c^2\left(10Cd^4-3d^2e(5Bd-7Ae)\right)\right)e^4}{\left(cd^2+ae^2\right)^5(d+ex)}+\frac{8a^2c^2\left(4cCd^3-ce(5Bd-6Ae)d-ae^2(2Cd-Be)\right)e^4}{\left(cd^2+ae^2\right)^4(d+ex)^2}+\frac{8a^2c^2\left(Cd^2-Bed+Ae^2\right)\cdots}{\left(cd^2+ae^2\right)^3(d+ex)^3}\cdots}{\cdots}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^3}$$

$$\Big\downarrow 2009$$

$$\frac{c(cx(3Acd(-11a^2e^4+6acd^2e^2+c^2d^4)-a(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)))+4a^2e(a^2Ce^4-2ace^2(4Cd^2-e(3Bd-Ae)\cdots}{2a(a+cx^2)(ae^2+cd^2)^4}$$

$$\frac{a\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-cx\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-3Be)-ae^2(3Cd-Be)\right)\right)}{4a\left(a+cx^2\right)^2\left(ae^2+cd^2\right)^3}$$

3.63. $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^3}\,dx$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

output
```
-1/4*(a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c*(A
*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*x)
/(a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + ((c*(4*a^2*e*(a^2*C*e^4 + c^2*(3*C*
d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e))) +
c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*C
*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x))/(2*a*
(c*d^2 + a*e^2)^4*(a + c*x^2)) + ((-4*a^2*c^2*e^3*(C*d^2 - B*d*e + A*e^2))
/((c*d^2 + a*e^2)^3*(d + e*x)^2) - (8*a^2*c^2*e^3*(4*c*C*d^3 - c*d*e*(5*B*
d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^4*(d + e*x)) + (c^(5/2
)*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a
*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^
3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a
]])/(Sqrt[a]*(c*d^2 + a*e^2)^5) + (8*a^2*c^2*e^3*(a^2*C*e^4 - a*c*e^2*(13*
C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d - 7*A*e)))*Log
[d + e*x])/(c*d^2 + a*e^2)^5 - (4*a^2*c^2*e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d
^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d - 7*A*e)))*Log[a
+ c*x^2])/(c*d^2 + a*e^2)^5)/(2*a*c))/(4*a*c)
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 1055, normalized size of antiderivative = 1.40

method	result	size
default	Expression too large to display	1055
risch	Expression too large to display	104197

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^2+c*d^2)^5*((1/8*c*(33*A*a^3*c*d*e^6+15*A*a^2*c^2*d^3*e^4-21*A*a*c
^3*d^5*e^2-3*A*c^4*d^7+7*B*a^4*e^7-31*B*a^3*c*d^2*e^5-35*B*a^2*c^2*d^4*e^3
+3*B*a*c^3*d^6*e-21*C*a^4*d*e^6+5*C*a^3*c*d^3*e^4+25*C*a^2*c^2*d^5*e^2-C*a
*c^3*d^7)/a^2*x^3+(A*a^2*c*e^7-4*A*a*c^2*d^2*e^5-5*A*c^3*d^4*e^3-3*B*a^2*c
*d*e^6+3*B*c^3*d^5*e^2-1/2*C*a^3*e^7+7/2*C*a^2*c*d^2*e^5+5/2*C*a*c^2*d^4*e
^3-3/2*C*c^3*d^6*e)*x^2+1/8*(39*A*a^3*c*d*e^6+25*A*a^2*c^2*d^3*e^4-19*A*a*
c^3*d^5*e^2-5*A*c^4*d^7+9*B*a^4*e^7-33*B*a^3*c*d^2*e^5-45*B*a^2*c^2*d^4*e^
3-3*B*a*c^3*d^6*e-27*C*a^4*d*e^6-5*C*a^3*c*d^3*e^4+23*C*a^2*c^2*d^5*e^2+C*
a*c^3*d^7)/a*x+1/4*(5*A*a^3*c*e^7-17*A*a^2*c^2*d^2*e^5-25*A*a*c^3*d^4*e^3-
3*A*c^4*d^6*e-15*B*a^3*c*d*e^6-5*B*a^2*c^2*d^3*e^4+11*B*a*c^3*d^5*e^2+B*c^
4*d^7-3*C*a^4*e^7+15*C*a^3*c*d^2*e^5+15*C*a^2*c^2*d^4*e^3-3*C*a*c^3*d^6*e)
/c)/(c*x^2+a)^2+1/8/a^2*(1/2*(-24*A*a^3*c*e^7+168*A*a^2*c^2*d^2*e^5+72*B*a
^3*c*d*e^6-120*B*a^2*c^2*d^3*e^4+8*C*a^4*e^7-104*C*a^3*c*d^2*e^5+80*C*a^2*
c^2*d^4*e^3)/c*ln(c*x^2+a)+(105*A*a^3*c*d*e^6-105*A*a^2*c^2*d^3*e^4-21*A*a
*c^3*d^5*e^2-3*A*c^4*d^7+15*B*a^4*e^7-135*B*a^3*c*d^2*e^5+45*B*a^2*c^2*d^4
*e^3+3*B*a*c^3*d^6*e-45*C*a^4*d*e^6+125*C*a^3*c*d^3*e^4-23*C*a^2*c^2*d^5*e
^2-C*a*c^3*d^7)/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))))-e^3*(6*A*c*d*e^2+B*a
*e^3-5*B*c*d^2*e-2*C*a*d*e^2+4*C*c*d^3)/(a*e^2+c*d^2)^4/(e*x+d)-1/2*e^3*(A
*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2-e^3*(3*A*a*c*e^4-21*A*c^2*d^2*
e^2-9*B*a*c*d*e^3+15*B*c^2*d^3*e-C*a^2*e^4+13*C*a*c*d^2*e^2-10*C*c^2*d^...
```

3.63.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

output
```
Timed out
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. $2(732) = 1464$.

Time = 0.33 (sec) , antiderivative size = 1835, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21
*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(c*x^2 + a)/(c^5*d^10 + 5*a*c^
4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^
5*e^10) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c
 - 21*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(e*x + d)/(c^5*d^10 + 5*a
*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 +
 a^5*e^10) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*c^2*d
^2*e^5 + 15*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 - (23*C*a^2*c^3 + 21*A*a
*c^4)*d^5*e^2 + 5*(25*C*a^3*c^2 - 21*A*a^2*c^3)*d^3*e^4 - 15*(3*C*a^4*c -
7*A*a^3*c^2)*d*e^6)*arctan(c*x/sqrt(a*c))/((a^2*c^5*d^10 + 5*a^3*c^4*d^8*e
^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^10)
*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74*B*a^4*c*d^3
*e^4 + 4*B*a^5*d*e^6 + 4*A*a^5*e^7 - 6*(C*a^3*c^2 + A*a^2*c^3)*d^6*e + 4*(
18*C*a^4*c - 11*A*a^3*c^2)*d^4*e^3 - 2*(9*C*a^5 - 31*A*a^4*c)*d^2*e^5 + (3
*B*a*c^4*d^4*e^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C*a*c^4 + 3*
A*c^5)*d^5*e^2 + 2*(29*C*a^2*c^3 - 9*A*a*c^4)*d^3*e^4 - (37*C*a^3*c^2 - 81
*A*a^2*c^3)*d*e^6)*x^5 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B
*a^3*c^2*d*e^6 - (C*a*c^4 + 3*A*c^5)*d^6*e + 2*(19*C*a^2*c^3 - 9*A*a*c^4)*
d^4*e^3 - (11*C*a^3*c^2 - 39*A*a^2*c^3)*d^2*e^5 - 2*(C*a^4*c - 3*A*a^3*c^2
)*e^7)*x^4 + (3*B*a*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2...
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. 2(732) = 1464.

Time = 0.28 (sec) , antiderivative size = 1614, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^
2*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10
+ 5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*
e^8 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6
+ 21*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(e*x
+ d))/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*
e^7 + 5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a
*c^4*d^6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*
e^3 - 125*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^
5 + 45*C*a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sq
rt(a*c))/((a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*
c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*e^
2*x^5 + 3*A*c^5*d^5*e^2*x^5 - 3*B*a*c^4*d^4*e^3*x^5 - 58*C*a^2*c^3*d^3*e^4
*x^5 + 18*A*a*c^4*d^3*e^4*x^5 + 78*B*a^2*c^3*d^2*e^5*x^5 + 37*C*a^3*c^2*d*
e^6*x^5 - 81*A*a^2*c^3*d*e^6*x^5 - 15*B*a^3*c^2*e^7*x^5 + 2*C*a*c^4*d^6*e*
x^4 + 6*A*c^5*d^6*e*x^4 - 6*B*a*c^4*d^5*e^2*x^4 - 76*C*a^2*c^3*d^4*e^3*x^4
 + 36*A*a*c^4*d^4*e^3*x^4 + 96*B*a^2*c^3*d^3*e^4*x^4 + 22*C*a^3*c^2*d^2*e^
5*x^4 - 78*A*a^2*c^3*d^2*e^5*x^4 + 6*B*a^3*c^2*d*e^6*x^4 + 4*C*a^4*c*e^7*x
^4 - 12*A*a^3*c^2*e^7*x^4 + C*a*c^4*d^7*x^3 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*
d^6*e*x^3 - 3*C*a^2*c^3*d^5*e^2*x^3 + 23*A*a*c^4*d^5*e^2*x^3 - 7*B*a^2*...
```

3.63.9 Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 8774, normalized size of antiderivative = 11.65

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3 (a+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^3),x)
```

output
```
((x^5*(3*A*c^5*d^5*e^2 - 15*B*a^3*c^2*e^7 + 18*A*a*c^4*d^3*e^4 - 81*A*a^2*
c^3*d*e^6 - 3*B*a*c^4*d^4*e^3 + C*a*c^4*d^5*e^2 + 37*C*a^3*c^2*d*e^6 + 78*
B*a^2*c^3*d^2*e^5 - 58*C*a^2*c^3*d^3*e^4))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a
*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (2*A*a^3*e^7 + B*c^
3*d^7 + 2*B*a^3*d*e^6 - 3*A*c^3*d^6*e - 9*C*a^3*d^2*e^5 - 22*A*a*c^2*d^4*e
^3 + 31*A*a^2*c*d^2*e^5 + 10*B*a*c^2*d^5*e^2 - 37*B*a^2*c*d^3*e^4 + 36*C*a
^2*c*d^4*e^3 - 3*C*a*c^2*d^6*e)/(4*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 +
4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x*(5*A*c^4*d^7 - 8*B*a^4*e^7 - C*
a*c^3*d^7 + 28*C*a^4*d*e^6 + 26*A*a*c^3*d^5*e^2 + 91*B*a^3*c*d^2*e^5 - 77*
C*a^3*c*d^3*e^4 + 49*A*a^2*c^2*d^3*e^4 + 2*B*a^2*c^2*d^4*e^3 - 10*C*a^2*c^
2*d^5*e^2 - 68*A*a^3*c*d*e^6 - B*a*c^3*d^6*e))/(8*a*(a^4*e^8 + c^4*d^8 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^2*(3*C*a^4*e^7
 - 9*A*a^3*c*e^7 + 5*A*c^4*d^6*e + 37*A*a*c^3*d^4*e^3 - 10*B*a*c^3*d^5*e^2
 + 23*C*a^3*c*d^2*e^5 - 73*A*a^2*c^2*d^2*e^5 + 88*B*a^2*c^2*d^3*e^4 - 71*C
*a^2*c^2*d^4*e^3 + 2*B*a^3*c*d*e^6 + 5*C*a*c^3*d^6*e))/(4*a*(a^4*e^8 + c^4
*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(3*A
*c^5*d^7 - 25*B*a^4*c*e^7 + C*a*c^4*d^7 + 23*A*a*c^4*d^5*e^2 - 151*A*a^3*c
^2*d*e^6 + 61*A*a^2*c^3*d^3*e^4 - 7*B*a^2*c^3*d^4*e^3 + 163*B*a^3*c^2*d^2*
e^5 - 3*C*a^2*c^3*d^5*e^2 - 129*C*a^3*c^2*d^3*e^4 - 3*B*a*c^4*d^6*e + 67*C
*a^4*c*d*e^6))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^...
```

3.63. $\int \frac{A+Bx+Cx^2}{(d+ex)^3(a+cx^2)^3}\,dx$

3.64 $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.64.1 Optimal result

Integrand size = 27, antiderivative size = 234

$$\begin{aligned}\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac(a+cx^2)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4aBe)x)}{24a^2c^2(a+cx^2)^2} \\ &\quad - \frac{(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe))(ae-cdx)(d+ex)}{16a^3c^3(a+cx^2)} \\ &\quad + \frac{(cd^2+ae^2)(a(Ac+5aC)e^2+cd(5Acd+aCd+4aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}\end{aligned}$$

output
```
-1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^4/a/c/(c*x^2+a)^3-1/24*(e*x+d)^3*(a*(A*c+5*
C*a)*e-c*(5*A*c*d+4*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/16*(a*(A*c+5*C*a
)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*(-c*d*x+a*e)*(e*x+d)/a^3/c^3/(c*x^2+a)+
1/16*(a*e^2+c*d^2)*(a*(A*c+5*C*a)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*arctan(
x*c^(1/2)/a^(1/2))/a^(7/2)/c^(7/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.87

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{5Ac^3d^4x+ac^2d^2(Cd^2+4Bde+6Ae^2)\,x+a^2ce^2(6Cd^2+e(4Bd+Ae))\,x-a^3e^3(32Cd+8Be+11Cex)}{16a^3c^3\,(a+cx^2)}$$
$$+\frac{Ac^3d^4x-a^3e^3(4Cd+Be+Cex)-ac^2d^2(4Ade+Cd^2x+6Ae^2x+Bd(d+4ex))+a^2ce(2Cd^2(2d+3}{6ac^3\,(a+cx^2)^3}$$
$$+\frac{5Ac^3d^4x+ac^2d^2(Cd^2+4Bde+6Ae^2)\,x+a^3e^3(48Cd+12Be+13Cex)-a^2ce(6Cd^2(4d+7ex)+e(4}{24a^2c^3\,(a+cx^2)^2}$$
$$+\frac{(cd^2+ae^2)\,(Ac(5cd^2+ae^2)+a(5aCe^2+cd(Cd+4Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{7/2}}$$

input
```
Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
(5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*
d^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^
3*(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*
(4*A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d
+ 3*e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2
)^3) + (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*
(48*C*d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(
9*d + 7*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2
+ a*e^2)*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

3.64.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2176, 25, 678, 487, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4 \left(A+Bx+Cx^2\right)}{(a+cx^2)^4}\,dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(d+ex)^3(5Acd+aCd+4aBe+(Ac+5aC)ex)}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(d+ex)^3(5Acd+aCd+4aBe+(Ac+5aC)ex)}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \text{678}$$

$$\frac{\frac{3(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))\int \frac{(d+ex)^2}{(cx^2+a)^2}dx}{4ac}-\frac{(d+ex)^3(ae(5aC+Ac)-cx(4aBe+aCd+5Acd))}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \text{487}$$

$$\frac{\frac{3(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))\left(\frac{(ae^2+cd^2)\int \frac{1}{cx^2+a}dx}{2ac}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)}{4ac}-\frac{(d+ex)^3(ae(5aC+Ac)-cx(4aBe+aCd+5Acd))}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(ae^2+cd^2)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(ae-cdx)}{2ac(a+cx^2)}\right)(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac))}{4ac}-\frac{(d+ex)^3(ae(5aC+Ac)-cx(4aBe+aCd+5Acd))}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)^4(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

input
```
Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
-1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(a*c*(a + c*x^2)^3) + (-1/4*((d +
 e*x)^3*(a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(a*c*(a + c
*x^2)^2) + (3*(a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(-1/
2*((a*e - c*d*x)*(d + e*x))/(a*c*(a + c*x^2)) + ((c*d^2 + a*e^2)*ArcTan[(S
qrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))))/(4*a*c))/(6*a*c)
```

3.64. $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 678
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[m*((c*d*f + a*e*g)/(2*a*c*(p + 1)))   Int[(d + e*x)^(
m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && EqQ[S
implify[m + 2*p + 3], 0] && LtQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(218) = 436$.

Time = 0.58 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.32

method	result
default	$\frac{\left(A\,a^{2}c\,e^{4}+6Aa\,c^{2}d^{2}e^{2}+5A\,d^{4}c^{3}+4B\,a^{2}cd\,e^{3}+4Ba\,c^{2}d^{3}e-11C\,a^{3}e^{4}+6C\,a^{2}c\,d^{2}e^{2}+Ca\,c^{2}d^{4}\right)x^{5}}{16a^{3}c}-\frac{e^{3}(Be+4Cd)x^{4}}{2c}-\frac{\left(A\,a^{2}c\,e^{4}-6Aa\,c^{2}d^{2}e^{2}-5A\,d\right.}{}$
risch	$\frac{\left(A\,a^{2}c\,e^{4}+6Aa\,c^{2}d^{2}e^{2}+5A\,d^{4}c^{3}+4B\,a^{2}cd\,e^{3}+4Ba\,c^{2}d^{3}e-11C\,a^{3}e^{4}+6C\,a^{2}c\,d^{2}e^{2}+Ca\,c^{2}d^{4}\right)x^{5}}{16a^{3}c}-\frac{e^{3}(Be+4Cd)x^{4}}{2c}-\frac{\left(A\,a^{2}c\,e^{4}-6Aa\,c^{2}d^{2}e^{2}-5A\,d\right.}{}$

input
```
int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2
*d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+
4*C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^
3-4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1
/2*e*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*
(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*
e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*
A*c^2*d^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*
e)/c^3)/(c*x^2+a)^3+1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^
2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c
^3/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(217) = 434$.

Time = 0.37 (sec) , antiderivative size = 1864, normalized size of antiderivative = 7.97

$$\int \frac{(d+ex)^4\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*B*a^4*c^3*d^4 + 48*B*a^5*c^2*d^2*e^2 + 16*B*a^6*c*e^4 - 6*(4*B*
a^2*c^5*d^3*e + 4*B*a^3*c^4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3
*c^4 + A*a^2*c^5)*d^2*e^2 - (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 32*(C*a^
5*c^2 + 2*A*a^4*c^3)*d^3*e + 32*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 48*(4*C*a^
4*c^3*d*e^3 + B*a^4*c^3*e^4)*x^4 - 16*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e
^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (
5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^3 + 48*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^
2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^5*c^2 + A*a^4*c^3)*d*e^3)*x^2 + 3*(4*B*a^
4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*
a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*
a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2
)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 + 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d
*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (
5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e
^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (
5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
a)/(c*x^2 + a)) + 6*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4*c^3 -
11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c + A*a^5
*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/
48*(8*B*a^4*c^3*d^4 + 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2...
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

output
```
Timed out
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(217) = 434$.

Time = 0.28 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4} dx = -\frac{8 Ba^3c^2d^4 + 24 Ba^4cd^2e^2 + 8 Ba^5e^4 - 3 (4 Bac^4d^3e + 4 Ba^2c^3de^3 + (Cac^4 + 5 Ac^5)d^4 + 6 (Ca^2c^3 + Aa}{} + \frac{(4 Bac^2d^3e + 4 Ba^2cde^3 + (Cac^2 + 5 Ac^3)d^4 + 6 (Ca^2c + Aac^2)d^2e^2 + (5 Ca^3 + Aa^2c)e^4) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16 \sqrt{ac}a^3c^3}$$

input
```
integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d
^3*e + 4*B*a^2*c^3*d*e^3 + (C*a*c^4 + 5*A*c^5)*d^4 + 6*(C*a^2*c^3 + A*a*c^
4)*d^2*e^2 - (11*C*a^3*c^2 - A*a^2*c^3)*e^4)*x^5 + 16*(C*a^4*c + 2*A*a^3*c
^2)*d^3*e + 16*(2*C*a^5 + A*a^4*c)*d*e^3 + 24*(4*C*a^3*c^2*d*e^3 + B*a^3*c
^2*e^4)*x^4 - 8*(4*B*a^2*c^3*d^3*e - 4*B*a^3*c^2*d*e^3 + (C*a^2*c^3 + 5*A*
a*c^4)*d^4 - 6*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2 - (5*C*a^4*c + A*a^3*c^2)*e
^4)*x^3 + 24*(2*C*a^3*c^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2
*C*a^4*c + A*a^3*c^2)*d*e^3)*x^2 + 3*(4*B*a^3*c^2*d^3*e + 4*B*a^4*c*d*e^3
+ (C*a^3*c^2 - 11*A*a^2*c^3)*d^4 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e^2 + (5*C*
a^5 + A*a^4*c)*e^4)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*
c^3) + 1/16*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2 + 5*A*c^3)*d^4 +
 6*(C*a^2*c + A*a*c^2)*d^2*e^2 + (5*C*a^3 + A*a^2*c)*e^4)*arctan(c*x/sqrt(
a*c))/(sqrt(a*c)*a^3*c^3)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(217) = 434$.

Time = 0.27 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.82

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4} dx = \frac{(Cac^2d^4 + 5 Ac^3d^4 + 4 Bac^2d^3e + 6 Ca^2cd^2e^2 + 6 Aac^2d^2e^2 + 4 Ba^2cde^3 + 5 Ca^3e^4 + Aa^2ce^4) \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16 \sqrt{ac}a^3c^3} + \frac{3 Cac^4d^4x^5 + 15 Ac^5d^4x^5 + 12 Bac^4d^3ex^5 + 18 Ca^2c^3d^2e^2x^5 + 18 Aac^4d^2e^2x^5 + 12 Ba^2c^3de^3x^5 - 33 Ca}{}$$

input
```
integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*
A*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/
sqrt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^
5 + 12*B*a*c^4*d^3*e*x^5 + 18*C*a^2*c^3*d^2*e^2*x^5 + 18*A*a*c^4*d^2*e^2*x
^5 + 12*B*a^2*c^3*d*e^3*x^5 - 33*C*a^3*c^2*e^4*x^5 + 3*A*a^2*c^3*e^4*x^5 -
 96*C*a^3*c^2*d*e^3*x^4 - 24*B*a^3*c^2*e^4*x^4 + 8*C*a^2*c^3*d^4*x^3 + 40*
A*a*c^4*d^4*x^3 + 32*B*a^2*c^3*d^3*e*x^3 - 48*C*a^3*c^2*d^2*e^2*x^3 + 48*A
*a^2*c^3*d^2*e^2*x^3 - 32*B*a^3*c^2*d*e^3*x^3 - 40*C*a^4*c*e^4*x^3 - 8*A*a
^3*c^2*e^4*x^3 - 48*C*a^3*c^2*d^3*e*x^2 - 72*B*a^3*c^2*d^2*e^2*x^2 - 96*C*
a^4*c*d*e^3*x^2 - 48*A*a^3*c^2*d*e^3*x^2 - 24*B*a^4*c*e^4*x^2 - 3*C*a^3*c^
2*d^4*x + 33*A*a^2*c^3*d^4*x - 12*B*a^3*c^2*d^3*e*x - 18*C*a^4*c*d^2*e^2*x
 - 18*A*a^3*c^2*d^2*e^2*x - 12*B*a^4*c*d*e^3*x - 15*C*a^5*e^4*x - 3*A*a^4*
c*e^4*x - 8*B*a^3*c^2*d^4 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a
^4*c*d^2*e^2 - 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 +
a)^3*a^3*c^3)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^4 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x\,(c\,d^2+a\,e^2)\,(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2)}{\sqrt{a}\,(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4)}\right)\,(c\,d^2+a\,e^2)\,(5\,C\,a^2\,e^2}{16\,a^{7/2}\,c^{7/2}}$$
$$-\frac{\frac{4\,C\,a^2\,d\,e^3+B\,a^2\,e^4+2\,C\,a\,c\,d^3\,e+3\,B\,a\,c\,d^2\,e^2+2\,A\,a\,c\,d\,e^3+B\,c^2\,d^4+4\,A\,c^2\,d^3\,e}{6\,c^3}+\frac{x^2\,(B\,a\,e^4+2\,A\,c\,d\,e^3+4\,C\,a\,d\,e^3+2\,C\,c\,d^3\,e+3\,B\,c\,d^2\,e^2)}{2\,c^2}}{}$$

input
```
int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

output
```
(atan((c^(1/2)*x*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 +
C*a*c*d^2 + 4*B*a*c*d*e))/(a^(1/2)*(5*A*c^3*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^
4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e
+ 4*B*a^2*c*d*e^3)))*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^
2 + C*a*c*d^2 + 4*B*a*c*d*e))/(16*a^(7/2)*c^(7/2)) - ((B*a^2*e^4 + B*c^2*d
^4 + 4*A*c^2*d^3*e + 4*C*a^2*d*e^3 + 2*A*a*c*d*e^3 + 2*C*a*c*d^3*e + 3*B*a
*c*d^2*e^2)/(6*c^3) + (x^2*(B*a*e^4 + 2*A*c*d*e^3 + 4*C*a*d*e^3 + 2*C*c*d^
3*e + 3*B*c*d^2*e^2))/(2*c^2) + (x^4*(B*e^4 + 4*C*d*e^3))/(2*c) + (x*(5*C*
a^3*e^4 - 11*A*c^3*d^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6
*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a*c^3) - (x^3*(
5*A*c^3*d^4 - 5*C*a^3*e^4 - A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2
- 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e - 4*B*a^2*c*d*e^3))/(6*a^2*c^2) - (x
^5*(5*A*c^3*d^4 - 11*C*a^3*e^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2
*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a^3*c))
/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.64. $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.65 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.65.1 Optimal result

Integrand size = 27, antiderivative size = 254

$$\begin{aligned}\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac(a+cx^2)^3}-\frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+3aBe)x)}{24a^2c^2(a+cx^2)^2}\\ &\quad-\frac{(d+ex)(ae(5Acd+aCd+3aBe)-(4a(Ac+2aC)e^2+3cd(5Acd+aCd+3aBe))x)}{48a^3c^2(a+cx^2)}\\ &\quad+\frac{(Acd(5cd^2+3ae^2)+a(ae^2(3Cd+Be)+cd^2(Cd+3Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}\end{aligned}$$

output
```
-1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^3-1/24*(e*x+d)^2*(2*a*(A*c+
2*C*a)*e-c*(5*A*c*d+3*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/48*(e*x+d)*(a*
e*(5*A*c*d+3*B*a*e+C*a*d)-(4*a*(A*c+2*C*a)*e^2+3*c*d*(5*A*c*d+3*B*a*e+C*a*
d))*x)/a^3/c^2/(c*x^2+a)+1/16*(A*c*d*(3*a*e^2+5*c*d^2)+a*(a*e^2*(B*e+3*C*d
)+c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/c^(5/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$
$$= \frac{-\frac{3\sqrt{a}\left(8a^3Ce^3-5Ac^3d^3x-a^2ce^2(3Cd+Be)x-ac^2d\left(Cd^2+3e(Bd+Ae)\right)x\right)}{a+cx^2}-\frac{8a^{5/2}\left(a^3Ce^3-Ac^3d^3x+ac^2d\left(Cd^2x+3Ae(d+ex)+Bd(d+3ex)\right)-\right.}{\left(a+cx^2\right)^3}}{}$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*
c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 -
A*c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*
c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/
2)*(12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a
^2*c*e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^
2 + 3*Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*
(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2176, 25, 685, 675, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$\Big\downarrow \ 2176$$

$$-\frac{\int -\frac{(d+ex)^2(5Acd+aCd+3aBe+2(Ac+2aC)ex)}{\left(cx^2+a\right)^3}dx}{6ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(d+ex)^2(5Acd+aCd+3aBe+2(Ac+2aC)ex)}{\left(cx^2+a\right)^3}dx}{6ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \; 685$$

$$\frac{\frac{\int \frac{(d+ex)\left(4a(Ac+2aC)e^2+c(5Acd+aCd+3aBe)xe+3cd(5Acd+aCd+3aBe)\right)}{\left(cx^2+a\right)^2}dx}{4ac}-\frac{(d+ex)^2(2ae(2aC+Ac)-cx(3aBe+aCd+5Acd))}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 675$$

$$\frac{\frac{\frac{3\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)\int \frac{1}{cx^2+a}dx}{2a}-\frac{2e\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)}{c\left(a+cx^2\right)}+\frac{x\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be)+\cdots\right)\right)}{2a\left(a+cx^2\right)}}{4ac}}{6ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{2a^{3/2}\sqrt{c}}-\frac{2e\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)}{c\left(a+cx^2\right)}+\frac{x\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be\cdots\right)\right)}{2a\left(a+cx^2\right)}}{4ac}}{6ac}-\frac{(d+ex)^3(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

input

```
Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output

```
-1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(a*c*(a + c*x^2)^3) + (-1/4*((d +
 e*x)^2*(2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(a*c*(a +
 c*x^2)^2) + ((-2*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B
*e))))/(c*(a + c*x^2)) + ((A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*
B*e) + 3*c*d^2*(C*d + 3*B*e)))*x)/(2*a*(a + c*x^2)) + (3*(A*c*d*(5*c*d^2 +
 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]))/(4*a*c))/(6*a*c)
```

3.65. $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 685
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[(d + e*x)^(m - 1)*(a + c*x^2)
^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m]
 || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.65. $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.65.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.51

method	result
default	$\frac{\left(3Aacd\,e^2+5A\,d^3c^2+a^2B\,e^3+3Bac\,d^2e+3C\,a^2d\,e^2+Cac\,d^3\right)x^5}{16a^3}-\frac{C\,e^3x^4}{2c}+\frac{\left(3Aacd\,e^2+5A\,d^3c^2-a^2B\,e^3+3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)x^3}{6c\,a^2}-$ e(
risch	$\frac{\left(3Aacd\,e^2+5A\,d^3c^2+a^2B\,e^3+3Bac\,d^2e+3C\,a^2d\,e^2+Cac\,d^3\right)x^5}{16a^3}-\frac{C\,e^3x^4}{2c}+\frac{\left(3Aacd\,e^2+5A\,d^3c^2-a^2B\,e^3+3Bac\,d^2e-3C\,a^2d\,e^2+Cac\,d^3\right)x^3}{6c\,a^2}-$ e(

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a
*c*d^3)/a^3*x^5-1/2*C*e^3*x^4/c+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3
*B*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/c/a^2*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2
*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B
*a*c*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a/c^2*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+
3*B*a*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+1/16
*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*c*d^
3)/a^3/c^2/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(237) = 474.

Time = 0.47 (sec) , antiderivative size = 1378, normalized size of antiderivative = 5.43

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(48*C*a^4*c^2*e^3*x^4 + 16*B*a^4*c^2*d^3 + 24*B*a^5*c*d*e^2 - 6*(3*
B*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c
^3 + A*a^2*c^4)*d*e^2)*x^5 + 24*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 8*(2*C*a^6
 + A*a^5*c)*e^3 - 16*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A
*a^2*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 24*(3*C*a^4*c^2*d^2
*e + 3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 + 3*(3*B*a^4*c*d
^2*e + B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*
d^3 + 3*(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^
2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4
 + (C*a^4*c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^
2*d^2*e + B*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3
*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
 + 6*(3*B*a^4*c^2*d^2*e + B*a^5*c*e^3 + (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3
*(C*a^5*c + A*a^4*c^2)*d*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*
x^2 + a^7*c^3), -1/48*(24*C*a^4*c^2*e^3*x^4 + 8*B*a^4*c^2*d^3 + 12*B*a^5*c
*d*e^2 - 3*(3*B*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^
3 + 3*(C*a^3*c^3 + A*a^2*c^4)*d*e^2)*x^5 + 12*(C*a^5*c + 2*A*a^4*c^2)*d^2*
e + 4*(2*C*a^6 + A*a^5*c)*e^3 - 8*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*
a^3*c^3 + 5*A*a^2*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 12*(3*
C*a^4*c^2*d^2*e + 3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 ...
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4} \, dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

output
```
Timed out
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx =$$
$$-\frac{24\,Ca^3c^2e^3x^4+8\,Ba^3c^2d^3+12\,Ba^4cde^2-3\,(3\,Bac^4d^2e+Ba^2c^3e^3+(Cac^4+5\,Ac^5)d^3+3\,(Ca^2c^3+A}{}$$
$$+\frac{(3\,Bacd^2e+Ba^2e^3+(Cac+5\,Ac^2)d^3+3\,(Ca^2+Aac)de^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(24*C*a^3*c^2*e^3*x^4 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 - 3*(3*B*
a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*(C*a^2*c^3 + A*a
*c^4)*d*e^2)*x^5 + 12*(C*a^4*c + 2*A*a^3*c^2)*d^2*e + 4*(2*C*a^5 + A*a^4*c
)*e^3 - 8*(3*B*a^2*c^3*d^2*e - B*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3
 - 3*(C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 12*(3*C*a^3*c^2*d^2*e + 3*B*a^3*
c^2*d*e^2 + (2*C*a^4*c + A*a^3*c^2)*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*e + B*a^
4*c*e^3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)*
x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(3*B*a*c
*d^2*e + B*a^2*e^3 + (C*a*c + 5*A*c^2)*d^3 + 3*(C*a^2 + A*a*c)*d*e^2)*arct
an(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(237) = 474.

Time = 0.27 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$=\frac{(Cacd^3+5\,Ac^2d^3+3\,Bacd^2e+3\,Ca^2de^2+3\,Aacde^2+Ba^2e^3)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$
$$+\frac{3\,Cac^4d^3x^5+15\,Ac^5d^3x^5+9\,Bac^4d^2ex^5+9\,Ca^2c^3de^2x^5+9\,Aac^4de^2x^5+3\,Ba^2c^3e^3x^5-24\,Ca^3c^2e^3x^4}{}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*
e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c
^4*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*e*x^5 + 9*C*a^2*c^3*d*e^2*x^
5 + 9*A*a*c^4*d*e^2*x^5 + 3*B*a^2*c^3*e^3*x^5 - 24*C*a^3*c^2*e^3*x^4 + 8*C
*a^2*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 24*B*a^2*c^3*d^2*e*x^3 - 24*C*a^3*
c^2*d*e^2*x^3 + 24*A*a^2*c^3*d*e^2*x^3 - 8*B*a^3*c^2*e^3*x^3 - 36*C*a^3*c^
2*d^2*e*x^2 - 36*B*a^3*c^2*d*e^2*x^2 - 24*C*a^4*c*e^3*x^2 - 12*A*a^3*c^2*e
^3*x^2 - 3*C*a^3*c^2*d^3*x + 33*A*a^2*c^3*d^3*x - 9*B*a^3*c^2*d^2*e*x - 9*
C*a^4*c*d*e^2*x - 9*A*a^3*c^2*d*e^2*x - 3*B*a^4*c*e^3*x - 8*B*a^3*c^2*d^3
- 12*C*a^4*c*d^2*e - 24*A*a^3*c^2*d^2*e - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 -
 4*A*a^4*c*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3)}{16\,a^{7/2}\,c^{5/2}}$$

$$-\frac{\frac{2\,C\,a^2\,e^3+3\,C\,a\,c\,d^2\,e+3\,B\,a\,c\,d\,e^2+A\,a\,c\,e^3+2\,B\,c^2\,d^3+6\,A\,c^2\,d^2\,e}{12\,c^3}+\frac{x^2\,(A\,c\,e^3+2\,C\,a\,e^3+3\,B\,c\,d\,e^2+3\,C\,c\,d^2\,e)}{4\,c^2}-\frac{x^5\,(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3 \cdots}{\cdots}}{\cdots}$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*
d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a^(7/2)*c^(5/2)) - ((2*B*c^2*d
^3 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e + 3*B*a*c*d*e^2 + 3*C*a*c*d^2
*e)/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d*e^2 + 3*C*c*d^2*e))/(4*
c^2) - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c
*d*e^2 + 3*B*a*c*d^2*e))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3
- B*a^2*e^3 + C*a*c*d^3 - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/
(6*a^2*c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A
*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a*c^2))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*
a*c^2*x^4)
```

3.66 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4} dx$

3.66.1 Optimal result

Integrand size = 27, antiderivative size = 225

$$\begin{aligned}\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx &= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac(a+cx^2)^3} \\ &\quad -\frac{2ae(4Acd+2aCd+aBe)+(3a(Ac+aC)e^2-cd(5Acd+aCd+2aBe))x}{24a^2c^2(a+cx^2)^2} \\ &\quad +\frac{(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe))x}{16a^3c^2(a+cx^2)} \\ &\quad +\frac{(a(Ac+aC)e^2+cd(5Acd+aCd+2aBe))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}\end{aligned}$$

output
```
-1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^3+1/24*(-2*a*e*(4*A*c*d+B*a
*e+2*C*a*d)-(3*a*(A*c+C*a)*e^2-c*d*(5*A*c*d+2*B*a*e+C*a*d))*x)/a^2/c^2/(c*
x^2+a)^2+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*x/a^3/c^2/(c*x
^2+a)+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/
a^(1/2))/a^(7/2)/c^(5/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{(Ac(5cd^2+ae^2)+a(aCe^2+cd(Cd+2Be)))\,x}{16a^3c^2\,(a+cx^2)}$$
$$+ \frac{5Ac^2d^2x+ac(Cd^2+e(2Bd+Ae))\,x-a^2e(12Cd+6Be+7Cex)}{24a^2c^2\,(a+cx^2)^2}$$
$$+ \frac{Ac^2d^2x+a^2e(2Cd+Be+Cex)-ac(Cd^2x+Ae(2d+ex)+Bd(d+2ex))}{6ac^2\,(a+cx^2)^3}$$
$$+ \frac{(Ac(5cd^2+ae^2)+a(aCe^2+cd(Cd+2Be)))\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{5/2}}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*
(a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(1
2*C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*
e*(2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)
))/(6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C
*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

3.66.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2176, 25, 675, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 \left(A+Bx+Cx^2\right)}{(a+cx^2)^4}dx$$
$$\Big\downarrow \text{ 2176}$$
$$-\frac{\int -\frac{(d+ex)(5Acd+aCd+2aBe+3(Ac+aC)ex)}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$\downarrow$ 25

$$\frac{\int \frac{(d+ex)(5Acd+aCd+2aBe+3(Ac+aC)ex)}{(cx^2+a)^3}dx}{6ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$\downarrow$ 675

$$\frac{\frac{3(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))\int \frac{1}{(cx^2+a)^2}dx}{4ac}-\frac{x(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd))}{4ac(a+cx^2)^2}-\frac{e(aBe+2aCd+4Acd)}{2c(a+cx^2)^2}}{6ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$\downarrow$ 215

$$\frac{\frac{3(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a\left(a+cx^2\right)}\right)}{4ac}-\frac{x(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd))}{4ac(a+cx^2)^2}-\frac{e(aBe+2aCd+4Acd)}{2c(a+cx^2)^2}}{6ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

$\downarrow$ 218

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}\right)(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac))}{4ac}-\frac{x(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd))}{4ac(a+cx^2)^2}-\frac{e(aBe+2aCd+4Acd)}{2c(a+cx^2)^2}}{6ac}-\frac{(d+ex)^2(aB-x(Ac-aC))}{6ac\,(a+cx^2)^3}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
-1/6*((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(a*c*(a + c*x^2)^3) + (-1/2*(e*(4
*A*c*d + 2*a*C*d + a*B*e))/(c*(a + c*x^2)^2) - ((3*a*(A*c + a*C)*e^2 - c*d
*(5*A*c*d + a*C*d + 2*a*B*e))*x)/(4*a*c*(a + c*x^2)^2) + (3*(a*(A*c + a*C)
*e^2 + c*d*(5*A*c*d + a*C*d + 2*a*B*e))*(x/(2*a*(a + c*x^2)) + ArcTan[(Sqr
t[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a*c))/(6*a*c)
```

3.66. $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.66. $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

3.66.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.19

method	result
default	$\frac{\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2Bacde+a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{5}}{16a^{3}}+\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2Bacde-a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{6c\,a^{2}}-\frac{e(Be+2Cd)x^{2}}{4c}-\frac{\left(Aac\,e^{2}-11A\,c^{2}d^{2}+2Bacd\right.}{16a\,c^{2}}}{\left(c\,x^{2}+a\right)^{3}}$
risch	$\frac{\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2Bacde+a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{5}}{16a^{3}}+\frac{\left(Aac\,e^{2}+5A\,c^{2}d^{2}+2Bacde-a^{2}C\,e^{2}+Cac\,d^{2}\right)x^{3}}{6c\,a^{2}}-\frac{e(Be+2Cd)x^{2}}{4c}-\frac{\left(Aac\,e^{2}-11A\,c^{2}d^{2}+2Bacd\right.}{16a\,c^{2}}}{\left(c\,x^{2}+a\right)^{3}}$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*
(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/c/a^2*x^3-1/4*e*(B
*e+2*C*d)*x^2/c-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d
^2)/a/c^2*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+
1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3/c^2/(a*c)
^(1/2)*arctan(c*x/(a*c)^(1/2))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(208) = 416$.

Time = 0.54 (sec) , antiderivative size = 1062, normalized size of antiderivative = 4.72

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \left[-\frac{16\,Ba^4c^2d^2 + 8\,Ba^5ce^2 - 6\,(2\,Ba^2c^4de + (Ca^2c^4 + 5\,Aac^5)d^2 + (Ca^3c^3 + Aa^2c^4)e^2)x^5 - 16\,(2\,Ba^3c^3de}{}\right.$$

$$-\frac{8\,Ba^4c^2d^2 + 4\,Ba^5ce^2 - 3\,(2\,Ba^2c^4de + (Ca^2c^4 + 5\,Aac^5)d^2 + (Ca^3c^3 + Aa^2c^4)e^2)x^5 - 8\,(2\,Ba^3c^3de +}{}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*B*a^4*c^2*d^2 + 8*B*a^5*c*e^2 - 6*(2*B*a^2*c^4*d*e + (C*a^2*c^4
 + 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 16*(2*B*a^3*c^3*d*e
 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 16*(
C*a^5*c + 2*A*a^4*c^2)*d*e + 24*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 + 3*
(2*B*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A
*a*c^4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a
^3*c^2 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^
4*c)*e^2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c +
 A*a^3*c^2)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 +
 a)) + 6*(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3*c^3)*d^2 + (C*a^5*c + A*
a^4*c^2)*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3),
-1/48*(8*B*a^4*c^2*d^2 + 4*B*a^5*c*e^2 - 3*(2*B*a^2*c^4*d*e + (C*a^2*c^4 +
 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 8*(2*B*a^3*c^3*d*e +
(C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 8*(C*a^
5*c + 2*A*a^4*c^2)*d*e + 12*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 - 3*(2*B
*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c
^4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c
^2 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)
*e^2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a
^3*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(2*B*a^4*c^2*d*e ...
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2\,(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx =$$
$$-\frac{8\,Ba^3cd^2 + 4\,Ba^4e^2 - 3\,(2\,Bac^3de + (Cac^3 + 5\,Ac^4)d^2 + (Ca^2c^2 + Aac^3)e^2)x^5 - 8\,(2\,Ba^2c^2de + (Ca^2c}{}$$
$$+\frac{(2\,Bacde + (Cac + 5\,Ac^2)d^2 + (Ca^2 + Aac)e^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(8*B*a^3*c*d^2 + 4*B*a^4*e^2 - 3*(2*B*a*c^3*d*e + (C*a*c^3 + 5*A*c^4
)*d^2 + (C*a^2*c^2 + A*a*c^3)*e^2)*x^5 - 8*(2*B*a^2*c^2*d*e + (C*a^2*c^2 +
 5*A*a*c^3)*d^2 - (C*a^3*c - A*a^2*c^2)*e^2)*x^3 + 8*(C*a^4 + 2*A*a^3*c)*d
*e + 12*(2*C*a^3*c*d*e + B*a^3*c*e^2)*x^2 + 3*(2*B*a^3*c*d*e + (C*a^3*c -
11*A*a^2*c^2)*d^2 + (C*a^4 + A*a^3*c)*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4
 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + (C*a*c + 5*A*c^2)*d^2 +
(C*a^2 + A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \frac{(Cacd^2 + 5\,Ac^2d^2 + 2\,Bacde + Ca^2e^2 + Aace^2)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c^2}$$
$$+\frac{3\,Cac^3d^2x^5 + 15\,Ac^4d^2x^5 + 6\,Bac^3dex^5 + 3\,Ca^2c^2e^2x^5 + 3\,Aac^3e^2x^5 + 8\,Ca^2c^2d^2x^3 + 40\,Aac^3d^2x^3 + 16}{}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*
d^2*x^5 + 6*B*a*c^3*d*e*x^5 + 3*C*a^2*c^2*e^2*x^5 + 3*A*a*c^3*e^2*x^5 + 8*
C*a^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*e*x^3 - 8*C*a^3*c*
e^2*x^3 + 8*A*a^2*c^2*e^2*x^3 - 24*C*a^3*c*d*e*x^2 - 12*B*a^3*c*e^2*x^2 -
3*C*a^3*c*d^2*x + 33*A*a^2*c^2*d^2*x - 6*B*a^3*c*d*e*x - 3*C*a^4*e^2*x - 3
*A*a^3*c*e^2*x - 8*B*a^3*c*d^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^
2)/((c*x^2 + a)^3*a^3*c^2)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(C\,a^2\,e^2 + C\,a\,c\,d^2 + 2\,B\,a\,c\,d\,e + A\,a\,c\,e^2 + 5\,A\,c^2\,d^2)}{16\,a^{7/2}\,c^{5/2}}$$

$$- \frac{\frac{B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e}{12\,c^2} - \frac{x^5\,(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2)}{16\,a^3} + \frac{x^2\,(B\,e^2+2\,C\,d\,e)}{4\,c} + \frac{x\,(C\,a^2\,e^2+C\,a\,c\,d^2+}{}}{a^3 + 3\,a^2\,c\,x^2 + 3\,a\,c^2\,x^4 + c^3\,x^6}$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^
2 + 2*B*a*c*d*e))/(16*a^(7/2)*c^(5/2)) - ((B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e
 + 2*C*a*d*e)/(12*c^2) - (x^5*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c
*d^2 + 2*B*a*c*d*e))/(16*a^3) + (x^2*(B*e^2 + 2*C*d*e))/(4*c) + (x*(C*a^2*
e^2 - 11*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(16*a*c^2) - (x
^3*(5*A*c^2*d^2 - C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(6*a^2
*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx$

3.67.1 Optimal result

Integrand size = 25, antiderivative size = 165

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Acd+aCd+aBe)x}{16a^3c\left(a+cx^2\right)} + \frac{(5Acd+aCd+aBe)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

output
```
-1/6*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^3+1/24*(-2*a*(2*A*c+C*a)*e+c*
(5*A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2+1/16*(5*A*c*d+B*a*e+C*a*d)*x/
a^3/c/(c*x^2+a)+1/16*(5*A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(7/
2)/c^(3/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$
$$=\frac{\frac{2a^{3/2}\left(-6a^2Ce+5Ac^2dx+ac(Cd+Be)x\right)}{\left(a+cx^2\right)^2}+\frac{3\sqrt{a}c(5Acd+aCd+aBe)x}{a+cx^2}+\frac{8a^{5/2}\left(a^2Ce+Ac^2dx-ac(Ae+Cdx+B(d+ex))\right)}{\left(a+cx^2\right)^3}+3\sqrt{c}(5Acd+}{48a^{7/2}c^2}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2
+ (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*
C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt
[c]*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2
)
```

3.67.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2176, 25, 454, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$$

$$\Big\downarrow \ 2176$$

$$-\frac{\int -\frac{5Acd+a(Cd+Be)+2(2Ac+aC)ex}{\left(cx^2+a\right)^3}dx}{6ac}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5Acd+aCd+aBe+2(2Ac+aC)ex}{\left(cx^2+a\right)^3}dx}{6ac}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \ 454$$

$$\frac{\frac{3(aBe+aCd+5Acd)\int \frac{1}{\left(cx^2+a\right)^2}dx}{4a}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$\Big\downarrow$ 215

$$\frac{\frac{3(aBe+aCd+5Acd)\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a\left(a+cx^2\right)}\right)}{4a}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}\right)(aBe+aCd+5Acd)}{4a}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{4ac(a+cx^2)^2}}{6ac}-\frac{(d+ex)(aB-x(Ac-aC))}{6ac\left(a+cx^2\right)^3}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

output
```
-1/6*((a*B - (A*c - a*C)*x)*(d + e*x))/(a*c*(a + c*x^2)^3) + (-1/4*(2*a*(2
*A*c + a*C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(a*c*(a + c*x^2)^2) + (3*(5
*A*c*d + a*C*d + a*B*e)*(x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]
/(2*a^(3/2)*Sqrt[c])))/(4*a))/(6*a*c)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.67. $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx$

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.67.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

method	result
default	$\frac{\frac{(5Acd+Bae+Cad)c\,x^5}{16a^3}+\frac{(5Acd+Bae+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-Bae-Cad)x}{16ac}-\frac{2Ace+2Bcd+Cae}{12c^2}}{(c\,x^2+a)^3}+\frac{(5Acd+Bae+Cad)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Acd+Bae+Cad)c\,x^5}{16a^3}+\frac{(5Acd+Bae+Cad)x^3}{6a^2}-\frac{Ce\,x^2}{4c}+\frac{(11Acd-Bae-Cad)x}{16ac}-\frac{2Ace+2Bcd+Cae}{12c^2}}{(c\,x^2+a)^3}-\frac{5\ln\left(cx+\sqrt{-ac}\right)Ad}{32\sqrt{-ac}\,a^3}-\frac{\ln\left(cx+\sqrt{-ac\right.}}{32\sqrt{-ac}\,c\,a}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/
4*C*e*x^2/c+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)
/c^2)/(c*x^2+a)^3+1/16*(5*A*c*d+B*a*e+C*a*d)/a^3/c/(a*c)^(1/2)*arctan(c*x/
(a*c)^(1/2))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(148) = 296.

Time = 0.40 (sec) , antiderivative size = 636, normalized size of antiderivative = 3.85

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$$
$$= \left[-\frac{24\,Ca^4cex^2 + 16\,Ba^4cd - 6\,(Ba^2c^3e + (Ca^2c^3 + 5\,Aac^4)d)x^5 - 16\,(Ba^3c^2e + (Ca^3c^2 + 5\,Aa^2c^3)d)x^3 +}{}\right.$$
$$-\frac{12\,Ca^4cex^2 + 8\,Ba^4cd - 3\,(Ba^2c^3e + (Ca^2c^3 + 5\,Aac^4)d)x^5 - 8\,(Ba^3c^2e + (Ca^3c^2 + 5\,Aa^2c^3)d)x^3 - 3}{}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(24*C*a^4*c*e*x^2 + 16*B*a^4*c*d - 6*(B*a^2*c^3*e + (C*a^2*c^3 + 5*
A*a*c^4)*d)*x^5 - 16*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 + 3*(
(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^
2*c^2 + 5*A*a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2
+ (C*a^4 + 5*A*a^3*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^
2 + a)) + 8*(C*a^5 + 2*A*a^4*c)*e + 6*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2
)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12
*C*a^4*c*e*x^2 + 8*B*a^4*c*d - 3*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A*a*c^4)*d)
*x^5 - 8*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 - 3*((B*a*c^3*e +
 (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*
a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C*a^4 + 5
*A*a^3*c)*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 4*(C*a^5 + 2*A*a^4*c)*e + 3
*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4
+ 3*a^6*c^3*x^2 + a^7*c^2)]
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

output
```
Timed out
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{(a+cx^2)^4}\,dx =$$
$$-\frac{12\,Ca^3cex^2+8\,Ba^3cd-3\,(Bac^3e+(Cac^3+5\,Ac^4)d)x^5-8\,(Ba^2c^2e+(Ca^2c^2+5\,Aac^3)d)x^3+4\,(Ca^4}{48\,(a^3c^5x^6+3\,a^4c^4x^4+3\,a^5c^3x^2+a^6c^2)}$$
$$+\frac{(Bae+(Ca+5\,Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(12*C*a^3*c*e*x^2 + 8*B*a^3*c*d - 3*(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)
*d)*x^5 - 8*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*x^3 + 4*(C*a^4 + 2*A
*a^3*c)*e + 3*(B*a^3*c*e + (C*a^3*c - 11*A*a^2*c^2)*d)*x)/(a^3*c^5*x^6 + 3
*a^4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(B*a*e + (C*a + 5*A*c)*d)*a
rctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{(a+cx^2)^4}\,dx = \frac{(Cad+5\,Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$
$$+\frac{3\,Cac^3dx^5+15\,Ac^4dx^5+3\,Bac^3ex^5+8\,Ca^2c^2dx^3+40\,Aac^3dx^3+8\,Ba^2c^2ex^3-12\,Ca^3cex^2-3\,Ca^3cd}{48\,(cx^2+a)^3a^3c^2}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1
/48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*e*x^5 + 8*C*a^2*c^2*d*x^
3 + 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*e*x^3 - 12*C*a^3*c*e*x^2 - 3*C*a^3*c*d*
x + 33*A*a^2*c^2*d*x - 3*B*a^3*c*e*x - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c
*e)/((c*x^2 + a)^3*a^3*c^2)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(5\,A\,c\,d+B\,a\,e+C\,a\,d)}{16\,a^{7/2}\,c^{3/2}} - \frac{\frac{2\,A\,c\,e+2\,B\,c\,d+C\,a\,e}{12\,c^2} - \frac{x^3\,(5\,A\,c\,d+B\,a\,e+C\,a\,d)}{6\,a^2} + \frac{C\,e\,x^2}{4\,c} + \frac{x\,(B\,a\,e-11\,A\,c\,d+C\,a\,d)}{16\,a\,c} - \frac{c\,x^5\,(5\,A\,c\,d+B\,a\,e+C\,a\,d)}{16\,a^3}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

output
```
(atan((c^(1/2)*x)/a^(1/2))*(5*A*c*d + B*a*e + C*a*d))/(16*a^(7/2)*c^(3/2))
 - ((2*A*c*e + 2*B*c*d + C*a*e)/(12*c^2) - (x^3*(5*A*c*d + B*a*e + C*a*d))
/(6*a^2) + (C*e*x^2)/(4*c) + (x*(B*a*e - 11*A*c*d + C*a*d))/(16*a*c) - (c*
x^5*(5*A*c*d + B*a*e + C*a*d))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*
a*c^2*x^4)
```

3.68 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx$

3.68.1 Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx = -\frac{aB-(Ac-aC)x}{6ac(a+cx^2)^3} + \frac{(5Ac+aC)x}{24a^2c(a+cx^2)^2} + \frac{(5Ac+aC)x}{16a^3c(a+cx^2)} + \frac{(5Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

output
```
1/6*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^3+1/24*(5*A*c+C*a)*x/a^2/c/(c*x^2+a)^
2+1/16*(5*A*c+C*a)*x/a^3/c/(c*x^2+a)+1/16*(5*A*c+C*a)*arctan(x*c^(1/2)/a^(
1/2))/a^(7/2)/c^(3/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4} dx = \frac{15Ac^3x^5 - a^3(8B+3Cx) + ac^2x^3(40A+3Cx^2) + a^2cx(33A+8Cx^2)}{48a^3c(a+cx^2)^3} + \frac{(5Ac+aC)\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

output
```
(15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(
33*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

3.68.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2345, 25, 27, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{5Ac+aC}{c\left(cx^2+a\right)^3}dx}{6a}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5Ac+aC}{c\left(cx^2+a\right)^3}dx}{6a}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+5Ac)\int \frac{1}{\left(cx^2+a\right)^3}dx}{6ac}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aC+5Ac)\left(\frac{3\int \frac{1}{\left(cx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+cx^2\right)^2}\right)}{6ac}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aC+5Ac)\left(\frac{3\left(\frac{\int \frac{1}{cx^2+a}dx}{2a}+\frac{x}{2a(a+cx^2)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}\right)}{6ac}-\frac{aB-x(Ac-aC)}{6ac\,(a+cx^2)^3}$$

$$\Big\downarrow \; 218$$

$$\frac{(aC+5Ac)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a(a+cx^2)}\right)}{4a}+\frac{x}{4a(a+cx^2)^2}\right)}{6ac}-\frac{aB-x(Ac-aC)}{6ac\,(a+cx^2)^3}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

output
```
-1/6*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*(x/(4*a*(a
 + c*x^2)^2) + (3*(x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^
(3/2)*Sqrt[c])))/(4*a)))/(6*a*c)
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{(5Ac+Ca)c\,x^5}{16a^3}+\frac{(5Ac+Ca)x^3}{6a^2}+\frac{(11Ac-Ca)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}+\frac{(5Ac+Ca)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16a^3c\sqrt{ac}}$
risch	$\frac{\frac{(5Ac+Ca)c\,x^5}{16a^3}+\frac{(5Ac+Ca)x^3}{6a^2}+\frac{(11Ac-Ca)x}{16ac}-\frac{B}{6c}}{(c\,x^2+a)^3}-\frac{5\ln(cx+\sqrt{-ac})A}{32\sqrt{-ac}\,a^3}-\frac{\ln(cx+\sqrt{-ac})C}{32\sqrt{-ac}\,c\,a^2}+\frac{5\ln(-cx+\sqrt{-ac})A}{32\sqrt{-ac}\,a^3}+\frac{\ln(-cx+\sqrt{-}}{32\sqrt{-ac}\,c}$

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*
x-1/6*B/c)/(c*x^2+a)^3+1/16*(5*A*c+C*a)/a^3/c/(a*c)^(1/2)*arctan(c*x/(a*c)
^(1/2))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.41

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx$$
$$= \left[-\frac{16\,Ba^4c-6\,(Ca^2c^3+5\,Aac^4)x^5-16\,(Ca^3c^2+5\,Aa^2c^3)x^3+3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3}{96\,(a^4c^5x^6+3\,a^5c^4x^4+}\right.$$
$$-\frac{8\,Ba^4c-3\,(Ca^2c^3+5\,Aac^4)x^5-8\,(Ca^3c^2+5\,Aa^2c^3)x^3-3\,((Cac^3+5\,Ac^4)x^6+Ca^4+5\,Aa^3c+3\,(C}{48\,(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a
^2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^
2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^
5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*
a^2*c^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 +
 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a
^3*c + 5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11
*A*a^3*c^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7c^3}}\cdot(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32} + \frac{-8Ba^3+x^5\cdot(15Ac^3+3Cac^2)+x^3\cdot(40Aac^2+8Ca^2c)+x(33Aa^2c-3Ca^3)}{48a^6c+144a^5c^2x^2+144a^4c^3x^4+48a^3c^4x^6}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

output
```
-sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/
32 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) +
x)/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8
*C*a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 +
 144*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{3\,(Cac^2+5\,Ac^3)x^5-8\,Ba^3+8\,(Ca^2c+5\,Aac^2)x^3-3\,(Ca^3-11\,Aa^2c)x}{48\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)} + \frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 -
3*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a
^6*c) + 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3c} + \frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\,(cx^2+a)^3a^3c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

output
```
1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c
^2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*
a^2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 12.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^4}\,dx = \frac{\frac{x^3\,(5\,A\,c+C\,a)}{6\,a^2}-\frac{B}{6\,c}+\frac{c\,x^5\,(5\,A\,c+C\,a)}{16\,a^3}+\frac{x\,(11\,A\,c-C\,a)}{16\,a\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(5\,A\,c+C\,a)}{16\,a^{7/2}\,c^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^4,x)
```

output
```
((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) +
(x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) +
 (atan((c^(1/2)*x)/a^(1/2))*(5*A*c + C*a))/(16*a^(7/2)*c^(3/2))
```

3.69 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx$

3.69.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2(1+x^2)}-\frac{3\arctan(x)}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
3/2*x+1/2*x^2-1/2*x^3/(x^2+1)-3/2*arctan(x)-1/2*ln(x^2+1)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\left(x\left(2+x+\frac{1}{1+x^2}\right)-3\arctan(x)-\log\left(1+x^2\right)\right)$$

input
```
Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
(x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

3.69.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2335, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(x^2+x+1)}{(x^2+1)^2}dx$$

$$\Big\downarrow \; 2335$$

$$-\frac{1}{2}\int -\frac{x^2(2x+3)}{x^2+1}dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\int \frac{x^2(2x+3)}{x^2+1}dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \; 523$$

$$\frac{1}{2}\int \left(2x-\frac{2x+3}{x^2+1}+3\right)dx-\frac{x^3}{2(x^2+1)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-3\arctan(x)+x^2-\log\left(x^2+1\right)+3x\right)-\frac{x^3}{2(x^2+1)}$$

input
```
Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
-1/2*x^3/(1 + x^2) + (3*x + x^2 - 3*ArcTan[x] - Log[1 + x^2])/2
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	siz
default	$x+\frac{x^{2}}{2}+\frac{x}{2x^{2}+2}-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{3\arctan\left(x\right)}{2}$	30
risch	$x+\frac{x^{2}}{2}+\frac{x}{2x^{2}+2}-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{3\arctan\left(x\right)}{2}$	30
meijerg	$\frac{x^{2}\left(3x^{2}+6\right)}{6x^{2}+6}-\frac{\ln\left(x^{2}+1\right)}{2}+\frac{x\left(10x^{2}+15\right)}{10x^{2}+10}-\frac{3\arctan\left(x\right)}{2}-\frac{x^{2}}{2\left(x^{2}+1\right)}$	62
parallelrisch	$\frac{3i\ln\left(x-i\right)x^{2}-3i\ln\left(x+i\right)x^{2}+2x^{4}-2\ln\left(x-i\right)x^{2}-2\ln\left(x+i\right)x^{2}+4x^{3}-2+3i\ln\left(x-i\right)-3i\ln\left(x+i\right)-2\ln\left(x-i\right)-2\ln\left(x+i\right)+6x}{4x^{2}+4}$	97

input
```
int(x^3*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+1/2/(x^2+1)*x-1/2*ln(x^2+1)-3/2*arctan(x)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx=\frac{x^4+2\,x^3+x^2-3\,(x^2+1)\arctan\left(x\right)-(x^2+1)\log\left(x^2+1\right)+3\,x}{2\,(x^2+1)}$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) +
3*x)/(x^2 + 1)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{x^2}{2} + x + \frac{x}{2x^2+2} - \frac{\log(x^2+1)}{2} - \frac{3\,\mathrm{atan}(x)}{2}$$

input
```
integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\,x^2 + x + \frac{x}{2\,(x^2+1)} - \frac{3}{2}\,\arctan(x) - \frac{1}{2}\,\log(x^2+1)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{1}{2}\,x^2 + x + \frac{x}{2\,(x^2+1)} - \frac{3}{2}\,\arctan(x) - \frac{1}{2}\,\log(x^2+1)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx = x - \frac{\ln(x^2+1)}{2} - \frac{3\,\mathrm{atan}(x)}{2} + \frac{x}{2\,(x^2+1)} + \frac{x^2}{2}$$

input
```
int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx$

3.70.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x-\frac{x^2}{2(1+x^2)}-\arctan(x)+\frac{1}{2}\log\left(1+x^2\right)$$

output
```
x-1/2*x^2/(x^2+1)-arctan(x)+1/2*ln(x^2+1)
```

3.70.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x+\frac{1}{2(1+x^2)}-\arctan(x)+\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

3.70.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(x^2+x+1)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{1}{2}\int -\frac{2x(x+1)}{x^2+1}dx-\frac{x^2}{2(x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x(x+1)}{x^2+1}dx-\frac{x^2}{2(x^2+1)}$$

$$\Big\downarrow \text{523}$$

$$\int \left(1-\frac{1-x}{x^2+1}\right)dx-\frac{x^2}{2(x^2+1)}$$

$$\Big\downarrow \text{2009}$$

$$-\arctan(x)-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+x$$

input
```
Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
default	$x+\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
risch	$x+\frac{1}{2x^2+2}+\frac{\ln(x^2+1)}{2}-\arctan(x)$	24
meijerg	$\frac{x(10x^2+15)}{10x^2+10}-\arctan(x)-\frac{x^2}{2(x^2+1)}+\frac{\ln(x^2+1)}{2}-\frac{x}{2(x^2+1)}$	53
parallelrisch	$\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+\ln(x-i)x^2+\ln(x+i)x^2+2x^3+1+i\ln(x-i)-i\ln(x+i)+\ln(x-i)+\ln(x+i)+2x}{2x^2+2}$	86

input
```
int(x^2*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx=\frac{2\,x^3-2\,(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)+2\,x+1}{2\,(x^2+1)}$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^
2 + 1)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{\log(x^2+1)}{2} - \operatorname{atan}(x) + \frac{1}{2x^2+2}$$

input
```
integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2(x^2+1)} - \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{1}{2(x^2+1)} - \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{x^2(1+x+x^2)}{(1+x^2)^2}\,dx = x + \frac{\ln(x^2+1)}{2} - \operatorname{atan}(x) + \frac{1}{2\,(x^2+1)}$$

input
```
int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

3.71 $\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$

3.71.1 Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} + \frac{1}{2}\log(1+x^2)$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx = \frac{1}{2}\left(-\frac{x}{1+x^2} + \arctan(x) + \log(1+x^2)\right)$$

input
```
Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
(-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

3.71.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2335, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+x+1)}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{1}{2}\int -\frac{2x+1}{x^2+1}dx-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2x+1}{x^2+1}dx-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{452}$$

$$\frac{1}{2}\left(\int \frac{1}{x^2+1}dx+2\int \frac{x}{x^2+1}dx\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\int \frac{x}{x^2+1}dx+\arctan(x)\right)-\frac{x}{2(x^2+1)}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2}\left(\arctan(x)+\log\left(x^2+1\right)\right)-\frac{x}{2(x^2+1)}$$

input
```
Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + (ArcTan[x] + Log[1 + x^2])/2
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2-2\ln(x-i)x^2-2\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)-2\ln(x-i)-2\ln(x+i)+2x}{4(x^2+1)}$	86

input
```
int(x*(x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)*x+1/2*arctan(x)+1/2*ln(x^2+1)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)+(x^2+1)\log(x^2+1)-x}{2(x^2+1)}$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)+\frac{1}{2}\log(x^2+1)$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = -\frac{x}{2\,(x^2+1)} + \frac{1}{2}\,\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

input
```
integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x(1+x+x^2)}{(1+x^2)^2}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,\left(x^2+1\right)}$$

input
```
int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

output
```
log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

3.72 $\int \frac{1+x+x^2}{(1+x^2)^2} \, dx$

3.72.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1+x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x)$$

output
```
-1/2/(x^2+1)+arctan(x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{2(1+x^2)} + \arctan(x)$$

input
```
Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x]
```

3.72.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{1}{2}\int -\frac{2}{x^2+1}dx-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^2+1}dx-\frac{1}{2(x^2+1)}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(x)-\frac{1}{2(x^2+1)}$$

input
```
Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*1/(1 + x^2) + ArcTan[x]
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{2(x^2+1)}+\arctan(x)$	13
risch	$-\frac{1}{2(x^2+1)}+\arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)}+\arctan(x)+\frac{x^2}{2x^2+2}+\frac{x}{2x^2+2}$	37
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+1+i\ln(x-i)-i\ln(x+i)}{2(x^2+1)}$	50

input
```
int((x^2+x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)+arctan(x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx=\frac{2\,(x^2+1)\arctan(x)-1}{2\,(x^2+1)}$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{2x^2+2}$$

input
```
integrate((x**2+x+1)/(x**2+1)**2,x)
```

output
```
atan(x) - 1/(2*x**2 + 2)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2(x^2+1)} + \arctan(x)$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2/(x^2 + 1) + arctan(x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{2(x^2+1)} + \arctan(x)$$

input
```
integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2/(x^2 + 1) + arctan(x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{2\ (x^2+1)}$$

input
```
int((x + x^2 + 1)/(x^2 + 1)^2,x)
```

output
```
atan(x) - 1/(2*(x^2 + 1))
```

3.73 $\int \frac{1+x+x^2}{x(1+x^2)^2} dx$

3.73.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1+x+x^2}{x(1+x^2)^2} dx = \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2} + \log(x) - \frac{1}{2}\log(1+x^2)$$

output
```
1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.73.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x(1+x^2)^2} dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x) + 2\log(x) - \log(1+x^2)\right)$$

input
```
Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

output
```
(x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

3.73.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2336, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \ 2336$$

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\int -\frac{x+2}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\int \frac{x+2}{x\left(x^2+1\right)}dx+\frac{x}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 523$$

$$\frac{1}{2}\int \left(\frac{1-2x}{x^2+1}+\frac{2}{x}\right)dx+\frac{x}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\arctan(x)-\log\left(x^2+1\right)+2\log(x)\right)+\frac{x}{2\left(x^2+1\right)}$$

input
```
Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

output
```
x/(2*(1 + x^2)) + (ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.73.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln(x^2+1)}{2}$
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{1}{2}+\ln(x)$
parallelrisch	$\frac{-i\ln(x-i)x^2+i\ln(x+i)x^2+4\ln(x)x^2-2\ln(x-i)x^2-2\ln(x+i)x^2-i\ln(x-i)+i\ln(x+i)+4\ln(x)-2\ln(x-i)-2\ln(x+i)+2x}{4x^2+4}$

input
```
int((x^2+x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x^2+1)*x+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)+2\,(x^2+1)\log(x)+x}{2\,(x^2+1)}$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x
)/(x^2 + 1)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{x}{2x^2+2}+\log(x)-\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

output
```
x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx=\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(|x|)$$

input
```
integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx = \ln\left(x\right)+\frac{x}{2\,\left(x^2+1\right)}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
 1))
```

3.74 $\int \frac{1+x+x^2}{x^2(1+x^2)^2}\,dx$

3.74.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.74.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\arctan(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

output
```
-x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.74.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2336, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^2\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \ 2336$$

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\int -\frac{2(x+1)}{x^2\left(x^2+1\right)}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x+1}{x^2\left(x^2+1\right)}dx+\frac{1}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 523$$

$$\int \left(\frac{-x-1}{x^2+1}+\frac{1}{x}+\frac{1}{x^2}\right)dx+\frac{1}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 2009$$

$$-\arctan(x)+\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)$$

input
```
Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

output
```
-x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.74.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result
default	$-\frac{1}{x}+\frac{1}{2x^2+2}-\arctan\left(x\right)+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$
risch	$\frac{-x^2+\frac{1}{2}x-1}{x(x^2+1)}-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(x\right)+\ln\left(x\right)$
meijerg	$\frac{x}{2x^2+2}-\arctan\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)-\frac{3x^2+2}{x(2x^2+2)}$
parallelrisch	$\frac{i\ln(x-i)x^3-i\ln(x+i)x^3+2\ln(x)x^3-\ln(x-i)x^3-\ln(x+i)x^3-2+i\ln(x-i)x-i\ln(x+i)x+2\ln(x)x-\ln(x-i)x-\ln(x+i)x-2x^2+}{2(x^2+1)x}$

input
```
int((x^2+x+1)/x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = -\frac{2\,x^2+2\,\left(x^3+x\right)\arctan\left(x\right)+\left(x^3+x\right)\log\left(x^2+1\right)-2\,\left(x^3+x\right)\log\left(x\right)-x+2}{2\,\left(x^3+x\right)}$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)
*log(x) - x + 2)/(x^3 + x)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^2}{x^2 (1+x^2)^2} dx = \log(x) - \frac{\log(x^2+1)}{2} - \operatorname{atan}(x) + \frac{-2x^2+x-2}{2x^3+2x}$$

input
```
integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
```

output
```
log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1+x+x^2}{x^2 (1+x^2)^2} dx = -\frac{2x^2-x+2}{2(x^3+x)} - \arctan(x) - \frac{1}{2}\log(x^2+1) + \log(x)$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^2}{x^2 (1+x^2)^2} dx = -\frac{2x^2-x+2}{2(x^3+x)} - \arctan(x) - \frac{1}{2}\log(x^2+1) + \log(|x|)$$

input
```
integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x)
)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx = \ln\left(x\right) - \frac{x^2-\frac{x}{2}+1}{x^3+x} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
```

output
```
log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2
+ 1)/(x + x^3)
```

3.75 $\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx$

3.75.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx = -\frac{1}{2x^2} - \frac{1}{x} - \frac{x}{2(1+x^2)} - \frac{3\arctan(x)}{2} - \log(x) + \frac{1}{2}\log(1+x^2)$$

output
```
-1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

3.75.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2} dx = \frac{1}{2}\left(-\frac{1}{x^2} - \frac{2}{x} - \frac{x}{1+x^2} - 3\arctan(x) - 2\log(x) + \log(1+x^2)\right)$$

input
```
Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

output
```
(-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

3.75.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^3 (x^2+1)^2} \, dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{1}{2} \int -\frac{-x^3+2x+2}{x^3 (x^2+1)} dx - \frac{x}{2 (x^2+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2} \int \frac{-x^3+2x+2}{x^3 (x^2+1)} dx - \frac{x}{2 (x^2+1)}$$

$$\Big\downarrow \; 2333$$

$$\frac{1}{2} \int \left(\frac{2x-3}{x^2+1} - \frac{2}{x} + \frac{2}{x^2} + \frac{2}{x^3}\right) dx - \frac{x}{2 (x^2+1)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-3\arctan(x) - \frac{1}{x^2} + \log\left(x^2+1\right) - \frac{2}{x} - 2\log(x)\right) - \frac{x}{2 (x^2+1)}$$

input
```
Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

output
```
-1/2*x/(1 + x^2) + (-x^(-2) - 2/x - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])
/2
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.75.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result
default	$-\frac{1}{2x^{2}}-\frac{1}{x}-\frac{x}{2(x^{2}+1)}-\frac{3\arctan(x)}{2}-\ln(x)+\frac{\ln(x^{2}+1)}{2}$
risch	$\frac{-\frac{3}{2}x^{3}-\frac{1}{2}x^{2}-x-\frac{1}{2}}{x^{2}(x^{2}+1)}+\frac{\ln(x^{2}+1)}{2}-\frac{3\arctan(x)}{2}-\ln(x)$
meijerg	$-\frac{x^{2}}{2x^{2}+2}+\frac{\ln(x^{2}+1)}{2}-\ln(x)-\frac{3x^{2}+2}{x(2x^{2}+2)}-\frac{3\arctan(x)}{2}+\frac{3x^{2}}{2(3x^{2}+3)}-\frac{1}{2x^{2}}$
parallelrisch	$-\frac{3i\ln(x+i)x^{4}+3i\ln(x+i)x^{2}+4\ln(x)x^{4}-2\ln(x-i)x^{4}-2\ln(x+i)x^{4}+2-3i\ln(x-i)x^{4}-3i\ln(x-i)x^{2}+4\ln(x)x^{2}-2\ln(x-i)x^{2}-}{4x^{2}(x^{2}+1)}$

input
```
int((x^2+x+1)/x^3/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-1/x-1/2/(x^2+1)*x-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\frac{3\,x^3+x^2+3\,(x^4+x^2)\arctan(x)-(x^4+x^2)\log(x^2+1)+2\,(x^4+x^2)\log(x)+2\,x+1}{2\,(x^4+x^2)}$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2
*(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\log(x)+\frac{\log(x^2+1)}{2}-\frac{3\operatorname{atan}(x)}{2}+\frac{-3x^3-x^2-2x-1}{2x^4+2x^2}$$

input
```
integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

output
```
-log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x*
*4 + 2*x**2)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1+x+x^2}{x^3(1+x^2)^2}\,dx = -\frac{3\,x^3+x^2+2\,x+1}{2\,(x^4+x^2)}-\frac{3}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)-\log(x)$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1
) - log(x)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\frac{3\,x^3+x^2+2\,x+1}{2\,(x^2+1)x^2} - \frac{3}{2}\,\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right) - \log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
 + 1) - log(abs(x))
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx = -\ln\left(x\right) - \frac{\frac{3\,x^3}{2}+\frac{x^2}{2}+x+\frac{1}{2}}{x^4+x^2} + \ln\left(x-\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

input
```
int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

output
```
log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2
+ (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

3.76 $\int \frac{1+2x+x^2}{(1+x^2)^2} dx$

3.76.1 Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1+2x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{1+x^2} + \arctan(x)$$

output
```
-1/(x^2+1)+arctan(x)
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2} \, dx = -\frac{1}{1+x^2} + \arctan(x)$$

input
```
Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

output
```
-(1 + x^2)^(-1) + ArcTan[x]
```

3.76.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2006, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x+1}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{(x+1)^2}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{487}$$

$$\int \frac{1}{x^2+1}dx-\frac{(1-x)(x+1)}{2(x^2+1)}$$

$$\Big\downarrow \text{216}$$

$$\arctan(x)-\frac{(1-x)(x+1)}{2(x^2+1)}$$

input
```
Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]
```

output
```
-1/2*((1 - x)*(1 + x))/(1 + x^2) + ArcTan[x]
```

3.76.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.76.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{x^2+1}+\arctan(x)$	13
risch	$-\frac{1}{x^2+1}+\arctan(x)$	13
meijerg	$-\frac{x}{2(x^2+1)}+\arctan(x)+\frac{x^2}{x^2+1}+\frac{x}{2x^2+2}$	36
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+2+i\ln(x-i)-i\ln(x+i)}{2(x^2+1)}$	50

input
```
int((x^2+2*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(x^2+1)+arctan(x)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx=\frac{(x^2+1)\arctan(x)-1}{x^2+1}$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{x^2+1}$$

input
```
integrate((x**2+2*x+1)/(x**2+1)**2,x)
```

output
```
atan(x) - 1/(x**2 + 1)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{x^2+1} + \arctan(x)$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/(x^2 + 1) + arctan(x)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = -\frac{1}{x^2+1} + \arctan(x)$$

input
```
integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/(x^2 + 1) + arctan(x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+2x+x^2}{(1+x^2)^2}\,dx = \operatorname{atan}(x) - \frac{1}{x^2+1}$$

input
```
int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
```

output
```
atan(x) - 1/(x^2 + 1)
```

3.77 $\int \frac{2+12x+3x^2}{(4+x^2)^2} \, dx$

3.77.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{2+12x+3x^2}{(4+x^2)^2} \, dx = -\frac{24+5x}{4(4+x^2)} + \frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

output
```
1/4*(-24-5*x)/(x^2+4)+7/8*arctan(1/2*x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{2+12x+3x^2}{(4+x^2)^2} \, dx = \frac{-24-5x}{4(4+x^2)} + \frac{7}{8}\arctan\left(\frac{x}{2}\right)$$

input
```
Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

output
```
(-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

3.77.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2345, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+12x+2}{(x^2+4)^2} \, dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{1}{8}\int -\frac{14}{x^2+4}dx-\frac{5x+24}{4(x^2+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}\int \frac{1}{x^2+4}dx-\frac{5x+24}{4(x^2+4)}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{8}\arctan\left(\frac{x}{2}\right)-\frac{5x+24}{4(x^2+4)}$$

input
```
Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

output
```
-1/4*(24 + 5*x)/(4 + x^2) + (7*ArcTan[x/2])/8
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.77.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
risch	$\frac{-\frac{5x}{4}-6}{x^2+4}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}$	21
meijerg	$\frac{x}{4x^2+16}+\frac{7\arctan\left(\frac{x}{2}\right)}{8}-\frac{3x}{8\left(\frac{x^2}{4}+1\right)}+\frac{3x^2}{8\left(\frac{x^2}{4}+1\right)}$	46
parallelrisch	$-\frac{7i\ln(x-2i)x^2-7i\ln(x+2i)x^2+28i\ln(x-2i)-28i\ln(x+2i)-24x^2+20x}{16(x^2+4)}$	57

input
```
int((3*x^2+12*x+2)/(x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx=\frac{7\,(x^2+4)\arctan\left(\frac{1}{2}\,x\right)-10\,x-48}{8\,(x^2+4)}$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
```

output
```
1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = \frac{-5x-24}{4x^2+16} + \frac{7\operatorname{atan}\left(\frac{x}{2}\right)}{8}$$

input
```
integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

output
```
(-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

output
```
-1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{5\,x+24}{4\,(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
```

output
```
-1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{2+12x+3x^2}{\left(4+x^2\right)^2}\,dx = \frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8} - \frac{\frac{5\,x}{4}+6}{x^2+4}$$

input
```
int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
```

output
```
(7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

3.78 $\int (g+hx)^3\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.78.1 Optimal result

Integrand size = 29, antiderivative size = 390

$$\begin{aligned}
&\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx \\
&= \frac{(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{16c^2} \\
&\quad -\frac{(8afh^2+c(3fg^2-7h(eg+2dh)))\,(g+hx)^2\,(a+cx^2)^{3/2}}{70c^2h} \\
&\quad -\frac{(3fg-7eh)(g+hx)^3\,(a+cx^2)^{3/2}}{42ch}+\frac{f(g+hx)^4\,(a+cx^2)^{3/2}}{7ch} \\
&\quad +\frac{(8(8a^2fh^4-2ach^2(15fg^2+7h(3eg+dh))-c^2g^2(3fg^2-7h(eg+12dh)))-3ch(ah^2(41fg+35eh)+}{840c^3h} \\
&\quad +\frac{a(8c^2dg^3+a^2h^2(3fg+eh)-2acg(fg^2+3h(eg+dh)))\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}
\end{aligned}$$

output
```
-1/70*(8*a*f*h^2+c*(3*f*g^2-7*h*(2*d*h+e*g)))*(h*x+g)^2*(c*x^2+a)^(3/2)/c^
2/h-1/42*(-7*e*h+3*f*g)*(h*x+g)^3*(c*x^2+a)^(3/2)/c/h+1/7*f*(h*x+g)^4*(c*x
^2+a)^(3/2)/c/h+1/840*(64*a^2*f*h^4-16*a*c*h^2*(15*f*g^2+7*h*(d*h+3*e*g))-
8*c^2*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-3*c*h*(a*h^2*(35*e*h+41*f*g)+2*c*g*(3
*f*g^2-7*h*(7*d*h+e*g)))*x)*(c*x^2+a)^(3/2)/c^3/h+1/16*a*(8*c^2*d*g^3+a^2*
h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)
^(1/2))/c^(5/2)+1/16*(8*c^2*d*g^3+a^2*h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(
d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

3.78.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.92

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{a+cx^2}(128a^3fh^3-a^2ch(7h(96eg+32dh+15ehx)+f(672g^2+315ghx+64h^2x^2))+2ac^2(7dh(120g}{}$$

input
```
Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(128*a^3*f*h^3 - a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x
) + f*(672*g^2 + 315*g*h*x + 64*h^2*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g
*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) +
 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(
10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*
h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x
^2 + 20*h^3*x^3)))) - 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) -
 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(
1680*c^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2185, 27, 687, 27, 687, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(g+hx)^3\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int h(g+hx)^3((7cd-4af)h-c(3fg-7eh)x)\sqrt{cx^2+a}dx}{7ch^2}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (g+hx)^3((7cd-4af)h-c(3fg-7eh)x)\sqrt{cx^2+a}dx}{7ch}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int 3c(g+hx)^2\left(h(14cdg-5afg-7aeh)-\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)x\right)\sqrt{cx^2+a}dx}{6c}-\frac{1}{6}\left(a+cx^2\right)^{3/2}(g+hx)^3(3fg-7eh)}{7ch}+$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\int(g+hx)^2\left(h(14cdg-5afg-7aeh)-\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)x\right)\sqrt{cx^2+a}dx-\frac{1}{6}\left(a+cx^2\right)^{3/2}(g+}{7ch}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$\downarrow$ 687

$$\frac{\frac{1}{2}\left(\frac{\int(g+hx)\left(h\left(70c^2dg^2+16a^2fh^2-ac\left(19fg^2+7h(7eg+4dh)\right)\right)-c\left(6cfg^3-14ch(eg+7dh)g+ah^2(41fg+35eh)\right)x\right)\sqrt{cx^2+a}dx}{5c}-\frac{(a+cx^2)^{3/2}(g+}{}\right.}{7ch}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$\downarrow$ 676

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}h\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)\int\sqrt{cx^2+a}dx+\frac{2\left(a+cx^2\right)^{3/2}\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+15fg^2\right)-c^2\left(3fg^4-7g^2h(12dh+e\right.\right.}{3c}}{5c}\right.}{}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$\downarrow$ 211

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}h\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{2\left(a+cx^2\right)^{3/2}\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+15fg^2\right)-c^2\right.}{3c}}{5c}\right.}{}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$\downarrow$ 224

3.78. $\int(g+hx)^3\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

$$\frac{1}{2}\left(\frac{\frac{35}{4}h\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{2\left(a+cx^2\right)^{3/2}\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+1\cdots\right.\right.}{3c}}{5c}\cdots\right.$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{\frac{35}{4}h\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)+\frac{2\left(a+cx^2\right)^{3/2}\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+15\cdots\right.\right.}{3c}}{5c}\cdots\right.$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^4}{7ch}$$

input
```
Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (-1/6*((3*f*g - 7*e*h)*(g + h*
x)^3*(a + c*x^2)^(3/2)) + (-1/5*((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d
*h))*(g + h*x)^2*(a + c*x^2)^(3/2))/c + ((2*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f
*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)))*(a + c
*x^2)^(3/2))/(3*c) - (h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h^2*(41*f*
g + 35*e*h))*x*(a + c*x^2)^(3/2))/4 + (35*h*(8*c^2*d*g^3 + a^2*h^2*(3*f*g
+ e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*((x*Sqrt[a + c*x^2])/2 + (a*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4)/(5*c))/2)/(7*c*h)
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.78. $\int (g+hx)^3\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.78.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.92

method	result
default	$d\,g^{3}\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)+f\,h^{3}\left(\frac{x^{4}\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{7c}-\frac{4a\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{5c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{15c^{2}}\right)}{7c}\right)+\left(e\,h^{3}+3\right.$
risch	$\frac{(240f\,h^{3}c^{3}x^{6}+280c^{3}e\,h^{3}x^{5}+840c^{3}f g\,h^{2}x^{5}+48a\,c^{2}f\,h^{3}x^{4}+336c^{3}d\,h^{3}x^{4}+1008c^{3}eg\,h^{2}x^{4}+1008c^{3}f\,g^{2}h\,x^{4}+70a\,c^{2}e\,h^{3}x^{3}+210a\,c^{2}fg}{}$

input
```
int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*g^3*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+
f*h^3*(1/7*x^4*(c*x^2+a)^(3/2)/c-4/7*a/c*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a
/c^2*(c*x^2+a)^(3/2)))+(e*h^3+3*f*g*h^2)*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/
c*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2)))))+1/3*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(3/2)/c+(d*
h^3+3*e*g*h^2+3*f*g^2*h)*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(
3/2))+(3*d*g*h^2+3*e*g^2*h+f*g^3)*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*(1/2*x*
(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.19

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[-\frac{105\left(6\,a^{2}ceg^{2}h-a^{3}eh^{3}-2\left(4\,ac^{2}d-a^{2}cf\right)g^{3}+3\left(2\,a^{2}cd-a^{3}f\right)gh^{2}\right)\sqrt{c}\log\left(-2\,cx^{2}-2\,\sqrt{cx^{2}+a}\sqrt{c}x\right.}{}\right.$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 +
 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqr
t(c)*x - a) - 2*(240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2*c*e*g*h^2 +
 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*c^3*e*g*h^2
 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(
7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c^2*e*h^3
+ 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e*g*h^2 +
 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105*(6*
a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a
^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/1680*(105*(6*a^2*c*e*g^2*h - a^3
*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 + 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(-
c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h^3*x^6 + 560*a*c^2*e*g
^3 - 672*a^2*c*e*g*h^2 + 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*
f*g^2*h + 21*c^3*e*g*h^2 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d -
 2*a^2*c*f)*g^2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^
3*e*g^2*h + a*c^2*e*h^3 + 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*
g^3 + 21*a*c^2*e*g*h^2 + 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2
*c*f)*h^3)*x^2 + 105*(6*a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f
)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

3.78.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(377) = 754$.

Time = 0.58 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.94

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^3x^6}{7} + \frac{x^5(ceh^3+3cfgh^2)}{6c} + \frac{x^4\left(\frac{afh^3}{7}+cdh^3+3cegh^2+3cfg^2h\right)}{5c} + \frac{x^3\left(aeh^3+3afgh^2-\frac{5a(ceh^3+3cfgh^2)}{6c}+3cdgh^2+3c\right.}{4c}\right. \\ \sqrt{a}\left(dg^3x + \frac{fh^3x^6}{6} + \frac{x^5(eh^3+3fgh^2)}{5} + \frac{x^4(dh^3+3egh^2+3fg^2h)}{4} + \frac{x^3\cdot(3dgh^2+3eg^2h+fg^3)}{3} + \frac{x^2\cdot(3dg^2h+eg^3)}{2}\right) \end{cases}$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h**3*x**6/7 + x**5*(c*e*h**3 + 3*c*f*g*h**2
)/(6*c) + x**4*(a*f*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c)
 + x**3*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2)/(6*c) + 3
*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + x**2*(a*d*h**3 + 3*a*e*g*h*
*2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2
*h)/(5*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + x*(3*a*d*g*h**2 + 3*a*e*g**2*
h + a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2
)/(6*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + c*d*g**3)/(2*c)
+ (3*a*d*g**2*h + a*e*g**3 - 2*a*(a*d*h**3 + 3*a*e*g*h**2 + 3*a*f*g**2*h -
 4*a*(a*f*h**3/7 + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(5*c) + 3*c*d*g
**2*h + c*e*g**3)/(3*c))/c) + (a*d*g**3 - a*(3*a*d*g*h**2 + 3*a*e*g**2*h +
 a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(c*e*h**3 + 3*c*f*g*h**2)/(
6*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + c*d*g**3)/(2*c))*Pi
ecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*lo
g(x)/sqrt(c*x**2), True)), Ne(c, 0)), (sqrt(a)*(d*g**3*x + f*h**3*x**6/6 +
 x**5*(e*h**3 + 3*f*g*h**2)/5 + x**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4
+ x**3*(3*d*g*h**2 + 3*e*g**2*h + f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2
), True))
```

3.78. $\int (g+hx)^3\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.78.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.12

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{(cx^2+a)^{\frac{3}{2}}fh^3x^4}{7\,c} - \frac{4\,(cx^2+a)^{\frac{3}{2}}afh^3x^2}{35\,c^2} + \frac{1}{2}\,\sqrt{cx^2+a}dg^3x + \frac{adg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}} + \frac{(cx^2+a)^{\frac{3}{2}}eg^3}{3\,c} + \frac{(cx^2+a)^{\frac{3}{2}}dg^2h}{c} + \frac{8\,(cx^2+a)^{\frac{3}{2}}a^2fh^3}{105\,c^3} + \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{3}{2}}x^3}{6\,c} + \frac{(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{3}{2}}x^2}{5\,c} - \frac{(3\,fgh^2+eh^3)(cx^2+a)^{\frac{3}{2}}ax}{8\,c^2} + \frac{(3\,fgh^2+eh^3)\sqrt{cx^2+a}a^2x}{16\,c^2} + \frac{(fg^3+3\,eg^2h+3\,dgh^2)(cx^2+a)^{\frac{3}{2}}x}{4\,c} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}ax}{8\,c} + \frac{(3\,fgh^2+eh^3)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{5}{2}}} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} - \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)(cx^2+a)^{\frac{3}{2}}a}{15\,c^2}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(c*x^2 + a)^(3/2)*f*h^3*x^4/c - 4/35*(c*x^2 + a)^(3/2)*a*f*h^3*x^2/c^2
 + 1/2*sqrt(c*x^2 + a)*d*g^3*x + 1/2*a*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c
) + 1/3*(c*x^2 + a)^(3/2)*e*g^3/c + (c*x^2 + a)^(3/2)*d*g^2*h/c + 8/105*(c
*x^2 + a)^(3/2)*a^2*f*h^3/c^3 + 1/6*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*
x^3/c + 1/5*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(3/2)*x^2/c - 1/8*
(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a*x/c^2 + 1/16*(3*f*g*h^2 + e*h^3)*s
qrt(c*x^2 + a)*a^2*x/c^2 + 1/4*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a)
^(3/2)*x/c - 1/8*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a*x/c + 1
/16*(3*f*g*h^2 + e*h^3)*a^3*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/8*(f*g^3 +
3*e*g^2*h + 3*d*g*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/15*(3*f*g^2*
h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(3/2)*a/c^2
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.19

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(6\,fh^3x+\frac{7\,(3\,c^5fgh^2+c^5eh^3)}{c^5}\right)x+\frac{6\,(21\,c^5fg^2h+21\,c^5egh^2+7\,c^5dh^3+a\right.}{c^5}\right.\right.\right.\right.\right.$$
$$-\frac{(8\,ac^2dg^3-2\,a^2cfg^3-6\,a^2ceg^2h-6\,a^2cdgh^2+3\,a^3fgh^2+a^3eh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*e*h^
3)/c^5)*x + 6*(21*c^5*f*g^2*h + 21*c^5*e*g*h^2 + 7*c^5*d*h^3 + a*c^4*f*h^3
)/c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*e*g^2*h + 18*c^5*d*g*h^2 + 3*a*c^4*f*g
*h^2 + a*c^4*e*h^3)/c^5)*x + 8*(35*c^5*e*g^3 + 105*c^5*d*g^2*h + 21*a*c^4*
f*g^2*h + 21*a*c^4*e*g*h^2 + 7*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3)/c^5)*x + 105
*(8*c^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*e*g^2*h + 6*a*c^4*d*g*h^2 - 3*a^2*
c^3*f*g*h^2 - a^2*c^3*e*h^3)/c^5)*x + 16*(35*a*c^4*e*g^3 + 105*a*c^4*d*g^2
*h - 42*a^2*c^3*f*g^2*h - 42*a^2*c^3*e*g*h^2 - 14*a^2*c^3*d*h^3 + 8*a^3*c^
2*f*h^3)/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*e*g^2*h - 6*
a^2*c*d*g*h^2 + 3*a^3*f*g*h^2 + a^3*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/c^(5/2)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.79 $\int (g+hx)^2\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.79.1 Optimal result

Integrand size = 29, antiderivative size = 280

$$\begin{aligned}
&\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx \\
&= \frac{(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{16c^2} \\
&\quad-\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch}+\frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} \\
&\quad-\frac{(8(2ah^2(2fg+eh)+cg(fg^2-2h(eg+5dh)))-3h(5(2cd-af)h^2-2cg(fg-2eh))\,x)\left(a+cx^2\right)^{3/2}}{120c^2h} \\
&\quad+\frac{a(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{5/2}}
\end{aligned}$$

output
```
-1/10*(-2*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h+1/6*f*(h*x+g)^3*(c*x^2+a)
^(3/2)/c/h-1/120*(16*a*h^2*(e*h+2*f*g)+8*c*g*(f*g^2-2*h*(5*d*h+e*g))-3*h*(
5*(-a*f+2*c*d)*h^2-2*c*g*(-2*e*h+f*g))*x)*(c*x^2+a)^(3/2)/c^2/h+1/16*a*(8*
c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+
a)^(1/2))/c^(5/2)+1/16*(8*c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))
*x*(c*x^2+a)^(1/2)/c^2
```

3.79.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.88

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{a+cx^2}(-a^2h(64fg+32eh+15fhx)+2ac(5dh(16g+3hx)+fx(15g^2+16ghx+5h^2x^2)+e(40g^2+\cdots}{\cdots}$$
$$-\frac{a(8c^2dg^2+a^2fh^2-2ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{16c^{5/2}}$$

input
```
Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5*d*h*(16
*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*g*h*x +
 8*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(10*g^2
 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))))/(240*c
^2) - (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(16*c^(5/2))
```

3.79.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2185, 27, 687, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(g+hx)^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int 3h(g+hx)^2((2cd-af)h-c(fg-2eh)x)\sqrt{cx^2+a}dx}{6ch^2}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (g+hx)^2((2cd-af)h-c(fg-2eh)x)\sqrt{cx^2+a}dx}{2ch}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{\int c(g+hx)\left(h(10cdg-3afg-4aeh)+\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)\sqrt{cx^2+a}dx}{5c}-\frac{1}{5}\left(a+cx^2\right)^{3/2}(g+hx)^2(fg-2eh)}{2ch}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int (g+hx)\left(h(10cdg-3afg-4aeh)+\left(5(2cd-af)h^2-2cg(fg-2eh)\right)x\right)\sqrt{cx^2+a}dx-\frac{1}{5}\left(a+cx^2\right)^{3/2}(g+h}{2ch}$$
$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{5}\left(\frac{5h\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)\int\sqrt{cx^2+a}dx}{4c}-\frac{2\left(a+cx^2\right)^{3/2}\left(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg^3\right)}{3c}+\frac{hx\left(a+cx^2\right)^{3/2}\left(5h^2(2c\right.}{4}\right.}{2ch}$$
$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{5}\left(\frac{5h\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)}{4c}-\frac{2\left(a+cx^2\right)^{3/2}\left(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg^3\right)}{3c}+hx\right.}{2ch}$$
$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{1}{5}\left(\frac{5h\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)}{4c}-\frac{2\left(a+cx^2\right)^{3/2}\left(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg\right.}{3c}\right.}{2ch}$$
$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^3}{6ch}$$

$$\Big\downarrow \; 219$$

3.79. $\int (g+hx)^2\sqrt{a+cx^2}(d+ex+fx^2)\, dx$

$$\frac{\frac{1}{5}\left(\frac{5h\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)(a^2fh^2-2ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{4c}-\frac{2(a+cx^2)^{3/2}(2ah^2(eh+2fg)-2cgh(5dh+eg)+cfg^3)}{3c}\right.}{2ch}$$

$$\frac{f(a+cx^2)^{3/2}(g+hx)^3}{6ch}$$

input
```
Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) + (-1/5*((f*g - 2*e*h)*(g + h*x)
^2*(a + c*x^2)^(3/2)) + ((-2*(c*f*g^3 - 2*c*g*h*(e*g + 5*d*h) + 2*a*h^2*(2
*f*g + e*h))*(a + c*x^2)^(3/2))/(3*c) + (h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f
*g - 2*e*h))*x*(a + c*x^2)^(3/2))/(4*c) + (5*h*(8*c^2*d*g^2 + a^2*f*h^2 -
2*a*c*(f*g^2 + h*(2*e*g + d*h)))*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt
[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/(4*c))/5)/(2*c*h)
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.79.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97

method	result
default	$d\,g^{2}\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)+f\,h^{2}\left(\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4c}\right)}{2c}\right.$
risch	$-\frac{\left(-40f\,h^{2}c^{2}x^{5}-48c^{2}e\,h^{2}x^{4}-96c^{2}f g h\,x^{4}-10af\,h^{2}c\,x^{3}-60c^{2}d\,h^{2}x^{3}-120c^{2}eg h\,x^{3}-60c^{2}f\,g^{2}x^{3}-16ae\,h^{2}c\,x^{2}-32afghc\,x^{2}-160c^{2}d\right.}{24}$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*g^2*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))+
f*h^2*(1/6*x^3*(c*x^2+a)^(3/2)/c-1/2*a/c*(1/4*x*(c*x^2+a)^(3/2)/c-1/4*a/c*
(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))))+(e*h
^2+2*f*g*h)*(1/5*x^2*(c*x^2+a)^(3/2)/c-2/15*a/c^2*(c*x^2+a)^(3/2))+1/3*(2*
d*g*h+e*g^2)*(c*x^2+a)^(3/2)/c+(d*h^2+2*e*g*h+f*g^2)*(1/4*x*(c*x^2+a)^(3/2
)/c-1/4*a/c*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1
/2))))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.12

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \left[-\frac{15\left(4\,a^2cegh-2\left(4\,ac^2d-a^2cf\right)g^2+\left(2\,a^2cd-a^3f\right)h^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(40\right.$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/480*(15*(4*a^2*c*e*g*h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^
3*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(40*
c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*
h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 +
 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3
*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (
2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*e*g*
h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan
(sqrt(-c)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2
*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g
*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5
*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*
g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2
 + a))/c^3]
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.73

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{fh^2x^5}{6}+\frac{x^4(ceh^2+2cfgh)}{5c}+\frac{x^3\left(\frac{afh^2}{6}+cdh^2+2cegh+cfg^2\right)}{4c}+\frac{x^2\left(aeh^2+2afgh-\frac{4a(ceh^2+2cfgh)}{5c}+2cdgh+ceg^2\right)}{3c}+\right.\\ \sqrt{a}\left(dg^2x+\frac{fh^2x^5}{5}+\frac{x^4(eh^2+2fgh)}{4}+\frac{x^3(dh^2+2egh+fg^2)}{3}+\frac{x^2\cdot(2dgh+eg^2)}{2}\right)\end{cases}$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h**2*x**5/6 + x**4*(c*e*h**2 + 2*c*f*g*h)/(
5*c) + x**3*(a*f*h**2/6 + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + x**2*(a
*e*h**2 + 2*a*f*g*h - 4*a*(c*e*h**2 + 2*c*f*g*h)/(5*c) + 2*c*d*g*h + c*e*g
**2)/(3*c) + x*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + c*d*h*
*2 + 2*c*e*g*h + c*f*g**2)/(4*c) + c*d*g**2)/(2*c) + (2*a*d*g*h + a*e*g**2
 - 2*a*(a*e*h**2 + 2*a*f*g*h - 4*a*(c*e*h**2 + 2*c*f*g*h)/(5*c) + 2*c*d*g*
h + c*e*g**2)/(3*c))/c) + (a*d*g**2 - a*(a*d*h**2 + 2*a*e*g*h + a*f*g**2 -
 3*a*(a*f*h**2/6 + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + c*d*g**2)/(2*c
))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)),
(x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (sqrt(a)*(d*g**2*x + f*h**2*x**
5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**3*(d*h**2 + 2*e*g*h + f*g**2)/3 + x**
2*(2*d*g*h + e*g**2)/2), True))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.09

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{(cx^2+a)^{\frac{3}{2}}fh^2x^3}{6\,c} + \frac{1}{2}\sqrt{cx^2+a}dg^2x - \frac{(cx^2+a)^{\frac{3}{2}}afh^2x}{8\,c^2} + \frac{\sqrt{cx^2+a}a^2fh^2x}{16\,c^2} + \frac{adg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}} + \frac{a^3fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{5}{2}}} + \frac{(cx^2+a)^{\frac{3}{2}}eg^2}{3\,c} + \frac{2\,(cx^2+a)^{\frac{3}{2}}dgh}{3\,c} + \frac{(2\,fgh+eh^2)(cx^2+a)^{\frac{3}{2}}x^2}{5\,c} + \frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{3}{2}}x}{4\,c} - \frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}ax}{8\,c} - \frac{(fg^2+2\,egh+dh^2)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} - \frac{2\,(2\,fgh+eh^2)(cx^2+a)^{\frac{3}{2}}a}{15\,c^2}$$

input

```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output

```
1/6*(c*x^2 + a)^(3/2)*f*h^2*x^3/c + 1/2*sqrt(c*x^2 + a)*d*g^2*x - 1/8*(c*x
^2 + a)^(3/2)*a*f*h^2*x/c^2 + 1/16*sqrt(c*x^2 + a)*a^2*f*h^2*x/c^2 + 1/2*a
*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/16*a^3*f*h^2*arcsinh(c*x/sqrt(a*
c))/c^(5/2) + 1/3*(c*x^2 + a)^(3/2)*e*g^2/c + 2/3*(c*x^2 + a)^(3/2)*d*g*h/
c + 1/5*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*x^2/c + 1/4*(f*g^2 + 2*e*g*h +
 d*h^2)*(c*x^2 + a)^(3/2)*x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 +
 a)*a*x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/
2) - 2/15*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*a/c^2
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.12

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,fh^2x+\frac{6\,(2\,c^4fgh+c^4eh^2)}{c^4}\right)x+\frac{5\,(6\,c^4fg^2+12\,c^4egh+6\,c^4dh^2+ac^3fh^2)}{c^4}\right)\right.\right.\right.$$
$$-\frac{(8\,ac^2dg^2-2\,a^2cfg^2-4\,a^2cegh-2\,a^2cdh^2+a^3fh^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*e*h^2)/c^4
)*x + 5*(6*c^4*f*g^2 + 12*c^4*e*g*h + 6*c^4*d*h^2 + a*c^3*f*h^2)/c^4)*x +
8*(5*c^4*e*g^2 + 10*c^4*d*g*h + 2*a*c^3*f*g*h + a*c^3*e*h^2)/c^4)*x + 15*(
8*c^4*d*g^2 + 2*a*c^3*f*g^2 + 4*a*c^3*e*g*h + 2*a*c^3*d*h^2 - a^2*c^2*f*h^
2)/c^4)*x + 16*(5*a*c^3*e*g^2 + 10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h - 2*a^2*c
^2*e*h^2)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 4*a^2*c*e*g*h - 2*a
^2*c*d*h^2 + a^3*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.80 $\int (g+hx)\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.80.1 Optimal result

Integrand size = 27, antiderivative size = 175

$$\begin{aligned}\int (g+hx)\sqrt{a+cx^2}(d+ex+fx^2)\,dx &= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}\\ &\quad-\frac{(4(2afh^2+c(3fg^2-5h(eg+dh)))+3ch(3fg-5eh)x)\left(a+cx^2\right)^{3/2}}{60c^2h}\\ &\quad+\frac{a(4cdg-afg-aeh)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}\end{aligned}$$

output
```
1/5*f*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h-1/60*(8*a*f*h^2+4*c*(3*f*g^2-5*h*(d*h+
e*g))+3*c*h*(-5*e*h+3*f*g)*x)*(c*x^2+a)^(3/2)/c^2/h+1/8*a*(-a*e*h-a*f*g+4*
c*d*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(4*c*d*g-a*(e*h+f*g)
)*x*(c*x^2+a)^(1/2)/c
```

3.80.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{a+cx^2}(-16a^2fh+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3g+2hx)+x(5e(4g+3h}{120c^2}$$

input
```
Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*
g + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(
5*g + 4*h*x)))) + 15*a*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2]])/(120*c^2)
```

3.80.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2185, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}(g+hx)\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int h(g+hx)((5cd-2af)h-c(3fg-5eh)x)\sqrt{cx^2+a}dx}{5ch^2}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^2}{5ch}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (g+hx)((5cd-2af)h-c(3fg-5eh)x)\sqrt{cx^2+a}dx}{5ch}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^2}{5ch}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{5}{4}h(-aeh-afg+4cdg)\int\sqrt{cx^2+a}dx-\frac{(a+cx^2)^{3/2}(2afh^2-5ch(dh+eg)+3cfg^2)}{3c}-\frac{1}{4}hx(a+cx^2)^{3/2}(3fg-5eh)}{5ch}+\frac{f(a+cx^2)^{3/2}(g+hx)^2}{5ch}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{4}h(-aeh-afg+4cdg)\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)-\frac{(a+cx^2)^{3/2}(2afh^2-5ch(dh+eg)+3cfg^2)}{3c}-\frac{1}{4}hx(a+cx^2)^{3/2}}{5ch}$$
$$\frac{f(a+cx^2)^{3/2}(g+hx)^2}{5ch}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{5}{4}h(-aeh-afg+4cdg)\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)-\frac{(a+cx^2)^{3/2}(2afh^2-5ch(dh+eg)+3cfg^2)}{3c}-\frac{1}{4}hx(a+}{5ch}$$
$$\frac{f(a+cx^2)^{3/2}(g+hx)^2}{5ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{4}h\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)(-aeh-afg+4cdg)-\frac{(a+cx^2)^{3/2}(2afh^2-5ch(dh+eg)+3cfg^2)}{3c}-\frac{1}{4}hx(a+cx}{5ch}$$
$$\frac{f(a+cx^2)^{3/2}(g+hx)^2}{5ch}$$

input
```
Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^2*(a + c*x^2)^(3/2))/(5*c*h) + (-1/3*((3*c*f*g^2 + 2*a*f*h^2
 - 5*c*h*(e*g + d*h))*(a + c*x^2)^(3/2))/c - (h*(3*f*g - 5*e*h)*x*(a + c*x^
2)^(3/2))/4 + (5*h*(4*c*d*g - a*f*g - a*e*h)*((x*Sqrt[a + c*x^2])/2 + (a*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4)/(5*c*h)
```

3.80. $\int(g+hx)\sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.80.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{(-24hfc^2x^4-30c^2ehx^3-30c^2fgx^3-8acfhx^2-40c^2dhx^2-40c^2egx^2-15aehxc-15afgxc-60c^2dgx+16a^2fh-40acdh-40aceg)}{120c^2}$
default	$dg\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)+hf\left(\frac{x^2(cx^2+a)^{\frac{3}{2}}}{5c}-\frac{2a(cx^2+a)^{\frac{3}{2}}}{15c^2}\right)+(eh+fg)\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4c}-\frac{a(}{}\right.$

input
```
int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*c^2*f*h*x^4-30*c^2*e*h*x^3-30*c^2*f*g*x^3-8*a*c*f*h*x^2-40*c^2
*d*h*x^2-40*c^2*e*g*x^2-15*a*c*e*h*x-15*a*c*f*g*x-60*c^2*d*g*x+16*a^2*f*h-
40*a*c*d*h-40*a*c*e*g)/c^2*(c*x^2+a)^(1/2)-1/8*a/c^(3/2)*(a*e*h+a*f*g-4*c*
d*g)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.88

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\left[\frac{15\,(a^2eh-(4\,acd-a^2f)g)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\,(24\,c^2fhx^4+40\,aceg+30\,(c^2fg+}{240}\right.$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c
*x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g +
c^2*e*h)*x^3 + 8*(5*c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^
2*f)*h + 15*(a*c*e*h + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/120
*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^2*e*h)*x^3 + 8*(5*
c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f)*h + 15*(a*c*e*h
 + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{fhx^4}{5}+\frac{x^3(ceh+cfg)}{4c}+\frac{x^2\left(\frac{afh}{5}+cdh+ceg\right)}{3c}+\frac{x\left(aeh+afg-\frac{3a(ceh+cfg)}{4c}+cdg\right)}{2c}+\frac{adh+aeg-\frac{2a\left(\frac{afh}{5}+cdh+ceg\right)}{3c}}{c}\right)+\\ \sqrt{a}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right)\end{cases}$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h*x**4/5 + x**3*(c*e*h + c*f*g)/(4*c) + x**
2*(a*f*h/5 + c*d*h + c*e*g)/(3*c) + x*(a*e*h + a*f*g - 3*a*(c*e*h + c*f*g)
/(4*c) + c*d*g)/(2*c) + (a*d*h + a*e*g - 2*a*(a*f*h/5 + c*d*h + c*e*g)/(3*
c))/c) + (a*d*g - a*(a*e*h + a*f*g - 3*a*(c*e*h + c*f*g)/(4*c) + c*d*g)/(2
*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0))
, (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (sqrt(a)*(d*g*x + f*h*x**4/4
+ x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{\left(cx^2+a\right)^{\frac{3}{2}}fhx^2}{5\,c}+\frac{1}{2}\,\sqrt{cx^2+a}dgx + \frac{adg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}eg}{3\,c} + \frac{\left(cx^2+a\right)^{\frac{3}{2}}dh}{3\,c}-\frac{2\,\left(cx^2+a\right)^{\frac{3}{2}}afh}{15\,c^2} + \frac{\left(cx^2+a\right)^{\frac{3}{2}}(fg+eh)x}{4\,c}-\frac{\sqrt{cx^2+a}(fg+eh)ax}{8\,c} - \frac{(fg+eh)a^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(c*x^2 + a)^(3/2)*f*h*x^2/c + 1/2*sqrt(c*x^2 + a)*d*g*x + 1/2*a*d*g*ar
csinh(c*x/sqrt(a*c))/sqrt(c) + 1/3*(c*x^2 + a)^(3/2)*e*g/c + 1/3*(c*x^2 +
a)^(3/2)*d*h/c - 2/15*(c*x^2 + a)^(3/2)*a*f*h/c^2 + 1/4*(c*x^2 + a)^(3/2)*
(f*g + e*h)*x/c - 1/8*sqrt(c*x^2 + a)*(f*g + e*h)*a*x/c - 1/8*(f*g + e*h)*
a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,fhx+\frac{5\,(c^3fg+c^3eh)}{c^3}\right)x+\frac{4\,(5\,c^3eg+5\,c^3dh+ac^2fh)}{c^3}\right)x+\frac{15\,(4\,c^3dg+ac^2f$$
$$-\frac{(4\,acdg-a^2fg-a^2eh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*e*h)/c^3)*x + 4*(
5*c^3*e*g + 5*c^3*d*h + a*c^2*f*h)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*
c^2*e*h)/c^3)*x + 8*(5*a*c^2*e*g + 5*a*c^2*d*h - 2*a^2*c*f*h)/c^3) - 1/8*(
4*a*c*d*g - a^2*f*g - a^2*e*h)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3
/2)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.81 $\int \sqrt{a+cx^2}(d+ex+fx^2)\,dx$

3.81.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \sqrt{a+cx^2}(d+ex+fx^2)\,dx = \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e(a+cx^2)^{3/2}}{3c}+\frac{fx(a+cx^2)^{3/2}}{4c}+\frac{a(4cd-af)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}$$

output
```
1/3*e*(c*x^2+a)^(3/2)/c+1/4*f*x*(c*x^2+a)^(3/2)/c+1/8*a*(-a*f+4*c*d)*arcta
nh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(-a*f+4*c*d)*x*(c*x^2+a)^(1/2)/c
```

3.81.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \sqrt{a+cx^2}(d+ex+fx^2)\,dx = \frac{\sqrt{a+cx^2}(8ae+12cdx+3afx+8cex^2+6cfx^3)}{24c}+\frac{a(-4cd+af)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{8c^{3/2}}$$

input
```
Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(8*a*e + 12*c*d*x + 3*a*f*x + 8*c*e*x^2 + 6*c*f*x^3))/(24
*c) + (a*(-4*c*d + a*f)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(3/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2346, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 2346}$$

$$\frac{\int (4cd-af+4cex)\sqrt{cx^2+a}dx}{4c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{(4cd-af)\int \sqrt{cx^2+a}dx+\frac{4}{3}e\left(a+cx^2\right)^{3/2}}{4c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(4cd-af)\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{4}{3}e\left(a+cx^2\right)^{3/2}}{4c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(4cd-af)\left(\frac{1}{2}a\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{4}{3}e\left(a+cx^2\right)^{3/2}}{4c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)(4cd-af)+\frac{4}{3}e\left(a+cx^2\right)^{3/2}}{4c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

input
```
Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + c*x^2)^(3/2))/(4*c) + ((4*e*(a + c*x^2)^(3/2))/3 + (4*c*d - a*f)
*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt
[c])))/(4*c)
```

3.81.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

3.81.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result	si
risch	$\frac{\left(6cf\,x^{3}+8ce\,x^{2}+3afx+12cdx+8ae\right)\sqrt{c\,x^{2}+a}}{24c}-\frac{a\left(fa-4cd\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}$	75
default	$d\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)+f\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4c}-\frac{a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4c}\right)+\frac{e\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{3c}$	11

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*c*f*x^3+8*c*e*x^2+3*a*f*x+12*c*d*x+8*a*e)/c*(c*x^2+a)^(1/2)-1/8*a*
(a*f-4*c*d)/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[-\frac{3\,(4\,acd-a^2f)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\,(4\,c^2d+acf)\right.}{48\,c^2}$$
$$\left.-\frac{3\,(4\,acd-a^2f)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\,(4\,c^2d+acf)x)\sqrt{cx^2+a}}{24\,c^2}\right]$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(
c)*x - a) - 2*(6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c*f)*x
)*sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) - (6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d +
 a*c*f)*x)*sqrt(c*x^2 + a))/c^2]
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.17

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+cx^2}\left(\frac{ae}{3c}+\frac{ex^2}{3}+\frac{fx^3}{4}+\frac{x\left(\frac{af}{4}+cd\right)}{2c}\right)+\left(ad-\frac{a\left(\frac{af}{4}+cd\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \text{for } c\\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) & \text{othe}\end{cases}$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(a*e/(3*c) + e*x**2/3 + f*x**3/4 + x*(a*f/4 +
c*d)/(2*c)) + (a*d - a*(a*f/4 + c*d)/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(
a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), N
e(c, 0)), (sqrt(a)*(d*x + e*x**2/2 + f*x**3/3), True))
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{1}{2}\sqrt{cx^2+a}dx + \frac{\left(cx^2+a\right)^{\frac{3}{2}}fx}{4\,c} - \frac{\sqrt{cx^2+a}afx}{8\,c} + \frac{ad\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}} - \frac{a^2f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}} + \frac{\left(cx^2+a\right)^{\frac{3}{2}}e}{3\,c}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + a)*d*x + 1/4*(c*x^2 + a)^(3/2)*f*x/c - 1/8*sqrt(c*x^2 + a
)*a*f*x/c + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/8*a^2*f*arcsinh(c*x
/sqrt(a*c))/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*e/c
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \frac{1}{24}\sqrt{cx^2+a}\left(\left(2\,(3\,fx+4\,e)x+\frac{3\,(4\,c^2d+acf)}{c^2}\right)x+\frac{8\,ae}{c}\right) - \frac{(4\,acd-a^2f)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*
a*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3
/2)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.82 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx} \, dx$

3.82.1 Optimal result

Integrand size = 29, antiderivative size = 206

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx} \, dx &= \frac{(2(fg^2-egh+dh^2)-h(fg-eh)x)\sqrt{a+cx^2}}{2h^3} \\ &\quad + \frac{f(a+cx^2)^{3/2}}{3ch} \\ &\quad - \frac{(2cdgh^2+(fg-eh)(2cg^2+ah^2))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} \\ &\quad - \frac{\sqrt{cg^2+ah^2}(fg^2-egh+dh^2)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4}\end{aligned}$$

output
```
1/3*f*(c*x^2+a)^(3/2)/c/h-1/2*(2*c*d*g*h^2+(-e*h+f*g)*(a*h^2+2*c*g^2))*arc
tanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)-(d*h^2-e*g*h+f*g^2)*arctanh((-
c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*h^2+c*g^2)^(1/2)/h^4+1/
2*(2*d*h^2-2*e*g*h+2*f*g^2-h*(-e*h+f*g)*x)*(c*x^2+a)^(1/2)/h^3
```

3.82.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$
$$= \frac{\frac{h\sqrt{a+cx^2}(2afh^2+3ch(-2eg+2dh+ehx)+cf(6g^2-3ghx+2h^2x^2))}{c}+12\sqrt{-cg^2-ah^2}(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx}{\sqrt{-c}}\right.}{6h^4}$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
((h*Sqrt[a + c*x^2]*(2*a*f*h^2 + 3*c*h*(-2*e*g + 2*d*h + e*h*x) + c*f*(6*g
^2 - 3*g*h*x + 2*h^2*x^2)))/c + 12*Sqrt[-(c*g^2) - a*h^2]*(f*g^2 + h*(-(e*
g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) -
a*h^2]] + (3*(a*h^2*(f*g - e*h) + 2*c*(f*g^3 + g*h*(-(e*g) + d*h)))*Log[-(
Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(6*h^4)
```

3.82.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$
$$\Big\downarrow \text{ 2185}$$
$$\frac{\int \frac{3ch(dh-(fg-eh)x)\sqrt{cx^2+a}}{g+hx}dx}{3ch^2}+\frac{f(a+cx^2)^{3/2}}{3ch}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{(dh-(fg-eh)x)\sqrt{cx^2+a}}{g+hx}dx}{h}+\frac{f(a+cx^2)^{3/2}}{3ch}$$
$$\Big\downarrow \text{ 682}$$

$$\frac{\frac{\int \frac{c\left(ah\left(fg^2-h(eg-2dh)\right)-\left(2cdgh^2+(fg-eh)\left(2cg^2+ah^2\right)\right)x\right)}{(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{ah\left(fg^2-h(eg-2dh)\right)-\left(2cdgh^2+(fg-eh)\left(2cg^2+ah^2\right)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 719

$$\frac{\frac{\frac{2\left(ah^2+cg^2\right)\left(dh^2-egh+fg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{h}}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{2\left(ah^2+cg^2\right)\left(dh^2-egh+fg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2\left(ah^2+cg^2\right)\left(dh^2-egh+fg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{\sqrt{c}h}}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 488

$$\frac{\frac{-\frac{2\left(ah^2+cg^2\right)\left(dh^2-egh+fg^2\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{\sqrt{c}h}}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^2}}{h}+\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

$\Big\downarrow$ 219

3.82. $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

$$\frac{\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{\sqrt{c}h}-\frac{2\sqrt{ah^2+cg^2}\left(dh^2-egh+fg^2\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h}}{2h^2}+\frac{\sqrt{a+cx^2}\left(2\left(dh^2-egh+fg^2\right)\right.}{2h^2}}{h}$$

$$\frac{f\left(a+cx^2\right)^{3/2}}{3ch}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + c*x^2)^(3/2))/(3*c*h) + (((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*
h)*x)*Sqrt[a + c*x^2])/(2*h^2) + (-(((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 +
 a*h^2))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h)) - (2*Sqrt[c*g^
2 + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h
^2]*Sqrt[a + c*x^2])])/h)/(2*h^2))/h
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.82.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.53

method	result
risch	$\frac{\left(2f\,h^{2}c\,x^{2}+3ce\,h^{2}x-3cfghx+2af\,h^{2}+6cd\,h^{2}-6cegh+6cf\,g^{2}\right)\sqrt{c\,x^{2}+a}}{6c\,h^{3}}+\frac{\frac{\left(ae\,h^{3}-afg\,h^{2}-2cdg\,h^{2}+2ce\,g^{2}h-2cf\,g^{3}\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right.}{h\sqrt{c}}}{}$
default	$\frac{eh\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)+\frac{fh\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{3c}-fg\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{h^{2}}+\frac{\left(d\,h^{2}-egh+f\,g^{2}\right)\left(\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\right.}\right.}{}$

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*f*h^2*x^2+3*c*e*h^2*x-3*c*f*g*h*x+2*a*f*h^2+6*c*d*h^2-6*c*e*g*h+6
*c*f*g^2)/c*(c*x^2+a)^(1/2)/h^3+1/2/h^3*((a*e*h^3-a*f*g*h^2-2*c*d*g*h^2+2*
c*e*g^2*h-2*c*f*g^3)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(2*a*d*h^4-2*
a*e*g*h^3+2*a*f*g^2*h^2+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)/h^2/((a*h^2+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2
)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x
+1/h*g)))
```

3.82.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

3.82.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = & -\frac{\sqrt{cx^2+a}fgx}{2\,h^2} + \frac{\sqrt{cx^2+a}ex}{2\,h} - \frac{\sqrt{c}fg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} \\ & + \frac{\sqrt{c}eg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3} - \frac{\sqrt{c}dg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2} \\ & - \frac{afg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}h^2} + \frac{ae\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}h} \\ & + \frac{\sqrt{a+\frac{cg^2}{h^2}}fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} \\ & - \frac{\sqrt{a+\frac{cg^2}{h^2}}eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} \\ & + \frac{\sqrt{a+\frac{cg^2}{h^2}}d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h} \\ & + \frac{\sqrt{cx^2+a}fg^2}{h^3} - \frac{\sqrt{cx^2+a}eg}{h^2} + \frac{\sqrt{cx^2+a}d}{h} + \frac{(cx^2+a)^{\frac{3}{2}}f}{3\,ch}\end{aligned}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

output
```
-1/2*sqrt(c*x^2 + a)*f*g*x/h^2 + 1/2*sqrt(c*x^2 + a)*e*x/h - sqrt(c)*f*g^3
*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^3 - s
qrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 1/2*a*f*g*arcsinh(c*x/sqrt(a*c))/(
sqrt(c)*h^2) + 1/2*a*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h) + sqrt(a + c*g^2
/h^2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/h^3 - sqrt(a + c*g^2/h^2)*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + sqrt(a + c*g^2/h^2)*d*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + sqrt(c*x^
2 + a)*f*g^2/h^3 - sqrt(c*x^2 + a)*e*g/h^2 + sqrt(c*x^2 + a)*d/h + 1/3*(c*
x^2 + a)^(3/2)*f/(c*h)
```

3.82.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.83 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.83.1 Optimal result

Integrand size = 29, antiderivative size = 308

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(2(ah^2(2fg-eh)+cg(3fg^2-h(2eg-dh)))-h(afh^2+c(3fg^2-2h(eg-dh)))x)\sqrt{a+cx^2}}{2h^3(cg^2+ah^2)} - \frac{(fg^2-egh+dh^2)(a+cx^2)^{3/2}}{h(cg^2+ah^2)(g+hx)} + \frac{(afh^2+2c(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}h^4} + \frac{(ah^2(2fg-eh)+cg(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^4\sqrt{cg^2+ah^2}}$$

output
```
-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/2*(a*f*h^2+
2*c*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/
2)+(a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh((-c*g*x+a*h)/
(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(1/2)-1/2*(2*a*h^2*
(-e*h+2*f*g)+2*c*g*(3*f*g^2-h*(-d*h+2*e*g))-h*(a*f*h^2+c*(3*f*g^2-2*h*(-d*
h+e*g)))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$
$$= \frac{\frac{h\sqrt{a+cx^2}(2h(2eg-dh+ehx)+f(-6g^2-3ghx+h^2x^2))}{g+hx} + \frac{4(3cfg^3+cgh(-2eg+dh)+ah^2(2fg-eh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}} - \frac{(6cfg^2+\cdots}{\cdots}}{2h^4}$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2
*x^2)))/(g + h*x) + (4*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g -
e*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2
]])/Sqrt[-(c*g^2) - a*h^2] - ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e*g + d*h))
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(2*h^4)
```

3.83.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2182, 25, 682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$
$$\Big\downarrow \text{ 2182}$$
$$-\frac{\int -\frac{\left(cdg-afg+aeh+\left(afh-c\left(-\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{ah^2+cg^2} - \frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$
$$\Big\downarrow \text{ 25}$$
$$\frac{\int \frac{\left(cdg-afg+aeh+\left(afh-c\left(-\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{ah^2+cg^2} - \frac{(a+cx^2)^{3/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$
$$\Big\downarrow \text{ 682}$$

$$\frac{\frac{\int -\frac{c\left(cg^2+ah^2\right)\left(ah(3fg-2eh)-\left(6cfg^2+afh^2-2ch(2eg-dh)\right)x\right)}{h(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-dh)+3c\right.\right.}{2h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{c\left(cg^2+ah^2\right)\left(ah(3fg-2eh)-\left(6cfg^2+afh^2-2ch(2eg-dh)\right)x\right)}{h(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-dh)+3c\right.\right.}{2h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\left(ah^2+cg^2\right)\int \frac{ah(3fg-2eh)-\left(6cfg^2+afh^2-2ch(2eg-dh)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^3}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-dh)+3cfg\right.\right.}{2h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{-\frac{\left(ah^2+cg^2\right)\left(\frac{2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(afh^2-2ch(2eg-dh)+6cfg^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{h}\right)}{2h^3}-\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)\right.\right.}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{-\frac{\left(ah^2+cg^2\right)\left(\frac{2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(afh^2-2ch(2eg-dh)+6cfg^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}\right)}{2h^3}-\sqrt{a+cx^2}\left(2\left(ah^2(2\right.\right.}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 219$$

3.83. $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}dx$

$$-\frac{\frac{(ah^2+cg^2)\left(\frac{2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(afh^2-2ch(2eg-dh)+6cfg^2\right)}{\sqrt{c}h}\right)}{2h^3}-\frac{\sqrt{a+cx^2}(2(ah^2(2\ldots}{}}{ah^2+cg^2}$$
$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$\downarrow$ 488

$$-\frac{\frac{(ah^2+cg^2)\left(-\frac{2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(afh^2-2ch(2eg-dh)+6cfg^2\right)}{\sqrt{c}h}\right)}{2h^3}-\frac{\sqrt{\ldots}}{}}{ah^2+cg^2}$$
$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{\frac{(ah^2+cg^2)\left(-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(afh^2-2ch(2eg-dh)+6cfg^2\right)}{\sqrt{c}h}-\frac{2\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)}{h\sqrt{ah^2+cg^2}}\right)}{2h^3}-\frac{\sqrt{\ldots}}{}}{ah^2+cg^2}$$
$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
-(((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
)) + (-1/2*((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h
*(3*c*f*g^2 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/h^3 - ((c*g
^2 + a*h^2)*(-(((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[
c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h)) - (2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h)
 + a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a
+ c*x^2])])/(h*Sqrt[c*g^2 + a*h^2])))/(2*h^3))/(c*g^2 + a*h^2)
```

3.83. $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.83. $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.67

method	result
risch	$\frac{(fxh+2eh-4fg)\sqrt{cx^2+a}}{2h^3}+\frac{\frac{(afh^2+2cdh^2-4cegh+6cfg^2)\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{h\sqrt{c}}-\frac{(2aeh^3-4afgh^2-4cdgh^2+6ceg^2h-8cfg^3)\ln\left(\frac{2ah^2+\cdots}{h^2}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{f\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{h^2}+\frac{(eh-2fg)\left(\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{ah^2+cg^2}{h^2}}-\frac{\sqrt{c}\,g\ln\left(\frac{-\frac{cg}{h}+c\left(x+\frac{g}{h}\right)}{\sqrt{c}}+\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg(\cdots}{h}\cdots}\right)}{h}\cdots\right.}{\cdots}$

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(f*h*x+2*e*h-4*f*g)*(c*x^2+a)^(1/2)/h^3+1/2/h^3*((a*f*h^2+2*c*d*h^2-4*
c*e*g*h+6*c*f*g^2)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(2*a*e*h^3-4*a*
f*g*h^2-4*c*d*g*h^2+6*c*e*g^2*h-8*c*f*g^3)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3*(2*
a*d*h^4-2*a*e*g*h^3+2*a*f*g^2*h^2+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)*(-1
/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*
g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*
g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

3.83.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.83.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{\sqrt{cx^2+a}fg^2}{h^4x+gh^3} + \frac{\sqrt{cx^2+a}eg}{h^3x+gh^2} - \frac{\sqrt{cx^2+a}d}{h^2x+gh} + \frac{\sqrt{cx^2+a}fx}{2h^2} + \frac{3\sqrt{c}fg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} - \frac{2\sqrt{c}eg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3} + \frac{\sqrt{c}d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2} + \frac{af\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}h^2} - \frac{cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^5} + \frac{ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^4} - \frac{cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3} - \frac{2\sqrt{a+\frac{cg^2}{h^2}}fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} + \frac{\sqrt{a+\frac{cg^2}{h^2}}e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} - \frac{2\sqrt{cx^2+a}fg}{h^3} + \frac{\sqrt{cx^2+a}e}{h^2}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

output
```
-sqrt(c*x^2 + a)*f*g^2/(h^4*x + g*h^3) + sqrt(c*x^2 + a)*e*g/(h^3*x + g*h^
2) - sqrt(c*x^2 + a)*d/(h^2*x + g*h) + 1/2*sqrt(c*x^2 + a)*f*x/h^2 + 3*sqr
t(c)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^4 - 2*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c
))/h^3 + sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 1/2*a*f*arcsinh(c*x/sqrt(a
*c))/(sqrt(c)*h^2) - c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/
(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + c*e*g^2*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2
/h^2)*h^4) - c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 2*sqrt(a + c*g^2/h^2)*f*g*arcs
inh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + s
qrt(a + c*g^2/h^2)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/h^2 - 2*sqrt(c*x^2 + a)*f*g/h^3 + sqrt(c*x^2 + a)*e/h^2
```

3.83.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{cx^2+a}(fx^2+ex+d)}{(hx+g)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.84 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.84.1 Optimal result

Integrand size = 29, antiderivative size = 296

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx &= \frac{(2(3fg-eh)(cg^2+ah^2)+h(2afh^2+c(3fg^2-h(eg-dh)))x)\sqrt{a+cx^2}}{2h^3(cg^2+ah^2)(g+hx)} \\ &\quad - \frac{(fg^2-egh+dh^2)(a+cx^2)^{3/2}}{2h(cg^2+ah^2)(g+hx)^2} - \frac{\sqrt{c}(3fg-eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4} \\ &\quad - \frac{(2a^2fh^4+2c^2g^3(3fg-eh)+ach^2(9fg^2-h(3eg-dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4(cg^2+ah^2)^{3/2}}\end{aligned}$$

output
```
-1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(2*
a^2*f*h^4+2*c^2*g^3*(-e*h+3*f*g)+a*c*h^2*(9*f*g^2-h*(-d*h+3*e*g)))*arctanh
((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(3/2)
-(-e*h+3*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4+1/2*(2*(-e*h+
3*f*g)*(a*h^2+c*g^2)+h*(2*a*f*h^2+c*(3*f*g^2-h*(-d*h+e*g)))*x)*(c*x^2+a)^(
1/2)/h^3/(a*h^2+c*g^2)/(h*x+g)
```

3.84.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$
$$= \frac{\frac{h\sqrt{a+cx^2}\left(ah^2\left(-h(eg+dh+2ehx)+f\left(5g^2+8ghx+2h^2x^2\right)\right)+cg\left(dh^3x-egh(2g+3hx)+fg\left(6g^2+9ghx+2h^2x^2\right)\right)\right)}{(cg^2+ah^2)(g+hx)^2}+\frac{2(2a^2fh^4+2c^2g^3(3fg-eh)-}{}}{2h^4}$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
((h*Sqrt[a + c*x^2]*(a*h^2*(-(h*(e*g + d*h + 2*e*h*x)) + f*(5*g^2 + 8*g*h*
x + 2*h^2*x^2)) + c*g*(d*h^3*x - e*g*h*(2*g + 3*h*x) + f*g*(6*g^2 + 9*g*h*
x + 2*h^2*x^2))))/((c*g^2 + a*h^2)*(g + h*x)^2) + (2*(2*a^2*f*h^4 + 2*c^2*
g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*ArcTan[(Sqrt[c]*
(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)
^(3/2) + 2*Sqrt[c]*(3*f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*h
^4)
```

3.84.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2182, 25, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(2(cdg-afg+aeh)+\left(2afh-c\left(-\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^2}dx}{2\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(2(cdg-afg+aeh)+\left(2afh-c\left(-\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^2}dx}{2\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 681

$$\frac{\frac{\sqrt{a+cx^2}\left(hx\left(2afh^2-ch(eg-dh)+3cfg^2\right)+2\left(ah^2+cg^2\right)(3fg-eh)\right)}{h^3(g+hx)}-\frac{\int -\frac{2\left(ah\left(3cfg^2+2afh^2-ch(eg-dh)\right)-2c(3fg-eh)\left(cg^2+ah^2\right)x\right)}{h(g+hx)\sqrt{cx^2+a}}dx}{2h^2}}{2\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{ah\left(3cfg^2+2afh^2-ch(eg-dh)\right)-2c(3fg-eh)\left(cg^2+ah^2\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{h^3}+\frac{\sqrt{a+cx^2}\left(hx\left(2afh^2-ch(eg-dh)+3cfg^2\right)+2\left(ah^2+cg^2\right)(3fg-eh)\right)}{h^3(g+hx)}}{2\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 719

$$\frac{\frac{\frac{\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2c\left(ah^2+cg^2\right)(3fg-eh)\int \frac{1}{\sqrt{cx^2+a}}dx}{h}}{h^3}+\frac{\sqrt{a+cx^2}\left(hx\left(2afh^2-ch(eg-dh)+3c\cdots\right.\right.}{h^3(g+h\cdots}}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2c\left(ah^2+cg^2\right)(3fg-eh)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}}{h^3}+\sqrt{a+cx^2}\left(hx\left(2afh^2-ch(eg-\cdots\right.\right.}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)(3fg-eh)}{h}}{h^3}+\sqrt{a+cx^2}\left(hx\left(2afh^2-ch(e\cdots\right.\right.}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

3.84. $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}dx$

$$\Big\downarrow \ 488$$

$$\frac{-\frac{\frac{\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)(3fg-eh)}{h}}{h^3}+\frac{\sqrt{a+cx^2}(h\cdots}{\cdots}}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{-\frac{\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{h\sqrt{ah^2+cg^2}}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)(3fg-eh)}{h}}{h^3}+\frac{\sqrt{a+cx^2}(h\cdots}{\cdots}}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
-1/2*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^2) + (((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c
*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(h^3*(g + h*x)) + ((-2*Sqrt[c]*(3*f*g
- e*h)*(c*g^2 + a*h^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h - ((2*a^2*f
*h^4 + 2*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 +
a*h^2]))/h^3)/(2*(c*g^2 + a*h^2))
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(274) = 548$.

Time = 0.70 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.20

method	result
risch	$\frac{f\sqrt{c\,x^2+a}}{h^3}+\frac{\frac{\sqrt{c}\,(eh-3fg)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{h}-\frac{\left(af\,h^2+cd\,h^2-3cegh+6cf\,g^2\right)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2c}{}}}{x+\frac{g}{h}}\right.}{h^2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}}}{}$
default	Expression too large to display

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
f/h^3*(c*x^2+a)^(1/2)+1/h^3*(c^(1/2)*(e*h-3*f*g)/h*ln(x*c^(1/2)+(c*x^2+a)^
(1/2))-1/h^2*(a*f*h^2+c*d*h^2-3*c*e*g*h+6*c*f*g^2)/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3
*(a*e*h^3-2*a*f*g*h^2-2*c*d*g*h^2+3*c*e*g^2*h-4*c*f*g^3)*(-1/(a*h^2+c*g^2)
*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c
*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g
/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(a*d*h^4-a*e*g*h^3+a*f*g^2*h^2
+c*d*g^2*h^2-c*e*g^3*h+c*f*g^4)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/
h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2
)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*g^
2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)
+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+1/h*g))))
```

3.84. $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.84.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 927 vs. $2(275) = 550$.

Time = 0.26 (sec) , antiderivative size = 927, normalized size of antiderivative = 3.13

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$
$$= -\frac{\sqrt{cx^2+a}cfg^3}{2\left(cg^2h^4x+ah^6x+cg^3h^3+agh^5\right)} + \frac{\sqrt{cx^2+a}ceg^2}{2\left(cg^2h^3x+ah^5x+cg^3h^2+agh^4\right)}$$
$$- \frac{(cx^2+a)^{\frac{3}{2}}fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4x+cg^4h+ag^2h^3\right)}$$
$$+ \frac{\sqrt{cx^2+a}cfg^2}{2\left(cg^2h^3+ah^5\right)} - \frac{\sqrt{cx^2+a}cdg}{2\left(cg^2h^2x+ah^4x+cg^3h+agh^3\right)}$$
$$+ \frac{(cx^2+a)^{\frac{3}{2}}eg}{2\left(cg^2h^2x^2+ah^4x^2+2\,cg^3hx+2\,agh^3x+cg^4+ag^2h^2\right)} - \frac{\sqrt{cx^2+a}ceg}{2\left(cg^2h^2+ah^4\right)}$$
$$- \frac{(cx^2+a)^{\frac{3}{2}}d}{2\left(cg^2hx^2+ah^3x^2+2\,cg^3x+2\,agh^2x+\frac{cg^4}{h}+ag^2h\right)} + \frac{\sqrt{cx^2+a}cd}{2\left(cg^2h+ah^3\right)}$$
$$+ \frac{2\sqrt{cx^2+a}fg}{h^4x+gh^3} - \frac{\sqrt{cx^2+a}e}{h^3x+gh^2} - \frac{3\sqrt{c}fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} + \frac{\sqrt{c}e\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}$$
$$- \frac{c^2fg^4\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^7} + \frac{c^2eg^3\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^6}$$
$$- \frac{c^2dg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5} + \frac{5\,cfg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^5}$$
$$- \frac{3\,ceg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^4} + \frac{cd\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\sqrt{a+\frac{cg^2}{h^2}}h^3}$$
$$+ \frac{\sqrt{a+\frac{cg^2}{h^2}}f\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} + \frac{\sqrt{cx^2+a}f}{h^3}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")
```

output
```
-1/2*sqrt(c*x^2 + a)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
 + 1/2*sqrt(c*x^2 + a)*c*e*g^2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^
4) - 1/2*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*
x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c*f*g^2/(c*g^
2*h^3 + a*h^5) - 1/2*sqrt(c*x^2 + a)*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*
h + a*g*h^3) + 1/2*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*
g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*
g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*
g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*sqrt(c*x^2 + a)*c*d/(c*g^2*
h + a*h^3) + 2*sqrt(c*x^2 + a)*f*g/(h^4*x + g*h^3) - sqrt(c*x^2 + a)*e/(h^
3*x + g*h^2) - 3*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*arcsin
h(c*x/sqrt(a*c))/h^3 - 1/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 1/2*c^2*e*
g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/((a + c*g^2/h^2)^(3/2)*h^6) - 1/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(
h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 5/
2*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/(sqrt(a + c*g^2/h^2)*h^5) - 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(
h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4) + 1/2*
c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g...
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(275) = 550$.

Time = 0.32 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$
$$= \frac{(6\,c^2fg^4 - 2\,c^2eg^3h + 9\,acfg^2h^2 - 3\,acegh^3 + acdh^4 + 2\,a^2fh^4)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{(cg^2h^4 + ah^6)\sqrt{-cg^2-ah^2}}$$
$$+ \frac{\sqrt{cx^2+a}f}{h^3}$$
$$+ \frac{6\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 c^2fg^4h - 4\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 c^2eg^3h^2 + 2\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 c^2dg^2h^3 + 5\left(\sqrt{c}x\right.}{}$$
$$+ \frac{\left(3\sqrt{c}fg - \sqrt{c}eh\right)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{h^4}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

output
```
(6*c^2*f*g^4 - 2*c^2*e*g^3*h + 9*a*c*f*g^2*h^2 - 3*a*c*e*g*h^3 + a*c*d*h^4
 + 2*a^2*f*h^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2
+ a)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h - 4*(sqrt(c)*x
 - sqrt(c*x^2 + a))^3*c^2*e*g^3*h^2 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^
2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(c)
*x - sqrt(c*x^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*
d*h^5 + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x -
sqrt(c*x^2 + a))^2*c^(5/2)*e*g^4*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(
5/2)*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - (
sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 - (sqrt(c)*x - sqrt(c*x
^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(
c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 14*(sqr
t(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*
a*c^2*e*g^3*h^2 - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a
))*a^2*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 5*a^2*c^(3/
2)*f*g^3*h^2 - 3*a^2*c^(3/2)*e*g^2*h^3 + a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(
c)*f*g*h^4 - 2*a^3*sqrt(c)*e*h^5)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(
c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) +...
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^3}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.85 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.85.1 Optimal result

Integrand size = 29, antiderivative size = 314

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$
$$-\frac{(2c^2fg^5+a^2eh^5+acgh^2(3fg^2+dh^2)+h(2a^2fh^4+acgh^2(6fg-eh)+c^2(3fg^4-dg^2h^2))x)\sqrt{a+cx^2}}{2h^3(cg^2+ah^2)^2(g+hx)^2}$$
$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{3/2}}{3h(cg^2+ah^2)(g+hx)^3}+\frac{\sqrt{c}f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^4}$$
$$+\frac{c(2c^2fg^5+a^2h^4(4fg-eh)+acgh^2(5fg^2-dh^2))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^4(cg^2+ah^2)^{5/2}}$$

output
```
-1/3*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(
2*c^2*f*g^5+a^2*h^4*(-e*h+4*f*g)+a*c*g*h^2*(-d*h^2+5*f*g^2))*arctanh((-c*g
*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(5/2)+f*arc
tanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4-1/2*(2*c^2*f*g^5+a^2*e*h^5+a*c
*g*h^2*(d*h^2+3*f*g^2)+h*(2*a^2*f*h^4+a*c*g*h^2*(-e*h+6*f*g)+c^2*(-d*g^2*h
^2+3*f*g^4))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^2
```

3.85.2 Mathematica [A] (verified)

Time = 10.53 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$
$$= \frac{\frac{h\sqrt{a+cx^2}\left(-2(fg^2+h(-eg+dh))+\frac{\left(7cfg^3+cgh(-4eg+dh)-3ah^2(-2fg+eh)\right)(g+hx)}{cg^2+ah^2}-\frac{\left(6a^2fh^4+c^2\left(11fg^4-g^2h(2eg+dh)\right)+ach^2\left(20fg^2+h(-5eg+2dh)\right)\cdots}{\left(cg^2+ah^2\right)^2}\cdots\right.}{(g+hx)^3}\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(
-4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a
^2*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-
5*e*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^
2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x
])/(c*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]]
+ (3*c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))
*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(
5/2))/(6*h^4)
```

3.85.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2182, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{3\left(cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{cx^2+a}}{(g+hx)^3}dx}{3\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{cx^2+a}}{(g+hx)^3}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 680

$$\frac{-\frac{\int \frac{2c\left(ah\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2f\left(cg^2+ah^2\right)^2x\right)}{h(g+hx)\sqrt{cx^2+a}}dx}{4h^2(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^4+acgh\left(dh^2+3f\right.\right.}{2h^2(g+hx)^2(ah^2+cg^2)}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{c\int \frac{ah\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2f\left(cg^2+ah^2\right)^2x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^4+acgh\left(dh^2+3fg^2\right)\right.}{2h^2(g+hx)^2(ah^2+cg^2)}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 719

$$\frac{-\frac{c\left(\frac{\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2f\left(ah^2+cg^2\right)^2\int \frac{1}{\sqrt{cx^2+a}}dx}{h}\right)}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\right.\right.}{2h^2(g\ldots}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 224

$$\frac{-\frac{c\left(\frac{\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2f\left(ah^2+cg^2\right)^2\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}\right)}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh^4+acgh^2(6fg-\right.\right.}{}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

3.85. $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}dx$

$$-\frac{\frac{c\left(\frac{\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{2f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)^2}{\sqrt{c}h}\right)}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh^4+acgh^2(6fg-\dots\right.\right.}{\dots}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 488

$$-\frac{\frac{c\left(-\frac{\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{2f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)^2}{\sqrt{c}h}\right)}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh\dots\right.\right.}{\dots}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{\frac{c\left(-\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{h\sqrt{ah^2+cg^2}}-\frac{2f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)^2}{\sqrt{c}h}\right)}{2h^3(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}\left(x\left(2a^2fh\dots\right.\right.}{\dots}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
-1/3*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^3) + (-1/2*(((2*c^2*f*g^5)/h + a^2*e*h^4 + a*c*g*h*(3*f*g^2 + d*h^2) +
 (2*a^2*f*h^4 + a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sq
rt[a + c*x^2])/(h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - (c*((-2*f*(c*g^2 + a*h^
2)^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h) - ((2*c^2*f*g^5 + a
^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/
(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 + a*h^2])))/(2*h^3*(
c*g^2 + a*h^2)))/(c*g^2 + a*h^2)
```

3.85. $\int\frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}dx$

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(292) = 584$.

Time = 0.74 (sec) , antiderivative size = 2448, normalized size of antiderivative = 7.80

method	result	size
default	Expression too large to display	2448

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
f/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*
h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x
+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/
((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*
h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((
x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*
g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))))+(e*h-2*f*g)/h^5*(-1/2/(
a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a*h^2+c*g^2)*(
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((
-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)
/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^
2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/
h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+c*g^2)*h^2*(1
/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2)*ln((-c*g...
```

3.85.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.85.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1772 vs. $2(293) = 586$.

Time = 0.28 (sec) , antiderivative size = 1772, normalized size of antiderivative = 5.64

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")
```

output
```
-1/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*
x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^2*e*g
^3/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*
h^4 + a^2*g*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*
g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^
6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/2*sqrt(c*x^2 + a)*c^2*f
*g^3/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^2*d*g
^2/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^
3 + a^2*g*h^5) + 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^
2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x
+ c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*sqrt(c*x^2 + a)*c^2*e*g^2/(
c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c^2*
g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*
x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*sqrt(c*x^
2 + a)*c^2*d*g/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(3/
2)*f*g^2/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*
c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + sqrt(c*x^2 + a)*c*f*g
^2/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) + 1/3*(c*x^2 + a)^(3/2)*e
*g/(c*g^2*h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*
h*x + 3*a*g^2*h^3*x + c*g^5 + a*g^3*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c...
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(293) = 586$.

Time = 0.37 (sec) , antiderivative size = 1701, normalized size of antiderivative = 5.42

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

output
```
-(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*
c*e*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2
 - a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2))
- sqrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*e*g^4*h^3 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 - 12*(sq
rt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*e*g^2*h^5 + 3*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 -
3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*e*h^7 + 54*(sqrt(c)*x - sqrt(c*x^2
 + a))^4*c^(7/2)*f*g^6*h - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*e*g^
5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*(sqrt(c)*
x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a*c^(5/2)*e*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*
g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 + 3*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*e*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x
^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(
c)*f*h^7 + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 8*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^4*e*g^6*h - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^
5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 8*(sqrt(c)*x
- sqrt(c*x^2 + a))^3*a*c^3*e*g^4*h^3 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))...
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^4}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.86 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

3.86.1 Optimal result

Integrand size = 29, antiderivative size = 313

$$\begin{aligned}\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx &= -\frac{(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^3\,(g+hx)^2}\\ &\quad-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4}\\ &\quad+\frac{(4ah^2(2fg-eh)+cg(3fg^2+h(eg-5dh)))\,(a+cx^2)^{3/2}}{12h\,(cg^2+ah^2)^2\,(g+hx)^3}\\ &\quad-\frac{ac(4c^2dg^2+4a^2fh^2-ac(fg^2-h(5eg-dh)))\,\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8\,(cg^2+ah^2)^{7/2}}\end{aligned}$$

output
```
-1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^4+1/12*(4
*a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-5*d*h+e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2
+c*g^2)^2/(h*x+g)^3-1/8*a*c*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*
e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*
g^2)^(7/2)-1/8*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g)))*(-c*g*
x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)^2
```

3.86.2 Mathematica [A] (verified)

Time = 10.75 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$
$$= \frac{1}{24}\left(-\frac{\sqrt{a+cx^2}\left(6(cg^2+ah^2)^3\left(fg^2+h(-eg+dh)\right)-2(cg^2+ah^2)^2\left(9cfg^3+cgh(-5eg+dh)-4ah^2(-\right.\right.}{}\right.$$
$$+\frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log(g+hx)}{(cg^2+ah^2)^{7/2}}$$
$$\left.-\frac{3ac(4c^2dg^2+4a^2fh^2-ac(fg^2+h(-5eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{7/2}}\right)$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
(-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c
*g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h)
)*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g
+ d*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*
h^4*(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3
*f*g^5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^
4)) + (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*
Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 -
a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt
[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/24
```

3.86.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2182, 25, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$\Big\downarrow$ 2182

$$-\frac{\int -\frac{\left(4(cdg-afg+aeh)+\left(4afh+c\left(\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^4}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\left(4(cdg-afg+aeh)+\left(4afh+c\left(\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^4}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 679

$$\frac{\frac{\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)\int \frac{\sqrt{cx^2+a}}{(g+hx)^3}dx}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4ah^2(2fg-eh)+cgh(eg-5dh)+3cfg^3\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 486

$$\frac{\frac{\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)\left(\frac{ac\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4ah^2(2fg-eh)+cgh(eg-5dh)+3cfg^3\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}}{4\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)\left(-\frac{ac\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4ah^2(2fg-eh)+cgh(eg-\right.}{3h(g+hx)^3\left(ah^2+cg^2\right)}}{4\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 219

3.86. $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

$$\frac{\frac{\left(-\frac{ac\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{2\left(ah^2+cg^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4ah^2(2fg-eh)+cgh(eg-\right.}{3h(g+hx)^3(ah^2+cg^2)}}{4\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^4) + (((3*c*f*g^3 + c*g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a +
c*x^2)^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + ((4*c^2*d*g^2 + 4*a^2*f*
h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(-1/2*((a*h - c*g*x)*Sqrt[a + c*x^2])
/((c*g^2 + a*h^2)*(g + h*x)^2) - (a*c*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 +
a*h^2]*Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)^(3/2))))/(c*g^2 + a*h^2))/(4*
(c*g^2 + a*h^2))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3902 vs. $2(293) = 586$.

Time = 0.77 (sec) , antiderivative size = 3903, normalized size of antiderivative = 12.47

method	result	size
default	Expression too large to display	3903

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x
+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)-c*g*h/(a
*h^2+c*g^2)*(((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(
1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((
2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*c/(a*h^2+
c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)-2*c*g/h)/c*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)
+(a*h^2+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/c^(3/2
)*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+
c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2+c*g^2)*h^2*(((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c^(1/2)*g/h*ln((-c*g/h+c*(x+1/h*g))/c^(1/2)+
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))-(a*h^2+c*g^2)/h
^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+1/h*g))))+(e*h-2*f*g)/h^6*(-1/3/(a*h^2+c*g^2)*h^2/(x+1/h*g)^3
*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(3/2)+c*g*h/(a*h^2+c*
g^2)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+
(a*h^2+c*g^2)/h^2)^(3/2)+1/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/...
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. 2(294) = 588.

Time = 61.13 (sec) , antiderivative size = 2552, normalized size of antiderivative = 8.15

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

output
```
[1/48*(3*(5*a^2*c^2*e*g^5*h + (4*a*c^3*d - a^2*c^2*f)*g^6 - (a^2*c^2*d - 4
*a^3*c*f)*g^4*h^2 + (5*a^2*c^2*e*g*h^5 + (4*a*c^3*d - a^2*c^2*f)*g^2*h^4 -
 (a^2*c^2*d - 4*a^3*c*f)*h^6)*x^4 + 4*(5*a^2*c^2*e*g^2*h^4 + (4*a*c^3*d -
a^2*c^2*f)*g^3*h^3 - (a^2*c^2*d - 4*a^3*c*f)*g*h^5)*x^3 + 6*(5*a^2*c^2*e*g
^3*h^3 + (4*a*c^3*d - a^2*c^2*f)*g^4*h^2 - (a^2*c^2*d - 4*a^3*c*f)*g^2*h^4
)*x^2 + 4*(5*a^2*c^2*e*g^4*h^2 + (4*a*c^3*d - a^2*c^2*f)*g^5*h - (a^2*c^2*
d - 4*a^3*c*f)*g^3*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2
- 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a
*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(8*a*c^3*e*g^7 - a^2*c
^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 - 2*a^4*e*g*h^6 - 6*a^4*d*h^7 - (28*a*c^
3*d - 13*a^2*c^2*f)*g^6*h - (47*a^2*c^2*d - 11*a^3*c*f)*g^4*h^3 - (25*a^3*
c*d + 2*a^4*f)*g^2*h^5 + (6*c^4*f*g^7 + 2*c^4*e*g^6*h + 11*a*c^3*e*g^4*h^3
 + a^2*c^2*e*g^2*h^5 - 8*a^3*c*e*h^7 + (2*c^4*d + 25*a*c^3*f)*g^5*h^2 - (1
1*a*c^3*d - 47*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 28*a^3*c*f)*g*h^6)*x^3
 + (8*c^4*e*g^7 + 44*a*c^3*e*g^5*h^2 + 19*a^2*c^2*e*g^3*h^4 - 17*a^3*c*e*g
*h^6 + 4*(2*c^4*d + a*c^3*f)*g^6*h - (32*a*c^3*d - 41*a^2*c^2*f)*g^4*h^3 -
 (43*a^2*c^2*d - 25*a^3*c*f)*g^2*h^5 - 3*(a^3*c*d + 4*a^4*f)*h^7)*x^2 + (1
7*a*c^3*e*g^6*h - 19*a^2*c^2*e*g^4*h^3 - 44*a^3*c*e*g^2*h^5 - 8*a^4*e*h^7
+ 3*(4*c^4*d + a*c^3*f)*g^7 - (25*a*c^3*d - 43*a^2*c^2*f)*g^5*h^2 - (41*a^
2*c^2*d - 32*a^3*c*f)*g^3*h^4 - 4*(a^3*c*d + 2*a^4*f)*g*h^6)*x)*sqrt(c*...
```

3.86.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{a+cx^2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3404 vs. 2(294) = 588.

Time = 0.35 (sec) , antiderivative size = 3404, normalized size of antiderivative = 10.88

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
```

output
```
-5/8*sqrt(c*x^2 + a)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*
c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7
 + a^3*g*h^9) + 5/8*sqrt(c*x^2 + a)*c^3*e*g^4/(c^3*g^6*h^3*x + 3*a*c^2*g^4
*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3
*a^2*c*g^3*h^6 + a^3*g*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^3
*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7
*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h
 + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*sqrt(c*x^2 + a)*
c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 5/
8*sqrt(c*x^2 + a)*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g
^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3
*g*h^7) + 5/8*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h
^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h
^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a
^2*c*g^4*h^4 + a^3*g^2*h^6) - 5/8*sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^2 +
 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*
d*g^2/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7
*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x
 + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*sqrt(c
*x^2 + a)*c^3*d*g^2/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^...
```

3.86.8 Giac [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{cx^2+a}(fx^2+ex+d)}{(hx+g)^5}\,dx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.87 $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.87.1 Optimal result

Integrand size = 29, antiderivative size = 433

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{c(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,(ah-cgx)\sqrt{a+cx^2}}{8\,(cg^2+ah^2)^4\,(g+hx)^2} - \frac{(fg^2-egh+dh^2)\,(a+cx^2)^{3/2}}{5h\,(cg^2+ah^2)\,(g+hx)^5} + \frac{(5ah^2(2fg-eh)+cg(3fg^2+h(2eg-7dh)))\,(a+cx^2)^{3/2}}{20h\,(cg^2+ah^2)^2\,(g+hx)^4} - \frac{(20a^2fh^4-c^2g^2(3fg^2+h(2eg-27dh))-ach^2(18fg^2-h(33eg-8dh)))\,(a+cx^2)^{3/2}}{60h\,(cg^2+ah^2)^3\,(g+hx)^3} - \frac{ac^2(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2-3h(2eg-dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8\,(cg^2+ah^2)^{9/2}}$$

output

```
-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+1/20*(5
*a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-7*d*h+2*e*g)))*(c*x^2+a)^(3/2)/h/(a*h
^2+c*g^2)^2/(h*x+g)^4-1/60*(20*a^2*f*h^4-c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g
))-a*c*h^2*(18*f*g^2-h*(-8*d*h+33*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)^3
/(h*x+g)^3-1/8*a*c^2*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-
d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*
h^2+c*g^2)^(9/2)-1/8*c*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*
(-d*h+2*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2
```

3.87.2 Mathematica [A] (verified)

Time = 10.88 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx =$$

$$-\frac{\sqrt{a+cx^2}\Big(24(cg^2+ah^2)^4\,(fg^2+h(-eg+dh))-6(cg^2+ah^2)^3\,(11cfg^3+cgh(-6eg+dh)-5ah^2(-2}{}$$

$$+\frac{ac^2(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2+3h(-2eg+dh)))\log(g+hx)}{8\,(cg^2+ah^2)^{9/2}}$$

$$-\frac{ac^2(4c^2dg^3+a^2h^2(6fg-eh)-acg(fg^2+3h(-2eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{8\,(cg^2+ah^2)^{9/2}}$$

input
```
Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/120*(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) -
 6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^
2*h*(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^
2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 +
h*(24*e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^
3 - c*(-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^
3*(6*f*g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g +
 16*d*h)))*(g + h*x)^4))/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c
^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log
[g + h*x])/(8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*
g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^
2 + a*h^2]*Sqrt[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

3.87.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2182, 25, 688, 25, 27, 679, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{\left(5(cdg-afg+aeh)+\left(5afh+c\left(\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^5}dx}{5\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(5(cdg-afg+aeh)+\left(5afh+c\left(\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^5}dx}{5\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 688$$

$$\frac{\frac{\left(a+cx^2\right)^{3/2}\left(5ah^2(2fg-eh)+cgh(2eg-7dh)+3cfg^3\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}-\frac{\int -\frac{\left(4h\left(5c^2dg^2+5a^2fh^2-ac\left(2fg^2-h(7eg-2dh)\right)\right)+c\left(3cfg^3+ch(2eg-7dh)g+5ah^2(2fg-eh)\right)x\right)}{h(g+hx)^4}}{4\left(ah^2+cg^2\right)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(4h\left(5c^2dg^2+5a^2fh^2-ac\left(2fg^2-h(7eg-2dh)\right)\right)+c\left(3cfg^3+ch(2eg-7dh)g+5ah^2(2fg-eh)\right)x\right)\sqrt{cx^2+a}}{h(g+hx)^4}dx}{4\left(ah^2+cg^2\right)}+\frac{\left(a+cx^2\right)^{3/2}\left(5ah^2(2fg-eh)+cgh(2eg-7dh)\right.}{4h(g+hx)^4\left(ah^2+cg^2\right)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(4h\left(5c^2dg^2+5a^2fh^2-ac\left(2fg^2-h(7eg-2dh)\right)\right)+c\left(3cfg^3+ch(2eg-7dh)g+5ah^2(2fg-eh)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^4}dx}{4h\left(ah^2+cg^2\right)}+\frac{\left(a+cx^2\right)^{3/2}\left(5ah^2(2fg-eh)+cgh(2eg-7dh)\right.}{4h(g+hx)^4\left(ah^2+cg^2\right)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 679$$

3.87. $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

$$\frac{\frac{\frac{5ch\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)\int\frac{\sqrt{cx^2+a}}{(g+hx)^3}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h(33eg-8dh)\right)-c^2\left(g^2h(2eg-27dh)+3fg^4\right)\right)}{3(g+hx)^3\left(ah^2+cg^2\right)}}{4h(ah^2+cg^2)}+\frac{(\cdots}{}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{486}$$

$$\frac{\frac{\frac{5ch\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)\left(\frac{ac\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h(33eg-8dh)\cdots\right.\right.}{3(g+hx)^3\left(ah^2+cg^2\right)}}{4h(ah^2+cg^2)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\frac{5ch\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)\left(-\frac{ac\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg\cdots\right.\right.}{3(g+\cdots}}{4h(ah^2+cg^2)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{5ch\left(-\frac{ac\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{2\left(ah^2+cg^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2\cdots\right.\right.}{3(g+\cdots}}{4h(ah^2+cg^2)}}{5\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

input
```
Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

3.87. $\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

output
```
-1/5*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^5) + (((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a
+ c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + (-1/3*((20*a^2*f*h^4 -
 c^2*(3*f*g^4 + g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g -
8*d*h)))*(a + c*x^2)^(3/2))/((c*g^2 + a*h^2)*(g + h*x)^3) + (5*c*h*(4*c^2*
d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*(-1/2*(
(a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x)^2) - (a*c*ArcTan
h[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)
^(3/2))))/(c*g^2 + a*h^2))/(4*h*(c*g^2 + a*h^2)))/(5*(c*g^2 + a*h^2))
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6084 vs. $2(409) = 818$.

Time = 0.86 (sec) , antiderivative size = 6085, normalized size of antiderivative = 14.05

method	result	size
default	Expression too large to display	6085

input
```
int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1918 vs. $2(410) = 820$.

Time = 165.64 (sec) , antiderivative size = 3862, normalized size of antiderivative = 8.92

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

output
```
[-1/240*(15*(6*a^2*c^3*e*g^7*h - a^3*c^2*e*g^5*h^3 + (4*a*c^4*d - a^2*c^3*
f)*g^8 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^6*h^2 + (6*a^2*c^3*e*g^2*h^6 - a^3*
c^2*e*h^8 + (4*a*c^4*d - a^2*c^3*f)*g^3*h^5 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*
g*h^7)*x^5 + 5*(6*a^2*c^3*e*g^3*h^5 - a^3*c^2*e*g*h^7 + (4*a*c^4*d - a^2*c
^3*f)*g^4*h^4 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^2*h^6)*x^4 + 10*(6*a^2*c^3*e
*g^4*h^4 - a^3*c^2*e*g^2*h^6 + (4*a*c^4*d - a^2*c^3*f)*g^5*h^3 - 3*(a^2*c^
3*d - 2*a^3*c^2*f)*g^3*h^5)*x^3 + 10*(6*a^2*c^3*e*g^5*h^3 - a^3*c^2*e*g^3*
h^5 + (4*a*c^4*d - a^2*c^3*f)*g^6*h^2 - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^4*h^
4)*x^2 + 5*(6*a^2*c^3*e*g^6*h^2 - a^3*c^2*e*g^4*h^4 + (4*a*c^4*d - a^2*c^3
*f)*g^7*h - 3*(a^2*c^3*d - 2*a^3*c^2*f)*g^5*h^3)*x)*sqrt(c*g^2 + a*h^2)*lo
g((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(
c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) -
 2*(40*a*c^4*e*g^9 - 46*a^2*c^3*e*g^7*h^2 - 113*a^3*c^2*e*g^5*h^4 - 33*a^4
*c*e*g^3*h^6 - 6*a^5*e*g*h^8 - 24*a^5*d*h^9 - 9*(20*a*c^4*d - 9*a^2*c^3*f)
*g^8*h - (329*a^2*c^3*d - 53*a^3*c^2*f)*g^6*h^3 - (247*a^3*c^2*d + 32*a^4*
c*f)*g^4*h^5 - 2*(61*a^4*c*d + 2*a^5*f)*g^2*h^7 + (6*c^5*f*g^8*h + 4*c^5*e
*g^7*h^2 + 32*a*c^4*e*g^5*h^4 - 53*a^2*c^3*e*g^3*h^6 - 81*a^3*c^2*e*g*h^8
+ 3*(2*c^5*d + 11*a*c^4*f)*g^6*h^3 - (77*a*c^4*d - 113*a^2*c^3*f)*g^4*h^5
- (67*a^2*c^3*d - 46*a^3*c^2*f)*g^2*h^7 + 8*(2*a^3*c^2*d - 5*a^4*c*f)*h^9)
*x^4 + 5*(6*c^5*f*g^9 + 4*c^5*e*g^8*h + 32*a*c^4*e*g^6*h^3 - 35*a^2*c^3...
```

3.87.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
```

output
```
Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5793 vs. $2(410) = 820$.

Time = 0.42 (sec) , antiderivative size = 5793, normalized size of antiderivative = 13.38

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")
```

output
```
-7/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*
c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^
7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 7/8*sqrt(c*x^2
 + a)*c^4*e*g^5/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x +
 4*a^3*c*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^
2*g^5*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*f*g^
5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c
*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2
*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3
*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7/8*sqrt(c
*x^2 + a)*c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4
*a^3*c*g^2*h^9 + a^4*h^11) - 7/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^4*g^8*h^2*x
+ 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x
 + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4
*g*h^9) + 7/8*(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h
^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^
4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x +
 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*
g^4*h^6 + a^4*g^2*h^8) - 7/8*sqrt(c*x^2 + a)*c^4*e*g^4/(c^4*g^8*h^2 + 4*a*
c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/8*(c*...
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4170 vs. $2(410) = 820$.

Time = 0.40 (sec) , antiderivative size = 4170, normalized size of antiderivative = 9.63

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

output
```
-1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 + 6*a^2*c^3*e*g^2*h - 3*a^2*c^3*d*g*h^
2 + 6*a^3*c^2*f*g*h^2 - a^3*c^2*e*h^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^
2*c^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(6
0*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c
*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^
3*e*g^2*h^9 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 90*(sq
rt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 - 15*(sqrt(c)*x - sqrt(c*x^2
 + a))^9*a^3*c^2*e*h^11 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g
^8*h^3 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7 + 810*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^2*c^(7/2)*e*g^3*h^8 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g
^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 135*(
sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*e*g*h^10 - 120*(sqrt(c)*x - sqr
t(c*x^2 + a))^8*a^4*c^(3/2)*f*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c
^6*f*g^9*h^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*e*g^8*h^3 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 640*(sqrt(c)*x - sqrt(c*x^2
 + a))^7*a*c^5*e*g^6*h^5 + 1880*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d*g^
5*h^6 - 1910*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 1860*(...
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

input
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.88.1 Optimal result

Integrand size = 29, antiderivative size = 462

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{a(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x\sqrt{a+cx^2}}{128c^2}$$
$$+\frac{(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2}$$
$$+\frac{(8(9cd-4af)h^2-3cg(5fg-9eh))\,(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h}$$
$$-\frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch}+\frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch}$$
$$+\frac{(4(32a^2fh^4-8ach^2(17fg^2+9h(3eg+dh))-3c^2g^2(5fg^2-3h(3eg+64dh)))-5ch(ah^2(61fg+63eh)+}{5040c^3h}$$
$$+\frac{a^2(48c^2dg^3+3a^2h^2(3fg+eh)-8acg(fg^2+3h(eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}$$

output
```
1/192*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x
*(c*x^2+a)^(3/2)/c^2+1/504*(8*(-4*a*f+9*c*d)*h^2-3*c*g*(-9*e*h+5*f*g))*(h*
x+g)^2*(c*x^2+a)^(5/2)/c^2/h-1/72*(-9*e*h+5*f*g)*(h*x+g)^3*(c*x^2+a)^(5/2)
/c/h+1/9*f*(h*x+g)^4*(c*x^2+a)^(5/2)/c/h+1/5040*(128*a^2*f*h^4-32*a*c*h^2*
(17*f*g^2+9*h*(d*h+3*e*g))-12*c^2*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-5*c*h*(
a*h^2*(63*e*h+61*f*g)+2*c*g*(5*f*g^2-9*h*(12*d*h+e*g)))*x)*(c*x^2+a)^(5/2)
/c^3/h+1/128*a^2*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d
*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^3
+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^
2
```

3.88.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.03

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{\sqrt{a+cx^2}(1024a^4fh^3 - a^3ch(9h(768eg+256dh+105ehx)+f(6912g^2+2835ghx+512h^2x^2))}{}$$

input
```
Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(1024*a^4*f*h^3 - a^3*c*h*(9*h*(768*e*g + 256*d*h + 105*e
*h*x) + f*(6912*g^2 + 2835*g*h*x + 512*h^2*x^2)) + 6*a^2*c^2*(12*d*h*(336*
g^2 + 105*g*h*x + 16*h^2*x^2) + 3*e*(448*g^3 + 420*g^2*h*x + 192*g*h^2*x^2
 + 35*h^3*x^3) + f*x*(420*g^3 + 576*g^2*h*x + 315*g*h^2*x^2 + 64*h^3*x^3))
 + 16*c^4*x^3*(18*d*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3) + x*
(9*e*(56*g^3 + 140*g^2*h*x + 120*g*h^2*x^2 + 35*h^3*x^3) + 5*f*x*(84*g^3 +
 216*g^2*h*x + 189*g*h^2*x^2 + 56*h^3*x^3))) + 8*a*c^3*x*(18*d*(175*g^3 +
336*g^2*h*x + 245*g*h^2*x^2 + 64*h^3*x^3) + x*(9*e*(224*g^3 + 490*g^2*h*x
+ 384*g*h^2*x^2 + 105*h^3*x^3) + f*x*(1470*g^3 + 3456*g^2*h*x + 2835*g*h^2
*x^2 + 800*h^3*x^3)))) - 315*a^2*Sqrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g
+ e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2]])/(40320*c^3)
```

3.88.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2185, 27, 687, 27, 687, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2} (g+hx)^3 \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int h(g+hx)^3((9cd-4af)h-c(5fg-9eh)x)\left(cx^2+a\right)^{3/2}dx}{9ch^2}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (g+hx)^3((9cd-4af)h-c(5fg-9eh)x)\left(cx^2+a\right)^{3/2}dx}{9ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int c(g+hx)^2\left(h(72cdg-17afg-27aeh)+\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)x\right)\left(cx^2+a\right)^{3/2}dx}{8c}-\frac{1}{8}\left(a+cx^2\right)^{5/2}(g+hx)^3(5fg-9eh)}{9ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{8}\int (g+hx)^2\left(h(72cdg-17afg-27aeh)+\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)x\right)\left(cx^2+a\right)^{3/2}dx-\frac{1}{8}\left(a+cx^2\right)^{\cdots}}{9ch} \cdots \frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{1}{8}\left(\frac{\int (g+hx)\left(h\left(504c^2dg^2+64a^2fh^2-ac\left(89fg^2+9h(27eg+16dh)\right)\right)-3c\left(a(61fg+63eh)h^2+2c\left(5fg^3-9gh(eg+12dh)\right)\right)x\right)\left(cx^2+a\right)^{3/2}dx}{7c}+\frac{(a+cx\cdots}{\cdots}\cdots\right.}{9ch} \cdots \frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{1}{8}\Bigg(\frac{\frac{21}{2}h\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)\int\left(cx^2+a\right)^{3/2}dx+\frac{2\left(a+cx^2\right)^{5/2}\left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-3c^2\left(5fg^4-3g^2h(\right.\right.}{5c}}{7c}$$

$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{8}\Bigg(\frac{\frac{21}{2}h\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)\left(\frac{3}{4}a\int\sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{2\left(a+cx^2\right)^{5/2}\left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg\right.\right.}{5c}}{7c}$$

$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{8}\Bigg(\frac{\frac{21}{2}h\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{2\left(a+cx^2\right)^{5/2}\left(32a^2fh^4-8ac\right.}{}}{7c}$$

$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{8}\Bigg(\frac{\frac{21}{2}h\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{2\left(a+cx^2\right)^{5/2}\left(32\right.}{}}{7c}$$

$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\Bigg(\frac{\frac{21}{2}h\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)+\frac{2\left(a+cx^2\right)^{5/2}\left(32a^2\right.}{}}{7c}$$

$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^4}{9ch}$$

3.88. $\int(g+hx)^3\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

input
```
Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + (-1/8*((5*f*g - 9*e*h)*(g + h*
x)^3*(a + c*x^2)^(5/2)) + (((8*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h)
)*(g + h*x)^2*(a + c*x^2)^(5/2))/(7*c) + ((2*(32*a^2*f*h^4 - 8*a*c*h^2*(17
*f*g^2 + 9*h*(3*e*g + d*h)) - 3*c^2*(5*f*g^4 - 3*g^2*h*(3*e*g + 64*d*h)))*
(a + c*x^2)^(5/2))/(5*c) - (h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*
g*h*(e*g + 12*d*h)))*x*(a + c*x^2)^(5/2))/2 + (21*h*(48*c^2*d*g^3 + 3*a^2*
h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*((x*(a + c*x^2)^(3/
2))/4 + (3*a*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*Sqrt[c])))/4))/2)/(7*c))/8)/(9*c*h)
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.88.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.88

method	result
default	$d\,g^{3}\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\,h^{3}\left(\frac{x^{4}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{9c}-\frac{4a\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{7c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{35c^{2}}\right)}{9c}\right.$
risch	$\frac{(4480c^{4}f\,h^{3}x^{8}+5040c^{4}e\,h^{3}x^{7}+15120c^{4}fg\,h^{2}x^{7}+6400a\,c^{3}f\,h^{3}x^{6}+5760c^{4}d\,h^{3}x^{6}+17280c^{4}eg\,h^{2}x^{6}+17280c^{4}f\,g^{2}h\,x^{6}+7560a\,c^{3}e\,h^{3}x^{7}}{}$

input
```
int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^3*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))))+f*h^3*(1/9*x^4*(c*x^2+a)^(5/2)/c-4/9*a/c*(1/
7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2)))+(e*h^3+3*f*g*h^2)*(1/
8*x^3*(c*x^2+a)^(5/2)/c-3/8*a/c*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c
*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x
^2+a)^(1/2))))))+1/5*(3*d*g^2*h+e*g^3)*(c*x^2+a)^(5/2)/c+(d*h^3+3*e*g*h^2+
3*f*g^2*h)*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+(3*d*g*h
^2+3*e*g^2*h+f*g^3)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2
)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
)))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.55

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/80640*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*c*f)
*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2
+ a)*sqrt(c)*x - a) - 2*(4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^
3*c*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h +
 27*c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*
c^4*e*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21
*c^4*e*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*
d + a^2*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^
3*c*d - 4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4
*d + 7*a*c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a
*c^3*e*g^3 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a
^2*c^2*d - 4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 +
 8*(10*a*c^3*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sq
rt(c*x^2 + a))/c^3, 1/40320*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^
2*c^2*d - a^3*c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(-c)*arctan(sq
rt(-c)*x/sqrt(c*x^2 + a)) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 691
2*a^3*c*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2
*h + 27*c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 +
 24*c^4*e*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384
*(21*c^4*e*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24...
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. 2(449) = 898.

Time = 0.66 (sec) , antiderivative size = 1428, normalized size of antiderivative = 3.09

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(c*f*h**3*x**8/9 + x**7*(c**2*e*h**3 + 3*c**2*
f*g*h**2)/(8*c) + x**6*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 +
3*c**2*f*g**2*h)/(7*c) + x**5*(2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(c**2*e
*h**3 + 3*c**2*f*g*h**2)/(8*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*
f*g**3)/(6*c) + x**4*(a**2*f*h**3 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*
f*g**2*h - 6*a*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f
*g**2*h)/(7*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c) + x**3*(a**2*e*h**3
+ 3*a**2*f*g*h**2 + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*a*(
2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c)
+ 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + c**2*d*g**3)/(4
*c) + x**2*(a**2*d*h**3 + 3*a**2*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*c*d*g**2
*h + 2*a*c*e*g**3 - 4*a*(a**2*f*h**3 + 2*a*c*d*h**3 + 6*a*c*e*g*h**2 + 6*a
*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**
2*f*g**2*h)/(7*c) + 3*c**2*d*g**2*h + c**2*e*g**3)/(5*c))/(3*c) + x*(3*a**
2*d*g*h**2 + 3*a**2*e*g**2*h + a**2*f*g**3 + 2*a*c*d*g**3 - 3*a*(a**2*e*h*
*3 + 3*a**2*f*g*h**2 + 6*a*c*d*g*h**2 + 6*a*c*e*g**2*h + 2*a*c*f*g**3 - 5*
a*(2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*
c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*f*g**3)/(6*c) + c**2*d*g**3)
/(4*c))/(2*c) + (3*a**2*d*g**2*h + a**2*e*g**3 - 2*a*(a**2*d*h**3 + 3*a**2
*e*g*h**2 + 3*a**2*f*g**2*h + 6*a*c*d*g**2*h + 2*a*c*e*g**3 - 4*a*(a**2...
```

3.88. $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.88.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.14

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{\left(cx^2+a\right)^{\frac{5}{2}} fh^3x^4}{9\,c} - \frac{4\left(cx^2+a\right)^{\frac{5}{2}} afh^3x^2}{63\,c^2} + \frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}} dg^3x + \frac{3}{8}\sqrt{cx^2+a}\,adg^3x + \frac{3\,a^2dg^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}} + \frac{\left(cx^2+a\right)^{\frac{5}{2}} eg^3}{5\,c} + \frac{3\left(cx^2+a\right)^{\frac{5}{2}} dg^2h}{5\,c} + \frac{8\left(cx^2+a\right)^{\frac{5}{2}} a^2fh^3}{315\,c^3} + \frac{(3\,fgh^2+eh^3)\left(cx^2+a\right)^{\frac{5}{2}} x^3}{8\,c} + \frac{(3\,fg^2h+3\,egh^2+dh^3)\left(cx^2+a\right)^{\frac{5}{2}} x^2}{7\,c} - \frac{(3\,fgh^2+eh^3)\left(cx^2+a\right)^{\frac{5}{2}} ax}{16\,c^2} + \frac{(3\,fgh^2+eh^3)\left(cx^2+a\right)^{\frac{3}{2}} a^2x}{64\,c^2} + \frac{3\,(3\,fgh^2+eh^3)\sqrt{cx^2+a}\,a^3x}{128\,c^2} + \frac{(fg^3+3\,eg^2h+3\,dgh^2)\left(cx^2+a\right)^{\frac{5}{2}} x}{6\,c} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)\left(cx^2+a\right)^{\frac{3}{2}} ax}{24\,c} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}\,a^2x}{16\,c} + \frac{3\,(3\,fgh^2+eh^3)a^4 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{5}{2}}} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)a^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}} - \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)\left(cx^2+a\right)^{\frac{5}{2}} a}{35\,c^2}$$

input
```
integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/9*(c*x^2 + a)^(5/2)*f*h^3*x^4/c - 4/63*(c*x^2 + a)^(5/2)*a*f*h^3*x^2/c^2
 + 1/4*(c*x^2 + a)^(3/2)*d*g^3*x + 3/8*sqrt(c*x^2 + a)*a*d*g^3*x + 3/8*a^2
*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2 + a)^(5/2)*e*g^3/c + 3/
5*(c*x^2 + a)^(5/2)*d*g^2*h/c + 8/315*(c*x^2 + a)^(5/2)*a^2*f*h^3/c^3 + 1/
8*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*x^3/c + 1/7*(3*f*g^2*h + 3*e*g*h^2
 + d*h^3)*(c*x^2 + a)^(5/2)*x^2/c - 1/16*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(
5/2)*a*x/c^2 + 1/64*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a^2*x/c^2 + 3/12
8*(3*f*g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a^3*x/c^2 + 1/6*(f*g^3 + 3*e*g^2*h +
 3*d*g*h^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(
c*x^2 + a)^(3/2)*a*x/c - 1/16*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 +
 a)*a^2*x/c + 3/128*(3*f*g*h^2 + e*h^3)*a^4*arcsinh(c*x/sqrt(a*c))/c^(5/2)
 - 1/16*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2)
 - 2/35*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(5/2)*a/c^2
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.38

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} (d+ex+fx^2) \, dx = \frac{1}{40320} \sqrt{cx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(2\left(7\left(8\,cfh^3x+\frac{9\,(3\,c^8fgh^2+c^8eh^3)}{c^7}\right)x+\frac{8\,(27\,c^8fg^2h+27\,c^8\right.\right.\right.\right.\right.\right.\right.\right. - \frac{\left(48\,a^2c^2dg^3-8\,a^3cfg^3-24\,a^3ceg^2h-24\,a^3cdgh^2+9\,a^4fgh^2+3\,a^4eh^3\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 +
 c^8*e*h^3)/c^7)*x + 8*(27*c^8*f*g^2*h + 27*c^8*e*g*h^2 + 9*c^8*d*h^3 + 10
*a*c^7*f*h^3)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*e*g^2*h + 24*c^8*d*g*h^2 +
 27*a*c^7*f*g*h^2 + 9*a*c^7*e*h^3)/c^7)*x + 48*(21*c^8*e*g^3 + 63*c^8*d*g^
2*h + 72*a*c^7*f*g^2*h + 72*a*c^7*e*g*h^2 + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3
)/c^7)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*e*g^2*h + 168*a*
c^7*d*g*h^2 + 9*a^2*c^6*f*g*h^2 + 3*a^2*c^6*e*h^3)/c^7)*x + 64*(126*a*c^7*
e*g^3 + 378*a*c^7*d*g^2*h + 27*a^2*c^6*f*g^2*h + 27*a^2*c^6*e*g*h^2 + 9*a^
2*c^6*d*h^3 - 4*a^3*c^5*f*h^3)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*
g^3 + 24*a^2*c^6*e*g^2*h + 24*a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 - 3*a^3*
c^5*e*h^3)/c^7)*x + 128*(63*a^2*c^6*e*g^3 + 189*a^2*c^6*d*g^2*h - 54*a^3*c
^5*f*g^2*h - 54*a^3*c^5*e*g*h^2 - 18*a^3*c^5*d*h^3 + 8*a^4*c^4*f*h^3)/c^7)
 - 1/128*(48*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*e*g^2*h - 24*a^3*c*d
*g*h^2 + 9*a^4*f*g*h^2 + 3*a^4*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)
))/c^(5/2)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3 \left(a+cx^2\right)^{3/2} (d+ex+fx^2) \, dx = \int (g+h\,x)^3\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,dx$$

input
```
int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.89.1 Optimal result

Integrand size = 29, antiderivative size = 346

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{a(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,x\sqrt{a+cx^2}}{128c^2} + \frac{(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,x(a+cx^2)^{3/2}}{192c^2} - \frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} - \frac{(12(8ah^2(2fg+eh)+cg(5fg^2-8h(eg+7dh)))-5h(7(8cd-3af)h^2-2cg(5fg-8eh))\,x)\left(a+cx^2\right)^{5/2}}{1680c^2h} + \frac{a^2(48c^2dg^2+3a^2fh^2-8ac(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{5/2}}$$

output

```
1/192*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(
3/2)/c^2-1/56*(-8*e*h+5*f*g)*(h*x+g)^2*(c*x^2+a)^(5/2)/c/h+1/8*f*(h*x+g)^3
*(c*x^2+a)^(5/2)/c/h-1/1680*(96*a*h^2*(e*h+2*f*g)+12*c*g*(5*f*g^2-8*h*(7*d
*h+e*g))-5*h*(7*(-3*a*f+8*c*d)*h^2-2*c*g*(-8*e*h+5*f*g))*x)*(c*x^2+a)^(5/2
)/c^2/h+1/128*a^2*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^2+3*a^2*f*h^
2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

3.89.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.96

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} (d+ex+fx^2) \, dx = \frac{\sqrt{c}\sqrt{a+cx^2}(-3a^3h(512fg+256eh+105fhx)+6a^2c(28dh(32g+5hx)+8e(56g^2+35ghx+$$

input
```
Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + c*x^2]*(-3*a^3*h*(512*f*g + 256*e*h + 105*f*h*x) + 6*a^2
*c*(28*d*h*(32*g + 5*h*x) + 8*e*(56*g^2 + 35*g*h*x + 8*h^2*x^2) + f*x*(140
*g^2 + 128*g*h*x + 35*h^2*x^2)) + 16*c^3*x^3*(14*d*(15*g^2 + 24*g*h*x + 10
*h^2*x^2) + x*(8*e*(21*g^2 + 35*g*h*x + 15*h^2*x^2) + 5*f*x*(28*g^2 + 48*g
*h*x + 21*h^2*x^2))) + 8*a*c^2*x*(14*d*(75*g^2 + 96*g*h*x + 35*h^2*x^2) +
x*(4*e*(168*g^2 + 245*g*h*x + 96*h^2*x^2) + f*x*(490*g^2 + 768*g*h*x + 315
*h^2*x^2)))) - 105*a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e
*g + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(13440*c^(5/2))
```

3.89.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2185, 27, 687, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2} (g+hx)^2 \left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int h(g+hx)^2((8cd-3af)h-c(5fg-8eh)x)\left(cx^2+a\right)^{3/2}dx}{8ch^2}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (g+hx)^2((8cd-3af)h-c(5fg-8eh)x)\left(cx^2+a\right)^{3/2}dx}{8ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int c(g+hx)\left(h(56cdg-11afg-16aeh)+\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)\left(cx^2+a\right)^{3/2}dx}{7c}-\frac{1}{7}\left(a+cx^2\right)^{5/2}(g+hx)^2(5fg-8eh)}{8ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\int(g+hx)\left(h(56cdg-11afg-16aeh)+\left(7(8cd-3af)h^2-2cg(5fg-8eh)\right)x\right)\left(cx^2+a\right)^{3/2}dx-\frac{1}{7}\left(a+cx^2\right)^{5/}}{8ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{7}\left(\frac{7h\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)\int\left(cx^2+a\right)^{3/2}dx}{6c}-\frac{2\left(a+cx^2\right)^{5/2}\left(8ah^2(eh+2fg)-8cgh(7dh+eg)+5cfg^3\right)}{5c}+\frac{hx\left(a+cx^2\right)^{5/2}(7}{}\right.}{8ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}\left(\frac{7h\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)\left(\frac{3}{4}a\int\sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)}{6c}-\frac{2\left(a+cx^2\right)^{5/2}\left(8ah^2(eh+2fg)-8cgh(7dh+eg)+5cfg^3\right)}{5c}\right.}{8ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}\left(\frac{7h\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)}{6c}-\frac{2\left(a+cx^2\right)^{5/2}\left(8ah^2(eh+2fg)-8cg\right.}{5c}\right.}{8ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^3}{8ch}$$

$$\Big\downarrow \; 224$$

3.89. $\int(g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

$$\frac{\frac{1}{7}\left(\frac{7h(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x(a+cx^2)^{3/2}\right)}{6c}-\frac{2(a+cx^2)^{5/2}(8ah^2(eh+2\ldots}{}\right.}{8ch}$$

$$\frac{f(a+cx^2)^{5/2}(g+hx)^3}{8ch}$$

$\downarrow$ 219

$$\frac{\frac{1}{7}\left(\frac{7h\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x(a+cx^2)^{3/2}\right)(3a^2fh^2-8ac(h(dh+2eg)+fg^2)+48c^2dg^2)}{6c}-\frac{2(a+cx^2)^{5/2}(8ah^2(eh+2f\ldots}{}\right.}{8ch}$$

$$\frac{f(a+cx^2)^{5/2}(g+hx)^3}{8ch}$$

input
```
Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) + (-1/7*((5*f*g - 8*e*h)*(g + h*
x)^2*(a + c*x^2)^(5/2)) + ((-2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^
2*(2*f*g + e*h))*(a + c*x^2)^(5/2))/(5*c) + (h*(7*(8*c*d - 3*a*f)*h^2 - 2*
c*g*(5*f*g - 8*e*h))*x*(a + c*x^2)^(5/2))/(6*c) + (7*h*(48*c^2*d*g^2 + 3*a
^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*((x*(a + c*x^2)^(3/2))/4 + (3*
a*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqr
t[c])))/4))/(6*c))/7)/(8*c*h)
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.89. $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.89.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.92

method	result
default	$d\,g^{2}\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\,h^{2}\left(\frac{x^{3}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{8c}-\frac{3a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\right.}{}\right.}{}\right.$
risch	$-\frac{\left(-1680c^{3}f\,h^{2}x^{7}-1920c^{3}e\,h^{2}x^{6}-3840c^{3}fgh\,x^{6}-2520a\,c^{2}f\,h^{2}x^{5}-2240c^{3}d\,h^{2}x^{5}-4480c^{3}egh\,x^{5}-2240c^{3}f\,g^{2}x^{5}-3072a\,c^{2}e\,h^{2}x^{4}-\right.}{}$

input
```
int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^2*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))))+f*h^2*(1/8*x^3*(c*x^2+a)^(5/2)/c-3/8*a/c*(1/
6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a
)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))))))+(e*h^2+2*f*g*h)*(1
/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x^2+a)^(5/2))+1/5*(2*d*g*h+e*g^2)*(
c*x^2+a)^(5/2)/c+(d*h^2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1
/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/
2)+(c*x^2+a)^(1/2)))))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.40

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}(d+ex+fx^2)\,dx=\left[-\frac{105\,(16\,a^3cegh-8\,(6\,a^2c^2d-a^3cf)g^2+(8\,a^3cd-3\,a^4f)h^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}\right.}{}\right.$$

input
```
integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/26880*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*
d - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a)
- 2*(1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1920*(2*c
^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^4*d + 9
*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d + 8*a*c
^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a
*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 384*(
14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 + 105
*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3
*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*e*g*h - 8*(6*a^
2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3
*c*e*h^2 + 1920*(2*c^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*
e*g*h + (8*c^4*d + 9*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2
+ 2*(7*c^4*d + 8*a*c^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*
a*c^3*f)*g^2 + (56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*
a^3*c*f)*g*h + 384*(14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c
^2*f)*g*h)*x^2 + 105*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 +
(8*a^2*c^2*d - 3*a^3*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(330) = 660$.

Time = 0.64 (sec) , antiderivative size = 933, normalized size of antiderivative = 2.70

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} (d+ex+fx^2)\, dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{cfh^2x^7}{8} + \frac{x^6(c^2eh^2+2c^2fgh)}{7c} + \frac{x^5\cdot\left(\frac{9acfh^2}{8}+c^2dh^2+2c^2egh+c^2fg^2\right)}{6c} + \frac{x^4\cdot\left(2aceh^2+4acfgh-\frac{6a(c^2eh^2+\cdots}{7c}\right.}{5c} \cdots\right. \\ a^{\frac{3}{2}}\left(dg^2x + \frac{fh^2x^5}{5} + \frac{x^4(eh^2+2fgh)}{4} + \frac{x^3(dh^2+2egh+fg^2)}{3} + \frac{x^2\cdot(2dgh+eg^2)}{2}\right) \end{cases}$$

input
```
integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(c*f*h**2*x**7/8 + x**6*(c**2*e*h**2 + 2*c**2*
f*g*h)/(7*c) + x**5*(9*a*c*f*h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*
g**2)/(6*c) + x**4*(2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(c**2*e*h**2 + 2*c**2
*f*g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5*c) + x**3*(a**2*f*h**2 + 2*
a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + c**2*d*h**
2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + c**2*d*g**2)/(4*c) + x**2*(a**2*e*
h**2 + 2*a**2*f*g*h + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*c*e*h**2 + 4*a
*c*f*g*h - 6*a*(c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*
g**2)/(5*c))/(3*c) + x*(a**2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2 + 2*a*c*d
*g**2 - 3*a*(a**2*f*h**2 + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a
*(9*a*c*f*h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + c**2*
d*g**2)/(4*c))/(2*c) + (2*a**2*d*g*h + a**2*e*g**2 - 2*a*(a**2*e*h**2 + 2*
a**2*f*g*h + 4*a*c*d*g*h + 2*a*c*e*g**2 - 4*a*(2*a*c*e*h**2 + 4*a*c*f*g*h
- 6*a*(c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + 2*c**2*d*g*h + c**2*e*g**2)/(5*
c))/(3*c))/c) + (a**2*d*g**2 - a*(a**2*d*h**2 + 2*a**2*e*g*h + a**2*f*g**2
 + 2*a*c*d*g**2 - 3*a*(a**2*f*h**2 + 2*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*
g**2 - 5*a*(9*a*c*f*h**2/8 + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*
c) + c**2*d*g**2)/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2)
+ 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a
**(3/2)*(d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**3*(d...
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.10

$$
\begin{aligned}
\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{(cx^2+a)^{\frac{5}{2}}fh^2x^3}{8\,c}+\frac{1}{4}\,(cx^2+a)^{\frac{3}{2}}dg^2x \\
&+\frac{3}{8}\,\sqrt{cx^2+a}adg^2x-\frac{(cx^2+a)^{\frac{5}{2}}afh^2x}{16\,c^2}+\frac{(cx^2+a)^{\frac{3}{2}}a^2fh^2x}{64\,c^2}+\frac{3\,\sqrt{cx^2+a}a^3fh^2x}{128\,c^2} \\
&+\frac{3\,a^2dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{3\,a^4fh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{5}{2}}}+\frac{(cx^2+a)^{\frac{5}{2}}eg^2}{5\,c}+\frac{2\,(cx^2+a)^{\frac{5}{2}}dgh}{5\,c} \\
&+\frac{(2\,fgh+eh^2)(cx^2+a)^{\frac{5}{2}}x^2}{7\,c}+\frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{5}{2}}x}{6\,c} \\
&-\frac{(fg^2+2\,egh+dh^2)(cx^2+a)^{\frac{3}{2}}ax}{24\,c}-\frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}a^2x}{16\,c} \\
&-\frac{(fg^2+2\,egh+dh^2)a^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}-\frac{2\,(2\,fgh+eh^2)(cx^2+a)^{\frac{5}{2}}a}{35\,c^2}
\end{aligned}
$$

input
```
integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/8*(c*x^2 + a)^(5/2)*f*h^2*x^3/c + 1/4*(c*x^2 + a)^(3/2)*d*g^2*x + 3/8*sq
rt(c*x^2 + a)*a*d*g^2*x - 1/16*(c*x^2 + a)^(5/2)*a*f*h^2*x/c^2 + 1/64*(c*x
^2 + a)^(3/2)*a^2*f*h^2*x/c^2 + 3/128*sqrt(c*x^2 + a)*a^3*f*h^2*x/c^2 + 3/
8*a^2*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/128*a^4*f*h^2*arcsinh(c*x/s
qrt(a*c))/c^(5/2) + 1/5*(c*x^2 + a)^(5/2)*e*g^2/c + 2/5*(c*x^2 + a)^(5/2)*
d*g*h/c + 1/7*(2*f*g*h + e*h^2)*(c*x^2 + a)^(5/2)*x^2/c + 1/6*(f*g^2 + 2*e
*g*h + d*h^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^2 + 2*e*g*h + d*h^2)*(c*x^
2 + a)^(3/2)*a*x/c - 1/16*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a^2*x/
c - 1/16*(f*g^2 + 2*e*g*h + d*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
35*(2*f*g*h + e*h^2)*(c*x^2 + a)^(5/2)*a/c^2
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.27

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{1}{13440}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\left(7cfh^2x+\frac{8\left(2c^7fgh+c^7eh^2\right)}{c^6}\right)x+\frac{7\left(8c^7fg^2+16c^7egh+\right.}{c^6}\right.\right.\right.\right.\right.\right.$$
$$-\frac{\left(48a^2c^2dg^2-8a^3cfg^2-16a^3cegh-8a^3cdh^2+3a^4fh^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{128c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*
e*h^2)/c^6)*x + 7*(8*c^7*f*g^2 + 16*c^7*e*g*h + 8*c^7*d*h^2 + 9*a*c^6*f*h^
2)/c^6)*x + 48*(7*c^7*e*g^2 + 14*c^7*d*g*h + 16*a*c^6*f*g*h + 8*a*c^6*e*h^
2)/c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 112*a*c^6*e*g*h + 56*a*c^6
*d*h^2 + 3*a^2*c^5*f*h^2)/c^6)*x + 192*(14*a*c^6*e*g^2 + 28*a*c^6*d*g*h +
2*a^2*c^5*f*g*h + a^2*c^5*e*h^2)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*
f*g^2 + 16*a^2*c^5*e*g*h + 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2)/c^6)*x + 384
*(7*a^2*c^5*e*g^2 + 14*a^2*c^5*d*g*h - 4*a^3*c^4*f*g*h - 2*a^3*c^4*e*h^2)/
c^6) - 1/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 16*a^3*c*e*g*h - 8*a^3*c*
d*h^2 + 3*a^4*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \int (g+h\,x)^2 \left(c\,x^2+a\right)^{3/2} \left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

3.90.1 Optimal result

Integrand size = 27, antiderivative size = 213

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c}+\frac{(6cdg-a(fg+eh))x(a+cx^2)^{3/2}}{24c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch}-\frac{\left(6(2afh^2+c(5fg^2-7h(eg+dh)))+5ch(5fg-7eh)x\right)\left(a+cx^2\right)^{5/2}}{210c^2h}+\frac{a^2(6cdg-afg-aeh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}$$

output
```
1/24*(6*c*d*g-a*(e*h+f*g))*x*(c*x^2+a)^(3/2)/c+1/7*f*(h*x+g)^2*(c*x^2+a)^(
5/2)/c/h-1/210*(12*a*f*h^2+6*c*(5*f*g^2-7*h*(d*h+e*g))+5*c*h*(-7*e*h+5*f*g
)*x)*(c*x^2+a)^(5/2)/c^2/h+1/16*a^2*(-a*e*h-a*f*g+6*c*d*g)*arctanh(x*c^(1/
2)/(c*x^2+a)^(1/2))/c^(3/2)+1/16*a*(-a*e*h-a*f*g+6*c*d*g)*x*(c*x^2+a)^(1/2
)/c
```

3.90.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{a+cx^2}(-96a^3fh+3a^2c(112dh+7e(16g+5hx)+fx(35g+16hx))+4c^3x^3(21d(5g+4hx)}{}$$

input
```
Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[a + c*x^2]*(-96*a^3*f*h + 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*
x*(35*g + 16*h*x)) + 4*c^3*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x
) + 5*f*x*(7*g + 6*h*x))) + 2*a*c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g
 + 35*h*x) + f*x*(245*g + 192*h*x)))) + 105*a^2*Sqrt[c]*(-6*c*d*g + a*f*g
+ a*e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(1680*c^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2185, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2}(g+hx)\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int h(g+hx)((7cd-2af)h-c(5fg-7eh)x)\left(cx^2+a\right)^{3/2}dx}{7ch^2}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (g+hx)((7cd-2af)h-c(5fg-7eh)x)\left(cx^2+a\right)^{3/2}dx}{7ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{7}{6}h(-aeh-afg+6cdg)\int\left(cx^2+a\right)^{3/2}dx-\frac{\left(a+cx^2\right)^{5/2}\left(2afh^2-7ch(dh+eg)+5cfg^2\right)}{5c}-\frac{1}{6}hx\left(a+cx^2\right)^{5/2}(5fg-7eh)}{7ch}+\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$\downarrow$ 211

$$\frac{\frac{7}{6}h(-aeh-afg+6cdg)\left(\frac{3}{4}a\int\sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)-\frac{\left(a+cx^2\right)^{5/2}\left(2afh^2-7ch(dh+eg)+5cfg^2\right)}{5c}-\frac{1}{6}hx\left(a+cx\right.}{7ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$\downarrow$ 211

$$\frac{\frac{7}{6}h(-aeh-afg+6cdg)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)-\frac{\left(a+cx^2\right)^{5/2}\left(2afh^2-7ch(dh+eg)+5c\right.}{5c}}{7ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$\downarrow$ 224

$$\frac{\frac{7}{6}h(-aeh-afg+6cdg)\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)-\frac{\left(a+cx^2\right)^{5/2}\left(2afh^2-7ch(d\right.}{5c}}{7ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

$\downarrow$ 219

$$\frac{\frac{7}{6}h\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)(-aeh-afg+6cdg)-\frac{\left(a+cx^2\right)^{5/2}\left(2afh^2-7ch(dh+\right.}{5c}}{7ch}$$
$$\frac{f\left(a+cx^2\right)^{5/2}(g+hx)^2}{7ch}$$

input

```
Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

3.90. $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

output
```
(f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7*c*h) + (-1/5*((5*c*f*g^2 + 2*a*f*h^2
- 7*c*h*(e*g + d*h))*(a + c*x^2)^(5/2))/c - (h*(5*f*g - 7*e*h)*x*(a + c*x^
2)^(5/2))/6 + (7*h*(6*c*d*g - a*f*g - a*e*h)*((x*(a + c*x^2)^(3/2))/4 + (3
*a*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sq
rt[c])))/4))/6)/(7*c*h)
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.90.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

method	result
default	$dg\left(\frac{x\left(c\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}\right)}{4}\right)+hf\left(\frac{x^{2}\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{7c}-\frac{2a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}{35c^{2}}\right)+\left(eh+fg\right)\left(\frac{x}{}\right.$
risch	$-\frac{\left(-240c^{3}fh\,x^{6}-280c^{3}eh\,x^{5}-280c^{3}fg\,x^{5}-384a\,c^{2}fh\,x^{4}-336c^{3}dh\,x^{4}-336c^{3}eg\,x^{4}-490a\,c^{2}eh\,x^{3}-490a\,c^{2}fg\,x^{3}-420c^{3}dg\,x^{3}-48a^{2}\right.}{1680c^{2}}$

input
```
int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g*(1/4*x*(c*x^2+a)^(3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x
*c^(1/2)+(c*x^2+a)^(1/2))))+h*f*(1/7*x^2*(c*x^2+a)^(5/2)/c-2/35*a/c^2*(c*x
^2+a)^(5/2))+(e*h+f*g)*(1/6*x*(c*x^2+a)^(5/2)/c-1/6*a/c*(1/4*x*(c*x^2+a)^(
3/2)+3/4*a*(1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/
2)))))+1/5*(d*h+e*g)*(c*x^2+a)^(5/2)/c
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.24

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \left[\frac{105\left(a^3eh-\left(6a^2cd-a^3f\right)g\right)\sqrt{c}\log\left(-2cx^2+2\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(240c^3fhx^6+280(c^3\right.}{}\right.$$

input
```
integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/3360*(105*(a^3*e*h - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sq
rt(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*(c^3*f*g + c^3*e*h
)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d + 8*a*c^2*f)*h)*x^4 + 70*
(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14*a*c^2*e*g + (14*a*c^2
*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 105*(a^2*c*e*h + (10*a
*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*e*h - (6*a^
2*c*d - a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f
*h*x^6 + 280*(c^3*f*g + c^3*e*h)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*
c^3*d + 8*a*c^2*f)*h)*x^4 + 70*(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3
 + 48*(14*a*c^2*e*g + (14*a*c^2*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^
3*f)*h + 105*(a^2*c*e*h + (10*a*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^
2]
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(196) = 392$.

Time = 0.59 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.38

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \begin{cases} \sqrt{a+cx^2}\left(\frac{cfhx^6}{7}+\frac{x^5\left(c^2eh+c^2fg\right)}{6c}+\frac{x^4\cdot\left(\frac{8acfh}{7}+c^2dh+c^2eg\right)}{5c}+\frac{x^3\cdot\left(2aceh+2acfg-\frac{5a\left(c^2eh+c^2fg\right)}{6c}+c^2dg\right)}{4c}+\frac{x^2}{}\right. \\ a^{\frac{3}{2}}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right) \end{cases}$$

input
```
integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(c*f*h*x**6/7 + x**5*(c**2*e*h + c**2*f*g)/(6*
c) + x**4*(8*a*c*f*h/7 + c**2*d*h + c**2*e*g)/(5*c) + x**3*(2*a*c*e*h + 2*
a*c*f*g - 5*a*(c**2*e*h + c**2*f*g)/(6*c) + c**2*d*g)/(4*c) + x**2*(a**2*f
*h + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + c**2*d*h + c**2*e*g)/(5*c)
)/(3*c) + x*(a**2*e*h + a**2*f*g + 2*a*c*d*g - 3*a*(2*a*c*e*h + 2*a*c*f*g
- 5*a*(c**2*e*h + c**2*f*g)/(6*c) + c**2*d*g)/(4*c))/(2*c) + (a**2*d*h + a
**2*e*g - 2*a*(a**2*f*h + 2*a*c*d*h + 2*a*c*e*g - 4*a*(8*a*c*f*h/7 + c**2*
d*h + c**2*e*g)/(5*c))/(3*c))/c) + (a**2*d*g - a*(a**2*e*h + a**2*f*g + 2*
a*c*d*g - 3*a*(2*a*c*e*h + 2*a*c*f*g - 5*a*(c**2*e*h + c**2*f*g)/(6*c) + c
**2*d*g)/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/
sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(3/2)*(
d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2), True))
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(cx^2+a\right)^{\frac{5}{2}}fhx^2}{7\,c} \\
&+\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dgx+\frac{3}{8}\sqrt{cx^2+a}\,adgx+\frac{3\,a^2dg\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}} \\
&+\frac{\left(cx^2+a\right)^{\frac{5}{2}}eg}{5\,c}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}dh}{5\,c}-\frac{2\left(cx^2+a\right)^{\frac{5}{2}}afh}{35\,c^2}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}(fg+eh)x}{6\,c} \\
&-\frac{\left(cx^2+a\right)^{\frac{3}{2}}(fg+eh)ax}{24\,c}-\frac{\sqrt{cx^2+a}(fg+eh)a^2x}{16\,c}-\frac{(fg+eh)a^3\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}
\end{aligned}
$$

input
```
integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/7*(c*x^2 + a)^(5/2)*f*h*x^2/c + 1/4*(c*x^2 + a)^(3/2)*d*g*x + 3/8*sqrt(c
*x^2 + a)*a*d*g*x + 3/8*a^2*d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^
2 + a)^(5/2)*e*g/c + 1/5*(c*x^2 + a)^(5/2)*d*h/c - 2/35*(c*x^2 + a)^(5/2)*
a*f*h/c^2 + 1/6*(c*x^2 + a)^(5/2)*(f*g + e*h)*x/c - 1/24*(c*x^2 + a)^(3/2)
*(f*g + e*h)*a*x/c - 1/16*sqrt(c*x^2 + a)*(f*g + e*h)*a^2*x/c - 1/16*(f*g
+ e*h)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,cfhx+\frac{7\left(c^6fg+c^6eh\right)}{c^5}\right)x+\frac{6\left(7\,c^6eg+7\,c^6dh+8\,ac^5fh\right)}{c^5}\right)x\right.\right.\right.\right.$$
$$-\frac{\left(6\,a^2cdg-a^3fg-a^3eh\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*e*h)/c^5)*
x + 6*(7*c^6*e*g + 7*c^6*d*h + 8*a*c^5*f*h)/c^5)*x + 35*(6*c^6*d*g + 7*a*c
^5*f*g + 7*a*c^5*e*h)/c^5)*x + 24*(14*a*c^5*e*g + 14*a*c^5*d*h + a^2*c^4*f
*h)/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*e*h)/c^5)*x + 48*(7
*a^2*c^4*e*g + 7*a^2*c^4*d*h - 2*a^3*c^3*f*h)/c^5) - 1/16*(6*a^2*c*d*g - a
^3*f*g - a^3*e*h)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.91 $\int (a+cx^2)^{3/2} (d+ex+fx^2)\, dx$

3.91.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int (a+cx^2)^{3/2} (d+ex+fx^2)\, dx = \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c} + \frac{(6cd-af)x(a+cx^2)^{3/2}}{24c} + \frac{e(a+cx^2)^{5/2}}{5c} + \frac{fx(a+cx^2)^{5/2}}{6c} + \frac{a^2(6cd-af)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}$$

output
```
1/24*(-a*f+6*c*d)*x*(c*x^2+a)^(3/2)/c+1/5*e*(c*x^2+a)^(5/2)/c+1/6*f*x*(c*x
^2+a)^(5/2)/c+1/16*a^2*(-a*f+6*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(
3/2)+1/16*a*(-a*f+6*c*d)*x*(c*x^2+a)^(1/2)/c
```

3.91.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int (a+cx^2)^{3/2} (d+ex +fx^2)\, dx = \frac{\sqrt{c}\sqrt{a+cx^2}(3a^2(16e+5fx)+4c^2x^3(15d+2x(6e+5fx))+2acx(75d+x(48e+35fx)))+1}{240c^{3/2}}$$

input
```
Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + c*x^2]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*
e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) + 15*a^2*(-6*c*d + a*f)*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(240*c^(3/2))
```

3.91.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2346, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int (6cd-af+6cex) \left(cx^2+a\right)^{3/2}dx}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{455}$$

$$\frac{(6cd-af)\int \left(cx^2+a\right)^{3/2}dx+\frac{6}{5}e\left(a+cx^2\right)^{5/2}}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6cd-af)\left(\frac{3}{4}a\int \sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{6}{5}e\left(a+cx^2\right)^{5/2}}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{211}$$

$$\frac{(6cd-af)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{6}{5}e\left(a+cx^2\right)^{5/2}}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6cd-af)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{6}{5}e\left(a+cx^2\right)^{5/2}}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)(6cd-af)+\frac{6}{5}e\left(a+cx^2\right)^{5/2}}{6c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

input
```
Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + c*x^2)^(5/2))/(6*c) + ((6*e*(a + c*x^2)^(5/2))/5 + (6*c*d - a*f)
*((x*(a + c*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt
[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4))/(6*c)
```

3.91.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.91.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(40c^2 f x^5+48e x^4c^2+70acf x^3+60c^2d x^3+96ace x^2+15a^2 fx+150acdx+48a^2e)\sqrt{c x^2+a}}{240c} - \frac{a^2(fa-6cd)\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{16c^{\frac{3}{2}}}$
default	$d\left(\frac{x(c x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+f\left(\frac{x(c x^2+a)^{\frac{5}{2}}}{6c}-\frac{a\left(\frac{x(c x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln(x\sqrt{c}+}{2\sqrt{}}\right.}{4}\right.}{6c}\right.$

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/240/c*(40*c^2*f*x^5+48*c^2*e*x^4+70*a*c*f*x^3+60*c^2*d*x^3+96*a*c*e*x^2+
15*a^2*f*x+150*a*c*d*x+48*a^2*e)*(c*x^2+a)^(1/2)-1/16*a^2*(a*f-6*c*d)/c^(3
/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.91

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \left[-\frac{15\left(6\,a^2cd-a^3f\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40\,c^3fx^5+48\,c^3ex^4+96\,ac^2ex\right.}{480\,c^2}\right.$$
$$-\frac{15\left(6\,a^2cd-a^3f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-\left(40\,c^3fx^5+48\,c^3ex^4+96\,ac^2ex^2+48\,a^2ce+10\left(6\,c^3d+7\,ac^2f\right)\right.}{240\,c^2}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*s
qrt(c)*x - a) - 2*(40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c
*e + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^
2 + a))/c^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqr
t(c*x^2 + a)) - (40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e
 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2
+ a))/c^2]
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\begin{cases}\sqrt{a+cx^2}\left(\frac{a^2e}{5c}+\frac{2aex^2}{5}+\frac{cex^4}{5}+\frac{cfx^5}{6}+\frac{x^3\cdot\left(\frac{7acf}{6}+c^2d\right)}{4c}+\frac{x\left(a^2f+2acd-\frac{3a\left(\frac{7acf}{6}+c^2d\right)}{4c}\right)}{2c}\right)+\left(a^2d-\frac{a\left(\cdots\right.}{\cdots}\right. \\ a^{\frac{3}{2}}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right)\end{cases}$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(a**2*e/(5*c) + 2*a*e*x**2/5 + c*e*x**4/5 + c*
f*x**5/6 + x**3*(7*a*c*f/6 + c**2*d)/(4*c) + x*(a**2*f + 2*a*c*d - 3*a*(7*
a*c*f/6 + c**2*d)/(4*c))/(2*c)) + (a**2*d - a*(a**2*f + 2*a*c*d - 3*a*(7*a
*c*f/6 + c**2*d)/(4*c))/(2*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) +
 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a*
*(3/2)*(d*x + e*x**2/2 + f*x**3/3), True))
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dx+\frac{3}{8}\sqrt{cx^2+a}adx+\frac{\left(cx^2+a\right)^{\frac{5}{2}}fx}{6\,c}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}afx}{24\,c}-\frac{\sqrt{cx^2+a}a^2fx}{16\,c}+\frac{3\,a^2d\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}-\frac{a^3f\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,c^{\frac{3}{2}}}+\frac{\left(cx^2+a\right)^{\frac{5}{2}}e}{5\,c}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + a)^(3/2)*d*x + 3/8*sqrt(c*x^2 + a)*a*d*x + 1/6*(c*x^2 + a)^(5
/2)*f*x/c - 1/24*(c*x^2 + a)^(3/2)*a*f*x/c - 1/16*sqrt(c*x^2 + a)*a^2*f*x/
c + 3/8*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/16*a^3*f*arcsinh(c*x/sqrt
(a*c))/c^(3/2) + 1/5*(c*x^2 + a)^(5/2)*e/c
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int (a+cx^2)^{3/2} (d+ex+fx^2)\,dx = \frac{1}{240}\sqrt{cx^2+a}\left(\frac{48\,a^2e}{c} + \left(2\left(48\,ae + \left(4\,(5\,cfx+6\,ce)x + \frac{5\,(6\,c^5d+7\,ac^4f)}{c^4}\right)x\right)x + \frac{15\,(10\,ac\right.\right.$$
$$-\frac{(6\,a^2cd-a^3f)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/240*sqrt(c*x^2 + a)*(48*a^2*e/c + (2*(48*a*e + (4*(5*c*f*x + 6*c*e)*x +
5*(6*c^5*d + 7*a*c^4*f)/c^4)*x)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x) -
1/16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (a+cx^2)^{3/2} (d+ex+fx^2)\,dx = \int (c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)\,dx$$

input
```
int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.92 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

3.92.1 Optimal result

Integrand size = 29, antiderivative size = 326

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2}{8h^5}$$
$$+\frac{(4(fg^2-egh+dh^2)-3h(fg-eh)x)(a+cx^2)^{3/2}}{12h^3}+\frac{f(a+cx^2)^{5/2}}{5ch}$$
$$-\frac{(3a^2h^4(fg-eh)+8c^2g^3(fg^2-h(eg-dh))+12acgh^2(fg^2-h(eg-dh)))\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6}$$
$$-\frac{(cg^2+ah^2)^{3/2}(fg^2-egh+dh^2)\,\mathrm{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}$$

output
```
1/12*(4*d*h^2-4*e*g*h+4*f*g^2-3*h*(-e*h+f*g)*x)*(c*x^2+a)^(3/2)/h^3+1/5*f*
(c*x^2+a)^(5/2)/c/h-(a*h^2+c*g^2)^(3/2)*(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*
x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*(3*a^2*h^4*(-e*h+f*g)+
8*c^2*g^3*(f*g^2-h*(-d*h+e*g))+12*a*c*g*h^2*(f*g^2-h*(-d*h+e*g)))*arctanh(
x*c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+1/8*(8*(a*h^2+c*g^2)*(d*h^2-e*g*h+f
*g^2)-h*(4*c*d*g*h^2+(-e*h+f*g)*(3*a*h^2+4*c*g^2))*x)*(c*x^2+a)^(1/2)/h^5
```

3.92.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{h\sqrt{a+cx^2}\left(24a^2fh^4+ach^2\left(5h(-32eg+32dh+15ehx)+f\left(160g^2-75ghx+48h^2x^2\right)\right)+2c^2\left(f\left(60g^4-\right.\right.\right.}{}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
((h*Sqrt[a + c*x^2]*(24*a^2*f*h^4 + a*c*h^2*(5*h*(-32*e*g + 32*d*h + 15*e*
h*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + 2*c^2*(f*(60*g^4 - 30*g^3*h*
x + 20*g^2*h^2*x^2 - 15*g*h^3*x^3 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*
h*x + 2*h^2*x^2) + e*(-12*g^3 + 6*g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3)))))/c
 - 240*(-(c*g^2) - a*h^2)^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]
*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] + (15*(3*a^2*h^4*(
f*g - e*h) + 12*a*c*g*h^2*(f*g^2 + h*(-(e*g) + d*h)) + 8*c^2*(f*g^5 + g^3*
h*(-(e*g) + d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(120*h^6)
```

3.92.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{5ch(dh-(fg-eh)x)(cx^2+a)^{3/2}}{g+hx}dx}{5ch^2}+\frac{f(a+cx^2)^{5/2}}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(dh-(fg-eh)x)(cx^2+a)^{3/2}}{g+hx}dx}{h}+\frac{f(a+cx^2)^{5/2}}{5ch}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{\int \frac{c\left(ah\left(fg^2-h(eg-4dh)\right)-\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{4ch^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(dh^2-egh+fg^2\right)-3hx(fg-eh)\right)}{12h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(ah\left(fg^2-h(eg-4dh)\right)-\left(4cdgh^2+(fg-eh)\left(4cg^2+3ah^2\right)\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{4h^2}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(dh^2-egh+fg^2\right)-3hx(fg-eh)\right)}{12h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 682

$$\frac{\frac{\frac{\int \frac{c\left(ah\left(a\left(5fg^2-h(5eg-8dh)\right)h^2+4c\left(fg^4-g^2h(eg-dh)\right)\right)-\left(3a^2(fg-eh)h^4+12acg\left(fg^2-h(eg-dh)\right)h^2+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)x\right)}{(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}+\sqrt{a+cx^2}\left(8\left(ah^2+cg^2\right)\right)}{4h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{ah\left(a\left(5fg^2-h(5eg-8dh)\right)h^2+4c\left(fg^4-g^2h(eg-dh)\right)\right)-\left(3a^2(fg-eh)h^4+12acg\left(fg^2-h(eg-dh)\right)h^2+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^2}+\sqrt{a+cx^2}\left(8\left(ah^2+cg^2\right)\left(d\right)\right)}{4h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 719

$$\frac{\frac{\frac{\frac{8\left(ah^2+cg^2\right)^2\left(dh^2-egh+fg^2\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+a}}dx}{h}}{2h^2}+\sqrt{a+cx^2}\left(8\left(ah^2+\right)\right)}{4h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\frac{8\left(ah^2+cg^2\right)^2\left(dh^2-egh+fg^2\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}}{2h^2}+\sqrt{a+cx^2}}{4h^2}}{h}+\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

3.92. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{8\left(ah^2+cg^2\right)^2\left(dh^2-egh+fg^2\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)}{\sqrt{c}h}}{2h^2}+\frac{\sqrt{a+cx^2}\ldots}{\ldots}}{4h^2}}{h}$$

$$\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 488

$$\frac{\frac{\frac{-\frac{8\left(ah^2+cg^2\right)^2\left(dh^2-egh+fg^2\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-d\ldots\right.\right.}{\sqrt{c}h}}{2h^2}}{4h^2}}{h}$$

$$\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

$\downarrow$ 219

$$\frac{\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)}{\sqrt{c}h}-\frac{8\left(ah^2+cg^2\right)^{3/2}\left(dh^2-egh+fg^2\right)\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2\ldots}}\right.}{h}}{2h^2}}{4h^2}}{h}$$

$$\frac{f\left(a+cx^2\right)^{5/2}}{5ch}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + c*x^2)^(5/2))/(5*c*h) + (((4*(f*g^2 - e*g*h + d*h^2) - 3*h*(f*g -
e*h)*x)*(a + c*x^2)^(3/2))/(12*h^2) + (((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h
+ d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c
*x^2])/(2*h^2) + (-(((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g
 - d*h)) + 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]])/(Sqrt[c]*h)) - (8*(c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*
ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h)/(2*h^2))/
(4*h^2))/h
```

3.92. $\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.92.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.73

method	result
risch	$\frac{(24c^2 f h^4 x^4+30c^2 e h^4 x^3-30c^2 fg h^3 x^3+48acf h^4 x^2+40c^2 d h^4 x^2-40c^2 eg h^3 x^2+40c^2 f g^2 h^2 x^2+75ace h^4 x-75acfg h^3 x-60c^2 dg h^3}{120c h}$
default	$\frac{eh\left(\frac{x\left(c x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)+\frac{fh\left(c x^2+a\right)^{\frac{5}{2}}}{5c}-fg\left(\frac{x\left(c x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{c x^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{h^2}$

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/120/c*(24*c^2*f*h^4*x^4+30*c^2*e*h^4*x^3-30*c^2*f*g*h^3*x^3+48*a*c*f*h^4
*x^2+40*c^2*d*h^4*x^2-40*c^2*e*g*h^3*x^2+40*c^2*f*g^2*h^2*x^2+75*a*c*e*h^4
*x-75*a*c*f*g*h^3*x-60*c^2*d*g*h^3*x+60*c^2*e*g^2*h^2*x-60*c^2*f*g^3*h*x+2
4*a^2*f*h^4+160*a*c*d*h^4-160*a*c*e*g*h^3+160*a*c*f*g^2*h^2+120*c^2*d*g^2*
h^2-120*c^2*e*g^3*h+120*c^2*f*g^4)*(c*x^2+a)^(1/2)/h^5+1/8/h^5*((3*a^2*e*h
^5-3*a^2*f*g*h^4-12*a*c*d*g*h^4+12*a*c*e*g^2*h^3-12*a*c*f*g^3*h^2-8*c^2*d*
g^3*h^2+8*c^2*e*g^4*h-8*c^2*f*g^5)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)
-(8*a^2*d*h^6-8*a^2*e*g*h^5+8*a^2*f*g^2*h^4+16*a*c*d*g^2*h^4-16*a*c*e*g^3*
h^3+16*a*c*f*g^4*h^2+8*c^2*d*g^4*h^2-8*c^2*e*g^5*h+8*c^2*f*g^6)/h^2/((a*h^
2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*
g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))
```

3.92.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

3.92.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(299) = 598.

Time = 0.26 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.94

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = -\frac{\sqrt{cx^2+a}cfg^3x}{2\,h^4} + \frac{\sqrt{cx^2+a}ceg^2x}{2\,h^3} - \frac{\sqrt{cx^2+a}cdgx}{2\,h^2} - \frac{(cx^2+a)^{\frac{3}{2}}fgx}{4\,h^2} - \frac{3\sqrt{cx^2+a}afgx}{8\,h^2} + \frac{(cx^2+a)^{\frac{3}{2}}ex}{4\,h} + \frac{3\sqrt{cx^2+a}aex}{8\,h} - \frac{c^{\frac{3}{2}}fg^5\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^6} + \frac{c^{\frac{3}{2}}eg^4\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^5} - \frac{c^{\frac{3}{2}}dg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} - \frac{3\,a\sqrt{c}fg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^4} + \frac{3\,a\sqrt{c}eg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^3} - \frac{3\,a\sqrt{c}dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,h^2} - \frac{3\,a^2fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}h^2} + \frac{3\,a^2e\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}h} + \frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}fg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} - \frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}eg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} + \frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}d\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h} + \frac{\sqrt{cx^2+a}cfg^4}{h^5} - \frac{\sqrt{cx^2+a}ceg^3}{h^4} + \frac{\sqrt{cx^2+a}cdg^2}{h^3} + \frac{(cx^2+a)^{\frac{3}{2}}fg^2}{3\,h^3} + \frac{\sqrt{cx^2+a}afg^2}{h^3} - \frac{(cx^2+a)^{\frac{3}{2}}eg}{3\,h^2} - \frac{\sqrt{cx^2+a}aeg}{h^2} + \frac{(cx^2+a)^{\frac{3}{2}}d}{3\,h} + \frac{\sqrt{cx^2+a}ad}{h} + \frac{(cx^2+a)^{\frac{5}{2}}f}{5\,ch}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

output
```
-1/2*sqrt(c*x^2 + a)*c*f*g^3*x/h^4 + 1/2*sqrt(c*x^2 + a)*c*e*g^2*x/h^3 - 1
/2*sqrt(c*x^2 + a)*c*d*g*x/h^2 - 1/4*(c*x^2 + a)^(3/2)*f*g*x/h^2 - 3/8*sqr
t(c*x^2 + a)*a*f*g*x/h^2 + 1/4*(c*x^2 + a)^(3/2)*e*x/h + 3/8*sqrt(c*x^2 +
a)*a*e*x/h - c^(3/2)*f*g^5*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*e*g^4*arcs
inh(c*x/sqrt(a*c))/h^5 - c^(3/2)*d*g^3*arcsinh(c*x/sqrt(a*c))/h^4 - 3/2*a*
sqrt(c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*g^2*arcsinh(c*x
/sqrt(a*c))/h^3 - 3/2*a*sqrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 3/8*a^2*f
*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + 3/8*a^2*e*arcsinh(c*x/sqrt(a*c))
/(sqrt(c)*h) + (a + c*g^2/h^2)^(3/2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - (a + c*g^2/h^2)^(3/2)*e*g*ar
csinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 +
 (a + c*g^2/h^2)^(3/2)*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/h + sqrt(c*x^2 + a)*c*f*g^4/h^5 - sqrt(c*x^2 + a)*c*
e*g^3/h^4 + sqrt(c*x^2 + a)*c*d*g^2/h^3 + 1/3*(c*x^2 + a)^(3/2)*f*g^2/h^3
+ sqrt(c*x^2 + a)*a*f*g^2/h^3 - 1/3*(c*x^2 + a)^(3/2)*e*g/h^2 - sqrt(c*x^2
 + a)*a*e*g/h^2 + 1/3*(c*x^2 + a)^(3/2)*d/h + sqrt(c*x^2 + a)*a*d/h + 1/5*
(c*x^2 + a)^(5/2)*f/(c*h)
```

3.92.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.93 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.93.1 Optimal result

Integrand size = 29, antiderivative size = 432

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(8(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))-h(20cfg^2-16cegh+12cdh^2+3afh^2)\,x)\sqrt{a+cx^2}}{8h^5}$$
$$-\frac{(4(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))-3h(afh^2+c(5fg^2-4h(eg-dh)))\,x)\,(a+cx^2)^{3/2}}{12h^3\,(cg^2+ah^2)}$$
$$-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{h\,(cg^2+ah^2)\,(g+hx)}$$
$$+\frac{(3a^2fh^4+8c^2g^2(5fg^2-h(4eg-3dh))+12ach^2(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}h^6}$$
$$+\frac{\sqrt{cg^2+ah^2}(ah^2(2fg-eh)+cg(5fg^2-h(4eg-3dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^6}$$

output
```
-1/12*(4*a*h^2*(-e*h+2*f*g)+4*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))-3*h*(a*f*h^2+
c*(5*f*g^2-4*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)-(d*h^2-e*
g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/8*(3*a^2*f*h^4+8*c^2*
g^2*(5*f*g^2-h*(-3*d*h+4*e*g))+12*a*c*h^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+(a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2-
h*(-3*d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2
))*(a*h^2+c*g^2)^(1/2)/h^6-1/8*(8*a*h^2*(-e*h+2*f*g)+8*c*g*(5*f*g^2-h*(-3*
d*h+4*e*g))-h*(3*a*f*h^2+12*c*d*h^2-16*c*e*g*h+20*c*f*g^2)*x)*(c*x^2+a)^(1
/2)/h^5
```

3.93.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{\frac{h\sqrt{a+cx^2}\left(-2cf\left(60g^4+30g^3hx-10g^2h^2x^2+5gh^3x^3-3h^4x^4\right)+ah^2\left(8h(7eg-3dh+4ehx)+f(-88g\right.\right.}{g+hx}}{}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
((h*Sqrt[a + c*x^2]*(-2*c*f*(60*g^4 + 30*g^3*h*x - 10*g^2*h^2*x^2 + 5*g*h^
3*x^3 - 3*h^4*x^4) + a*h^2*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 4
9*g*h*x + 15*h^2*x^2)) + 4*c*h*(3*d*h*(-6*g^2 - 3*g*h*x + h^2*x^2) + 2*e*(
12*g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))))/(g + h*x) - 48*Sqrt[-(c*g^2
) - a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*ArcT
an[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]] - (3*(3
*a^2*f*h^4 + 12*a*c*h^2*(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^
2*h*(-4*e*g + 3*d*h)))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/(24*h
^6)
```

3.93.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2182, 25, 682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(cdg-afg+aeh+\left(afh-c\left(-\frac{5fg^2}{h}+4eg-4dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{g+hx}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(cdg-afg+aeh+\left(afh-c\left(-\frac{5fg^2}{h}+4eg-4dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{g+hx}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{682}$$

$$\frac{\frac{\int -\frac{c\left(cg^2+ah^2\right)\left(ah(5fg-4eh)-\left(20cfg^2-16cehg+12cdh^2+3afh^2\right)x\right)\sqrt{cx^2+a}}{h(g+hx)}dx}{4ch^2}-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3\right)-3hx\left(afh^2 \cdots\right.\right.}{12h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c\left(cg^2+ah^2\right)\left(ah(5fg-4eh)-\left(20cfg^2-16cehg+12cdh^2+3afh^2\right)x\right)\sqrt{cx^2+a}}{h(g+hx)}dx}{4ch^2}-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3\right)-3hx\left(afh^2 \cdots\right.\right.}{12h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\left(ah^2+cg^2\right)\int \frac{\left(ah(5fg-4eh)-\left(20cfg^2-16cehg+12cdh^2+3afh^2\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{4h^3}-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3\right)-3hx\left(afh^2 \cdots\right.\right.}{12h^3}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{682}$$

3.93. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}dx$

$$-\frac{(ah^2+cg^2)\left(\frac{\int \frac{c\left(ah\left(a(13fg-8eh)h^2+4c\left(5fg^3-gh(4eg-3dh)\right)\right)-\left(2acg(5fg-4eh)h^2+\left(2cg^2+ah^2\right)\left(3afh^2+4c\left(5fg^2-h(4eg-3dh)\right)\right)\right)x\right)}{(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}+\frac{\sqrt{a+cx^2}\left(8\left(a \cdots\right.\right.}{\cdots}\right.}{4h^3 \cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{(ah^2+cg^2)\left(\frac{\int \frac{ah\left(a(13fg-8eh)h^2+4c\left(5fg^3-gh(4eg-3dh)\right)\right)-\left(2acg(5fg-4eh)h^2+\left(2cg^2+ah^2\right)\left(3afh^2+4c\left(5fg^2-h(4eg-3dh)\right)\right)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^2}+\frac{\sqrt{a+cx^2}\left(8\left(ah^2(\cdots\right.\right.}{\cdots}\right.}{4h^3 \cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 719$$

$$-\frac{(ah^2+cg^2)\left(\frac{\frac{8\left(ah^2+cg^2\right)\left(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right) \cdots}{h}}{2h^2 \cdots}\right.}{4h^3 \cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 224$$

$$-\frac{(ah^2+cg^2)\left(\frac{\frac{8\left(ah^2+cg^2\right)\left(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right) \cdots}{h}}{2h^2 \cdots}\right.}{4h^3 \cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 219$$

3.93. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

$$-\frac{(ah^2+cg^2)\left(\frac{\frac{8(ah^2+cg^2)(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2(\cdots\right)}{\sqrt{c}h}}{2h^2}\cdots\right)}{4h^3}\cdots$$

$$\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$

$\big\downarrow$ 488

$$-\frac{(ah^2+cg^2)\left(\frac{-\frac{8(ah^2+cg^2)(ah^2(2fg-eh)-cgh(4eg-3dh)+5cfg^3)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(\cdots\right)\right)}{\sqrt{c}h}}{2h^2}\cdots\right)}{4h^3}\cdots$$

$$\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$

$\big\downarrow$ 219

$$-\frac{(ah^2+cg^2)\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right)}{\sqrt{c}h}-\frac{8\sqrt{ah^2+cg^2}\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\cdots}{h}}{2h^2}\cdots\right)}{4h^3}\cdots$$

$$\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{h(g+hx)(ah^2+cg^2)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
-(((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
)) + (-1/12*((4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h))
- 3*h*(5*c*f*g^2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/h^3
- ((c*g^2 + a*h^2)*(((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g
- e*h)) - h*(20*c*f*g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a +
 c*x^2])/(2*h^2) + (-(((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*
h)) + 12*a*c*h^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]])/(Sqrt[c]*h)) - (8*Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g
- 3*d*h) + a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]
*Sqrt[a + c*x^2])])/h)/(2*h^2)))/(4*h^3))/(c*g^2 + a*h^2)
```

3.93. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.93. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.93.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.70

method	result
risch	$\frac{(6f x^3 h^3 c+8ce h^3 x^2-16cfg h^2 x^2+15af h^3 x+12cd h^3 x-24ceg h^2 x+36cf g^2 hx+32ae h^3-64afg h^2-48cdg h^2+72ce g^2 h-96cf g^3)\sqrt{}}{24h^5}$
default	Expression too large to display

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*c*f*h^3*x^3+8*c*e*h^3*x^2-16*c*f*g*h^2*x^2+15*a*f*h^3*x+12*c*d*h^3
*x-24*c*e*g*h^2*x+36*c*f*g^2*h*x+32*a*e*h^3-64*a*f*g*h^2-48*c*d*g*h^2+72*c
*e*g^2*h-96*c*f*g^3)*(c*x^2+a)^(1/2)/h^5+1/8/h^5*((3*a^2*f*h^4+12*a*c*d*h^
4-24*a*c*e*g*h^3+36*a*c*f*g^2*h^2+24*c^2*d*g^2*h^2-32*c^2*e*g^3*h+40*c^2*f
*g^4)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-(8*a^2*e*h^5-16*a^2*f*g*h^4-
32*a*c*d*g*h^4+48*a*c*e*g^2*h^3-64*a*c*f*g^3*h^2-32*c^2*d*g^3*h^2+40*c^2*e
*g^4*h-48*c^2*f*g^5)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^3*(8*a^2*d*h^6-8*a^2*e*g*h^
5+8*a^2*f*g^2*h^4+16*a*c*d*g^2*h^4-16*a*c*e*g^3*h^3+16*a*c*f*g^4*h^2+8*c^2
*d*g^4*h^2-8*c^2*e*g^5*h+8*c^2*f*g^6)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+
1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/
((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*
h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+1/h*g))))
```

3.93.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.93.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.64

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = -\frac{(cx^2+a)^{\frac{3}{2}}fg^2}{h^4x+gh^3} + \frac{(cx^2+a)^{\frac{3}{2}}eg}{h^3x+gh^2} - \frac{(cx^2+a)^{\frac{3}{2}}d}{h^2x+gh} + \frac{5\sqrt{cx^2+a}cfg^2x}{2h^4} - \frac{2\sqrt{cx^2+a}cegx}{h^3} + \frac{3\sqrt{cx^2+a}cdx}{2h^2} + \frac{(cx^2+a)^{\frac{3}{2}}fx}{4h^2} + \frac{3\sqrt{cx^2+a}afx}{8h^2} + \frac{5c^{\frac{3}{2}}fg^4\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^6} - \frac{4c^{\frac{3}{2}}eg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^5} + \frac{3c^{\frac{3}{2}}dg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4} + \frac{9a\sqrt{c}fg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2h^4} - \frac{3a\sqrt{c}eg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3} + \frac{3a\sqrt{c}d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2h^2} + \frac{3a^2f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}h^2} - \frac{3\sqrt{a+\frac{cg^2}{h^2}}cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^5} + \frac{3\sqrt{a+\frac{cg^2}{h^2}}ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^4} - \frac{3\sqrt{a+\frac{cg^2}{h^2}}cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} - \frac{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^3} + \frac{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|} - \frac{ah}{\sqrt{ac}|hx+g|}\right)}{h^2} - \frac{5\sqrt{cx^2+a}cfg^3}{h^5} + \frac{4\sqrt{cx^2+a}ceg^2}{h^4} - \frac{3\sqrt{cx^2+a}cdg}{h^3} - \frac{2(cx^2+a)^{\frac{3}{2}}fg}{3h^3} - \frac{2\sqrt{cx^2+a}afg}{h^3} + \frac{(cx^2+a)^{\frac{3}{2}}e}{3h^2} + \frac{\sqrt{cx^2+a}ae}{h^2}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")
```

output
```
-(c*x^2 + a)^(3/2)*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^(3/2)*e*g/(h^3*x +
g*h^2) - (c*x^2 + a)^(3/2)*d/(h^2*x + g*h) + 5/2*sqrt(c*x^2 + a)*c*f*g^2*x
/h^4 - 2*sqrt(c*x^2 + a)*c*e*g*x/h^3 + 3/2*sqrt(c*x^2 + a)*c*d*x/h^2 + 1/4
*(c*x^2 + a)^(3/2)*f*x/h^2 + 3/8*sqrt(c*x^2 + a)*a*f*x/h^2 + 5*c^(3/2)*f*g
^4*arcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g^3*arcsinh(c*x/sqrt(a*c))/h^5
 + 3*c^(3/2)*d*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + 9/2*a*sqrt(c)*f*g^2*arcsin
h(c*x/sqrt(a*c))/h^4 - 3*a*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/h^3 + 3/2*a*
sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 3/8*a^2*f*arcsinh(c*x/sqrt(a*c))/(s
qrt(c)*h^2) - 3*sqrt(a + c*g^2/h^2)*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3*sqrt(a + c*g^2/h^2)*c*e*g
^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
h^4 - 3*sqrt(a + c*g^2/h^2)*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - 2*(a + c*g^2/h^2)^(3/2)*f*g*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + (a + c
*g^2/h^2)^(3/2)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/h^2 - 5*sqrt(c*x^2 + a)*c*f*g^3/h^5 + 4*sqrt(c*x^2 + a)*c*e
*g^2/h^4 - 3*sqrt(c*x^2 + a)*c*d*g/h^3 - 2/3*(c*x^2 + a)^(3/2)*f*g/h^3 - 2
*sqrt(c*x^2 + a)*a*f*g/h^3 + 1/3*(c*x^2 + a)^(3/2)*e/h^2 + sqrt(c*x^2 + a)
*a*e/h^2
```

3.93.8 Giac [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\int \frac{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)}{(hx+g)^2}\,dx$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.93. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.94 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.94.1 Optimal result

Integrand size = 29, antiderivative size = 488

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{(2a^2fh^4+2c^2g^2(10fg^2-3h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh)\cdots}{2h^5(cg^2+\cdots}$$

$$-\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-(2afh^2+c(5fg^2-3h(eg-dh)))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{2h(cg^2+ah^2)(g+hx)^2}$$

$$-\frac{\sqrt{c}(3ah^2(3fg-eh)+2cg(10fg^2-3h(2eg-dh)))\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6}$$

$$-\frac{(2a^2fh^4+2c^2g^2(10fg^2-3h(2eg-dh))+ach^2(19fg^2-3h(3eg-dh)))\,\mathrm{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2h^6\sqrt{cg^2+ah^2}}$$

output
```
-1/6*(2*c*g*(6*e*g-10*f*g^2/h-3*d*h)-2*a*h*(-3*e*h+7*f*g)-(2*a*f*h^2+c*(5*
f*g^2-3*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)-1/2*(d
*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(3*a*h^2*(
-e*h+3*f*g)+2*c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)
^(1/2))*c^(1/2)/h^6-1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))
+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(
1/2)/(c*x^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(1/2)+1/2*(2*a^2*f*h^4+2*c^2*g^2*(
10*f*g^2-3*h*(-d*h+2*e*g))+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g))-c*h*(a*h^2*
(-3*e*h+7*f*g)+c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*
h^2+c*g^2)
```

3.94.2 Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.74

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{\frac{h\sqrt{a+cx^2}\left(ah^2(-3h(eg+dh+2ehx)+f(17g^2+28ghx+8h^2x^2))+c(f(60g^4+90g^3hx+20g^2h^2x^2-5\right.}{(g+hx)^2}}{}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*
h*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x
^3 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18
*g^2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 - (6*(2*a^2*f*h^4 + a*c*
h^2*(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g +
d*h)))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^
2]])/Sqrt[-(c*g^2) - a*h^2] + 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e*g + d*
h) - 3*a*h^2*(-3*f*g + e*h))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(6*h^6)
```

3.94.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2182, 25, 681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(2(cdg-afg+aeh)+\left(2afh-c\left(-\frac{5fg^2}{h}+3eg-3dh\right)\right)x\right)(cx^2+a)^{3/2}}{(g+hx)^2}dx}{2(ah^2+cg^2)}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{2h(g+hx)^2(ah^2+cg^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(2(cdg-afg+aeh)+\left(2afh-c\left(-\frac{5fg^2}{h}+3eg-3dh\right)\right)x\right)(cx^2+a)^{3/2}}{(g+hx)^2}dx}{2(ah^2+cg^2)}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{2h(g+hx)^2(ah^2+cg^2)}$$

$$\Big\downarrow \; 681$$

$$\frac{-\frac{\int -\frac{2\left(ah\left(5cfg^2+2afh^2-3ch(eg-dh)\right)-2c\left(10cfg^3-3ch(2eg-dh)g+ah^2(7fg-3eh)\right)x\right)\sqrt{cx^2+a}}{h(g+hx)}dx}{2h^2}-\frac{\left(a+cx^2\right)^{3/2}\left(2\left(cg\left(-3dh+6eg-\frac{10fg^2}{h}\right)-ah(7f\cdots\right.\right.}{3h^2(g+\cdots}}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(ah\left(5cfg^2+2afh^2-3ch(eg-dh)\right)-2c\left(10cfg^3-3ch(2eg-dh)g+ah^2(7fg-3eh)\right)x\right)\sqrt{cx^2+a}}{g+hx}dx}{h^3}-\frac{\left(a+cx^2\right)^{3/2}\left(2\left(cg\left(-3dh+6eg-\frac{10fg^2}{h}\right)-ah(7fg-3e\cdots\right.\right.}{3h^2(g+hx)}}{2\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{\frac{\int \frac{2c\left(cg^2+ah^2\right)\left(ah\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)-c\left(20cfg^3-6ch(2eg-dh)g+3ah^2(3fg-eh)\right)x\right)}{(g+hx)\sqrt{cx^2+a}}dx}{2ch^2}+\frac{\sqrt{a+cx^2}\left(2a^2fh^4-chx\left(ah^2(7fg-3eh)-3cgh(2eg-dh)+\cdots\right.\right.}{\cdots}}{h^3}}{2\cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\left(ah^2+cg^2\right)\int \frac{ah\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)-c\left(20cfg^3-6ch(2eg-dh)g+3ah^2(3fg-eh)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{h^2}+\frac{\sqrt{a+cx^2}\left(2a^2fh^4-chx\left(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cf\cdots\right.\right.}{\cdots}}{h^3}}{2(\cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{\left(ah^2+cg^2\right)\left(\frac{\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2\left(10fg^4-3g^2h(2eg-dh)\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{c\left(3ah^2(3fg-eh)-6cgh(2eg-dh)+20cfg^3\right)\int \frac{1}{\sqrt{cx^2+a}}\cdots}{h}\right.}{h^2}\cdots}{h^3}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 224$$

3.94. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

$$\frac{\frac{\left(ah^2+cg^2\right)\left(\frac{\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2\left(10fg^4-3g^2h(2eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{c\left(3ah^2(3fg-eh)-6cgh(2eg-dh)+20cfg^3\right)\int\frac{1}{1-\frac{cx^2}{cx^2+c}}}{h}\right.}{h^2}}{h^3}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\left(ah^2+cg^2\right)\left(\frac{\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2\left(10fg^4-3g^2h(2eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3ah^2(3fg-eh)-6cgh(2eg-\right.}{h}\right.}{h^2}}{h^3}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 488

$$\frac{\frac{\left(ah^2+cg^2\right)\left(-\frac{\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2\left(10fg^4-3g^2h(2eg-dh)\right)\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3ah^2(3fg-\right.}{h}\right.}{h^2}}{h^3}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\left(ah^2+cg^2\right)\left(-\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2\left(10fg^4-3g^2h(2eg-dh)\right)\right)}{h\sqrt{ah^2+cg^2}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3ah^2(3fg-\right.}{h}\right.}{h^2}}{h^3}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

3.94. $\int\frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

output
```
-1/2*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^2) + (-1/3*((2*(c*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*
h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(h
^2*(g + h*x)) + (((2*a^2*f*h^4 + 2*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h))
+ a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)) - c*h*(10*c*f*g^3 - 3*c*g*h*(2*e*
g - d*h) + a*h^2*(7*f*g - 3*e*h))*x)*Sqrt[a + c*x^2])/h^2 + ((c*g^2 + a*h^
2)*(-((Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h)
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h) - ((2*a^2*f*h^4 + 2*c^2*(10*f*g
^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 +
a*h^2])))/h^2)/h^3)/(2*(c*g^2 + a*h^2))
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.94. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(458) = 916.

Time = 0.70 (sec) , antiderivative size = 1167, normalized size of antiderivative = 2.39

method	result	size
risch	Expression too large to display	1167
default	Expression too large to display	2863

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*f*h^2*x^2+3*c*e*h^2*x-9*c*f*g*h*x+8*a*f*h^2+6*c*d*h^2-18*c*e*g*h+
36*c*f*g^2)*(c*x^2+a)^(1/2)/h^5+1/2/h^5*(c^(1/2)*(3*a*e*h^3-9*a*f*g*h^2-6*
c*d*g*h^2+12*c*e*g^2*h-20*c*f*g^3)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-(2*a^2*
f*h^4+4*a*c*d*h^4-12*a*c*e*g*h^3+24*a*c*f*g^2*h^2+12*c^2*d*g^2*h^2-20*c^2*
e*g^3*h+30*c^2*f*g^4)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^
2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(2*a^2*e*h^5-4*a^2*f*g*h^4-8*a
*c*d*g*h^4+12*a*c*e*g^2*h^3-16*a*c*f*g^3*h^2-8*c^2*d*g^3*h^2+10*c^2*e*g^4*
h-12*c^2*f*g^5)/h^3*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h
*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1
/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
+1/h^4*(2*a^2*d*h^6-2*a^2*e*g*h^5+2*a^2*f*g^2*h^4+4*a*c*d*g^2*h^4-4*a*c*e*
g^3*h^3+4*a*c*f*g^4*h^2+2*c^2*d*g^4*h^2-2*c^2*e*g^5*h+2*c^2*f*g^6)*(-1/2/(
a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h
^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(
1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln(...
```

3.94.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.94.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(459) = 918.

Time = 0.29 (sec) , antiderivative size = 1299, normalized size of antiderivative = 2.66

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

output
```
3/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*sqrt(c*x^2 + a)*c^
2*f*g^3*x/(c*g^2*h^4 + a*h^6) - 3/2*sqrt(c*x^2 + a)*c^2*e*g^3/(c*g^2*h^4 +
 a*h^6) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3
 + a*g*h^5) + 3/2*sqrt(c*x^2 + a)*c^2*e*g^2*x/(c*g^2*h^3 + a*h^5) + 3/2*sq
rt(c*x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^(3/2)*c*e*g^
2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f*
g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a
*g^2*h^3) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c
*x^2 + a)*c^2*d*g*x/(c*g^2*h^2 + a*h^4) + 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c*g
^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^(5/2)*e*g/(c*g^2
*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/
2*(c*x^2 + a)^(3/2)*c*e*g/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(5/2)*d/(c
*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/
2*(c*x^2 + a)^(3/2)*c*d/(c*g^2*h + a*h^3) + 2*(c*x^2 + a)^(3/2)*f*g/(h^4*x
 + g*h^3) - (c*x^2 + a)^(3/2)*e/(h^3*x + g*h^2) - 7/2*sqrt(c*x^2 + a)*c*f*
g*x/h^4 + 3/2*sqrt(c*x^2 + a)*c*e*x/h^3 - 10*c^(3/2)*f*g^3*arcsinh(c*x/sqr
t(a*c))/h^6 + 6*c^(3/2)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^5 - 3*c^(3/2)*d*g*a
rcsinh(c*x/sqrt(a*c))/h^4 - 9/2*a*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 +
 3/2*a*sqrt(c)*e*arcsinh(c*x/sqrt(a*c))/h^3 + 3/2*c^2*f*g^4*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^...
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(459) = 918$.

Time = 0.35 (sec) , antiderivative size = 1021, normalized size of antiderivative = 2.09

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2cfx}{h^3} - \frac{3(3c^2fgh^{14}-c^2eh^{15})}{ch^{18}}\right) + \frac{2(18c^2fg^2h^{13}-9\ldots}{\ldots}\right.$$
$$+ \frac{\left(20c^{\frac{3}{2}}fg^3 - 12c^{\frac{3}{2}}eg^2h + 6c^{\frac{3}{2}}dgh^2 + 9a\sqrt{c}fgh^2 - 3a\sqrt{c}eh^3\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2h^6}$$
$$+ \frac{(20c^2fg^4 - 12c^2eg^3h + 6c^2dg^2h^2 + 19acfg^2h^2 - 9acegh^3 + 3acdh^4 + 2a^2fh^4)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h\ldots}{\sqrt{-cg^2-ah^2}}\ldots\right.}{\sqrt{-cg^2-ah^2}\,h^6}$$
$$+ \frac{10\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2fg^4h - 8\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2eg^3h^2 + 6\left(\sqrt{c}x-\sqrt{cx^2+a}\right)^3c^2dg^2h^3 + 5\left(\sqrt{c}x-\ldots\right.}{\ldots}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*e*h^15)/(c*h
^18)) + 2*(18*c^2*f*g^2*h^13 - 9*c^2*e*g*h^14 + 3*c^2*d*h^15 + 4*a*c*f*h^1
5)/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 - 12*c^(3/2)*e*g^2*h + 6*c^(3/2)*d*g*
h^2 + 9*a*sqrt(c)*f*g*h^2 - 3*a*sqrt(c)*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c
*x^2 + a)))/h^6 + (20*c^2*f*g^4 - 12*c^2*e*g^3*h + 6*c^2*d*g^2*h^2 + 19*a*
c*f*g^2*h^2 - 9*a*c*e*g*h^3 + 3*a*c*d*h^4 + 2*a^2*f*h^4)*arctan(-((sqrt(c)
*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 -
a*h^2)*h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h - 8*(sqrt(c)
*x - sqrt(c*x^2 + a))^3*c^2*e*g^3*h^2 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*
c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*
c*d*h^5 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 14*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*c^(5/2)*e*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 +
 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 - 5*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2
*sqrt(c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 2
6*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h + 20*(sqrt(c)*x - sqrt(c*x^2
 + a))*a*c^2*e*g^3*h^2 - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3
- 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqr...
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^3}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.95 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.95.1 Optimal result

Integrand size = 29, antiderivative size = 475

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{((cg^2+ah^2)(3afh^2+2c(10fg^2-h(4eg-dh)))+ch(3ah^2(3fg-eh)+cg(10fg^2-h(4eg-dh)))x)\sqrt{c}}{2h^5(cg^2+ah^2)(g+hx)}$$
$$-\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-(3afh^2+c(5fg^2-2h(eg-dh)))x\right)(a+cx^2)^{3/2}}{6h^2(cg^2+ah^2)(g+hx)^2}$$
$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{3h(cg^2+ah^2)(g+hx)^3}+\frac{\sqrt{c}(3afh^2+2c(10fg^2-h(4eg-dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2h^6}$$
$$+\frac{c(3a^2h^4(4fg-eh)+2c^2g^3(10fg^2-h(4eg-dh))+3acgh^2(11fg^2-h(4eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+c}}\right)}{2h^6(cg^2+ah^2)^{3/2}}$$

output
```
-1/6*(c*g*(4*e*g-10*f*g^2/h-d*h)-3*a*h*(-e*h+3*f*g)-(3*a*f*h^2+c*(5*f*g^2-
2*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)^2-1/3*(d*h^2
-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(3*a^2*h^4*(
-e*h+4*f*g)+2*c^2*g^3*(10*f*g^2-h*(-d*h+4*e*g))+3*a*c*g*h^2*(11*f*g^2-h*(-
d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6
/(a*h^2+c*g^2)^(3/2)+1/2*(3*a*f*h^2+2*c*(10*f*g^2-h*(-d*h+4*e*g)))*arctanh
(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^6-1/2*((a*h^2+c*g^2)*(3*a*f*h^2+2*c*
(10*f*g^2-h*(-d*h+4*e*g)))+c*h*(3*a*h^2*(-e*h+3*f*g)+c*g*(10*f*g^2-h*(-d*h
+4*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)/(h*x+g)
```

3.95.2 Mathematica [A] (verified)

Time = 11.03 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \frac{-\frac{h\sqrt{a+cx^2}\left(2(cg^2+ah^2)^2(fg^2+h(-eg+dh))-(cg^2+ah^2)(13cfg^3+cgh(-10eg+7dh)-3ah^2(-2}{}}{}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
(-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c
*g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*
h))*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) +
c^2*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)
*(c*g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c
*g^2 + a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^
2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))
*Log[g + h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 +
 2*c*h*(-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h
^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*
f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[
a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

3.95.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2182, 25, 681, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(3(cdg-afg+aeh)+\left(3afh-c\left(-\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)(cx^2+a)^{3/2}}{(g+hx)^3}dx}{3(ah^2+cg^2)}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{3h(g+hx)^3(ah^2+cg^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(3(cdg-afg+aeh)+\left(3afh-c\left(-\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^3}dx}{3\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 681

$$\frac{\frac{\left(a+cx^2\right)^{3/2}\left(x\left(3afh^2-2ch(eg-dh)+5cfg^2\right)+3ah(3fg-eh)-\frac{1}{2}c\left(-2dgh+8eg^2-\frac{20fg^3}{h}\right)\right)}{2h^2(g+hx)^2}-\frac{3\int -\frac{4\left(ah\left(5cfg^2+3afh^2-2ch(eg-dh)\right)-c\left(10cfg^3-c\right.\right.}{h(g+hx)^2}}{8h^2}}{3\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\left(ah\left(5cfg^2+3afh^2-2ch(eg-dh)\right)-c\left(10cfg^3-ch(4eg-dh)g+3ah^2(3fg-eh)\right)x\right)\sqrt{cx^2+a}}{(g+hx)^2}dx}{2h^3}+\frac{\left(a+cx^2\right)^{3/2}\left(x\left(3afh^2-2ch(eg-dh)+5cfg^2\right)+3ah(3fg\right.\right.}{2h^2(g+hx)^2}}{3\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 681

$$\frac{\frac{3\left(-\frac{\int \frac{2c\left(ah\left(10cfg^3-ch(4eg-dh)g+3ah^2(3fg-eh)\right)-\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)x\right)}{(g+hx)\sqrt{cx^2+a}}dx}{2h^2}-\frac{\sqrt{a+cx^2}\left(chx\left(3ah^2(3fg-eh)-cgh(4eg-dh)+10cfg^3\right.\right.}{h^2(g+\right.}{2h^3}}{3\left(ah^2+\right.}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{3\left(-\frac{c\int \frac{ah\left(10cfg^3-ch(4eg-dh)g+3ah^2(3fg-eh)\right)-\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{h^2}-\frac{\sqrt{a+cx^2}\left(chx\left(3ah^2(3fg-eh)-cgh(4eg-dh)+10cfg^3\right)+\right.}{h^2(g+hx)}\right.}{2h^3}}{3\left(ah^2+\right.}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$\downarrow$ 719

3.95. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

$$3\left(-\frac{c\left(\frac{\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(ah^2+cg^2\right)\left(3afh^2-2ch(4eg-dh)+20cfg^2\right)\int\frac{1}{\sqrt{cx^2\cdots}}\cdots}{h}\cdots\right.}{h^2}\cdots\right.$$
$$\cdots\frac{}{2h^3}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\Big\downarrow\ 224$$

$$3\left(-\frac{c\left(\frac{\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\left(ah^2+cg^2\right)\left(3afh^2-2ch(4eg-dh)+20cfg^2\right)\int\frac{\cdots}{1-\frac{\cdots}{c\cdots}}\cdots}{h}\cdots\right.}{h^2}\cdots\right.$$
$$\cdots\frac{}{2h^3}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\Big\downarrow\ 219$$

$$3\left(-\frac{c\left(\frac{\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)\left(3afh^2-2ch(4e\cdots\right.}{\sqrt{c}h}\cdots\right.}{h^2}\cdots\right.$$
$$\cdots\frac{}{2h^3}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\Big\downarrow\ 488$$

$$3\left(-\frac{c\left(-\frac{\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)\cdots}{\sqrt{c}\cdots}\cdots\right.}{h^2}\cdots\right.$$
$$\cdots\frac{}{2h^3}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

3.95. $\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

$$\Big\downarrow \; 219$$

$$\frac{3\left(-\frac{c\left(-\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)}{h\sqrt{ah^2+cg^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)\cdots}{\sqrt{c}h\cdots}\right)}{h^2}\cdots\right)}{2h^3}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{3h(g+hx)^3\left(ah^2+cg^2\right)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
-1/3*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^3) + (((3*a*h*(3*f*g - e*h) - (c*(8*e*g^2 - (20*f*g^3)/h - 2*d*g*h))/2
 + (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(2*h^
2*(g + h*x)^2) + (3*(-((((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(
4*e*g - d*h)) + c*h*(10*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e
*h))*x)*Sqrt[a + c*x^2])/(h^2*(g + h*x))) - (c*(-(((c*g^2 + a*h^2)*(20*c*f
*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h)) - ((3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(
4*e*g - d*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g
*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 + a*h^2])))/h^2)
)/(2*h^3))/(3*(c*g^2 + a*h^2))
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] && !RationalQ[m])) && NeQ[m, -1] && !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(447) = 894$.

Time = 0.70 (sec) , antiderivative size = 1917, normalized size of antiderivative = 4.04

method	result	size
risch	Expression too large to display	1917
default	Expression too large to display	4813

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(f*h*x+2*e*h-8*f*g)*(c*x^2+a)^(1/2)/h^5+1/2/h^5*(c^(1/2)*(3*a*f*h^2+
2*c*d*h^2-8*c*e*g*h+20*c*f*g^2)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-4*c/h^2*(a
*e*h^3-4*a*f*g*h^2-2*c*d*g*h^2+5*c*e*g^2*h-10*c*f*g^3)/((a*h^2+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/
2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(
2*a^2*f*h^4+4*a*c*d*h^4-12*a*c*e*g*h^3+24*a*c*f*g^2*h^2+12*c^2*d*g^2*h^2-2
0*c^2*e*g^3*h+30*c^2*f*g^4)/h^3*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^
2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*
g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+(2*a^2*e*h^5-4*a^2*f*g*h^4-8*a*c*d*g*h^4+12*a*c*e*g^2*h^3-16*
a*c*f*g^3*h^2-8*c^2*d*g^3*h^2+10*c^2*e*g^4*h-12*c^2*f*g^5)/h^4*(-1/2/(a*h^
2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h
^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c
*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c
-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^5*(2*a^2*d...
```

3.95.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

output
```
Timed out
```

3.95.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2415 vs. $2(448) = 896$.

Time = 0.33 (sec) , antiderivative size = 2415, normalized size of antiderivative = 5.08

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + a)*c^3*f*g^5/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/
2*sqrt(c*x^2 + a)*c^3*f*g^4*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) - 1/
2*sqrt(c*x^2 + a)*c^3*e*g^4/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/6*
(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x +
 c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*e*g^3*
x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*d*g^3/
(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^3/
(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4
 + a^2*g*h^6) - 1/6*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2
*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x
 + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/6*(c*x^2 + a)^(3/2)*c^2*f*
g^3/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*d*g^
2*x/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*
g^2/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h
^3 + a^2*g*h^5) + 1/6*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g
^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x
 + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^
2/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 9/2*sqrt(c*x^2 + a)*c^2*f*g^3/
(c*g^2*h^5 + a*h^7) + 4*sqrt(c*x^2 + a)*c^2*f*g^2*x/(c*g^2*h^4 + a*h^6) -
1/6*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^...
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1878 vs. $2(448) = 896$.

Time = 0.39 (sec) , antiderivative size = 1878, normalized size of antiderivative = 3.95

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*e*h^11)/h^15) - (20*c
^3*f*g^5 - 8*c^3*e*g^4*h + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 - 12*a*c^2
*e*g^2*h^3 + 3*a*c^2*d*g*h^4 + 12*a^2*c*f*g*h^4 - 3*a^2*c*e*h^5)*arctan(-(
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2
*h^6 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*c^3*f*g^5*h^2 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*e*g^4*h^3 + 18*
(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(c)*x - sqrt(c*x^2
 + a))^5*a*c^2*f*g^3*h^4 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*e*g^2*
h^5 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*
e*h^7 + 210*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h - 120*(sqrt(c)
*x - sqrt(c*x^2 + a))^4*c^(7/2)*e*g^5*h^2 + 54*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*c^(7/2)*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g
^4*h^3 - 84*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*e*g^3*h^4 + 27*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*(sqrt(c)*x - sqrt(c*x^
2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*e*g*h^6 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 + 188*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*c^4*f*g^7 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*e*g^6*h
+ 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x - sq...
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^4}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.95. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.96 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

3.96.1 Optimal result

Integrand size = 29, antiderivative size = 511

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{c\Big(8(5fg-eh)\,(cg^2+ah^2)^2+h(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2(35}{8h^5\,(cg^2+ah^2)^2\,(g+hx)}$$
$$+\frac{(4a^2h^4(fg-2eh)-4c^2g^4(5fg-eh)-acgh^2(25fg^2-h(5eg-9dh))-3h(4a^2fh^4+ach^2(17fg^2-h(5eg}{24h^3\,(cg^2+ah^2)^2\,(g+hx)^3}$$
$$-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{4h\,(cg^2+ah^2)\,(g+hx)^4}-\frac{c^{3/2}(5fg-eh)\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}$$
$$-\frac{c(12a^3fh^6+8c^3g^5(5fg-eh)+20ac^2g^3h^2(5fg-eh)+3a^2ch^4(25fg^2-h(5eg-dh)))\,\mathrm{arctanh}\Big(\frac{ah-c}{\sqrt{cg^2+ah^2}}}{8h^6\,(cg^2+ah^2)^{5/2}}$$

output
```
1/24*(4*a^2*h^4*(-2*e*h+f*g)-4*c^2*g^4*(-e*h+5*f*g)-a*c*g*h^2*(25*f*g^2-h*
(-9*d*h+5*e*g))-3*h*(4*a^2*f*h^4+a*c*h^2*(17*f*g^2-h*(-d*h+5*e*g))+2*c^2*g
^2*(5*f*g^2-h*(d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^3
-1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^4-c^(3/2)
*(-e*h+5*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*c*(12*a^3*f*h^6+8
*c^3*g^5*(-e*h+5*f*g)+20*a*c^2*g^3*h^2*(-e*h+5*f*g)+3*a^2*c*h^4*(25*f*g^2-
h*(-d*h+5*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))
/h^6/(a*h^2+c*g^2)^(5/2)+1/8*c*(8*(-e*h+5*f*g)*(a*h^2+c*g^2)^2+h*(12*a^2*f
*h^4+4*c^2*g^3*(-e*h+5*f*g)+a*c*h^2*(35*f*g^2-h*(-3*d*h+7*e*g)))*x)*(c*x^2
+a)^(1/2)/h^5/(a*h^2+c*g^2)^2/(h*x+g)
```

3.96.2 Mathematica [A] (verified)

Time = 11.55 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.13

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = -\frac{h\sqrt{a+cx^2}\left(6(cg^2+ah^2)^3(fg^2+h(-eg+dh))-2(cg^2+ah^2)^2(17cfg^3+cgh(-13eg+9dh)-4ah^2(-2fg+eh))(g+hx)+(cg^2+ah^2)(12a^2fh^4+2c^2(43\ldots}{\ldots}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/24*((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h))
- 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*
f*g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 +
g^2*h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g +
 h*x)^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g
 + 3*d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 2
4*c*f*(c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3
*c*(12*a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h
) + 3*a^2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^
2)^(5/2) + 24*c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (
3*c*(12*a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*
h) + 3*a^2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g
^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/h^6
```

3.96.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2182, 25, 680, 27, 681, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{\left(4(cdg-afg+aeh)+\left(4afh-c\left(-\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^4}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(4(cdg-afg+aeh)+\left(4afh-c\left(-\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^4}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 680$$

$$\frac{\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right.}{6h^2(g+hx)^3\left(ah^2+cg^2\right)}}{4\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right.}{6h^2(g+hx)^3\left(ah^2+cg^2\right)}}{4\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 681$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right.}{6h^2(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right.}{6h^2(g+hx)^3\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

3.96. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

$\downarrow$ 719

$$\frac{\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right)}{6h^2(g+hx)^3(ah^2+cg^2)}}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\downarrow$ 224

$$\frac{\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right)}{6h^2(g+hx)^3(ah^2+cg^2)}}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right)}{6h^2(g+hx)^3(ah^2+cg^2)}}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\downarrow$ 488

$$\frac{\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right)}{6h^2(g+hx)^3(ah^2+cg^2)}}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

3.96. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

$$\frac{\frac{(a+cx^2)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(25fg^2-h(5eg-9dh)\right)-\frac{4c^2g^4(5fg-eh)}{h}\right)}{6h^2(g+hx)^3(ah^2+cg^2)}}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{4h(g+hx)^4\left(ah^2+cg^2\right)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^4) + (((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f*g - e*h))/h - a*c*g*
h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*c*h^2*(17*f*g^2 - h*
(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x)*(a + c*x^2)^(3/2)
)/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^3) - (c*(-(((8*(5*f*g - e*h)*(c*g^2 + a
*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g - e*h) + a*c*h^2*(35*f*g^2 -
h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(h^2*(g + h*x))) - ((-8*Sqrt[c]*(5
*f*g - e*h)*(c*g^2 + a*h^2)^2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h - ((
12*a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) +
3*a^2*c*h^4*(25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^
2 + a*h^2]*Sqrt[a + c*x^2]])]/(h*Sqrt[c*g^2 + a*h^2]))/h^2))/(2*h^3*(c*g^2
 + a*h^2)))/(4*(c*g^2 + a*h^2))
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.96. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3213 vs. $2(485) = 970$.

Time = 0.72 (sec) , antiderivative size = 3214, normalized size of antiderivative = 6.29

method	result	size
risch	Expression too large to display	3214
default	Expression too large to display	7961

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*c*(c*x^2+a)^(1/2)+1/h^5*(c^(3/2)*(e*h-5*f*g)/h*ln(x*c^(1/2)+(c*x^2+a
)^(1/2))-c/h^2*(2*a*f*h^2+c*d*h^2-5*c*e*g*h+15*c*f*g^2)/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1
/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g))+
2*c/h^3*(a*e*h^3-4*a*f*g*h^2-2*c*d*g*h^2+5*c*e*g^2*h-10*c*f*g^3)*(-1/(a*h^
2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)
^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h
^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(a^2*f*h^4+2*a*c*d*h^4
-6*a*c*e*g*h^3+12*a*c*f*g^2*h^2+6*c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^
4)*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/
h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^
2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*
g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g
^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h
^5*(a^2*e*h^5-2*a^2*f*g*h^4-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-
4*c^2*d*g^3*h^2+5*c^2*e*g^4*h-6*c^2*f*g^5)*(-1/3/(a*h^2+c*g^2)*h^2/(x+1...
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4326 vs. 2(485) = 970.

Time = 0.42 (sec) , antiderivative size = 4326, normalized size of antiderivative = 8.47

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
```

output
```
3/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2
*h^9 + a^3*h^11) - 3/8*sqrt(c*x^2 + a)*c^4*f*g^5*x/(c^3*g^6*h^4 + 3*a*c^2*
g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 3/8*sqrt(c*x^2 + a)*c^4*e*g^5/(c^3
*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)
^(3/2)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x +
a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9)
+ 3/8*sqrt(c*x^2 + a)*c^4*e*g^4*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
*g^2*h^7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^3*g^6*h^3 + 3*a*c^2
*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^3*e*g^4/(c
^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7
*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/8*(c*x^2 + a)^(5
/2)*c^2*f*g^4/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2
 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x +
 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h
^7) + 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a
^2*c*g^2*h^7 + a^3*h^9) - 3/8*sqrt(c*x^2 + a)*c^4*d*g^3*x/(c^3*g^6*h^2 + 3
*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*
g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c
^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 1/8*(c*x^2 + a
)^(5/2)*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*...
```

3.96.8 Giac [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)}{(hx+g)^5}\,dx$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.97 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.97.1 Optimal result

Integrand size = 29, antiderivative size = 507

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx =$$

$$-\frac{c(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4(13fg^2+3dh^2)+h(12c^3fg^6+8a^3fh^6+a^2cgh^4(34f}{8h^5(cg^2+ah^2)^3(g+hx)^2}$$

$$-\frac{(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2(5fg^2+3dh^2)+h(4a^2fh^4+acgh^2(14fg-3eh)+c^2(7fg^4-3dg^2h^2}{12h^3(cg^2+ah^2)^2(g+hx)^4}$$

$$-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{5h(cg^2+ah^2)(g+hx)^5}+\frac{c^{3/2}f\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{h^6}$$

$$+\frac{c^2(8c^3fg^7+28ac^2fg^5h^2+3a^3h^6(6fg-eh)+a^2cgh^4(35fg^2-3dh^2))\,\mathrm{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{8h^6(cg^2+ah^2)^{7/2}}$$

output
```
-1/12*(4*c^2*f*g^5-a^2*h^4*(-3*e*h+2*f*g)+a*c*g*h^2*(3*d*h^2+5*f*g^2)+h*(4
*a^2*f*h^4+a*c*g*h^2*(-3*e*h+14*f*g)+c^2*(-3*d*g^2*h^2+7*f*g^4))*x)*(c*x^2
+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^4-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^
(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2
))/h^6+1/8*c^2*(8*c^3*f*g^7+28*a*c^2*f*g^5*h^2+3*a^3*h^6*(-e*h+6*f*g)+a^2*
c*g*h^4*(-3*d*h^2+35*f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x
^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(7/2)-1/8*c*(8*c^3*f*g^7+20*a*c^2*f*g^5*h^2
-a^3*h^6*(-3*e*h+2*f*g)+a^2*c*g*h^4*(3*d*h^2+13*f*g^2)+h*(12*c^3*f*g^6+8*a
^3*f*h^6+a^2*c*g*h^4*(-3*e*h+34*f*g)+a*c^2*g^2*h^2*(-3*d*h^2+35*f*g^2))*x)
*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)^3/(h*x+g)^2
```

3.97.2 Mathematica [A] (verified)

Time = 11.29 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.26

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \frac{-\frac{h\sqrt{a+cx^2}\left(24(cg^2+ah^2)^4(fg^2+h(-eg+dh))-6(cg^2+ah^2)^3(21cfg^3+cgh(-16eg+11dh)-5ah}{}}{}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
(-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6
*(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*
g + e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 +
 9*g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*
(g + h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*
g^5 + 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d
*h)))*(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d
*h)) + 3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(26
1*f*g^2 - h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^
5)) - (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h)
 + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) +
 120*c^(3/2)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 +
 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3
*d*h^2))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 +
a*h^2)^(7/2))/(120*h^6)
```

3.97.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2182, 27, 680, 27, 680, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{5\left(cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^5}dx}{5\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\left(cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^5}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 680

$$\frac{-\frac{\int \frac{2c\left(3ah\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-4f\left(cg^2+ah^2\right)^2x\right)\sqrt{cx^2+a}}{h(g+hx)^3}dx}{8h^2(ah^2+cg^2)}-\frac{(a+cx^2)^{3/2}\left(x\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^3(\cdots}{12h^2(g+hx)^4(ah^2+cg^2)}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 27

$$\frac{-\frac{c\int \frac{\left(3ah\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-4f\left(cg^2+ah^2\right)^2x\right)\sqrt{cx^2+a}}{(g+hx)^3}dx}{4h^3(ah^2+cg^2)}-\frac{(a+cx^2)^{3/2}\left(x\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^3(2\cdots}{12h^2(g+hx)^4(ah^2+cg^2)}}{ah^2+cg^2}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 680

$$-\frac{c\left(\frac{\sqrt{a+cx^2}\left(-a^3h^6(2fg-3eh)+a^2cgh^4\left(3dh^2+13fg^2\right)+hx\left(8a^3fh^6+a^2cgh^4(34fg-3eh)+ac^2g^2h^2\left(35fg^2-3dh^2\right)+12c^3fg^6\right)+20ac^2fg^5h^2+8c^3fg^7\right)}{2h^2(g+hx)^2\left(ah^2+cg^2\right)}-\frac{\int -\cdots}{\cdots}\cdots\right.}{4h^3(ah^2+cg^2)}\cdots$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 27

3.97. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

$$-\frac{c\left(\frac{c\int\frac{ah\left(4c^2fg^5+ach^2\left(11fg^2-3dh^2\right)g+a^2h^4(10fg-3eh)\right)-8f\left(cg^2+ah^2\right)^3x}{(g+hx)\sqrt{cx^2+a}}dx}{2h^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\left(-a^3h^6(2fg-3eh)+a^2cgh^4\left(3dh^2+13fg^2\right)+hx\left(8a^3fh^6+a^2cgh^4\right.\right.}{2h^2(g+hx)^2}\right.}{4h^3(ah^2+cg^2)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\downarrow$ 719

$$-\frac{c\left(\frac{c\left(\frac{\left(3a^3h^6(6fg-eh)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{8f\left(ah^2+cg^2\right)^3\int\frac{1}{\sqrt{cx^2+a}}dx}{h}\right)}{2h^2\left(ah^2+cg^2\right)}+\sqrt{a+cx^2}\left(-a^3h^6(2fg-3eh)\right.\right.}{4h^3(ah^2+cg^2)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\downarrow$ 224

$$-\frac{c\left(\frac{c\left(\frac{\left(3a^3h^6(6fg-eh)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{8f\left(ah^2+cg^2\right)^3\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}\right)}{2h^2\left(ah^2+cg^2\right)}+\sqrt{a+cx^2}\left(-a^3h^6(2\right.\right.}{4h^3(ah^2+cg^2)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{c\left(\frac{c\left(\frac{\left(3a^3h^6(6fg-eh)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{8f\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)^3}{\sqrt{c}h}\right)}{2h^2\left(ah^2+cg^2\right)}+\sqrt{a+cx^2}\left(-a^3h^6(2\right.\right.}{4h^3(ah^2+cg^2)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\downarrow$ 488

3.97. $\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}dx$

$$-\frac{c\left(\frac{c\left(-\frac{\left(3a^3h^6(6fg-eh)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{8f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)^3}{\sqrt{c}h}\right)}{2h^2\left(ah^2+cg^2\right)}+\sqrt{}\right)}{4h^3(ah^2+cg}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{\left(a+cx^2\right)^{3/2}\left(x\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^3(2fg-3eh)+acgh\left(3dh^2+5fg^2\right)+\frac{4c^2fg^5}{h}\right)}{12h^2(g+hx)^4\left(ah^2+cg^2\right)}-\frac{c\left(\frac{c\left(-\frac{\text{arctanh}\left(\frac{}{\sqrt{a+c}}\right)}{}\right)}{}\right)}{}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/5*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^5) + (-1/12*(((4*c^2*f*g^5)/h - a^2*h^3*(2*f*g - 3*e*h) + a*c*g*h*(5*f
*g^2 + 3*d*h^2) + (4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*h) + c^2*(7*f*g^4
 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(h^2*(c*g^2 + a*h^2)*(g + h*x)^4) -
 (c*(((8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c*
g*h^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(
34*f*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/
(2*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) + (c*((-8*f*(c*g^2 + a*h^2)^3*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h) - ((8*c^3*f*g^7 + 28*a*c^2*f*g^5
*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh
[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 + a*h
^2]))/(2*h^2*(c*g^2 + a*h^2))))/(4*h^3*(c*g^2 + a*h^2)))/(c*g^2 + a*h^2)
```

3.97. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 680
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Sim
p[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2))   Int[(d + e*x)^(m + 2)*(a + c*x^
2)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f
*(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
 g}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3
, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.97. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10597 vs. $2(481) = 962$.

Time = 0.75 (sec) , antiderivative size = 10598, normalized size of antiderivative = 20.90

method	result	size
default	Expression too large to display	10598

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.97.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^6} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

output
```
Timed out
```

3.97.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

output
```
Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6650 vs. $2(482) = 964$.

Time = 0.49 (sec) , antiderivative size = 6650, normalized size of antiderivative = 13.12

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

output
```
3/8*sqrt(c*x^2 + a)*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g
^4*h^9 + 4*a^3*c*g^2*h^11 + a^4*h^13) - 3/8*sqrt(c*x^2 + a)*c^5*f*g^6*x/(c
^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*
h^12) - 3/8*sqrt(c*x^2 + a)*c^5*e*g^6/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a
^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) + 1/8*(c*x^2 + a)^(3/2)*c^4*
f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g
^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7
 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 3/8*sqrt(c*x^2 + a)*c^5*e*g^5*x/(c^4*g^
8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11)
+ 3/8*sqrt(c*x^2 + a)*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2
*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^5/(
c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*
x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3
*c*g^3*h^8 + a^4*g*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^3*x^
2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^
4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x +
8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*
c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 1/8*(c*x^2 + a)^(3/2)*c^4*f
*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9
+ a^4*h^11) - 3/8*sqrt(c*x^2 + a)*c^5*d*g^4*x/(c^4*g^8*h^2 + 4*a*c^3*g^...
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4363 vs. 2(482) = 964.

Time = 0.55 (sec) , antiderivative size = 4363, normalized size of antiderivative = 8.61

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

output
```
-1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*
d*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*e*h^7)*arctan(-((sqrt(c)*x - sqrt
(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*
g^4*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*l
og(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*
x^2 + a))^9*c^5*f*g^7*h^4 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*e*g^6*
h^5 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6 - 360*(sqrt(c)*
x - sqrt(c*x^2 + a))^9*a*c^4*e*g^4*h^7 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a)
)^9*a^2*c^3*f*g^3*h^8 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*e*g^2*
h^9 + 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x
 - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^
9*a^3*c^2*e*h^11 + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3
 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*e*g^7*h^4 - 120*(sqrt(c)*x
 - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a*c^(9/2)*f*g^6*h^5 - 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2
)*e*g^5*h^6 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 + 85
95*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7 - 1440*(sqrt(c)*x
 - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*e*g^3*h^8 + 45*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^
(5/2)*f*g^2*h^9 - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*e*g*h^...
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^6}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.97. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.98 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.98.1 Optimal result

Integrand size = 29, antiderivative size = 404

$$\begin{aligned}\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx =& -\frac{ac(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)\sqrt{a+cx^2}}{16(cg^2+ah^2)^4(g+hx)^2}\\ &-\frac{(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))(ah-cgx)(a+cx^2)^{3/2}}{24(cg^2+ah^2)^3(g+hx)^4}\\ &-\frac{(fg^2-egh+dh^2)(a+cx^2)^{5/2}}{6h(cg^2+ah^2)(g+hx)^6}\\ &+\frac{(6ah^2(2fg-eh)+cg(5fg^2+h(eg-7dh)))(a+cx^2)^{5/2}}{30h(cg^2+ah^2)^2(g+hx)^5}\\ &-\frac{a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2-h(7eg-dh)))\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16(cg^2+ah^2)^{9/2}}\end{aligned}$$

output
```
-1/24*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c
*x^2+a)^(3/2)/(a*h^2+c*g^2)^3/(h*x+g)^4-1/6*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^
(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^6+1/30*(6*a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2+h*
(-7*d*h+e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^5-1/16*a^2*c^2*(6
*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*arctanh((-c*g*x+a*h)/(a
*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(9/2)-1/16*a*c*(6*c^2*d*g
^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a
*h^2+c*g^2)^4/(h*x+g)^2
```

3.98.2 Mathematica [A] (verified)

Time = 11.41 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.72

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx=\frac{1}{240}\left(-\frac{\sqrt{a+cx^2}\left(40(cg^2+ah^2)^5(fg^2+h(-eg+dh))-8(cg^2+ah^2)\right.}{}\right.$$
$$+\frac{15a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2+h(-7eg+dh)))\log(g+hx)}{(cg^2+ah^2)^{9/2}}$$
$$\left.-\frac{15a^2c^2(6c^2dg^2+6a^2fh^2-ac(fg^2+h(-7eg+dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{(cg^2+ah^2)^{9/2}}\right)$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output
```
(-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(
c*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 +
 g^2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*
(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(1
00*f*g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g +
3*d*h)))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6
- g^4*h*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) +
3*a^2*c*h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^
6*(41*f*g - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^
3*(10*f*g^7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g
 + 14*d*h)))*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*
c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h
*x])/(c*g^2 + a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*
(f*g^2 + h*(-7*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a +
 c*x^2]])/(c*g^2 + a*h^2)^(9/2))/240
```

3.98.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2182, 25, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{\left(6(cdg-afg+aeh)+\left(6afh+c\left(\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^6}dx}{6\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(6(cdg-afg+aeh)+\left(6afh+c\left(\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^6}dx}{6\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 679$$

$$\frac{\frac{\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)\int \frac{\left(cx^2+a\right)^{3/2}}{(g+hx)^5}dx}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{5/2}\left(6ah^2(2fg-eh)+cgh(eg-7dh)+5cfg^3\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}}{6\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)\left(\frac{3ac\int \frac{\sqrt{cx^2+a}}{(g+hx)^3}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{5/2}\left(6ah^2(2fg-eh)+cgh(eg-7dh)+5cfg^3\right)}{5h(g+hx)^5\left(ah^2+cg^2\right)}}{6\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 486$$

$$\frac{\frac{\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)\left(\frac{3ac\left(\frac{ac\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}+\frac{\left(a+cx^2\right)^{5/2}\left(6ah^2(2fg \ldots\right.}{5h(g+h \ldots}}{6\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 488$$

3.98. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}dx$

$$\frac{\frac{(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)\left(\frac{3ac\left(-\frac{ac\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{2(ah^2+cg^2)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2(ah^2+cg^2)}\right)}{4(ah^2+cg^2)}-\frac{(a+cx^2)^{3/2}(ah-cgx)}{4(g+hx)^4(ah^2+cg^2)}\right)}{ah^2+cg^2}+\frac{(a+cx^2)}{}}{6(ah^2+cg^2)}$$

$$\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{6h(g+hx)^6(ah^2+cg^2)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{3ac\left(-\frac{ac\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{2(ah^2+cg^2)^{3/2}}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2(ah^2+cg^2)}\right)}{4(ah^2+cg^2)}-\frac{(a+cx^2)^{3/2}(ah-cgx)}{4(g+hx)^4(ah^2+cg^2)}\right)(6a^2fh^2-ac(fg^2-h(7eg-dh))+6c^2dg^2)}{ah^2+cg^2}+\frac{(a+cx^2)}{}}{6(ah^2+cg^2)}$$

$$\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{6h(g+hx)^6(ah^2+cg^2)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output
```
-1/6*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^6) + (((5*c*f*g^3 + c*g*h*(e*g - 7*d*h) + 6*a*h^2*(2*f*g - e*h))*(a +
c*x^2)^(5/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) + ((6*c^2*d*g^2 + 6*a^2*f*
h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(-1/4*((a*h - c*g*x)*(a + c*x^2)^(3/2
))/((c*g^2 + a*h^2)*(g + h*x)^4) + (3*a*c*(-1/2*((a*h - c*g*x)*Sqrt[a + c*
x^2])/((c*g^2 + a*h^2)*(g + h*x)^2) - (a*c*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g
^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*(c*g^2 + a*h^2)^(3/2))))/(4*(c*g^2 + a*h
^2))))/(c*g^2 + a*h^2))/(6*(c*g^2 + a*h^2))
```

3.98. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.98. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 16382 vs. $2(380) = 760$.

Time = 0.85 (sec) , antiderivative size = 16383, normalized size of antiderivative = 40.55

method	result	size
default	Expression too large to display	16383

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.98.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

output
```
Timed out
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

output
```
Timed out
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10724 vs. 2(381) = 762.

Time = 0.64 (sec) , antiderivative size = 10724, normalized size of antiderivative = 26.54

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

output
```
7/16*sqrt(c*x^2 + a)*c^6*f*g^8/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^
3*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) - 7/16*sqrt
(c*x^2 + a)*c^6*f*g^7*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h
^8 + 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 7/16*sqrt(c*x^2
+ a)*c^6*e*g^7/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a
^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 7/48*(c*x^2 + a)^(3/2)*c^
5*f*g^7/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^
3*c^2*g^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a*c^
4*g^9*h^5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 + a
^5*g*h^13) + 7/16*sqrt(c*x^2 + a)*c^6*e*g^6*x/(c^5*g^10*h^3 + 5*a*c^4*g^8*
h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^1
3) + 7/16*sqrt(c*x^2 + a)*c^6*d*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a
^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/48*
(c*x^2 + a)^(3/2)*c^5*e*g^6/(c^5*g^10*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c
^3*g^6*h^7*x + 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^11*x + a^5*h^13*x + c^
5*g^11*h^2 + 5*a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5
*a^4*c*g^3*h^10 + a^5*g*h^12) - 7/48*(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10
*h^3*x^2 + 5*a*c^4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7*x^2 + 10*a^3*c^2*g^4*h
^9*x^2 + 5*a^4*c*g^2*h^11*x^2 + a^5*h^13*x^2 + 2*c^5*g^11*h^2*x + 10*a*c^4
*g^9*h^4*x + 20*a^2*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^8*x + 10*a^4*c*g^3...
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6061 vs. 2(381) = 762.

Time = 0.42 (sec) , antiderivative size = 6061, normalized size of antiderivative = 15.00

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

output
```
1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 + 7*a^3*c^3*e*g*h - a^3*c^3*d*h^2 + 6
*a^4*c^2*f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*
c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(
c*x^2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*
f*g^6*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(
sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt
(c*x^2 + a))^11*a^3*c^3*f*g^2*h^11 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^11*
a^3*c^3*e*g*h^12 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d*h^13 + 15
0*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 + 1200*(sqrt(c)*x - sqrt
(c*x^2 + a))^10*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*
c^(13/2)*e*g^8*h^5 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*f*g^
7*h^6 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*e*g^6*h^7 + 7200*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 + 1440*(sqrt(c)*x -
sqrt(c*x^2 + a))^10*a^2*c^(9/2)*e*g^4*h^9 - 990*(sqrt(c)*x - sqrt(c*x^2 +
a))^10*a^2*c^(9/2)*d*g^3*h^10 + 4965*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*
c^(7/2)*f*g^3*h^10 - 195*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*e*g^
2*h^11 + 165*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sq
rt(c*x^2 + a))^10*a^4*c^(5/2)*e*h^13 + 3200*(sqrt(c)*x - sqrt(c*x^2 + a...
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx=\int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^7}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.99 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

3.99.1 Optimal result

Integrand size = 29, antiderivative size = 532

$$\begin{aligned}
&\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \\
&-\frac{ac^2(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\sqrt{a+cx^2}}{16\,(cg^2+ah^2)^5\,(g+hx)^2} \\
&-\frac{c(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,(ah-cgx)\,(a+cx^2)^{3/2}}{24\,(cg^2+ah^2)^4\,(g+hx)^4} \\
&-\frac{(fg^2-egh+dh^2)\,(a+cx^2)^{5/2}}{7h\,(cg^2+ah^2)\,(g+hx)^7} \\
&+\frac{(7ah^2(2fg-eh)+cg(5fg^2+h(2eg-9dh)))\,(a+cx^2)^{5/2}}{42h\,(cg^2+ah^2)^2\,(g+hx)^6} \\
&-\frac{(42a^2fh^4-c^2g^2(5fg^2+h(2eg-51dh))-ach^2(26fg^2-h(61eg-12dh)))\,(a+cx^2)^{5/2}}{210h\,(cg^2+ah^2)^3\,(g+hx)^5} \\
&-\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2-h(8eg-3dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{16\,(cg^2+ah^2)^{11/2}}
\end{aligned}$$

output
```
-1/24*c*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*
(-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^4/(h*x+g)^4-1/7*(d*h^2-e*g*h+f*
g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^7+1/42*(7*a*h^2*(-e*h+2*f*g)+
c*g*(5*f*g^2+h*(-9*d*h+2*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^
6-1/210*(42*a^2*f*h^4-c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-a*c*h^2*(26*f*g^
2-h*(-12*d*h+61*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^3/(h*x+g)^5-1/16*a^
2*c^3*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*ar
ctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(11/
2)-1/16*a*c^2*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e
*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^5/(h*x+g)^2
```

3.99.2 Mathematica [A] (verified)

Time = 11.31 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.62

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx =$$

$$-\frac{\sqrt{a+cx^2}\Big(240(cg^2+ah^2)^6(fg^2+h(-eg+dh))-40(cg^2+ah^2)^5(29cfg^3+cgh(-22eg+15dh)-7ah^2(}{}$$

$$+\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2+h(-8eg+3dh)))\log(g+hx)}{16(cg^2+ah^2)^{11/2}}$$

$$-\frac{a^2c^3(6c^2dg^3+a^2h^2(8fg-eh)-acg(fg^2+h(-8eg+3dh)))\log\left(ah-cgx+\sqrt{cg^2+ah^2}\sqrt{a+cx^2}\right)}{16(cg^2+ah^2)^{11/2}}$$

input
```
Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

output
```
-1/1680*(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h))
 - 40*(c*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(
-2*f*g + e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(31
4*f*g^2 + h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d
*h)))*(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) +
2*c^2*(500*f*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(
-544*e*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 +
c^3*(400*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-2
9*e*g + 8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h
*x)^4 - c^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2
*(89*f*g^2 + 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g -
 73*d*h)) + 4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-
336*a^4*f*h^8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^
2*g^2*h^4*(505*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4
*e*g + 3*d*h)) + 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x
)^6))/((c*g^2*h + a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*
(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g
^2 + a*h^2)^(11/2)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*
g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt
[a + c*x^2]])/(16*(c*g^2 + a*h^2)^(11/2))
```

3.99.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2182, 25, 688, 25, 27, 679, 486, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}dx$$

$$\Big\downarrow \text{ 2182}$$

$$-\frac{\int -\frac{\left(7(cdg-afg+aeh)+\left(7afh+c\left(\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)(cx^2+a)^{3/2}}{(g+hx)^7}dx}{7(ah^2+cg^2)}-\frac{(a+cx^2)^{5/2}(dh^2-egh+fg^2)}{7h(g+hx)^7(ah^2+cg^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(7(cdg-afg+aeh)+\left(7afh+c\left(\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^7}dx}{7\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\downarrow$ 688

$$\frac{\frac{\left(a+cx^2\right)^{5/2}\left(7ah^2(2fg-eh)+cgh(2eg-9dh)+5cfg^3\right)}{6h(g+hx)^6\left(ah^2+cg^2\right)}-\frac{\int -\frac{\left(6h\left(7c^2dg^2+7a^2fh^2-ac\left(2fg^2-h(9eg-2dh)\right)\right)+c\left(5cfg^3+ch(2eg-9dh)g+7ah^2(2fg-eh)\right)x\right)}{h(g+hx)^6}}{6\left(ah^2+cg^2\right)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\left(6h\left(7c^2dg^2+7a^2fh^2-ac\left(2fg^2-h(9eg-2dh)\right)\right)+c\left(5cfg^3+ch(2eg-9dh)g+7ah^2(2fg-eh)\right)x\right)\left(cx^2+a\right)^{3/2}}{h(g+hx)^6}dx}{6\left(ah^2+cg^2\right)}+\frac{\left(a+cx^2\right)^{5/2}\left(7ah^2(2fg-eh)+cgh(2eg-9\right.}{6h(g+hx)^6\left(ah^2+cg^2\right)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(6h\left(7c^2dg^2+7a^2fh^2-ac\left(2fg^2-h(9eg-2dh)\right)\right)+c\left(5cfg^3+ch(2eg-9dh)g+7ah^2(2fg-eh)\right)x\right)\left(cx^2+a\right)^{3/2}}{(g+hx)^6}dx}{6h\left(ah^2+cg^2\right)}+\frac{\left(a+cx^2\right)^{5/2}\left(7ah^2(2fg-eh)+cgh(2eg-9\right.}{6h(g+hx)^6\left(ah^2+cg^2\right)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\downarrow$ 679

$$\frac{\frac{\frac{7ch\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)\int \frac{\left(cx^2+a\right)^{3/2}}{(g+hx)^5}dx}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(42a^2fh^4-ach^2\left(26fg^2-h(61eg-12dh)\right)-c^2\left(g^2h(2eg-51dh)+5fg^4\right)\right)}{5(g+hx)^5\left(ah^2+cg^2\right)}}{6h\left(ah^2+cg^2\right)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\downarrow$ 486

3.99. $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

$$\frac{\frac{\frac{7ch\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)\left(\frac{3ac\int\frac{\sqrt{cx^2+a}}{(g+hx)^3}dx}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(42a^2fh^4-ach^2\left(26fg^2-h(61eg-12dh)\right)-c\right.}{5(g+hx)^5\left(ah^2+cg^2\right)}}{6h(ah^2+cg^2)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 486

$$\frac{\frac{\frac{7ch\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)\left(\frac{3ac\left(\frac{ac\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}-\frac{\left(a+cx^2\right)^{5/2}\left(42a^2\right.}{}}{6h(ah^2+cg^2)}}{7\left(ah^2+cg^2\right)}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 488

$$\frac{\frac{\frac{7ch\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)\left(\frac{3ac\left(-\frac{ac\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)}{ah^2+cg^2}-\frac{\left(a\right.}{}}{6h(ah^2+cg^2)}}{7\left(ah^2+cg^2\right.}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

$\big\downarrow$ 219

3.99. $\int\frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

$$\frac{\frac{7ch\left(\frac{3ac\left(-\frac{ac\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{2\left(ah^2+cg^2\right)^{3/2}}-\frac{\sqrt{a+cx^2}(ah-cgx)}{2(g+hx)^2\left(ah^2+cg^2\right)}\right)}{4\left(ah^2+cg^2\right)}-\frac{\left(a+cx^2\right)^{3/2}(ah-cgx)}{4(g+hx)^4\left(ah^2+cg^2\right)}\right)\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)}{ah^2+cg^2}-\frac{(a-\cdots}{\cdots}}{6h(ah^2+cg^2)}}{7\left(ah^2+cg^2\right)\cdots}$$

$$\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-egh+fg^2\right)}{7h(g+hx)^7\left(ah^2+cg^2\right)}$$

input
```
Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

output
```
-1/7*((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g + h
*x)^7) + (((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a
+ c*x^2)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + (-1/5*((42*a^2*f*h^4 -
 c^2*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g -
12*d*h)))*(a + c*x^2)^(5/2))/((c*g^2 + a*h^2)*(g + h*x)^5) + (7*c*h*(6*c^2
*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*(-1/4*
((a*h - c*g*x)*(a + c*x^2)^(3/2))/((c*g^2 + a*h^2)*(g + h*x)^4) + (3*a*c*(
-1/2*((a*h - c*g*x)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x)^2) - (a*c*
ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*(c*g^2 +
a*h^2)^(3/2))))/(4*(c*g^2 + a*h^2))))/(c*g^2 + a*h^2))/(6*h*(c*g^2 + a*h^2
)))/(7*(c*g^2 + a*h^2))
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.99. $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24804 vs. 2(504) = 1008.

Time = 1.08 (sec) , antiderivative size = 24805, normalized size of antiderivative = 46.63

method	result	size
default	Expression too large to display	24805

input
```
int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.99.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

output
```
Timed out
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

output
```
Timed out
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16249 vs. 2(505) = 1010.

Time = 0.83 (sec) , antiderivative size = 16249, normalized size of antiderivative = 30.54

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
```

output
```
9/16*sqrt(c*x^2 + a)*c^7*f*g^9/(c^6*g^12*h^5 + 6*a*c^5*g^10*h^7 + 15*a^2*c
^4*g^8*h^9 + 20*a^3*c^3*g^6*h^11 + 15*a^4*c^2*g^4*h^13 + 6*a^5*c*g^2*h^15
+ a^6*h^17) - 9/16*sqrt(c*x^2 + a)*c^7*f*g^8*x/(c^6*g^12*h^4 + 6*a*c^5*g^1
0*h^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*c^2*g^4*h^12 + 6
*a^5*c*g^2*h^14 + a^6*h^16) - 9/16*sqrt(c*x^2 + a)*c^7*e*g^8/(c^6*g^12*h^4
 + 6*a*c^5*g^10*h^6 + 15*a^2*c^4*g^8*h^8 + 20*a^3*c^3*g^6*h^10 + 15*a^4*c^
2*g^4*h^12 + 6*a^5*c*g^2*h^14 + a^6*h^16) + 3/16*(c*x^2 + a)^(3/2)*c^6*f*g
^8/(c^6*g^12*h^4*x + 6*a*c^5*g^10*h^6*x + 15*a^2*c^4*g^8*h^8*x + 20*a^3*c^
3*g^6*h^10*x + 15*a^4*c^2*g^4*h^12*x + 6*a^5*c*g^2*h^14*x + a^6*h^16*x + c
^6*g^13*h^3 + 6*a*c^5*g^11*h^5 + 15*a^2*c^4*g^9*h^7 + 20*a^3*c^3*g^7*h^9 +
 15*a^4*c^2*g^5*h^11 + 6*a^5*c*g^3*h^13 + a^6*g*h^15) + 9/16*sqrt(c*x^2 +
a)*c^7*e*g^7*x/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*g^8*h^7 + 20*
a^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^6*h^15) + 9/1
6*sqrt(c*x^2 + a)*c^7*d*g^7/(c^6*g^12*h^3 + 6*a*c^5*g^10*h^5 + 15*a^2*c^4*
g^8*h^7 + 20*a^3*c^3*g^6*h^9 + 15*a^4*c^2*g^4*h^11 + 6*a^5*c*g^2*h^13 + a^
6*h^15) - 3/16*(c*x^2 + a)^(3/2)*c^6*e*g^7/(c^6*g^12*h^3*x + 6*a*c^5*g^10*
h^5*x + 15*a^2*c^4*g^8*h^7*x + 20*a^3*c^3*g^6*h^9*x + 15*a^4*c^2*g^4*h^11*
x + 6*a^5*c*g^2*h^13*x + a^6*h^15*x + c^6*g^13*h^2 + 6*a*c^5*g^11*h^4 + 15
*a^2*c^4*g^9*h^6 + 20*a^3*c^3*g^7*h^8 + 15*a^4*c^2*g^5*h^10 + 6*a^5*c*g^3*
h^12 + a^6*g*h^14) - 3/16*(c*x^2 + a)^(5/2)*c^5*f*g^7/(c^6*g^12*h^3*x^2...
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7857 vs. 2(505) = 1010.

Time = 0.47 (sec) , antiderivative size = 7857, normalized size of antiderivative = 14.77

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

output
```
-1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 + 8*a^3*c^4*e*g^2*h - 3*a^3*c^4*d*g*
h^2 + 8*a^4*c^3*f*g*h^2 - a^4*c^3*e*h^3)*arctan(((sqrt(c)*x - sqrt(c*x^2 +
 a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 1
0*a^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(
-c*g^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g
^3*h^12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 + 840*(s
qrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*e*g^2*h^13 - 315*(sqrt(c)*x - sqrt(
c*x^2 + a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4
*c^3*f*g*h^14 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*e*h^15 - 1680
*(sqrt(c)*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x -
sqrt(c*x^2 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 +
 a))^12*a^2*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*d*g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*
f*g^4*h^11 + 10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*e*g^3*h^12
 - 4095*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sq
rt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1365*(sqrt(c)*x - s
qrt(c*x^2 + a))^12*a^4*c^(7/2)*e*g*h^14 - 1680*(sqrt(c)*x - sqrt(c*x^2 + a
))^12*a^5*c^(5/2)*f*h^15 - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^1
1*h^4 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*e*g^10*h^5 - 28000*(sqrt
(c)*x - sqrt(c*x^2 + a))^11*a*c^7*f*g^9*h^6 - 11200*(sqrt(c)*x - sqrt(c...
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \int \frac{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^8}\,dx$$

input
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

output
```
int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)
```

3.100 $\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\, dx$

3.100.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\, dx = \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c} + \frac{5a(8Ac-aC)x(a+cx^2)^{3/2}}{192c} + \frac{(8Ac-aC)x(a+cx^2)^{5/2}}{48c} + \frac{B(a+cx^2)^{7/2}}{7c} + \frac{Cx(a+cx^2)^{7/2}}{8c} + \frac{5a^3(8Ac-aC)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}$$

output
```
5/192*a*(8*A*c-C*a)*x*(c*x^2+a)^(3/2)/c+1/48*(8*A*c-C*a)*x*(c*x^2+a)^(5/2)
/c+1/7*B*(c*x^2+a)^(7/2)/c+1/8*C*x*(c*x^2+a)^(7/2)/c+5/128*a^3*(8*A*c-C*a)
*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+5/128*a^2*(8*A*c-C*a)*x*(c*x^2
+a)^(1/2)/c
```

3.100.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\, dx = \frac{\sqrt{c}\sqrt{a+cx^2}(3a^3(128B+35Cx)+16c^3x^5(28A+3x(8B+7Cx))+8ac^2x^3(182A+x(144B+}{2688}$$

input
```
Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + c*x^2]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*
(8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(92
4*A + x*(576*B + 413*C*x))) + 105*a^3*(-8*A*c + a*C)*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2]])/(2688*c^(3/2))
```

3.100.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2346, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^{5/2} \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int (8Ac+8Bxc-aC) \left(cx^2+a\right)^{5/2}dx}{8c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$$\Big\downarrow \text{455}$$

$$\frac{(8Ac-aC)\int \left(cx^2+a\right)^{5/2}dx+\frac{8}{7}B\left(a+cx^2\right)^{7/2}}{8c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ac-aC)\left(\frac{5}{6}a\int \left(cx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+cx^2\right)^{5/2}\right)+\frac{8}{7}B\left(a+cx^2\right)^{7/2}}{8c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ac-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{cx^2+a}dx+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}\right)+\frac{8}{7}B\left(a+cx^2\right)^{7/2}}{8c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$$\Big\downarrow \text{211}$$

$$\frac{(8Ac-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{cx^2+a}}dx+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}\right)+\frac{8}{7}B\left(a+cx^2\right)^{7/2}}{8c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$\downarrow$ 224

$$\frac{(8Ac-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}\right)+\frac{8}{7}B\left(a+c\right.}{8c}\quad\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

$\downarrow$ 219

$$\frac{(8Ac-aC)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{1}{2}x\sqrt{a+cx^2}\right)+\frac{1}{4}x\left(a+cx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+cx^2\right)^{5/2}\right)+\frac{8}{7}B\left(a+cx\right.}{8c}\quad\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}$$

input
```
Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

output
```
(C*x*(a + c*x^2)^(7/2))/(8*c) + ((8*B*(a + c*x^2)^(7/2))/7 + (8*A*c - a*C)
*((x*(a + c*x^2)^(5/2))/6 + (5*a*((x*(a + c*x^2)^(3/2))/4 + (3*a*((x*Sqrt[
a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c])))/4))
/6))/(8*c)
```

3.100.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.100.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

method	result
risch	$\frac{(336c^3Cx^7+384Bc^3x^6+448Ac^3x^5+952ac^2Cx^5+1152aBc^2x^4+1456aAc^2x^3+826Ca^2cx^3+1152a^2Bcx^2+1848a^2Acx+105a^3Cx}{2688c}$
default	$A\left(\frac{x(cx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(cx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{cx^2+a}}{2}+\frac{a\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{2\sqrt{c}}\right)}{4}\right)}{6}\right)+C\left(\frac{x(cx^2+a)^{\frac{7}{2}}}{8c}-\frac{a\left(\frac{x(cx^2+a)^{\frac{5}{2}}}{6}+\frac{5a(}{}\right.}{}\right.$

input
```
int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/2688/c*(336*C*c^3*x^7+384*B*c^3*x^6+448*A*c^3*x^5+952*C*a*c^2*x^5+1152*B
*a*c^2*x^4+1456*A*a*c^2*x^3+826*C*a^2*c*x^3+1152*B*a^2*c*x^2+1848*A*a^2*c*
x+105*C*a^3*x+384*B*a^3)*(c*x^2+a)^(1/2)+5/128*a^3*(8*A*c-C*a)/c^(3/2)*ln(
x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.98

$$\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\,dx = \left[-\frac{105\,(Ca^4-8\,Aa^3c)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(336\,Cc^4x^7+384\,Bc^4x^6+11\right.$$

input
```
integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 11
52*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*
a^2*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 +
 a))/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(
c*x^2 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a
^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^
2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c
^2]
```

3.100.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(150) = 300$.

Time = 0.48 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.95

$$\int (a + cx^2)^{5/2} (A + Bx + Cx^2)\, dx = \begin{cases} \sqrt{a + cx^2} \left(\frac{Ba^3}{7c} + \frac{3Ba^2x^2}{7} + \frac{3Bacx^4}{7} + \frac{Bc^2x^6}{7} + \frac{Cc^2x^7}{8} + \frac{x^5\left(Ac^3 + \frac{17Cac^2}{8}\right)}{6c} + \frac{x^3 \cdot \left(3Aac^2 + 3Ca^2c - \frac{5a\left(Ac^3 + 1\right.}{6c}\right.}{4c} \right. \\ a^{\frac{5}{2}} \left(Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3} \right) \end{cases}$$

input

```
integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

output

```
Piecewise((sqrt(a + c*x**2)*(B*a**3/(7*c) + 3*B*a**2*x**2/7 + 3*B*a*c*x**4
/7 + B*c**2*x**6/7 + C*c**2*x**7/8 + x**5*(A*c**3 + 17*C*a*c**2/8)/(6*c) +
 x**3*(3*A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 17*C*a*c**2/8)/(6*c))/(4*c)
 + x*(3*A*a**2*c + C*a**3 - 3*a*(3*A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 1
7*C*a*c**2/8)/(6*c))/(4*c))/(2*c)) + (A*a**3 - a*(3*A*a**2*c + C*a**3 - 3*
a*(3*A*a*c**2 + 3*C*a**2*c - 5*a*(A*c**3 + 17*C*a*c**2/8)/(6*c))/(4*c))/(2
*c))*Piecewise((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0))
, (x*log(x)/sqrt(c*x**2), True)), Ne(c, 0)), (a**(5/2)*(A*x + B*x**2/2 + C
*x**3/3), True))
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int (a + cx^2)^{5/2} (A + Bx + Cx^2)\, dx = \frac{1}{6} (cx^2 + a)^{\frac{5}{2}} Ax + \frac{5}{24} (cx^2 + a)^{\frac{3}{2}} Aax + \frac{5}{16} \sqrt{cx^2 + a} Aa^2x + \frac{(cx^2 + a)^{\frac{7}{2}} Cx}{8\,c} - \frac{(cx^2 + a)^{\frac{5}{2}} Cax}{48\,c} - \frac{5\,(cx^2 + a)^{\frac{3}{2}} Ca^2x}{192\,c} - \frac{5\,\sqrt{cx^2 + a} Ca^3x}{128\,c} - \frac{5\,Ca^4 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{128\,c^{\frac{3}{2}}} + \frac{5\,Aa^3 \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{c}} + \frac{(cx^2 + a)^{\frac{7}{2}} B}{7\,c}$$

input
```
integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + a)^(5/2)*A*x + 5/24*(c*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(c*x^2
 + a)*A*a^2*x + 1/8*(c*x^2 + a)^(7/2)*C*x/c - 1/48*(c*x^2 + a)^(5/2)*C*a*x
/c - 5/192*(c*x^2 + a)^(3/2)*C*a^2*x/c - 5/128*sqrt(c*x^2 + a)*C*a^3*x/c -
 5/128*C*a^4*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 5/16*A*a^3*arcsinh(c*x/sqrt(
a*c))/sqrt(c) + 1/7*(c*x^2 + a)^(7/2)*B/c
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00

$$\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\,dx = \frac{1}{2688}\left(\frac{384\,Ba^3}{c} + \left(2\left(576\,Ba^2 + \left(4\left(144\,Bac + \left(6\,(7\,Cc^2x + 8\,Bc^2)x + \frac{7\,(17\,Cac^7 + 8\,Ac^8}{c^6}\right.\right.\right.\right.\right.\right. + \frac{5\,(Ca^4 - 8\,Aa^3c)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+a}\right|\right)}{128\,c^{\frac{3}{2}}}$$

input
```
integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*
c^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^
7)/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5
/128*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (a+cx^2)^{5/2} (A+Bx+Cx^2)\,dx = \int (c\,x^2+a)^{5/2}\,(C\,x^2+B\,x+A)\,dx$$

input
```
int((a + c*x^2)^(5/2)*(A + B*x + C*x^2),x)
```

output
```
int((a + c*x^2)^(5/2)*(A + B*x + C*x^2), x)
```

3.101 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.101.1 Optimal result

Integrand size = 29, antiderivative size = 325

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{(4(5cd-4af)h^2-3cg(fg-5eh))(g+hx)^2\sqrt{a+cx^2}}{60c^2h}$$
$$-\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch}+\frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch}$$
$$+\frac{(4(16a^2fh^4-4ach^2(13fg^2+5h(3eg+dh))-c^2g^2(3fg^2-5h(3eg+16dh)))-ch(ah^2(71fg+45eh)-}{120c^3h}$$
$$+\frac{(8c^2dg^3+3a^2h^2(3fg+eh)-4acg(fg^2+3h(eg+dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-4*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arct
anh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/60*(4*(-4*a*f+5*c*d)*h^2-3*c*g*(-
5*e*h+f*g))*(h*x+g)^2*(c*x^2+a)^(1/2)/c^2/h-1/20*(-5*e*h+f*g)*(h*x+g)^3*(c
*x^2+a)^(1/2)/c/h+1/5*f*(h*x+g)^4*(c*x^2+a)^(1/2)/c/h+1/120*(64*a^2*f*h^4-
16*a*c*h^2*(13*f*g^2+5*h*(d*h+3*e*g))-4*c^2*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g
))-c*h*(a*h^2*(45*e*h+71*f*g)+2*c*g*(3*f*g^2-5*h*(10*d*h+3*e*g)))*x)*(c*x^
2+a)^(1/2)/c^3/h
```

3.101.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+cx^2}} \, dx$$
$$= \frac{\sqrt{a+cx^2}(64a^2fh^3 - ach(5h(48eg+16dh+9ehx)+f(240g^2+135ghx+32h^2x^2))+2c^2(10dh(18g^2+9\cdots}{}$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[a + c*x^2]*(64*a^2*f*h^3 - a*c*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) +
f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + 2*c^2*(10*d*h*(18*g^2 + 9*g*h*x +
2*h^2*x^2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*
g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3))) - 15*Sqrt[c]*(8*c^2*d*g^3 +
 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2]])/(120*c^3)
```

3.101.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2185, 27, 687, 27, 687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+cx^2}} \, dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{h(g+hx)^3((5cd-4af)h-c(fg-5eh)x)}{\sqrt{cx^2+a}}dx}{5ch^2} + \frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(g+hx)^3((5cd-4af)h-c(fg-5eh)x)}{\sqrt{cx^2+a}}dx}{5ch} + \frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\int \frac{c(g+hx)^2\left(h(20cdg-13afg-15aeh)+\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)x\right)}{\sqrt{cx^2+a}}dx}{4c}-\frac{1}{4}\sqrt{a+cx^2}(g+hx)^3(fg-5eh)}{5ch}+\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{4}\int \frac{(g+hx)^2\left(h(20cdg-13afg-15aeh)+(4(5cd-4af)h^2-3cg(fg-5eh))x\right)}{\sqrt{cx^2+a}}dx-\frac{1}{4}\sqrt{a+cx^2}(g+hx)^3(fg-5eh)}{5ch}+\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$\Big\downarrow$ 687

$$\frac{\frac{1}{4}\left(\frac{\int \frac{(g+hx)\left(h\left(60c^2dg^2+32a^2fh^2-ac\left(33fg^2+5h(15eg+8dh)\right)\right)-c\left(6cfg^3-10ch(3eg+10dh)g+ah^2(71fg+45eh)\right)x\right)}{\sqrt{cx^2+a}}dx}{3c}+\frac{\sqrt{a+cx^2}(g+hx)^2(4h^2(5cd-4af}{3c}\right.}{5ch}$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$\Big\downarrow$ 676

$$\frac{1}{4}\left(\frac{\frac{15}{2}h\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)\int \frac{1}{\sqrt{cx^2+a}}dx+\frac{2\sqrt{a+cx^2}\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2\left(3fg^4-5g^2h(16dh+3eg\right.\right.}{c}}{3c}\right.$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$\Big\downarrow$ 224

$$\frac{1}{4}\left(\frac{\frac{15}{2}h\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+\frac{2\sqrt{a+cx^2}\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2\left(3fg^4-5g^2h(\right.\right.}{c}}{3c}\right.$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

$\Big\downarrow$ 219

3.101. $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

$$\frac{1}{4}\left(\frac{\frac{15h\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)}{2\sqrt{c}}+\frac{2\sqrt{a+cx^2}\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2\left(3fg^4-5g^2h(16dh+\right.\right.}{c}}{3c}\right.$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^4}{5ch}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(f*(g + h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + (-1/4*((f*g - 5*e*h)*(g + h*x)^3
*Sqrt[a + c*x^2]) + (((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h
*x)^2*Sqrt[a + c*x^2])/(3*c) + ((2*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2 + 5
*h*(3*e*g + d*h)) - c^2*(3*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)))*Sqrt[a + c*x
^2])/c - (h*(6*c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*
h))*x*Sqrt[a + c*x^2])/2 + (15*h*(8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) -
4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(
2*Sqrt[c]))/(3*c))/4)/(5*c*h)
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.101.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.87

method	result
risch	$\frac{(24f\,h^3c^2x^4+30c^2e\,h^3x^3+90c^2fg\,h^2x^3-32acf\,h^3x^2+40c^2d\,h^3x^2+120c^2eg\,h^2x^2+120c^2f\,g^2h\,x^2-45ace\,h^3x-135acfg\,h^2x+180c^2d}{120c^3}$
default	$\frac{d\,g^3\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+f\,h^3\left(\frac{x^4\sqrt{c\,x^2+a}}{5c}-\frac{4a\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}-\frac{2a\sqrt{c\,x^2+a}}{3c^2}\right)}{5c}\right)+(e\,h^3+3fg\,h^2)\left(\frac{x^3\sqrt{c\,x^2+a}}{4c}-\right.$

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*c^2*f*h^3*x^4+30*c^2*e*h^3*x^3+90*c^2*f*g*h^2*x^3-32*a*c*f*h^3*x
^2+40*c^2*d*h^3*x^2+120*c^2*e*g*h^2*x^2+120*c^2*f*g^2*h*x^2-45*a*c*e*h^3*x
-135*a*c*f*g*h^2*x+180*c^2*d*g*h^2*x+180*c^2*e*g^2*h*x+60*c^2*f*g^3*x+64*a
^2*f*h^3-80*a*c*d*h^3-240*a*c*e*g*h^2-240*a*c*f*g^2*h+360*c^2*d*g^2*h+120*
c^2*e*g^3)/c^3*(c*x^2+a)^(1/2)+1/8/c^(5/2)*(3*a^2*e*h^3+9*a^2*f*g*h^2-12*a
*c*d*g*h^2-12*a*c*e*g^2*h-4*a*c*f*g^3+8*c^2*d*g^3)*ln(x*c^(1/2)+(c*x^2+a)^
(1/2))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.72

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$

$$= \left[-\frac{15\,(12\,aceg^2h-3\,a^2eh^3-4\,(2\,c^2d-acf)g^3+3\,(4\,acd-3\,a^2f)gh^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x\right.}{}\right.$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4
*a*c*d - 3*a^2*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^2 + 120*(3*c^
2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*f*g*h^2 + c^
2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d - 4*a*c*f)*h^
3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c^2*d - 3*a
*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/120*(15*(12*a*c*e*g^2*h - 3*a^2*e*
h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4*a*c*d - 3*a^2*f)*g*h^2)*sqrt(-c)*arct
an(sqrt(-c)*x/sqrt(c*x^2 + a)) + (24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a
*c*e*g*h^2 + 120*(3*c^2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 +
30*(3*c^2*f*g*h^2 + c^2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 +
(5*c^2*d - 4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*
h^3 + 3*(4*c^2*d - 3*a*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.21

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^3x^4}{5c} + \frac{x^3(eh^3+3fgh^2)}{4c} + \frac{x^2\left(-\frac{4afh^3}{5c}+dh^3+3egh^2+3fg^2h\right)}{3c} + \frac{x\left(-\frac{3a\left(eh^3+3fgh^2\right)}{4c}+3dgh^2+3eg^2h+fg^3\right)}{2c} + \frac{-\frac{2a(}{}}{} \right. \\ \frac{dg^3x+\frac{fh^3x^6}{6}+\frac{x^5\left(eh^3+3fgh^2\right)}{5}+\frac{x^4\left(dh^3+3egh^2+3fg^2h\right)}{4}+\frac{x^3\cdot\left(3dgh^2+3eg^2h+fg^3\right)}{3}+\frac{x^2\cdot\left(3dg^2h+eg^3\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h**3*x**4/(5*c) + x**3*(e*h**3 + 3*f*g*h**2
)/(4*c) + x**2*(-4*a*f*h**3/(5*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c
) + x*(-3*a*(e*h**3 + 3*f*g*h**2)/(4*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3
)/(2*c) + (-2*a*(-4*a*f*h**3/(5*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*
c) + 3*d*g**2*h + e*g**3)/c) + (-a*(-3*a*(e*h**3 + 3*f*g*h**2)/(4*c) + 3*d
*g*h**2 + 3*e*g**2*h + f*g**3)/(2*c) + d*g**3)*Piecewise((log(2*sqrt(c)*sq
rt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True))
, Ne(c, 0)), ((d*g**3*x + f*h**3*x**6/6 + x**5*(e*h**3 + 3*f*g*h**2)/5 + x
**4*(d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/4 + x**3*(3*d*g*h**2 + 3*e*g**2*h +
 f*g**3)/3 + x**2*(3*d*g**2*h + e*g**3)/2)/sqrt(a), True))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}fh^3x^4}{5\,c} - \frac{4\sqrt{cx^2+a}afh^3x^2}{15\,c^2} + \frac{dg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+a}eg^3}{c} + \frac{3\sqrt{cx^2+a}dg^2h}{c} + \frac{8\sqrt{cx^2+a}a^2fh^3}{15\,c^3} + \frac{(3\,fgh^2+eh^3)\sqrt{cx^2+a}x^3}{4\,c} + \frac{(3\,fg^2h+3\,egh^2+dh^3)\sqrt{cx^2+a}x^2}{3\,c} - \frac{3\,(3\,fgh^2+eh^3)\sqrt{cx^2+a}ax}{8\,c^2} + \frac{(fg^3+3\,eg^2h+3\,dgh^2)\sqrt{cx^2+a}x}{2\,c} + \frac{3\,(3\,fgh^2+eh^3)a^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}} - \frac{(fg^3+3\,eg^2h+3\,dgh^2)a\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}} - \frac{2\,(3\,fg^2h+3\,egh^2+dh^3)\sqrt{cx^2+a}a}{3\,c^2}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(c*x^2 + a)*f*h^3*x^4/c - 4/15*sqrt(c*x^2 + a)*a*f*h^3*x^2/c^2 + d
*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c*x^2 + a)*e*g^3/c + 3*sqrt(c*x
^2 + a)*d*g^2*h/c + 8/15*sqrt(c*x^2 + a)*a^2*f*h^3/c^3 + 1/4*(3*f*g*h^2 +
e*h^3)*sqrt(c*x^2 + a)*x^3/c + 1/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*
x^2 + a)*x^2/c - 3/8*(3*f*g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a*x/c^2 + 1/2*(f*
g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*x/c + 3/8*(3*f*g*h^2 + e*h^3)
*a^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/2*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*
a*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqr
t(c*x^2 + a)*a/c^2
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4fh^3x}{c}+\frac{5(3c^4fgh^2+c^4eh^3)}{c^5}\right)x+\frac{4(15c^4fg^2h+15c^4egh^2+5c^4dh^3-4ac^3fh\right.\right.\right.\right.$$
$$-\frac{(8c^2dg^3-4acfg^3-12aceg^2h-12acdgh^2+9a^2fgh^2+3a^2eh^3)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*e*h^3)/
c^5)*x + 4*(15*c^4*f*g^2*h + 15*c^4*e*g*h^2 + 5*c^4*d*h^3 - 4*a*c^3*f*h^3)
/c^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*e*g^2*h + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*
h^2 - 3*a*c^3*e*h^3)/c^5)*x + 8*(15*c^4*e*g^3 + 45*c^4*d*g^2*h - 30*a*c^3*
f*g^2*h - 30*a*c^3*e*g*h^2 - 10*a*c^3*d*h^3 + 8*a^2*c^2*f*h^3)/c^5) - 1/8*
(8*c^2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*e*g^2*h - 12*a*c*d*g*h^2 + 9*a^2*f*g*h
^2 + 3*a^2*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.102 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.102.1 Optimal result

Integrand size = 29, antiderivative size = 223

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch}+\frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{(4(4ah^2(2fg+eh)+cg(fg^2-4h(eg+3dh)))-h(3(4cd-3af)h^2-2cg(fg-4eh))x)\sqrt{a+cx^2}}{24c^2h} + \frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*c^2*d*g^2+3*a^2*f*h^2-4*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2
)/(c*x^2+a)^(1/2))/c^(5/2)-1/12*(-4*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h
+1/4*f*(h*x+g)^3*(c*x^2+a)^(1/2)/c/h-1/24*(16*a*h^2*(e*h+2*f*g)+4*c*g*(f*g
^2-4*h*(3*d*h+e*g))-h*(3*(-3*a*f+4*c*d)*h^2-2*c*g*(-4*e*h+f*g))*x)*(c*x^2+
a)^(1/2)/c^2/h
```

3.102.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a+cx^2}(-ah(32fg+16eh+9fhx)+2c(6dh(4g+hx)+4e(3g^2+3ghx+h^2x^2)+fx(6g^2+8ghx+3}{24c^2} - \frac{(8c^2dg^2+3a^2fh^2-4ac(fg^2+h(2eg+dh)))\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{8c^{5/2}}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(4*g + h
*x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2))
))/(24*c^2) - ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)
))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(8*c^(5/2))
```

3.102.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2185, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\sqrt{a+cx^2}} \, dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int \frac{h(g+hx)^2((4cd-3af)h-c(fg-4eh)x)}{\sqrt{cx^2+a}}dx}{4ch^2}+\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(g+hx)^2((4cd-3af)h-c(fg-4eh)x)}{\sqrt{cx^2+a}}dx}{4ch}+\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\frac{\int \frac{c(g+hx)\left(h(12cdg-7afg-8aeh)+\left(3(4cd-3af)h^2-2cg(fg-4eh)\right)x\right)}{\sqrt{cx^2+a}}dx}{3c}-\frac{1}{3}\sqrt{a+cx^2}(g+hx)^2(fg-4eh)}{4ch}+\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{3}\int \frac{(g+hx)\left(h(12cdg-7afg-8aeh)+\left(3(4cd-3af)h^2-2cg(fg-4eh)\right)x\right)}{\sqrt{cx^2+a}}dx-\frac{1}{3}\sqrt{a+cx^2}(g+hx)^2(fg-4eh)}{4ch}+\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{1}{3}\left(\frac{3h(3a^2fh^2-4ac(h(dh+2eg)+fg^2)+8c^2dg^2)\int\frac{1}{\sqrt{cx^2+a}}dx}{2c}-\frac{2\sqrt{a+cx^2}(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3)}{c}+\frac{hx\sqrt{a+cx^2}(3h^2(4cd-3a}{2c}\right.}{4ch}$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$\downarrow$ 224

$$\frac{\frac{1}{3}\left(\frac{3h(3a^2fh^2-4ac(h(dh+2eg)+fg^2)+8c^2dg^2)\int\frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{2c}-\frac{2\sqrt{a+cx^2}(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3)}{c}+\frac{hx\sqrt{a+cx^2}(3h^2}{}\right.}{4ch}$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

$\downarrow$ 219

$$\frac{\frac{1}{3}\left(\frac{3h\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3a^2fh^2-4ac(h(dh+2eg)+fg^2)+8c^2dg^2)}{2c^{3/2}}-\frac{2\sqrt{a+cx^2}(4ah^2(eh+2fg)-4cgh(3dh+eg)+cfg^3)}{c}+\frac{hx\sqrt{a+cx^2}(3h^2}{}\right.}{4ch}$$

$$\frac{f\sqrt{a+cx^2}(g+hx)^3}{4ch}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(f*(g + h*x)^3*Sqrt[a + c*x^2])/(4*c*h) + (-1/3*((f*g - 4*e*h)*(g + h*x)^2
*Sqrt[a + c*x^2]) + ((-2*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*f*g
 + e*h))*Sqrt[a + c*x^2])/c + (h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e
*h))*x*Sqrt[a + c*x^2])/(2*c) + (3*h*(8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f
*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
)/3)/(4*c*h)
```

3.102. $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.102. $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.102.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-6f\,h^2c\,x^3-8ce\,h^2x^2-16cfgh\,x^2+9xaf\,h^2-12d\,h^2xc-24eghxc-12f\,g^2xc+16ae\,h^2+32afgh-48cdgh-24ce\,g^2)\sqrt{c\,x^2+a}}{24c^2}+\frac{(3$
default	$\frac{d\,g^2\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+f\,h^2\left(\frac{x^3\sqrt{c\,x^2+a}}{4c}-\frac{3a\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+(e\,h^2+2fgh)\left(\frac{x^2\sqrt{c\,x^2+a}}{3c}\right.$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*c*f*h^2*x^3-8*c*e*h^2*x^2-16*c*f*g*h*x^2+9*a*f*h^2*x-12*c*d*h^2*
x-24*c*e*g*h*x-12*c*f*g^2*x+16*a*e*h^2+32*a*f*g*h-48*c*d*g*h-24*c*e*g^2)/c
^2*(c*x^2+a)^(1/2)+1/8*(3*a^2*f*h^2-4*a*c*d*h^2-8*a*c*e*g*h-4*a*c*f*g^2+8*
c^2*d*g^2)/c^(5/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.71

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$=\left[-\frac{3\,(8\,acegh-4\,(2\,c^2d-acf)g^2+(4\,acd-3\,a^2f)h^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(6\,c^2fh\right.$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2
)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^2*f*h^2
*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 8*(2*c^2
*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d - 3*a*c*
f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f
)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a
)) + (6*c^2*f*h^2*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*
f)*g*h + 8*(2*c^2*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h +
(4*c^2*d - 3*a*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.19

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fh^2x^3}{4c}+\frac{x^2(eh^2+2fgh)}{3c}+\frac{x\left(-\frac{3afh^2}{4c}+dh^2+2egh+fg^2\right)}{2c}+\frac{-\frac{2a(eh^2+2fgh)}{3c}+2dgh+eg^2}{c}\right)+\left(-\frac{a\left(-\frac{3afh^2}{4c}+dh^2+2\right.}{2c}\right. \\ \frac{dg^2x+\frac{fh^2x^5}{5}+\frac{x^4(eh^2+2fgh)}{4}+\frac{x^3(dh^2+2egh+fg^2)}{3}+\frac{x^2\cdot(2dgh+eg^2)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h**2*x**3/(4*c) + x**2*(e*h**2 + 2*f*g*h)/(
3*c) + x*(-3*a*f*h**2/(4*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) + (-2*a*(e*
h**2 + 2*f*g*h)/(3*c) + 2*d*g*h + e*g**2)/c) + (-a*(-3*a*f*h**2/(4*c) + d*
h**2 + 2*e*g*h + f*g**2)/(2*c) + d*g**2)*Piecewise((log(2*sqrt(c)*sqrt(a +
 c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**2), True)), Ne(c
, 0)), ((d*g**2*x + f*h**2*x**5/5 + x**4*(e*h**2 + 2*f*g*h)/4 + x**3*(d*h*
*2 + 2*e*g*h + f*g**2)/3 + x**2*(2*d*g*h + e*g**2)/2)/sqrt(a), True))
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}\,fh^2x^3}{4\,c} - \frac{3\,\sqrt{cx^2+a}\,afh^2x}{8\,c^2} + \frac{dg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} + \frac{3\,a^2fh^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}} + \frac{\sqrt{cx^2+a}\,eg^2}{c} + \frac{2\,\sqrt{cx^2+a}\,dgh}{c} + \frac{(2\,fgh+eh^2)\sqrt{cx^2+a}\,x^2}{3\,c} + \frac{(fg^2+2\,egh+dh^2)\sqrt{cx^2+a}\,x}{2\,c} - \frac{(fg^2+2\,egh+dh^2)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}} - \frac{2\,(2\,fgh+eh^2)\sqrt{cx^2+a}\,a}{3\,c^2}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^2 + a)*f*h^2*x^3/c - 3/8*sqrt(c*x^2 + a)*a*f*h^2*x/c^2 + d*g^
2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/8*a^2*f*h^2*arcsinh(c*x/sqrt(a*c))/c^
(5/2) + sqrt(c*x^2 + a)*e*g^2/c + 2*sqrt(c*x^2 + a)*d*g*h/c + 1/3*(2*f*g*h
 + e*h^2)*sqrt(c*x^2 + a)*x^2/c + 1/2*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2
 + a)*x/c - 1/2*(f*g^2 + 2*e*g*h + d*h^2)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2)
 - 2/3*(2*f*g*h + e*h^2)*sqrt(c*x^2 + a)*a/c^2
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3\,fh^2x}{c}+\frac{4\,(2\,c^3fgh+c^3eh^2)}{c^4}\right)x+\frac{3\,(4\,c^3fg^2+8\,c^3egh+4\,c^3dh^2-3\,ac^2fh^2)}{c^4}\right)x+\frac{8}{}\right.$$
$$-\frac{(8\,c^2dg^2-4\,acfg^2-8\,acegh-4\,acdh^2+3\,a^2fh^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*e*h^2)/c^4)*x
 + 3*(4*c^3*f*g^2 + 8*c^3*e*g*h + 4*c^3*d*h^2 - 3*a*c^2*f*h^2)/c^4)*x + 8*
(3*c^3*e*g^2 + 6*c^3*d*g*h - 4*a*c^2*f*g*h - 2*a*c^2*e*h^2)/c^4) - 1/8*(8*
c^2*d*g^2 - 4*a*c*f*g^2 - 8*a*c*e*g*h - 4*a*c*d*h^2 + 3*a^2*f*h^2)*log(abs
(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \int \frac{{(g+h\,x)}^2\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+a}}\,dx$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

3.103 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.103.1 Optimal result

Integrand size = 27, antiderivative size = 136

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch}-\frac{\left(2\left(2afh^2+c\left(fg^2-3h(eg+dh)\right)\right)+ch(fg-3eh)x\right)\sqrt{a+cx^2}}{6c^2h}+\frac{(2cdg-a(fg+eh))\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

output
```
1/2*(2*c*d*g-a*(e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/3*f
*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h-1/6*(4*a*f*h^2+2*c*(f*g^2-3*h*(d*h+e*g))+c*
h*(-3*e*h+f*g)*x)*(c*x^2+a)^(1/2)/c^2/h
```

3.103.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.71

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a+cx^2}\left(-4afh+c\left(6eg+6dh+3fgx+3ehx+2fhx^2\right)\right)+3\sqrt{c}(-2cdg+afg+aeh)\log\left(-\sqrt{c}x+\sqrt{a}\right.}{6c^2}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*
x^2)) + 3*Sqrt[c]*(-2*c*d*g + a*f*g + a*e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2]])/(6*c^2)
```

3.103.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{h(g+hx)((3cd-2af)h-c(fg-3eh)x)}{\sqrt{cx^2+a}}dx}{3ch^2}+\frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(g+hx)((3cd-2af)h-c(fg-3eh)x)}{\sqrt{cx^2+a}}dx}{3ch}+\frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{3}{2}h(-aeh-afg+2cdg)\int \frac{1}{\sqrt{cx^2+a}}dx-\frac{\sqrt{a+cx^2}\left(2afh^2-3ch(dh+eg)+cfg^2\right)}{c}-\frac{1}{2}hx\sqrt{a+cx^2}(fg-3eh)}{3ch}+\frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{2}h(-aeh-afg+2cdg)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}-\frac{\sqrt{a+cx^2}\left(2afh^2-3ch(dh+eg)+cfg^2\right)}{c}-\frac{1}{2}hx\sqrt{a+cx^2}(fg-3eh)}{3ch}+\frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3h\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(-aeh-afg+2cdg)}{2\sqrt{c}} - \frac{\sqrt{a+cx^2}(2afh^2-3ch(dh+eg)+cfg^2)}{c} - \frac{1}{2}hx\sqrt{a+cx^2}(fg-3eh)}{3ch} + \frac{f\sqrt{a+cx^2}(g+hx)^2}{3ch}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

output
```
(f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) + (-(((c*f*g^2 + 2*a*f*h^2 - 3*c*h
*(e*g + d*h))*Sqrt[a + c*x^2])/c) - (h*(f*g - 3*e*h)*x*Sqrt[a + c*x^2])/2
+ (3*h*(2*c*d*g - a*f*g - a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*
Sqrt[c]))/(3*c*h)
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.103.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{\left(-2 f h c \,x^{2}-3 e h x c -3 c f x g +4 a f h -6 c d h -6 c e g \right) \sqrt{c \,x^{2}+a}}{6 c^{2}}-\frac{\left(a e h +a f g -2 c d g \right) \ln \left(x \sqrt{c}+\sqrt{c \,x^{2}+a}\right)}{2 c^{\frac{3}{2}}}$
default	$\frac{d g \ln \left(x \sqrt{c}+\sqrt{c \,x^{2}+a}\right)}{\sqrt{c}}+f h \left(\frac{x^{2} \sqrt{c \,x^{2}+a}}{3 c}-\frac{2 a \sqrt{c \,x^{2}+a}}{3 c^{2}}\right)+\left(e h +f g \right) \left(\frac{x \sqrt{c \,x^{2}+a}}{2 c}-\frac{a \ln \left(x \sqrt{c}+\sqrt{c \,x^{2}+a}\right)}{2 c^{\frac{3}{2}}}\right)+\frac{(dh+$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*c*f*h*x^2-3*c*e*h*x-3*c*f*g*x+4*a*f*h-6*c*d*h-6*c*e*g)/c^2*(c*x^2
+a)^(1/2)-1/2/c^(3/2)*(a*e*h+a*f*g-2*c*d*g)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{3\left(aeh-(2\,cd-af)g\right)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)+2\left(2\,cfhx^2+6\,ceg+2\,(3\,cd-2\,af)h+\right.}{12\,c^2}\right.$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a
)*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d - 2*a*f)*h + 3*(c*f
*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt
(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*
d - 2*a*f)*h + 3*(c*f*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2]
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{fhx^2}{3c}+\frac{x(eh+fg)}{2c}+\frac{-\frac{2afh}{3c}+dh+eg}{c}\right)+\left(-\frac{a(eh+fg)}{2c}+dg\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \text{fo} \\ \frac{dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}}{\sqrt{a}} & \text{o} \end{cases}$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(f*h*x**2/(3*c) + x*(e*h + f*g)/(2*c) + (-2*a*
f*h/(3*c) + d*h + e*g)/c) + (-a*(e*h + f*g)/(2*c) + d*g)*Piecewise((log(2*
sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sqrt(c*x**
2), True)), Ne(c, 0)), ((d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d
*h + e*g)/2)/sqrt(a), True))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}fhx^2}{3\,c}+\frac{dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+a}eg}{c}+\frac{\sqrt{cx^2+a}dh}{c}-\frac{2\sqrt{cx^2+a}afh}{3\,c^2} + \frac{\sqrt{cx^2+a}(fg+eh)x}{2\,c}-\frac{(fg+eh)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(c*x^2 + a)*f*h*x^2/c + d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(
c*x^2 + a)*e*g/c + sqrt(c*x^2 + a)*d*h/c - 2/3*sqrt(c*x^2 + a)*a*f*h/c^2 +
 1/2*sqrt(c*x^2 + a)*(f*g + e*h)*x/c - 1/2*(f*g + e*h)*a*arcsinh(c*x/sqrt(
a*c))/c^(3/2)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fhx}{c}+\frac{3\,(c^2fg+c^2eh)}{c^3}\right)x+\frac{2\,(3\,c^2eg+3\,c^2dh-2\,acfh)}{c^3}\right) - \frac{(2\,cdg-afg-aeh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*e*h)/c^3)*x + 2*(3*c^2*
e*g + 3*c^2*d*h - 2*a*c*f*h)/c^3) - 1/2*(2*c*d*g - a*f*g - a*e*h)*log(abs(
-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.67

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx = \begin{cases} \frac{2\,f\,g\,x^3+3\,e\,g\,x^2+6\,d\,g\,x}{6\,\sqrt{a}}+\frac{3\,f\,h\,x^4+4\,e\,h\,x^3+6\,d\,h\,x^2}{12\,\sqrt{a}} \\ \frac{d\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{d\,h\,\sqrt{c\,x^2+a}}{c}+\frac{e\,g\,\sqrt{c\,x^2+a}}{c}+\frac{e\,h\,x\,\sqrt{c\,x^2+a}}{2\,c}+\frac{f\,g\,x\,\sqrt{c\,x^2+a}}{2\,c}-\frac{f\,h\,\sqrt{c\,x^2+a}\,(2\,a-c\,x^2)}{3\,c^2}-\frac{a\,e\,h\,\ln\left(2\,\sqrt{c}\right.}{2} \end{cases}$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

output
```
piecewise(c == 0, (3*e*g*x^2 + 2*f*g*x^3 + 6*d*g*x)/(6*a^(1/2)) + (6*d*h*x
^2 + 4*e*h*x^3 + 3*f*h*x^4)/(12*a^(1/2)), c ~= 0, (d*g*log(c^(1/2)*x + (a
+ c*x^2)^(1/2)))/c^(1/2) + (d*h*(a + c*x^2)^(1/2))/c + (e*g*(a + c*x^2)^(1
/2))/c + (e*h*x*(a + c*x^2)^(1/2))/(2*c) + (f*g*x*(a + c*x^2)^(1/2))/(2*c)
 - (f*h*(a + c*x^2)^(1/2)*(2*a - c*x^2))/(3*c^2) - (a*e*h*log(2*c^(1/2)*x
+ 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) - (a*f*g*log(2*c^(1/2)*x + 2*(a + c*x^
2)^(1/2)))/(2*c^(3/2)))
```

3.103. $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+cx^2}}\,dx$

3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

3.104.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}$$

output
```
1/2*(-a*f+2*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+e*(c*x^2+a)^(1
/2)/c+1/2*f*x*(c*x^2+a)^(1/2)/c
```

3.104.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{(2e+fx)\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

output
```
((2*e + f*x)*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/(
-Sqrt[a] + Sqrt[a + c*x^2])])/c^(3/2)
```

3.104.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2346, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{2cd-af+2cex}{\sqrt{cx^2+a}}dx}{2c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

$$\Big\downarrow \text{455}$$

$$\frac{(2cd-af)\int \frac{1}{\sqrt{cx^2+a}}dx+2e\sqrt{a+cx^2}}{2c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2cd-af)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}+2e\sqrt{a+cx^2}}{2c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd-af)}{\sqrt{c}}+2e\sqrt{a+cx^2}}{2c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

output
```
(f*x*Sqrt[a + c*x^2])/(2*c) + (2*e*Sqrt[a + c*x^2] + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c])/(2*c)
```

3.104.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

3.104.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{(fx+2e)\sqrt{c\,x^2+a}}{2c}-\frac{(fa-2cd)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}$	52
default	$\frac{d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+f\left(\frac{x\sqrt{c\,x^2+a}}{2c}-\frac{a\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{e\sqrt{c\,x^2+a}}{c}$	77

input
```
int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(f*x+2*e)/c*(c*x^2+a)^(1/2)-1/2*(a*f-2*c*d)/c^(3/2)*ln(x*c^(1/2)+(c*x^
2+a)^(1/2))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$$
$$= \left[-\frac{(2\,cd-af)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(cfx+2\,ce)\sqrt{cx^2+a}}{4\,c^2}, -\frac{(2\,cd-af)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(cfx+2\,ce)\sqrt{cx^2+a}}{2\,c^2}\right]$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x -
a) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*
arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+cx^2}\left(\frac{e}{c}+\frac{fx}{2c}\right)+\left(-\frac{af}{2c}+d\right)\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{a+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right) & \text{for } c\neq 0\\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**2)*(e/c + f*x/(2*c)) + (-a*f/(2*c) + d)*Piecewise
((log(2*sqrt(c)*sqrt(a + c*x**2) + 2*c*x)/sqrt(c), Ne(a, 0)), (x*log(x)/sq
rt(c*x**2), True)), Ne(c, 0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True)
)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{\sqrt{cx^2+a}fx}{2c} + \frac{d\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}} - \frac{af\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{3}{2}}} + \frac{\sqrt{cx^2+a}e}{c}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + a)*f*x/c + d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/2*a*f*arc
sinh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2 + a)*e/c
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2e}{c}\right) - \frac{(2cd-af)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
 + sqrt(c*x^2 + a)))/c^(3/2)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx = \begin{cases} \frac{2fx^3+3ex^2+6dx}{6\sqrt{a}} & \text{if } c=0 \\ \frac{e\sqrt{cx^2+a}}{c} + \frac{d\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)}{\sqrt{c}} - \frac{af\ln\left(2\sqrt{c}x+2\sqrt{cx^2+a}\right)}{2c^{3/2}} + \frac{fx\sqrt{cx^2+a}}{2c} & \text{if } c\neq 0 \end{cases}$$

input
```
int((d + e*x + f*x^2)/(a + c*x^2)^(1/2),x)
```

output
```
piecewise(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^(1/2)), c ~= 0, (e*(a +
 c*x^2)^(1/2))/c + (d*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(1/2) - (a*f*l
og(2*c^(1/2)*x + 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) + (f*x*(a + c*x^2)^(1/2
))/(2*c))
```

3.104. $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

3.105.1 Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=\frac{f\sqrt{a+cx^2}}{ch}-\frac{(fg-eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}-\frac{(fg^2-egh+dh^2)\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}$$

output
```
-(-e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*
g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+
c*g^2)^(1/2)+f*(c*x^2+a)^(1/2)/c/h
```

3.105.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx=\frac{\frac{fh\sqrt{a+cx^2}}{c}-\frac{2(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}}+\frac{(fg-eh)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

output
```
((f*h*Sqrt[a + c*x^2])/c - (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(
g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/Sqrt[-(c*g^2) - a*h
^2] + ((f*g - e*h)*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2
```

3.105.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}(g+hx)}dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int \frac{ch(dh-(fg-eh)x)}{(g+hx)\sqrt{cx^2+a}}dx}{ch^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{dh-(fg-eh)x}{(g+hx)\sqrt{cx^2+a}}dx}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\frac{(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{(fg-eh)\int \frac{1}{\sqrt{cx^2+a}}dx}{h}}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{(fg-eh)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h}}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h}}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{-\frac{(dh^2-egh+fg^2)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{h}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h}}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{(dh^2-egh+fg^2)\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h\sqrt{ah^2+cg^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h}}{h}+\frac{f\sqrt{a+cx^2}}{ch}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

output
```
(f*Sqrt[a + c*x^2])/(c*h) + (-(((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c
*x^2]])/(Sqrt[c]*h)) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqr
t[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h*Sqrt[c*g^2 + a*h^2]))/h
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.105.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

method	result
risch	$\frac{f\sqrt{c\,x^2+a}}{c h}+\frac{\frac{(eh-fg)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{h\sqrt{c}}-\frac{\left(d\,h^2-egh+f\,g^2\right)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2+}{h^2}}}{x+\frac{g}{h}}}{h^2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}}}{h}$
default	$\frac{\frac{eh\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{fh\sqrt{c\,x^2+a}}{c}-\frac{fg\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}}{h^2}-\frac{\left(d\,h^2-egh+f\,g^2\right)\ln\left(\frac{\frac{2a\,h^2+2c\,g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2}}{x+\frac{g}{h}}}{h^3\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*(c*x^2+a)^(1/2)/c/h+1/h*((e*h-f*g)/h*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/
2)-(d*h^2-e*g*h+f*g^2)/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h
^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 114.78 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.78

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$$

$$= \left[-\frac{(cfg^3-ceg^2h+afgh^2-aeh^3)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-(cfg^2-cegh+cdh^2)\sqrt{cg^2+}}{2\,(c^2g^2h^2+}\right.$$

$$-\frac{2\,(cfg^2-cegh+cdh^2)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right)+(cfg^3-ceg^2h+afgh^2-aeh}{2\,(c^2g^2h^2+ach^4)}$$

$$-\frac{(cfg^2-cegh+cdh^2)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+a}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^2}\right)-(cfg^3-ceg^2h+afgh^2-aeh^3}{c^2g^2h^2+ach^4}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((c*f*g^3 - c*e*g^2*h + a*f*g*h^2 - a*e*h^3)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - (c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt(c*g^
2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*
x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*
h*x + g^2)) - 2*(c*f*g^2*h + a*f*h^3)*sqrt(c*x^2 + a))/(c^2*g^2*h^2 + a*c*
h^4), -1/2*(2*(c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt(-c*g^2 - a*h^2)*arctan(sq
rt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2
*g^2 + a*c*h^2)*x^2)) + (c*f*g^3 - c*e*g^2*h + a*f*g*h^2 - a*e*h^3)*sqrt(c
)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(c*f*g^2*h + a*f*h^3
)*sqrt(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4), 1/2*(2*(c*f*g^3 - c*e*g^2*h +
a*f*g*h^2 - a*e*h^3)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c*f*g^
2 - c*e*g*h + c*d*h^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*
a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*
sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(c*f*g^2*h + a*f*h^3)*sqrt
(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4), -((c*f*g^2 - c*e*g*h + c*d*h^2)*sqrt
(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)
/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (c*f*g^3 - c*e*g^2*h + a
*f*g*h^2 - a*e*h^3)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*g^2
*h + a*f*h^3)*sqrt(c*x^2 + a))/(c^2*g^2*h^2 + a*c*h^4)]
```

3.105.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = &-\frac{fg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h^2}+\frac{e\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h}\\&+\frac{fg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3}\\&-\frac{eg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^2}\\&+\frac{d\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h}+\frac{\sqrt{cx^2+a}f}{ch}\end{aligned}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + e*arcsinh(c*x/sqrt(a*c))/(sqrt
(c)*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^2) + d*arc
sinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(
a + c*g^2/h^2)*h) + sqrt(c*x^2 + a)*f/(c*h)
```

3.105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)), x)
```

3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

3.106.1 Optimal result

Integrand size = 29, antiderivative size = 168

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{h(cg^2+ah^2)(g+hx)}+\frac{f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2} + \frac{(ah^2(2fg-eh)+c(fg^3-dgh^2))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{h^2(cg^2+ah^2)^{3/2}}$$

output
```
(a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)
^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*g^2)^(3/2)+f*arctanh(x*c^(1/2)/(c*x^2
+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)
/(h*x+g)
```

3.106.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+cx^2}}{(cg^2+ah^2)(g+hx)}+\frac{2(ah^2(2fg-eh)+c(fg^3-dgh^2))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{3/2}}+\frac{f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{\sqrt{c}}}{h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*
x)) + (2*(a*h^2*(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*ArcTan[(Sqrt[c]*(g +
h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)^(3/2
) + (f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/Sqrt[c])/h^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2182, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}(g+hx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{cdg-afg+aeh+f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{cx^2+a}}dx}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(aeh-2afg+cdg-\frac{cfg^3}{h^2}\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx+\frac{f\left(ah^2+cg^2\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{h^2}}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\left(aeh-2afg+cdg-\frac{cfg^3}{h^2}\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx+\frac{f\left(ah^2+cg^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{h^2}}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\left(aeh-2afg+cdg-\frac{cfg^3}{h^2}\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx+\frac{f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)}{\sqrt{c}h^2}}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 488

$$\frac{\frac{f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)}{\sqrt{c}h^2}-\left(aeh-2afg+cdg-\frac{cfg^3}{h^2}\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(ah^2+cg^2\right)}{\sqrt{c}h^2}-\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(aeh-2afg+cdg-\frac{cfg^3}{h^2}\right)}{\sqrt{ah^2+cg^2}}}{ah^2+cg^2}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

output
```
-(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
 + ((f*(c*g^2 + a*h^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2)
 - ((c*d*g - 2*a*f*g - (c*f*g^3)/h^2 + a*e*h)*ArcTanh[(a*h - c*g*x)/(Sqrt[
c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2])/(c*g^2 + a*h^2)
```

3.106. $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(154) = 308$.

Time = 0.60 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.32

method	result
default	$\frac{f\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{h^{2}\sqrt{c}}-\frac{(eh-2fg)\ln\left(\frac{\frac{2a\,h^{2}+2c\,g^{2}}{h^{2}}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}\sqrt{\left(x+\frac{g}{h}\right)^{2}c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}{x+\frac{g}{h}}\right)}{h^{3}\sqrt{\frac{a\,h^{2}+c\,g^{2}}{h^{2}}}}+\frac{(d\,h^{2}-egh+}{}$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g)/((a*h^2+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h
^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/
h*g))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)
^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^
2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*
g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))
```

3.106.5 Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.106.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)^2}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

3.106.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(155) = 310.

Time = 0.23 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.49

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = & -\frac{\sqrt{cx^2+a}fg^2}{cg^2h^2x+ah^4x+cg^3h+agh^3}+\frac{\sqrt{cx^2+a}eg}{cg^2hx+ah^3x+cg^3+agh^2} \\ & -\frac{\sqrt{cx^2+a}d}{cg^2x+ah^2x+\frac{cg^3}{h}+agh}+\frac{f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h^2} \\ & +\frac{cfg^3\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5} \\ & -\frac{ceg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^4} \\ & +\frac{cdg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} \\ & -\frac{2\,fg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3} \\ & +\frac{e\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^2}\end{aligned}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + sqrt(
c*x^2 + a)*e*g/(c*g^2*h*x + a*h^3*x + c*g^3 + a*g*h^2) - sqrt(c*x^2 + a)*d
/(c*g^2*x + a*h^2*x + c*g^3/h + a*g*h) + f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)
*h^2) + c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) - c*e*g^2*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4
) + c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(3/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + e*arcs
inh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a
 + c*g^2/h^2)*h^2)
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

3.107.1 Optimal result

Integrand size = 29, antiderivative size = 225

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2h(cg^2+ah^2)(g+hx)^2}+\frac{(2ah^2(2fg-eh)+cg(fg^2+h(eg-3dh)))\sqrt{a+cx^2}}{2h(cg^2+ah^2)^2(g+hx)} - \frac{(2c^2dg^2+2a^2fh^2-ac(fg^2-h(3eg-dh)))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2(cg^2+ah^2)^{5/2}}$$

output
```
-1/2*(2*c^2*d*g^2+2*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+3*e*g)))*arctanh((-c*g*x+
a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)-1/2*(d*h^2-e
*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h*x+g)^2+1/2*(2*a*h^2*(-e*h+2
*f*g)+c*g*(f*g^2+h*(-3*d*h+e*g)))*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)^2/(h*x+g
)
```

3.107.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.90

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = \frac{\sqrt{a+cx^2}(cg(fg^2x+eg(2g+hx)-dh(4g+3hx))-ah(-fg(3g+4hx)+h(dh+e(g+2hx))))}{2\left(cg^2+ah^2\right)^2(g+hx)^2} - \frac{\left(2c^2dg^2+2a^2fh^2-ac(fg^2+h(-3eg+dh))\right)\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\left(-cg^2-ah^2\right)^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

output
```
(Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*(4*g + 3*h*x)) - a*
h*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))))/(2*(c*g^2 + a*h^2)^2*
(g + h*x)^2) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)
))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])
/(-(c*g^2) - a*h^2)^(5/2)
```

3.107.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}(g+hx)^3}\,dx$$

$$\Big\downarrow \; 2182$$

$$-\frac{\int -\frac{2(cdg-afg+aeh)+\left(2afh+c\left(\frac{fg^2}{h}+eg-dh\right)\right)x}{(g+hx)^2\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2(cdg-afg+aeh)+\left(2afh+c\left(\frac{fg^2}{h}+eg-dh\right)\right)x}{(g+hx)^2\sqrt{cx^2+a}}dx}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{ah^2+cg^2}+\frac{\sqrt{a+cx^2}\left(2ah^2(2fg-eh)+cgh(eg-3dh)+cfg^3\right)}{h(g+hx)\left(ah^2+cg^2\right)}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{\sqrt{a+cx^2}\left(2ah^2(2fg-eh)+cgh(eg-3dh)+cfg^3\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{ah^2+cg^2}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\sqrt{a+cx^2}\left(2ah^2(2fg-eh)+cgh(eg-3dh)+cfg^3\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{\left(ah^2+cg^2\right)^{3/2}}}{2\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

output
```
-1/2*((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x
)^2) + (((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a +
c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*
(f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt
[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2))/(2*(c*g^2 + a*h^2))
```

3.107. $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.107. $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(209) = 418$.

Time = 0.64 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.60

method	result
default	$-\frac{f \ln\left(\frac{\frac{2a h^2+2c g^2}{h^2}-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a h^2+c g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a h^2+c g^2}{h^2}}}{x+\frac{g}{h}}\right)}{h^3\sqrt{\frac{a h^2+c g^2}{h^2}}}+\frac{(eh-2fg)\left(-\frac{h^2\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+a h \cdots}}{\left(a h^2+c g^2\right)\left(x+\frac{g}{h}\right)} \cdots\right.}{\cdots}$

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)
+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*(
(x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^
2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1
/h*g)^2*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3/2*c*g*
h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c-2*c*g/h*(x+
1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-c*g*h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*
((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2
*c/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c
*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(210) = 420.

Time = 4.71 (sec) , antiderivative size = 1088, normalized size of antiderivative = 4.84

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$$
$$=\left[\frac{(3\,aceg^3h+(2\,c^2d-acf)g^4-(acd-2\,a^2f)g^2h^2+(3\,acegh^3+(2\,c^2d-acf)g^2h^2-(acd-2\,a^2f)h^4)x^2}{}\right.$$
$$-\frac{(3\,aceg^3h+(2\,c^2d-acf)g^4-(acd-2\,a^2f)g^2h^2+(3\,acegh^3+(2\,c^2d-acf)g^2h^2-(acd-2\,a^2f)h^4)x}{}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 +
 (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 +
 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x
)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2
+ a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2
*x^2 + 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^
2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f
*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3
*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g
^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 +
3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*
g^3*h^5 + a^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a
*c*d - 2*a^2*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*
c*d - 2*a^2*f)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (
a*c*d - 2*a^2*f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2
)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x
^2)) - (2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d -
 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h -
 a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*
a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*...
```

3.107.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = \int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,(g+hx)^3}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(210) = 420$.

Time = 0.24 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.98

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{3\sqrt{cx^2+a}\,cfg^3}{2\left(c^2g^4h^2x+2\,acg^2h^4x+a^2h^6x+c^2g^5h+2\,acg^3h^3+a^2gh^5\right)} + \frac{3\sqrt{cx^2+a}\,ceg^2}{2\left(c^2g^4hx+2\,acg^2h^3x+a^2h^5x+c^2g^5+2\,acg^3h^2+a^2gh^4\right)} - \frac{3\sqrt{cx^2+a}\,cdg}{2\left(c^2g^4x+2\,acg^2h^2x+a^2h^4x+\frac{c^2g^5}{h}+2\,acg^3h+a^2gh^3\right)} - \frac{\sqrt{cx^2+a}\,fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4x+cg^4h+ag^2h^3\right)} + \frac{\sqrt{cx^2+a}\,eg}{2\left(cg^2h^2x^2+ah^4x^2+2\,cg^3hx+2\,agh^3x+cg^4+ag^2h^2\right)} + \frac{2\sqrt{cx^2+a}\,fg}{cg^2h^2x+ah^4x+cg^3h+agh^3} - \frac{\sqrt{cx^2+a}\,d}{2\left(cg^2hx^2+ah^3x^2+2\,cg^3x+2\,agh^2x+\frac{cg^4}{h}+ag^2h\right)} - \frac{\sqrt{cx^2+a}\,e}{cg^2hx+ah^3x+cg^3+agh^2} + \frac{3\,c^2fg^4\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^7} - \frac{3\,c^2eg^3\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^6} + \frac{3\,c^2dg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^5} - \frac{5\,cfg^2\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^5} + \frac{3\,ceg\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^4} - \frac{cd\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{2\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} + \frac{f\,\mathrm{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}\,h^3}$$

3.107. $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*sqrt(c*x^2 + a)*c*f*g^3/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x
+ c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) + 3/2*sqrt(c*x^2 + a)*c*e*g^2/(c^
2*g^4*h*x + 2*a*c*g^2*h^3*x + a^2*h^5*x + c^2*g^5 + 2*a*c*g^3*h^2 + a^2*g*
h^4) - 3/2*sqrt(c*x^2 + a)*c*d*g/(c^2*g^4*x + 2*a*c*g^2*h^2*x + a^2*h^4*x
+ c^2*g^5/h + 2*a*c*g^3*h + a^2*g*h^3) - 1/2*sqrt(c*x^2 + a)*f*g^2/(c*g^2*
h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) +
 1/2*sqrt(c*x^2 + a)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*
h^3*x + c*g^4 + a*g^2*h^2) + 2*sqrt(c*x^2 + a)*f*g/(c*g^2*h^2*x + a*h^4*x
+ c*g^3*h + a*g*h^3) - 1/2*sqrt(c*x^2 + a)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*
c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) - sqrt(c*x^2 + a)*e/(c*g^2*h*x
+ a*h^3*x + c*g^3 + a*g*h^2) + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(
h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 3/
2*c^2*e*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/((a + c*g^2/h^2)^(5/2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a
*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h
^5) - 5/2*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 3/2*c*e*g*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)
*h^4) - 1/2*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + f*arcsinh(c*g*x/(sqrt(a*c)*a...
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(210) = 420$.

Time = 0.30 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.73

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$$
$$= -\frac{(2\,c^2dg^2 - acfg^2 + 3\,acegh - acdh^2 + 2\,a^2fh^2)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{(c^2g^4 + 2\,acg^2h^2 + a^2h^4)\sqrt{-cg^2-ah^2}}$$
$$+ \frac{2\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 c^2fg^4h - 2\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 c^2dg^2h^3 + 5\left(\sqrt{c}x - \sqrt{cx^2+a}\right)^3 acfg^2h^3 - 3\left(\sqrt{c}x\right.}{}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(2*c^2*d*g^2 - a*c*f*g^2 + 3*a*c*e*g*h - a*c*d*h^2 + 2*a^2*f*h^2)*arctan(
((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*
g^4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*
h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 - 3*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 +
2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*c^(5/2)*e*g^4*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^
3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^2 - 5*(sqrt(c)
*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*e*g^2*h^3 + 3*(sqrt(c)*x - sqrt(c*x^2 +
a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*
g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*e*h^5 - 2*(sqrt(c)*x
 - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*
e*g^3*h^2 + 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^
2*c*e*g*h^4 + (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + a^2*c^(3/2)*f*g^
3*h^2 + a^2*c^(3/2)*e*g^2*h^3 - 3*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*
h^4 - 2*a^3*sqrt(c)*e*h^5)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt
(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g -
 a*h)^2)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)), x)
```

3.108 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 29, antiderivative size = 229

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h(4(3c^2dg^2+4a^2fh^2-ac(7fg^2+3h(3eg+dh)))+ch(6cdg-11afg-9aeh)x)\sqrt{a+cx^2}}{6ac^3} - \frac{(3ah^2(3fg+eh)-2cg(fg^2+3h(eg+dh)))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}$$

output
```
-1/2*(3*a*h^2*(e*h+3*f*g)-2*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(
c*x^2+a)^(1/2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^3/a/c/(c*x^2+a)^(1/2)-1
/3*(-4*a*f+3*c*d)*h*(h*x+g)^2*(c*x^2+a)^(1/2)/a/c^2-1/6*h*(12*c^2*d*g^2+16
*a^2*f*h^2-4*a*c*(7*f*g^2+3*h*(d*h+3*e*g))+c*h*(-9*a*e*h-11*a*f*g+6*c*d*g)
*x)*(c*x^2+a)^(1/2)/a/c^3
```

3.108.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \frac{-16a^3fh^3+6c^3dg^3x+ac^2(6dh(-3g^2-3ghx+h^2x^2)-3e(2g^3+6g^2hx$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
(-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
 - 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*
h*x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^
2) + 3*h*(4*d*h + 3*e*(4*g + h*x))) + 3*a*Sqrt[c]*(3*a*h^2*(3*f*g + e*h) -
 2*c*(f*g^3 + 3*g*h*(e*g + d*h)))*Sqrt[a + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2]])/(6*a*c^3*Sqrt[a + c*x^2])
```

3.108.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2176, 25, 687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{(g+hx)^2(a(fg+3eh)-(3cd-4af)hx)}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(g+hx)^2(a(fg+3eh)-(3cd-4af)hx)}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{\int \frac{(g+hx)\left(a\left(2(3cd-4af)h^2+3cg(fg+3eh)\right)-ch(6cdg-11afg-9aeh)x\right)}{\sqrt{cx^2+a}}dx}{3c}-\frac{h\sqrt{a+cx^2}(g+hx)^2(3cd-4af)}{3c}}{ac}-\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{\frac{3}{2}a\left(-3ah^2(eh+3fg)+6cgh(dh+eg)+2cfg^3\right)\int \frac{1}{\sqrt{cx^2+a}}dx-\frac{2h\sqrt{a+cx^2}\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)}{c}-\frac{1}{2}h^2x\sqrt{a+cx^2}(-9aeh-11afg+6c}{3c}}{ac}$$
$$\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{3}{2}a\left(-3ah^2(eh+3fg)+6cgh(dh+eg)+2cfg^3\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}-\frac{2h\sqrt{a+cx^2}\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)}{c}-\frac{1}{2}h^2x\sqrt{a+cx^2}(-9aeh-11}{3c}}{ac}$$
$$\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{-\frac{2h\sqrt{a+cx^2}\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)}{c}+\frac{3a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(-3ah^2(eh+3fg)+6cgh(dh+eg)+2cfg^3\right)}{2\sqrt{c}}-\frac{1}{2}h^2x\sqrt{a+cx^2}(-9aeh-11af}{3c}}{ac}$$
$$\frac{(g+hx)^3(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
-(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) + (-1/3*((3*c
*d - 4*a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/c + ((-2*h*(3*c^2*d*g^2 + 4*a^2
*f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h)))*Sqrt[a + c*x^2])/c - (h^2*(6*c
*d*g - 11*a*f*g - 9*a*e*h)*x*Sqrt[a + c*x^2])/2 + (3*a*(2*c*f*g^3 + 6*c*g*
h*(e*g + d*h) - 3*a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/(2*Sqrt[c]))/(3*c))/(a*c)
```

3.108. $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.108. $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx$

3.108.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{h(-2fh^2cx^2-3ceh^2x-9cfghx+10afh^2-6cdh^2-18cegh-18cfg^2)\sqrt{cx^2+a}}{6c^3}-\frac{\frac{aeh^3x}{\sqrt{cx^2+a}}-\frac{2c^2dg^3x}{a\sqrt{cx^2+a}}+\frac{3afgh^2x}{\sqrt{cx^2+a}}+(3aceh^3+9a$
default	$\frac{dg^3x}{a\sqrt{cx^2+a}}+fh^3\left(\frac{x^4}{3c\sqrt{cx^2+a}}-\frac{4a\left(\frac{x^2}{c\sqrt{cx^2+a}}+\frac{2a}{c^2\sqrt{cx^2+a}}\right)}{3c}\right)+(eh^3+3fgh^2)\left(\frac{x^3}{2c\sqrt{cx^2+a}}-\frac{3a\left(-\frac{x}{c\sqrt{cx^2+a}}+\right.}{}\right.$

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*h*(-2*c*f*h^2*x^2-3*c*e*h^2*x-9*c*f*g*h*x+10*a*f*h^2-6*c*d*h^2-18*c*e
*g*h-18*c*f*g^2)/c^3*(c*x^2+a)^(1/2)-1/2/c^2*(a*e*h^3*x/(c*x^2+a)^(1/2)-2*
c^2*d*g^3*x/a/(c*x^2+a)^(1/2)+3*a*f*g*h^2*x/(c*x^2+a)^(1/2)+(3*a*c*e*h^3+9
*a*c*f*g*h^2-6*c^2*d*g*h^2-6*c^2*e*g^2*h-2*c^2*f*g^3)*(-x/c/(c*x^2+a)^(1/2
)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(-2*a^2*f*h^3+2*a*c*d*h^3+6*a*c
*e*g*h^2+6*a*c*f*g^2*h-6*c^2*d*g^2*h-2*c^2*e*g^3)/c/(c*x^2+a)^(1/2))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 758, normalized size of antiderivative = 3.31

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \left[-\frac{3\,(2\,a^2cfg^3+6\,a^2ceg^2h-3\,a^3eh^3+3\,(2\,a^2cd-3\,a^3f)gh^2+(2\,ac^2fg^3\right.$$

$$-\frac{3\,(2\,a^2cfg^3+6\,a^2ceg^2h-3\,a^3eh^3+3\,(2\,a^2cd-3\,a^3f)gh^2+(2\,ac^2fg^3+6\,ac^2eg^2h-3\,a^2ceh^3+3\,(2\,ac^2d}{}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d -
3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a
*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*s
qrt(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 -
18*(a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*
f*g*h^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c
^2*d - 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d
 - a*c^2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*
c^4*x^2 + a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3
 + 3*(2*a^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^
2*c*e*h^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)
*x/sqrt(c*x^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^
2 - 18*(a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*
c^2*f*g*h^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3
*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c
^3*d - a*c^2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))
/(a*c^4*x^2 + a^2*c^3)]
```

3.108.6 Sympy [F]

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx=\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx &= \frac{fh^3x^4}{3\sqrt{cx^2+a}c} - \frac{4afh^3x^2}{3\sqrt{cx^2+a}c^2} \\ &+ \frac{dg^3x}{\sqrt{cx^2+a}a} - \frac{eg^3}{\sqrt{cx^2+a}c} - \frac{3dg^2h}{\sqrt{cx^2+a}c} - \frac{8a^2fh^3}{3\sqrt{cx^2+a}c^3} \\ &+ \frac{(3fgh^2+eh^3)x^3}{2\sqrt{cx^2+a}c} + \frac{(3fg^2h+3egh^2+dh^3)x^2}{\sqrt{cx^2+a}c} + \frac{3(3fgh^2+eh^3)ax}{2\sqrt{cx^2+a}c^2} \\ &- \frac{(fg^3+3eg^2h+3dgh^2)x}{\sqrt{cx^2+a}c} - \frac{3(3fgh^2+eh^3)a\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{5}{2}}} \\ &+ \frac{(fg^3+3eg^2h+3dgh^2)\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} + \frac{2(3fg^2h+3egh^2+dh^3)a}{\sqrt{cx^2+a}c^2}\end{aligned}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*f*h^3*x^4/(sqrt(c*x^2 + a)*c) - 4/3*a*f*h^3*x^2/(sqrt(c*x^2 + a)*c^2)
+ d*g^3*x/(sqrt(c*x^2 + a)*a) - e*g^3/(sqrt(c*x^2 + a)*c) - 3*d*g^2*h/(sqr
t(c*x^2 + a)*c) - 8/3*a^2*f*h^3/(sqrt(c*x^2 + a)*c^3) + 1/2*(3*f*g*h^2 + e
*h^3)*x^3/(sqrt(c*x^2 + a)*c) + (3*f*g^2*h + 3*e*g*h^2 + d*h^3)*x^2/(sqrt(
c*x^2 + a)*c) + 3/2*(3*f*g*h^2 + e*h^3)*a*x/(sqrt(c*x^2 + a)*c^2) - (f*g^3
 + 3*e*g^2*h + 3*d*g*h^2)*x/(sqrt(c*x^2 + a)*c) - 3/2*(3*f*g*h^2 + e*h^3)*
a*arcsinh(c*x/sqrt(a*c))/c^(5/2) + (f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*arcsinh
(c*x/sqrt(a*c))/c^(3/2) + 2*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*a/(sqrt(c*x^2
+ a)*c^2)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.45

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \frac{\left(\left(\left(\frac{2fh^3x}{c} + \frac{3\left(3ac^4fgh^2+ac^4eh^3\right)}{ac^5}\right)x + \frac{2\left(9ac^4fg^2h+9ac^4egh^2+3ac^4dh^3-4a^2c^3fh^3\right)}{ac^5}\right)x \cdots}{\cdots} - \frac{\left(2cfg^3+6ceg^2h+6cdgh^2-9afgh^2-3aeh^3\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*e*h^3)/(a*c^5))*x + 2*(9*
a*c^4*f*g^2*h + 9*a*c^4*e*g*h^2 + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3)/(a*c^5)
)*x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*e*g^2*h - 6*a*c^4*d*g*h^2 +
 9*a^2*c^3*f*g*h^2 + 3*a^2*c^3*e*h^3)/(a*c^5))*x - 2*(3*a*c^4*e*g^3 + 9*a*
c^4*d*g^2*h - 18*a^2*c^3*f*g^2*h - 18*a^2*c^3*e*g*h^2 - 6*a^2*c^3*d*h^3 +
8*a^3*c^2*f*h^3)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*e*g^2*h +
 6*c*d*g*h^2 - 9*a*f*g*h^2 - 3*a*e*h^3)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
a)))/c^(5/2)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{(c\,x^2+a)^{3/2}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.109 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

3.109.1 Optimal result

Integrand size = 29, antiderivative size = 149

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx=-\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}}-\frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx^2}}{2ac^2}+\frac{((2cd-3af)h^2+2cg(fg+2eh))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2c^{5/2}}$$

output
```
1/2*((-3*a*f+2*c*d)*h^2+2*c*g*(2*e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^2/a/c/(c*x^2+a)^(1/2)-1/2*h*(4*c*d*
g-4*a*(e*h+2*f*g)+(-3*a*f+2*c*d)*h*x)*(c*x^2+a)^(1/2)/a/c^2
```

3.109.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{c}\left(2c^2dg^2x+a^2h(8fg+4eh+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x\right.\right.\right.}{a\sqrt{a+cx^2}}}{2c^{5/2}}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
((Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(
2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x
^2))))/(a*Sqrt[a + c*x^2]) + (3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*L
og[-(Sqrt[c]*x) + Sqrt[a + c*x^2]])/(2*c^(5/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2176, 25, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2176}$$

$$-\frac{\int -\frac{(g+hx)(a(fg+2eh)-(2cd-3af)hx)}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(g+hx)(a(fg+2eh)-(2cd-3af)hx)}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{a\left(h^2(2cd-3af)+2cg(2eh+fg)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{2c}-\frac{2h\sqrt{a+cx^2}(-aeh-2afg+cdg)}{c}-\frac{h^2x\sqrt{a+cx^2}(2cd-3af)}{2c}}{ac}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{a\left(h^2(2cd-3af)+2cg(2eh+fg)\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{2c}-\frac{2h\sqrt{a+cx^2}(-aeh-2afg+cdg)}{c}-\frac{h^2x\sqrt{a+cx^2}(2cd-3af)}{2c}}{ac}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(h^2(2cd-3af)+2cg(2eh+fg))}{2c^{3/2}}-\frac{2h\sqrt{a+cx^2}(-aeh-2afg+cdg)}{c}-\frac{h^2x\sqrt{a+cx^2}(2cd-3af)}{2c}}{ac}-\frac{(g+hx)^2(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
-(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) + ((-2*h*(c*d
*g - 2*a*f*g - a*e*h)*Sqrt[a + c*x^2])/c - ((2*c*d - 3*a*f)*h^2*x*Sqrt[a +
 c*x^2])/(2*c) + (a*((2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(S
qrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2)))/(a*c)
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.109.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

method	result
risch	$\frac{h(fxh+2eh+4fg)\sqrt{cx^2+a}}{2c^2} - \frac{\frac{afh^2x}{\sqrt{cx^2+a}} - \frac{2c^2dg^2x}{a\sqrt{cx^2+a}} + (3acfh^2-2c^2dh^2-4c^2egh-2c^2fg^2)\left(-\frac{x}{c\sqrt{cx^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{c^{\frac{3}{2}}}\right) - \frac{2e}{}}{2c^2}$
default	$\frac{dg^2x}{a\sqrt{cx^2+a}} + fh^2\left(\frac{x^3}{2c\sqrt{cx^2+a}} - \frac{3a\left(-\frac{x}{c\sqrt{cx^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right) + (eh^2+2fgh)\left(\frac{x^2}{c\sqrt{cx^2+a}} + \frac{2a}{c^2\sqrt{cx^2+a}}\right.$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*h*(f*h*x+2*e*h+4*f*g)/c^2*(c*x^2+a)^(1/2)-1/2/c^2*(a*f*h^2*x/(c*x^2+a)
^(1/2)-2*c^2*d*g^2*x/a/(c*x^2+a)^(1/2)+(3*a*c*f*h^2-2*c^2*d*h^2-4*c^2*e*g*
h-2*c^2*f*g^2)*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2
)))-(2*a*c*e*h^2+4*a*c*f*g*h-4*c^2*d*g*h-2*c^2*e*g^2)/c/(c*x^2+a)^(1/2))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.56

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \left[-\frac{(2\,a^2cfg^2+4\,a^2cegh+(2\,a^2cd-3\,a^3f)h^2+(2\,ac^2fg^2+4\,ac^2egh+(2\right.$$

$$-\frac{(2\,a^2cfg^2+4\,a^2cegh+(2\,a^2cd-3\,a^3f)h^2+(2\,ac^2fg^2+4\,ac^2egh+(2\,ac^2d-3\,a^2cf)h^2)x^2)\sqrt{-c}\arctan\left(\right.}{}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c
^2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(c)*log(-
2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*
e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a
*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d -
3*a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*
f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c
^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sq
rt(c*x^2 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c
^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g
*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*sqrt(c*x^2
+ a))/(a*c^4*x^2 + a^2*c^3)]
```

3.109.6 Sympy [F]

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.52

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \frac{fh^2x^3}{2\sqrt{cx^2+a}c} + \frac{dg^2x}{\sqrt{cx^2+a}a} + \frac{3\,afh^2x}{2\sqrt{cx^2+a}c^2} - \frac{3\,afh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}} - \frac{eg^2}{\sqrt{cx^2+a}c} - \frac{2\,dgh}{\sqrt{cx^2+a}c} + \frac{(2\,fgh+eh^2)x^2}{\sqrt{cx^2+a}c} - \frac{(fg^2+2\,egh+dh^2)x}{\sqrt{cx^2+a}c} + \frac{(fg^2+2\,egh+dh^2)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} + \frac{2\,(2\,fgh+eh^2)a}{\sqrt{cx^2+a}c^2}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*f*h^2*x^3/(sqrt(c*x^2 + a)*c) + d*g^2*x/(sqrt(c*x^2 + a)*a) + 3/2*a*f*
h^2*x/(sqrt(c*x^2 + a)*c^2) - 3/2*a*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) -
 e*g^2/(sqrt(c*x^2 + a)*c) - 2*d*g*h/(sqrt(c*x^2 + a)*c) + (2*f*g*h + e*h^
2)*x^2/(sqrt(c*x^2 + a)*c) - (f*g^2 + 2*e*g*h + d*h^2)*x/(sqrt(c*x^2 + a)*
c) + (f*g^2 + 2*e*g*h + d*h^2)*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 2*(2*f*g*h
 + e*h^2)*a/(sqrt(c*x^2 + a)*c^2)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.44

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx=\frac{\left(\left(\frac{fh^2x}{c}+\frac{2\left(2\,ac^3fgh+ac^3eh^2\right)}{ac^4}\right)x+\frac{2\,c^4dg^2-2\,ac^3fg^2-4\,ac^3egh-2\,ac^3dh^2+3\,a^2c^2fh^2}{ac^4}\right)x-}{2\sqrt{cx^2+a}}$$
$$-\frac{\left(2\,cfg^2+4\,cegh+2\,cdh^2-3\,afh^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*e*h^2)/(a*c^4))*x + (2*c^4*d*g
^2 - 2*a*c^3*f*g^2 - 4*a*c^3*e*g*h - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2)/(a*c
^4))*x - 2*(a*c^3*e*g^2 + 2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h - 2*a^2*c^2*e*h^
2)/(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 4*c*e*g*h + 2*c*d*h^2 - 3*a
*f*h^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

3.110 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

3.110.1 Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

output
```
(e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)-(a*e-(-a*f+c*d)*x)*(h
*x+g)/a/c/(c*x^2+a)^(1/2)-(-2*a*f+c*d)*h*(c*x^2+a)^(1/2)/a/c^2
```

3.110.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{-aceg-acdh+2a^2fh+c^2dgx-acfgx-acehx+acfhx^2}{ac^2\sqrt{a+cx^2}} + \frac{(-fg-eh)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
(-(a*c*e*g) - a*c*d*h + 2*a^2*f*h + c^2*d*g*x - a*c*f*g*x - a*c*e*h*x + a*
c*f*h*x^2)/(a*c^2*Sqrt[a + c*x^2]) + ((-(f*g) - e*h)*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2]])/c^(3/2)
```

3.110.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2176, 25, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{(a+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 2176}$$

$$-\frac{\int -\frac{a(fg+eh)-(cd-2af)hx}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{a(fg+eh)-(cd-2af)hx}{\sqrt{cx^2+a}}dx}{ac}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{a(eh+fg)\int \frac{1}{\sqrt{cx^2+a}}dx-\frac{h\sqrt{a+cx^2}(cd-2af)}{c}}{ac}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{a(eh+fg)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}-\frac{h\sqrt{a+cx^2}(cd-2af)}{c}}{ac}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh+fg)}{\sqrt{c}}-\frac{h\sqrt{a+cx^2}(cd-2af)}{c}}{ac}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

output
```
-(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) + (-(((c*d - 2*
a*f)*h*Sqrt[a + c*x^2])/c) + (a*(f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c
*x^2]])/Sqrt[c])/(a*c)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.110.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{fh\sqrt{c\,x^2+a}}{c^2}+\frac{\frac{cdgx}{a\sqrt{c\,x^2+a}}+(ehc+cfg)\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{-afh+cdh+ceg}{c\sqrt{c\,x^2+a}}}{c}$	113
default	$\frac{dgx}{a\sqrt{c\,x^2+a}}+fh\left(\frac{x^2}{c\sqrt{c\,x^2+a}}+\frac{2a}{c^2\sqrt{c\,x^2+a}}\right)+(eh+fg)\left(-\frac{x}{c\sqrt{c\,x^2+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{dh+eg}{c\sqrt{c\,x^2+a}}$	118

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*f*h*(c*x^2+a)^(1/2)+1/c*(c*d*g*x/a/(c*x^2+a)^(1/2)+(c*e*h+c*f*g)*(-x
/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2)))-(-a*f*h+c*d*h+
c*e*g)/c/(c*x^2+a)^(1/2))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.78

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{(a^2fg+a^2eh+(acfg+aceh)x^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)}{2\,(ac^3x\right.$$

$$-\frac{(a^2fg+a^2eh+(acfg+aceh)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)-(acfhx^2-aceg-(acd-2\,a^2f)h-(aceh-(c}{ac^3x^2+a^2c^2}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(c)*log(-2*c*x^2 -
 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*
a^2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 +
a^2*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(-c)*arctan(
sqrt(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h
 - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)
]
```

3.110.6 Sympy [A] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.09

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = dh\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + eg\left(\begin{cases} -\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right) + fg\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}} - \frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right) + fh\left(\begin{cases} \frac{2a}{c^2\sqrt{a+cx^2}} + \frac{x^2}{c\sqrt{a+cx^2}} & \text{for } c\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{dgx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), Tru
e)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)
), True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
 c*x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(
1 + c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqr
t(a + c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sq
rt(1 + c*x**2/a))
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+cx^2)^{3/2}}\,dx = \frac{fhx^2}{\sqrt{cx^2+a}c} + \frac{dgx}{\sqrt{cx^2+a}a} - \frac{eg}{\sqrt{cx^2+a}c} - \frac{dh}{\sqrt{cx^2+a}c} + \frac{2\,afh}{\sqrt{cx^2+a}c^2} - \frac{(fg+eh)x}{\sqrt{cx^2+a}c} + \frac{(fg+eh)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
f*h*x^2/(sqrt(c*x^2 + a)*c) + d*g*x/(sqrt(c*x^2 + a)*a) - e*g/(sqrt(c*x^2
+ a)*c) - d*h/(sqrt(c*x^2 + a)*c) + 2*a*f*h/(sqrt(c*x^2 + a)*c^2) - (f*g +
 e*h)*x/(sqrt(c*x^2 + a)*c) + (f*g + e*h)*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{\left(\frac{fhx}{c}+\frac{c^3dg-ac^2fg-ac^2eh}{ac^3}\right)x-\frac{ac^2eg+ac^2dh-2\,a^2cfh}{ac^3}}{\sqrt{cx^2+a}} - \frac{(fg+eh)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*e*h)/(a*c^3))*x - (a*c^2*e*g + a*
c^2*d*h - 2*a^2*c*f*h)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + e*h)*log(abs(-sqr
t(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = \frac{e\,h\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} + \frac{f\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} - \frac{d\,h}{c\,\sqrt{c\,x^2+a}} - \frac{e\,g}{c\,\sqrt{c\,x^2+a}} + \frac{d\,g\,x}{a\,\sqrt{c\,x^2+a}} - \frac{e\,h\,x}{c\,\sqrt{c\,x^2+a}} - \frac{f\,g\,x}{c\,\sqrt{c\,x^2+a}} + \frac{f\,h\,\left(c\,x^2+2\,a\right)}{c^2\,\sqrt{c\,x^2+a}}$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

output
```
(e*h*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) + (f*g*log(c^(1/2)*x + (a
 + c*x^2)^(1/2)))/c^(3/2) - (d*h)/(c*(a + c*x^2)^(1/2)) - (e*g)/(c*(a + c*
x^2)^(1/2)) + (d*g*x)/(a*(a + c*x^2)^(1/2)) - (e*h*x)/(c*(a + c*x^2)^(1/2)
) - (f*g*x)/(c*(a + c*x^2)^(1/2)) + (f*h*(2*a + c*x^2))/(c^2*(a + c*x^2)^(
1/2))
```

3.111 $\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx$

3.111.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}$$

output
```
f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+(-a*e+(-a*f+c*d)*x)/a/c/(c*x^
2+a)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = \frac{-ae+cdx-afx}{ac\sqrt{a+cx^2}}-\frac{f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

output
```
(-(a*e) + c*d*x - a*f*x)/(a*c*Sqrt[a + c*x^2]) - (f*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2]])/c^(3/2)
```

3.111.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2345, 25, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{af}{c\sqrt{cx^2+a}}dx}{a}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{af}{c\sqrt{cx^2+a}}dx}{a}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\int \frac{1}{\sqrt{cx^2+a}}dx}{c}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{f\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{c}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{f\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

output
```
-((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sq
rt[a + c*x^2]])/c^(3/2)
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.111.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{dx}{a\sqrt{c\,x^{2}+a}}+f\left(-\frac{x}{c\sqrt{c\,x^{2}+a}}+\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{c^{\frac{3}{2}}}\right)-\frac{e}{c\sqrt{c\,x^{2}+a}}$	70

input
```
int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d*x/a/(c*x^2+a)^(1/2)+f*(-x/c/(c*x^2+a)^(1/2)+1/c^(3/2)*ln(x*c^(1/2)+(c*x^
2+a)^(1/2)))-e/c/(c*x^2+a)^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = \left[\frac{(acfx^2+a^2f)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\,(ace-(c^2d-acf)x)\sqrt{cx^2}}{2\,(ac^3x^2+a^2c^2)}\right.$$

$$\left.-\frac{(acfx^2+a^2f)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+(ace-(c^2d-acf)x)\sqrt{cx^2+a}}{ac^3x^2+a^2c^2}\right]$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)
*x - a) - 2*(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*
c^2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) +
(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

3.111.6 Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = e\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}c\sqrt{1+\frac{cx^2}{a}}}\right)+\frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

output
```
e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True)
) + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)
)) + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = \frac{dx}{\sqrt{cx^2+a}a} - \frac{fx}{\sqrt{cx^2+a}c} + \frac{f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{cx^2+a}c}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
d*x/(sqrt(c*x^2 + a)*a) - f*x/(sqrt(c*x^2 + a)*c) + f*arcsinh(c*x/sqrt(a*c
))/c^(3/2) - e/(sqrt(c*x^2 + a)*c)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = -\frac{\frac{e}{c} - \frac{(c^2d-acf)x}{ac^2}}{\sqrt{cx^2+a}} - \frac{f\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}} - \frac{e}{c\sqrt{c\,x^2+a}} + \frac{d\,x}{a\sqrt{c\,x^2+a}} - \frac{f\,x}{c\sqrt{c\,x^2+a}}$$

input
```
int((d + e*x + f*x^2)/(a + c*x^2)^(3/2),x)
```

output
```
(f*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) - e/(c*(a + c*x^2)^(1/2)) +
 (d*x)/(a*(a + c*x^2)^(1/2)) - (f*x)/(c*(a + c*x^2)^(1/2))
```

3.112 $\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx$

3.112.1 Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac(cg^2+ah^2)\sqrt{a+cx^2}} - \frac{(fg^2-egh+dh^2)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{(cg^2+ah^2)^{3/2}}$$

output
```
-(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1
/2))/(a*h^2+c*g^2)^(3/2)+(-a*(a*f*h-c*d*h+c*e*g)+c*(a*e*h-a*f*g+c*d*g)*x)/
a/c/(a*h^2+c*g^2)/(c*x^2+a)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx = \frac{-a^2fh+c^2dgx+ac(-eg+dh-fgx+ehx)}{ac(cg^2+ah^2)\sqrt{a+cx^2}} + \frac{2(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

output
```
(-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2
+ a*h^2)*Sqrt[a + c*x^2]) + (2*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*
(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/(-(c*g^2) - a*h^2)
^(3/2)
```

3.112.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2178, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}(g+hx)}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{ac(fg^2-ehg+dh^2)}{(cg^2+ah^2)(g+hx)\sqrt{cx^2+a}}dx}{ac}-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}(ah^2+cg^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ac(fg^2-ehg+dh^2)}{(cg^2+ah^2)(g+hx)\sqrt{cx^2+a}}dx}{ac}-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}(ah^2+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{ah^2+cg^2}-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}(ah^2+cg^2)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{(dh^2-egh+fg^2)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}}{ah^2+cg^2}-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}(ah^2+cg^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(dh^2-egh+fg^2)\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{(ah^2+cg^2)^{3/2}}-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}(ah^2+cg^2)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

output
```
-((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 +
a*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/
(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(129) = 258$.

Time = 0.51 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.80

method	result
default	$\frac{\frac{ehx}{a\sqrt{cx^2+a}}-\frac{fh}{c\sqrt{cx^2+a}}-\frac{fgx}{a\sqrt{cx^2+a}}}{h^2}+\frac{(dh^2-egh+fg^2)\left(\frac{h^2}{(ah^2+cg^2)\sqrt{\left(x+\frac{g}{h}\right)^2c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{ah^2+cg^2}{h^2}}}+\frac{2c\ldots}{(ah^2+cg^2)\left(\frac{4c(ah^2+cg^2)}{h^2}\ldots\right.}\ldots\right.}{\ldots}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/h^2*(e*h*x/a/(c*x^2+a)^(1/2)-f*h/c/(c*x^2+a)^(1/2)-f*g*x/a/(c*x^2+a)^(1/
2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x
+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*
g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1
/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(130) = 260$.

Time = 1.06 (sec) , antiderivative size = 721, normalized size of antiderivative = 5.22

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = \left[\frac{(a^2cfg^2-a^2cegh+a^2cdh^2+(ac^2fg^2-ac^2egh+ac^2dh^2)x^2)\sqrt{cg^2+ah^2}\,\log\ldots}{\ldots}\right.$$

$$-\frac{(a^2cfg^2-a^2cegh+a^2cdh^2+(ac^2fg^2-ac^2egh+ac^2dh^2)x^2)\sqrt{-cg^2-ah^2}\arctan\left(\frac{\sqrt{-cg^2-ah^2}(cgx-ah)\sqrt{cx^2+\ldots}}{acg^2+a^2h^2+(c^2g^2+ach^2)x^{\ldots}}\right.}{a^2c^3g^4+2a^3c^2g^{\ldots}}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a*c^2*f*g^2 - a*c^2*e*g*
h + a*c^2*d*h^2)*x^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a
^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*s
qrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a*c^2*e*g^3 + a^2*c*e*g*h^
2 - (a*c^2*d - a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a
^2*c*e*h^3 + (c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*
x^2 + a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^
2*c^3*g^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*
h^2 + (a*c^2*f*g^2 - a*c^2*e*g*h + a*c^2*d*h^2)*x^2)*sqrt(-c*g^2 - a*h^2)*
arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h
^2 + (c^2*g^2 + a*c*h^2)*x^2)) + (a*c^2*e*g^3 + a^2*c*e*g*h^2 - (a*c^2*d -
 a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*e*h^3 + (
c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a^2
*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2
+ a^3*c^2*h^4)*x^2)]
```

3.112.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{\frac{3}{2}}(g+hx)}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(130) = 260$.

Time = 0.24 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.28

$$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx = \frac{cfg^3x}{\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4} - \frac{ceg^2x}{\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3} + \frac{cdgx}{\sqrt{cx^2+a}acg^2+\sqrt{cx^2+a}a^2h^2} + \frac{fg^2}{\sqrt{cx^2+a}cg^2h+\sqrt{cx^2+a}ah^3} - \frac{eg}{\sqrt{cx^2+a}cg^2+\sqrt{cx^2+a}ah^2} + \frac{d}{\frac{\sqrt{cx^2+a}cg^2}{h}+\sqrt{cx^2+a}ah} - \frac{fgx}{\sqrt{cx^2+a}ah^2} + \frac{ex}{\sqrt{cx^2+a}ah} + \frac{fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3} - \frac{eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^2} + \frac{d\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h} - \frac{f}{\sqrt{cx^2+a}ch}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
c*f*g^3*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - c*e*g^
2*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + c*d*g*x/(sqrt(
c*x^2 + a)*a*c*g^2 + sqrt(c*x^2 + a)*a^2*h^2) + f*g^2/(sqrt(c*x^2 + a)*c*g
^2*h + sqrt(c*x^2 + a)*a*h^3) - e*g/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 +
a)*a*h^2) + d/(sqrt(c*x^2 + a)*c*g^2/h + sqrt(c*x^2 + a)*a*h) - f*g*x/(sqr
t(c*x^2 + a)*a*h^2) + e*x/(sqrt(c*x^2 + a)*a*h) + f*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2
)*h^3) - e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/((a + c*g^2/h^2)^(3/2)*h^2) + d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
 + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) - f/(sqrt
(c*x^2 + a)*c*h)
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(130) = 260$.

Time = 0.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx = \frac{\frac{(c^3dg^3-ac^2fg^3+ac^2eg^2h+ac^2dgh^2-a^2cfgh^2+a^2ceh^3)x}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4} - \frac{ac^2eg^3-ac^2dg^2h+a^2cfg^2h+a^2cegh^2-a^2cd}{ac^3g^4+2\,a^2c^2g^2h^2+a^3ch^4}}{\sqrt{cx^2+a}}$$

$$-\frac{2\,(fg^2-egh+dh^2)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2-ah^2}}\right)}{(cg^2+ah^2)\sqrt{-cg^2-ah^2}}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*e*g^2*h + a*c^2*d*g*h^2 - a^2*c*f*g*h^2
+ a^2*c*e*h^3)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) - (a*c^2*e*g^
3 - a*c^2*d*g^2*h + a^2*c*f*g^2*h + a^2*c*e*g*h^2 - a^2*c*d*h^3 + a^3*f*h^
3)/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2
 - e*g*h + d*h^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqr
t(-c*g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)(a+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\left(c\,x^2+a\right)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x)
```

3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx$

3.113.1 Optimal result

Integrand size = 29, antiderivative size = 239

$$\begin{aligned}\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac(fg^2-h(2eg-dh))\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}}\\ &\quad-\frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{\left(cg^2+ah^2\right)^2(g+hx)}\\ &\quad+\frac{\left(ah^2(2fg-eh)-cg(fg^2-h(2eg-3dh))\right)\operatorname{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{\left(cg^2+ah^2\right)^{5/2}}\end{aligned}$$

output
```
(a*h^2*(-e*h+2*f*g)-c*g*(f*g^2-h*(-3*d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*
h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)+(-a*(c*g*(-2*d*h+e*g
)+a*h*(-e*h+2*f*g))+(c^2*d*g^2+a^2*f*h^2-a*c*(f*g^2-h*(-d*h+2*e*g)))*x)/a/
(a*h^2+c*g^2)^2/(c*x^2+a)^(1/2)-h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/(a*h
^2+c*g^2)^2/(h*x+g)
```

3.113.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2}{(g+hx)^2 (a+cx^2)^{3/2}}\,dx = \frac{c^2dg^2x(g+hx)+a^2h(h(2eg-dh+ehx)+f(-3g^2-ghx+h^2x^2))+ac(-}{a\,(cg^2+ah^2)^2\,(g+h}$$
$$-\frac{2(cfg^3+cgh(-2eg+3dh)+ah^2(-2fg+eh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

output
```
(c^2*d*g^2*x*(g + h*x) + a^2*h*(h*(2*e*g - d*h + e*h*x) + f*(-3*g^2 - g*h*
x + h^2*x^2)) + a*c*(-(f*g^2*x*(g + 2*h*x)) + d*h*(2*g^2 + g*h*x - 2*h^2*x
^2) + e*g*(-g^2 + g*h*x + 3*h^2*x^2)))/(a*(c*g^2 + a*h^2)^2*(g + h*x)*Sqrt
[a + c*x^2]) - (2*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*h)
)*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + c*x^2])/Sqrt[-(c*g^2) - a*h^2]])/
(-(c*g^2) - a*h^2)^(5/2)
```

3.113.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}(g+hx)^2}dx$$
$$\Big\downarrow \text{2178}$$
$$-\frac{\int \frac{ac\left(a\left(fg^2-dh^2\right)h^2+(cg(eg-2dh)+ah(2fg-eh))xh^2-c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{\left(cg^2+ah^2\right)^2(g+hx)^2\sqrt{cx^2+a}}dx}{ac}-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}$$
$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a(fg^2-dh^2)h^2+(cg(eg-2dh)+ah(2fg-eh))xh^2-c(fg^4-g^2h(2eg-3dh))}{(g+hx)^2\sqrt{cx^2+a}}dx}{(ah^2+cg^2)^2}-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}$$

$$\Big\downarrow \text{679}$$

$$-\frac{\frac{h\sqrt{a+cx^2}(dh^2-egh+fg^2)}{g+hx}-\left(-ah^2(2fg-eh)-cgh(2eg-3dh)+cfg^3\right)\int \frac{1}{(g+hx)\sqrt{cx^2+a}}dx}{(ah^2+cg^2)^2}-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{\left(-ah^2(2fg-eh)-cgh(2eg-3dh)+cfg^3\right)\int \frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}+\frac{h\sqrt{a+cx^2}(dh^2-egh+fg^2)}{g+hx}}{(ah^2+cg^2)^2}-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x(a^2fh^2-ac(fg^2-h(2eg-dh))+c^2dg^2)}{a\sqrt{a+cx^2}\,(ah^2+cg^2)^2}-\frac{\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(-ah^2(2fg-eh)-cgh(2eg-3dh)+cfg^3\right)}{\sqrt{ah^2+cg^2}}+\frac{h\sqrt{a+cx^2}(dh^2-egh+fg^2)}{g+hx}}{(ah^2+cg^2)^2}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

output
```
-((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*
c*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (
(h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(g + h*x) + ((c*f*g^3 - c*g*h*
(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 +
 a*h^2]*Sqrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2])/(c*g^2 + a*h^2)^2
```

3.113. $\int \frac{d+ex+fx^2}{(g+hx)^2(a+cx^2)^{3/2}}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(228) = 456$.

Time = 0.53 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.65

method	result
default	$\frac{fx}{h^2 a\sqrt{c\,x^2+a}} + \frac{(eh-2fg)\left(\frac{h^2}{\left(a\,h^2+c\,g^2\right)\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\frac{g}{h}\right)}{h}+\frac{a\,h^2+c\,g^2}{h^2}}}+\frac{2cgh\left(2c\left(x+\frac{g}{h}\right)-\frac{2cg}{h}\right)}{\left(a\,h^2+c\,g^2\right)\left(\frac{4c\left(a\,h^2+c\,g^2\right)}{h^2}-\frac{4c^2g^2}{h^2}\right)\sqrt{\left(x+\frac{g}{h}\right)^2 c-\frac{2cg\left(x+\right.}{h}}}\right.}{h^3}$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^2*x/a/(c*x^2+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*
c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-
2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/
h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1
/h*g)))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h^2+c*g^2)*h^2/(x+1/h*g)/((x+1/h*
g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h/(a*h^2+c*g^2)*(1
/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/
2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*
c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)-1/(
a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h
*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c*g^2)*h^2*(2*c*(x+1/h*g)-2
*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/
h*g)+(a*h^2+c*g^2)/h^2)^(1/2))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(230) = 460.

Time = 1.82 (sec) , antiderivative size = 1573, normalized size of antiderivative = 6.58

$$\int \frac{d+ex+fx^2}{(g+hx)^2\,(a+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f
)*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d
- 2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 +
 (3*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2
 + a^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*
a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2
 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a
*c^2*e*g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*
a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*
c*e*g*h^4 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a
^2*c*d - a^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5
+ (c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f
)*g*h^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^
3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 +
 a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4
*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5
*h^7)*x), -((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*
a^3*f)*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c
^2*d - 2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*
h^3 + (3*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e...
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\,(a+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
```

output
```
Timed out
```

3.113.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(230) = 460$.

Time = 0.26 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.54

$$\int \frac{d+ex+fx^2}{(g+hx)^2(a+cx^2)^{3/2}}\,dx = \frac{3\,c^2fg^4x}{\sqrt{cx^2+a}ac^2g^4h^2+2\sqrt{cx^2+a}a^2cg^2h^4+\sqrt{cx^2+a}a^3h^6}$$
$$-\frac{3\,c^2eg^3x}{\sqrt{cx^2+a}ac^2g^4h+2\sqrt{cx^2+a}a^2cg^2h^3+\sqrt{cx^2+a}a^3h^5}$$
$$+\frac{3\,c^2dg^2x}{\sqrt{cx^2+a}ac^2g^4+2\sqrt{cx^2+a}a^2cg^2h^2+\sqrt{cx^2+a}a^3h^4}$$
$$+\frac{3\,cfg^3}{\sqrt{cx^2+a}c^2g^4h+2\sqrt{cx^2+a}acg^2h^3+\sqrt{cx^2+a}a^2h^5}$$
$$-\frac{4\,cfg^2x}{\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4}-\frac{3\,ceg^2}{\sqrt{cx^2+a}c^2g^4+2\sqrt{cx^2+a}acg^2h^2+\sqrt{cx^2+a}a^2h^4}$$
$$+\frac{3\,cegx}{\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3}+\frac{3\,cdg}{\frac{\sqrt{cx^2+a}c^2g^4}{h}+2\sqrt{cx^2+a}acg^2h+\sqrt{cx^2+a}a^2h^3}$$
$$-\frac{fg^2}{\sqrt{cx^2+a}cg^2h^2x+\sqrt{cx^2+a}ah^4x+\sqrt{cx^2+a}cg^3h+\sqrt{cx^2+a}agh^3}$$
$$-\frac{2\,cdx}{\sqrt{cx^2+a}acg^2+\sqrt{cx^2+a}a^2h^2}$$
$$+\frac{eg}{\sqrt{cx^2+a}cg^2hx+\sqrt{cx^2+a}ah^3x+\sqrt{cx^2+a}cg^3+\sqrt{cx^2+a}agh^2}$$
$$-\frac{2\,fg}{\sqrt{cx^2+a}cg^2h+\sqrt{cx^2+a}ah^3}$$
$$-\frac{d}{\sqrt{cx^2+a}cg^2x+\sqrt{cx^2+a}ah^2x+\frac{\sqrt{cx^2+a}cg^3}{h}+\sqrt{cx^2+a}agh}$$
$$+\frac{e}{\sqrt{cx^2+a}cg^2+\sqrt{cx^2+a}ah^2}+\frac{fx}{\sqrt{cx^2+a}ah^2}+\frac{3\,cfg^3\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^5}$$
$$-\frac{3\,ceg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^4}+\frac{3\,cdg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{5}{2}}h^3}$$
$$-\frac{2\,fg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^3}+\frac{e\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\left(a+\frac{cg^2}{h^2}\right)^{\frac{3}{2}}h^2}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
3*c^2*f*g^4*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2
*h^4 + sqrt(c*x^2 + a)*a^3*h^6) - 3*c^2*e*g^3*x/(sqrt(c*x^2 + a)*a*c^2*g^4
*h + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 3*c^2*d*
g^2*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(
c*x^2 + a)*a^3*h^4) + 3*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2
+ a)*a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) - 4*c*f*g^2*x/(sqrt(c*x^2 + a)
*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 3*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g
^4 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 3*c*e*g*x/
(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 3*c*d*g/(sqrt(c*x^
2 + a)*c^2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3)
- f*g^2/(sqrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^
2 + a)*c*g^3*h + sqrt(c*x^2 + a)*a*g*h^3) - 2*c*d*x/(sqrt(c*x^2 + a)*a*c*g
^2 + sqrt(c*x^2 + a)*a^2*h^2) + e*g/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c*x^
2 + a)*a*h^3*x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) - 2*f*g/
(sqrt(c*x^2 + a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) - d/(sqrt(c*x^2 + a)*c*g
^2*x + sqrt(c*x^2 + a)*a*h^2*x + sqrt(c*x^2 + a)*c*g^3/h + sqrt(c*x^2 + a)
*a*g*h) + e/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a*h^2) + f*x/(sqrt(c*
x^2 + a)*a*h^2) + 3*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 3*c*e*g^2*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g...
```

3.113.8 Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{fx^2+ex+d}{\left(cx^2+a\right)^{\frac{3}{2}}(hx+g)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\,(a+cx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

3.114.1 Optimal result

Integrand size = 29, antiderivative size = 374

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a(a^2fh^3-c^2g^2(eg-3dh)-ach(3fg^2-h(3eg-dh)))+c(c^2dg^3+a^2h^2(3f}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$
$$-\frac{h(fg^2-egh+dh^2)\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^2(g+hx)^2}+\frac{h(2ah^2(2fg-eh)-cg(3fg^2-h(5eg-7dh)))\sqrt{a+cx^2}}{2\left(cg^2+ah^2\right)^3(g+hx)}$$
$$-\frac{(2a^2fh^4-ach^2(11fg^2-9egh+3dh^2)+2c^2g^2(fg^2-3egh+6dh^2))\,\text{arctanh}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\sqrt{a+cx^2}}\right)}{2\left(cg^2+ah^2\right)^{7/2}}$$

output
```
-1/2*(2*a^2*f*h^4-a*c*h^2*(3*d*h^2-9*e*g*h+11*f*g^2)+2*c^2*g^2*(6*d*h^2-3*
e*g*h+f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a
*h^2+c*g^2)^(7/2)+(a*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(3*f*g^2-h*(-d*
h+3*e*g)))+c*(c^2*d*g^3+a^2*h^2*(-e*h+3*f*g)-a*c*g*(f*g^2-3*h*(-d*h+e*g)))
*x)/a/(a*h^2+c*g^2)^3/(c*x^2+a)^(1/2)-1/2*h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^
(1/2)/(a*h^2+c*g^2)^2/(h*x+g)^2+1/2*h*(2*a*h^2*(-e*h+2*f*g)-c*g*(3*f*g^2-h
*(-7*d*h+5*e*g)))*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)
```

3.114.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1537 vs. $2(374) = 748$.

Time = 11.11 (sec) , antiderivative size = 1537, normalized size of antiderivative = 4.11

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+cx^2)^{3/2}}\,dx = \frac{a^4h^3(f(5g^2+8ghx+2h^2x^2)-h(dh+e(g+2hx)))-4c^{7/2}g^2x^2(-\sqrt{c}x+\sqrt{}}{}$$

$$-\frac{15afh^2\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{5/2}}+\frac{21aeh^3\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g(-cg^2-ah^2)^{5/2}}$$
$$-\frac{2f\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{(-cg^2-ah^2)^{3/2}}+\frac{6eh\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g(-cg^2-ah^2)^{3/2}}$$
$$-\frac{12dh^2\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g^2(-cg^2-ah^2)^{3/2}}-\frac{27adh^4\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\,(cg^3+agh^2)^2}$$
$$-\frac{15a^2h^4(fg^2+h(-eg+dh))\arctan\left(\frac{-\sqrt{c}(g+hx)+h\sqrt{a+cx^2}}{\sqrt{-cg^2-ah^2}}\right)}{g^2(-cg^2-ah^2)^{7/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

output
```
(a^4*h^3*(f*(5*g^2 + 8*g*h*x + 2*h^2*x^2) - h*(d*h + e*(g + 2*h*x))) - 4*c
^(7/2)*g^2*x^2*(-(Sqrt[c]*x) + Sqrt[a + c*x^2])*(d*(-g^3 + 4*g^2*h*x + 18*
g*h^2*x^2 + 12*h^3*x^3) + g*x*(f*g*x*(3*g + 2*h*x) - e*(2*g^2 + 9*g*h*x +
6*h^2*x^2))) - a^3*(Sqrt[c]*h^2*Sqrt[a + c*x^2]*(-3*d*h^3*x + e*h*(4*g^2 +
 5*g*h*x - 2*h^2*x^2) + f*(-10*g^3 - 5*g^2*h*x + 14*g*h^2*x^2 + 6*h^3*x^3)
) + c*h*(f*(10*g^4 + 39*g^3*h*x + 26*g^2*h^2*x^2 - 20*g*h^3*x^3 - 10*h^4*x
^4) + h*(d*h*(10*g^2 + 11*g*h*x + 8*h^2*x^2) - e*(12*g^3 + 27*g^2*h*x + 20
*g*h^2*x^2 - 2*h^3*x^3)))) + a^2*(c^2*(d*h*(6*g^4 + 45*g^3*h*x + 14*g^2*h^
2*x^2 - 29*g*h^3*x^3 - 19*h^4*x^4) + e*g*(-2*g^4 - 31*g^3*h*x + 10*g^2*h^2
*x^2 + 81*g*h^3*x^3 + 57*h^4*x^4) + f*x*(13*g^5 - 28*g^4*h*x - 105*g^3*h^2
*x^2 - 75*g^2*h^3*x^3 + 12*g*h^4*x^4 + 8*h^5*x^5)) + c^(3/2)*Sqrt[a + c*x^
2]*(f*(-5*g^5 + 20*g^4*h*x + 72*g^3*h^2*x^2 + 53*g^2*h^3*x^3 - 12*g*h^4*x^
4 - 8*h^5*x^5) + h*(e*g*(11*g^3 - 14*g^2*h*x - 54*g*h^2*x^2 - 39*h^3*x^3)
+ d*h*(-13*g^3 + 4*g^2*h*x + 20*g*h^2*x^2 + 13*h^3*x^3)))) + a*(c^(5/2)*Sq
rt[a + c*x^2]*(d*(2*g^5 - 14*g^4*h*x - 81*g^3*h^2*x^2 - 48*g^2*h^3*x^3 + 1
8*g*h^4*x^4 + 12*h^5*x^5) + g*x*(f*g*x*(-19*g^3 + 14*g^2*h*x + 66*g*h^2*x^
2 + 44*h^3*x^3) + e*(6*g^4 + 49*g^3*h*x + 14*g^2*h^2*x^2 - 54*g*h^3*x^3 -
36*h^4*x^4))) - c^3*x*(d*(4*g^5 - 22*g^4*h*x - 117*g^3*h^2*x^2 - 72*g^2*h^
3*x^3 + 18*g*h^4*x^4 + 12*h^5*x^5) + g*x*(f*g*x*(-25*g^3 + 10*g^2*h*x + 66
*g*h^2*x^2 + 44*h^3*x^3) + e*(10*g^4 + 67*g^3*h*x + 26*g^2*h^2*x^2 - 54...
```

3.114.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2178, 25, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+cx^2)^{3/2}(g+hx)^3}dx$$

$$\Big\downarrow \ 2178$$

$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$
$$\frac{\int -\frac{\frac{ac\left(a^2fh^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)\right)x^2h^3}{\left(cg^2+ah^2\right)^3}+\frac{ac\left(a^2eh^4-2acg^2(4fg-3eh)h-c^2g^3(3eg-8dh)\right)xh^2}{\left(cg^2+ah^2\right)^3}+\frac{ac\left(a^2dh^6-acg^2\left(3fg^2-h(eg+3dh)\right)h^2+\right.}{\left(cg^2+ah^2\right)^3}}{(g+hx)^3\sqrt{cx^2+a}}}{ac}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\frac{ac\left(a^2fh^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)\right)x^2h^3}{\left(cg^2+ah^2\right)^3}+\frac{ac\left(a^2eh^4-2acg^2(4fg-3eh)h-c^2g^3(3eg-8dh)\right)xh^2}{\left(cg^2+ah^2\right)^3}+\frac{ac\left(a^2dh^6-acg^2\left(3fg^2-h(eg+3dh)\right)h^2+c^2\right.}{\left(cg^2+ah^2\right)^3}}{(g+hx)^3\sqrt{cx^2+a}}}{ac}$$
$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

$$\Big\downarrow \ 2182$$

$$\frac{-\frac{\int \frac{ac\left(2\left(a^2(fg-eh)h^4+2acg\left(2fg^2-h(eg+dh)\right)h^2-c^2g^3\left(fg^2-3ehg+6dh^2\right)\right)-h\left(2a^2fh^4-ac\left(7fg^2-h(7eg-3dh)\right)h^2-c^2\left(fg^4+h(eg-5dh)g^2\right)\right)x\right)}{\left(cg^2+ah^2\right)^2(g+hx)^2\sqrt{cx^2+a}}dx}{2(ah^2+cg^2)}-\frac{ach}{2}}{ac}$$
$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{ac\int \frac{2\left(a^2(fg-eh)h^4+2acg\left(2fg^2-h(eg+dh)\right)h^2-c^2g^3\left(fg^2-3ehg+6dh^2\right)\right)-h\left(2a^2fh^4-ac\left(7fg^2-h(7eg-3dh)\right)h^2-c^2\left(fg^4+h(eg-5dh)g^2\right)\right)x}{(g+hx)^2\sqrt{cx^2+a}}dx}{2(ah^2+cg^2)^3}-\frac{ach\sqrt{}}{2(}}{ac}$$
$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

$\downarrow$ 679

$$-\frac{ac\left(\frac{h\sqrt{a+cx^2}\left(-2ah^2(2fg-eh)-cgh(5eg-7dh)+3cfg^3\right)}{g+hx}-\left(2a^2fh^4-ach^2\left(3dh^2-9egh+11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg^2\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+a}}dx\right)}{2(ah^2+cg^2)^3} - \ldots$$

$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\ldots\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

$\downarrow$ 488

$$-\frac{ac\left(\left(2a^2fh^4-ach^2\left(3dh^2-9egh+11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg^2\right)\right)\int\frac{1}{cg^2+ah^2-\frac{(ah-cgx)^2}{cx^2+a}}d\frac{ah-cgx}{\sqrt{cx^2+a}}+\frac{h\sqrt{a+cx^2}\left(-2ah^2(2fg-eh)-cgh(5eg-7dh)\ldots\right.}{g+hx}\right.}{2(ah^2+cg^2)^3}$$

$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\ldots\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

$\downarrow$ 219

$$-\frac{ac\left(\frac{\text{arctanh}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4-ach^2\left(3dh^2-9egh+11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg^2\right)\right)}{\sqrt{ah^2+cg^2}}+\frac{h\sqrt{a+cx^2}\left(-2ah^2(2fg-eh)-cgh(5eg-7dh)+3cfg\ldots\right.}{g+hx}\right.}{2(ah^2+cg^2)^3}$$

$$\frac{a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)+cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\ldots\right.}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

output
```
(a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
 + c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))
*x)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) + (-1/2*(a*c*h*(f*g^2 - e*g*h +
d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)^2) - (a*c*((h*(3*c*f*
g^3 - c*g*h*(5*e*g - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(g +
 h*x) + ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g^2
*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*S
qrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2]))/(2*(c*g^2 + a*h^2)^3))/(a*c)
```

3.114. $\int \frac{d+ex+fx^2}{(g+hx)^3(a+cx^2)^{3/2}}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.114. $\int \frac{d+ex+fx^2}{(g+hx)^3(a+cx^2)^{3/2}}\,dx$

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(356) = 712$.

Time = 0.66 (sec) , antiderivative size = 1787, normalized size of antiderivative = 4.78

method	result	size
default	Expression too large to display	1787

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^3*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/
h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2
)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(
1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2
-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c-2*c*g/h*(x+1
/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h-2*f*g)/h^4*(-1/(a*h^2+c*g
^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2
)+3*c*g*h/(a*h^2+c*g^2)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h
*g)+(a*h^2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)
/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a
*h^2+c*g^2)/h^2)^(1/2)-1/(a*h^2+c*g^2)*h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2
*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+1/h*g)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*c/(a*h^2+c
*g^2)*h^2*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*(a*h^2+c*g^2)/h^2-4*c^2*g^2/h^2)/((
x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2))+(d*h^2-e*g*h+f*g^
2)/h^5*(-1/2/(a*h^2+c*g^2)*h^2/(x+1/h*g)^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g
)+(a*h^2+c*g^2)/h^2)^(1/2)+5/2*c*g*h/(a*h^2+c*g^2)*(-1/(a*h^2+c*g^2)*h^2/(
x+1/h*g)/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^2+c*g^2)/h^2)^(1/2)+3*c*g*h
/(a*h^2+c*g^2)*(1/(a*h^2+c*g^2)*h^2/((x+1/h*g)^2*c-2*c*g/h*(x+1/h*g)+(a*h^
2+c*g^2)/h^2)^(1/2)+2*c*g*h/(a*h^2+c*g^2)*(2*c*(x+1/h*g)-2*c*g/h)/(4*c*...
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. $2(357) = 714$.

Time = 8.79 (sec) , antiderivative size = 2853, normalized size of antiderivative = 7.63

$$\int \frac{d + ex + fx^2}{(g + hx)^3 \left(a + cx^2\right)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((2*a^2*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c
^2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^
4*h^2 - 6*a*c^3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f
)*g^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*
c^3*e*g^4*h^2 + 9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3
- (3*a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h
+ 3*a^2*c^2*e*g^3*h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*
h^2 + 9*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2
*(2*a^2*c^2*f*g^5*h - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^
2*d - 11*a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(c*g^2 + a
*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 -
 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x +
 g^2)) - 2*(2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 + a^
4*e*g*h^6 + a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d +
 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*h^3
 + 7*a^2*c^2*e*g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (
11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3
 - (16*a*c^3*e*g^5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d -
 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*
a^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + ...
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(357) = 714$.

Time = 0.31 (sec) , antiderivative size = 2254, normalized size of antiderivative = 6.03

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+cx^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
15/2*c^3*f*g^5*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h^2 + 3*sqrt(c*x^2 + a)*a^2*c^
2*g^4*h^4 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^6 + sqrt(c*x^2 + a)*a^4*h^8) - 1
5/2*c^3*e*g^4*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a^2*c^2*g
^4*h^3 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^5 + sqrt(c*x^2 + a)*a^4*h^7) + 15/2
*c^3*d*g^3*x/(sqrt(c*x^2 + a)*a*c^3*g^6 + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*h^
2 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^4 + sqrt(c*x^2 + a)*a^4*h^6) + 15/2*c^2*
f*g^4/(sqrt(c*x^2 + a)*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h^3 + 3*sqr
t(c*x^2 + a)*a^2*c*g^2*h^5 + sqrt(c*x^2 + a)*a^3*h^7) - 25/2*c^2*f*g^3*x/(
sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x
^2 + a)*a^3*h^6) - 15/2*c^2*e*g^3/(sqrt(c*x^2 + a)*c^3*g^6 + 3*sqrt(c*x^2
+ a)*a*c^2*g^4*h^2 + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a^3
*h^6) + 19/2*c^2*e*g^2*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h + 2*sqrt(c*x^2 + a)*
a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 15/2*c^2*d*g^2/(sqrt(c*x^2 + a)
*c^3*g^6/h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h
^3 + sqrt(c*x^2 + a)*a^3*h^5) - 5/2*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h^2*x
 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^4*x + sqrt(c*x^2 + a)*a^2*h^6*x + sqrt(c*x^
2 + a)*c^2*g^5*h + 2*sqrt(c*x^2 + a)*a*c*g^3*h^3 + sqrt(c*x^2 + a)*a^2*g*h
^5) - 13/2*c^2*d*g*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*
g^2*h^2 + sqrt(c*x^2 + a)*a^3*h^4) + 5/2*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4*
h*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^3*x + sqrt(c*x^2 + a)*a^2*h^5*x + sqr...
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(357) = 714$.

Time = 0.31 (sec) , antiderivative size = 1421, normalized size of antiderivative = 3.80

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+cx^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((c^6*d*g^9 - a*c^5*f*g^9 + 3*a*c^5*e*g^8*h + 8*a^2*c^4*e*g^6*h^3 - 6*a^2*
c^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 + 6*a^3*c^3*e*g^4*h^5 - 8*a^3*c^3*d*g^
3*h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^8 - a^5*c*
e*h^9)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^
3*g^6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) - (a*c^5*e*g
^9 - 3*a*c^5*d*g^8*h + 3*a^2*c^4*f*g^8*h - 8*a^2*c^4*d*g^6*h^3 + 8*a^3*c^3
*f*g^6*h^3 - 6*a^3*c^3*e*g^5*h^4 - 6*a^3*c^3*d*g^4*h^5 + 6*a^4*c^2*f*g^4*h
^5 - 8*a^4*c^2*e*g^3*h^6 - 3*a^5*c*e*g*h^8 + a^5*c*d*h^9 - a^6*f*h^9)/(a*c
^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 1
5*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2
*f*g^4 - 6*c^2*e*g^3*h + 12*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 + 9*a*c*e*g*h
^3 - 3*a*c*d*h^4 + 2*a^2*f*h^4)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h +
sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2
*h^4 + a^3*h^6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^2*f*g^4*h - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*e*g^3*h^2 + 6*(sqrt(c
)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*a*c*f*g^2*h^3 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*e*g*h^4 - (sqrt(c)*
x - sqrt(c*x^2 + a))^3*a*c*d*h^5 + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/
2)*f*g^5 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*e*g^4*h + 14*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2...
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)), x)
```

3.115 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx$

3.115.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = -\frac{aB-(Ac-aC)x}{3ac\,(a+cx^2)^{3/2}}+\frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}$$

output
```
1/3*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(3/2)+1/3*(2*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{-a^2B+3aAcx+2Ac^2x^3+acCx^3}{3a^2c\,(a+cx^2)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

output
```
(-(a^2*B) + 3*a*A*c*x + 2*A*c^2*x^3 + a*c*C*x^3)/(3*a^2*c*(a + c*x^2)^(3/2
))
```

3.115.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2345, 25, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 2345$$

$$-\frac{\int -\frac{2Ac+aC}{c\left(cx^2+a\right)^{3/2}}dx}{3a}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2Ac+aC}{c\left(cx^2+a\right)^{3/2}}dx}{3a}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(aC+2Ac)\int \frac{1}{\left(cx^2+a\right)^{3/2}}dx}{3ac}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 208$$

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

output
```
-1/3*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*
a^2*c*Sqrt[a + c*x^2])
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.115.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{2A\,c^{2}x^{3}+Cac\,x^{3}+3aAcx-B\,a^{2}}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}c\,a^{2}}$	47
trager	$\frac{2A\,c^{2}x^{3}+Cac\,x^{3}+3aAcx-B\,a^{2}}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}c\,a^{2}}$	47
default	$A\left(\frac{x}{3a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{c\,x^{2}+a}}\right)+C\left(-\frac{x}{2c\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{c\,x^{2}+a}}\right)}{2c}\right)-\frac{B}{3c\left(c\,x^{2}+a\right)^{\frac{3}{2}}}$	105

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^(3/2)/c/a^2
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^{5/2}} \, dx = \frac{(3\,Aacx + (Cac + 2\,Ac^2)x^3 - Ba^2)\sqrt{cx^2 + a}}{3\,(a^2c^3x^4 + 2\,a^3c^2x^2 + a^4c)}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x
^4 + 2*a^3*c^2*x^2 + a^4*c)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^{5/2}} \, dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1 + \frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1 + \frac{cx^2}{a}}} + \frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1 + \frac{cx^2}{a}} + 3a^{\frac{5}{2}}cx^2\sqrt{1 + \frac{cx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ac\sqrt{a+cx^2} + 3c^2x^2\sqrt{a+cx^2}} & \text{for } c \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Cx^3}{3a^{\frac{5}{2}}\sqrt{1 + \frac{cx^2}{a}} + 3a^{\frac{3}{2}}cx^2\sqrt{1 + \frac{cx^2}{a}}}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**
2/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1
 + c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqr
t(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)
*sqrt(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\sqrt{cx^2+a}a^2}+\frac{Ax}{3\,(cx^2+a)^{\frac{3}{2}}a} - \frac{Cx}{3\,(cx^2+a)^{\frac{3}{2}}c}+\frac{Cx}{3\sqrt{cx^2+a}ac}-\frac{B}{3\,(cx^2+a)^{\frac{3}{2}}c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((
c*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(
3/2)*c)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{x\left(\frac{3\,A}{a}+\frac{(Cac+2\,Ac^2)x^2}{a^2c}\right)-\frac{B}{c}}{3\,(cx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{5/2}}\,dx = \frac{2\,A\,c\,x\,(c\,x^2+a)-C\,a^2\,x-B\,a^2+C\,a\,x\,(c\,x^2+a)+A\,a\,c\,x}{3\,a^2\,c\,(c\,x^2+a)^{3/2}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^(5/2),x)
```

output
```
(2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a
^2*c*(a + c*x^2)^(3/2))
```

3.116 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx$

3.116.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = -\frac{aB-(Ac-aC)x}{5ac\,(a+cx^2)^{5/2}} + \frac{(4Ac+aC)x}{15a^2c\,(a+cx^2)^{3/2}} + \frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}$$

output
```
1/5*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(5/2)+1/15*(4*A*c+C*a)*x/a^2/c/(c*x^2
+a)^(3/2)+2/15*(4*A*c+C*a)*x/a^3/c/(c*x^2+a)^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{-3a^3B+8Ac^3x^5+5a^2cx(3A+Cx^2)+2ac^2x^3(10A+Cx^2)}{15a^3c\,(a+cx^2)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

output
```
(-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*
x^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

3.116.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2345, 25, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{4Ac+aC}{c\left(cx^2+a\right)^{5/2}}dx}{5a}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4Ac+aC}{c\left(cx^2+a\right)^{5/2}}dx}{5a}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aC+4Ac)\int \frac{1}{\left(cx^2+a\right)^{5/2}}dx}{5ac}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{(aC+4Ac)\left(\frac{2\int \frac{1}{\left(cx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)}{5ac}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\left(\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)(aC+4Ac)}{5ac}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

output
```
-1/5*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*(x/(3*
a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])))/(5*a*c)
```

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.116.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{8A\,c^{3}x^{5}+2a\,c^{2}C\,x^{5}+20aA\,c^{2}x^{3}+5C\,a^{2}c\,x^{3}+15a^{2}Acx-3B\,a^{3}}{15\left(c\,x^{2}+a\right)^{\frac{5}{2}}a^{3}c}$
trager	$\frac{8A\,c^{3}x^{5}+2a\,c^{2}C\,x^{5}+20aA\,c^{2}x^{3}+5C\,a^{2}c\,x^{3}+15a^{2}Acx-3B\,a^{3}}{15\left(c\,x^{2}+a\right)^{\frac{5}{2}}a^{3}c}$
default	$A\left(\frac{x}{5a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{c\,x^{2}+a}}}{a}\right)+C\left(-\frac{x}{4c\left(c\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{a\left(\frac{x}{5a\left(c\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(c\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{c\,x^{2}+a}}}{a}\right)}{4c}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-
3*B*a^3)/(c*x^2+a)^(5/2)/a^3/c
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{(2\,(Cac^2+4\,Ac^3)x^5+15\,Aa^2cx-3\,Ba^3+5\,(Ca^2c+4\,Aac^2)x^3)\sqrt{cx^2+a}}{15\,(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c)}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*
A*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2
 + a^6*c)
```

3.116.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(87) = 174.

Time = 12.06 (sec) , antiderivative size = 638, normalized size of antiderivative = 6.58

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{c}{}}}\right.$$

$$+\frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

$$+\frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

$$\left.+\frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}\right)$$

$$+B\left(\begin{cases}-\frac{1}{5a^2c\sqrt{a+cx^2}+10ac^2x^2\sqrt{a+cx^2}+5c^3x^4\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{7}{2}}} & \text{otherwise}\end{cases}\right)$$

$$+C\left(\frac{5ax^3}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}\right.$$

$$\left.+\frac{2cx^5}{15a^{\frac{9}{2}}\sqrt{1+\frac{cx^2}{a}}+30a^{\frac{7}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{5}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}}\right)$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

3.116. $\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx$

output
```
A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c*
*3*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2
/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt
(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2
*x**5/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x*
*2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6
*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) +
 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/
(5*a**2*c*sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4
*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(1
5*a**(9/2)*sqrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15
*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c
*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*s
qrt(1 + c*x**2/a)))
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{7/2}}\,dx = \frac{8\,Ax}{15\,\sqrt{cx^2+a}a^3}+\frac{4\,Ax}{15\,(cx^2+a)^{\frac{3}{2}}a^2}+\frac{Ax}{5\,(cx^2+a)^{\frac{5}{2}}a}-\frac{Cx}{5\,(cx^2+a)^{\frac{5}{2}}c}+\frac{2\,Cx}{15\,\sqrt{cx^2+a}a^2c}+\frac{Cx}{15\,(cx^2+a)^{\frac{3}{2}}ac}-\frac{B}{5\,(cx^2+a)^{\frac{5}{2}}c}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*
x/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c
*x^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(
5/2)*c)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(Cac^3+4\,Ac^4)x^2}{a^3c^2}+\frac{5\,(Ca^2c^2+4\,Aac^3)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\,(cx^2+a)^{\frac{5}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3
)/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx = \frac{8\,A\,c\,x\,(c\,x^2+a)^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,(c\,x^2+a)^2+C\,a^2\,x\,(c\,x^2+a)+3\,A}{15\,a^3\,c\,(c\,x^2+a)^{5/2}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^(7/2),x)
```

output
```
(8*A*c*x*(a + c*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + c*x^2)^2 + C*a
^2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x
^2)^(5/2))
```

3.117 $\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx$

3.117.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx = -\frac{aB-(Ac-aC)x}{7ac\,(a+cx^2)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\,(a+cx^2)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\,(a+cx^2)^{3/2}} + \frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}$$

output
```
1/7*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(7/2)+1/35*(6*A*c+C*a)*x/a^2/c/(c*x^2
+a)^(5/2)+4/105*(6*A*c+C*a)*x/a^3/c/(c*x^2+a)^(3/2)+8/105*(6*A*c+C*a)*x/a^
4/c/(c*x^2+a)^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx = \frac{-15a^4B+48Ac^4x^7+35a^3cx(3A+Cx^2)+8ac^3x^5(21A+Cx^2)+14a^2c^2x^3(15A+}{105a^4c\,(a+cx^2)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

output
```
(-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
 C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

3.117.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2345, 25, 27, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{6Ac+aC}{c\left(cx^2+a\right)^{7/2}}dx}{7a}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ac+aC}{c\left(cx^2+a\right)^{7/2}}dx}{7a}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+6Ac)\int \frac{1}{\left(cx^2+a\right)^{7/2}}dx}{7ac}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ac)\left(\frac{4\int \frac{1}{\left(cx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}\right)}{7ac}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ac)\left(\frac{4\left(\frac{2\int \frac{1}{\left(cx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+cx^2\right)^{5/2}}\right)}{7ac}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\left(a+cx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+cx^2)^{5/2}}\right)(aC+6Ac)}{7ac}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

output
```
-1/7*(a*B - (A*c - a*C)*x)/(a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*(x/(5*
a*(a + c*x^2)^(5/2)) + (4*(x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a
 + c*x^2])))/(5*a)))/(7*a*c)
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.117.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168aA\,c^3x^5+28C\,a^2c^2x^5+210a^2A\,c^2x^3+35C\,a^3c\,x^3+105a^3Acx-15B\,a^4}{105(c\,x^2+a)^{\frac{7}{2}}a^4c}$
trager	$\frac{48A\,c^4x^7+8Ca\,c^3x^7+168aA\,c^3x^5+28C\,a^2c^2x^5+210a^2A\,c^2x^3+35C\,a^3c\,x^3+105a^3Acx-15B\,a^4}{105(c\,x^2+a)^{\frac{7}{2}}a^4c}$
default	$A\left(\frac{x}{7a(c\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(c\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(c\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{c\,x^2+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x}{6c(c\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(c\,x^2+a\right)^{\frac{7}{2}}}+\frac{6}{35a\left(c\,x\right.}\right.}{}\right.$

input
```
int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a
^2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx = \frac{(8\,(Cac^3+6\,Ac^4)x^7+105\,Aa^3cx+28\,(Ca^2c^2+6\,Aac^3)x^5-15\,Ba^4+35\,(Ca^3c+6}{105\,(a^4c^5x^8+4\,a^5c^4x^6+6\,a^6c^3x^4+4\,a^7c^2x^2+a^8c)}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c
^3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*
c^5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

3.117.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(117) = 234$.

Time = 26.21 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A + Bx + Cx^2}{(a + cx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt
(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2
)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a
) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*
sqrt(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) +
210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c*
*4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) +
 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**
(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*
a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 +
c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**
5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) +
 429*a**11*c**3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x*
*2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a
**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c
*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6
*x**12*sqrt(1 + c*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*
x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**
4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525...
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{A + Bx + Cx^2}{(a + cx^2)^{9/2}}\, dx &= \frac{16\, Ax}{35\, \sqrt{cx^2 + a}a^4} + \frac{8\, Ax}{35\, (cx^2 + a)^{\frac{3}{2}}a^3} \\ &+ \frac{6\, Ax}{35\, (cx^2 + a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\, (cx^2 + a)^{\frac{7}{2}}a} - \frac{Cx}{7\, (cx^2 + a)^{\frac{7}{2}}c} + \frac{8\, Cx}{105\, \sqrt{cx^2 + a}a^3c} \\ &+ \frac{4\, Cx}{105\, (cx^2 + a)^{\frac{3}{2}}a^2c} + \frac{Cx}{35\, (cx^2 + a)^{\frac{5}{2}}ac} - \frac{B}{7\, (cx^2 + a)^{\frac{7}{2}}c}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*
x^2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2
+ a)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^
(7/2)*c)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Cac^5+6\,Ac^6)x^2}{a^4c^3}+\frac{7\,(Ca^2c^4+6\,Aac^5)}{a^4c^3}\right)+\frac{35\,(Ca^3c^3+6\,Aa^2c^4)}{a^4c^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\,(cx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a
*c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*
x - 15*B/c)/(c*x^2 + a)^(7/2)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(a+cx^2)^{9/2}}\,dx = \frac{x\,(6\,A\,c+C\,a)}{35\,a^2\,c\,(c\,x^2+a)^{5/2}}-\frac{\frac{B}{7\,c}-x\left(\frac{A}{7\,a}-\frac{C}{7\,c}\right)}{(c\,x^2+a)^{7/2}}+\frac{x\,(24\,A\,c+4\,C\,a)}{105\,a^3\,c\,(c\,x^2+a)^{3/2}}+\frac{x\,(48\,A\,c+8\,C\,a)}{105\,a^4\,c\,\sqrt{c\,x^2+a}}$$

input
```
int((A + B*x + C*x^2)/(a + c*x^2)^(9/2),x)
```

output
```
(x*(6*A*c + C*a))/(35*a^2*c*(a + c*x^2)^(5/2)) - (B/(7*c) - x*(A/(7*a) - C
/(7*c)))/(a + c*x^2)^(7/2) + (x*(24*A*c + 4*C*a))/(105*a^3*c*(a + c*x^2)^(
3/2)) + (x*(48*A*c + 8*C*a))/(105*a^4*c*(a + c*x^2)^(1/2))
```

3.118 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

3.118.1 Optimal result

Integrand size = 29, antiderivative size = 106

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{810}(3937+2073x)\sqrt{2+3x^2} + \frac{5\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)-19/540*(1+2*x)^2*(3*x^2+2)^(1/2)+13/60*
(1+2*x)^3*(3*x^2+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2+2)^(1/2)-1/810*(3937+2073*
x)*(3*x^2+2)^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\sqrt{2+3x^2}(-1841-135x+2292x^2+2430x^3+864x^4) - \frac{5\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4))/405 - (5
*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

3.118.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2185, 27, 687, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{60}\int -\frac{4(17-39x)(2x+1)^3}{\sqrt{3x^2+2}}dx+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}(2x+1)^4\sqrt{3x^2+2}-\frac{1}{15}\int \frac{(17-39x)(2x+1)^3}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{13}{4}(2x+1)^3\sqrt{3x^2+2}-\frac{1}{12}\int \frac{3(2x+1)^2(19x+224)}{\sqrt{3x^2+2}}dx\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{13}{4}(2x+1)^3\sqrt{3x^2+2}-\frac{1}{4}\int \frac{(2x+1)^2(19x+224)}{\sqrt{3x^2+2}}dx\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{1}{9}\int \frac{2(2x+1)(2073x+932)}{\sqrt{3x^2+2}}dx-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+2}(2x+1)^3\right)+\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\int \frac{(2x+1)(2073x+932)}{\sqrt{3x^2+2}}dx-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+2}(2x+1)^3\right)+$$
$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\left(-450\int \frac{1}{\sqrt{3x^2+2}}dx+691\sqrt{3x^2+2}x+\frac{3937}{3}\sqrt{3x^2+2}\right)-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x^2+\right.$$
$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(\frac{1}{4}\left(-\frac{2}{9}\left(-150\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+691\sqrt{3x^2+2}x+\frac{3937}{3}\sqrt{3x^2+2}\right)-\frac{19}{9}\sqrt{3x^2+2}(2x+1)^2\right)+\frac{13}{4}\sqrt{3x\right.$$
$$\frac{2}{15}\sqrt{3x^2+2}(2x+1)^4$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 + ((13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/4 +
 ((-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (2*((3937*Sqrt[2 + 3*x^2])/3 + 691
*x*Sqrt[2 + 3*x^2] - 150*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9)/4)/15
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.118.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(864x^4+2430x^3+2292x^2-135x-1841)\sqrt{3x^2+2}}{405}+\frac{5\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\left(\frac{32}{15}x^4+6x^3+\frac{764}{135}x^2-\frac{1}{3}x-\frac{1841}{405}\right)\sqrt{3x^2+2}+\frac{5\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{5\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{32x^4\sqrt{3x^2+2}}{15}+\frac{764x^2\sqrt{3x^2+2}}{135}+6x^3\sqrt{3x^2+2}-\frac{x\sqrt{3x^2+2}}{3}$
meijerg	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)}{3}+\frac{34\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}\,\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{3x^2}{2}+1}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}\right.}{}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/405*(864*x^4+2430*x^3+2292*x^2-135*x-1841)*(3*x^2+2)^(1/2)+5/9*arcsinh(1
/2*x*6^(1/2))*3^(1/2)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2} + \frac{5}{18}\,\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/1
8*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{32x^4\sqrt{3x^2+2}}{15}+6x^3\sqrt{3x^2+2}+\frac{764x^2\sqrt{3x^2+2}}{135} - \frac{x\sqrt{3x^2+2}}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x*
*2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)
*asinh(sqrt(6)*x/2)/9
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{32}{15}\sqrt{3\,x^2+2}x^4 + 6\sqrt{3\,x^2+2}x^3 + \frac{764}{135}\sqrt{3\,x^2+2}x^2 - \frac{1}{3}\sqrt{3\,x^2+2}x + \frac{5}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{1841}{405}\sqrt{3\,x^2+2}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2
)*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/
405*sqrt(3*x^2 + 2)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{405}\left(3\left(2\left(9\left(16\,x+45\right)x+382\right)x-45\right)x-1841\right)\sqrt{3\,x^2+2} - \frac{5}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/
9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{5\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)
/45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

3.118. $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

3.119.1 Optimal result

Integrand size = 29, antiderivative size = 82

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2} - \frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)$$

output
```
-arcsinh(1/2*x*6^(1/2))*3^(1/2)+5/18*(1+2*x)^2*(3*x^2+2)^(1/2)+1/6*(1+2*x)
^3*(3*x^2+2)^(1/2)-1/27*(61+3*x)*(3*x^2+2)^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\sqrt{2+3x^2}\left(-49+54x+84x^2+36x^3\right) + \sqrt{3}\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 + Sqrt[3]*Log[-(Sqrt[3
]*x) + Sqrt[2 + 3*x^2]]
```

3.119.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2185, 27, 687, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{\sqrt{3x^2+2}} \, dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{48} \int -\frac{24(2-5x)(2x+1)^2}{\sqrt{3x^2+2}} dx + \frac{1}{6} \sqrt{3x^2+2} (2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}(2x+1)^3 \sqrt{3x^2+2} - \frac{1}{2} \int \frac{(2-5x)(2x+1)^2}{\sqrt{3x^2+2}} dx$$

$$\Big\downarrow \text{687}$$

$$\frac{1}{2} \left(\frac{5}{9}(2x+1)^2 \sqrt{3x^2+2} - \frac{1}{9} \int \frac{2(2x+1)(3x+29)}{\sqrt{3x^2+2}} dx \right) + \frac{1}{6} \sqrt{3x^2+2} (2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{5}{9}(2x+1)^2 \sqrt{3x^2+2} - \frac{2}{9} \int \frac{(2x+1)(3x+29)}{\sqrt{3x^2+2}} dx \right) + \frac{1}{6} \sqrt{3x^2+2} (2x+1)^3$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{2} \left(\frac{5}{9}(2x+1)^2 \sqrt{3x^2+2} - \frac{2}{9} \left(27 \int \frac{1}{\sqrt{3x^2+2}} dx + \sqrt{3x^2+2} x + \frac{61}{3} \sqrt{3x^2+2} \right) \right) + \frac{1}{6} \sqrt{3x^2+2} (2x+1)^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2} \left(\frac{5}{9}(2x+1)^2 \sqrt{3x^2+2} - \frac{2}{9} \left(9\sqrt{3} \text{arcsinh}\left(\sqrt{\frac{3}{2}} x \right) + \sqrt{3x^2+2} x + \frac{61}{3} \sqrt{3x^2+2} \right) \right) + \frac{1}{6} \sqrt{3x^2+2} (2x+1)^3$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 + ((5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (2*
((61*Sqrt[2 + 3*x^2])/3 + x*Sqrt[2 + 3*x^2] + 9*Sqrt[3]*ArcSinh[Sqrt[3/2]*
x]))/9)/2
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.119. $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

3.119.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

method	result
risch	$\frac{(36x^3+84x^2+54x-49)\sqrt{3x^2+2}}{27} - \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}$
trager	$\left(\frac{4}{3}x^3 + \frac{28}{9}x^2 + 2x - \frac{49}{27}\right)\sqrt{3x^2+2} + \operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2} + 3x\right.$
default	$-\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3} - \frac{49\sqrt{3x^2+2}}{27} + \frac{4x^3\sqrt{3x^2+2}}{3} + 2x\sqrt{3x^2+2} + \frac{28x^2\sqrt{3x^2+2}}{9}$
meijerg	$\frac{\sqrt{3}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3} + \frac{20\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2} - \sqrt{\pi}\operatorname{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}} + \frac{7\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}} + \frac{28\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}\right.}{}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*(36*x^3+84*x^2+54*x-49)*(3*x^2+2)^(1/2)-arcsinh(1/2*x*6^(1/2))*3^(1/2
)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(36\,x^3+84\,x^2+54\,x-49\right)\sqrt{3\,x^2+2} + \frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x - 3\,x^2 - 1\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(
3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{4x^3\sqrt{3x^2+2}}{3}+\frac{28x^2\sqrt{3x^2+2}}{9}+2x\sqrt{3x^2+2} - \frac{49\sqrt{3x^2+2}}{27}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
 2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{4}{3}\sqrt{3\,x^2+2}x^3+\frac{28}{9}\sqrt{3\,x^2+2}x^2+2\sqrt{3\,x^2+2}x - \sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{49}{27}\sqrt{3\,x^2+2}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
 sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(6\left(2\left(3\,x+7\right)x+9\right)x-49\right)\sqrt{3\,x^2+2} + \sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)
*x + sqrt(3*x^2 + 2))
```

3.119.9 Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(4\,x^3+\frac{28\,x^2}{3}+6\,x-\frac{49}{9}\right)}{3}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*
asinh((6^(1/2)*x)/2)
```

3.120 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

3.120.1 Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx=\frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2+2)^(1/2)+7/27*(1+
3*x)*(3*x^2+2)^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx=\frac{1}{27}\sqrt{2+3x^2}(13+45x+24x^2)+\frac{7\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2))/27 + (7*Log[-(Sqrt[3]*x) + Sqrt[2 +
 3*x^2]])/(3*Sqrt[3])
```

3.120.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2185, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \ 2185$$

$$\frac{1}{36}\int -\frac{28(1-3x)(2x+1)}{\sqrt{3x^2+2}}dx+\frac{2}{9}\sqrt{3x^2+2}(2x+1)^2$$

$$\Big\downarrow \ 27$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\int \frac{(1-3x)(2x+1)}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \ 676$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\left(3\int \frac{1}{\sqrt{3x^2+2}}dx-\sqrt{3x^2+2}x-\frac{1}{3}\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \ 222$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2+2}-\frac{7}{9}\left(\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-\sqrt{3x^2+2}x-\frac{1}{3}\sqrt{3x^2+2}\right)$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 - (7*(-1/3*Sqrt[2 + 3*x^2] - x*Sqrt[2 +
3*x^2] + Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.120.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

method	result
risch	$\frac{(24x^2+45x+13)\sqrt{3x^2+2}}{27}-\frac{7\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
default	$-\frac{7\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}+\frac{13\sqrt{3x^2+2}}{27}+\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}$
trager	$\left(\frac{8}{9}x^2+\frac{5}{3}x+\frac{13}{27}\right)\sqrt{3x^2+2}+\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
meijerg	$\frac{\sqrt{3}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)}{3}+\frac{10\sqrt{3}\left(\frac{\sqrt{\pi}\,x\sqrt{3}\sqrt{2}\sqrt{\frac{3x^2}{2}+1}}{2}-\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{\frac{3x^2}{2}+1}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}\right.}{}$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/27*(24*x^2+45*x+13)*(3*x^2+2)^(1/2)-7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}\left(24\,x^2+45\,x+13\right)\sqrt{3\,x^2+2} + \frac{7}{18}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(
3*x^2 + 2)*x - 3*x^2 - 1)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{8x^2\sqrt{3x^2+2}}{9} + \frac{5x\sqrt{3x^2+2}}{3} + \frac{13\sqrt{3x^2+2}}{27} - \frac{7\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

output
```
8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/2
7 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{8}{9}\sqrt{3x^2+2}x^2 + \frac{5}{3}\sqrt{3x^2+2}x - \frac{7}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{13}{27}\sqrt{3x^2+2}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*
sqrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{1}{27}(3(8x+15)x+13)\sqrt{3x^2+2} + \frac{7}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x +
sqrt(3*x^2 + 2))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{8x^2}{3}+5x+\frac{13}{9}\right)}{3} - \frac{7\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{9}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh(
(6^(1/2)*x)/2))/9
```

3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$

3.121.1 Optimal result

Integrand size = 29, antiderivative size = 67

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{2\sqrt{11}}$$

output
```
1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)+2/3*(3*x^2+2)^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{2}{3}\sqrt{2+3x^2}+\frac{\operatorname{arctanh}\left(\sqrt{\frac{3}{11}}+2\sqrt{\frac{3}{11}}x-\frac{2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{2\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
(2*Sqrt[2 + 3*x^2])/3 + ArcTanh[Sqrt[3/11] + 2*Sqrt[3/11]*x - (2*Sqrt[2 +
3*x^2])/Sqrt[11]]/Sqrt[11] - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]]/(2*Sqrt[3
])
```

3.121.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2185, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{1}{12}\int \frac{12(x+1)}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{1}{2}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{488}$$

$$-\frac{1}{2}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}+\frac{2}{3}\sqrt{3x^2+2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{2\sqrt{3}}-\frac{\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{2}{3}\sqrt{3x^2+2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

output
```
(2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*
x)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.121.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result
default	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^{2}+2}}{3}-\frac{\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^{2}-12x+5}}\right)}{22}$
risch	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{6}+\frac{2\sqrt{3x^{2}+2}}{3}-\frac{\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^{2}-12x+5}}\right)}{22}$
trager	$\frac{2\sqrt{3x^{2}+2}}{3}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}-3\right)\sqrt{3x^{2}+2}+3x\right)}{6}+\frac{\operatorname{RootOf}\left(\_Z^{2}-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}-11\right)x+11\sqrt{3x^{2}+}}{1+2x}\right.}{22}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/3*(3*x^2+2)^(1/2)-1/22*11^(1/2)*arcta
nh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int\frac{1+3x+4x^{2}}{(1+2x)\sqrt{2+3x^{2}}}\,dx=\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^{2}+2}x-3\,x^{2}-1\right)+\frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3\,x^{2}+2}(3\,x-4)+21\,x^{2}-12\,x+19}{4\,x^{2}+4\,x+1}\right)+\frac{2}{3}\sqrt{3\,x^{2}+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*l
og(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x
 + 1)) + 2/3*sqrt(3*x^2 + 2)
```

3.121.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{1}{6}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{1}{22}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|} - \frac{2\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{2}{3}\sqrt{3\,x^2+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2
*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(1
1) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 13.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx = \frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{44} + \frac{2\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{6}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

output
```
(11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)
*3^(1/2)*x)/2))/6
```

3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

3.122.1 Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{11(1+2x)}+\frac{\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

output
```
1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*
x^2+2)^(1/2))*11^(1/2)-1/11*(3*x^2+2)^(1/2)/(1+2*x)
```

3.122.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{11+22x}-\frac{8\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{11\sqrt{11}} -\frac{\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
-(Sqrt[2 + 3*x^2]/(11 + 22*x)) - (8*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqr
t[2 + 3*x^2])/Sqrt[11]])/(11*Sqrt[11]) - Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2
]]/Sqrt[3]
```

3.122.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2182, 25, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{1}{11}\int -\frac{22x+7}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{11}\int \frac{22x+7}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{11}\left(11\int \frac{1}{\sqrt{3x^2+2}}dx-4\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{11}\left(\frac{11\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}-4\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{11}\left(4\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{11\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{11}\left(\frac{11\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}\right)-\frac{\sqrt{3x^2+2}}{11(2x+1)}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

output
```
-1/11*Sqrt[2 + 3*x^2]/(1 + 2*x) + ((11*ArcSinh[Sqrt[3/2]*x])/Sqrt[3] + (4*
ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/Sqrt[11])/11
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.122.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}+\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$
default	$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$
trager	$-\frac{\sqrt{3x^2+2}}{11(1+2x)}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}-\frac{4\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{\cdots}}{1+\cdots}\right.}{121}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/11*(3*x^2+2)^(1/2)/(1+2*x)+1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*11^
(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$
$$=\frac{121\sqrt{3}(2x+1)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+12\sqrt{11}(2x+1)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)-\cdots}{726\,(2x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 +
12*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

3.122.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{4}{121}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|} - \frac{2\sqrt{6}}{3\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2+2}}{11\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/
abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(56) = 112.

Time = 0.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.69

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}+\frac{\sqrt{11}}{2\,x+1}\right)-3\right)}{121\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}+\frac{2\sqrt{11}}{2\,x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}+\frac{\sqrt{11}}{2\,x+1}\right)}\right)}{3\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}}{22\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
4/121*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqr
t(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3
) + 2*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + 2*sqrt(11)/(2*x + 1))/(sqr
t(3) + sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)))/sgn(
1/(2*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1
))
```

3.122.9 Mupad [B] (verification not implemented)

Time = 13.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{3} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{121} + \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{22\,\left(x+\frac{1}{2}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

output
```
(3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
 (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 -
(3^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

3.123.1 Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
-103/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)-1/22*(3*
x^2+2)^(1/2)/(1+2*x)^2+13/242*(3*x^2+2)^(1/2)/(1+2*x)
```

3.123.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{\frac{11(1+13x)\sqrt{2+3x^2}}{(1+2x)^2}+206\sqrt{11}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{1331}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 + 206*Sqrt[11]*ArcTanh[(Sqrt[
3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/1331
```

3.123.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{1}{22}\int -\frac{41x+14}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{22}\int \frac{41x+14}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{22}\left(\frac{206}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{13\sqrt{3x^2+2}}{11(2x+1)}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{22}\left(\frac{13\sqrt{3x^2+2}}{11(2x+1)}-\frac{206}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{22}\left(\frac{13\sqrt{3x^2+2}}{11(2x+1)}-\frac{206\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}\right)-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

output
```
-1/22*Sqrt[2 + 3*x^2]/(1 + 2*x)^2 + ((13*Sqrt[2 + 3*x^2])/(11*(1 + 2*x)) -
 (206*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11]))/22
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.123. $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

3.123.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{39x^3+3x^2+26x+2}{121(1+2x)^2\sqrt{3x^2+2}}-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$	65
trager	$\frac{(13x+1)\sqrt{3x^2+2}}{121(1+2x)^2}-\frac{103\,\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$	71
default	$-\frac{103\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}+\frac{13\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}$	74

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/121*(39*x^3+3*x^2+26*x+2)/(1+2*x)^2/(3*x^2+2)^(1/2)-103/1331*11^(1/2)*ar
ctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\sqrt{11}(4x^2+4x+1)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+22\sqrt{3x^2+2}(13x+1)}{2662\,(4x^2+4x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
 - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x
+ 1))/(4*x^2 + 4*x + 1)
```

3.123.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 + 2)), x)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103}{1331}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2+2}}{22\,(4\,x^2+4\,x+1)} + \frac{13\,\sqrt{3\,x^2+2}}{242\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(
2*x + 1)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.34

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right) + \frac{72\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-13\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2-168\,\sqrt{3}x+104\,\sqrt{3}+168\,\sqrt{3\,x^2+2}}{484\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(7
2*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2
))^2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sq
rt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx = \frac{103\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} - \frac{103\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{88\,\left(x^2+x+\frac{1}{4}\right)} + \frac{13\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{484\,\left(x+\frac{1}{2}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

output
```
(103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x +
x^2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

3.124.1 Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
-38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(398+279*x)/(3*x^2+2)^(1/2)+292/
81*(3*x^2+2)^(1/2)+4*x*(3*x^2+2)^(1/2)+32/27*x^2*(3*x^2+2)^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{2362+2133x+2136x^2+1944x^3+576x^4}{162\sqrt{2+3x^2}}+\frac{38\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4)/(162*Sqrt[2 + 3*x^2]) + (3
8*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

3.124.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2345, 27, 2346, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{1}{2}\int \frac{4(-48x^3-108x^2-70x+21)}{9\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\int \frac{-48x^3-108x^2-70x+21}{\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{9}\int \frac{3(-324x^2-146x+63)}{\sqrt{3x^2+2}}dx-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\int \frac{-324x^2-146x+63}{\sqrt{3x^2+2}}dx-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{2346}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(\frac{1}{6}\int \frac{6(171-146x)}{\sqrt{3x^2+2}}dx-54x\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(\int \frac{171-146x}{\sqrt{3x^2+2}}dx-54x\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(171\int \frac{1}{\sqrt{3x^2+2}}dx-54\sqrt{3x^2+2}x-\frac{146}{3}\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{2}{9}\left(\frac{1}{3}\left(57\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)-54\sqrt{3x^2+2}x-\frac{146}{3}\sqrt{3x^2+2}\right)-\frac{16}{3}x^2\sqrt{3x^2+2}\right)$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(398 + 279*x)/(54*Sqrt[2 + 3*x^2]) - (2*((-16*x^2*Sqrt[2 + 3*x^2])/3 + ((-
146*Sqrt[2 + 3*x^2])/3 - 54*x*Sqrt[2 + 3*x^2] + 57*Sqrt[3]*ArcSinh[Sqrt[3/
2]*x])/3))/9
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.124.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

method	result
risch	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}-\frac{38\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{576x^4+1944x^3+2136x^2+2133x+2362}{162\sqrt{3x^2+2}}-\frac{38\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$\frac{79x}{6\sqrt{3x^2+2}}+\frac{1181}{81\sqrt{3x^2+2}}+\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}-\frac{38\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{34\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{3\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+\frac{16\cdots}{}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/162*(576*x^4+1944*x^3+2136*x^2+2133*x+2362)/(3*x^2+2)^(1/2)-38/9*arcsinh
(1/2*x*6^(1/2))*3^(1/2)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{342\sqrt{3}(3\,x^2+2)\log\left(\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+\left(576\,x^4+1944\,x^3+21\cdots\right.}{162\left(3\,x^2+2\right)}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
+ (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2
+ 2)
```

3.124.6 Sympy [F]

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{32\,x^4}{9\sqrt{3\,x^2+2}} + \frac{12\,x^3}{\sqrt{3\,x^2+2}} + \frac{356\,x^2}{27\sqrt{3\,x^2+2}} - \frac{38}{9}\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) + \frac{79\,x}{6\sqrt{3\,x^2+2}} + \frac{1181}{81\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2
+ 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181
/81/sqrt(3*x^2 + 2)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}x + \sqrt{3\,x^2+2}\right) + \frac{3\,(8\,(3\,(8\,x+27)x+89)x+711)x+2362}{162\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)
*x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3} - \frac{38\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{9} - \frac{\sqrt{3}\sqrt{6}\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(1194+\sqrt{6}\,279\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*a
sinh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x
^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2
)*279i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

3.125 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

3.125.1 Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(70-47*x)/(3*x^2+2)^(1/2)+28/9*(3*
x^2+2)^(1/2)+8/9*x*(3*x^2+2)^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx=\frac{182-15x+168x^2+48x^3}{18\sqrt{2+3x^2}}-\frac{4\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(182 - 15*x + 168*x^2 + 48*x^3)/(18*Sqrt[2 + 3*x^2]) - (4*Log[-(Sqrt[3]*x)
 + Sqrt[2 + 3*x^2]])/(3*Sqrt[3])
```

3.125.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2345, 27, 2346, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \; 2345$$

$$\frac{70-47x}{18\sqrt{3x^2+2}} - \frac{1}{2}\int -\frac{8(12x^2+21x+7)}{9\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{4}{9}\int \frac{12x^2+21x+7}{\sqrt{3x^2+2}}dx + \frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \; 2346$$

$$\frac{4}{9}\left(\frac{1}{6}\int \frac{18(7x+1)}{\sqrt{3x^2+2}}dx + 2\sqrt{3x^2+2}x\right) + \frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{4}{9}\left(3\int \frac{7x+1}{\sqrt{3x^2+2}}dx + 2\sqrt{3x^2+2}x\right) + \frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \; 455$$

$$\frac{4}{9}\left(3\left(\int \frac{1}{\sqrt{3x^2+2}}dx + \frac{7}{3}\sqrt{3x^2+2}\right) + 2\sqrt{3x^2+2}x\right) + \frac{70-47x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \; 222$$

$$\frac{4}{9}\left(3\left(\frac{\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}} + \frac{7}{3}\sqrt{3x^2+2}\right) + 2\sqrt{3x^2+2}x\right) + \frac{70-47x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (4*(2*x*Sqrt[2 + 3*x^2] + 3*((7*Sqrt[2
+ 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/Sqrt[3])))/9
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.125.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result
risch	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$
trager	$\frac{48x^3+168x^2-15x+182}{18\sqrt{3x^2+2}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\mathrm{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$
default	$-\frac{5x}{6\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}+\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{4\,\mathrm{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}+\frac{28x^2}{3\sqrt{3x^2+2}}$
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{20\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{7\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{28\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(6x^2+8\right)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}+ 32$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(48*x^3+168*x^2-15*x+182)/(3*x^2+2)^(1/2)+4/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{4\,\sqrt{3}(3\,x^2+2)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(48\,x^3+168\,x^2-15\,x}{18\,(3\,x^2+2)}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

3.125.6 Sympy [F]

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{8\,x^3}{3\,\sqrt{3\,x^2+2}} + \frac{28\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{4}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right) - \frac{5\,x}{6\,\sqrt{3\,x^2+2}} + \frac{91}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1
/2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = -\frac{4}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(8\,(2\,x+7)x-5)x+182}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x -
5)*x + 182)/sqrt(3*x^2 + 2)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 13.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{4\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\left(\frac{8\,x}{3}+\frac{28}{3}\right)\sqrt{x^2+\frac{2}{3}}}{3} + \frac{\sqrt{3}\sqrt{6}\left(-630+\sqrt{6}\,141\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(630+\sqrt{6}\,141\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^
2 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2
)*1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*
(x^2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

3.125. $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

3.126 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

3.126.1 Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

output
```
10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(2-51*x)/(3*x^2+2)^(1/2)+8/9*(3*x
^2+2)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx=\frac{34-51x+48x^2}{18\sqrt{2+3x^2}}-\frac{10\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(34 - 51*x + 48*x^2)/(18*Sqrt[2 + 3*x^2]) - (10*Log[-(Sqrt[3]*x) + Sqrt[2
+ 3*x^2]])/(3*Sqrt[3])
```

3.126.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2345, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{3/2}}dx$$

$$\Big\downarrow \ 2345$$

$$\frac{2-51x}{18\sqrt{3x^2+2}}-\frac{1}{2}\int -\frac{4(4x+5)}{3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}\int \frac{4x+5}{\sqrt{3x^2+2}}dx+\frac{2-51x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \ 455$$

$$\frac{2}{3}\left(5\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{4}{3}\sqrt{3x^2+2}\right)+\frac{2-51x}{18\sqrt{3x^2+2}}$$

$$\Big\downarrow \ 222$$

$$\frac{2}{3}\left(\frac{5\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}+\frac{4}{3}\sqrt{3x^2+2}\right)+\frac{2-51x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

output
```
(2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (2*((4*Sqrt[2 + 3*x^2])/3 + (5*ArcSinh[S
qrt[3/2]*x])/Sqrt[3]))/3
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.126.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$	35
default	$-\frac{17x}{6\sqrt{3x^2+2}}+\frac{17}{9\sqrt{3x^2+2}}+\frac{8x^2}{3\sqrt{3x^2+2}}+\frac{10\,\operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{9}$	51
trager	$\frac{48x^2-51x+34}{18\sqrt{3x^2+2}}-\frac{10\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{9}$	53
meijerg	$\frac{\sqrt{2}\,x}{4\sqrt{\frac{3x^2}{2}+1}}+\frac{10\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{3}\,\sqrt{2}}{2\sqrt{\frac{3x^2}{2}+1}}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x\sqrt{3}\,\sqrt{2}}{2}\right)\right)}{9\sqrt{\pi}}+\frac{5\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{\frac{3x^2}{2}+1}}\right)}{6\sqrt{\pi}}+\frac{8\sqrt{2}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\,(6x^2+8)}{4\sqrt{\frac{3x^2}{2}+1}}\right)}{9\sqrt{\pi}}$	122

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(48*x^2-51*x+34)/(3*x^2+2)^(1/2)+10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{10\sqrt{3}(3x^2+2)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)+(48x^2-51x+34)\sqrt{3}}{18\left(3x^2+2\right)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

3.126.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.

Time = 5.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{30\sqrt{3}x^2\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18} + \frac{8x^2}{3\sqrt{3x^2+2}} - \frac{30x\sqrt{3x^2+2}}{27x^2+18} + \frac{x}{2\sqrt{3x^2+2}} + \frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}x}{2}\right)}{27x^2+18} + \frac{17}{9\sqrt{3x^2+2}}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

output
```
30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2
+ 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20
*sqrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{8\,x^2}{3\,\sqrt{3\,x^2+2}} + \frac{10}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right) - \frac{17\,x}{6\,\sqrt{3\,x^2+2}} + \frac{17}{9\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqr
t(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = -\frac{10}{9}\,\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right) + \frac{3\,(16\,x-17)x+34}{18\,\sqrt{3\,x^2+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x +
34)/sqrt(3*x^2 + 2)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx = \frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9} + \frac{10\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9} + \frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

output
```
(8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (
6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)
/(648*(x + (6^(1/2)*1i)/3))
```

3.126. $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{3/2}} \, dx$

3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

3.127.1 Optimal result

Integrand size = 29, antiderivative size = 53

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-38+21x}{66\sqrt{2+3x^2}}-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{11\sqrt{11}}$$

output
```
-2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/66*(-38+2
1*x)/(3*x^2+2)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx=\frac{-418+231x-12\sqrt{22+33x^2}\text{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{726\sqrt{2+3x^2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
(-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])
/(726*Sqrt[2 + 3*x^2])
```

3.127.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2178, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 2178}$$

$$-\frac{1}{6}\int -\frac{12}{11(2x+1)\sqrt{3x^2+2}}dx-\frac{38-21x}{66\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{38-21x}{66\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{ 488}$$

$$-\frac{2}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{38-21x}{66\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}-\frac{38-21x}{66\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

output
```
-1/66*(38 - 21*x)/Sqrt[2 + 3*x^2] - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2
+ 3*x^2])])/(11*Sqrt[11])
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.127.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$	48
trager	$\frac{-38+21x}{66\sqrt{3x^2+2}}-\frac{2\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{121}$	64
default	$\frac{x}{4\sqrt{3x^2+2}}-\frac{2}{3\sqrt{3x^2+2}}+\frac{1}{11\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{3x}{44\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{121}$	88

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/66*(-38+21*x)/(3*x^2+2)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/
2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{6\sqrt{11}(3\,x^2+2)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,\sqrt{3\,x^2+2}(21\,x-3}{726\,(3\,x^2+2)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 2
1*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3
*x^2 + 2)
```

3.127.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{7\,x}{22\,\sqrt{3\,x^2+2}}-\frac{19}{33\,\sqrt{3\,x^2+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{2}{121}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{21\,x-38}{66\,\sqrt{3\,x^2+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x
- 38)/sqrt(3*x^2 + 2)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx = \frac{\sqrt{11}\left(2\,\ln\left(x+\frac{1}{2}\right)-2\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,\left(114+\sqrt{6}\,21\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

output
```
(11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1
i)/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2
 + 2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

3.127. $\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{3/2}}\,dx$

3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx$

3.128.1 Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx=\frac{-10+97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
4/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/242*(-10+
97*x)/(3*x^2+2)^(1/2)-4/121*(3*x^2+2)^(1/2)/(1+2*x)
```

3.128.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx=\frac{11(-26+77x+170x^2)+8(1+2x)\sqrt{22+33x^2}\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{2662(1+2x)\sqrt{2+3x^2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
(11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*
x)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

3.128.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2 \left(3x^2+2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{6} \int -\frac{24(3-5x)}{121(2x+1)^2\sqrt{3x^2+2}}dx - \frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{121} \int \frac{3-5x}{(2x+1)^2\sqrt{3x^2+2}}dx - \frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{4}{121}\left(-\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx - \frac{\sqrt{3x^2+2}}{2x+1}\right) - \frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{4}{121}\left(\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}} - \frac{\sqrt{3x^2+2}}{2x+1}\right) - \frac{10-97x}{242\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{121}\left(\frac{\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}} - \frac{\sqrt{3x^2+2}}{2x+1}\right) - \frac{10-97x}{242\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

output
```
-1/242*(10 - 97*x)/Sqrt[2 + 3*x^2] + (4*(-(Sqrt[2 + 3*x^2]/(1 + 2*x)) + Ar
cTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/Sqrt[11]))/121
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.128. $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx$

3.128.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result
risch	$\frac{170x^2+77x-26}{242(1+2x)\sqrt{3x^2+2}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{1331}$
trager	$\frac{(170x^2+77x-26)\sqrt{3x^2+2}}{1452x^3+726x^2+968x+484}-\frac{4\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\operatorname{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{2\sqrt{3x^2+2}}-\frac{1}{22\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{2}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{18x}{121\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x}{11\sqrt{12\left(x+\frac{1}{2}\right.}}\right.}{1331}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/242*(170*x^2+77*x-26)/(1+2*x)/(3*x^2+2)^(1/2)+4/1331*11^(1/2)*arctanh(2/
11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\frac{4\sqrt{11}(6x^3+3x^2+4x+2)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)+11\left(170x^2\right.}{2662\left(6x^3+3x^2+4x+2\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)
*(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x -
 26)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

3.128.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{\left(2x+1\right)^2\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 + 2)**(3/2)), x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx=-\frac{4}{1331}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{85\,x}{242\,\sqrt{3\,x^2+2}}-\frac{2}{121\,\sqrt{3\,x^2+2}}-\frac{1}{11\,\left(2\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
-4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*
x^2 + 2)*x + sqrt(3*x^2 + 2))
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx = -\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3}+24\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{\frac{\frac{93}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{44}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{85}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}} + \frac{4\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{1331\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
-1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(
1/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)
)))/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 +
3) + 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
 + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

3.128.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx = \frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331} - \frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{97\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{121\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,5\mathrm{i}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

output
```
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 -
(4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x -
 (6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)
/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x
^2 + 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 +
2/3)^(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

3.128. $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{3/2}}\,dx$

3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{3/2}}\,dx$

3.129.1 Optimal result

Integrand size = 29, antiderivative size = 97

$$\int \frac{1+3x+4x^2}{(1+2x)^3\,(2+3x^2)^{3/2}}\,dx=\frac{358+351x}{2662\sqrt{2+3x^2}}-\frac{2\sqrt{2+3x^2}}{121(1+2x)^2}+\frac{2\sqrt{2+3x^2}}{1331(1+2x)}-\frac{322\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

output
```
-322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/2662*
(358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2/1331*(3*x^2+
2)^(1/2)/(1+2*x)
```

3.129.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{1+3x+4x^2}{(1+2x)^3\,(2+3x^2)^{3/2}}\,dx=\frac{\frac{11(278+1799x+2716x^2+1428x^3)}{(1+2x)^2\sqrt{2+3x^2}}+1288\sqrt{11}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2+3x^2}}{\sqrt{11}}\right)}{29282}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
((11*(278 + 1799*x + 2716*x^2 + 1428*x^3))/((1 + 2*x)^2*Sqrt[2 + 3*x^2]) +
 1288*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11
]])/29282
```

3.129.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2178, 27, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{351x+358}{2662\sqrt{3x^2+2}}-\frac{1}{6}\int -\frac{12\left(716x^2+606x+245\right)}{1331(2x+1)^3\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{716x^2+606x+245}{(2x+1)^3\sqrt{3x^2+2}}dx}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\left(-\frac{1}{22}\int -\frac{22(325x+157)}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\int \frac{325x+157}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{2\left(161\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{2\left(-161\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(-\frac{161\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}+\frac{\sqrt{3x^2+2}}{2x+1}-\frac{11\sqrt{3x^2+2}}{(2x+1)^2}\right)}{1331}+\frac{351x+358}{2662\sqrt{3x^2+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

output
```
(358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) + (2*((-11*Sqrt[2 + 3*x^2])/(1 + 2*x)
^2 + Sqrt[2 + 3*x^2]/(1 + 2*x) - (161*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/Sqrt[11]))/1331
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.129.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

method	result
risch	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{1428x^3+2716x^2+1799x+278}{2662(1+2x)^2\sqrt{3x^2+2}}-\frac{322\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{14641}$
default	$\frac{161}{1331\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{357x}{2662\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}+\frac{7}{484\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-3x}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2662*(1428*x^3+2716*x^2+1799*x+278)/(1+2*x)^2/(3*x^2+2)^(1/2)-322/14641*
11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{322\sqrt{11}(12\,x^4+12\,x^3+11\,x^2+8\,x+2)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right.}{29282\,(12\,x^4+12\,x^3+11\,x^2+8}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

output
```
1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*
sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(1
428*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*
x^2 + 8*x + 2)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx &= \frac{322}{14641}\sqrt{11}\,\mathrm{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) \\ &+ \frac{357\,x}{2662\,\sqrt{3\,x^2+2}}+\frac{161}{1331\,\sqrt{3\,x^2+2}} \\ &- \frac{1}{22\left(4\,\sqrt{3\,x^2+2}x^2+4\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}+\frac{7}{242\left(2\,\sqrt{3\,x^2+2}x+\sqrt{3\,x^2+2}\right)}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

output
```
322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*
x + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*
sqrt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sq
rt(3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.02

$$\int \frac{1+3x+4x^2}{(1+2x)^3\,(2+3x^2)^{3/2}}\,dx = \frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3\,x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3\,x^2+2}}\right) + \frac{351\,x+358}{2662\sqrt{3\,x^2+2}} + \frac{36\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+48\sqrt{3}x+8\sqrt{3}-48\sqrt{3\,x^2+2}}{1331\left(\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2}\right)-2\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

output
```
322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x
^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*
(351*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3
 - sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48
*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x -
sqrt(3*x^2 + 2)) - 2)^2
```

3.129.9 Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.86

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{3/2}}\,dx = \frac{322\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{14641} - \frac{322\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{117\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{242\left(x^2+x+\frac{1}{4}\right)} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,179\mathrm{i}}{15972\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

output
```
(322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2
)*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(53
24*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(
1/2)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2
)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)
*179i)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*1
79i)/(15972*(x + (6^(1/2)*1i)/3))
```

3.130 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.130.1 Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{398+279x}{162(2+3x^2)^{3/2}} - \frac{152+465x}{54\sqrt{2+3x^2}} + \frac{32}{27}\sqrt{2+3x^2} + \frac{8\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

output
```
1/162*(398+279*x)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*
(-152-465*x)/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{254-2511x+936x^2-4185x^3+1728x^4}{162(2+3x^2)^{3/2}} - \frac{8\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4)/(162*(2 + 3*x^2)^(3/2)) - (
8*Log[-(Sqrt[3]*x) + Sqrt[2 + 3*x^2]])/Sqrt[3]
```

3.130.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2345, 27, 2345, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{279x+398}{162(3x^2+2)^{3/2}}-\frac{1}{6}\int \frac{2(-96x^3-216x^2-140x+11)}{3(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{279x+398}{162(3x^2+2)^{3/2}}-\frac{1}{9}\int \frac{-96x^3-216x^2-140x+11}{(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{9}\left(\frac{1}{2}\int \frac{16(4x+9)}{\sqrt{3x^2+2}}dx-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(8\int \frac{4x+9}{\sqrt{3x^2+2}}dx-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{9}\left(8\left(9\int \frac{1}{\sqrt{3x^2+2}}dx+\frac{4}{3}\sqrt{3x^2+2}\right)-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9}\left(8\left(3\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)+\frac{4}{3}\sqrt{3x^2+2}\right)-\frac{465x+152}{6\sqrt{3x^2+2}}\right)+\frac{279x+398}{162(3x^2+2)^{3/2}}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) + (-1/6*(152 + 465*x)/Sqrt[2 + 3*x^2
] + 8*((4*Sqrt[2 + 3*x^2])/3 + 3*Sqrt[3]*ArcSinh[Sqrt[3/2]*x]))/9
```

3.130. $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.130.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

method	result
risch	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}$
trager	$\frac{1728x^4-4185x^3+936x^2-2511x+254}{162(3x^2+2)^{\frac{3}{2}}}+\frac{8\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{3}$
default	$-\frac{65x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}+\frac{127}{81(3x^2+2)^{\frac{3}{2}}}+\frac{32x^4}{3(3x^2+2)^{\frac{3}{2}}}+\frac{52x^2}{9(3x^2+2)^{\frac{3}{2}}}-\frac{8x^3}{(3x^2+2)^{\frac{3}{2}}}+\frac{8\ \operatorname{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{3}$
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{17\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{2\sqrt{\pi}}+\frac{68\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{16\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+15\right)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right.}{9\sqrt{\pi}}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/162*(1728*x^4-4185*x^3+936*x^2-2511*x+254)/(3*x^2+2)^(3/2)+8/3*arcsinh(1
/2*x*6^(1/2))*3^(1/2)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{216\,\sqrt{3}(9\,x^4+12\,x^2+4)\log\left(-\sqrt{3}\sqrt{3\,x^2+2}x-3\,x^2-1\right)+(1728\,x^4-4}{162\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3
*x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2)
)/(9*x^4 + 12*x^2 + 4)
```

3.130.6 Sympy [F]

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \int \frac{(2x+1)^3\cdot\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{32\,x^4}{3\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{8}{3}\,x\left(\frac{9\,x^2}{(3\,x^2+2)^{\frac{3}{2}}}+\frac{4}{(3\,x^2+2)^{\frac{3}{2}}}\right) + \frac{8}{3}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{6}x\right)-\frac{11\,x}{18\,\sqrt{3\,x^2+2}}+\frac{52\,x^2}{9\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{65\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}}+\frac{127}{81\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2
)^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) +
52/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2
)^(3/2)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3\,x^2+2}\right)+\frac{9\left(\left(3\left(64\,x-155\right)x+104\right)x-279\right)x+254}{162\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)
*x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.90

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{32\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27} + \frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{3} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(1824+\sqrt{6}\,1953\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{7776\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2
))/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1
/2)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i
)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x
 + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^
(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)
^(1/2)*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i +
 1824)*(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

3.130. $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.131 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.131.1 Optimal result

Integrand size = 29, antiderivative size = 60

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{70-47x}{54(2+3x^2)^{3/2}} - \frac{168+59x}{54\sqrt{2+3x^2}} + \frac{16\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

output
```
1/54*(70-47*x)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(
-168-59*x)/(3*x^2+2)^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{-266-165x-504x^2-177x^3}{54(2+3x^2)^{3/2}} - \frac{16\log\left(-\sqrt{3}x+\sqrt{2+3x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(-266 - 165*x - 504*x^2 - 177*x^3)/(54*(2 + 3*x^2)^(3/2)) - (16*Log[-(Sqrt
[3]*x) + Sqrt[2 + 3*x^2]])/(9*Sqrt[3])
```

3.131.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2345, 27, 2345, 27, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2+2)^{5/2}} dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{70-47x}{54(3x^2+2)^{3/2}} - \frac{1}{6}\int -\frac{2(144x^2+252x+37)}{9(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int \frac{144x^2+252x+37}{(3x^2+2)^{3/2}}dx + \frac{70-47x}{54(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{1}{27}\left(-\frac{1}{2}\int -\frac{96}{\sqrt{3x^2+2}}dx - \frac{59x+168}{2\sqrt{3x^2+2}}\right) + \frac{70-47x}{54(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\left(48\int \frac{1}{\sqrt{3x^2+2}}dx - \frac{59x+168}{2\sqrt{3x^2+2}}\right) + \frac{70-47x}{54(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{27}\left(16\sqrt{3}\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right) - \frac{59x+168}{2\sqrt{3x^2+2}}\right) + \frac{70-47x}{54(3x^2+2)^{3/2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) + (-1/2*(168 + 59*x)/Sqrt[2 + 3*x^2] +
16*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.131.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\ \text{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}$
trager	$-\frac{177x^3+504x^2+165x+266}{54(3x^2+2)^{\frac{3}{2}}}+\frac{16\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(\text{RootOf}\left(\_Z^2-3\right)\sqrt{3x^2+2}+3x\right)}{27}$
default	$-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}-\frac{133}{27(3x^2+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2+2)^{\frac{3}{2}}}+\frac{16\ \text{arcsinh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{3}}{27}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}$
meijerg	$\frac{\sqrt{2}\,x\left(3x^2+3\right)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{6\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{7\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{28\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\left(18x^2+8\right)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}+\frac{32\sqrt{3}\left(-\frac{\sqrt{\pi}\,x\sqrt{2}\,\sqrt{3}\left(30x^2+15\right)}{20\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\cdots\right)}{81\sqrt{\pi}}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/54*(177*x^3+504*x^2+165*x+266)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1
/2))*3^(1/2)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{16\sqrt{3}(9x^4+12x^2+4)\log\left(-\sqrt{3}\sqrt{3x^2+2}x-3x^2-1\right)-(177x^3+504}{54(9x^4+12x^2+4)}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x
^2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x
^2 + 4)
```

3.131.6 Sympy [F]

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{16}{27}x\left(\frac{9x^2}{(3x^2+2)^{\frac{3}{2}}}+\frac{4}{(3x^2+2)^{\frac{3}{2}}}\right) + \frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)+\frac{37x}{54\sqrt{3x^2+2}}-\frac{28x^2}{3(3x^2+2)^{\frac{3}{2}}}-\frac{37x}{18(3x^2+2)^{\frac{3}{2}}}-\frac{133}{27(3x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
-16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*a
rcsinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/
2) - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = -\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right) - \frac{3\left(\left(59x+168\right)x+55\right)x+266}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
 + 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.33

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{16\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{6}\left(-672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\left(672+\sqrt{6}\,63\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*
(((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)
/72 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1
/2)*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*
35i)/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6
^(1/2)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(
1/2)*6^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)
*1i)/3))
```

3.131. $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

3.132.1 Optimal result

Integrand size = 27, antiderivative size = 41

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{2-51x}{54(2+3x^2)^{3/2}}-\frac{16-13x}{18\sqrt{2+3x^2}}$$

output
```
1/54*(2-51*x)/(3*x^2+2)^(3/2)+1/18*(-16+13*x)/(3*x^2+2)^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx=\frac{-94+27x-144x^2+117x^3}{54(2+3x^2)^{3/2}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2345, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)(4x^2+3x+1)}{(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{2-51x}{54(3x^2+2)^{3/2}}-\frac{1}{6}\int -\frac{2(24x+13)}{3(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{24x+13}{(3x^2+2)^{3/2}}dx+\frac{2-51x}{54(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{ 453}$$

$$\frac{2-51x}{54(3x^2+2)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

output
```
(2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.132.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
trager	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
risch	$\frac{117x^3-144x^2+27x-94}{54(3x^2+2)^{\frac{3}{2}}}$	27
default	$-\frac{17x}{18(3x^2+2)^{\frac{3}{2}}}+\frac{13x}{18\sqrt{3x^2+2}}-\frac{47}{27(3x^2+2)^{\frac{3}{2}}}-\frac{8x^2}{3(3x^2+2)^{\frac{3}{2}}}$	51
meijerg	$\frac{\sqrt{2}\,x(3x^2+3)}{24\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,x^3}{12\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}+\frac{5\sqrt{2}\left(\frac{\sqrt{\pi}}{2}-\frac{\sqrt{\pi}}{2\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{18\sqrt{\pi}}+\frac{8\sqrt{2}\left(\sqrt{\pi}-\frac{\sqrt{\pi}\,(18x^2+8)}{8\left(\frac{3x^2}{2}+1\right)^{\frac{3}{2}}}\right)}{27\sqrt{\pi}}$	102

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{(2+3x^2)^{5/2}}\,dx=\frac{(117\,x^3-144\,x^2+27\,x-94)\sqrt{3\,x^2+2}}{54\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

3.132.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(36) = 72.

Time = 18.99 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.39

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}} + \frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} + \frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}} - \frac{32}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}} - \frac{5}{27x^2\sqrt{3x^2+2}+18\sqrt{3x^2+2}}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

output
```
10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sq
rt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) +
 54*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) -
 32/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x
**2 + 2) + 18*sqrt(3*x**2 + 2))
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{13\,x}{18\,\sqrt{3\,x^2+2}} - \frac{8\,x^2}{3\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{17\,x}{18\,\left(3\,x^2+2\right)^{\frac{3}{2}}} - \frac{47}{27\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^
(3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{9\,((13\,x-16)x+3)x-94}{54\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.51

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx = \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(192+\sqrt{6}\,69\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2592\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

output
```
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
 + (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27
- (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/
3) - (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/2
7 - (3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x -
(6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*
1i)/(2592*(x + (6^(1/2)*1i)/3))
```

3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

3.133.1 Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-38+21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}$$

output
```
1/198*(-38+21*x)/(3*x^2+2)^(3/2)-8/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{-274+801x+216x^2+855x^3}{2178\left(2+3x^2\right)^{3/2}}-\frac{8\text{arctanh}\left(\frac{4-3x}{\sqrt{22+33x^2}}\right)}{121\sqrt{11}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
(-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(
4 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

3.133.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2178, 27, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2178}$$

$$-\frac{1}{18}\int -\frac{6(14x+13)}{11(2x+1)\left(3x^2+2\right)^{3/2}}dx-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{33}\int \frac{14x+13}{(2x+1)\left(3x^2+2\right)^{3/2}}dx-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{1}{66}\int -\frac{144}{(2x+1)\sqrt{3x^2+2}}dx\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{33}\left(\frac{24}{11}\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx+\frac{95x+24}{22\sqrt{3x^2+2}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{24}{11}\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{33}\left(\frac{95x+24}{22\sqrt{3x^2+2}}-\frac{24\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}\right)-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

output
```
-1/198*(38 - 21*x)/(2 + 3*x^2)^(3/2) + ((24 + 95*x)/(22*Sqrt[2 + 3*x^2]) -
 (24*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11]))/33
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.133. $\int \frac{1+3x+4x^2}{(1+2x)(2+3x^2)^{5/2}} \, dx$

3.133.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

method	result
trager	$\frac{855x^3+216x^2+801x-274}{2178(3x^2+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{1331}$
default	$\frac{x}{12(3x^2+2)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}-\frac{2}{9(3x^2+2)^{\frac{3}{2}}}+\frac{1}{33\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{x}{44\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2178*(855*x^3+216*x^2+801*x-274)/(3*x^2+2)^(3/2)+8/1331*RootOf(_Z^2-11)*
ln((3*RootOf(_Z^2-11)*x+11*(3*x^2+2)^(1/2)-4*RootOf(_Z^2-11))/(1+2*x))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx = \frac{72\sqrt{11}(9\,x^4+12\,x^2+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+11\,(855\,x^3+}{23958\,(9\,x^4+12\,x^2+4)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(
3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2
+ 801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{8}{1331}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{95\,x}{726\,\sqrt{3\,x^2+2}}+\frac{4}{121\,\sqrt{3\,x^2+2}}+\frac{7\,x}{66\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{19}{99\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 +
2)^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx=\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\,\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\,\sqrt{3\,x^2+2}\right|}{2\,\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\,\sqrt{3\,x^2+2}}\right)+\frac{9\,((95\,x+24)x+89)x-274}{2178\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
+ 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*
((95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.99

$$\int \frac{1+3x+4x^2}{(1+2x)\,(2+3x^2)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\,\ln\left(x+\frac{1}{2}\right)-8\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331}$$
$$-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}$$
$$-\frac{\sqrt{3}\,\sqrt{6}\left(-288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\left(288+\sqrt{6}\,303\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{104544\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

output
```
(11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176
)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (
6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 +
21/176)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*
(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2
+ 2/3)^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)
*303i + 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx$

3.134.1 Optimal result

Integrand size = 29, antiderivative size = 95

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{-10+97x}{726(2+3x^2)^{3/2}} + \frac{24+887x}{7986\sqrt{2+3x^2}} - \frac{16\sqrt{2+3x^2}}{1331(1+2x)} - \frac{32\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

output
```
1/726*(-10+97*x)/(3*x^2+2)^(3/2)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3
*x^2+2)^(1/2))*11^(1/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16/1331*(3*x^2+2
)^(1/2)/(1+2*x)
```

3.134.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{11(-446+2717x+4602x^2+2805x^3+4458x^4)-192\sqrt{22+33x^2}(2+4x+}{87846(1+2x)(2+3x^2)^{3/2}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
(11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^
2]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*
(1 + 2*x)*(2 + 3*x^2)^(3/2))
```

3.134.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2178, 27, 2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2 \left(3x^2+2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{1}{18} \int -\frac{6 \left(388x^2+328x+133\right)}{121(2x+1)^2 \left(3x^2+2\right)^{3/2}}dx-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{363} \int \frac{388x^2+328x+133}{(2x+1)^2 \left(3x^2+2\right)^{3/2}}dx-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{2178}$$

$$\frac{1}{363} \left(\frac{887x+24}{22\sqrt{3x^2+2}}-\frac{1}{6} \int -\frac{288(x+6)}{11(2x+1)^2\sqrt{3x^2+2}}dx\right)-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{363} \left(\frac{48}{11} \int \frac{x+6}{(2x+1)^2\sqrt{3x^2+2}}dx+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{1}{363} \left(\frac{48}{11} \left(2 \int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{\sqrt{3x^2+2}}{2x+1}\right)+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{363} \left(\frac{48}{11} \left(-2 \int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{363} \left(\frac{48}{11} \left(-\frac{2\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}-\frac{\sqrt{3x^2+2}}{2x+1}\right)+\frac{887x+24}{22\sqrt{3x^2+2}}\right)-\frac{10-97x}{726 \left(3x^2+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

output
```
-1/726*(10 - 97*x)/(2 + 3*x^2)^(3/2) + ((24 + 887*x)/(22*Sqrt[2 + 3*x^2])
+ (48*(-(Sqrt[2 + 3*x^2]/(1 + 2*x)) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[
2 + 3*x^2])])/Sqrt[11]))/11)/363
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
risch	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)} - \frac{32\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{14641}$
trager	$\frac{4458x^4+2805x^3+4602x^2+2717x-446}{7986(3x^2+2)^{\frac{3}{2}}(1+2x)} + \frac{32\,\mathrm{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3x^2+2}-4\,\mathrm{RootOf}\left(\_Z^2-11\right)}{1+2x}\right)}{14641}$
default	$\frac{x}{6(3x^2+2)^{\frac{3}{2}}} + \frac{x}{6\sqrt{3x^2+2}} - \frac{1}{22\left(x+\frac{1}{2}\right)\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}} + \frac{4}{363\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}} - \frac{10x}{121\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}} - \frac{}{1331}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7986*(4458*x^4+2805*x^3+4602*x^2+2717*x-446)/(3*x^2+2)^(3/2)/(1+2*x)-32/
14641*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(x+1/2)^2-12*x+5)^(1/2))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2+3x^2)^{5/2}}\,dx = \frac{96\,\sqrt{11}(18\,x^5+9\,x^4+24\,x^3+12\,x^2+8\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}(3\,x-4)+21\,x^2-1}{4\,x^2+4\,x+1}\right.}{87846\,(18\,x^5+9\,x^4+24\,x^3}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sq
rt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1))
+ 11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*
x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{32}{14641}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right) + \frac{743\,x}{7986\,\sqrt{3\,x^2+2}} + \frac{16}{1331\,\sqrt{3\,x^2+2}} + \frac{61\,x}{726\,(3\,x^2+2)^{\frac{3}{2}}} - \frac{1}{11\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)} + \frac{4}{363\,(3\,x^2+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(
3*x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/36
3/(3*x^2 + 2)^(3/2)
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(76) = 152.

Time = 0.30 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = -\frac{1}{263538}\sqrt{11}\left(743\sqrt{11}\sqrt{3}-576\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{32\sqrt{11}\log\left(\sqrt{11}\left(\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}+\frac{\sqrt{11}}{2x+1}\right)-3\right)}{14641\operatorname{sgn}\left(\frac{1}{2x+1}\right)} + \frac{\frac{\frac{\frac{11\left(\frac{731}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{528}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{14163}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{6111}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{2229}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{7986\left(\frac{6}{2x+1}-\frac{11}{(2x+1)^2}-3\right)\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
-1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*
sgn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/
(2*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((1
1*(731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14
163/sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/
sgn(1/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) +
11/(2*x + 1)^2 + 3))
```

3.134. $\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx$

3.134.9 Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.84

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2+3x^2)^{5/2}}\,dx = \frac{\sqrt{11}\left(8\,\ln\left(x+\frac{1}{2}\right)-8\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641} + \frac{\sqrt{11}\left(\frac{48\,\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22} - \frac{8\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} - \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} + \frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{291}{1936}+\frac{\sqrt{6}\,15\mathrm{i}}{1936}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{97}{968}+\frac{\sqrt{6}\,5\mathrm{i}}{968}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27} - \frac{\sqrt{3}\sqrt{6}\left(-288+\sqrt{6}\,2481\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\left(288+\sqrt{6}\,2481\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1149984\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

input

```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

output

```
(11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))
/3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/
2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3
)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/193
6 - 291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*
1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2
)*15i)/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968
+ 97/968)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*
2481i - 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/
2)*6^(1/2)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1
/2)*1i)/3))
```

3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}}\,dx$

3.135.1 Optimal result

Integrand size = 29, antiderivative size = 117

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}}\,dx = \frac{358+351x}{7986(2+3x^2)^{3/2}}+\frac{1216+2133x}{29282\sqrt{2+3x^2}} -\frac{8\sqrt{2+3x^2}}{1331(1+2x)^2}-\frac{8\sqrt{2+3x^2}}{1331(1+2x)}-\frac{1216\operatorname{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{14641\sqrt{11}}$$

output
```
1/7986*(358+351*x)/(3*x^2+2)^(3/2)-1216/161051*arctanh(1/11*(4-3*x)*11^(1/
2)/(3*x^2+2)^(1/2))*11^(1/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8/1331*
(3*x^2+2)^(1/2)/(1+2*x)^2-8/1331*(3*x^2+2)^(1/2)/(1+2*x)
```

3.135.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2+3x^2)^{5/2}}\,dx = \frac{\frac{11(7010+57371x+109844x^2+116937x^3+111060x^4+67284x^5)}{(1+2x)^2(2+3x^2)^{3/2}}+14592\sqrt{11}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x}{\sqrt{}}\right.}{966306}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/
((1 + 2*x)^2*(2 + 3*x^2)^(3/2)) + 14592*Sqrt[11]*ArcTanh[(Sqrt[3] + 2*Sqrt
[3]*x - 2*Sqrt[2 + 3*x^2])/Sqrt[11]])/966306
```

3.135.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2178, 27, 2178, 27, 2182, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3(3x^2+2)^{5/2}}dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{351x+358}{7986(3x^2+2)^{3/2}}-\frac{1}{18}\int -\frac{18(936x^3+2836x^2+1914x+607)}{1331(2x+1)^3(3x^2+2)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{936x^3+2836x^2+1914x+607}{(2x+1)^3(3x^2+2)^{3/2}}dx}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2178}$$

$$\frac{\frac{2133x+1216}{22\sqrt{3x^2+2}}-\frac{1}{6}\int -\frac{96(304x^2+315x+142)}{11(2x+1)^3\sqrt{3x^2+2}}dx}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{11}\int \frac{304x^2+315x+142}{(2x+1)^3\sqrt{3x^2+2}}dx+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\frac{16}{11}\left(-\frac{1}{22}\int -\frac{11(271x+196)}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\int \frac{271x+196}{(2x+1)^2\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(152\int \frac{1}{(2x+1)\sqrt{3x^2+2}}dx-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986(3x^2+2)^{3/2}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(-152\int \frac{1}{11-\frac{(4-3x)^2}{3x^2+2}}d\frac{4-3x}{\sqrt{3x^2+2}}-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{16}{11}\left(\frac{1}{2}\left(-\frac{152\text{arctanh}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{\sqrt{11}}-\frac{11\sqrt{3x^2+2}}{2x+1}\right)-\frac{11\sqrt{3x^2+2}}{2(2x+1)^2}\right)+\frac{2133x+1216}{22\sqrt{3x^2+2}}}{1331}+\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

output
```
(358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + ((1216 + 2133*x)/(22*Sqrt[2 + 3*x
^2]) + (16*((-11*Sqrt[2 + 3*x^2])/(2*(1 + 2*x)^2) + ((-11*Sqrt[2 + 3*x^2])
/(1 + 2*x) - (152*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/Sqrt[11])
/2))/11)/1331
```

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.135.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

method	result
risch	$\frac{67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010}{87846(3x^2+2)^{\frac{3}{2}}(1+2x)^2}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(4-3x)\sqrt{11}}{11\sqrt{12\left(x+\frac{1}{2}\right)^2-12x+5}}\right)}{161051}$
trager	$\frac{(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010)\sqrt{3x^2+2}}{87846(6x^3+3x^2+4x+2)^2}+\frac{1216\,\operatorname{RootOf}\left(\_Z^2-11\right)\ln\left(\frac{3\,\operatorname{RootOf}\left(\_Z^2-11\right)x+11\sqrt{3\cdots}}{1+2\cdots}\right.}{161051}$
default	$\frac{152}{3993\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{87x}{2662\left(3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{1869x}{29282\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}+\frac{608}{14641\sqrt{3\left(x+\frac{1}{2}\right)^2-3x+\frac{5}{4}}}-\frac{1216\sqrt{11}\cdots}{\cdots}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/87846*(67284*x^5+111060*x^4+116937*x^3+109844*x^2+57371*x+7010)/(3*x^2+2
)^(3/2)/(1+2*x)^2-1216/161051*11^(1/2)*arctanh(2/11*(4-3*x)*11^(1/2)/(12*(
x+1/2)^2-12*x+5)^(1/2))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{5/2}}\,dx = \frac{3648\sqrt{11}(36\,x^6+36\,x^5+57\,x^4+48\,x^3+28\,x^2+16\,x+4)\log\left(-\frac{\sqrt{11}\sqrt{3\,x^2+2}}{}\right.}{966306\,(36\,x^6+}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

output
```
1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^
2 + 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 573
71*x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2
+ 16*x + 4)
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

output
```
Timed out
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3 (2+3x^2)^{5/2}}\,dx &= \frac{1216}{161051}\sqrt{11}\,\text{arsinh}\left(\frac{\sqrt{6}x}{2\,|2\,x+1|}-\frac{2\sqrt{6}}{3\,|2\,x+1|}\right) \\ &+ \frac{1869\,x}{29282\sqrt{3\,x^2+2}} + \frac{608}{14641\sqrt{3\,x^2+2}} + \frac{87\,x}{2662\,(3\,x^2+2)^{\frac{3}{2}}} \\ &- \frac{1}{22\left(4\,(3\,x^2+2)^{\frac{3}{2}}x^2+4\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)} \\ &+ \frac{1}{242\left(2\,(3\,x^2+2)^{\frac{3}{2}}x+(3\,x^2+2)^{\frac{3}{2}}\right)} + \frac{152}{3993\,(3\,x^2+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

output
```
1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(
2*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/
2662*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(
3/2)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(
3/2)) + 152/3993/(3*x^2 + 2)^(3/2)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.56

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)$$
$$+\frac{9\left((2133x+1216)x+1851\right)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+24\sqrt{3}x-8\sqrt{3}-24\sqrt{3x^2+2}\right)}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)-2\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

output
```
1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3
*x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/878
46*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt
(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

3.135.9 Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.57

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{1216\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{161051} - \frac{1216\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051} - \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{711\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{58564\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} - \frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x^2+x+\frac{1}{4}\right)} + \frac{179\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{95832\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)} - \frac{4\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}-\frac{2}{3}\right)} - \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,9265\mathrm{i}}{2108304\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)} + \frac{\sqrt{3}\sqrt{6}\sqrt{x^2+\frac{2}{3}}\,13\mathrm{i}}{21296\left(-x^2+\frac{2\mathrm{i}\sqrt{6}\,x}{3}+\frac{2}{3}\right)}$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

output
```
(1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(
1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))
/(95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/
(58564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x +
 (6^(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) +
 (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) -
(4*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2
/3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(
x^2 + 2/3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*
(x^2 + 2/3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)
*(x^2 + 2/3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

3.136 $\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$

3.136.1 Optimal result

Integrand size = 27, antiderivative size = 420

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \frac{f(g+hx)^{1+m} \left(a+cx^2\right)^{1+p}}{ch(3+m+2p)}$$
$$-\frac{\left(afh^2(1+m)-c\left(2fg^2(1+p)-h(eg-dh)(3+m+2p)\right)\right)(g+hx)^{1+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(\right.}{ch^3(1+m)(3+m+2p)}$$
$$-\frac{(2fg(1+p)-eh(3+m+2p))(g+hx)^{2+m}\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\text{AppellF1}\left(2\right.}{h^3(2+m)(3+m+2p)}$$

output
```
f*(h*x+g)^(1+m)*(c*x^2+a)^(p+1)/c/h/(3+m+2*p)-(a*f*h^2*(1+m)-c*(2*f*g^2*(p
+1)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,
2+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/c/h
^3/(1+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/
(g+h*(-a)^(1/2)/c^(1/2)))^p)-(2*f*g*(p+1)-e*h*(3+m+2*p))*(h*x+g)^(2+m)*(c*
x^2+a)^p*AppellF1(2+m,-p,-p,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(
g+h*(-a)^(1/2)/c^(1/2)))/h^3/(2+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/
c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)
```

3.136.2 Mathematica [F]

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$$

input
```
Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

3.136.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2185, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p \left(d+ex+fx^2\right) (g+hx)^m \, dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -h(g+hx)^m(h(af(m+1)-cd(m+2p+3))+c(2fg(p+1)-eh(m+2p+3))x)\left(cx^2+a\right)^p dx}{ch^2(m+2p+3)}+\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}-\frac{\int h(g+hx)^m(h(af(m+1)-cd(m+2p+3))+c(2fg(p+1)-eh(m+2p+3))x)\left(cx^2+a\right)^p dx}{ch^2(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}-\frac{\int (g+hx)^m(h(af(m+1)-cd(m+2p+3))+c(2fg(p+1)-eh(m+2p+3))x)\left(cx^2+a\right)^p dx}{ch(m+2p+3)}$$

$$\Big\downarrow \text{719}$$

$$\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}-\frac{\frac{\left(afh^2(m+1)-c\left(2fg^2(p+1)-h(m+2p+3)(eg-dh)\right)\right)\int(g+hx)^m\left(cx^2+a\right)^pdx}{h}+\frac{c(2fg(p+1)-eh(m+2p+3))\int(g+hx)^{m+1}\left(cx^2+a\right)^pdx}{h}}{ch(m+2p+3)}$$

$$\Big\downarrow \; 514$$

$$\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}-\frac{\frac{\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\left(afh^2(m+1)-c\left(2fg^2(p+1)-h(m+2p+3)(eg-dh)\right)\right)\int(g+hx)^m\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^p\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right.}{h^2}}{ch(m}$$

$$\Big\downarrow \; 150$$

$$\frac{f\left(a+cx^2\right)^{p+1}(g+hx)^{m+1}}{ch(m+2p+3)}-\frac{\left(a+cx^2\right)^p(g+hx)^{m+1}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\text{AppellF1}\left(m+1,-p,-p,m+2,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)-c\left(2fg^2(p+1)-h(\right.\right.}{h^2(m+1)}$$

input
```
Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - (((a*f*h^2
*(1 + m) - c*(2*f*g^2*(1 + p) - h*(e*g - d*h)*(3 + m + 2*p)))*(g + h*x)^(1
 + m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a
]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^2*(1 + m)*(1 - (g
 + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqr
t[c]))^p) + (c*(2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m)*(a +
c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c
]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^2*(2 + m)*(1 - (g + h*x)/(g
- (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p))/
(c*h*(3 + m + 2*p))
```

3.136. $\int (g+hx)^m\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.136.4 Maple [F]

$$\int (hx+g)^m \left(c\,x^2+a\right)^p \left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

3.136.5 Fricas [F]

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int (g+hx)^m (a+cx^2)^p (d+ex+fx^2)\,dx = \int (fx^2+ex+d)(cx^2+a)^p(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

3.136.8 Giac [F]

$$\int (g+hx)^m (a+cx^2)^p (d+ex+fx^2)\,dx = \int (fx^2+ex+d)(cx^2+a)^p(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m (a+cx^2)^p (d+ex+fx^2)\,dx = \int (g+h\,x)^m\,(c\,x^2+a)^p\,(f\,x^2+e\,x+d)\,dx$$

input
```
int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.137 $\int (g+hx)^m \sqrt{a+cx^2} (d+ex+fx^2) \, dx$

3.137.1 Optimal result

Integrand size = 29, antiderivative size = 403

$$\int (g+hx)^m \sqrt{a+cx^2} (d+ex+fx^2) \, dx = \frac{f(g+hx)^{1+m} (a+cx^2)^{3/2}}{ch(4+m)} - \frac{(afh^2(1+m)-c(3fg^2-h(eg-dh)(4+m)))(g+hx)^{1+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(1+m,-\frac{1}{2},-\frac{1}{2},2+n\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}} - \frac{(3fg-eh(4+m))(g+hx)^{2+m}\sqrt{a+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{h^3(2+m)(4+m)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}}$$

output
```
f*(h*x+g)^(1+m)*(c*x^2+a)^(3/2)/c/h/(4+m)-(a*f*h^2*(1+m)-c*(3*f*g^2-h*(-d*
h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,(h*x+g)/(g-h*(-a)^
(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^2+a)^(1/2)/c/h^3/(1+
m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(-h*x-g)/(g+h*(-a)
^(1/2)/c^(1/2)))^(1/2)-(3*f*g-e*h*(4+m))*(h*x+g)^(2+m)*AppellF1(2+m,-1/2,-
1/2,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))
*(c*x^2+a)^(1/2)/h^3/(2+m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/
2)/(1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^(1/2)
```

3.137.2 Mathematica [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

3.137.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2185, 25, 27, 719, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)(g+hx)^m\,dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -h(g+hx)^m(h(af(m+1)-cd(m+4))+c(3fg-eh(m+4))x)\sqrt{cx^2+a}dx}{ch^2(m+4)}+\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\int h(g+hx)^m(h(af(m+1)-cd(m+4))+c(3fg-eh(m+4))x)\sqrt{cx^2+a}dx}{ch^2(m+4)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\int (g+hx)^m(h(af(m+1)-cd(m+4))+c(3fg-eh(m+4))x)\sqrt{cx^2+a}dx}{ch(m+4)}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{c(3fg-eh(m+4))\int (g+hx)^{m+1}\sqrt{cx^2+a}dx}{h}-\frac{\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)\int (g+hx)^m\sqrt{cx^2+a}dx}{h}}{ch(m+4)}$$

$\Big\downarrow$ 514

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{c\sqrt{a+cx^2}(3fg-eh(m+4))\int (g+hx)^{m+1}\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}}d(g+hx)}{h^2\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}-\frac{\sqrt{a+cx^2}\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)\int (g+hx)^{m}\ldots}{h^2\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+\ldots}{\ldots}}}}{ch(m+4)}$$

$\Big\downarrow$ 150

$$\frac{f\left(a+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{c\sqrt{a+cx^2}(g+hx)^{m+2}(3fg-eh(m+4))\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{h^2(m+2)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}}}-\frac{\sqrt{a+cx^2}(g+hx)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m-\ldots\right.}{h^2(m+1)\ldots}}{ch(m+4)}$$

input
```
Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) - (-(((3*c*f*g^2 - a
*f*h^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2
]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]),
 (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^2*(1 + m)*Sqrt[1 - (g + h*x)/(g
 - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]))
 + (c*(3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 +
 m, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g
+ (Sqrt[-a]*h)/Sqrt[c])])/(h^2*(2 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-a]*h
)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]))/(c*h*(4 + m))
```

3.137. $\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

3.137.4 Maple [F]

$$\int (hx+g)^m \left(f\,x^2+ex+d\right) \sqrt{c\,x^2+a}dx$$

input
```
int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

output
```
int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

3.137.5 Fricas [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.137.6 Sympy [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{a+cx^2}(g+hx)^m\left(d+ex+fx^2\right)\,dx$$

input
```
integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

3.137.7 Maxima [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.137.8 Giac [F]

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^m\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.138 $\int (g+hx)^{-3-2p} (a+cx^2)^p (d+ex+fx^2)\, dx$

3.138.1 Optimal result

Integrand size = 31, antiderivative size = 474

$$\int (g+hx)^{-3-2p} (a+cx^2)^p (d+ex+fx^2)\, dx = -\frac{(fg^2-egh+dh^2)(g+hx)^{-2(1+p)}(a+cx^2)^{1+p}}{2h(cg^2+ah^2)(1+p)} - \frac{f(g+hx)^{-2p}(a+cx^2)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g}{g+}\right.}{2h^3p} + \frac{(ah^2(2fg-eh)+c(fg^3-dgh^2))(\sqrt{-a}-\sqrt{c}x)\left(-\frac{(\sqrt{c}g+\sqrt{-a}h)(\sqrt{-a}+\sqrt{c}x)}{(\sqrt{c}g-\sqrt{-a}h)(\sqrt{-a}-\sqrt{c}x)}\right)^{-p}(g+hx)^{-1-2p}(a+cx^2)^p\,\text{I}}{h^2(\sqrt{c}g+\sqrt{-a}h)(cg^2+ah^2)(1+2p)}$$

output

```
-1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(p+1)/h/(a*h^2+c*g^2)/(p+1)/((h*x+g)^(2
+2*p))-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(h*x+g)/(g-h*(-a)^(1/2)
/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/h^3/p/((h*x+g)^(2*p))/((1+(-h*
x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p
)+(a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*(h*x+g)^(-1-2*p)*(c*x^2+a)^p*hyp
ergeom([-p, -1-2*p],[-2*p],2*(h*x+g)*(-a)^(1/2)*c^(1/2)/(-h*(-a)^(1/2)+g*c
^(1/2))/((-a)^(1/2)-x*c^(1/2)))*((-a)^(1/2)-x*c^(1/2))/h^2/(a*h^2+c*g^2)/(
1+2*p)/(h*(-a)^(1/2)+g*c^(1/2))/((-(h*(-a)^(1/2)+g*c^(1/2))*((-a)^(1/2)+x*
c^(1/2))/(-h*(-a)^(1/2)+g*c^(1/2))/((-a)^(1/2)-x*c^(1/2)))^p)
```

3.138.2 Mathematica [F]

$$\int (g+hx)^{-3-2p} \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \int (g+hx)^{-3-2p} \left(a+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$$

input
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

3.138.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2187, 25, 514, 150, 679, 489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p \left(d+ex+fx^2\right) (g+hx)^{-2p-3} \, dx$$

$$\Big\downarrow \text{2187}$$

$$\frac{\int -(g+hx)^{-2p-3} \left(fg^2-dh^2+h(2fg-eh)x\right) \left(cx^2+a\right)^p dx}{h^2}+\frac{f\int (g+hx)^{-2p-1} \left(cx^2+a\right)^p dx}{h^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\int (g+hx)^{-2p-1} \left(cx^2+a\right)^p dx}{h^2}-\frac{\int (g+hx)^{-2p-3} \left(fg^2-dh^2+h(2fg-eh)x\right) \left(cx^2+a\right)^p dx}{h^2}$$

$$\Big\downarrow \text{514}$$

$$\frac{f\left(a+cx^2\right)^p \left(1-\frac{g+hx}{g-\frac{\sqrt{-ah}}{\sqrt{c}}}\right)^{-p} \left(1-\frac{g+hx}{\frac{\sqrt{-ah}}{\sqrt{c}}+g}\right)^{-p} \int (g+hx)^{-2p-1} \left(1-\frac{g+hx}{g-\frac{\sqrt{-ah}}{\sqrt{c}}}\right)^p \left(1-\frac{g+hx}{g+\frac{\sqrt{-ah}}{\sqrt{c}}}\right)^p d(g+hx)}{h^3}-\frac{\int (g+hx)^{-2p-3} \left(fg^2-dh^2+h(2fg-eh)x\right) \left(cx^2+a\right)^p dx}{h^2}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\int (g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+a\right)^p\,dx}{h^2}-$$
$$\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}$$

$$\Big\downarrow \text{679}$$

$$-\frac{\frac{\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)\int (g+hx)^{-2(p+1)}\left(cx^2+a\right)^p dx}{ah^2+cg^2}+\frac{h\left(a+cx^2\right)^{p+1}(g+hx)^{-2(p+1)}\left(dh^2-egh+fg^2\right)}{2(p+1)\left(ah^2+cg^2\right)}}{h^2}-$$
$$\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}$$

$$\Big\downarrow \text{489}$$

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)}{2h^3p}$$
$$\frac{\frac{h\left(a+cx^2\right)^{p+1}(g+hx)^{-2(p+1)}\left(dh^2-egh+fg^2\right)}{2(p+1)\left(ah^2+cg^2\right)}-\frac{\left(\sqrt{-a}-\sqrt{c}x\right)\left(a+cx^2\right)^p(g+hx)^{-2p-1}\left(-\frac{\left(\sqrt{-a}+\sqrt{c}x\right)\left(\sqrt{-a}h+\sqrt{c}g\right)}{\left(\sqrt{-a}-\sqrt{c}x\right)\left(\sqrt{c}g-\sqrt{-a}h\right)}\right)^{-p}\left(ah^2(2fg-eh)+c\left(fg^3-d\right.\right.}{(2p+1)\left(\sqrt{-a}h+\sqrt{c}g\right)\left(ah^2+c\right.}}{h^2}$$

input
```
Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
-1/2*(f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (g + h*x)/(g - (Sqrt
[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*p*(g + h*x)^
(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sq
rt[-a]*h)/Sqrt[c]))^p) - ((h*(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/
(2*(c*g^2 + a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - ((a*h^2*(2*f*g - e*h)
+ c*(f*g^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*
x^2)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x)
)/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/((Sqrt[c]*g + Sqrt[-
a]*h)*(c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + S
qrt[c]*x))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p))/h^2
```

3.138. $\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 489
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*b, 2]}, Simp[(q - b*x)*(c + d*x)^(n + 1)*((a + b*x^2)^p/((n +
1)*(b*c + d*q)*((b*c + d*q)*((q + b*x)/((b*c - d*q)*(-q + b*x))))^p))*Hyper
geometric2F1[n + 1, -p, n + 2, 2*b*q*((c + d*x)/((b*c - d*q)*(q - b*x)))],
x]] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 2, 0]
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2187
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Simp[Coeff[Pq, x, q]/e^q   Int[(d + e*x)^(m + q
)*(a + b*x^2)^p, x], x] + Simp[1/e^q   Int[(d + e*x)^m*(a + b*x^2)^p*Expand
ToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, b, d,
e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.138.4 Maple [F]

$$\int (hx+g)^{-3-2p} \left(c\,x^{2}+a\right)^{p} \left(f\,x^{2}+ex+d\right) dx$$

input
```
int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

3.138.5 Fricas [F]

$$\int (g+hx)^{-3-2p} \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \int \left(fx^{2}+ex+d\right) \left(cx^{2}+a\right)^{p} (hx+g)^{-2p-3} dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p} \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \text{Timed out}$$

input
```
integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.138.8 Giac [F]

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \frac{\left(c\,x^2+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

input
```
int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

output
```
int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.139 $\int (d+ex)^m (-cd^2 + bde + be^2x + ce^2x^2)^p ((-cd + be$

3.139.1 Optimal result

Integrand size = 69, antiderivative size = 222

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$=\frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)}$$
$$-\frac{(beg(1+m+p)+c(dg(1-m)-ef(3+m+2p)))(d+ex)^m\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(cd-be-cex)^2\left(-d(cd\right.}{c^2e^2(2+p)(3+m+2p)}$$

output
```
g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)-(
b*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^m*(c*(e*x+d)/(-b*e+2*c*
d))^(-m-p)*(-c*e*x-b*e+c*d)^2*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^p*hypergeo
m([-m-p, 2+p],[3+p],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c^2/e^2/(2+p)/(3+m+2*p)
```

3.139.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$=\frac{(d+ex)^m(-cd+be+cex)^2(-((d+ex)(-be+c(d-ex))))^p\left(ceg(d+ex)+\frac{e(cdg(-1+m)-beg(1+m+p)+cef(3}{}\right.}{c^2e^3(3+m+2p)}$$

input
```
Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d)
+ b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

output
```
((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x)))
)^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 +
 m + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p
, 2 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3
*(3 + m + 2*p))
```

3.139.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.101$, Rules used = {2163, 27, 1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(bde+be^2x-cd^2+ce^2x^2\right)^p \left(x(beg-cdg+cef)+f(be-cd)+cegx^2\right)dx$$

$$\Big\downarrow \text{2163}$$

$$de\int \frac{(d+ex)^{m-1}(f+gx)\left(cx^2e^2+bxe^2-d(cd-be)\right)^{p+1}}{de}dx$$

$$\Big\downarrow \text{27}$$

$$\int (f+gx)(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+1}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(beg(m+p+1)+cdg(1-m)-cef(m+2p+3))\int (d+ex)^{m-1}\left(cx^2e^2+bxe^2-d(cd-be)\right)^{p+1}dx}{ce(m+2p+3)}$$

$$\Big\downarrow \text{1139}$$

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(m+p+1)+cdg(1-m)-cef(m+2p+3))\int \left(\frac{ex}{d}+1\right)^{m-1}\left(cx^2e^2+bxe^2-d(cd-\right.}{cde(m+2p+3)}$$

$\Big\downarrow$ 1138

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-p}(cdex-d(cd-be))^{-p}\left(-d(cd-be)+be^2x+ce^2x^2\right)^p(beg(m+p+1)+cdg(1-m)-c}{cde(m+2p+3)}$$

$\Big\downarrow$ 80

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(cdex-d(cd-be))^{-p}\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+cdg(1-m)-}{cde(m+2p+3)}$$

$\Big\downarrow$ 79

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(cdex-d(cd-be))^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+cdg(1-m)-ce}{c^2d^2e^2(p+2)(m+2p+3)}$$

input
```
Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) + b*e)
*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

output
```
(g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c
*e^2*(3 + m + 2*p)) - ((c*d*g*(1 - m) + b*e*g*(1 + m + p) - c*e*f*(3 + m +
 2*p))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(-(d*(c*d - b*e)
) + c*d*e*x)^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F
1[-m - p, 2 + p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*d^2*e^2*(
2 + p)*(3 + m + 2*p))
```

3.139.

$\int(d+ex)^m\left(-cd^2+bde+be^2x+ce^2x^2\right)^p\left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)dx$

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

3.139.
$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) dx$

rule 2163
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*
e + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

3.139.4 Maple [F]

$$\int (ex+d)^m \left(c\,e^2x^2+b\,e^2x+bde-c\,d^2\right)^p \left(-(-be+cd)\,f+(beg-cdg+cef)\,x+ceg\,x^2\right)dx$$

input
```
int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*
g+c*e*f)*x+c*e*g*x^2),x)
```

output
```
int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*
g+c*e*f)*x+c*e*g*x^2),x)
```

3.139.5 Fricas [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$=\int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*
g-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

output
```
integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

3.139.6 Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f
+(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.139.7 Maxima [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*
g-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

output
```
integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x
^2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

3.139.8 Giac [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*
g-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

output
```
integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x
^2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$=\int (d+e\,x)^m\,\left(c\,e\,g\,x^2+(b\,e\,g-c\,d\,g+c\,e\,f)\,x+f\,(b\,e-c\,d)\right)\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2+b\,e^2\,x\right)^p\,dx$$

input
```
int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c
*e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

output
```
int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c
*e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

3.140 $\int (a+bx+cx^2)^4 (A+Cx^2)\, dx$

3.140.1 Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int (a+bx+cx^2)^4 (A+Cx^2)\, dx = {} & a^4Ax+2a^3Abx^2+\frac{1}{3}a^2(6Ab^2+4aAc+a^2C)\,x^3 \\ & +ab\left(A(b^2+3ac)+a^2C\right)x^4 \\ & +\frac{1}{5}\left(A(b^4+12ab^2c+6a^2c^2)+2a^2(3b^2+2ac)\,C\right)x^5 \\ & +\frac{2}{3}b(b^2+3ac)\,(Ac+aC)x^6 \\ & +\frac{1}{7}\left(2Ac^2(3b^2+2ac)+(b^4+12ab^2c+6a^2c^2)\,C\right)x^7 \\ & +\frac{1}{2}bc\left(Ac^2+(b^2+3ac)\,C\right)x^8 \\ & +\frac{1}{9}c^2(Ac^2+6b^2C+4acC)\,x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}\end{aligned}$$

output
```
a^4*A*x+2*a^3*A*b*x^2+1/3*a^2*(4*A*a*c+6*A*b^2+C*a^2)*x^3+a*b*(A*(3*a*c+b^
2)+C*a^2)*x^4+1/5*(A*(6*a^2*c^2+12*a*b^2*c+b^4)+2*a^2*(2*a*c+3*b^2)*C)*x^5
+2/3*b*(3*a*c+b^2)*(A*c+C*a)*x^6+1/7*(2*A*c^2*(2*a*c+3*b^2)+(6*a^2*c^2+12*
a*b^2*c+b^4)*C)*x^7+1/2*b*c*(A*c^2+(3*a*c+b^2)*C)*x^8+1/9*c^2*(A*c^2+4*C*a
*c+6*C*b^2)*x^9+2/5*b*c^3*C*x^10+1/11*c^4*C*x^11
```

3.140.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01

$$\int (a+bx+cx^2)^4 (A+Cx^2)\, dx = a^4Ax+2a^3Abx^2+\frac{1}{3}a^2(6Ab^2+4aAc+a^2C)\,x^3 + ab(Ab^2+3aAc+a^2C)\,x^4 + \frac{1}{5}(Ab^4+12aAb^2c+6a^2Ac^2+6a^2b^2C+4a^3cC)\,x^5 + \frac{2}{3}b(b^2+3ac)\,(Ac+aC)x^6 + \frac{1}{7}(6Ab^2c^2+4aAc^3+b^4C+12ab^2cC+6a^2c^2C)\,x^7 + \frac{1}{2}bc(Ac^2+b^2C+3acC)\,x^8 + \frac{1}{9}c^2(Ac^2+6b^2C+4acC)\,x^9+\frac{2}{5}bc^3Cx^{10}+\frac{1}{11}c^4Cx^{11}$$

input
```
Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

output
```
a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A
*b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2
*b^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A
*b^2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(
A*c^2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9
+ (2*b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

3.140.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Cx^2)\,(a+bx+cx^2)^4\, dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(a^4A+4a^3Abx+4abx^3(a^2C+A(3ac+b^2))+a^2x^2(a^2C+4aAc+6Ab^2)+x^6(C(6a^2c^2+12ab^2c+b^4)+2A\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4Ax + 2a^3Abx^2 + abx^4(a^2C + A(3ac + b^2)) + \frac{1}{3}a^2x^3(a^2C + 4aAc + 6Ab^2) + \frac{1}{7}x^7(C(6a^2c^2 + 12ab^2c + b^4) + 2Ac^2(2ac + 3b^2)) + \frac{1}{5}x^5(A(6a^2c^2 + 12ab^2c + b^4) + 2a^2C(2ac + 3b^2)) + \frac{1}{9}c^2x^9(4acC + Ac^2 + 6b^2C) + \frac{1}{2}bcx^8(C(3ac + b^2) + Ac^2) + \frac{2}{3}bx^6(3ac + b^2)(aC + Ac) + \frac{2}{5}bc^3Cx^{10} + \frac{1}{11}c^4Cx^{11}$$

input
```
Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

output
```
a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A
*(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(
3*b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c
^2*(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^
2 + (b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2
*b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

3.140.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.140.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.05

method	result
norman	$\frac{c^4 C x^{11}}{11}+\frac{2 b c^3 C x^{10}}{5}+\left(\frac{1}{9} c^4 A+\frac{4}{9} C a c^3+\frac{2}{3} C b^2 c^2\right) x^9+\left(\frac{1}{2} A b c^3+\frac{3}{2} C a b c^2+\frac{1}{2} C b^3 c\right) x^8+\left(\frac{4}{7} A\right.$
gosper	$\frac{1}{3} x^3 a^4 C+\frac{1}{7} x^7 C b^4+\frac{4}{3} a^3 A c x^3+\frac{6}{5} a^2 A c^2 x^5+\frac{4}{7} a A c^3 x^7+x^4 A b^3 a+2 x^3 A a^2 b^2+2 x^2 A a^3 b$ -
risch	$\frac{1}{3} x^3 a^4 C+\frac{1}{7} x^7 C b^4+\frac{4}{3} a^3 A c x^3+\frac{6}{5} a^2 A c^2 x^5+\frac{4}{7} a A c^3 x^7+x^4 A b^3 a+2 x^3 A a^2 b^2+2 x^2 A a^3 b$ -
parallelrisch	$\frac{1}{3} x^3 a^4 C+\frac{1}{7} x^7 C b^4+\frac{4}{3} a^3 A c x^3+\frac{6}{5} a^2 A c^2 x^5+\frac{4}{7} a A c^3 x^7+x^4 A b^3 a+2 x^3 A a^2 b^2+2 x^2 A a^3 b$ -
default	$\frac{c^4 C x^{11}}{11}+\frac{2 b c^3 C x^{10}}{5}+\frac{\left(\left(2\left(2 a c+b^2\right) c^2+4 b^2 c^2\right) C+c^4 A\right) x^9}{9}+\frac{\left(\left(4 b a c^2+4\left(2 a c+b^2\right) b c\right) C+4 A b c^3\right) x^8}{8}+\frac{\left(\left(2 a^2 c^2+8 a b^2 c\right.\right.}{}$

input
```
int((c*x^2+b*x+a)^4*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+(1/9*c^4*A+4/9*C*a*c^3+2/3*C*b^2*c^2)*x^9
+(1/2*A*b*c^3+3/2*C*a*b*c^2+1/2*C*b^3*c)*x^8+(4/7*A*a*c^3+6/7*A*b^2*c^2+6/
7*C*a^2*c^2+12/7*C*a*b^2*c+1/7*C*b^4)*x^7+(2*a*A*b*c^2+2/3*A*b^3*c+2*C*a^2
*b*c+2/3*C*a*b^3)*x^6+(6/5*A*a^2*c^2+12/5*A*a*b^2*c+1/5*A*b^4+4/5*C*a^3*c+
6/5*C*a^2*b^2)*x^5+(3*A*a^2*b*c+A*a*b^3+C*a^3*b)*x^4+(4/3*A*a^3*c+2*A*a^2*
b^2+1/3*a^4*C)*x^3+2*x^2*A*a^3*b+a^4*A*x
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \left(a+bx+cx^2\right)^4\left(A+Cx^2\right)\,dx = {} & \frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{1}{9}\left(6\,Cb^2c^2+4\,Cac^3+Ac^4\right)x^9 \\ & +\frac{1}{2}\left(Cb^3c+3\,Cabc^2+Abc^3\right)x^8 \\ & +\frac{1}{7}\left(Cb^4+12\,Cab^2c+4\,Aac^3+6\left(Ca^2+Ab^2\right)c^2\right)x^7 \\ & +2\,Aa^3bx^2 \\ & +\frac{2}{3}\left(Cab^3+3\,Aabc^2+\left(3\,Ca^2b+Ab^3\right)c\right)x^6+Aa^4x \\ & +\frac{1}{5}\left(6\,Ca^2b^2+Ab^4+6\,Aa^2c^2+4\left(Ca^3+3\,Aab^2\right)c\right)x^5 \\ & +\left(Ca^3b+Aab^3+3\,Aa^2bc\right)x^4 \\ & +\frac{1}{3}\left(Ca^4+6\,Aa^2b^2+4\,Aa^3c\right)x^3\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

output
```
1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)
*x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2
*c + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3
 + 3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 +
 A*b^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 +
 3*A*a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.26

$$\int (a+bx+cx^2)^4 (A+Cx^2)\,dx = Aa^4x + 2Aa^3bx^2 + \frac{2Cbc^3x^{10}}{5} + \frac{Cc^4x^{11}}{11} + x^9\left(\frac{Ac^4}{9} + \frac{4Cac^3}{9} + \frac{2Cb^2c^2}{3}\right) + x^8\left(\frac{Abc^3}{2} + \frac{3Cabc^2}{2} + \frac{Cb^3c}{2}\right) + x^7 \cdot \left(\frac{4Aac^3}{7} + \frac{6Ab^2c^2}{7} + \frac{6Ca^2c^2}{7} + \frac{12Cab^2c}{7} + \frac{Cb^4}{7}\right) + x^6 \cdot \left(2Aabc^2 + \frac{2Ab^3c}{3} + 2Ca^2bc + \frac{2Cab^3}{3}\right) + x^5 \cdot \left(\frac{6Aa^2c^2}{5} + \frac{12Aab^2c}{5} + \frac{Ab^4}{5} + \frac{4Ca^3c}{5} + \frac{6Ca^2b^2}{5}\right) + x^4 \cdot (3Aa^2bc + Aab^3 + Ca^3b) + x^3 \cdot \left(\frac{4Aa^3c}{3} + 2Aa^2b^2 + \frac{Ca^4}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

output
```
A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(
A*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c*
*2/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/
7 + 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*
a**2*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**
4/5 + 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*
a**3*b) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx &= \frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{1}{9}\left(6\,Cb^2c^2+4\,Cac^3+Ac^4\right)x^9\\
&\quad+\frac{1}{2}\left(Cb^3c+3\,Cabc^2+Abc^3\right)x^8\\
&\quad+\frac{1}{7}\left(Cb^4+12\,Cab^2c+4\,Aac^3+6\left(Ca^2+Ab^2\right)c^2\right)x^7\\
&\quad+2\,Aa^3bx^2\\
&\quad+\frac{2}{3}\left(Cab^3+3\,Aabc^2+\left(3\,Ca^2b+Ab^3\right)c\right)x^6+Aa^4x\\
&\quad+\frac{1}{5}\left(6\,Ca^2b^2+Ab^4+6\,Aa^2c^2+4\left(Ca^3+3\,Aab^2\right)c\right)x^5\\
&\quad+\left(Ca^3b+Aab^3+3\,Aa^2bc\right)x^4\\
&\quad+\frac{1}{3}\left(Ca^4+6\,Aa^2b^2+4\,Aa^3c\right)x^3
\end{aligned}
$$

input

```
integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

output

```
1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)
*x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2
*c + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3
 + 3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 +
 A*b^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 +
 3*A*a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.21

$$\int (a+bx+cx^2)^4 (A+Cx^2)\, dx = \frac{1}{11} Cc^4x^{11} + \frac{2}{5} Cbc^3x^{10} + \frac{2}{3} Cb^2c^2x^9 + \frac{4}{9} Cac^3x^9 + \frac{1}{9} Ac^4x^9 + \frac{1}{2} Cb^3cx^8 + \frac{3}{2} Cabc^2x^8 + \frac{1}{2} Abc^3x^8 + \frac{1}{7} Cb^4x^7 + \frac{12}{7} Cab^2cx^7 + \frac{6}{7} Ca^2c^2x^7 + \frac{6}{7} Ab^2c^2x^7 + \frac{4}{7} Aac^3x^7 + \frac{2}{3} Cab^3x^6 + 2\, Ca^2bcx^6 + \frac{2}{3} Ab^3cx^6 + 2\, Aabc^2x^6 + \frac{6}{5} Ca^2b^2x^5 + \frac{1}{5} Ab^4x^5 + \frac{4}{5} Ca^3cx^5 + \frac{12}{5} Aab^2cx^5 + \frac{6}{5} Aa^2c^2x^5 + Ca^3bx^4 + Aab^3x^4 + 3\, Aa^2bcx^4 + \frac{1}{3} Ca^4x^3 + 2\, Aa^2b^2x^3 + \frac{4}{3} Aa^3cx^3 + 2\, Aa^3bx^2 + Aa^4x$$

input
```
integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

output
```
1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
 1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1
/7*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7
+ 4/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 +
2*A*a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12
/5*A*a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2
*b*c*x^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x
^2 + A*a^4*x
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96

$$\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right)\,dx = x^5\left(\frac{4\,C\,a^3\,c}{5}+\frac{6\,C\,a^2\,b^2}{5}+\frac{6\,A\,a^2\,c^2}{5}+\frac{12\,A\,a\,b^2\,c}{5}+\frac{A\,b^4}{5}\right) + x^7\left(\frac{6\,C\,a^2\,c^2}{7}+\frac{12\,C\,a\,b^2\,c}{7}+\frac{4\,A\,a\,c^3}{7}+\frac{C\,b^4}{7} + \frac{6\,A\,b^2\,c^2}{7}\right)+x^3\left(\frac{C\,a^4}{3}+\frac{4\,A\,c\,a^3}{3}+2\,A\,a^2\,b^2\right) + x^9\left(\frac{2\,C\,b^2\,c^2}{3}+\frac{A\,c^4}{9}+\frac{4\,C\,a\,c^3}{9}\right)+\frac{C\,c^4\,x^{11}}{11} + A\,a^4\,x+\frac{2\,b\,x^6\,\left(b^2+3\,a\,c\right)\,\left(A\,c+C\,a\right)}{3} + a\,b\,x^4\,\left(C\,a^2+3\,A\,c\,a+A\,b^2\right) + \frac{b\,c\,x^8\,\left(C\,b^2+A\,c^2+3\,C\,a\,c\right)}{2}+2\,A\,a^3\,b\,x^2+\frac{2\,C\,b\,c^3\,x^{10}}{5}$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^4,x)
```

output
```
x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A
*a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a
*c^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3
) + x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 +
A*a^4*x + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 +
 3*A*a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b
*c^3*x^10)/5
```

3.141 $\int (a+bx+cx^2)^3 (A+Cx^2)\,dx$

3.141.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\int (a+bx+cx^2)^3 (A+Cx^2)\,dx = a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a(3A(b^2+ac)+a^2C)x^3+\frac{1}{4}b(A(b^2+6ac)+3a^2C)x^4+\frac{3}{5}(b^2+ac)(Ac+aC)x^5+\frac{1}{6}b(3Ac^2+(b^2+6ac)C)x^6+\frac{1}{7}c(Ac^2+3(b^2+ac)C)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9$$

output
```
a^3*A*x+3/2*a^2*A*b*x^2+1/3*a*(3*A*(a*c+b^2)+C*a^2)*x^3+1/4*b*(A*(6*a*c+b^
2)+3*C*a^2)*x^4+3/5*(a*c+b^2)*(A*c+C*a)*x^5+1/6*b*(3*A*c^2+(6*a*c+b^2)*C)*
x^6+1/7*c*(A*c^2+3*(a*c+b^2)*C)*x^7+3/8*b*c^2*C*x^8+1/9*c^3*C*x^9
```

3.141.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int (a+bx+cx^2)^3 (A+Cx^2)\,dx = a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a(3Ab^2+3aAc+a^2C)x^3+\frac{1}{4}b(Ab^2+6aAc+3a^2C)x^4+\frac{3}{5}(b^2+ac)(Ac+aC)x^5+\frac{1}{6}b(3Ac^2+b^2C+6acC)x^6+\frac{1}{7}c(Ac^2+3b^2C+3acC)x^7+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9$$

input
```
Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

output
```
a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(
A*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b
*(3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/
7 + (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

3.141.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Cx^2\right)\left(a+bx+cx^2\right)^3dx$$

$$\Big\downarrow \ 2188$$

$$\int \left(a^3A+bx^3\left(3a^2C+A\left(6ac+b^2\right)\right)+ax^2\left(a^2C+3A\left(ac+b^2\right)\right)+3a^2Abx+cx^6\left(3C\left(ac+b^2\right)+Ac^2\right)+bx^5\left(C\left(6\right.\right.\right.$$

$$\Big\downarrow \ 2009$$

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\left(6ac+b^2\right)+3Ac^2\right)+\frac{3}{5}x^5\left(ac+b^2\right)(aC+Ac)+\frac{3}{8}bc^2Cx^8+\frac{1}{9}c^3Cx^9$$

input
```
Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

output
```
a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*
(b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 +
(3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.141.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

method	result
norman	$\frac{c^3 C\,x^9}{9}+\frac{3b\,c^2 C\,x^8}{8}+\left(\frac{1}{7}A\,c^3+\frac{3}{7}a\,c^2 C+\frac{3}{7}C\,b^2 c\right)x^7+\left(\frac{1}{2}Ab\,c^2+Cabc+\frac{1}{6}C\,b^3\right)x^6+\left(\frac{3}{5}Aa\,c^2+\right.$
gosper	$\frac{1}{9}c^3 C\,x^9+\frac{3}{8}b\,c^2 C\,x^8+\frac{1}{7}A\,c^3 x^7+\frac{3}{7}x^7 a\,c^2 C+\frac{3}{7}x^7 C\,b^2 c+\frac{1}{2}x^6 Ab\,c^2+x^6 Cabc+\frac{1}{6}x^6 C\,b^3+\frac{3}{5}$
risch	$\frac{1}{9}c^3 C\,x^9+\frac{3}{8}b\,c^2 C\,x^8+\frac{1}{7}A\,c^3 x^7+\frac{3}{7}x^7 a\,c^2 C+\frac{3}{7}x^7 C\,b^2 c+\frac{1}{2}x^6 Ab\,c^2+x^6 Cabc+\frac{1}{6}x^6 C\,b^3+\frac{3}{5}$
parallelrisch	$\frac{1}{9}c^3 C\,x^9+\frac{3}{8}b\,c^2 C\,x^8+\frac{1}{7}A\,c^3 x^7+\frac{3}{7}x^7 a\,c^2 C+\frac{3}{7}x^7 C\,b^2 c+\frac{1}{2}x^6 Ab\,c^2+x^6 Cabc+\frac{1}{6}x^6 C\,b^3+\frac{3}{5}$
default	$\frac{c^3 C\,x^9}{9}+\frac{3b\,c^2 C\,x^8}{8}+\frac{\left(\left(a\,c^2+2b^2 c+c\left(2ac+b^2\right)\right)C+A\,c^3\right)x^7}{7}+\frac{\left(\left(4abc+b\left(2ac+b^2\right)\right)C+3Ab\,c^2\right)x^6}{6}+\frac{\left(\left(a\left(2ac+b^2\right)+2b^2 a\right.\right.}{}$

input
```
int((c*x^2+b*x+a)^3*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*C*x^9+3/8*b*c^2*C*x^8+(1/7*A*c^3+3/7*a*c^2*C+3/7*C*b^2*c)*x^7+(1/2
*A*b*c^2+C*a*b*c+1/6*C*b^3)*x^6+(3/5*A*a*c^2+3/5*A*b^2*c+3/5*c*a^2*C+3/5*C
*a*b^2)*x^5+(3/2*A*a*b*c+1/4*A*b^3+3/4*b*a^2*C)*x^4+(A*a^2*c+A*a*b^2+1/3*a
^3*C)*x^3+3/2*a^2*A*b*x^2+a^3*A*x
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx = \frac{1}{9} Cc^3x^9 + \frac{3}{8} Cbc^2x^8 + \frac{1}{7} \left(3 Cb^2c + 3 Cac^2 + Ac^3\right)x^7 + \frac{1}{6} \left(Cb^3 + 6 Cabc + 3 Abc^2\right)x^6 + \frac{3}{2} Aa^2bx^2 + \frac{3}{5} \left(Cab^2 + Aac^2 + \left(Ca^2 + Ab^2\right)c\right)x^5 + Aa^3x + \frac{1}{4} \left(3 Ca^2b + Ab^3 + 6 Aabc\right)x^4 + \frac{1}{3} \left(Ca^3 + 3 Aab^2 + 3 Aa^2c\right)x^3$$

input
```
integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

output
```
1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7
+ 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2
 + A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6
*A*a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx = Aa^3x + \frac{3Aa^2bx^2}{2} + \frac{3Cbc^2x^8}{8} + \frac{Cc^3x^9}{9} + x^7 \left(\frac{Ac^3}{7} + \frac{3Cac^2}{7} + \frac{3Cb^2c}{7}\right) + x^6 \left(\frac{Abc^2}{2} + Cabc + \frac{Cb^3}{6}\right) + x^5 \cdot \left(\frac{3Aac^2}{5} + \frac{3Ab^2c}{5} + \frac{3Ca^2c}{5} + \frac{3Cab^2}{5}\right) + x^4 \cdot \left(\frac{3Aabc}{2} + \frac{Ab^3}{4} + \frac{3Ca^2b}{4}\right) + x^3 \left(Aa^2c + Aab^2 + \frac{Ca^3}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

output
```
A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A
*c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b*
*3/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) +
 x**4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2
+ C*a**3/3)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx = \frac{1}{9}\,Cc^3x^9 + \frac{3}{8}\,Cbc^2x^8 + \frac{1}{7}\left(3\,Cb^2c + 3\,Cac^2 + Ac^3\right)x^7 + \frac{1}{6}\left(Cb^3 + 6\,Cabc + 3\,Abc^2\right)x^6 + \frac{3}{2}\,Aa^2bx^2 + \frac{3}{5}\left(Cab^2 + Aac^2 + \left(Ca^2 + Ab^2\right)c\right)x^5 + Aa^3x + \frac{1}{4}\left(3\,Ca^2b + Ab^3 + 6\,Aabc\right)x^4 + \frac{1}{3}\left(Ca^3 + 3\,Aab^2 + 3\,Aa^2c\right)x^3$$

input
```
integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

output
```
1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7
+ 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2
 + A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6
*A*a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.16

$$\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx = \frac{1}{9}\,Cc^3x^9 + \frac{3}{8}\,Cbc^2x^8 + \frac{3}{7}\,Cb^2cx^7 + \frac{3}{7}\,Cac^2x^7 + \frac{1}{7}\,Ac^3x^7 + \frac{1}{6}\,Cb^3x^6 + Cabcx^6 + \frac{1}{2}\,Abc^2x^6 + \frac{3}{5}\,Cab^2x^5 + \frac{3}{5}\,Ca^2cx^5 + \frac{3}{5}\,Ab^2cx^5 + \frac{3}{5}\,Aac^2x^5 + \frac{3}{4}\,Ca^2bx^4 + \frac{1}{4}\,Ab^3x^4 + \frac{3}{2}\,Aabcx^4 + \frac{1}{3}\,Ca^3x^3 + Aab^2x^3 + Aa^2cx^3 + \frac{3}{2}\,Aa^2bx^2 + Aa^3x$$

input
```
integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

output
```
1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*
A*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^
5 + 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4
+ 1/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*
x^3 + 3/2*A*a^2*b*x^2 + A*a^3*x
```

3.141.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int (a+bx+cx^2)^3 (A+Cx^2)\,dx = x^3\left(\frac{C\,a^3}{3}+A\,c\,a^2+A\,a\,b^2\right) + x^7\left(\frac{3\,C\,b^2\,c}{7}+\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right) + \frac{b\,x^4\,(3\,C\,a^2+6\,A\,c\,a+A\,b^2)}{4} + \frac{b\,x^6\,(C\,b^2+3\,A\,c^2+6\,C\,a\,c)}{6} + \frac{C\,c^3\,x^9}{9} + A\,a^3\,x + \frac{3\,x^5\,(b^2+a\,c)\,(A\,c+C\,a)}{5} + \frac{3\,A\,a^2\,b\,x^2}{2} + \frac{3\,C\,b\,c^2\,x^8}{8}$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^3,x)
```

output
```
x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*
C*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*
b^2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*
a))/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8
```

3.142 $\int (a+bx+cx^2)^2 (A+Cx^2) \, dx$

3.142.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int (a+bx+cx^2)^2 (A+Cx^2) \, dx = a^2Ax+aAbx^2+\frac{1}{3}\left(A(b^2+2ac)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4 +\frac{1}{5}\left(Ac^2+\left(b^2+2ac\right)C\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

output
```
a^2*A*x+a*A*b*x^2+1/3*(A*(2*a*c+b^2)+C*a^2)*x^3+1/2*b*(A*c+C*a)*x^4+1/5*(A
*c^2+(2*a*c+b^2)*C)*x^5+1/3*b*c*C*x^6+1/7*c^2*C*x^7
```

3.142.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)^2 (A+Cx^2) \, dx = a^2Ax+aAbx^2+\frac{1}{3}\left(Ab^2+2aAc+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4 +\frac{1}{5}\left(Ac^2+b^2C+2acC\right)x^5+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

input
```
Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

output
```
a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x
^4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

3.142.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Cx^2)\left(a+bx+cx^2\right)^2 dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^2\left(a^2C+A\left(2ac+b^2\right)\right)+a^2A+x^4\left(C\left(2ac+b^2\right)+Ac^2\right)+2bx^3(aC+Ac)+2aAbx+2bcCx^5+c^2Cx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

input
```
Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

output
```
a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x
^4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.142.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

method	result
default	$\frac{c^2 C\, x^7}{7}+\frac{bcC\, x^6}{3}+\frac{(A\, c^2+(2ac+b^2) C) x^5}{5}+\frac{(2Abc+2baC) x^4}{4}+\frac{(A(2ac+b^2)+C\, a^2) x^3}{3}+x^2 Aba+a^2 Ax$
norman	$\frac{c^2 C\, x^7}{7}+\frac{bcC\, x^6}{3}+\left(\frac{1}{5}A\, c^2+\frac{2}{5}Cac+\frac{1}{5}C\, b^2\right) x^5+\left(\frac{1}{2}Abc+\frac{1}{2}baC\right) x^4+\left(\frac{2}{3}Aac+\frac{1}{3}A\, b^2+\frac{1}{3}C\, a^2$
gosper	$\frac{1}{7}c^2 C\, x^7+\frac{1}{3}bcC\, x^6+\frac{1}{5}A\, c^2 x^5+\frac{2}{5}x^5 Cac+\frac{1}{5}x^5 C\, b^2+\frac{1}{2}x^4 Abc+\frac{1}{2}x^4 baC+\frac{2}{3}aAc\, x^3+\frac{1}{3}A\, b^2$
risch	$\frac{1}{7}c^2 C\, x^7+\frac{1}{3}bcC\, x^6+\frac{1}{5}A\, c^2 x^5+\frac{2}{5}x^5 Cac+\frac{1}{5}x^5 C\, b^2+\frac{1}{2}x^4 Abc+\frac{1}{2}x^4 baC+\frac{2}{3}aAc\, x^3+\frac{1}{3}A\, b^2$
parallelrisch	$\frac{1}{7}c^2 C\, x^7+\frac{1}{3}bcC\, x^6+\frac{1}{5}A\, c^2 x^5+\frac{2}{5}x^5 Cac+\frac{1}{5}x^5 C\, b^2+\frac{1}{2}x^4 Abc+\frac{1}{2}x^4 baC+\frac{2}{3}aAc\, x^3+\frac{1}{3}A\, b^2$

input
```
int((c*x^2+b*x+a)^2*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*C*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C
*a*b)*x^4+1/3*(A*(2*a*c+b^2)+C*a^2)*x^3+x^2*A*b*a+a^2*A*x
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+bx+cx^2)^2 (A+Cx^2)\, dx = \frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}(Cb^2+2\,Cac+Ac^2)x^5+Aabx^2 + \frac{1}{2}(Cab+Abc)x^4+Aa^2x+\frac{1}{3}(Ca^2+Ab^2+2\,Aac)x^3$$

input
```
integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

output
```
1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*
x^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^
3
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right)\,dx = Aa^2x+Aabx^2+\frac{Cbcx^6}{3}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Abc}{2}+\frac{Cab}{2}\right)+x^3\cdot\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

output
```
A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*
C*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/
3 + C*a**2/3)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right)\,dx = \frac{1}{7}\,Cc^2x^7+\frac{1}{3}\,Cbcx^6+\frac{1}{5}\,(Cb^2+2\,Cac+Ac^2)x^5+Aabx^2+\frac{1}{2}\,(Cab+Abc)x^4+Aa^2x+\frac{1}{3}\,(Ca^2+Ab^2+2\,Aac)x^3$$

input
```
integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

output
```
1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*
x^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^
3
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right)\,dx = \frac{1}{7}\,Cc^2x^7+\frac{1}{3}\,Cbcx^6+\frac{1}{5}\,Cb^2x^5+\frac{2}{5}\,Cacx^5+\frac{1}{5}\,Ac^2x^5+\frac{1}{2}\,Cabx^4+\frac{1}{2}\,Abcx^4+\frac{1}{3}\,Ca^2x^3+\frac{1}{3}\,Ab^2x^3+\frac{2}{3}\,Aacx^3+Aabx^2+Aa^2x$$

input
```
integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

output
```
1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*
x^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*
A*a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

3.142.9 Mupad [B] (verification not implemented)

Time = 13.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int (a+bx+cx^2)^2 (A+Cx^2)\,dx = x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}+\frac{A\,b^2}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right) +\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{b\,x^4\,(A\,c+C\,a)}{2}+A\,a\,b\,x^2+\frac{C\,b\,c\,x^6}{3}$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^2,x)
```

output
```
x^3*((A*b^2)/3 + (C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (C*b^2)/5 + (
2*C*a*c)/5) + (C*c^2*x^7)/7 + A*a^2*x + (b*x^4*(A*c + C*a))/2 + A*a*b*x^2
+ (C*b*c*x^6)/3
```

3.143 $\int (a+bx+cx^2)(A+Cx^2)\,dx$

3.143.1 Optimal result

Integrand size = 18, antiderivative size = 46

$$\int (a+bx+cx^2)(A+Cx^2)\,dx = aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

output
```
a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

3.143.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)(A+Cx^2)\,dx = aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

input
```
Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
```

output
```
a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

3.143.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Cx^2\right)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^2(aC+Ac)+aA+Abx+bCx^3+cCx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

input
```
Int[(a + b*x + c*x^2)*(A + C*x^2),x]
```

output
```
a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.143.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ab\,x^{2}}{2}+\frac{(Ac+Ca)x^{3}}{3}+\frac{bC\,x^{4}}{4}+\frac{cC\,x^{5}}{5}$	39
norman	$\frac{cC\,x^{5}}{5}+\frac{bC\,x^{4}}{4}+\left(\frac{Ac}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ab\,x^{2}}{2}+aAx$	40
gosper	$\frac{1}{5}cC\,x^{5}+\frac{1}{4}bC\,x^{4}+\frac{1}{3}Ac\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ab\,x^{2}+aAx$	41
risch	$\frac{1}{5}cC\,x^{5}+\frac{1}{4}bC\,x^{4}+\frac{1}{3}Ac\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ab\,x^{2}+aAx$	41
parallelrisch	$\frac{1}{5}cC\,x^{5}+\frac{1}{4}bC\,x^{4}+\frac{1}{3}Ac\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ab\,x^{2}+aAx$	41

input
```
int((c*x^2+b*x+a)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=\frac{1}{5}\,Ccx^{5}+\frac{1}{4}\,Cbx^{4}+\frac{1}{2}\,Abx^{2}+\frac{1}{3}\,(Ca+Ac)x^{3}+Aax$$

input
```
integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
```

output
```
1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx=Aax+\frac{Abx^{2}}{2}+\frac{Cbx^{4}}{4}+\frac{Ccx^{5}}{5}+x^{3}\left(\frac{Ac}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((c*x**2+b*x+a)*(C*x**2+A),x)
```

output
```
A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)(A+Cx^2)\,dx=\frac{1}{5}Ccx^5+\frac{1}{4}Cbx^4+\frac{1}{2}Abx^2+\frac{1}{3}(Ca+Ac)x^3+Aax$$

input
```
integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
```

output
```
1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int (a+bx+cx^2)(A+Cx^2)\,dx=\frac{1}{5}Ccx^5+\frac{1}{4}Cbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Acx^3+\frac{1}{2}Abx^2+Aax$$

input
```
integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
```

output
```
1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*
x
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int (a+bx+cx^2)(A+Cx^2)\,dx=\frac{C\,c\,x^5}{5}+\frac{C\,b\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{C\,a}{3}\right)x^3+\frac{A\,b\,x^2}{2}+A\,a\,x$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2),x)
```

output
```
x^3*((A*c)/3 + (C*a)/3) + A*a*x + (A*b*x^2)/2 + (C*b*x^4)/4 + (C*c*x^5)/5
```

3.144 $\int \frac{A+Cx^2}{a+bx+cx^2}\,dx$

3.144.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx=\frac{Cx}{c}-\frac{(2Ac^2+(b^2-2ac)\,C)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2-(2*A*c^2+(-2*a*c+b^2)*C)*arctanh((2*c*x+
b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx=\frac{Cx}{c}+\frac{(2Ac^2+b^2C-2acC)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

output
```
(C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

3.144.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{-aC+Ac-bCx}{c\left(a+bx+cx^2\right)}+\frac{C}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

output
```
(C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.144.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
default	$\frac{Cx}{c}+\frac{-\frac{bC\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(Ac-Ca+\frac{b^{2}C}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{Cx}{c}-\frac{2\ln\left(8Aa\,c^{3}-2A\,b^{2}c^{2}-8C\,a^{2}c^{2}+6Ca\,b^{2}c-C\,b^{4}-2\sqrt{-(4ac-b^{2})(2A\,c^{2}-2Cac+C\,b^{2})^{2}}\,cx-\sqrt{-(4ac-b^{2})(2A\,c^{2}-2Cac+C\,b^{2})}\right.}{c(4ac-b^{2})}$

input
```
int((C*x^2+A)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/c*(-1/2*b*C/c*ln(c*x^2+b*x+a)+2*(A*c-C*a+1/2*b^2*C/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.27

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx$$
$$=\left[\frac{(Cb^2-2\,Cac+2\,Ac^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(Cb^2c-4\,Cac^2)x-(Cb^3-}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(Cb^2-2\,Cac+2\,Ac^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(Cb^2c-4\,Cac^2)x+(Cb^3-4\,Cab}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(
C*b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^
2 - 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)
*x + (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(76) = 152$.

Time = 0.60 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.10

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \frac{Cx}{c} + \left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc - Cab - 4ac^2\left(-\frac{Cb}{2c^2} - \frac{\sqrt{-4ac+b^2}(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right.}{-2Ac^2+2Cac-}\right. + \left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}\,(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x + \frac{-Abc - Cab - 4ac^2\left(-\frac{Cb}{2c^2} + \frac{\sqrt{-4ac+b^2}(-2Ac^2+2Cac-Cb^2)}{2c^2\cdot(4ac-b^2)}\right.}{-2Ac^2+2Cac-}\right.$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

output
```
C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2
)/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**
2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b
**2))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c
- C*b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4
*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4
*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**
2*c*(-C*b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2
*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{A+Cx^2}{a+bx+cx^2}\,dx = \frac{Cx}{c} - \frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arc
tan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \frac{2\,A\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}} + \frac{C\,x}{c} \\ &\quad + \frac{C\,b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} - \frac{2\,C\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} \\ &\quad + \frac{C\,b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}} - \frac{2\,C\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}\end{aligned}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2),x)
```

output
```
(2*A*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b
^2)^(1/2) + (C*x)/c + (C*b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2))
 - (2*C*a*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4
*a*c - b^2)^(1/2)) + (C*b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c -
b^2)^(1/2)))/(c^2*(4*a*c - b^2)^(1/2)) - (2*C*a*b*c*log(a + b*x + c*x^2))/
(4*a*c^3 - b^2*c^2)
```

3.145 $\int \frac{A+Cx^2}{(a+bx+cx^2)^2} dx$

3.145.1 Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+(2Ac^2+(b^2-2ac)\,C)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{4(Ac+aC)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4
*(A*c+C*a)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx = \frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\,(-b^2+4ac)\,(a+x(b+cx))} + \frac{4(Ac+aC)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b
+ c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 +
4*a*c)^(3/2)
```

3.145.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$-\frac{\int \frac{2(Ac+aC)}{cx^2+bx+a}dx}{b^2-4ac}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2(aC+Ac)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4(aC+Ac)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4(aC+Ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
-((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.145.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{\left(2A\,c^{2}-2Cac+C\,b^{2}\right)x}{c\left(4ac-b^{2}\right)}+\frac{b\left(Ac+Ca\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{4\left(Ac+Ca\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{\frac{\left(2A\,c^{2}-2Cac+C\,b^{2}\right)x}{c\left(4ac-b^{2}\right)}+\frac{b\left(Ac+Ca\right)}{c\left(4ac-b^{2}\right)}}{c\,x^{2}+bx+a}+\frac{2\ln\left(\left(-8a\,c^{2}+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)Ac}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}+\frac{2\ln\left(\left(-8a\,c^{2}+2b^{2}c\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}\right.}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2
+b*x+a)+4*(A*c+C*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(96) = 192$.

Time = 0.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.11

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx$$
$$= \left[-\frac{Cab^3 - 4\,Aabc^2 + 2\,(Ca^2c + Aac^2 + (Cac^2 + Ac^3)x^2 + (Cabc + Abc^2)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-}{cx}\right.}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,c}\right.$$
$$-\frac{Cab^3 - 4\,Aabc^2 - 4\,(Ca^2c + Aac^2 + (Cac^2 + Ac^3)x^2 + (Cabc + Abc^2)x)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4}}{b^2}\right.}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 +
(C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A
*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(
a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 -
 4*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4
*C*a^2*b - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^
2)*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(92) = 184.

Time = 0.59 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.76

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx = -2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)+16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)-2b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)}{4Ac^2+4Cac}\right)+2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)\log\left(x+\frac{2Abc+2Cab+32a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)-16ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)+2b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}(Ac+Ca)}{4Ac^2+4Cac}\right)+\frac{Abc+Cab+x(2Ac^2-2Cac+Cb^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

output
```
-2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*
a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*
a*c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a
))/(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x
+ (2*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)
 - 16*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*
a*c - b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) + (A*b*c + C*a*b + x*(2
*A*c**2 - 2*C*a*c + C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b*
*2*c**2) + x*(4*a*b*c**2 - b**3*c))
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c
 - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.145.9 Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = \frac{\frac{A\,b\,c+C\,a\,b}{c\,(4\,a\,c-b^2)}+\frac{x\,(C\,b^2+2\,A\,c^2-2\,C\,a\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,\mathrm{atan}\left(\frac{\left(\frac{2\,(A\,c+C\,a)\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{4\,c\,x\,(A\,c+C\,a)}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,A\,c+2\,C\,a}\right)\,(A\,c+C\,a)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^2,x)
```

output
```
((A*b*c + C*a*b)/(c*(4*a*c - b^2)) + (x*(2*A*c^2 + C*b^2 - 2*C*a*c))/(c*(4
*a*c - b^2)))/(a + b*x + c*x^2) - (4*atan((((2*(A*c + C*a)*(b^3 - 4*a*b*c)
)/(4*a*c - b^2)^(5/2) - (4*c*x*(A*c + C*a))/(4*a*c - b^2)^(3/2))*(4*a*c -
b^2))/(2*A*c + 2*C*a))*(A*c + C*a))/(4*a*c - b^2)^(3/2)
```

3.145. $\int \frac{A+Cx^2}{(a+bx+cx^2)^2} \, dx$

3.146 $\int \frac{A+Cx^2}{(a+bx+cx^2)^3} dx$

3.146.1 Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3} dx = -\frac{bc\left(A+\frac{aC}{c}\right)+(2Ac^2+(b^2-2ac)C)x}{2c(b^2-4ac)(a+bx+cx^2)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2(b^2-4ac)^2(a+bx+cx^2)} - \frac{2(6Ac^2+(b^2+2ac)C)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/2*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)^2+1/2*(6*A*c+2*C*a+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(6
*A*c^2+(2*a*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(
5/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = \frac{1}{2}\left(\frac{(6Ac^2+(b^2+2ac)\,C)\,(b+2cx)}{c\,(b^2-4ac)^2\,(a+x(b+cx))} + \frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c\,(-b^2+4ac)\,(a+x(b+cx))^2} + \frac{4(6Ac^2+(b^2+2ac)\,C)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

output
```
(((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b +
c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(
a + x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

3.146.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2191, 27, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{2191}$$

$$-\frac{\int \frac{\frac{Cb^2}{c}+6Ac+2aC}{(cx^2+bx+a)^2}dx}{2\,(b^2-4ac)} - \frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(2aC+6Ac+\frac{b^2C}{c}\right)\int \frac{1}{(cx^2+bx+a)^2}dx}{2\,(b^2-4ac)} - \frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{1086}$$

$$-\frac{\left(2aC+6Ac+\frac{b^2C}{c}\right)\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\left(2aC+6Ac+\frac{b^2C}{c}\right)\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(2aC+6Ac+\frac{b^2C}{c}\right)\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*
(a + b*x + c*x^2)^2) - ((6*A*c + 2*a*C + (b^2*C)/c)*(-((b + 2*c*x)/((b^2 -
 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c))
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.146. $\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.146.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.67

method	result
default	$\frac{\frac{c\left(6A\,c^{2}+2Cac+C\,b^{2}\right)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(6A\,c^{2}+2Cac+C\,b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(10Aa\,c^{2}+2A\,b^{2}c-2c\,a^{2}C+5Ca\,b^{2}\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{b\left(10Aac-A\,b^{2}+6C\,a^{2}\right)}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}}{\left(c\,x^{2}+bx+a\right)^{2}}+\frac{2(6A\,c^{2}+2Cac+C\cdots}{(16a^{2}c^{2}-8a\cdots}$
risch	$\frac{\frac{c\left(6A\,c^{2}+2Cac+C\,b^{2}\right)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(6A\,c^{2}+2Cac+C\,b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(10Aa\,c^{2}+2A\,b^{2}c-2c\,a^{2}C+5Ca\,b^{2}\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{b\left(10Aac-A\,b^{2}+6C\,a^{2}\right)}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}}{\left(c\,x^{2}+bx+a\right)^{2}}-\frac{6\ln\left(\left(32a^{2}c^{3}-16a\cdots\right.\right.}{\cdots}$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2
*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2
*c+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/
(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(6*A*c^2+2*C*a*c+C*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(153) = 306$.

Time = 0.29 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.45

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^3}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*
A*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c +
6*A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b
*c^2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3
*A*b^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x +
b))/(c*x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*
A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b
^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*x), 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*
C*a*b^2*c^2 - 24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C
*a*b^3*c - 24*A*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2
+ 2*C*a^3*c + 6*A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b
^3*c + 2*C*a*b*c^2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 +
2*(2*C*a^2 + 3*A*b^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*...
```

3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(150) = 300$.

Time = 1.17 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.81

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^3}\, dx = -\sqrt{-\frac{1}{(4ac - b^2)^5}} \cdot (6Ac^2 + 2Cac$$
$$+Cb^2) \log\left(x + \frac{6Abc^2 + 2Cabc + Cb^3 - 64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}} \cdot (6Ac^2 + 2Cac + Cb^2) + 48a^2b^2c^2\sqrt{-\frac{1}{(4ac-}}}{}\right.$$
$$+\sqrt{-\frac{1}{(4ac - b^2)^5}} \cdot (6Ac^2 + 2Cac$$
$$+Cb^2) \log\left(x + \frac{6Abc^2 + 2Cabc + Cb^3 + 64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}} \cdot (6Ac^2 + 2Cac + Cb^2) - 48a^2b^2c^2\sqrt{-\frac{1}{(4ac-}}}{}\right.$$
$$+\frac{10Aabc - Ab^3 + 6Ca^2b + x^3 \cdot (12Ac^3 + 4Cac^2 + 2Cb^2c) + x^2 \cdot (18Abc^2 + 6Cabc + 3Cb^3) -}{32a^4c^2 - 16a^3b^2c + 2a^2b^4 + x^4 \cdot (32a^2c^4 - 16ab^2c^3 + 2b^4c^2) + x^3 \cdot (64a^2bc^3 - 32ab^3c^2 + 4b^5c) + x^2 \cdot (64}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c
**2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A
*c**2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a
*c + C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b*
*2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*
b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**
2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*
b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**
2) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*
c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**
3*(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c +
3*C*b**3) + x*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a
**4*c**2 - 16*a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c
**3 + 2*b**4*c**2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x
**2*(64*a**3*c**3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b*
*3*c + 4*a*b**5))
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.35

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = \frac{2\,(Cb^2+2\,Cac+6\,Ac^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,Cb^2cx^3+4\,Cac^2x^3+12\,Ac^3x^3+3\,Cb^3x^2+6\,Cabcx^2+18\,Abc^2x^2+10\,Cab^2x-4\,Ca^2cx+4\,Ab^2cx+}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*
a*c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 +
10*C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^
3 + 10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.49

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx = \frac{\frac{6\,C\,a^2\,b+10\,A\,c\,a\,b-A\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,(-2\,C\,a^2\,c+5\,C\,a\,b^2+10\,A\,a\,c^2+2\,A\,b^2\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{c\,x^3\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,(b^2+2\,a\,c)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3} + \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5)\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{(4\,a\,c-b^2)^{5/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{2\,c\,x\,(C\,b^2+6\,A\,c^2+2\,C\,a\,c)}{(4\,a\,c-b^2)^{5/2}}\right)\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}{C\,b^2+6\,A\,c^2+2\,C\,a\,c}\right)\,(C\,b^2+6\,A\,c^2+2\,C}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^3,x)
```

output
```
((6*C*a^2*b - A*b^3 + 10*A*a*b*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*
(10*A*a*c^2 + 2*A*b^2*c + 5*C*a*b^2 - 2*C*a^2*c))/(b^4 + 16*a^2*c^2 - 8*a*
b^2*c) + (3*b*x^2*(6*A*c^2 + C*b^2 + 2*C*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (c*x^3*(6*A*c^2 + C*b^2 + 2*C*a*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((
((b^5 + 16*a^2*b*c^2 - 8*a*b^3*c)*(6*A*c^2 + C*b^2 + 2*C*a*c))/((4*a*c - b
^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (2*c*x*(6*A*c^2 + C*b^2 + 2*C*
a*c))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*A*c^2 + C*b^
2 + 2*C*a*c))*(6*A*c^2 + C*b^2 + 2*C*a*c))/(4*a*c - b^2)^(5/2)
```

3.147 $\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx$

3.147.1 Optimal result

Integrand size = 20, antiderivative size = 206

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x}{3c\,(b^2-4ac)\,(a+bx+cx^2)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\,(b^2-4ac)^2\,(a+bx+cx^2)^2} - \frac{2(5Ac^2+(b^2+ac)\,C)\,(b+2cx)}{(b^2-4ac)^3\,(a+bx+cx^2)} + \frac{8c(5Ac^2+(b^2+ac)\,C)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{7/2}}$$

output
```
1/3*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)^3+1/3*(5*A*c+(a+b^2/c)*C)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(5
*A*c^2+(a*c+b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*c*(5*A*c^2+(a
*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = \frac{1}{3}\left(\frac{(5Ac^2+(b^2+ac)C)(b+2cx)}{c(b^2-4ac)^2(a+x(b+cx))^2} - \frac{6(5Ac^2+(b^2+ac)C)(b+2cx)}{(b^2-4ac)^3(a+x(b+cx))} + \frac{b^2Cx+aC(b-2cx)+Ac(b+2cx)}{c(-b^2+4ac)(a+x(b+cx))^3} + \frac{24c(5Ac^2+(b^2+ac)C)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{7/2}}\right)$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

output
```
(((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*
x))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x
*(b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*
a*c)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*
c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

3.147.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2191, 27, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx$$

$$\Big\downarrow \text{2191}$$

$$-\frac{\int \frac{2\left(5Ac+\left(\frac{b^2}{c}+a\right)C\right)}{(cx^2+bx+a)^3}dx}{3(b^2-4ac)} - \frac{x\left(C(b^2-2ac)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c(b^2-4ac)(a+bx+cx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)\int \frac{1}{(cx^2+bx+a)^3}dx}{3\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1086$$

$$-\frac{2\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)\left(-\frac{3c\int \frac{1}{\left(cx^2+bx+a\right)^2}dx}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1086$$

$$-\frac{2\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)\left(-\frac{3c\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 1083$$

$$-\frac{2\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)\left(-\frac{3c\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\left(C\left(a+\frac{b^2}{c}\right)+5Ac\right)\left(-\frac{3c\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{b^2-4ac}-\frac{b+2cx}{2(b^2-4ac)(a+bx+cx^2)^2}\right)}{3\left(b^2-4ac\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

3.147. $\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx$

output
```
-1/3*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*
(a + b*x + c*x^2)^3) - (2*(5*A*c + (a + b^2/c)*C)*(-1/2*(b + 2*c*x)/((b^2
- 4*a*c)*(a + b*x + c*x^2)^2) - (3*c*(-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*
x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^
(3/2)))/(b^2 - 4*a*c)))/(3*(b^2 - 4*a*c))
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.147.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(200) = 400$.

Time = 0.65 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.48

method	result
default	$\frac{\frac{4c^{3}\left(5A\,c^{2}+Cac+C\,b^{2}\right)x^{5}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}+\frac{10c^{2}\left(5A\,c^{2}+Cac+C\,b^{2}\right)b\,x^{4}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}+\frac{2\left(16ac+11b^{2}\right)c\left(5A\,c^{2}+Cac+C\,b^{2}\right)x^{3}}{3\left(64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}\right)}+\frac{b\left(16ac+b^{2}\right)\left(5A\,c^{2}+Cac+C\,b^{2}\right)x^{2}}{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12a\,b^{4}c-b^{6}}+}{\left(c\,x^{2}+bx+a\right)^{3}}$
risch	Expression too large to display

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^
5+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*
b*x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48
*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*
a^3*c^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
*x+1/3*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+8*c*(5*A*c^2+C*a*c+C*b^2
)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(198) = 396$.

Time = 0.32 (sec) , antiderivative size = 2103, normalized size of antiderivative = 10.21

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^4} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

output
```
[-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
 - 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c
^3 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*
a*b^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2
- 55*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C
*a*b^5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b
^3 - 5*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^
2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5
 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 +
 (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3
+ 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*
x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (
18*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256
*a^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 25
6*a^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*
c^4 - 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80
*a^2*b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c ...
```

3.147.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. 2(196) = 392.

Time = 1.99 (sec) , antiderivative size = 1224, normalized size of antiderivative = 5.94

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -4c\sqrt{-\frac{1}{(4ac-b^2)^7}}\cdot(5Ac^2+Cac$$
$$+Cb^2)\log\left(x+\frac{20Abc^3+4Cabc^2+4Cb^3c-1024a^4c^5\sqrt{-\frac{1}{(4ac-b^2)^7}}\cdot(5Ac^2+Cac+Cb^2)+1024a^3b^2c^4\sqrt{}\right.$$
$$+4c\sqrt{-\frac{1}{(4ac-b^2)^7}}\cdot(5Ac^2+Cac$$
$$+Cb^2)\log\left(x+\frac{20Abc^3+4Cabc^2+4Cb^3c+1024a^4c^5\sqrt{-\frac{1}{(4ac-b^2)^7}}\cdot(5Ac^2+Cac+Cb^2)-1024a^3b^2c^4\sqrt{}\right.$$
$$+\frac{66Aa^2bc^2-13Aab^3c+Ab^5+26Ca^3bc+Ca^2b^3+x^5\cdot(60Ac^5+12Cac^4+12Cb^2}{192a^6c^3-144a^5b^2c^2+36a^4b^4c-3a^3b^6+x^6\cdot(192a^3c^6-144a^2b^2c^5+36ab^4c^4-3b^6c^3)+x^5\cdot(576a^3bc^5}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

output
```
-4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*
b*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b*
*2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c -
b**2)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b*
*2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*
(5*A*c**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4
*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*
c**3 + 4*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b*
*2)**7)*(5*A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5
*A*c**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66
*A*a**2*b*c**2 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x*
*5*(60*A*c**5 + 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*
a*b*c**3 + 30*C*b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a
**2*c**3 + 54*C*a*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b
**3*c**2 + 48*C*a**2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c*
*3 + 54*A*a*b**2*c**2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c ...
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(198) = 396.

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.98

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx = -\frac{8\,(Cb^2c+Cac^2+5\,Ac^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3)\sqrt{-b^2+4\,ac}}$$
$$-\frac{12\,Cb^2c^3x^5+12\,Cac^4x^5+60\,Ac^5x^5+30\,Cb^3c^2x^4+30\,Cabc^3x^4+150\,Abc^4x^4+22\,Cb^4cx^3+54\,Cab^2c^2}{}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

output
```
-8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((
b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*
(12*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*
C*a*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32
*C*a^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*
a*b^3*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 +
3*C*a*b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2
*c^2*x + 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c
 + 66*A*a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^
2 + b*x + a)^3)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.39

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^4}\,dx =$$
$$-\frac{\frac{26\,C\,a^3\,b\,c+C\,a^2\,b^3+66\,A\,a^2\,b\,c^2-13\,A\,a\,b^3\,c+A\,b^5}{3\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}+\frac{x\,(-4\,C\,a^3\,c^2+22\,C\,a^2\,b^2\,c+44\,A\,a^2\,c^3+C\,a\,b^4+18\,A\,a\,b^2\,c^2-A\,b^4\,c)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{2\,x^3\,(11\,b^2\,c+}{3\,(-64\,a^3\,c^}}{x^2\,(3\,c\,a^2+3\,a\,b^2)+x^4\,(3\,b^2\,c+3\,a\,c^2)+}$$
$$-\frac{8\,c\,\mathrm{atan}\left(\frac{\left(\frac{8\,c^2\,x\,(C\,b^2+5\,A\,c^2+C\,a\,c)}{(4\,a\,c-b^2)^{7/2}}+\frac{4\,c\,(C\,b^2+5\,A\,c^2+C\,a\,c)\,(-64\,a^3\,b\,c^3+48\,a^2\,b^3\,c^2-12\,a\,b^5\,c+b^7)}{(4\,a\,c-b^2)^{7/2}\,(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}\right)(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6)}{4\,C\,b^2\,c+20\,A\,c^3+4\,C\,a\,c^2}\right.}{(4\,a\,c-b^2)^{7/2}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^4,x)
```

output
```
- ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*
a^3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A
*c^2 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + (x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^
2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^
2 + 3*a^2*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x
^6 + 3*b*c^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*
a*c))/(4*a*c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b
*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c))/(20*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c))*(5*A*c^2 + C*b^2 + C*a*c))/
(4*a*c - b^2)^(7/2)
```

3.147. $\int \frac{A+Cx^2}{(a+bx+cx^2)^4} \, dx$

3.148 $\int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2}\,dx$

3.148.1 Optimal result

Integrand size = 30, antiderivative size = 591

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx =$$

$$-\frac{(b^3e^3h-c^3d(3e^2f+3deg+d^2h)-bce^2(beg+3bdh+2aeh)+c^2e(ae(eg+3dh)+b(e^2f+3deg+3d^2}{c^4}$$

$$+\frac{e(b^2e^2h+c^2(e^2f+3deg+3d^2h)-ce(beg+3bdh+aeh))\,x^2}{2c^3}$$

$$+\frac{e^2(ceg+3cdh-beh)x^3}{3c^2}+\frac{e^3hx^4}{4c}$$

$$-\frac{(2c^5d^3f-b^5e^3h+b^3ce^2(beg+3bdh+5aeh)-c^4d(bd(3ef+dg)+2a(3e^2f+3deg+d^2h))-bc^2e(5a^2}{}$$

$$+\frac{(c^4d^2(3ef+dg)+b^4e^3h-b^2ce^2(beg+3bdh+3aeh)+c^2e(a^2e^2h+2abe(eg+3dh)+b^2(e^2f+3deg+}{2c^5}$$

output
```
-(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+
c^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+
c^2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b
*e*h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e
^3*h-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g
)+b^2*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2
*h+3*d*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e^2*(
5*a*e*h+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2*f))
-b*c^2*e*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))+c^3
*(2*a^2*e^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h+3*d
*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$

$$= \frac{12c(-b^3e^3h + c^3d(3e^2f + 3deg + d^2h) + bce^2(beg + 3bdh + 2aeh) - c^2e(ae(eg + 3dh) + b(e^2f + 3deg + 3}{}$$

input
```
Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
 3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^
2*h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e
*g + 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c
^4*e^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h +
 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
 3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2
*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) +
a*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

3.148.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{x\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3d^2h+3deg\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^3\left(2a^2e^2(3dh+eg)+3abe\left(3d^2h+3deg+e^2f\right)+b^2d\left(d^2h+3deg+3e^2f\right)\right)-bc^2e\left(5a^2e^2h\right.\right.}{}$$

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae(3\right.\right.}{2c^5}$$

$$\frac{x\left(c^2e\left(ae(3dh+eg)+b\left(3d^2h+3deg+e^2f\right)\right)-bce^2(2aeh+3bdh+beg)+b^3e^3h+c^3(-d)\left(d^2h+3deg+3e^2f\right)\right)}{c^4}$$

$$\frac{ex^2\left(-ce(aeh+3bdh+beg)+b^2e^2h+c^2\left(3d^2h+3deg+e^2f\right)\right)}{2c^3}+\frac{e^2x^3(-beh+3cdh+ceg)}{3c^2}+\frac{e^3hx^4}{4c}$$

input
```
Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
-(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d
*h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))
*x)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g +
3*b*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^
2) + (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*
b*d*h + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2
*h)) - b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g
 + 3*d^2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d
^2*h) + 3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h
- b^2*c*e^2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g
+ 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g
+ d^2*h) + a*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.148.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.47

method	result
default	$\frac{1}{4}h\,e^3x^4c^3-\frac{1}{3}b\,c^2e^3h\,x^3+c^3d\,e^2h\,x^3+\frac{1}{3}g\,e^3x^3c^3-\frac{1}{2}a\,c^2e^3h\,x^2+\frac{1}{2}b^2c\,e^3h\,x^2-\frac{3}{2}b\,c^2d\,e^2h\,x^2-\frac{1}{2}b\,c^2e^3g\,x^2+\frac{3}{2}c^3d^2eh\,x^2+\frac{3}{2}c^3d\,e^2g\,x^2+$
risch	Expression too large to display

input
```
int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*h*e^3*x^4*c^3-1/3*b*c^2*e^3*h*x^3+c^3*d*e^2*h*x^3+1/3*g*e^3*x^3
*c^3-1/2*a*c^2*e^3*h*x^2+1/2*b^2*c*e^3*h*x^2-3/2*b*c^2*d*e^2*h*x^2-1/2*b*c
^2*e^3*g*x^2+3/2*c^3*d^2*e*h*x^2+3/2*c^3*d*e^2*g*x^2+1/2*c^3*e^3*f*x^2+2*a
*b*c*e^3*h*x-3*a*c^2*d*e^2*h*x-a*c^2*e^3*g*x-b^3*e^3*h*x+3*b^2*c*d*e^2*h*x
+b^2*c*e^3*g*x-3*b*c^2*d^2*e*h*x-3*b*c^2*d*e^2*g*x-b*c^2*e^3*f*x+c^3*d^3*h
*x+3*c^3*d^2*e*g*x+3*c^3*d*e^2*f*x)+1/c^4*(1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^
3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^
3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*
d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*
g+3*c^4*d^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c*e^3*h+3*a^2*c^2*d*e^2*h+a
^2*c^2*e^3*g+a*b^3*e^3*h-3*a*b^2*c*d*e^2*h-a*b^2*c*e^3*g+3*a*b*c^2*d^2*e*h
+3*a*b*c^2*d*e^2*g+a*b*c^2*e^3*f-a*c^3*d^3*h-3*a*c^3*d^2*e*g-3*a*c^3*d*e^2
*f+c^4*d^3*f-1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^
2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^
2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^
3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d^2*e*f)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.148.5 Fricas [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 2150, normalized size of antiderivative = 3.64

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3
*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c
^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)
*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (
b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c
^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^
3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*
c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^
4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^
3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*(
(3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4
- 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3
+ 4*a^2*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d
^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2
+ 8*a^2*b*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 -
4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4
- 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3
+ 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^
3 - 4*a*b*c^4)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b...
```

3.148.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4972 vs. $2(619) = 1238$.

Time = 57.19 (sec) , antiderivative size = 4972, normalized size of antiderivative = 8.41

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2
*c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2)
+ 3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c*
*3 - 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*
h/c**3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3
*f/c**2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*
(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b*
*3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*
d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a
*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**
4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*
f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**
4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a
**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**
3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h
- 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d
*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e
**3*h - 4*a**2*b**2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3
*g - 6*a**2*c**3*d**2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f +...
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2}\, dx$$
$$= \frac{3\,c^3e^3hx^4 + 4\,c^3e^3gx^3 + 12\,c^3de^2hx^3 - 4\,bc^2e^3hx^3 + 6\,c^3e^3fx^2 + 18\,c^3de^2gx^2 - 6\,bc^2e^3gx^2 + 18\,c^3d^2ehx^2 -}{}$$
$$+ \frac{(3\,c^4d^2ef - 3\,bc^3de^2f + b^2c^2e^3f - ac^3e^3f + c^4d^3g - 3\,bc^3d^2eg + 3\,b^2c^2de^2g - 3\,ac^3de^2g - b^3ce^3g + 2\,ab}{}$$
$$+ \frac{(2\,c^5d^3f - 3\,bc^4d^2ef + 3\,b^2c^3de^2f - 6\,ac^4de^2f - b^3c^2e^3f + 3\,abc^3e^3f - bc^4d^3g + 3\,b^2c^3d^2eg - 6\,ac^4d^2eg}{}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*c^3*e^3*h*x^4 + 4*c^3*e^3*g*x^3 + 12*c^3*d*e^2*h*x^3 - 4*b*c^2*e^3
*h*x^3 + 6*c^3*e^3*f*x^2 + 18*c^3*d*e^2*g*x^2 - 6*b*c^2*e^3*g*x^2 + 18*c^3
*d^2*e*h*x^2 - 18*b*c^2*d*e^2*h*x^2 + 6*b^2*c*e^3*h*x^2 - 6*a*c^2*e^3*h*x^
2 + 36*c^3*d*e^2*f*x - 12*b*c^2*e^3*f*x + 36*c^3*d^2*e*g*x - 36*b*c^2*d*e^
2*g*x + 12*b^2*c*e^3*g*x - 12*a*c^2*e^3*g*x + 12*c^3*d^3*h*x - 36*b*c^2*d^
2*e*h*x + 36*b^2*c*d*e^2*h*x - 36*a*c^2*d*e^2*h*x - 12*b^3*e^3*h*x + 24*a*
b*c*e^3*h*x)/c^4 + 1/2*(3*c^4*d^2*e*f - 3*b*c^3*d*e^2*f + b^2*c^2*e^3*f -
a*c^3*e^3*f + c^4*d^3*g - 3*b*c^3*d^2*e*g + 3*b^2*c^2*d*e^2*g - 3*a*c^3*d*
e^2*g - b^3*c*e^3*g + 2*a*b*c^2*e^3*g - b*c^3*d^3*h + 3*b^2*c^2*d^2*e*h -
3*a*c^3*d^2*e*h - 3*b^3*c*d*e^2*h + 6*a*b*c^2*d*e^2*h + b^4*e^3*h - 3*a*b^
2*c*e^3*h + a^2*c^2*e^3*h)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - 3*b*c
^4*d^2*e*f + 3*b^2*c^3*d*e^2*f - 6*a*c^4*d*e^2*f - b^3*c^2*e^3*f + 3*a*b*c
^3*e^3*f - b*c^4*d^3*g + 3*b^2*c^3*d^2*e*g - 6*a*c^4*d^2*e*g - 3*b^3*c^2*d
*e^2*g + 9*a*b*c^3*d*e^2*g + b^4*c*e^3*g - 4*a*b^2*c^2*e^3*g + 2*a^2*c^3*e
^3*g + b^2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b^3*c^2*d^2*e*h + 9*a*b*c^3*d^2*e
*h + 3*b^4*c*d*e^2*h - 12*a*b^2*c^2*d*e^2*h + 6*a^2*c^3*d*e^2*h - b^5*e^3*
h + 5*a*b^3*c*e^3*h - 5*a^2*b*c^2*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 967, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx = x^3 \left(\frac{g\,e^3+3\,d\,h\,e^2}{3\,c} - \frac{b\,e^3\,h}{3\,c^2}\right) + x \left(\frac{h\,d^3+3\,g\,d^2\,e+3\,f\,d\,e^2}{c} + \frac{b\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c} - \frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{c} + \frac{a\,e^3\,h}{c^2}\right)}{c} - \frac{a\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}\right) - x^2 \left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{2\,c} - \frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{2\,c} + \frac{a\,e^3\,h}{2\,c^2}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\left(-4\,h\,a^3\,c^3\,e^3+13\,h\,a^2\,b^2\,c^2\,e^3-24\,h\,a^2\,b\,c^3\,d\,e^2-8\,g\,a^2\,b\,c^3\,e^3+12\,h\,a^2\,c^4\,d^2\,e+12\,g\right.}{} + \frac{e^3\,h\,x^4}{4\,c} + \frac{\operatorname{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(-5\,h\,a^2\,b\,c^2\,e^3+6\,h\,a^2\,c^3\,d\,e^2+2\,g\,a^2\,c^3\,e^3+5\,h\,a\,b^3\,c\,e^3-12\,h\,a\,b^2\,c^2\,d\,e^2-\right.}{}$$

input
```
int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*
f + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^
3*f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*
h)/c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2
))/(2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (l
og(a + b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2
*e^3*f - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4
*a*b*c^4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*
a*b^2*c^3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2
*g + 3*b^2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^
3*d^2*e*g + 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h +
 12*a*b*c^4*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2
*c^3*d^2*e*h + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 -
 b^2*c^5)) + (e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*
a*c - b^2)^(1/2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3
*f + b^2*c^3*d^3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3
*e^3*f + 5*a*b^3*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2
*e*f + 3*b^4*c*d*e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3
*d*e^2*f + 6*a^2*c^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b
^3*c^2*d^2*e*h + 9*a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d...
```

3.148. $\int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2}\,dx$

3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

3.149.1 Optimal result

Integrand size = 30, antiderivative size = 348

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx = \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))x}{c^3}$$
$$+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2hx^3}{3c}$$
$$-\frac{(2c^4d^2f+b^4e^2h-b^2ce(beg+2bdh+4aeh)-c^3(bd(2ef+dg)+2a(e^2f+2deg+d^2h))+c^2(2a^2e^2h+}{c^4\sqrt{b^2-4ac}}$$
$$+\frac{(c^3d(2ef+dg)-b^3e^2h+bce(beg+2bdh+2aeh)-c^2(ae(eg+2dh)+b(e^2f+2deg+d^2h)))\log(a+}{2c^4}$$

output
```
(b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*
e*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c+1/2*(c^3*d*(d*g+2*e*f)-b^
3*e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h+2*d*e*
g+e^2*f)))*ln(c*x^2+b*x+a)/c^4-(2*c^4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b
*d*h+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2
*h+3*a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a
*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$

$$= \frac{6c(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x+3c^2e(ceg+2cdh-beh)x^2+2c^3e^2hx^3+\frac{6(}{}}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a
*e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2
*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*
(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*
c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*
e*g + d^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

3.149.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{-ce(aeh+2bdh+beg)+b^2e^2h+c^2(d^2h+2deg+e^2f)}{c^3}+\frac{x(-c^2(ae(2dh+eg)+b(d^2h+2deg+e^2f))+b}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3(2a(c}{c^4\sqrt{b^2-4ac}}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(-c^2\left(ae(2dh+eg)+b\left(d^2h+2deg+e^2f\right)\right)+bce(2aeh+2bdh+beg)+b^3\left(-e^2\right)h+c^3d(dg+}{2c^4}$$
$$\frac{x\left(-ce(aeh+2bdh+beg)+b^2e^2h+c^2\left(d^2h+2deg+e^2f\right)\right)}{c^3}+\frac{ex^2(-beh+2cdh+ceg)}{2c^2}+\frac{e^2hx^3}{3c}$$

input
```
Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h
))*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c)
- ((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b
*d*(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a
*b*e*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e
^2*h + b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2
*f + 2*d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

3.149.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.149.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.30

method	result
default	$-\frac{-\frac{1}{3}h\,e^2x^3c^2+\frac{1}{2}bc\,e^2h\,x^2-c^2deh\,x^2-\frac{1}{2}g\,x^2c^2e^2+ac\,e^2hx-b^2e^2hx+2bcdehx+bc\,e^2gx-c^2d^2hx-2c^2degx-c^2e^2fx}{c^3}+\frac{\left(2abc\,e^2h-2\right.}{}$
risch	Expression too large to display

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*h*e^2*x^3*c^2+1/2*b*c*e^2*h*x^2-c^2*d*e*h*x^2-1/2*g*x^2*c^2*e
^2+a*c*e^2*h*x-b^2*e^2*h*x+2*b*c*d*e*h*x+b*c*e^2*g*x-c^2*d^2*h*x-2*c^2*d*e
*g*x-c^2*e^2*f*x)+1/c^3*(1/2*(2*a*b*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*
e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*
d^2*g+2*c^3*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*c*e^2*h-a*b^2*e^2*h+2*a*b*c*d*
e*h+a*b*c*e^2*g-a*c^2*d^2*h-2*a*c^2*d*e*g-a*c^2*e^2*f+c^3*d^2*f-1/2*(2*a*b
*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c
^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*g+2*c^3*d*e*f)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.66

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*
(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 -
 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^
2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2
*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x +
b))/(c*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*
c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^
3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*(
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*
a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^
3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^
3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^
2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a
*c^4)*e^2*g + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x
^2 - 6*sqrt(-b^2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*
e^2)*f - (b*c^3*d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)
*g + ((b^2*c^2 - 2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2
*c + 2*a^2*c^2)*e^2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*
c)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*...
```

3.149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2839 vs. $2(359) = 718$.

Time = 20.77 (sec) , antiderivative size = 2839, normalized size of antiderivative = 8.16

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b
**2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c +
e**2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h
+ 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g
- 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c*
*2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**
3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c
**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g -
 b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e
*f)/(2*c**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**
2*e**2*g + a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d
**2*h + 2*a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2
)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*
e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h
- 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g +
b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4
*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3
*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g
- b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g
- 4*a*c**3*d*e*f - b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h ...
```

3.149.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$
$$= \frac{2\,c^2e^2hx^3 + 3\,c^2e^2gx^2 + 6\,c^2dehx^2 - 3\,bce^2hx^2 + 6\,c^2e^2fx + 12\,c^2degx - 6\,bce^2gx + 6\,c^2d^2hx - 12\,bcdehx}{6\,c^3}$$
$$+ \frac{(2\,c^3def - bc^2e^2f + c^3d^2g - 2\,bc^2deg + b^2ce^2g - ac^2e^2g - bc^2d^2h + 2\,b^2cdeh - 2\,ac^2deh - b^3e^2h + 2\,ab}{2\,c^4}$$
$$+ \frac{(2\,c^4d^2f - 2\,bc^3def + b^2c^2e^2f - 2\,ac^3e^2f - bc^3d^2g + 2\,b^2c^2deg - 4\,ac^3deg - b^3ce^2g + 3\,abc^2e^2g + b^2c^2d}{\sqrt{-b^2 + 4\,ac}c}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*c^2*e^2*h*x^3 + 3*c^2*e^2*g*x^2 + 6*c^2*d*e*h*x^2 - 3*b*c*e^2*h*x^2
 + 6*c^2*e^2*f*x + 12*c^2*d*e*g*x - 6*b*c*e^2*g*x + 6*c^2*d^2*h*x - 12*b*c
*d*e*h*x + 6*b^2*e^2*h*x - 6*a*c*e^2*h*x)/c^3 + 1/2*(2*c^3*d*e*f - b*c^2*e
^2*f + c^3*d^2*g - 2*b*c^2*d*e*g + b^2*c*e^2*g - a*c^2*e^2*g - b*c^2*d^2*h
 + 2*b^2*c*d*e*h - 2*a*c^2*d*e*h - b^3*e^2*h + 2*a*b*c*e^2*h)*log(c*x^2 +
b*x + a)/c^4 + (2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*
f - b*c^3*d^2*g + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^
2*e^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h
+ b^4*e^2*h - 4*a*b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*arctan((2*c*x + b)/sqrt(-
b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$
$$= x^2 \left(\frac{g\,e^2 + 2\,d\,h\,e}{2\,c} - \frac{b\,e^2\,h}{2\,c^2}\right) - x \left(\frac{b\left(\frac{g\,e^2+2\,d\,h\,e}{c} - \frac{b\,e^2\,h}{c^2}\right)}{c} - \frac{h\,d^2 + 2\,g\,d\,e + f\,e^2}{c} + \frac{a\,e^2\,h}{c^2}\right)$$
$$- \frac{\ln(c\,x^2 + b\,x + a)\,(-8\,h\,a^2\,b\,c^2\,e^2 + 8\,h\,a^2\,c^3\,d\,e + 4\,g\,a^2\,c^3\,e^2 + 6\,h\,a\,b^3\,c\,e^2 - 10\,h\,a\,b^2\,c^2\,d\,e - 5\,g\,a\,b^2\,c^2}{}$$
$$+ \frac{e^2\,h\,x^3}{3\,c}$$
$$+ \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)(2\,h\,a^2\,c^2\,e^2 - 4\,h\,a\,b^2\,c\,e^2 + 6\,h\,a\,b\,c^2\,d\,e + 3\,g\,a\,b\,c^2\,e^2 - 2\,h\,a\,c^3\,d^2 - 4\,g\,a\,c^3}{}$$

input
```
int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*
h)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) -
(log(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c
^2*e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f +
 4*a*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2
*b^3*c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2
*b^4*c*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^
4)) + (e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b
^2)^(1/2))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^
2*c^2*d^2*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g +
3*a*b*c^2*e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*
c^3*d*e*f - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

3.149. $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

3.150 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

3.150.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c} - \frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh)\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(a+bx+cx^2\right)}{2c^3}$$

output

```
(-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e
*h+b*d*h+b*e*g))*ln(c*x^2+b*x+a)/c^3-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2*a*e
*g+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \frac{2c(ceg+cdh-beh)x+c^2ehx^2-\frac{2\left(-2c^3df+b^3eh+c^2(bef+bdg+2aeg+2adh)-bc(beg+bdh+3aeh)\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2(\ldots}{2c^3}$$

input
```
Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h +
c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*
g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

3.150.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{x\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)+abeh-ac(dh+eg)+c^2df}{c^2\left(a+bx+cx^2\right)}+\frac{-beh+cdh+ceg}{c^2}+\frac{ehx}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef)+bc(3aeh+bdh+beg)+b^3(-e)h+2c^3df\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}+\frac{x(-beh+cdh+ceg)}{c^2}+\frac{ehx^2}{2c}$$

input
```
Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h
- c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(
e*f + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2
*c^3)
```

3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.150.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

method	result
default	$-\frac{-\frac{1}{2}cehx^2+behx-cdhx-cegx}{c^2}+\frac{\frac{\left(-aehc+b^2eh-bcdh-bceg+c^2dg+c^2ef\right)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(baeh-acdh-aceg+c^2df-\frac{\left(-aehc+b^2eh-b\right.}{}\right.}{\sqrt{4c}}}{c^2}$
risch	Expression too large to display

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*e*h*x^2+b*e*h*x-c*d*h*x-c*e*g*x)+1/c^2*(1/2*(-a*c*e*h+b^2*e
*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(b*a*e*h-a*c*d*h-a
*c*e*g+c^2*d*f-1/2*(-a*c*e*h+b^2*e*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.69

$$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$=\left[\frac{(b^2c^2-4\,ac^3)ehx^2+\sqrt{b^2-4\,ac}((2\,c^3d-bc^2e)f-(bc^2d-(b^2c-2\,ac^2)e)g+((b^2c-2\,ac^2)d-(b^3-3\,a}{}\right.$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)
*f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a
*b*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*
a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*
c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^
4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)
, 1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^
2*e)*f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 -
 3*a*b*c)*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*
((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)
*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*
log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

3.150.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(182) = 364$.

Time = 6.20 (sec) , antiderivative size = 1265, normalized size of antiderivative = 7.15

$$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx = x\left(-\frac{beh}{c^2}+\frac{dh}{c}+\frac{eg}{c}\right)$$
$$+\left(-\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(-\frac{\sqrt{-4a}}{}\right.}{}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df)}{2c^3\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{aceh-b^2eh+bcdh+bceg-c^2dg-c^2ef}{2c^3}\right)\log\left(x+\frac{2a^2ceh-ab^2eh+abcdh+abceg+4ac^3\left(\frac{\sqrt{-4ac+}}{}\right.}{}\right.$$
$$+\frac{ehx^2}{2c}$$

input
```
integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h
+ b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h
 - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*
a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e
*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*
e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*
c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2
*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d
*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*
h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*
b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g
 - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*
e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h -
b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a
**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b
**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h +
b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**...
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \frac{cehx^2+2\,cegx+2\,cdhx-2\,behx}{2\,c^2} + \frac{\left(c^2ef+c^2dg-bceg-bcdh+b^2eh-aceh\right)\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{\left(2\,c^3df-bc^2ef-bc^2dg+b^2ceg-2\,ac^2eg+b^2cdh-2\,ac^2dh-b^3eh+3\,abceh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(c*e*h*x^2 + 2*c*e*g*x + 2*c*d*h*x - 2*b*e*h*x)/c^2 + 1/2*(c^2*e*f + c
^2*d*g - b*c*e*g - b*c*d*h + b^2*e*h - a*c*e*h)*log(c*x^2 + b*x + a)/c^3 +
 (2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d*h
- 2*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*
c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = x\left(\frac{d\,h+e\,g}{c}-\frac{b\,e\,h}{c^2}\right) - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(b^4\,e\,h-4\,a\,c^3\,d\,g-4\,a\,c^3\,e\,f-b^3\,c\,d\,h-b^3\,c\,e\,g+b^2\,c^2\,d\,g+b^2\,c^2\,e\,f+4\,a^2\,c^2\,e\,h+\right.}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\right.}{c^3\,\sqrt{4\,a\,c-b^2}} + \frac{e\,h\,x^2}{2\,c}$$

input
```
int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3
*d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4
*a^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4
 - b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*
(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f -
 b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^
2)/(2*c)
```

3.150. $\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx$

3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2} dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} - \frac{(2c^2f-bcg+b^2h-2ach)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} + \frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
h*x/c+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*ar
ctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} + \frac{(2c^2f-bcg+b^2h-2ach)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}} + \frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}$$

input
```
Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

output
```
(h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^
2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])
/(2*c^2)
```

3.151.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \ 2188$$

$$\int \left(\frac{-ah+x(cg-bh)+cf}{c\left(a+bx+cx^2\right)}+\frac{h}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

input
```
Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

output
```
(h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/
(2*c^2)
```

3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.151.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{hx}{c}+\frac{\frac{(-bh+cg)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-ah+cf-\frac{(-bh+cg)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$	93
risch	Expression too large to display	1649

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
h*x/c+1/c*(1/2*(-b*h+c*g)/c*ln(c*x^2+b*x+a)+2*(-a*h+c*f-1/2*(-b*h+c*g)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.28

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)hx-(2\,c^2f-bcg+(b^2-2\,ac)h)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((b^2}{2\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2
*(2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*
c)*h)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

3.151.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(88) = 176$.

Time = 1.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.30

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)} - \frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach\ldots}{2c^2\cdot(4a\ldots}\right.}{2ach-b^2h+bcg-2c^2f}\right) + \left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)} - \frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h\ldots}{2c^2\cdot(4ac-b^2\ldots}\right.}{2ach-b^2h+bcg-2c^2f}\right) + \frac{hx}{c}$$

input
```
integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
 - b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c
 + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (
b*h - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b
**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2))
- b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*
a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(
2*c**2))*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h
 + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a
*c*g + b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(
2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h
+ b*c*g - 2*c**2*f)) + h*x/c
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} + \frac{(cg-bh)\log(cx^2+bx+a)}{2\,c^2} + \frac{(2\,c^2f-bcg+b^2h-2\,ach)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*
h - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^
2)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.43

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{h\,x}{c} + \frac{\ln(c\,x^2+b\,x+a)\,(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2)}{2\,(4\,a\,c^3-b^2\,c^2)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

output
```
(h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))
/(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c -
b^2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

3.151. $\int \frac{f+gx+hx^2}{a+bx+cx^2} \, dx$

3.152 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$

3.152.1 Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx = -\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\,(cd^2-bde+ae^2)}$$

output
```
(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*
g+c*e*f)*ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)-(2*c^2*d*f+b*(-a*e+b*d)*h-c
*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(a
*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

3.152.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx = \frac{-2e(-2c^2df+b(-bd+ae)h+c(bef+bdg-2aeg+2adh))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2c\sqrt{-b^2+4ac}(e^2f-d}{2c\sqrt{-b^2+4ac}e\,(cd^2+e(-bd}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

output
```
(-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*
d*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2
*f - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b
*d*h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(
-(b*d) + a*e)))
```

3.152.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-x(-aeh+bdh-cdg+cef)-adh+aeg-bef+cdf}{(a+bx+cx^2)(ae^2-bde+cd^2)}+\frac{d^2h-deg+e^2f}{(d+ex)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\,(ae^2-bde+cd^2)}+\frac{\log(d+ex)\left(d^2h-deg+e^2f\right)}{e\,(ae^2-bde+cd^2)}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

output
```
-(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e
 + a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a
*e^2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^
2 - b*d*e + a*e^2))
```

3.152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.152.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

method	result
default	$\frac{(d^2h-deg+e^2f)\ln(ex+d)}{e(e^2a-bde+cd^2)}+\frac{\frac{(aeh-bdh+cdg-cef)\ln\left(cx^2+bx+a\right)}{2c}+\frac{2\left(-adh+aeg-bef+cdf-\frac{(aeh-bdh+cdg-cef)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{e^2a-bde+cd^2}$
risch	$\frac{\ln(ex+d)d^2h}{e(e^2a-bde+cd^2)}-\frac{\ln(ex+d)dg}{e^2a-bde+cd^2}+\frac{\ln(ex+d)ef}{e^2a-bde+cd^2}+\Bigg(\sum_{\_R=\text{RootOf}\left((4a^2c^2e^2-ab^2ce^2-4abc^2de+4ac^3d^2+b^3cde-b^2c^2d^2)\_\right.}$

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)*
(1/2*(a*e*h-b*d*h+c*d*g-c*e*f)/c*ln(c*x^2+b*x+a)+2*(-a*d*h+a*e*g-b*e*f+c*d
*f-1/2*(a*e*h-b*d*h+c*d*g-c*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2)))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 30.41 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.19

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$$
$$=\left[-\frac{\sqrt{b^2-4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-}}{cx^2+bx+a}\right.}{}\right.$$
$$-\frac{2\sqrt{-b^2+4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b}{b^2-4\,ac}\right.}{}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2 - 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*
g - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*
c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)
*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c
)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a
*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*
d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*sq
rt(-b^2 + 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b
*e^2 - (b^2 - 2*a*c)*d*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*
a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c -
4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*
x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c
- 4*a^2*c^2)*e^3)]
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx = -\frac{(cef-cdg+bdh-aeh)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(e^2f-deg+d^2h\right)\log\left(|ex+d|\right)}{cd^2e-bde^2+ae^3}+\frac{\left(2\,c^2df-bcef-bcdg+2\,aceg+b^2dh-2\,acdh-abeh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(c*e*f - c*d*g + b*d*h - a*e*h)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d
*e + a*c*e^2) + (e^2*f - d*e*g + d^2*h)*log(abs(e*x + d))/(c*d^2*e - b*d*e
^2 + a*e^3) + (2*c^2*d*f - b*c*e*f - b*c*d*g + 2*a*c*e*g + b^2*d*h - 2*a*c
*d*h - a*b*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.152.9 Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 2467, normalized size of antiderivative = 12.59

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)
```

output
```
(log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d
^4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)
^(1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h
 - 10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*
a^2*c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a
*c)^(1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1
/2) - 2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2)
- 2*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2
*h + b^2*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*
(b^2 - 4*a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*
a*b*c*d^3*e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) -
b^2*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*
a*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^3*e*g*(b^2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^
4*f*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2
*c*d*e^3*f*x - 10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x +
3*b^2*c*d^3*e*h*x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*
(b^2 - 4*a*c)^(1/2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b
^2 - 4*a*c)^(1/2) + 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2
*g*x - 3*b*c^2*d^2*e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b...
```

3.152. $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$

3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx$

3.153.1 Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = -\frac{e^2f-deg+d^2h}{e(cd^2-bde+ae^2)(d+ex)}$$
$$-\frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))\operatorname{arctan}}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^2}$$
$$+\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^2}$$
$$-\frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(a+bx+cx^2)}{2(cd^2-bde+ae^2)^2}$$

output
```
(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+(c*d*(-d*g+2*e*f)+a*e*(
-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d
*g+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+
c*d^2)^2-(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b
*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/(a*e^2-b*d*e+c*d^2)^2/(-4*a*c+b^2)^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx$$
$$= \frac{-\frac{2\left(cd^2+e(-bd+ae)\right)\left(e^2f-deg+d^2h\right)}{e(d+ex)}+\frac{2\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+d^2h\right)-c\left(bd(2ef+dg)+2a\left(e^2f-2deg+d^2h\right)\right)\right)\arctan\left(\frac{b}{\sqrt{-}}\right.}{\sqrt{-b^2+4ac}}}{}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

output
```
((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (
2*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) -
 c*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g
 - 2*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e
*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e
*(-(b*d) + a*e))^2)
```

3.153.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{e^2\left(a^2h-abg+b^2f\right)-cx\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)-c\left(a\left(d^2h-2deg+e^2f\right)+2bdef\right)}{\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)^2}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^2}-\frac{d^2h-deg+e^2f}{e(d+ex)\left(ae^2-bde+cd^2\right)}+$$
$$\frac{\log(d+ex)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^2}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

output
```
-((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2
*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f
 - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*
d*e + a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*
h))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.153.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.09

method	result
default	$-\frac{d^2h-deg+e^2f}{(e^2a-bde+c\,d^2)e(ex+d)}-\frac{\left(2adeh-a\,e^2g-b\,d^2h+b\,e^2f+c\,d^2g-2cdef\right)\ln(ex+d)}{(e^2a-bde+c\,d^2)^2}+\frac{\left(2acdhe-ac\,e^2g-bc\,d^2h+bc\,e^2f+c^2d^2g-2c^2d\right.}{2c}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)-(2*a*d*e*h-a*e^2*g-b*d^
2*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)+1/(a*e^2-b*
d*e+c*d^2)^2*(1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b*c*e^2*f+c^2*d^2*g-2*c
^2*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*e^2*h-a*b*e^2*g-a*c*d^2*h+2*a*c*d*e*g-a
*c*e^2*f+b^2*e^2*f-2*b*c*d*e*f+c^2*d^2*f-1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^
2*h+b*c*e^2*f+c^2*d^2*g-2*c^2*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2)))
```

3.153.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.45

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = -\frac{(2\,cdef-be^2f-cd^2g+ae^2g+bd^2h-2\,adeh)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{be}{ex+d}-\frac{bde}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)} - \frac{\frac{e^3f}{ex+d}-\frac{de^2g}{ex+d}+\frac{d^2eh}{ex+d}}{cd^2e^2-bde^3+ae^4} + \frac{(2\,c^2d^2e^2f-2\,bcde^3f+b^2e^4f-2\,ace^4f-bcd^2e^2g+4\,acde^3g-abe^4g+b^2d^2e^2h-2\,acd^2e^2h-2\,abde^3}{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\sqrt{-b^2+4}}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(2*c*d*e*f - b*e^2*f - c*d^2*g + a*e^2*g + b*d^2*h - 2*a*d*e*h)*log(c
 - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2
 + a*e^2/(e*x + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*a*b*d*e^3 + a^2*e^4) - (e^3*f/(e*x + d) - d*e^2*g/(e*x + d) + d^2*e*h
/(e*x + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + (2*c^2*d^2*e^2*f - 2*b*c*d*e^3
*f + b^2*e^4*f - 2*a*c*e^4*f - b*c*d^2*e^2*g + 4*a*c*d*e^3*g - a*b*e^4*g +
 b^2*d^2*e^2*h - 2*a*c*d^2*e^2*h - 2*a*b*d*e^3*h + 2*a^2*e^4*h)*arctan((2*
c*d - 2*c*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sq
rt(-b^2 + 4*a*c)*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 3991, normalized size of antiderivative = 12.63

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

output
```
(log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(
a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^
2) + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*
g + b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*
f*x + 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a
*c)^(1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*
c*d*e^3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^
4*h*x + 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1
/2) - a^2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) +
2*b*c^2*d^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*
b^3*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a
^2*c^2*d*e^3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f -
 b^3*c*d^2*e^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*
x + b^2*c^2*d^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^
2*e^2*f*(b^2 - 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*
d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b
^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d
^2*e^2*h*x - 6*a*b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6
*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h
*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3...
```

3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$

3.154.1 Optimal result

Integrand size = 30, antiderivative size = 509

$$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$$
$$= -\frac{e^2f-deg+d^2h}{2e(cd^2-bde+ae^2)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{(cd^2-bde+ae^2)^2(d+ex)}$$
$$-\frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-3dh)-}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^3}$$
$$+\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-c(ae(e^2f-3deg+3d^2h)+b(3de^2f-d^3h)))\log(d+ex)}{(cd^2-bde+ae^2)^3}$$
$$-\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-c(ae(e^2f-3deg+3d^2h)+b(3de^2f-d^3h)))\log(a+bx+cx}{2(cd^2-bde+ae^2)^3}$$

output
```
1/2*(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+(-c*d*(-d*g+2*e*f
)-a*e*(-2*d*h+e*g)+b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(c^2*d^
2*(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-
d^3*h+3*d*e^2*f)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*(c^2*d^2*(-d*g+3*e*
f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3*h+3*d*e^
2*f)))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3-(2*c^3*d^3*f-b*e^3*(a^2*h-a*b
*g+b^2*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3*e^2*f))-c*(2*a^2*e^2
*(-3*d*h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h+3*d*e^2*f)))*arctanh
((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.99

$$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx = -\frac{e^2f-deg+d^2h}{2e(cd^2+e(-bd+ae))(d+ex)^2} + \frac{cd(-2ef+dg)+ae(-eg+2dh)+b(e^2f-d^2h)}{(cd^2+e(-bd+ae))^2(d+ex)} + \frac{(-2c^3d^3f+be^3(b^2f-abg+a^2h)+c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(-2a^2e^2(eg-3dh ...}{\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))^3} - \frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(d+ex)}{(cd^2+e(-bd+ae))^3} + \frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(a+x(b+ ...}{2(cd^2+e(-bd+ae))^3}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) +
(c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 +
e*(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a
^2*h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-
2*a^2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f
 + d^3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*
g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*
Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^
3*(b^2*f - a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e
^2*f - d^3*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

3.154.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{e\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{(d+ex)\left(ae^2-bde+cd^2\right)^3}+\frac{-cx\left(e^3\left(a^2h-ab\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c\left(2a^2e^2(eg-3dh)-3abe\left(d^2(-h)-deg+e^2f\right)-\left(b^2\left(d^3h+3de^2f\right)\right)\right)-be^3\left(a^2h-abg+\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+$$
$$\frac{\log(d+ex)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^3}-$$
$$\frac{d^2h-deg+e^2f}{2e(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)}{(d+ex)\left(ae^2-bde+cd^2\right)^2}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*
d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e +
 a*e^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2
*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e
*g - 3*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*
ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e
+ a*e^2)^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*
e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*
d^2 - b*d*e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^
2*h) - a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a
+ b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

3.154. $\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}dx$

3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.154.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.24

method	result
default	$-\frac{d^2h-deg+e^2f}{2(e^2a-bde+c\,d^2)e(ex+d)^2}+\frac{(a^2e^3h-ab\,e^3g-3ac\,d^2eh+3acd\,e^2g-ac\,e^3f+b^2e^3f+bc\,d^3h-3bcd\,e^2f-c^2d^3g+3c^2d^2ef)\ln(ex+d)}{(e^2a-bde+c\,d^2)^3}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)/e/(e*x+d)^2+(a^2*e^3*h-a*b*e^
3*g-3*a*c*d^2*e*h+3*a*c*d*e^2*g-a*c*e^3*f+b^2*e^3*f+b*c*d^3*h-3*b*c*d*e^2*
f-c^2*d^3*g+3*c^2*d^2*e*f)/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)+(2*a*d*e*h-a*e^
2*g-b*d^2*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+1/(a*
e^2-b*d*e+c*d^2)^3*(1/2*(-a^2*c*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*
d*e^2*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^
3*d^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-a^2*b*e^3*h+3*a^2*c*d*e^2*h-a^2*c*e^3*g+a
*b^2*e^3*g-3*a*b*c*d*e^2*g+2*a*b*c*e^3*f-a*c^2*d^3*h+3*a*c^2*d^2*e*g-3*a*c
^2*d*e^2*f-b^3*e^3*f+3*b^2*c*d*e^2*f-3*b*c^2*d^2*e*f+c^3*d^3*f-1/2*(-a^2*c
*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2*g+a*c^2*e^3*f-b^2*c*e^3*f
-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*e*f)*b/c)/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.154.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.154.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(501) = 1002$.

Time = 0.27 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.09

$$\int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} dx =$$
$$-\frac{(3c^2d^2ef - 3bcde^2f + b^2e^3f - ace^3f - c^2d^3g + 3acde^2g - abe^3g + bcd^3h - 3acd^2eh + a^2e^3h)\log(cx^2}{2(c^3d^6 - 3bc^2d^5e + 3b^2cd^4e^2 + 3ac^2d^4e^2 - b^3d^3e^3 - 6abcd^3e^3 + 3ab^2d^2e^4 + 3a^2cd^2e^4 - 3a^2bde^5 +}$$
$$+\frac{(3c^2d^2e^2f - 3bcde^3f + b^2e^4f - ace^4f - c^2d^3eg + 3acde^3g - abe^4g + bcd^3eh - 3acd^2e^2h + a^2e^4h)\log}{c^3d^6e - 3bc^2d^5e^2 + 3b^2cd^4e^3 + 3ac^2d^4e^3 - b^3d^3e^4 - 6abcd^3e^4 + 3ab^2d^2e^5 + 3a^2cd^2e^5 - 3a^2bde^6 +}$$
$$+\frac{(2c^3d^3f - 3bc^2d^2ef + 3b^2cde^2f - 6ac^2de^2f - b^3e^3f + 3abce^3f - bc^2d^3g + 6ac^2d^2eg - 3abcde^2g + a}{(c^3d^6 - 3bc^2d^5e + 3b^2cd^4e^2 + 3ac^2d^4e^2 - b^3d^3e^3 - 6abcd^3e^3 + 3}$$
$$-\frac{5c^2d^4e^2f - 8bcd^3e^3f + 3b^2d^2e^4f + 6acd^2e^4f - 4abde^5f + a^2e^6f - 3c^2d^5eg + 4bcd^4e^2g - b^2d^3e^3g - 2}{}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(3*c^2*d^2*e*f - 3*b*c*d*e^2*f + b^2*e^3*f - a*c*e^3*f - c^2*d^3*g +
3*a*c*d*e^2*g - a*b*e^3*g + b*c*d^3*h - 3*a*c*d^2*e*h + a^2*e^3*h)*log(c*x
^2 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2
 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a
^2*b*d*e^5 + a^3*e^6) + (3*c^2*d^2*e^2*f - 3*b*c*d*e^3*f + b^2*e^4*f - a*c
*e^4*f - c^2*d^3*e*g + 3*a*c*d*e^3*g - a*b*e^4*g + b*c*d^3*e*h - 3*a*c*d^2
*e^2*h + a^2*e^4*h)*log(abs(e*x + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2
*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2
*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - 3*b*c^2
*d^2*e*f + 3*b^2*c*d*e^2*f - 6*a*c^2*d*e^2*f - b^3*e^3*f + 3*a*b*c*e^3*f -
 b*c^2*d^3*g + 6*a*c^2*d^2*e*g - 3*a*b*c*d*e^2*g + a*b^2*e^3*g - 2*a^2*c*e
^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*a*b*c*d^2*e*h + 6*a^2*c*d*e^2*h - a
^2*b*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5
*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3
*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*
a*c)) - 1/2*(5*c^2*d^4*e^2*f - 8*b*c*d^3*e^3*f + 3*b^2*d^2*e^4*f + 6*a*c*d
^2*e^4*f - 4*a*b*d*e^5*f + a^2*e^6*f - 3*c^2*d^5*e*g + 4*b*c*d^4*e^2*g - b
^2*d^3*e^3*g - 2*a*c*d^3*e^3*g + a^2*d*e^5*g + c^2*d^6*h - b^2*d^4*e^2*h -
 2*a*c*d^4*e^2*h + 4*a*b*d^3*e^3*h - 3*a^2*d^2*e^4*h + 2*(2*c^2*d^3*e^3*f
- 3*b*c*d^2*e^4*f + b^2*d*e^5*f + 2*a*c*d*e^5*f - a*b*e^6*f - c^2*d^4*e...
```

3.154.9 Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 12784, normalized size of antiderivative = 25.12

$$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

output
```
symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*
c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 +
120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*
e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5
*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a
^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8
*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*
a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a
^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*
c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^
5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*
a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 +
15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6
*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a
*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z
+ 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e
^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^
4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z
- 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3
*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93...
```

3.155 $\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

3.155.1 Optimal result

Integrand size = 30, antiderivative size = 288

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{e^2(2c^2f-bcg+2b^2h-6ach)\,x}{c^2(b^2-4ac)}$$
$$+\frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x\right)}{c(b^2-4ac)(a+bx+cx^2)}$$
$$+\frac{(4c^4d^2f-2b^4e^2h-6ac^2e(beg+2bdh+2aeh)+b^2ce(beg+2bdh+12aeh)-c^3(2bd(2ef+dg)-4a(e}{c^3(b^2-4ac)^{3/2}}$$
$$+\frac{e(ceg+2cdh-2beh)\log(a+bx+cx^2)}{2c^3}$$

output
```
e^2*(2*c^2*f+2*b^2*h-c*(6*a*h+b*g))*x/c^2/(-4*a*c+b^2)+(e*x+d)^2*(c*(2*a*g
-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)+(4*c^4*d^2*f-2*b^4*e^2*h-6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*
a*e*h+2*b*d*h+b*e*g)-c^3*(2*b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*ar
ctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h
+2*c*d*h+c*e*g)*ln(c*x^2+b*x+a)/c^3
```

3.155.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx$$

$$= \frac{2ce^2hx - \frac{2(b^4e^2hx+b^3e(aeh-c(eg+2dh)x)+b^2c(c(e^2f+2deg+d^2h)x-ae(eg+2dh+4ehx))+2c^2(c^2d^2fx-ac(e^2fx+2de(f+gx)+d^2(g+hx))}{(b^2-4ac)(a+x(b+cx))}}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
(2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c
*(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^
2*d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h
 + e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*
c*(d^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h +
2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g
) + 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)
])/(2*c^3)
```

3.155.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2175, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx$$

$$\Big\downarrow \; 2175$$

$$\frac{(d+ex)^2 \left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \frac{(d+ex)\left(2cdf-2bef-bdg+4aeg+2adh-\frac{2abeh}{c}-e\left(\frac{2hb^2}{c}-gb+2cf-6ah\right)x\right)}{cx^2+bx+a}dx}{b^2-4ac}$$

$\Big\downarrow$ 1200

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-$$
$$\frac{\int \left(\frac{2d^2fc^3-\left(bd(2ef+dg)-2a\left(hd^2+2egd+e^2f\right)\right)c^2-ae(beg+2bdh+6aeh)c+2ab^2e^2h-\left(b^2-4ac\right)e(ceg+2cdh-2beh)x}{c^2\left(cx^2+bx+a\right)}-\frac{e^2\left(2hb^2-cgb+2c^2f-6a\right.}{c^2}\right.}{b^2-4ac}$$

$\Big\downarrow$ 2009

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-$$
$$\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bdh+beg)-2b^4e^2h+4c^4d^2f\right)}{c^3\sqrt{b^2-4ac}}-\frac{e\left(b^2-\right.}{}}{b^2-4ac}$$

input
```
Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a
*c*h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (-((e^2*(2*c^2*f - b*c*g +
 2*b^2*h - 6*a*c*h)*x)/c^2) - ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e
*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c^3*(2*b*
d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*e*(c*e*g + 2*c*d*
h - 2*b*e*h)*Log[a + b*x + c*x^2])/(2*c^3))/(b^2 - 4*a*c)
```

3.155.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.155. $\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(283) = 566$.

Time = 0.77 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.18

method	result
default	$\frac{h e^2 x}{c^2} - \frac{-\frac{\left(2a^2c^2e^2h-4a\,b^2c\,e^2h+6ab\,c^2deh+3ab\,c^2e^2g-2a\,c^3d^2h-4a\,c^3deg-2a\,c^3e^2f+b^4e^2h-2b^3cdeh-b^3c\,e^2g+b^2c^2d^2h+2b^2c^2deg+b^2c^2e^2}{c\left(4ac-b^2\right)}}{}$
risch	Expression too large to display

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
h*e^2/c^2*x-1/c^2*((-(2*a^2*c^2*e^2*h-4*a*b^2*c*e^2*h+6*a*b*c^2*d*e*h+3*a*
b*c^2*e^2*g-2*a*c^3*d^2*h-4*a*c^3*d*e*g-2*a*c^3*e^2*f+b^4*e^2*h-2*b^3*c*d*
e*h-b^3*c*e^2*g+b^2*c^2*d^2*h+2*b^2*c^2*d*e*g+b^2*c^2*e^2*f-b*c^3*d^2*g-2*
b*c^3*d*e*f+2*c^4*d^2*f)/c/(4*a*c-b^2)*x+(3*a^2*b*c*e^2*h-4*a^2*c^2*d*e*h-
2*a^2*c^2*e^2*g-a*b^3*e^2*h+2*a*b^2*c*d*e*h+a*b^2*c*e^2*g-a*b*c^2*d^2*h-2*
a*b*c^2*d*e*g-a*b*c^2*e^2*f+2*a*c^3*d^2*g+4*a*c^3*d*e*f-b*c^3*d^2*f)/c/(4*
a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*e^2*h-8*a*c^2*d*e*h-4*
a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)/c*ln(c*x^2+b*x+a)+2*(6*
a^2*c*e^2*h-2*a*b^2*e^2*h+2*a*b*c*d*e*h+a*b*c*e^2*g-2*a*c^2*d^2*h-4*a*c^2*
d*e*g-2*a*c^2*e^2*f+b*c^2*d^2*g+2*b*c^2*d*e*f-2*c^3*d^2*f-1/2*(8*a*b*c*e^2
*h-8*a*c^2*d*e*h-4*a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(281) = 562$.

Time = 0.55 (sec) , antiderivative size = 2771, normalized size of antiderivative = 9.62

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f
- (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d
^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)
*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^
2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c -
 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d
^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (
b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3
*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c
^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*(
(a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*
e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5
)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*
e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4
)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^...
```

3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(291) = 582$.

Time = 156.09 (sec) , antiderivative size = 2966, normalized size of antiderivative = 10.30

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a*
*2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*
g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2
*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*
d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*
log(x + (-10*a**2*b*c*e**2*h - 16*a**2*c**4*(-e*(2*b*e*h - 2*c*d*h - c*e*g
)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e
**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3
*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g
+ 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a**2*c**2*d*e*h + 8*a**2*
c**2*e**2*g + 2*a*b**3*e**2*h + 8*a*b**2*c**3*(-e*(2*b*e*h - 2*c*d*h - c*e
*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c
*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c*
*3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*
g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 2*a*b**2*c*d*e*h - a*b**2*
c*e**2*g - 2*a*b*c**2*d**2*h - 4*a*b*c**2*d*e*g - 2*a*b*c**2*e**2*f - b**4
*c**2*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*
(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c...
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.90

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{e^2hx}{c^2}$$
$$-\frac{(4\,c^4d^2f-4\,bc^3def+4\,ac^3e^2f-2\,bc^3d^2g+8\,ac^3deg+b^3ce^2g-6\,abc^2e^2g+4\,ac^3d^2h+2\,b^3cdeh-12\,a}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}$$
$$+\frac{(ce^2g+2\,cdeh-2\,be^2h)\log(cx^2+bx+a)}{2\,c^3}$$
$$-\frac{\frac{(2\,c^4d^2f-2\,bc^3def+b^2c^2e^2f-2\,ac^3e^2f-bc^3d^2g+2\,b^2c^2deg-4\,ac^3deg-b^3ce^2g+3\,abc^2e^2g+b^2c^2d^2h-2\,ac^3d^2h-2\,b^3cdeh+6\,abc^2deh+b^4e^2h-4}{c}}{(cx^2+bx}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
e^2*h*x/c^2 - (4*c^4*d^2*f - 4*b*c^3*d*e*f + 4*a*c^3*e^2*f - 2*b*c^3*d^2*g
 + 8*a*c^3*d*e*g + b^3*c*e^2*g - 6*a*b*c^2*e^2*g + 4*a*c^3*d^2*h + 2*b^3*c
*d*e*h - 12*a*b*c^2*d*e*h - 2*b^4*e^2*h + 12*a*b^2*c*e^2*h - 12*a^2*c^2*e^
2*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2
 + 4*a*c)) + 1/2*(c*e^2*g + 2*c*d*e*h - 2*b*e^2*h)*log(c*x^2 + b*x + a)/c^
3 - ((2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*f - b*c^3*
d^2*g + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g +
b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h + b^4*e^2*
h - 4*a*b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*x/c + (b*c^3*d^2*f - 4*a*c^3*d*e*f
+ a*b*c^2*e^2*f - 2*a*c^3*d^2*g + 2*a*b*c^2*d*e*g - a*b^2*c*e^2*g + 2*a^2*
c^2*e^2*g + a*b*c^2*d^2*h - 2*a*b^2*c*d*e*h + 4*a^2*c^2*d*e*h + a*b^3*e^2*
h - 3*a^2*b*c*e^2*h)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx$$

$$= \frac{\frac{-3\,h\,a^2\,b\,c\,e^2+4\,h\,a^2\,c^2\,d\,e+2\,g\,a^2\,c^2\,e^2+h\,a\,b^3\,e^2-2\,h\,a\,b^2\,c\,d\,e-g\,a\,b^2\,c\,e^2+h\,a\,b\,c^2\,d^2+2\,g\,a\,b\,c^2\,d\,e+f\,a\,b\,c^2\,e^2-2\,g\,a\,c^3\,d^2-4\,f\,a\,c^3\,d\,e+f\,b\,c^3\,d^2}{c\,(4\,a\,c-b^2)}}{}$$

$$+ \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,h\,a^3\,b\,c^3\,e^2+64\,g\,a^3\,c^4\,e^2+128\,d\,h\,a^3\,c^4\,e+96\,h\,a^2\,b^3\,c^2\,e^2-48\,g\,a^2\,b^2\,c^3\,e^2-\right.}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+\right.}$$

$$+ \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(-12\,h\,a^2\,c^2\,e^2+12\,h\,a\,b^2\,c\,e^2-12\,h\,a\,b\,c^2\,d\,e-6\,g\,a\,b\,c^2\,e^2+4\,h\,a\,c^3\,d^2\right.}{c^3\,(4\,a\,c-b^2}$$

$$+ \frac{e^2\,h\,x}{c^2}$$

input
```
int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

output
```
((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^
2*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h -
4*a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*
(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h
 - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e
^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b
*c^2*x) + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^
2*g - 24*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*
b*c^3*e^2*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h
+ 24*a*b^4*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*
c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 1
2*a^2*c^2*e^2*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^
2*g - 6*a*b*c^2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f +
 2*b^3*c*d*e*h - 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/
c^2
```

3.156 $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

3.156.1 Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh)))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\,(b^2-4ac)^{3/2}} + \frac{eh\log(a+bx+cx^2)}{2c^2}$$

output
```
(e*x+d)*(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c
+b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*f)-2*a*
(d*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1
/2*e*h*ln(c*x^2+b*x+a)/c^2
```

3.156.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{-\frac{2\left(-b^3ehx+b^2(-aeh+c(eg+dh)x)+bc(adh-cefx+cd(f-gx)+ae(g+3hx))+2c\left(a^2eh+c^2dfx-ac(e(f+gx)+d(g+hx))\right)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2\left(4c^3df+b^3eh-6\right.}{}}{2c^2}$$

input
```
Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f
*x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*
(f + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*
d*f + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*
(b + c*x)])/(2*c^2)
```

3.156.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2175, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$\Big\downarrow \ 2175$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{2cdf-b(ef+dg)-\frac{abeh}{c}+2a(eg+dh)+\left(4a-\frac{b^2}{c}\right)ehx}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1142$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\frac{\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)\int\frac{1}{cx^2+bx+a}dx}{2c^2}-\frac{eh\left(b^2-4ac\right)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}}{b^2-4ac}$$

$$\Big\downarrow \; 1083$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{-\frac{eh\left(b^2-4ac\right)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}-\frac{\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c^2}}{b^2-4ac}$$

$$\Big\downarrow \; 219$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{-\frac{eh\left(b^2-4ac\right)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\sqrt{b^2-4ac}}}{b^2-4ac}$$

$$\Big\downarrow \; 1103$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\sqrt{b^2-4ac}}-\frac{eh\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)}{2c^2}}{b^2-4ac}$$

input
```
Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c
*h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (-(((4*c^3*d*f + b^3*e*h - 6
*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*e*h*Log[a +
b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

3.156. $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

3.156.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.156. $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2} dx$

3.156.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.73

method	result
default	$\frac{\frac{\left(3abceh-2a\,c^{2}dh-2a\,c^{2}eg-b^{3}eh+b^{2}cdh+b^{2}ceg-b\,c^{2}dg-b\,c^{2}ef+2c^{3}df\right)x}{c^{2}\left(4ac-b^{2}\right)}+\frac{2a^{2}ceh-a\,b^{2}eh+abcdh+abceg-2a\,c^{2}dg-2a\,c^{2}ef+b\,c^{2}df}{\left(4ac-b^{2}\right)c^{2}}}{c\,x^{2}+bx+a}+\frac{\left(4aehc\right.}{}$
risch	Expression too large to display

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*
g-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+
a*b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a
)+1/c/(4*a*c-b^2)*(1/2*(4*a*c*e*h-b^2*e*h)/c*ln(c*x^2+b*x+a)+2*(-b*a*e*h+2
*a*c*d*h+2*a*c*e*g-b*c*d*g-b*c*e*f+2*c^2*d*f-1/2*(4*a*c*e*h-b^2*e*h)*b/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(172) = 344$.

Time = 0.37 (sec) , antiderivative size = 1413, normalized size of antiderivative = 7.94

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
 (b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*
d - 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3
*d - b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 -
6*a*b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a
*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*
d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 -
 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 -
4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*
c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a
^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*l
og(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
 1/2*(2*((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d
 + (b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^
2*d - 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c
^3*d - b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4
- 6*a*b^2*c)*e)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*...
```

3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(170) = 340$.

Time = 19.64 (sec) , antiderivative size = 1535, normalized size of antiderivative = 8.62

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a
*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2
*c**3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h
 - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*
c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*
e*h + 8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h
- 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4
*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
)) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(
-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h
+ 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d
*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g +
 2*b*c**2*e*f - 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6
*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c
**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)))*log(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)*
*3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g +
 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +...
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{eh\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{\left(4\,c^3df-2\,bc^2ef-2\,bc^2dg+4\,ac^2eg+4\,ac^2dh+b^3eh-6\,abceh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}} - \frac{bc^2df-2\,ac^2ef-2\,ac^2dg+abceg+abcdh-ab^2eh+2\,a^2ceh+\left(2\,c^3df-bc^2ef-bc^2dg+b^2ceg-2\,ac\right.}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*e*h*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*e*f - 2*b*c^2*d*g
+ 4*a*c^2*e*g + 4*a*c^2*d*h + b^3*e*h - 6*a*b*c*e*h)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f -
2*a*c^2*e*f - 2*a*c^2*d*g + a*b*c*e*g + a*b*c*d*h - a*b^2*e*h + 2*a^2*c*e*
h + (2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d
*h - 2*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a
*c)*c^2)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 14.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.11

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{\frac{b\,c^2\,d\,f-2\,a\,c^2\,e\,f-2\,a\,c^2\,d\,g-a\,b^2\,e\,h+2\,a^2\,c\,e\,h+a\,b\,c\,d\,h+a\,b\,c\,e\,g}{c^2\,\left(4\,a\,c-b^2\right)}-\frac{x\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g-\right.}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,e\,h\,a^3\,c^3+48\,e\,h\,a^2\,b^2\,c^2-12\,e\,h\,a\,b^4\,c+e\,h\,b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c-4\,a\,b\,c^2}{c\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,c^3\,d\,f+b^3\,e\,h+4\,a\,c^2\,d\,h+4\,a\,c^2\,e\,g-2\,b\,c^2\,d\,g-2\,b\,c^2\,e\,f-6\,a\,b\,c\,e\,h\right.}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

output
```
((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*
d*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d
*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c
*e*h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6
*e*h - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c
^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^
2)^(1/2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e
*h + 4*a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))
/(c^2*(4*a*c - b^2)^(3/2))
```

3.156. $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

3.157 $\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2} \, dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{2(2cf-bg+2ah)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c
*x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{abh+2c^2fx+b^2hx+bc(f-gx)-2ac(g+hx)}{c\,(-b^2+4ac)\,(a+x(b+cx))} - \frac{2(-2cf+bg-2ah)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
 4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

3.157.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \ 2191$$

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{\int \frac{2cf-bg+2ah}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{(2ah-bg+2cf)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \ 1083$$

$$\frac{2(2ah-bg+2cf)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \ 219$$

$$\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{(b^2-4ac)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}$$

input
```
Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(
b^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

3.157.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.157.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
default	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2(2ah-bg+2cf)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2\ln\left(\left(-8a\,c^2+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)ah}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-8a\,c^2+2b^2c\right)x+(-4ac+\right.}{(-4ac+b^2)}$

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(113) = 226$.

Time = 0.31 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.36

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$$
$$= \left[-\frac{(2\,ac^2f - abcg + 2\,a^2ch + (2\,c^3f - bc^2g + 2\,ac^2h)x^2 + (2\,bc^2f - b^2cg + 2\,abch)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2}{}\right.}{ab^4c - 8\,a^2b^2c^2 +}\right.$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2
+ (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
+ (b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*
h + (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c +
8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^
2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a
*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*
c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^
2*c^2)*g - (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a
*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*x)]
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(107) = 214$.

Time = 1.00 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.89

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh}{4ach-2bcg+4c^2f}\right.$$
$$+\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh+}{4ach-2bcg+4c^2f}\right.$$
$$+\frac{abh-2acg+bcf+x(-2ach+b^2h-bcg+2c^2f)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

input
```
integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*s
qrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2
)**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**
2*f)) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*
c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g +
 4*c**2*f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*
c**2*f))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*
c**2 - b**3*c))
```

3.157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{2\,(2\,cf-bg+2\,ah)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*
a*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*
c*f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.157.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$$
$$=\frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,(4\,a\,c-b^2)}+\frac{x\,(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^3-4\,a\,b\,c\right)\,(2\,a\,h-b\,g+2\,c\,f)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{2\,c\,x\,(2\,a\,h-b\,g+2\,c\,f)}{\left(4\,a\,c-b^2\right)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,a\,h-b\,g+2\,c\,f}\right)\,(2\,a\,h-b\,g+2\,c\,f)}{\left(4\,a\,c-b^2\right)^{3/2}}$$

input
```
int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

output
```
((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c
*h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*
b*c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*
c*f))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h -
b*g + 2*c*f))/(4*a*c - b^2)^(3/2)
```

3.158 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\,dx$

3.158.1 Optimal result

Integrand size = 30, antiderivative size = 407

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$
$$+\frac{(4c^3d^3f+be(4abdeh-2a^2e^2h+b^2(e^2f-deg-d^2h))-2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))}{(b^2-4ac)^{3/2}\left(cd^2-bde+ae^2\right.}$$
$$+\frac{e(e^2f-deg+d^2h)\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2}-\frac{e(e^2f-deg+d^2h)\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2}$$

output
```
(b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-(2*c^2*d*f+b*(-a*e
+b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^2+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e
*g+e^2*f))-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^
2*d^2*g+2*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g
+e^2*f)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2
+b*x+a)/(a*e^2-b*d*e+c*d^2)^2
```

3.158.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$
$$= \frac{-2a^2eh+2c^2dfx+b^2(-ef+dhx)+bc(-efx+d(f-gx))+ab(dh+e(g-hx))+2ac(e(f+gx)-d(}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)(a+x(b+cx))}$$
$$-\frac{\left(-4c^3d^3f+2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))+be(-4abdeh+2a^2e^2h+b^2(-e^2f+deg+d\right.}{\left(-b^2+4ac\right)^{3/2}\left(cd^2+e(-bc\right.}$$
$$+\frac{e(e^2f-deg+d^2h)\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^2}-\frac{e(e^2f-deg+d^2h)\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^2}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

output
```
(-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f -
g*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2
 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f +
 2*c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b
*d*e*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2
*g + 2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b +
2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e
))^2) + (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e)
)^2 - (e*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

3.158.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$
$$\Big\downarrow \text{2177}$$

$$\frac{-x\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2ef}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$
$$\frac{\int \frac{2c^2fd^2-bc(ef+dg)d-be(bef-bdg+adh)+2ac\left(hd^2-egd+2e^2f\right)+e\left(2dfc^2-(bef+bdg-2aeg+2adh)c+b(bd-ae)h\right)x}{\left(cd^2-bed+ae^2\right)(d+ex)\left(cx^2+bx+a\right)}dx}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{-x\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2ef}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$
$$\frac{\int \frac{2c^2fd^2-bc(ef+dg)d-be(bef-bdg+adh)+2ac\left(hd^2-egd+2e^2f\right)+e\left(2dfc^2-(bef+bdg-2aeg+2adh)c+b(bd-ae)h\right)x}{(d+ex)\left(cx^2+bx+a\right)}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 1200

$$\frac{-x\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2ef}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$
$$\frac{\int \left(\frac{2c^3fd^3-c^2\left(bd(3ef+dg)-2a\left(hd^2-egd+3e^2f\right)\right)d+be^2\left(-eha^2+2bdha+b^2(ef-dg)\right)+ce\left(2e(eg-dh)a^2-b\left(3hd^2-egd+5e^2f\right)a+2b^2d^2g\right)+c\left(b^2-\right.}{\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)}\right.}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{-x\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2ef}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$
$$\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-deg+e^2f\right)\right)-2c^2d(bd(dg+3ef)-2\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}}{\left(b^2-4ac\right)\left(ae^2-bde+\right.}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^
2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2
- 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) - (-(((4*c^3*d^3*f + b
*e*(4*a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*
d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*
a^2*e*(e*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2))) - ((b^2 - 4
*a*c)*e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2) + ((
b^2 - 4*a*c)*e*(e^2*f - d*e*g + d^2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b
*d*e + a*e^2)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
```

3.158. $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\,dx$

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(401) = 802$.

Time = 0.96 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.99

method	result
default	$\frac{e\left(d^2h-deg+e^2f\right)\ln(ex+d)}{\left(e^2a-bde+c\,d^2\right)^2}-\frac{\frac{\left(a^2b\,e^3h+2a^2cd\,e^2h-2a^2c\,e^3g-2a\,b^2d\,e^2h-abc\,d^2eh+3abcd\,e^2g+abc\,e^3f+2a\,c^2d^3h-2a\,c^2d^2eg-2a\,c^2d\,e^2f+\right.}{4ac-b^2}}{}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e*(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/(a*e^2-b*d*e+c*d^2
)^2*(((a^2*b*e^3*h+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h-a*b*c*d^2
*e*h+3*a*b*c*d*e^2*g+a*b*c*e^3*f+2*a*c^2*d^3*h-2*a*c^2*d^2*e*g-2*a*c^2*d*e
^2*f+b^3*d^2*e*h-b^2*c*d^3*h-b^2*c*d^2*e*g-b^2*c*d*e^2*f+b*c^2*d^3*g+3*b*c
^2*d^2*e*f-2*c^3*d^3*f)/(4*a*c-b^2)*x+(2*a^3*e^3*h-3*a^2*b*d*e^2*h-a^2*b*e
^3*g+2*a^2*c*d^2*e*h+2*a^2*c*d*e^2*g-2*a^2*c*e^3*f+a*b^2*d^2*e*h+a*b^2*d*e
^2*g+a*b^2*e^3*f-a*b*c*d^3*h-3*a*b*c*d^2*e*g+a*b*c*d*e^2*f+2*a*c^2*d^3*g-2
*a*c^2*d^2*e*f-b^3*d*e^2*f+2*b^2*c*d^2*e*f-b*c^2*d^3*f)/(4*a*c-b^2))/(c*x^
2+b*x+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2*d^2*e*h-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f
-b^2*c*d^2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)/c*ln(c*x^2+b*x+a)+2*(a^2*b*e^3*h
+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h+3*a*b*c*d^2*e*h-a*b*c*d*e^2
*g+5*a*b*c*e^3*f-2*a*c^2*d^3*h+2*a*c^2*d^2*e*g-6*a*c^2*d*e^2*f+b^3*d*e^2*g
-b^3*e^3*f-2*b^2*c*d^2*e*g+b*c^2*d^3*g+3*b*c^2*d^2*e*f-2*c^3*d^3*f-1/2*(4*
a*c^2*d^2*e*h-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^2*e*h+b^2*c*d*e^2*g-b^
2*c*e^3*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.158.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

3.158.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(401) = 802.

Time = 0.27 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.18

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)^2}\,dx = -\frac{(e^3f-de^2g+d^2eh)\log(cx^2+bx+a)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)}$$
$$+\frac{(e^4f-de^3g+d^2e^2h)\log(|ex+d|)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5}$$
$$-\frac{(4\,c^3d^3f-6\,bc^2d^2ef+12\,ac^2de^2f+b^3e^3f-6\,abce^3f-2\,bc^2d^3g+4\,b^2cd^2eg-4\,ac^2d^2eg-b^3de^2g-2}{(b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2}$$
$$-\frac{bc^2d^3f-2\,b^2cd^2ef+2\,ac^2d^2ef+b^3de^2f-abcde^2f-ab^2e^3f+2\,a^2ce^3f-2\,ac^2d^3g+3\,abcd^2eg-ab^2}{}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(e^3*f - d*e^2*g + d^2*e*h)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3
*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (e^4*f - d*e^3
*g + d^2*e^2*h)*log(abs(e*x + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3
 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 6*b*c^2*d^2*e*f
 + 12*a*c^2*d*e^2*f + b^3*e^3*f - 6*a*b*c*e^3*f - 2*b*c^2*d^3*g + 4*b^2*c*
d^2*e*g - 4*a*c^2*d^2*e*g - b^3*d*e^2*g - 2*a*b*c*d*e^2*g + 4*a^2*c*e^3*g
+ 4*a*c^2*d^3*h - b^3*d^2*e*h - 2*a*b*c*d^2*e*h + 4*a*b^2*d*e^2*h - 4*a^2*
c*d*e^2*h - 2*a^2*b*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^
2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*
b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*
b^2*e^4 - 4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*b^2*c*d^2*e*
f + 2*a*c^2*d^2*e*f + b^3*d*e^2*f - a*b*c*d*e^2*f - a*b^2*e^3*f + 2*a^2*c*
e^3*f - 2*a*c^2*d^3*g + 3*a*b*c*d^2*e*g - a*b^2*d*e^2*g - 2*a^2*c*d*e^2*g
+ a^2*b*e^3*g + a*b*c*d^3*h - a*b^2*d^2*e*h - 2*a^2*c*d^2*e*h + 3*a^2*b*d*
e^2*h - 2*a^3*e^3*h + (2*c^3*d^3*f - 3*b*c^2*d^2*e*f + b^2*c*d*e^2*f + 2*a
*c^2*d*e^2*f - a*b*c*e^3*f - b*c^2*d^3*g + b^2*c*d^2*e*g + 2*a*c^2*d^2*e*g
 - 3*a*b*c*d*e^2*g + 2*a^2*c*e^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - b^3*d^2
*e*h + a*b*c*d^2*e*h + 2*a*b^2*d*e^2*h - 2*a^2*c*d*e^2*h - a^2*b*e^3*h)*x)
/((c*d^2 - b*d*e + a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 13698, normalized size of antiderivative = 33.66

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)
```

output
```
symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 19
2*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6
*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*
d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b
^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 44
0*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*
d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 -
192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2
*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4
*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d
^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e
^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d
^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c
^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 +
b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*
c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a
^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*
h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*
d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^
3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^...
```

3.158. $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\, dx$

3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx$

3.159.1 Optimal result

Integrand size = 30, antiderivative size = 673

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{e(e^2f-deg+d^2h)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$
$$-\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b(c^2d^2f+a^2e^2h-ac(3e^2f-2deg-d}{(b^2-4ac)\left(cd^2-bde\right.}$$
$$+\frac{(4c^4d^4f-b^3e^3(2bef-bdg-aeg+2adh)-2c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))-6c^2e(4ab}{}$$
$$-\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^3}$$
$$+\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^3}$$

output
```
-e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(-b^3*e^2*f+b^2*e*(a*
e*g+2*c*d*f)-2*a*c*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g))-b*(c^2*d^2*f+a^2*e^
2*h-a*c*(-d^2*h-2*d*e*g+3*e^2*f))-c*(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+
e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*x)/(
-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*
d*h-a*e*g-b*d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e
^2*f))-6*c^2*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c
*e*(6*a^2*b*e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(
2*d^2*h-d*e*g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(3/2)/(a*e^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*
d^2*h-3*d*e*g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*
h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e
^2-b*d*e+c*d^2)^3
```

3.159.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{e(e^2f-deg+d^2h)}{\left(cd^2+e(-bd+ae)\right)^2(d+ex)}$$
$$+\frac{-b^3e^2f+b^2(ae^2g-c(-2def+e^2fx+d^2hx))+b(-a^2e^2h+c^2d(-df+2efx+dgx)+ac(-d^2h+e^2(\cdots}{(b^2-4ac)\left(cd^2+e(-\cdots\right.}$$
$$-\frac{(4c^4d^4f+b^3e^3(-2bef+bdg+aeg-2adh)-2c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))-6c^2e(4a\cdots}{\cdots}$$
$$+\frac{(e^3(-2bef+bdg+aeg-2adh)+cde(4e^2f-3deg+2d^2h))\log(d+ex)}{\left(cd^2+e(-bd+ae)\right)^3}$$
$$-\frac{(e^3(-2bef+bdg+aeg-2adh)+cde(4e^2f-3deg+2d^2h))\log(a+x(b+cx))}{2\left(cd^2+e(-bd+ae)\right)^3}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

output
```
-((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) +
(-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^
2*e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g
*x) - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f +
g*x) + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d
^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*
b*e*f + b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e
^2*f - 2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e
^2*f - 2*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*
e^2*f - 3*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*
d - a*e))^3) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f
 - 3*d*e*g + 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*
(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h)
)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

3.159.3 Rubi [A] (verified)

Time = 3.85 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2177, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx$$

$$\Big\downarrow \text{2177}$$

$$-\frac{\int \frac{\frac{ce^2\left(\left(hd^2+e^2f\right)b^2-ae(eg+2dh)b+2c^2d^2f+2a^2e^2h-c\left(bd(2ef+dg)+2a\left(hd^2-2egd+e^2f\right)\right)\right)x^2}{\left(cd^2-bed+ae^2\right)^2}+\frac{e\left(4c^3fd^3-2c^2\left(bd(2ef+dg)-2a\left(-hd^2+egd+e^2f\right)\right)d-c\left(-\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-\cdots\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)(ae\cdots}$$

$$\Big\downarrow \text{2159}$$

$$-\frac{\int \left(\frac{(b^2-4ac)\left(e^2(2bef-bdg-aeg+2adh)-cd(2hd^2-3egd+4e^2f)\right)e^2}{(cd^2-bed+ae^2)^3(d+ex)} - \frac{(b^2-4ac)(hd^2-egd+e^2f)e^2}{(cd^2-bed+ae^2)^2(d+ex)^2} + \frac{2c^4fd^4-c^3\left(bd(4ef+dg)-2a(hd^2-2e\right.}{}\right.}{}$$

$$\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-\right.}{(b^2-4ac)\left(a+bx+cx^2\right)(ae}$$

$\downarrow$ 2009

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-\right.}{(b^2-4ac)\left(a+bx+cx^2\right)(}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-6c^2e\left(2a^2e\left(2d^2h-2deg+e^2f\right)+4abde^2f-b^2d^3g\right)-ce\left(-4a^3e^3h+6a^2be^3g-6ab^2e\left(2d^2h-deg+2e^2f\right)+b^3(-d)\left(-2d^2h-3\right.\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

output
```
-((b^3*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e
*g - 2*d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h))
 + c*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h
) - c*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2))) - (((b^2 - 4*a*c)*e*(e^2*
f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)) - ((4*c^4*d^4*f
 - b^3*e^3*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*
g) - 2*a*(6*e^2*f - 2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g
 + 2*a^2*e*(e^2*f - 2*d*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h
 - b^3*d*(4*e^2*f - 3*d*e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^
2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 -
 b*d*e + a*e^2)^3) + ((b^2 - 4*a*c)*e*(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*
d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e + a
*e^2)^3 - ((b^2 - 4*a*c)*e*(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*
(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a
*e^2)^3))/(b^2 - 4*a*c)
```

3.159. $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.159.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(668) = 1336$.

Time = 0.84 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.00

method	result	size
default	Expression too large to display	1345
risch	Expression too large to display	8771

input
```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-e*(2*a*d*e^2*h-a*e^3*
g-b*d*e^2*g+2*b*e^3*f-2*c*d^3*h+3*c*d^2*e*g-4*c*d*e^2*f)/(a*e^2-b*d*e+c*d^
2)^3*ln(e*x+d)+1/(a*e^2-b*d*e+c*d^2)^3*((c*(2*a^3*e^4*h-4*a^2*b*d*e^3*h-a^
2*b*e^4*g+4*a^2*c*d*e^3*g-2*a^2*c*e^4*f+3*a*b^2*d^2*e^2*h+a*b^2*d*e^3*g+a*
b^2*e^4*f-6*a*b*c*d^2*e^2*g-2*a*c^2*d^4*h+4*a*c^2*d^3*e*g-b^3*d^3*e*h-b^3*
d*e^3*f+b^2*c*d^4*h+b^2*c*d^3*e*g+3*b^2*c*d^2*e^2*f-b*c^2*d^4*g-4*b*c^2*d^
3*e*f+2*c^3*d^4*f)/(4*a*c-b^2)*x+(a^3*b*e^4*h-4*a^3*c*d*e^3*h+2*a^3*c*e^4*
g-a^2*b^2*d*e^3*h-a^2*b^2*e^4*g+6*a^2*b*c*d^2*e^2*h-3*a^2*b*c*e^4*f-4*a^2*
c^2*d^3*e*h+4*a^2*c^2*d*e^3*f+a*b^3*d*e^3*g+a*b^3*e^4*f-a*b^2*c*d^3*e*h-3*
a*b^2*c*d^2*e^2*g+a*b^2*c*d*e^3*f+a*b*c^2*d^4*h+4*a*b*c^2*d^3*e*g-6*a*b*c^
2*d^2*e^2*f-2*a*c^3*d^4*g+4*a*c^3*d^3*e*f-b^4*d*e^3*f+3*b^3*c*d^2*e^2*f-3*
b^2*c^2*d^3*e*f+b*c^3*d^4*f)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2
*(8*a^2*c^2*d*e^3*h-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*
c^2*d*e^3*g+8*a*b*c^2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*d*
e^3*f+b^3*c*d*e^3*g-2*b^3*c*e^4*f+2*b^2*c^2*d^3*e*h-3*b^2*c^2*d^2*e^2*g+4*
b^2*c^2*d*e^3*f)/c*ln(c*x^2+b*x+a)+2*(a*b^3*e^4*g+b^4*d*e^3*g-b*c^3*d^4*g+
2*a^3*c*e^4*h-6*a^2*c^2*e^4*f+2*a*c^3*d^4*h-4*a*c^3*d^3*e*g+4*b^3*c*d*e^3*
f-4*b*c^3*d^3*e*f-5*a^2*b*c*e^4*g+12*a^2*c^2*d*e^3*g+10*a*b^2*c*e^4*f-1/2*
(8*a^2*c^2*d*e^3*h-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c
^2*d*e^3*g+8*a*b*c^2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*...
```

3.159.5 Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

3.159.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.159.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(666) = 1332$.

Time = 0.29 (sec) , antiderivative size = 1506, normalized size of antiderivative = 2.24

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*c*d*e^3*f - 2*b*e^4*f - 3*c*d^2*e^2*g + b*d*e^3*g + a*e^4*g + 2*c*
d^3*e*h - 2*a*d*e^3*h)*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(
e*x + d) - b*d*e/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e
 + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (e^7*f/(e*x +
d) - d*e^6*g/(e*x + d) + d^2*e^5*h/(e*x + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5
 + b^2*d^2*e^6 + 2*a*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - (4*c^4*d^4*e^2*f
 - 8*b*c^3*d^3*e^3*f + 24*a*c^3*d^2*e^4*f + 4*b^3*c*d*e^5*f - 24*a*b*c^2*d
*e^5*f - 2*b^4*e^6*f + 12*a*b^2*c*e^6*f - 12*a^2*c^2*e^6*f - 2*b*c^3*d^4*e
^2*g + 6*b^2*c^2*d^3*e^3*g - 8*a*c^3*d^3*e^3*g - 3*b^3*c*d^2*e^4*g + b^4*d
*e^5*g - 6*a*b^2*c*d*e^5*g + 24*a^2*c^2*d*e^5*g + a*b^3*e^6*g - 6*a^2*b*c*
e^6*g + 4*a*c^3*d^4*e^2*h - 2*b^3*c*d^3*e^3*h + 12*a*b^2*c*d^2*e^4*h - 24*
a^2*c^2*d^2*e^4*h - 2*a*b^3*d*e^5*h + 4*a^3*c*e^6*h)*arctan((2*c*d - 2*c*d
^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sqrt(-b^2 + 4
*a*c)*e))/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e + 12*a*b*c^3*d^5*e
 + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^4*e^2 - b^5*d^3*e^
3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^2*e^4 - 9*a^2*b^2
*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3*b*c*d*e^5 + a^3
*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)*e^2) - ((2*c^4*d^3*e*f - 3*b*c^
3*d^2*e^2*f + 3*b^2*c^2*d*e^3*f - 6*a*c^3*d*e^3*f - b^3*c*e^4*f + 3*a*b...
```

3.159.9 Mupad [B] (verification not implemented)

Time = 21.29 (sec) , antiderivative size = 26278, normalized size of antiderivative = 39.05

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

output
```
((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f
- 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^
2*c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*
e^2*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^
2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e -
8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f +
 2*c^3*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2
*a^2*c*e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f +
 2*a*c^2*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b
*c*e^3*f + 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4
- a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*
e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^
2) - (x^2*(6*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f -
 8*a*c^2*d*e^2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2
*c*d*e^2*g - 2*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^
4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*
e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 +
2*a*b^2*c*d^2*e^2))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + sy
msum(log((x*(36*a^2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 +
4*c^7*d^4*e^3*f^2 + a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b...
```

3.159. $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx$

3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$

3.160.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = 3x+\frac{x^2}{2}+\frac{2(2-x)}{3(1-x+x^2)}+\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

output
```
3*x+1/2*x^2+2/3*(2-x)/(x^2-x+1)+2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2*x)*3^(1
/2))*3^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = 3x+\frac{x^2}{2}-\frac{2(-2+x)}{3(1-x+x^2)}-\frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

input
```
Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[
3]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

3.160.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(x^2+x+1\right)}{\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3}\int -\frac{-3x^3-6x^2-6x+2}{x^2-x+1}dx+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{3}\int \frac{-3x^3-6x^2-6x+2}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{3}\int \left(\frac{11-12x}{x^2-x+1}-3x-9\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3x^2}{2}+6\log\left(x^2-x+1\right)+9x\right)+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

input
```
Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(2*(2 - x))/(3*(1 - x + x^2)) + (9*x + (3*x^2)/2 + (10*ArcTan[(1 - 2*x)/Sq
rt[3]])/Sqrt[3] + 6*Log[1 - x + x^2])/3
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.160.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
default	$3x+\frac{x^{2}}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^{2}-x+1}+2\ln\left(x^{2}-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	53
risch	$3x+\frac{x^{2}}{2}+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^{2}-x+1}+2\ln\left(4x^{2}-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	55

input
```
int(x^3*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
3*x+1/2*x^2+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(
-1+2*x)*3^(1/2))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{9\,x^4 + 45\,x^3 - 20\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - 45\,x^2 + 36\,(x^2-x+1)\log\left(x^2-x+1\right) + 42\,...}{18\,(x^2-x+1)}$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x -
1)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1
)
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{x^2}{2} + 3x + \frac{4-2x}{3x^2-3x+3} + 2\log\left(x^2-x+1\right) - \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqr
t(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,(x^2-x+1)} + 2\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(
x^2 - x + 1) + 2*log(x^2 - x + 1)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,(x^2-x+1)} + 2\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(
x^2 - x + 1) + 2*log(x^2 - x + 1)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = 3\,x + 2\,\ln\left(x^2-x+1\right) - \frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1} - \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9} + \frac{x^2}{2}$$

input
```
int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx$

3.161.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{2(1-2x)}{3(1-x+x^2)}-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
x+2/3*(1-2*x)/(x^2-x+1)+3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^
(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x-\frac{2(-1+2x)}{3(1-x+x^2)}+\frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*S
qrt[3]) + (3*Log[1 - x + x^2])/2
```

3.161.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(x^2+x+1)}{(x^2-x+1)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3}\int \frac{3x^2+6x+2}{x^2-x+1}dx+\frac{2(1-2x)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{1}{3}\int \left(3-\frac{1-9x}{x^2-x+1}\right)dx+\frac{2(1-2x)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{9}{2}\log\left(x^2-x+1\right)+3x\right)+\frac{2(1-2x)}{3(x^2-x+1)}$$

input
```
Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(2*(1 - 2*x))/(3*(1 - x + x^2)) + (3*x - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqr
t[3] + (9*Log[1 - x + x^2])/2)/3
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.161.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
default	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	46
risch	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	48

input
```
int(x^2*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(-1+2*x)*3
^(1/2))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx$$
$$=\frac{18\,x^3+14\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-18\,x^2+27\,(x^2-x+1)\log\left(x^2-x+1\right)-6\,x+12}{18\,(x^2-x+1)}$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18
*x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(
2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1
) + 3/2*log(x^2 - x + 1)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1
) + 3/2*log(x^2 - x + 1)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x + \frac{3\,\ln\left(x^2-x+1\right)}{2} - \frac{\frac{4\,x}{3}-\frac{2}{3}}{x^2-x+1} + \frac{7\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*at
an((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx$

3.162.1 Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{2(1+x)}{3(1-x+x^2)} - \frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
-2/3*(1+x)/(x^2-x+1)+1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1
/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = -\frac{2(1+x)}{3(1-x+x^2)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[1 - x + x^2]/2
```

3.162.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2191, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+x+1)}{(x^2-x+1)^2}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3}\int \frac{3x+4}{x^2-x+1}dx-\frac{2(x+1)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{3}\left(\frac{11}{2}\int \frac{1}{x^2-x+1}dx+\frac{3}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(\frac{11}{2}\int \frac{1}{x^2-x+1}dx-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx-11\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{2(x+1)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(\frac{11\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{3}\left(\frac{11\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-x+1\right)\right)-\frac{2(x+1)}{3(x^2-x+1)}$$

input
```
Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(-2*(1 + x))/(3*(1 - x + x^2)) + ((11*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
+ (3*Log[1 - x + x^2])/2)/3
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.162. $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx$

3.162.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(x^2-x+1)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	45
risch	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(4x^2-4x+4)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	47

input
```
int(x*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^
(1/2))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+9(x^2-x+1)\log(x^2-x+1)-12x-12}{18(x^2-x+1)}$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x
+ 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{-2x-2}{3x^2-3x+3}+\frac{\log(x^2-x+1)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
(-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/9
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1
/2*log(x^2 - x + 1)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1
/2*log(x^2 - x + 1)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{\ln(x^2-x+1)}{2} - \frac{2\,x}{3\,(x^2-x+1)} - \frac{2}{3\,(x^2-x+1)} + \frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3
^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

3.163 $\int \frac{1+x+x^2}{(1-x+x^2)^2} \, dx$

3.163.1 Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1+x+x^2}{(1-x+x^2)^2} \, dx = -\frac{2(2-x)}{3(1-x+x^2)} - \frac{10 \arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-2/3*(2-x)/(x^2-x+1)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1+x+x^2}{(1-x+x^2)^2} \, dx = \frac{2(-2+x)}{3(1-x+x^2)} + \frac{10 \arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

output
```
(2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
])
```

3.163.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3}\int \frac{5}{x^2-x+1}dx-\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{3}\int \frac{1}{x^2-x+1}dx-\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{10}{3}\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{10\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

input
```
Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

output
```
(-2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
])
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.163.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	34
risch	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}$	34

input
```
int((x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1/2))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{2\left(5\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
 - x + 1)
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{2x-4}{3x^2-3x+3} + \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
(2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/
9
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2\,(x-2)}{3\,(x^2-x+1)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(x-2)}{3(x^2-x+1)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1} + \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
```

output
```
((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)
/3))/9
```

3.164 $\int \frac{1+x+x^2}{x(1-x+x^2)^2} \, dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{1+x+x^2}{x(1-x+x^2)^2} \, dx = -\frac{2(1-2x)}{3(1-x+x^2)} - \frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
-2/3*(1-2*x)/(x^2-x+1)+ln(x)-1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/
2))*3^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x(1-x+x^2)^2} \, dx = \frac{2(-1+2x)}{3(1-x+x^2)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

output
```
(2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + Log[x] - Log[1 - x + x^2]/2
```

3.164.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2177, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{1}{3}\int \frac{4x+3}{x\left(x^2-x+1\right)}dx-\frac{2(1-2x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}\int \left(\frac{7-3x}{x^2-x+1}+\frac{3}{x}\right)dx-\frac{2(1-2x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)\right)-\frac{2(1-2x)}{3\left(x^2-x+1\right)}$$

input
```
Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

output
```
(-2*(1 - 2*x))/(3*(1 - x + x^2)) + ((-11*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3
] + 3*Log[x] - (3*Log[1 - x + x^2])/2)/3
```

3.164.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.164.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}-\frac{\ln\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}+\ln\left(x\right)$	48
risch	$\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{11\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{\ln\left(4x^2-4x+4\right)}{2}+\ln\left(x\right)$	49

input
```
int((x^2+x+1)/x/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(-1+2*x)*3
^(1/2))+ln(x)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{22\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-9\,(x^2-x+1)\log\left(x^2-x+1\right)+18\,(x^2-x+1)\log\left(x\right)+24}{18\,(x^2-x+1)}$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x
+ 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{4x-2}{3x^2-3x+3}+\log(x)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

output
```
(4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*a
tan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(2x-1)}{3(x^2-x+1)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
 1/2*log(x^2 - x + 1) + log(x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2(2x-1)}{3\left(x^2-x+1\right)} - \frac{1}{2}\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
 1/2*log(x^2 - x + 1) + log(abs(x))
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \ln(x)+\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,11i}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,11i}{18}\right)$$

input
```
int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

output
```
log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 -
1/2)
```

3.165 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx$

3.165.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
-1/x+2/3*(1+x)/(x^2-x+1)+3*ln(x)-3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^
(1/2))*3^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx=-\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

output
```
-x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

3.165.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2177, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^2\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3}\int \frac{2x^2+6x+3}{x^2\left(x^2-x+1\right)}dx+\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3}\int \left(\frac{8-9x}{x^2-x+1}+\frac{9}{x}+\frac{3}{x^2}\right)dx+\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{9}{2}\log\left(x^2-x+1\right)-\frac{3}{x}+9\log(x)\right)+\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

input
```
Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

output
```
(2*(1 + x))/(3*(1 - x + x^2)) + (-3/x - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt
[3] + 9*Log[x] - (9*Log[1 - x + x^2])/2)/3
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^{2}-x+1}-\frac{3\ln\left(x^{2}-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{1}{x}+3\ln\left(x\right)$	55
risch	$\frac{-\frac{1}{3}x^{2}+\frac{5}{3}x-1}{x\left(x^{2}-x+1\right)}+\frac{7\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{3\ln\left(4x^{2}-4x+4\right)}{2}+3\ln\left(x\right)$	59

input
```
int((x^2+x+1)/x^2/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^
(1/2))-1/x+3*ln(x)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int\frac{1+x+x^{2}}{x^{2}\left(1-x+x^{2}\right)^{2}}\,dx$$
$$=\frac{14\sqrt{3}(x^{3}-x^{2}+x)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^{2}-27\left(x^{3}-x^{2}+x\right)\log\left(x^{2}-x+1\right)+54\left(x^{3}-x^{2}+x\right)}{18\left(x^{3}-x^{2}+x\right)}$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 2
7*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18
)/(x^3 - x^2 + x)
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log\left(x\right) - \frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

output
```
(-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)
/2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)} - \frac{3}{2}\log\left(x^2-x+1\right)+3\log\left(x\right)$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
 + x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x^2-5x+3}{3\left(x^3-x^2+x\right)} - \frac{3}{2}\log\left(x^2-x+1\right) + 3\log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
 + x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = 3\ln(x) - \frac{\frac{x^2}{3}-\frac{5x}{3}+1}{x^3-x^2+x} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

input
```
int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

output
```
3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7
i)/18 - 3/2)
```

3.166 $\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx$

3.166.1 Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = -\frac{1}{2x^2} - \frac{3}{x} + \frac{2(2-x)}{3(1-x+x^2)} + \frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x) - 2\log\left(1-x+x^2\right)$$

output
```
-1/2/x^2-3/x+2/3*(2-x)/(x^2-x+1)+4*ln(x)-2*ln(x^2-x+1)+10/9*arctan(1/3*(1-
2*x)*3^(1/2))*3^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = -\frac{1}{2x^2} - \frac{3}{x} - \frac{2(-2+x)}{3(1-x+x^2)} - \frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x) - 2\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

output
```
-1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

3.166.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2177, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^3\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3}\int \frac{-2x^3+6x^2+6x+3}{x^3\left(x^2-x+1\right)}dx+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3}\int \left(\frac{1-12x}{x^2-x+1}+\frac{12}{x}+\frac{9}{x^2}+\frac{3}{x^3}\right)dx+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2x^2}-6\log\left(x^2-x+1\right)-\frac{9}{x}+12\log(x)\right)+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

input
```
Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

output
```
(2*(2 - x))/(3*(1 - x + x^2)) + (-3/(2*x^2) - 9/x + (10*ArcTan[(1 - 2*x)/S
qrt[3]])/Sqrt[3] + 12*Log[x] - 6*Log[1 - x + x^2])/3
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.166.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}-2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}-\frac{1}{2x^2}-\frac{3}{x}+4\ln\left(x\right)$	60
risch	$\frac{-\frac{11}{3}x^3+\frac{23}{6}x^2-\frac{5}{2}x-\frac{1}{2}}{x^2\left(x^2-x+1\right)}-2\ln\left(4x^2-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{9}+4\ln\left(x\right)$	64

input
```
int((x^2+x+1)/x^3/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(-1+2*x)*3^(1
/2))-1/2/x^2-3/x+4*ln(x)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx =$$
$$-\frac{66\,x^3+20\,\sqrt{3}(x^4-x^3+x^2)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)-69\,x^2+36\,(x^4-x^3+x^2)\log\left(x^2-x+1\right)-72\,(}{18\,(x^4-x^3+x^2)}$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
-1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
 - 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*l
og(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = 4\log(x) - 2\log(x^2-x+1) - \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{-22x^3+23x^2-15x-3}{6x^4-6x^3+6x^2}$$

input
```
integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

output
```
4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3
)/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = -\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3-23x^2+15x+3}{6(x^4-x^3+x^2)} - 2\log(x^2-x+1) + 4\log(x)$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
-10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x
+ 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3-23x^2+15x+3}{6\left(x^2-x+1\right)x^2} - 2\log\left(x^2-x+1\right) + 4\log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
-10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x
+ 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = 4\ln(x) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \frac{\frac{11x^3}{3}-\frac{23x^2}{6}+\frac{5x}{2}+\frac{1}{2}}{x^4-x^3+x^2}$$

input
```
int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

output
```
4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3
)/3 + 1/2)/(x^2 - x^3 + x^4)
```

3.167 $\int \frac{1-x^2}{(1+x+x^2)^2} dx$

3.167.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{1-x^2}{(1+x+x^2)^2} dx = \frac{x}{1+x+x^2}$$

output
```
x/(x^2+x+1)
```

3.167.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x+x^2)^2} dx = \frac{x}{1+x+x^2}$$

input
```
Integrate[(1 - x^2)/(1 + x + x^2)^2,x]
```

output
```
x/(1 + x + x^2)
```

3.167.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+x+1)^2}\,dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{x}{x^2+x+1}$$

input
```
Int[(1 - x^2)/(1 + x + x^2)^2,x]
```

output
```
x/(1 + x + x^2)
```

3.167.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.167.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{x}{x^2+x+1}$	11
default	$\frac{x}{x^2+x+1}$	11
norman	$\frac{x}{x^2+x+1}$	11
risch	$\frac{x}{x^2+x+1}$	11
parallelrisch	$\frac{x}{x^2+x+1}$	11

input
```
int((-x^2+1)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+x+1)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x+x^2)^2}\,dx = \frac{x}{x^2+x+1}$$

input
```
integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
```

output
```
x/(x^2 + x + 1)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1-x^2}{(1+x+x^2)^2}\,dx = \frac{x}{x^2+x+1}$$

input
```
integrate((-x**2+1)/(x**2+x+1)**2,x)
```

output
```
x/(x**2 + x + 1)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x+x^2)^2}\,dx = \frac{x}{x^2+x+1}$$

input
```
integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
```

output
```
x/(x^2 + x + 1)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx = \frac{1}{x+\frac{1}{x}+1}$$

input
```
integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
```

output
```
1/(x + 1/x + 1)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx = \frac{x}{x^2+x+1}$$

input
```
int(-(x^2 - 1)/(x + x^2 + 1)^2,x)
```

output
```
x/(x + x^2 + 1)
```

3.168 $\int \frac{1+x^2}{1+x+x^2} dx$

3.168.1 Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1+x^2}{1+x+x^2} dx = x+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{1+x+x^2} dx = x+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + x^2)/(1 + x + x^2),x]
```

output
```
x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

3.168.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^2+x+1} \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(1-\frac{x}{x^2+x+1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2+x+1\right)+x$$

input
```
Int[(1 + x^2)/(1 + x + x^2),x]
```

output
```
x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.168.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$x-\frac{\ln\left(x^{2}+x+1\right)}{2}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	28
risch	$x-\frac{\ln\left(4x^{2}+4x+4\right)}{2}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	32

input
```
int((x^2+1)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^{2}+x+1\right)$$

input
```
integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x-\frac{\log\left(x^{2}+x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**2+1)/(x**2+x+1),x)
```

output
```
x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1+x^2}{1+x+x^2}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x - \frac{1}{2}\log\left(x^2+x+1\right)$$

input
```
integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1+x^2}{1+x+x^2}\,dx = x - \frac{\ln\left(x^2+x+1\right)}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((x^2 + 1)/(x + x^2 + 1),x)
```

output
```
x - log(x + x^2 + 1)/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3
```

3.169 $\int \frac{-1+x^2}{25-6x+x^2} \, dx$

3.169.1 Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{-1+x^2}{25-6x+x^2} \, dx = x-2\arctan\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

output
```
x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

3.169.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{25-6x+x^2} \, dx = x-2\arctan\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)$$

input
```
Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
```

output
```
x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

3.169.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^2-6x+25}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(1-\frac{2(13-3x)}{x^2-6x+25}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2\arctan\left(\frac{x-3}{4}\right)+3\log\left(x^2-6x+25\right)+x$$

input
```
Int[(-1 + x^2)/(25 - 6*x + x^2),x]
```

output
```
x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.169.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$x-2\arctan\left(-\frac{3}{4}+\frac{x}{4}\right)+3\ln\left(x^{2}-6x+25\right)$	22
risch	$x-2\arctan\left(-\frac{3}{4}+\frac{x}{4}\right)+3\ln\left(x^{2}-6x+25\right)$	22
parallelrisch	$i\ln\left(x-3-4i\right)-i\ln\left(x-3+4i\right)+3\ln\left(x-3-4i\right)+3\ln\left(x-3+4i\right)+x$	37

input
```
int((x^2-1)/(x^2-6*x+25),x,method=_RETURNVERBOSE)
```

output
```
x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x-2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\,\log\left(x^{2}-6\,x+25\right)$$

input
```
integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
```

output
```
x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x+3\log\left(x^{2}-6x+25\right)-2\,\mathrm{atan}\left(\frac{x}{4}-\frac{3}{4}\right)$$

input
```
integrate((x**2-1)/(x**2-6*x+25),x)
```

output
```
x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x - 2\,\arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\,\log\left(x^2 - 6\,x + 25\right)$$

input
```
integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
```

output
```
x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x - 2\,\arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\,\log\left(x^2 - 6\,x + 25\right)$$

input
```
integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
```

output
```
x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{-1+x^2}{25-6x+x^2}\,dx = x + 3\,\ln\left(x^2 - 6\,x + 25\right) - 2\,\mathrm{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

input
```
int((x^2 - 1)/(x^2 - 6*x + 25),x)
```

output
```
x + 3*log(x^2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

3.170 $\int \frac{-10+3x^2}{4-4x+x^2} dx$

3.170.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{-10+3x^2}{4-4x+x^2} \, dx = \frac{2}{2-x} + 3x + 12\log(2-x)$$

output
```
2/(2-x)+3*x+12*ln(2-x)
```

3.170.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{-10+3x^2}{4-4x+x^2} \, dx = -\frac{2}{-2+x} + 3(-2+x) + 12\log(-2+x)$$

input
```
Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

output
```
-2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

3.170.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-10}{x^2-4x+4}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(3-\frac{2(11-6x)}{x^2-4x+4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3x+\frac{2}{2-x}+12\log(2-x)$$

input
```
Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
```

output
```
2/(2 - x) + 3*x + 12*Log[2 - x]
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.170.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$3x+12\ln\left(-2+x\right)-\frac{2}{-2+x}$	18
risch	$3x+12\ln\left(-2+x\right)-\frac{2}{-2+x}$	18
norman	$\frac{3x^2-14}{-2+x}+12\ln\left(-2+x\right)$	21
parallelrisch	$\frac{12\ln(-2+x)x+3x^2-14-24\ln(-2+x)}{-2+x}$	27
meijerg	$-\frac{5x}{2\left(1-\frac{x}{2}\right)}+\frac{x\left(-\frac{3x}{2}+6\right)}{1-\frac{x}{2}}+12\ln\left(1-\frac{x}{2}\right)$	34

input
```
int((3*x^2-10)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
```

output
```
3*x+12*ln(-2+x)-2/(-2+x)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx=\frac{3\,x^2+12\,(x-2)\log\left(x-2\right)-6\,x-2}{x-2}$$

input
```
integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")
```

output
```
(3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx=3x+12\log\left(x-2\right)-\frac{2}{x-2}$$

input
```
integrate((3*x**2-10)/(x**2-4*x+4),x)
```

output
```
3*x + 12*log(x - 2) - 2/(x - 2)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x - \frac{2}{x-2} + 12\,\log\left(x-2\right)$$

input
```
integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
```

output
```
3*x - 2/(x - 2) + 12*log(x - 2)
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x - \frac{2}{x-2} + 12\,\log\left(|x-2|\right)$$

input
```
integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")
```

output
```
3*x - 2/(x - 2) + 12*log(abs(x - 2))
```

3.170.9 Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{-10+3x^2}{4-4x+x^2}\,dx = 3\,x + 12\,\ln\left(x-2\right) - \frac{2}{x-2}$$

input
```
int((3*x^2 - 10)/(x^2 - 4*x + 4),x)
```

output
```
3*x + 12*log(x - 2) - 2/(x - 2)
```

3.171 $\int \frac{8+x^2}{6-5x+x^2} \, dx$

3.171.1 Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{8+x^2}{6-5x+x^2} \, dx = x - 12\log(2-x) + 17\log(3-x)$$

output
```
x-12*ln(2-x)+17*ln(3-x)
```

3.171.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{8+x^2}{6-5x+x^2} \, dx = x - 12\log(2-x) + 17\log(3-x)$$

input
```
Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
```

output
```
x - 12*Log[2 - x] + 17*Log[3 - x]
```

3.171.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+8}{x^2-5x+6} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{5x+2}{x^2-5x+6}+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x-12 \log (2-x)+17 \log (3-x)$$

input
```
Int[(8 + x^2)/(6 - 5*x + x^2),x]
```

output
```
x - 12*Log[2 - x] + 17*Log[3 - x]
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.171.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
norman	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
risch	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15
parallelrisch	$x-12\ln\left(-2+x\right)+17\ln\left(-3+x\right)$	15

input
```
int((x^2+8)/(x^2-5*x+6),x,method=_RETURNVERBOSE)
```

output
```
x-12*ln(-2+x)+17*ln(-3+x)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x-12\,\log\left(x-2\right)+17\,\log\left(x-3\right)$$

input
```
integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
```

output
```
x - 12*log(x - 2) + 17*log(x - 3)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x+17\log\left(x-3\right)-12\log\left(x-2\right)$$

input
```
integrate((x**2+8)/(x**2-5*x+6),x)
```

output
```
x + 17*log(x - 3) - 12*log(x - 2)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\log(x-2) + 17\log(x-3)$$

input
```
integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
```

output
```
x - 12*log(x - 2) + 17*log(x - 3)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\log(|x-2|) + 17\log(|x-3|)$$

input
```
integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
```

output
```
x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

3.171.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{8+x^2}{6-5x+x^2}\,dx = x - 12\ln(x-2) + 17\ln(x-3)$$

input
```
int((x^2 + 8)/(x^2 - 5*x + 6),x)
```

output
```
x - 12*log(x - 2) + 17*log(x - 3)
```

3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 14

$$\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx = x+4\log(4-x)+\log(2+x)$$

output
```
x+4*ln(4-x)+ln(2+x)
```

3.172.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx = x+4\log(4-x)+\log(2+x)$$

input
```
Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

output
```
x + 4*Log[4 - x] + Log[2 + x]
```

3.172.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x-4}{x^2-2x-8}\,dx$$
$$\Big\downarrow \; 2188$$
$$\int \left(\frac{5x+4}{x^2-2x-8}+1\right)dx$$
$$\Big\downarrow \; 2009$$
$$x+4\log(4-x)+\log(x+2)$$

input
```
Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

output
```
x + 4*Log[4 - x] + Log[2 + x]
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.172.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$x+4\ln(x-4)+\ln(2+x)$	13
norman	$x+4\ln(x-4)+\ln(2+x)$	13
risch	$x+4\ln(x-4)+\ln(2+x)$	13
parallelrisch	$x+4\ln(x-4)+\ln(2+x)$	13

input
```
int((x^2+3*x-4)/(x^2-2*x-8),x,method=_RETURNVERBOSE)
```

output
```
x+4*ln(x-4)+ln(2+x)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log(x+2)+4\,\log(x-4)$$

input
```
integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
```

output
```
x + log(x + 2) + 4*log(x - 4)
```

3.172.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+4\log(x-4)+\log(x+2)$$

input
```
integrate((x**2+3*x-4)/(x**2-2*x-8),x)
```

output
```
x + 4*log(x - 4) + log(x + 2)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log(x+2)+4\,\log(x-4)$$

input
```
integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
```

output
```
x + log(x + 2) + 4*log(x - 4)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\log(|x+2|)+4\,\log(|x-4|)$$

input
```
integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
```

output
```
x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

3.172.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx = x+\ln(x+2)+4\,\ln(x-4)$$

input
```
int(-(3*x + x^2 - 4)/(2*x - x^2 + 8),x)
```

output
```
x + log(x + 2) + 4*log(x - 4)
```

3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2} dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} dx = x + \frac{3}{8}\arctan\left(\frac{1}{2}+x\right) + \frac{1}{8}\log\left(5+4x+4x^2\right)$$

output
```
x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

3.173.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} dx = x + \frac{3}{8}\arctan\left(\frac{1}{2}(1+2x)\right) + \frac{1}{8}\log\left(5+4x+4x^2\right)$$

input
```
Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

output
```
x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

3.173.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+5x+7}{4x^2+4x+5}\,dx$$
$$\Big\downarrow \text{2188}$$
$$\int \left(\frac{x+2}{4x^2+4x+5}+1\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)+x$$

input
```
Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

output
```
x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.173.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^{2}+4x+5\right)}{8}$	22
risch	$x+\frac{3\arctan\left(x+\frac{1}{2}\right)}{8}+\frac{\ln\left(4x^{2}+4x+5\right)}{8}$	22
parallelrisch	$x+\frac{\ln\left(x+\frac{1}{2}-i\right)}{8}-\frac{3i\ln\left(x+\frac{1}{2}-i\right)}{16}+\frac{\ln\left(x+\frac{1}{2}+i\right)}{8}+\frac{3i\ln\left(x+\frac{1}{2}+i\right)}{16}$	37

input
```
int((4*x^2+5*x+7)/(4*x^2+4*x+5),x,method=_RETURNVERBOSE)
```

output
```
x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4\,x^2+4\,x+5\right)$$

input
```
integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
```

output
```
x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx = x+\frac{\log\left(x^2+x+\frac{5}{4}\right)}{8}+\frac{3\,\mathrm{atan}\left(x+\frac{1}{2}\right)}{8}$$

input
```
integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

output
```
x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx = x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log\left(4\,x^2 + 4\,x + 5\right)$$

input
```
integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

output
```
x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx = x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log\left(4\,x^2 + 4\,x + 5\right)$$

input
```
integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
```

output
```
x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx = x + \frac{\ln\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

input
```
int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
```

output
```
x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

3.174 $\int \frac{2-x+x^2}{-5+2x+x^2} \, dx$

3.174.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{2-x+x^2}{-5+2x+x^2} \, dx = x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

output
```
x-1/6*ln(1+x-6^(1/2))*(9-5*6^(1/2))-1/6*ln(1+x+6^(1/2))*(9+5*6^(1/2))
```

3.174.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{2-x+x^2}{-5+2x+x^2} \, dx = x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\log\left(-1+\sqrt{6}-x\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

input
```
Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

output
```
x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
 Sqrt[6] + x])/6
```

3.174.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-x+2}{x^2+2x-5} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{7-3x}{x^2+2x-5}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

input
```
Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

output
```
x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sq
rt[6] + x])/6
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.174.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

method	result	size
default	$x-\frac{3\ln\left(x^{2}+2x-5\right)}{2}-\frac{5\sqrt{6}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3}$	30
risch	$x-\frac{3\ln\left(1+x-\sqrt{6}\right)}{2}+\frac{5\ln\left(1+x-\sqrt{6}\right)\sqrt{6}}{6}-\frac{3\ln\left(1+x+\sqrt{6}\right)}{2}-\frac{5\ln\left(1+x+\sqrt{6}\right)\sqrt{6}}{6}$	49

input
```
int((x^2-x+2)/(x^2+2*x-5),x,method=_RETURNVERBOSE)
```

output
```
x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right) + x - \frac{3}{2}\log\left(x^2+2x-5\right)$$

input
```
integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

output
```
5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2
+ 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x+\left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right)+\left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

input
```
integrate((x**2-x+2)/(x**2+2*x-5),x)
```

output
```
x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
 - sqrt(6) + 1)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\,\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)+x-\frac{3}{2}\,\log\left(x^2+2\,x-5\right)$$

input
```
integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

output
```
5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2
*x - 5)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \frac{5}{6}\,\sqrt{6}\log\left(\frac{\left|2\,x-2\,\sqrt{6}+2\right|}{\left|2\,x+2\,\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\,\log\left(\left|x^2+2\,x-5\right|\right)$$

input
```
integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

output
```
5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3
/2*log(abs(x^2 + 2*x - 5))
```

3.174.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = x-\ln\left(x+\sqrt{6}+1\right)\,\left(\frac{5\,\sqrt{6}}{6}+\frac{3}{2}\right)+\ln\left(x-\sqrt{6}+1\right)\,\left(\frac{5\,\sqrt{6}}{6}-\frac{3}{2}\right)$$

input
```
int((x^2 - x + 2)/(2*x + x^2 - 5),x)
```

output
```
x - log(x + 6^(1/2) + 1)*((5*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)*((5*
6^(1/2))/6 - 3/2)
```

3.175 $\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{2+3x}{2(4+7x+2x^2)}$$

output
```
1/2*(-2-3*x)/(2*x^2+7*x+4)
```

3.175.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = \frac{-2-3x}{2(4+7x+2x^2)}$$

input
```
Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

output
```
(-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

3.175.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2191, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+4x+1}{(2x^2+7x+4)^2} \, dx$$
$$\Big\downarrow \text{2191}$$
$$-\frac{\int 0dx}{17}-\frac{3x+2}{2(2x^2+7x+4)}$$
$$\Big\downarrow \text{24}$$
$$-\frac{3x+2}{2(2x^2+7x+4)}$$

input
```
Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

output
```
-1/2*(2 + 3*x)/(4 + 7*x + 2*x^2)
```

3.175.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = 
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + 
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.175.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
risch	$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$	17
norman	$\frac{-\frac{3x}{2}-1}{2x^2+7x+4}$	19
gosper	$-\frac{2+3x}{2(2x^2+7x+4)}$	20
parallelrisch	$\frac{-2-3x}{4x^2+14x+8}$	20

input
```
int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x,method=_RETURNVERBOSE)
```

output
```
(-3/4*x-1/2)/(x^2+7/2*x+2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

input
```
integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = \frac{-3x-2}{4x^2+14x+8}$$

input
```
integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
```

output
```
(-3*x - 2)/(4*x**2 + 14*x + 8)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

input
```
integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{3\,x+2}{2\,(2\,x^2+7\,x+4)}$$

input
```
integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
```

output
```
-1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx = -\frac{\frac{3\,x}{4}+\frac{1}{2}}{x^2+\frac{7\,x}{2}+2}$$

input
```
int((4*x + 3*x^2 + 1)/(7*x + 2*x^2 + 4)^2,x)
```

output
```
-((3*x)/4 + 1/2)/((7*x)/2 + x^2 + 2)
```

3.176 $\int \frac{1+x+x^2}{(3+2x+x^2)^2} \, dx$

3.176.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2} \, dx = \frac{1-x}{4(3+2x+x^2)} + \frac{3\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
1/4*(1-x)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2} \, dx = \frac{1-x}{4(3+2x+x^2)} + \frac{3\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

output
```
(1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

3.176.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{(x^2+2x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{8}\int \frac{6}{x^2+2x+3}dx+\frac{1-x}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}\int \frac{1}{x^2+2x+3}dx+\frac{1-x}{4(x^2+2x+3)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1-x}{4(x^2+2x+3)}-\frac{3}{2}\int \frac{1}{-(2x+2)^2-8}d(2x+2)$$

$$\Big\downarrow \text{217}$$

$$\frac{3\arctan\left(\frac{2x+2}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1-x}{4(x^2+2x+3)}$$

input
```
Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

output
```
(1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(2 + 2*x)/(2*Sqrt[2])])/(4*Sqrt[2]
)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.176.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{8}$	32
default	$\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}+\frac{3\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8}$	34

input
```
int((x^2+x+1)/(x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*x+1/4)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{3\sqrt{2}(x^2+2x+3)\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-2x+2}{8(x^2+2x+3)}$$

input
```
integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2
 + 2*x + 3)
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{1-x}{4x^2+8x+12} + \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

input
```
integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
```

output
```
(1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \frac{x-1}{4(x^2+2x+3)}$$

input
```
integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \frac{x-1}{4\,(x^2+2\,x+3)}$$

input
```
integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x+x^2}{(3+2x+x^2)^2}\,dx = \frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8} - \frac{\frac{x}{4}-\frac{1}{4}}{x^2+2\,x+3}$$

input
```
int((x + x^2 + 1)/(2*x + x^2 + 3)^2,x)
```

output
```
(3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3
)
```

3.177 $\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx$

3.177.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(1+x+x^2)^3}$$

output
```
-x/(x^2+x+1)^3
```

3.177.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(1+x+x^2)^3}$$

input
```
Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

output
```
-(x/(1 + x + x^2)^3)
```

3.177.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+2x-1}{(x^2+x+1)^4} dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{x}{(x^2+x+1)^3}$$

input
```
Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

output
```
-(x/(1 + x + x^2)^3)
```

3.177.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.177.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{(x^2+x+1)^3}$	12
default	$-\frac{x}{(x^2+x+1)^3}$	12
norman	$-\frac{x}{(x^2+x+1)^3}$	12
risch	$-\frac{x}{(x^2+x+1)^3}$	12
parallelrisch	$-\frac{x}{(x^2+x+1)^3}$	12

input
```
int((5*x^2+2*x-1)/(x^2+x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-x/(x^2+x+1)^3
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1}$$

input
```
integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")
```

output
```
-x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

3.177.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

input
```
integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
```

output
```
-x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

3.177.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(11) = 22.

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{x^6+3\,x^5+6\,x^4+7\,x^3+6\,x^2+3\,x+1}$$

input
```
integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
```

output
```
-x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(x^2+x+1)^3}$$

input
```
integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
```

output
```
-x/(x^2 + x + 1)^3
```

3.177.9 Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+2x+5x^2}{(1+x+x^2)^4}\,dx = -\frac{x}{(x^2+x+1)^3}$$

input
```
int((2*x + 5*x^2 - 1)/(x + x^2 + 1)^4,x)
```

output
```
-x/(x + x^2 + 1)^3
```

3.178 $\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) \, dx$

3.178.1 Optimal result

Integrand size = 22, antiderivative size = 267

$$\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) \, dx = \frac{5(b^2-4ac)^2 \left(32Ac^2+9b^2C-4acC\right) (b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5(b^2-4ac) \left(32Ac^2+9b^2C-4acC\right) (b+2cx) \left(a+bx+cx^2\right)^{3/2}}{6144c^4} + \frac{\left(32Ac^2+9b^2C-4acC\right) (b+2cx) \left(a+bx+cx^2\right)^{5/2}}{384c^3} - \frac{9bC(a+bx+cx^2)^{7/2}}{112c^2} + \frac{Cx(a+bx+cx^2)^{7/2}}{8c} - \frac{5(b^2-4ac)^3 \left(32Ac^2+9b^2C-4acC\right) \operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}$$

output
```
-5/6144*(-4*a*c+b^2)*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3
/2)/c^4+1/384*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^3
-9/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-5/32768*(
-4*a*c+b^2)^3*(32*A*c^2-4*C*a*c+9*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(11/2)+5/16384*(-4*a*c+b^2)^2*(32*A*c^2-4*C*a*c+9*C*b^
2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

3.178.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.35

$$\int (a+bx+cx^2)^{5/2} (A+Cx^2) \, dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(224Ac^2(b+2cx)\,(15b^4-40b^3cx+32bc^2x(13a+8cx^2)+8b^2c(-20a+11cx$$

input
```
Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(224*A*c^2*(b + 2*c*x)*(15*b^4 - 40*b^3*c*x
 + 32*b*c^2*x*(13*a + 8*c*x^2) + 8*b^2*c*(-20*a + 11*c*x^2) + 16*c^2*(33*a
^2 + 26*a*c*x^2 + 8*c^2*x^4)) + C*(945*b^7 - 630*b^6*c*x + 8*b^4*c^2*x*(79
1*a - 54*c*x^2) + 84*b^5*c*(-125*a + 6*c*x^2) + 16*b^3*c^2*(2359*a^2 - 284
*a*c*x^2 + 24*c^2*x^4) + 96*b^2*c^3*x*(-199*a^2 + 36*a*c*x^2 + 648*c^2*x^4
) + 896*c^4*x*(15*a^3 + 118*a^2*c*x^2 + 136*a*c^2*x^4 + 48*c^3*x^6) + 64*b
*c^3*(-663*a^3 + 174*a^2*c*x^2 + 2456*a*c^2*x^4 + 1584*c^3*x^6))) - 105*(b
^2 - 4*a*c)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a]
 + Sqrt[a + x*(b + c*x)])])/(344064*c^(11/2))
```

3.178.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2192, 27, 1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Cx^2)\,(a+bx+cx^2)^{5/2}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{1}{2}(16Ac-2aC-9bCx)\,(cx^2+bx+a)^{5/2}\,dx}{8c}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (2(8Ac-aC)-9bCx)\,(cx^2+bx+a)^{5/2}\,dx}{16c}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\int (cx^2+bx+a)^{5/2}dx}{2c}-\frac{9bC(a+bx+cx^2)^{7/2}}{7c}}{16c}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\left(\frac{(b+2cx)(a+bx+cx^2)^{5/2}}{12c}-\frac{5(b^2-4ac)\int (cx^2+bx+a)^{3/2}dx}{24c}\right)}{2c}-\frac{9bC(a+bx+cx^2)^{7/2}}{7c}}{16c}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\left(\frac{(b+2cx)(a+bx+cx^2)^{5/2}}{12c}-\frac{5(b^2-4ac)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c}\right)}{2c}-\frac{9bC(a+bx+cx^2)^{7/2}}{7c}}{16c}+\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\left(\frac{(b+2cx)(a+bx+cx^2)^{5/2}}{12c}-\frac{5(b^2-4ac)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right.}{16c}\right.}{24c}\right.}{2c}}{16c}$$

$$\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$$\Big\downarrow \text{1092}$$

3.178. $\int (a+bx+cx^2)^{5/2}(A+Cx^2)\,dx$

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\left(\frac{(b+2cx)(a+bx+cx^2)^{5/2}}{12c}-\frac{5(b^2-4ac)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}}{4c}\right.}{16c}\right.}{24c}\right.}{2c}}{16c}$$

$$\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

$\downarrow$ 219

$$\frac{\frac{(-4acC+32Ac^2+9b^2C)\left(\frac{(b+2cx)(a+bx+cx^2)^{5/2}}{12c}-\frac{5(b^2-4ac)\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{}{2\sqrt{c}}\right.}{8c^{3/2}}\right.}{16c}\right.}{24c}\right.}{2c}}{16c}$$

$$\frac{Cx(a+bx+cx^2)^{7/2}}{8c}$$

input
```
Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

output
```
(C*x*(a + b*x + c*x^2)^(7/2))/(8*c) + ((-9*b*C*(a + b*x + c*x^2)^(7/2))/(7
*c) + ((32*A*c^2 + 9*b^2*C - 4*a*c*C)*(((b + 2*c*x)*(a + b*x + c*x^2)^(5/2
))/(12*c) - (5*(b^2 - 4*a*c)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c)
- (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))
/(16*c)))/(24*c)))/(2*c))/(16*c)
```

3.178. $\int (a+bx+cx^2)^{5/2}(A+Cx^2)\,dx$

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.178. $\int (a+bx+cx^2)^{5/2} (A+Cx^2)\, dx$

3.178.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(237) = 474$.

Time = 0.88 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.79

method	result
risch	$\frac{(43008C\,c^7x^7+101376Cb\,c^6x^6+57344A\,c^7x^5+121856Ca\,c^6x^5+62208C\,b^2c^5x^5+143360Ab\,c^6x^4+157184Cab\,c^5x^4+384C\,b^3c^4x^4+18\ldots}{}$
default	$A\left(\frac{(2cx+b)(c\,x^2+bx+a)^{\frac{5}{2}}}{12c}+\frac{5(4ac-b^2)\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b\ldots}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.$

3.178. $\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx$

input
```
int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/344064/c^5*(43008*C*c^7*x^7+101376*C*b*c^6*x^6+57344*A*c^7*x^5+121856*C*
a*c^6*x^5+62208*C*b^2*c^5*x^5+143360*A*b*c^6*x^4+157184*C*a*b*c^5*x^4+384*
C*b^3*c^4*x^4+186368*A*a*c^6*x^3+96768*A*b^2*c^5*x^3+105728*C*a^2*c^5*x^3+
3456*C*a*b^2*c^4*x^3-432*C*b^4*c^3*x^3+279552*A*a*b*c^5*x^2+1792*A*b^3*c^4
*x^2+11136*C*a^2*b*c^4*x^2-4544*C*a*b^3*c^3*x^2+504*C*b^5*c^2*x^2+236544*A
*a^2*c^5*x+21504*A*a*b^2*c^4*x-2240*A*b^4*c^3*x+13440*C*a^3*c^4*x-19104*C*
a^2*b^2*c^3*x+6328*C*a*b^4*c^2*x-630*C*b^6*c*x+118272*A*a^2*b*c^4-35840*A*
a*b^3*c^3+3360*A*b^5*c^2-42432*C*a^3*b*c^3+37744*C*a^2*b^3*c^2-10500*C*a*b
^5*c+945*C*b^7)*(c*x^2+b*x+a)^(1/2)+5/32768*(2048*A*a^3*c^5-1536*A*a^2*b^2
*c^4+384*A*a*b^4*c^3-32*A*b^6*c^2-256*C*a^4*c^4+768*C*a^3*b^2*c^3-480*C*a^
2*b^4*c^2+112*C*a*b^6*c-9*C*b^8)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(237) = 474$.

Time = 0.36 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.57

$$\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) \, dx = \left[\frac{105\,(9\,Cb^8-112\,Cab^6c-2048\,Aa^3c^5+256\,(Ca^4+6\,Aa^2b^2)c^4-384\,(2\,Ca^3b^2+Aab^4)c^3+32}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

output
```
[1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6
*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b
^6)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 9
45*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 +
 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(
3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30
*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59
*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*
a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*
a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*
a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C
*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 38
4*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) + 2*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5
*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*
x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c
^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4
*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)...
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. $2(270) = 540$.

Time = 0.58 (sec) , antiderivative size = 2691, normalized size of antiderivative = 10.08

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(33*C*b*c*x**6/112 + C*c**2*x**7/8 + x**
5*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) + x**4*(3*A*b*c**2 + 2
37*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/
(12*c))/(5*c) + x**3*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 -
5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 +
237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)
/(12*c))/(10*c))/(4*c) + x**2*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*
b*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b*
*2*c/224)/(12*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*
a*b**2 - 5*a*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*
b*c**2 + 237*C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b*
*2*c/224)/(12*c))/(10*c))/(8*c))/(3*c) + x*(3*A*a**2*c + 3*A*a*b**2 + C*a*
*3 - 3*a*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*(A*c**3
+ 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*C*a*b*c/
56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12*c))/(10
*c))/(4*c) - 5*b*(6*A*a*b*c + A*b**3 + 3*C*a**2*b - 4*a*(3*A*b*c**2 + 237*
C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(12
*c))/(5*c) - 7*b*(3*A*a*c**2 + 3*A*b**2*c + 3*C*a**2*c + 3*C*a*b**2 - 5*a*
(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/(6*c) - 9*b*(3*A*b*c**2 + 237*
C*a*b*c/56 + C*b**3 - 11*b*(A*c**3 + 17*C*a*c**2/8 + 243*C*b**2*c/224)/...
```

3.178.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(237) = 474$.

Time = 0.30 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.80

$$\int (a+bx+cx^2)^{5/2}(A+Cx^2)\,dx = \frac{1}{344064}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Cc^2x+33\,Cbc\right)x+\frac{243\,Cb^2c^7+476\,Cac^8+22\cdots}{c^7}\right.\right.\right.\right.\right.\right.$$
$$+\frac{5\left(9\,Cb^8-112\,Cab^6c+480\,Ca^2b^4c^2+32\,Ab^6c^2-768\,Ca^3b^2c^3-384\,Aab^4c^3+256\,Ca^4c^4+1536\,Aa^2b^2c^4\cdots\right.}{32768\,c^{\frac{11}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

output
```
1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*
x + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228
*C*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608
*C*a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*
C*a*b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x -
 (315*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 -
 6720*C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^
7*c^2 - 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C
*a^3*b*c^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b
^8 - 112*C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3
- 384*A*a*b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^{5/2}(A+Cx^2)\,dx = \int (C\,x^2+A)\,\left(c\,x^2+b\,x+a\right)^{5/2}\,dx$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)
```

3.179 $\int (a+bx+cx^2)^{3/2} (A+Cx^2)\,dx$

3.179.1 Optimal result

Integrand size = 22, antiderivative size = 212

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (A+Cx^2)\,dx = &-\frac{(b^2-4ac)(24Ac^2+7b^2C-4acC)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}\\&+\frac{(24Ac^2+7b^2C-4acC)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}-\frac{7bC(a+bx+cx^2)^{5/2}}{60c^2}\\&+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}+\frac{(b^2-4ac)^2(24Ac^2+7b^2C-4acC)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

output
```
1/192*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-7/60*b*
C*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2
)^2*(24*A*c^2-4*C*a*c+7*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c
*x^2+b*x+a)^(1/2)/c^4
```

3.179.2 Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.08

$$\int (a+bx+cx^2)^{3/2} (A+Cx^2) \, dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(120Ac^2(b+2cx)(-3b^2+8bcx+4c(5a+2cx^2))+C(-105b^5+70b^4cx+8\cdots}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(120*A*c^2*(b + 2*c*x)*(-3*b^2 + 8*b*c*x +
4*c*(5*a + 2*c*x^2)) + C*(-105*b^5 + 70*b^4*c*x + 8*b^3*c*(95*a - 7*c*x^2)
 + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4
) + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*c^2*x^4))) + 15*(b^2 - 4*a*c)^2*(
24*A*c^2 + 7*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(
b + c*x)])])/(7680*c^(9/2))
```

3.179.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Cx^2) (a+bx+cx^2)^{3/2} \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{1}{2}(12Ac-2aC-7bCx)(cx^2+bx+a)^{3/2}dx}{6c}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (2(6Ac-aC)-7bCx)(cx^2+bx+a)^{3/2}dx}{12c}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{(-4acC+24Ac^2+7b^2C)\int (cx^2+bx+a)^{3/2}dx}{2c}-\frac{7bC(a+bx+cx^2)^{5/2}}{5c}}{12c}+\frac{Cx(a+bx+cx^2)^{5/2}}{6c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(-4acC+24Ac^2+7b^2C\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}-\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{5c}}{12c}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\left(-4acC+24Ac^2+7b^2C\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}-\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{5c}}{12c}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\left(-4acC+24Ac^2+7b^2C\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}-\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{5c}}{12c}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(-4acC+24Ac^2+7b^2C\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}-\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{5c}}{12c}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

3.179. $\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx$

output
```
(C*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((-7*b*C*(a + b*x + c*x^2)^(5/2))/(5
*c) + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2
))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - (
(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c
^(3/2))))/(16*c)))/(2*c))/(12*c)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.179.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.34

method	result
risch	$\frac{(1280c^5C\,x^5+1664bC\,c^4x^4+1920A\,c^5x^3+2240Ca\,c^4x^3+48C\,b^2c^3x^3+2880Ab\,c^4x^2+288Cab\,c^3x^2-56C\,b^3c^2x^2+4800Aa\,c^4x+240A\,b}{7680c^4}$
default	$A\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+C\left(\frac{x\left(c\,x^2+bx+a\right)}{6c}\right.$

input
```
int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7680*(1280*C*c^5*x^5+1664*C*b*c^4*x^4+1920*A*c^5*x^3+2240*C*a*c^4*x^3+48
*C*b^2*c^3*x^3+2880*A*b*c^4*x^2+288*C*a*b*c^3*x^2-56*C*b^3*c^2*x^2+4800*A*
a*c^4*x+240*A*b^2*c^3*x+480*C*a^2*c^3*x-432*C*a*b^2*c^2*x+70*C*b^4*c*x+240
0*A*a*b*c^3-360*A*b^3*c^2-1296*C*a^2*b*c^2+760*C*a*b^3*c-105*C*b^5)/c^4*(c
*x^2+b*x+a)^(1/2)+1/1024*(384*A*a^2*c^4-192*A*a*b^2*c^3+24*A*b^4*c^2-64*C*
a^3*c^3+144*C*a^2*b^2*c^2-60*C*a*b^4*c+7*C*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.85

$$\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right)\,dx = \left[\frac{15\,(7\,Cb^6-60\,Cab^4c+384\,Aa^2c^4-64\,(Ca^3+3\,Aab^2)c^3+24\,(6\,Ca^2b^2+Ab^4)c^2)\sqrt{c}\log\left(-8\,\ldots\right.}{} \right.$$

$$-\frac{15\,(7\,Cb^6-60\,Cab^4c+384\,Aa^2c^4-64\,(Ca^3+3\,Aab^2)c^3+24\,(6\,Ca^2b^2+Ab^4)c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(\ldots}{2\,(c^2x^2+\ldots}\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

output
```
[1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b
^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
 b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^
6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4
- 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^
6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^
2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*
x^2 + b*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4
- 64*(C*a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arct
an(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c))
 - 2*(1280*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 +
2400*A*a*b*c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a
*c^5 + 120*A*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 +
 2*(35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*
c^4)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(207) = 414$.

Time = 0.51 (sec) , antiderivative size = 775, normalized size of antiderivative = 3.66

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx = \begin{cases} \sqrt{a+bx+cx^2}\cdot\left(\frac{13Cbx^4}{60}+\frac{Ccx^5}{6}+\frac{x^3\left(Ac^2+\frac{7Cac}{6}+\frac{Cb^2}{40}\right)}{4c}+\frac{x^2\cdot\left(2Abc+\frac{17Cab}{15}-\frac{7b\left(Ac^2+\frac{7Cac}{6}+\frac{Cb^2}{40}\right)}{8c}\right)}{3c}+\frac{x\left(\right.}{} \right. \\ \frac{2\left(-\frac{2Ca(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{C(a+bx)^{\frac{9}{2}}}{9b^2}+\frac{(a+bx)^{\frac{5}{2}}\left(Ab^2+Ca^2\right)}{5b^2}\right)}{b} \\ a^{\frac{3}{2}}\left(Ax+\frac{Cx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(13*C*b*x**4/60 + C*c*x**5/6 + x**3*(A*c
**2 + 7*C*a*c/6 + C*b**2/40)/(4*c) + x**2*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*
c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*c) + x*(2*A*a*c + A*b**2 + C*a**2
- 3*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c) - 5*b*(2*A*b*c + 17*C*a*b/15
- 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(6*c))/(2*c) + (2*A*a*b - 2*
a*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*
c) - 3*b*(2*A*a*c + A*b**2 + C*a**2 - 3*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)
/(4*c) - 5*b*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)
/(8*c))/(6*c))/(4*c))/c) + (A*a**2 - a*(2*A*a*c + A*b**2 + C*a**2 - 3*a*(A
*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c) - 5*b*(2*A*b*c + 17*C*a*b/15 - 7*b*(A
*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(6*c))/(2*c) - b*(2*A*a*b - 2*a*(2*A
*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c))/(3*c) - 3
*b*(2*A*a*c + A*b**2 + C*a**2 - 3*a*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(4*c)
 - 5*b*(2*A*b*c + 17*C*a*b/15 - 7*b*(A*c**2 + 7*C*a*c/6 + C*b**2/40)/(8*c)
)/(6*c))/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
 + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)
/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(-2*C*a*(a + b*x)**(7/2)/
(7*b**2) + C*(a + b*x)**(9/2)/(9*b**2) + (a + b*x)**(5/2)*(A*b**2 + C*a**2
)/(5*b**2))/b, Ne(b, 0)), (a**(3/2)*(A*x + C*x**3/3), True))
```

3.179.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.39

$$\int \left(a+bx+cx^2\right)^{3/2} (A+Cx^2)\ dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,Ccx+13\,Cb)x+\frac{3\,Cb^2c^4+140\,Cac^5+120\,Ac^6}{c^5}\right)x-\frac{7\,C\cdots}{} \right.\right.\right.$$
$$-\frac{(7\,Cb^6-60\,Cab^4c+144\,Ca^2b^2c^2+24\,Ab^4c^2-64\,Ca^3c^3-192\,Aab^2c^3+384\,Aa^2c^4)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2\cdots}\right.\right.\right.}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*
c^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*
A*b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*
b^2*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a
^2*b*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a
*b^4*c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3
 + 384*A*a^2*c^4)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) +
b))/c^(9/2)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right)\ dx = \int \left(C\,x^2+A\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,dx$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)
```

3.180 $\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$

3.180.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx = \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}-\frac{\left(b^2-4ac\right)\left(16Ac^2+5b^2C-4acC\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}$$

output
```
-5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-1/128*(-4*
a*c+b^2)*(16*A*c^2-4*C*a*c+5*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))/c^(7/2)+1/64*(16*A*c^2-4*C*a*c+5*C*b^2)*(2*c*x+b)*(c*x^2+b*x+
a)^(1/2)/c^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}\left(48Ac^2(b+2cx)+C\left(15b^3-10b^2cx+24c^2x\left(a+2cx^2\right)+b\left(-52ac+8c^2x^2\right)\right)\right)-3(b^2 -}{192c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2*
c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)
*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x
*(b + c*x)])])/(192*c^(7/2))
```

3.180.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{1}{2}(8Ac-2aC-5bCx)\sqrt{cx^2+bx+a}dx}{4c}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (2(4Ac-aC)-5bCx)\sqrt{cx^2+bx+a}dx}{8c}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\left(-4acC+16Ac^2+5b^2C\right)\int \sqrt{cx^2+bx+a}dx}{2c}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{3c}}{8c}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(-4acC+16Ac^2+5b^2C\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}-\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{3c}}{8c}+\frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{(-4acC+16Ac^2+5b^2C)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}-\frac{5bC(a+bx+cx^2)^{3/2}}{3c}}{8c}+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(-4acC+16Ac^2+5b^2C)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{2c}-\frac{5bC(a+bx+cx^2)^{3/2}}{3c}}{8c}+\frac{Cx(a+bx+cx^2)^{3/2}}{4c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

output
```
(C*x*(a + b*x + c*x^2)^(3/2))/(4*c) + ((-5*b*C*(a + b*x + c*x^2)^(3/2))/(3
*c) + ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])
/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(8*c^(3/2))))/(2*c))/(8*c)
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.180.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
risch	$\frac{(48c^3 C x^3+8b c^2 C x^2+96A c^3 x+24a c^2 C x-10C b^2 c x+48A b c^2-52C a b c+15C b^3)\sqrt{c x^2+b x+a}}{192c^3}+\frac{(64A a c^3-16A b^2 c^2-16C a^2 c^2+ \cdots}{\cdots}$
default	$A\left(\frac{(2cx+b)\sqrt{c x^2+b x+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+b x+a}\right)}{8c^{\frac{3}{2}}}\right)+C\left(\frac{x(c x^2+b x+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(c x^2+b x+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b \cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right.$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*C*c^3*x^3+8*C*b*c^2*x^2+96*A*c^3*x+24*C*a*c^2*x-10*C*b^2*c*x+48*
A*b*c^2-52*C*a*b*c+15*C*b^3)/c^3*(c*x^2+b*x+a)^(1/2)+1/128*(64*A*a*c^3-16*
A*b^2*c^2-16*C*a^2*c^2+24*C*a*b^2*c-5*C*b^4)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.26

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$
$$=\left[-\frac{3\left(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\,(Ca^2+Ab^2)c^2\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*
a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*
a^2 + A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b
^3*c - 52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)
*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(151) = 302$.

Time = 0.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.96

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$
$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{Cbx^2}{24c}+\frac{Cx^3}{4}+\frac{x\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{2c}+\frac{Ab-\frac{Cab}{12c}-\frac{3b\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{4c}}{c}\right)+\left(Aa-\frac{a\left(Ac+\frac{Ca}{4}-\frac{5Cb^2}{48c}\right)}{2c}-\frac{b\left(\cdots\right.}{\cdots}\right. \\ \frac{2\left(-\frac{2Ca(a+bx)^{\frac{5}{2}}}{5b^2}+\frac{C(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(Ab^2+Ca^2\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(Ax+\frac{Cx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(C*b*x**2/(24*c) + C*x**3/4 + x*(A*c + C
*a/4 - 5*C*b**2/(48*c))/(2*c) + (A*b - C*a*b/(12*c) - 3*b*(A*c + C*a/4 - 5
*C*b**2/(48*c))/(4*c))/c) + (A*a - a*(A*c + C*a/4 - 5*C*b**2/(48*c))/(2*c)
 - b*(A*b - C*a*b/(12*c) - 3*b*(A*c + C*a/4 - 5*C*b**2/(48*c))/(4*c))/(2*c
))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), N
e(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x
)**2), True)), Ne(c, 0)), (2*(-2*C*a*(a + b*x)**(5/2)/(5*b**2) + C*(a + b*
x)**(7/2)/(7*b**2) + (a + b*x)**(3/2)*(A*b**2 + C*a**2)/(3*b**2))/b, Ne(b,
 0)), (sqrt(a)*(A*x + C*x**3/3), True))
```

3.180.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc^2}{c^3}\right)$$
$$+\frac{\left(5\,Cb^4-24\,Cab^2c+16\,Ca^2c^2+16\,Ab^2c^2-64\,Aac^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*
c^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128
*(5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.53

$$\int \sqrt{a+bx+cx^2}(A+Cx^2)\,dx$$
$$= A\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a} - \frac{Ca\left(\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c} + \frac{A\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}} - \frac{5Cb\left(\frac{\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)(b^3-4abc)}{16c^{5/2}}+\frac{(-3b^2+2cxb+8c(cx^2+a))\sqrt{cx^2+bx+a}}{24c^2}\right)}{8c} + \frac{Cx(cx^2+bx+a)^{3/2}}{4c}$$

input
```
int((A + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (C*a*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(4*c) + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*C*b*((log((b + 2*c*x)/c^(1/2)
 + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c
*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (C*x*
(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}} \, dx$

3.181.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}} \, dx = -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{(8Ac^2+3b^2C-4acC)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*A*c^2-4*C*a*c+3*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/c^(5/2)-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+1/2*C*x*(c*x^2+b*x+a)^(1/2)
/c
```

3.181.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}} \, dx = \frac{C(-3b+2cx)\sqrt{a+x(b+cx)}}{4c^2} + \frac{(8Ac^2+3b^2C-4acC)\,\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

input
```
Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*A*c^2 + 3*b^2*C - 4
*a*c*C)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4*c^(5/2
))
```

3.181.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{4Ac-2aC-3bCx}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2(2Ac-aC)-3bCx}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{(-4acC+8Ac^2+3b^2C)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}-\frac{3bC\sqrt{a+bx+cx^2}}{c}}{4c}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{(-4acC+8Ac^2+3b^2C)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{3bC\sqrt{a+bx+cx^2}}{c}}{4c}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(-4acC+8Ac^2+3b^2C)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{c}}{4c}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(C*x*Sqrt[a + b*x + c*x^2])/(2*c) + ((-3*b*C*Sqrt[a + b*x + c*x^2])/c + ((
8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*c^(3/2)))/(4*c)
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.181.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{C(-2cx+3b)\sqrt{c\,x^2+bx+a}}{4c^2}+\frac{(8A\,c^2-4Cac+3C\,b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{A\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+C\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx\right.}}{2c^{\frac{3}{2}}}\right.$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*C*(-2*c*x+3*b)/c^2*(c*x^2+b*x+a)^(1/2)+1/8*(8*A*c^2-4*C*a*c+3*C*b^2)/
c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+4\,(2\,Cc^2\right.}{16\,c^3}$$
$$-\frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,Cc^2x-3\,Cbc)\sqrt{cx^2+bx+a}}{8\,c^3}\Bigg]$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^
2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x -
3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*s
qrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b
*c*x + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

$$\int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}}\,dx = \begin{cases} \left(-\frac{3Cb}{4c^2} + \frac{Cx}{2c}\right)\sqrt{a + bx + cx^2} + \left(A - \frac{Ca}{2c} + \frac{3Cb^2}{8c^2}\right)\left(\begin{cases} \frac{\log\left(b + 2\sqrt{c}\sqrt{a+bx+cx^2} + 2cx\right)}{\sqrt{c}} & \text{for } a - \frac{b^2}{4c} \neq 0 \\ \frac{\left(\frac{b}{2c} + x\right)\log\left(\frac{b}{2c} + x\right)}{\sqrt{c\left(\frac{b}{2c} + x\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } c \neq \\ \frac{2A\sqrt{a+bx} + \frac{2C\left(a^2\sqrt{a+bx} - \frac{2a(a+bx)^{\frac{3}{2}}}{3} + \frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} & \text{for } b \neq \\ \frac{Ax + \frac{Cx^3}{3}}{\sqrt{a}} & \text{other} \end{cases}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-3*C*b/(4*c**2) + C*x/(2*c))*sqrt(a + b*x + c*x**2) + (A - C*a
/(2*c) + 3*C*b**2/(8*c**2))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*A*sqrt(a + b*x) + 2
*C*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2
)/b, Ne(b, 0)), ((A*x + C*x**3/3)/sqrt(a), True))
```

3.181.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Cx^2}{\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,Cx}{c}-\frac{3\,Cb}{c^2}\right) - \frac{(3\,Cb^2-4\,Cac+8\,Ac^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
 8*A*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5
/2)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+A}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((A + C*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.182 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+(2Ac^2+(b^2-2ac)\,C)\,x\right)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{C\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)-2*(b*c*(A+a*C
/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2Cx+aC(b-2cx)+Ac(b+2cx))}{c\,(-b^2+4ac)\,\sqrt{a+x(b+cx)}} + \frac{2C\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/(c*(-b^2 + 4*a*c)*Sqrt[a
 + x*(b + c*x)]) + (2*C*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*
x)])])/c^(3/2)
```

3.182.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2191$$

$$-\frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{C\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1092$$

$$\frac{2C\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{C\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*
Sqrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/c^(3/2)
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.182.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

method	result
default	$\frac{2A(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+C\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*A*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+C*(-x/c/(c*x^2+b*x+a)^(1/2)-
1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

3.182.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 403, normalized size of antiderivative = 4.11

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{3/2}}\, dx = \left[\frac{(Cab^2 - 4\,Ca^2c + (Cb^2c - 4\,Cac^2)x^2 + (Cb^3 - 4\,Cabc)x)\sqrt{c}\log\left(-8\,c^2x^2 - 8\,bc\right.}{2\,(ab^2c^2 - 4\,c} \right.$$

$$-\frac{(Cab^2 - 4\,Ca^2c + (Cb^2c - 4\,Cac^2)x^2 + (Cb^3 - 4\,Cabc)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right) + 2\,(Cabc +}{ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^2 + (b^3c^2 - 4\,abc^3)x}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*
c)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 +
2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a
*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c -
4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c
^2 + (C*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2
- 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

3.182.6 Sympy [F]

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{3/2}}\, dx = \int \frac{A + Cx^2}{(a + bx + cx^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.182.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(Cb^2-2\,Cac+2\,Ac^2)x}{b^2c-4\,ac^2}+\frac{Cab+Abc}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{C\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2
*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{C\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} + \frac{A\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\sqrt{c\,x^2+b\,x+a}} + \frac{C\left(\frac{a\,b}{2}-x\left(a\,c-\frac{b^2}{2}\right)\right)}{c\left(a\,c-\frac{b^2}{4}\right)\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(C*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) + (A*(b/2 +
 c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^(1/2)) + (C*((a*b)/2 - x*(a*c - b^
2/2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.182. $\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}}\,dx$

3.183 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx$

3.183.1 Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)^(3/2)+2/3*(8*A*c+4*C*a+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^
(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{-2A(b+2cx)\left(b^2-8bcx-4c(3a+2cx^2)\right)+2C\left(8a^2b+b^2x^2(3b+2cx)+4ax(3b\right.}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b
^2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a
*c)^2*(a + x*(b + c*x))^(3/2))
```

3.183.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2191, 27, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \ 2191$$

$$-\frac{2\int \frac{\frac{Cb^2}{c}+8Ac+4aC}{2(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(4aC+8Ac+\frac{b^2C}{c}\right)\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c
)*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(
3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.183.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

method	result
trager	$\frac{\frac{32}{3}A\,c^3x^3+\frac{16}{3}Ca\,c^2x^3+\frac{4}{3}C\,b^2c\,x^3+16Ab\,c^2x^2+8Cabc\,x^2+2C\,b^3x^2+16aA\,c^2x+4A\,b^2cx+8Ca\,b^2x+8Aabc-\frac{2}{3}A\,b^3+\frac{16}{3}b\,a^2C}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{32}{3}A\,c^3x^3+\frac{16}{3}Ca\,c^2x^3+\frac{4}{3}C\,b^2c\,x^3+16Ab\,c^2x^2+8Cabc\,x^2+2C\,b^3x^2+16aA\,c^2x+4A\,b^2cx+8Ca\,b^2x+8Aabc-\frac{2}{3}A\,b^3+\frac{16}{3}b\,a^2C}{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(16a^2c^2-8a\,b^2c+b^4\right)}$
default	$A\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)+C\left(-\frac{x}{2c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{4c\ldots}{3}\ldots\right)}{\left(4ac-b^2\right)\left(\ldots\right.}\ldots\right.}{\ldots}\right.$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(16*A*c^3*x^3+8*C*a*c^2*x^3+2*C*b^2*c*x^3+24*A*b*c^2*x^2+12*C*a*b*c*x^
2+3*C*b^3*x^2+24*A*a*c^2*x+6*A*b^2*c*x+12*C*a*b^2*x+12*A*a*b*c-A*b^3+8*C*a
^2*b)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(106) = 212$.

Time = 0.67 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.12

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{5/2}}\,dx = \frac{2\,(8\,Ca^2b - Ab^3 + 12\,Aabc + 2\,(Cb^2c + 4\,Cac^2 + 8\,Ac^3)x^3 + 3\,(Cb^3 + 4\,Ca}{3\,(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^
3 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a
*c^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*
a^3*b*c^2)*x)
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.183.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.69

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{2\,(Cb^2c+4\,Cac^2+8\,Ac^3)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\,(Cb^3+4\,Cabc+8\,Abc^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{6\,(2\,Cab^2+Ab^2c+4\,Aac^2)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{8\,Ca^2\cdots}{b^4-\cdots}\right.}{3\,(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2)
+ 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 6*
(2*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (8*C
*a^2*b - A*b^3 + 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x
+ a)^(3/2)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,C\,a^2\,b+12\,C\,a\,b^2\,x+12\,C\,a\,b\,c\,x^2+12\,A\,a\,b\,c+8\,C\,a\,c^2\,x^3+24\,A\,a\,c^2\,x+3\,C\cdots}{3\,(4\,a\,c-b^2)^2\,(c\,x^2+b\,x\cdots}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

output
```
(2*(16*A*c^3*x^3 - A*b^3 + 3*C*b^3*x^2 + 8*C*a^2*b + 24*A*a*c^2*x + 6*A*b^
2*c*x + 12*C*a*b^2*x + 24*A*b*c^2*x^2 + 8*C*a*c^2*x^3 + 2*C*b^2*c*x^3 + 12
*A*a*b*c + 12*C*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

3.184 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx$

3.184.1 Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}} + \frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}} - \frac{16\left(16Ac^2+3b^2C+4acC\right)(b+2cx)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}$$

output
```
-2/5*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)^(5/2)+2/15*(16*A*c+4*C*a+3*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x
+a)^(3/2)-16/15*(16*A*c^2+4*C*a*c+3*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2
+b*x+a)^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.40

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx =$$

$$-\frac{2(A(b+2cx)\left(3b^4-16b^3cx+64bc^2x(5a+4cx^2)+8b^2c(-5a+14cx^2)+16c^2(15a^2+20acx^2+8c^2x^4)\right)+}{}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(A*(b + 2*c*x)*(3*b^4 - 16*b^3*c*x + 64*b*c^2*x*(5*a + 4*c*x^2) + 8*b^
2*c*(-5*a + 14*c*x^2) + 16*c^2*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4)) + C*(96*
a^3*b*c + 3*b^2*x^2*(5*b^3 + 30*b^2*c*x + 40*b*c^2*x^2 + 16*c^3*x^3) + 8*a
^2*(b^3 + 30*b^2*c*x + 30*b*c^2*x^2 + 20*c^3*x^3) + 4*a*x*(5*b^4 + 50*b^3*
c*x + 60*b^2*c^2*x^2 + 40*b*c^3*x^3 + 16*c^4*x^4))))/(15*(b^2 - 4*a*c)^3*(
a + x*(b + c*x))^(5/2))
```

3.184.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2191, 27, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{2191}$$

$$-\frac{2\int \frac{\frac{3Cb^2}{c}+16Ac+4aC}{2(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(4aC+16Ac+\frac{3b^2C}{c}\right)\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{\left(4aC+16Ac+\frac{3b^2C}{c}\right)\left(-\frac{8c\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{5(b^2-4ac)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c(b^2-4ac)(a+bx+cx^2)^{5/2}}-\frac{\left(\frac{16c(b+2cx)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)\left(4aC+16Ac+\frac{3b^2C}{c}\right)}{5(b^2-4ac)}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c
)*(a + b*x + c*x^2)^(5/2)) - ((16*A*c + 4*a*C + (3*b^2*C)/c)*((-2*(b + 2*c
*x))/(3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(b^
2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])))/(5*(b^2 - 4*a*c))
```

3.184.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.184.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.76

method	result
trager	$\frac{\frac{512}{15}A\,c^5x^5+128Aab\,c^3x^2+32C\,a^2b\,c^2x^2+32A\,a^2b\,c^2-\frac{16}{3}Aa\,b^3c+\frac{2}{5}A\,b^5+\frac{64}{5}C\,a^3bc+12C\,b^4c\,x^3+\frac{32}{3}A\,b^3c^2x^2+64A\,a^2c^3x-\frac{4}{3}A\,b^4cx+}{}$
gosper	$\frac{\frac{512}{15}A\,c^5x^5+128Aab\,c^3x^2+32C\,a^2b\,c^2x^2+32A\,a^2b\,c^2-\frac{16}{3}Aa\,b^3c+\frac{2}{5}A\,b^5+\frac{64}{5}C\,a^3bc+12C\,b^4c\,x^3+\frac{32}{3}A\,b^3c^2x^2+64A\,a^2c^3x-\frac{4}{3}A\,b^4cx+}{}$
default	$A\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{c\,x^2+bx+a}}\right)}{5(4ac-b^2)}\right)+C\left(-\frac{x}{4c(c\,x^2+bx+a)^{\frac{5}{2}}}-\frac{3b \cdots}{\cdots}\right.$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(256*A*c^5*x^5+64*C*a*c^4*x^5+48*C*b^2*c^3*x^5+640*A*b*c^4*x^4+160*C*
a*b*c^3*x^4+120*C*b^3*c^2*x^4+640*A*a*c^4*x^3+480*A*b^2*c^3*x^3+160*C*a^2*
c^3*x^3+240*C*a*b^2*c^2*x^3+90*C*b^4*c*x^3+960*A*a*b*c^3*x^2+80*A*b^3*c^2*
x^2+240*C*a^2*b*c^2*x^2+200*C*a*b^3*c*x^2+15*C*b^5*x^2+480*A*a^2*c^3*x+240
*A*a*b^2*c^2*x-10*A*b^4*c*x+240*C*a^2*b^2*c*x+20*C*a*b^4*x+240*A*a^2*b*c^2
-40*A*a*b^3*c+3*A*b^5+96*C*a^3*b*c+8*C*a^2*b^3)/(4*a*c-b^2)^3/(c*x^2+b*x+a
)^(5/2)
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(155) = 310.

Time = 5.16 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.37

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = -\frac{2\,(8\,Ca^2b^3+3\,Ab^5+240\,Aa^2bc^2+16\,(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5)x^5+40\,(3\,Cb^3c^2+4\,Cabc^3+16\,A}{15\,(a^3b^6-12\,a^4b^4c+48\,a^5b^2c^2-64\,a^6c^3+(b^6c^3-12\,ab^4c^4+48\,a^2b^2c^5-64\,a^3c^6)x^6+3\,(b^7c^2-12\,ab^5c^3}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c
^4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*
(9*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 +
 5*(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x
^2 + 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*
a^2*c^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^
2*b^2*c^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2
*c^4 - 64*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c
^3 - 384*a^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^
4*b^2*c^3 - 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 -
 64*a^5*b*c^3)*x)
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.184.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.71

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{2\left(\left(\left(2\left(4\left(\frac{2\,(3\,Cb^2c^3+4\,Cac^4+16\,Ac^5)x}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}+\frac{5\,(3\,Cb^3c^2+4\,Cabc^3+16\,Abc^4)}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right)x+\frac{5\,(9\,Cb^4c+24\,Cab^2c^2+16\,Ca^2c^3+48\,Ab^2c^3+64\,Aac^{}}{b^6-12\,ab^4c+48\,a^2b^2c^2-64\,a^3c^3}\right.\right.\right.}{}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6 - 12*a*b^4*c
+ 48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4
)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(9*C*b^4*c + 24*
C*a*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6 - 12*a*b^4*c
+ 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(3*C*b^5 + 40*C*a*b^3*c + 48*C*a^2*b
*c^2 + 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 -
64*a^3*c^3))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c + 24*A*a*b^2*c^2
 + 48*A*a^2*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + (8*
C*a^2*b^3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 240*A*a^2*b*c^2)/(b^6
- 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))/(c*x^2 + b*x + a)^(5/2)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.46

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{7/2}}\,dx = \frac{\frac{b\,c\,(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}+\frac{2\,c^2\,x\,(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)^2}}{\sqrt{c\,x^2+b\,x+a}}$$
$$+\frac{\frac{8\,C\,b\,c}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{16\,C\,c^2\,x}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}}{\sqrt{c\,x^2+b\,x+a}}-\frac{\frac{4\,C\,x}{15\,(4\,a\,c-b^2)}-\frac{2\,C\,b}{15\,c\,(4\,a\,c-b^2)}}{(c\,x^2+b\,x+a)^{3/2}}$$
$$+\frac{x\left(\frac{4\,A\,c^2}{5\,(4\,a\,c^2-b^2\,c)}+\frac{2\,C\,b^2}{5\,(4\,a\,c^2-b^2\,c)}-\frac{4\,C\,a\,c}{5\,(4\,a\,c^2-b^2\,c)}\right)+\frac{2\,A\,b\,c}{5\,(4\,a\,c^2-b^2\,c)}+\frac{2\,C\,a\,b}{5\,(4\,a\,c^2-b^2\,c)}}{(c\,x^2+b\,x+a)^{5/2}}$$
$$+\frac{x\left(\frac{2\,c\,(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}+\frac{16\,C\,a\,c^2}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,C\,b^2\,c}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}\right)+\frac{b\,(8\,C\,b^2+32\,A\,c^2+8\,C\,a\,c)}{15\,(4\,a\,c^2-b^2\,c)\,(4\,a\,c-b^2)}-\frac{8\,C\,a}{15\,(4\,a\,c^2-b^2}}{(c\,x^2+b\,x+a)^{3/2}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)
```

output
```
((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^
2)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*
c)*(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a
+ b*x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c
- b^2)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (
2*C*b^2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b
*c)/(5*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*
x^2)^(5/2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c
)*(4*a*c - b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*
C*b^2*c)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 +
8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2
- b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

3.185 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

3.185.1 Optimal result

Integrand size = 22, antiderivative size = 220

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx=-\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+(b^2-2ac)\,C\right)x\right)}{7c\,(b^2-4ac)\,(a+bx+cx^2)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\,(b^2-4ac)^2\,(a+bx+cx^2)^{5/2}}-\frac{32(24Ac^2+5b^2C+4acC)\,(b+2cx)}{105\,(b^2-4ac)^3\,(a+bx+cx^2)^{3/2}}+\frac{256c(24Ac^2+5b^2C+4acC)\,(b+2cx)}{105\,(b^2-4ac)^4\,\sqrt{a+bx+cx^2}}$$

output
```
-2/7*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+
a)^(7/2)+2/35*(24*A*c+4*C*a+5*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x
+a)^(5/2)-32/105*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^
2+b*x+a)^(3/2)+256/105*c*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)
^4/(c*x^2+b*x+a)^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.86

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \frac{-6A(b+2cx)\left(5b^6-24b^5cx+64b^3c^2x(7a-12cx^2)+4b^4c(-21a+26cx^2)-128\right.}{}$$

input
```
Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

output
```
(-6*A*(b + 2*c*x)*(5*b^6 - 24*b^5*c*x + 64*b^3*c^2*x*(7*a - 12*c*x^2) + 4*
b^4*c*(-21*a + 26*c*x^2) - 128*b*c^3*x*(35*a^2 + 56*a*c*x^2 + 24*c^2*x^4)
- 16*b^2*c^2*(-35*a^2 + 196*a*c*x^2 + 184*c^2*x^4) - 64*c^3*(35*a^3 + 70*a
^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6)) + 2*C*(1920*a^4*b*c^2 + 320*a^3*c*(
b^3 + 21*b^2*c*x + 21*b*c^2*x^2 + 14*c^3*x^3) + 5*b^2*x^2*(-7*b^5 + 70*b^4
*c*x + 560*b^3*c^2*x^2 + 1120*b^2*c^3*x^3 + 896*b*c^4*x^4 + 256*c^5*x^5) +
 8*a^2*(-b^5 + 140*b^4*c*x + 1190*b^3*c^2*x^2 + 1540*b^2*c^3*x^3 + 1120*b*
c^4*x^4 + 448*c^5*x^5) + 4*a*x*(-7*b^6 + 343*b^5*c*x + 2170*b^4*c^2*x^2 +
3360*b^3*c^3*x^3 + 2240*b^2*c^4*x^4 + 896*b*c^5*x^5 + 256*c^6*x^6)))/(105*
(b^2 - 4*a*c)^4*(a + x*(b + c*x))^(7/2))
```

3.185.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2191, 27, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}dx$$

$$\Big\downarrow \ 2191$$

$$-\frac{2\int \frac{\frac{5Cb^2}{c}+24Ac+4aC}{2(cx^2+bx+a)^{7/2}}dx}{7(b^2-4ac)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(4aC+24Ac+\frac{5b^2C}{c}\right)\int \frac{1}{(cx^2+bx+a)^{7/2}}dx}{7(b^2-4ac)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{\left(4aC+24Ac+\frac{5b^2C}{c}\right)\left(-\frac{16c\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{7\left(b^2-4ac\right)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{\left(4aC+24Ac+\frac{5b^2C}{c}\right)\left(-\frac{16c\left(-\frac{8c\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}\right)}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{7\left(b^2-4ac\right)}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{\left(-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}\right)}{5(b^2-4ac)}\right)\left(4aC+24Ac+\frac{5b^2C}{c}\right)}{7\left(b^2-4ac\right)}$$

input
```
Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

output
```
(-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c
)*(a + b*x + c*x^2)^(7/2)) - ((24*A*c + 4*a*C + (5*b^2*C)/c)*((-2*(b + 2*c
*x))/(5*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(
3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(b^2 - 4*
a*c)^2*Sqrt[a + b*x + c*x^2])))/(5*(b^2 - 4*a*c))))/(7*(b^2 - 4*a*c))
```

3.185. $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.185.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(204) = 408$.

Time = 0.76 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.38

method	result
trager	$-\frac{2}{7}A\,b^7+\frac{128}{21}C\,a^3b^3c+\frac{256}{7}C\,a^4b\,c^2+\frac{4}{5}A\,b^6cx-\frac{8}{15}Ca\,b^6x+256A\,a^3c^4x+\frac{160}{3}C\,b^5c^2x^4+512A\,a^2c^5x^3+32A\,b^4c^3x^3+\frac{256}{3}C\,a^3c^4x^3+\frac{20}{3}$
default	$A\left(\frac{\frac{4cx}{7}+\frac{2b}{7}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{7}{2}}}+\frac{24c\left(\frac{\frac{4cx}{5}+\frac{2b}{5}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}+\frac{16c\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3\left(4ac-b^2\right)^2\sqrt{c\,x^2+bx+a}}\right)}{5\left(4ac-b^2\right)}\right)}{7(4ac-b^2)}\right)+C$
gosper	$-\frac{2}{7}A\,b^7+\frac{128}{21}C\,a^3b^3c+\frac{256}{7}C\,a^4b\,c^2+\frac{4}{5}A\,b^6cx-\frac{8}{15}Ca\,b^6x+256A\,a^3c^4x+\frac{160}{3}C\,b^5c^2x^4+512A\,a^2c^5x^3+32A\,b^4c^3x^3+\frac{256}{3}C\,a^3c^4x^3+\frac{20}{3}$

input
```
int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

3.185. $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

output
```
2/105*(6144*A*c^7*x^7+1024*C*a*c^6*x^7+1280*C*b^2*c^5*x^7+21504*A*b*c^6*x^
6+3584*C*a*b*c^5*x^6+4480*C*b^3*c^4*x^6+21504*A*a*c^6*x^5+26880*A*b^2*c^5*
x^5+3584*C*a^2*c^5*x^5+8960*C*a*b^2*c^4*x^5+5600*C*b^4*c^3*x^5+53760*A*a*b
*c^5*x^4+13440*A*b^3*c^4*x^4+8960*C*a^2*b*c^4*x^4+13440*C*a*b^3*c^3*x^4+28
00*C*b^5*c^2*x^4+26880*A*a^2*c^5*x^3+40320*A*a*b^2*c^4*x^3+1680*A*b^4*c^3*
x^3+4480*C*a^3*c^4*x^3+12320*C*a^2*b^2*c^3*x^3+8680*C*a*b^4*c^2*x^3+350*C*
b^6*c*x^3+40320*A*a^2*b*c^4*x^2+6720*A*a*b^3*c^3*x^2-168*A*b^5*c^2*x^2+672
0*C*a^3*b*c^3*x^2+9520*C*a^2*b^3*c^2*x^2+1372*C*a*b^5*c*x^2-35*C*b^7*x^2+1
3440*A*a^3*c^4*x+10080*A*a^2*b^2*c^3*x-840*A*a*b^4*c^2*x+42*A*b^6*c*x+6720
*C*a^3*b^2*c^2*x+1120*C*a^2*b^4*c*x-28*C*a*b^6*x+6720*A*a^3*b*c^3-1680*A*a
^2*b^3*c^2+252*A*a*b^5*c-15*A*b^7+1920*C*a^4*b*c^2+320*C*a^3*b^3*c-8*C*a^2
*b^5)/(4*a*c-b^2)^4/(c*x^2+b*x+a)^(7/2)
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(204) = 408$.

Time = 14.68 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.45

$$\int \frac{A + Cx^2}{(a + bx + cx^2)^{9/2}}\, dx =$$

$$-\frac{2\,(8\,Ca^2b^5 + 15\,Ab^7 - 6720\,Aa^3bc^3 - 256\,(5\,Cb^2c^5 + 4\,Cac^6 + 24\,Ac^7)x^7 - 896\,(5\,Cb^3c^4 + 4\,C}{105\,(a^4b^8 - 16\,a^5b^6c + 96\,a^6b^4c^2 - 256\,a^7b^2c^3 + 256\,a^8c^4 + (b^8c^4 - 16\,ab^6c^5 + 96\,a^2b^4c^6 - 256\,a^3b^2c^7 + 256}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
-2/105*(8*C*a^2*b^5 + 15*A*b^7 - 6720*A*a^3*b*c^3 - 256*(5*C*b^2*c^5 + 4*C
*a*c^6 + 24*A*c^7)*x^7 - 896*(5*C*b^3*c^4 + 4*C*a*b*c^5 + 24*A*b*c^6)*x^6
- 224*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 96*A*a*c^6 + 8*(2*C*a^2 + 15*A*b^2)
*c^5)*x^5 - 560*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 96*A*a*b*c^5 + 8*(2*C*a^2*
b + 3*A*b^3)*c^4)*x^4 - 70*(5*C*b^6*c + 124*C*a*b^4*c^2 + 384*A*a^2*c^5 +
64*(C*a^3 + 9*A*a*b^2)*c^4 + 8*(22*C*a^2*b^2 + 3*A*b^4)*c^3)*x^3 - 240*(8*
C*a^4*b - 7*A*a^2*b^3)*c^2 + 7*(5*C*b^7 - 196*C*a*b^5*c - 5760*A*a^2*b*c^4
 - 960*(C*a^3*b + A*a*b^3)*c^3 - 8*(170*C*a^2*b^3 - 3*A*b^5)*c^2)*x^2 - 4*
(80*C*a^3*b^3 + 63*A*a*b^5)*c + 14*(2*C*a*b^6 - 720*A*a^2*b^2*c^3 - 960*A*
a^3*c^4 - 60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*
sqrt(c*x^2 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b
^2*c^3 + 256*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*
b^2*c^7 + 256*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 -
256*a^3*b^3*c^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*
a^2*b^6*c^4 - 576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^
11*c - 13*a*b^9*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 +
768*a^5*b*c^6)*x^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3
 - 2240*a^4*b^4*c^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 1
3*a^2*b^9*c + 48*a^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*
b*c^5)*x^3 + 2*(3*a^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b...
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
```

output
```
Timed out
```

3.185.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(204) = 408$.

Time = 0.29 (sec) , antiderivative size = 805, normalized size of antiderivative = 3.66

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \frac{2\left(\left(\left(\left(2\left(8\left(2\left(4\left(\frac{2\left(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7\right)x}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4}+\frac{7\left(5\,Cb^3c^4+4\,Cabc^5+24\,Abc^6\right.}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+}\right.\right.\right.\right.\right.\right.\right.}{}$$

input
```
integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

output
```
2/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8 - 16*a*
b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*C*b^3*c^4 +
 4*C*a*b*c^5 + 24*A*b*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^
2*c^3 + 256*a^4*c^4))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*a^2*c^5
+ 120*A*b^2*c^5 + 96*A*a*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3
*b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 16*C*a^2*b
*c^4 + 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^6*c + 124*C*a*b^4*c^2 + 176*C
*a^2*b^2*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^4 + 384*A*a^2*c
^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x
 - 7*(5*C*b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24*A*b^5*c^2 - 960*C*
a^3*b*c^3 - 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2
*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 14*(2*C*a*b^6 - 80*C*a^2*b^
4*c - 3*A*b^6*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2 - 720*A*a^2*b^2*c^3 -
 960*A*a^3*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256
*a^4*c^4))*x - (8*C*a^2*b^5 + 15*A*b^7 - 320*C*a^3*b^3*c - 252*A*a*b^5*c -
 1920*C*a^4*b*c^2 + 1680*A*a^2*b^3*c^2 - 6720*A*a^3*b*c^3)/(b^8 - 16*a*b^6
*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7
/2)
```

3.185. $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

3.185.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 1018, normalized size of antiderivative = 4.63

$$\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx = \frac{x\left(\frac{2c^2(160Cb^2+768Ac^2+96Cac)}{105(4ac^2-b^2c)(4ac-b^2)^2}-\frac{64Cac^3}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{32Cb^2c^2}{105(4ac^2-b^2c)(4ac-b^2)^2}\right)+\frac{b}{\cdots}}{(cx^2+bx+a)^{3/2}}$$
$$-\frac{\frac{8Cb}{105(4ac-b^2)^2}-\frac{16Ccx}{105(4ac-b^2)^2}}{(cx^2+bx+a)^{3/2}}+\frac{\frac{8Cbc}{105(4ac^2-b^2c)(4ac-b^2)}+\frac{16Cc^2x}{105(4ac^2-b^2c)(4ac-b^2)}}{(cx^2+bx+a)^{3/2}}$$
$$-\frac{\frac{4Cx}{35(4ac-b^2)}-\frac{2Cb}{35c(4ac-b^2)}}{(cx^2+bx+a)^{5/2}}+\frac{\frac{bc(1312Cb^2c+6144Ac^3+896Cac^2)}{105(4ac^2-b^2c)(4ac-b^2)^3}+\frac{2c^2x(1312Cb^2c+6144Ac^3+896Cac^2)}{105(4ac^2-b^2c)(4ac-b^2)^3}}{\sqrt{cx^2+bx+a}}$$
$$+\frac{x\left(\frac{4Ac^2}{7(4ac^2-b^2c)}+\frac{2Cb^2}{7(4ac^2-b^2c)}-\frac{4Cac}{7(4ac^2-b^2c)}\right)+\frac{2Abc}{7(4ac^2-b^2c)}+\frac{2Cab}{7(4ac^2-b^2c)}}{(cx^2+bx+a)^{7/2}}$$
$$+\frac{x\left(\frac{2c(12Cb^2+48Ac^2+8Cac)}{35(4ac^2-b^2c)(4ac-b^2)}+\frac{16Cac^2}{35(4ac^2-b^2c)(4ac-b^2)}-\frac{8Cb^2c}{35(4ac^2-b^2c)(4ac-b^2)}\right)+\frac{b(12Cb^2+48Ac^2+8Cac)}{35(4ac^2-b^2c)(4ac-b^2)}-\frac{8C\cdots}{35(4ac^2-\cdots}}{(cx^2+bx+a)^{5/2}}$$
$$-\frac{\frac{32Cbc^2}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{64Cc^3x}{105(4ac^2-b^2c)(4ac-b^2)^2}}{\sqrt{cx^2+bx+a}}$$
$$+\frac{\frac{64Cbc^2}{105(4ac^2-b^2c)(4ac-b^2)^2}+\frac{128Cc^3x}{105(4ac^2-b^2c)(4ac-b^2)^2}}{\sqrt{cx^2+bx+a}}$$

input
```
int((A + C*x^2)/(a + b*x + c*x^2)^(9/2),x)
```

output
```
(x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a
*c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*
C*b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*A*c^2 + 16
0*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c
^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) - ((8
*C*b)/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x +
 c*x^2)^(3/2) + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c
^2*x)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) - ((4
*C*x)/(35*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)
^(5/2) + ((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 -
b^2*c)*(4*a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*
c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/2) + (x*
((4*A*c^2)/(7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*
a*c)/(7*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/
(7*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(7/2) + (x*((2*c*(48*A*c^2 + 12*C
*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(
4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c
 - b^2))) + (b*(48*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a
*c - b^2)) - (8*C*a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x +
c*x^2)^(5/2) - ((32*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + ...
```

3.185. $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

3.186 $\int (g+hx)^3\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.186.1 Optimal result

Integrand size = 32, antiderivative size = 930

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\left(256c^5dg^3-33b^5fh^3+6b^3ch^2(20afh+7b(3fg+eh))-8bc^2h(10a^2fh^2+14abh(3fg+eh)+7b^2(3fg^2\right.}{}$$
$$+\frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2(3fg^2-7h(eg+2dh))\right)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{280c^3h}$$
$$-\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}$$
$$+\frac{\left(1155b^4fh^4-128c^4g^2(3fg^2-7h(eg+12dh))-42b^2ch^3(78afh+35b(3fg+eh))+8c^2h^2(128a^2fh^2+\right.}{}$$
$$-\frac{(b^2-4ac)\left(256c^5dg^3-33b^5fh^3+6b^3ch^2(20afh+7b(3fg+eh))-8bc^2h(10a^2fh^2+14abh(3fg+eh)\right.}{}$$

output
```
1/280*(33*b^2*f*h^2-2*c*h*(16*a*f*h+21*b*e*h+8*b*f*g)-4*c^2*(3*f*g^2-7*h*(
2*d*h+e*g)))*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^3/h-1/84*(11*b*f*h-14*c*e*h+6
*c*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2/h+1/7*f*(h*x+g)^4*(c*x^2+b*x+a)^
(3/2)/c/h+1/13440*(1155*b^4*f*h^4-128*c^4*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-4
2*b^2*c*h^3*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+343*a*b*h
*(e*h+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(15*f*g^2
+7*h*(d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*c*h*(231*b^3*f*h^
3-6*b*c*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*g*(3*f*g^2-7*h*(7*d*h+e*g)
)+8*c^2*h*(a*h*(35*e*h+41*f*g)+b*(5*f*g^2+7*h*(7*d*h+9*e*g))))*x)*(c*x^2+b
*x+a)^(3/2)/c^5/h-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*
h^2*(20*a*f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g
)+7*b^2*(d*h^2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*
(d*h+e*g)))+16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*
a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/c^(13/2)+1/1024*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*f*h
+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^
2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+1
6*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^
2+h*(d*h+3*e*g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

3.186.2 Mathematica [A] (verified)

Time = 11.64 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.18

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^6fh^3+210b^5ch^2(63fg+21eh+11fhx)-84b^4ch(-260afh^2+35ch(6eg+\ldots}{\ldots}$$

input
```
Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g
+ 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h +
e*h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*
h^3 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h
*x + 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*
(20*g^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*
h*x + 189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*
(35*g + 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3
) + 7*h*(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 3
2*b*c^3*(a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 60
9*g^2*h*x + 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(1
95*g^2 + 87*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g
*h^2*x^2 + h^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^
3) + f*x*(35*g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(12
8*a^3*f*h^3 - a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315
*g*h*x + 64*h^2*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) +
7*e*(40*g^3 + 45*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*
g^2*h*x + 35*g*h^2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x
+ 15*g*h^2*x^2 + 4*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 +
 10*h^3*x^3) + 3*f*x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))...
```

3.186.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^3(3bfg-14cdh+8afh+(6cfg-14ceh+11bfh)x)\sqrt{cx^2+bx+a}dx}{7ch^2}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{\int (g+hx)^3(3bfg-14cdh+8afh+(6cfg-14ceh+11bfh)x)\sqrt{cx^2+bx+a}dx}{14ch}$$

$\Big\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{\frac{\int -\frac{3}{2}(g+hx)^2\left(11fghb^2+22afh^2b-2cg(3fg+7eh)b+4ch(14cdg-5afg-7aeh)+\left(-4\left(3fg^2-7h(eg+2dh)\right)c^2-2h(8bfg+21beh+16afh)c+33b^2fh^2\right)\right.}{6c}}{14ch}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{\int (g+hx)^2\left(11fghb^2+22afh^2b-2cg(3fg+7eh)b+4ch(14cdg-5afg-7aeh)+\left(-4\left(3fg^2-7h(eg+\right.\right.\right.}{4c}}{14ch}$$

$\Big\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{\int -\frac{1}{2}(g+hx)\left(99fgh^2b^3+2\left(66afh^3-cgh(79fg+63eh)\right)b^2+4c\left(6cfg^3+14ch(4eg+3dh)g-ah^2(95fg+42eh\right.\right.\right.}{}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{\int (g+hx)\left(\right.}{}$$

$\Big\downarrow$ 1225

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{-\frac{35h(16c}{}}{}$$

$\Big\downarrow$ 1087

3.186. $\int (g+hx)^3\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\left(16c\right.}{}}{}}{}$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\left(16c\right.}{}}{}}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\left(\frac{(b\cdots}{}\right.}{}}{}}{}$$

input
```
Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

3.186. $\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

output
```
(f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) - (((6*c*f*g - 14*c*e*h +
11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (((33*b^2*f*h^2 - 2
*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3*f*g^2 - 7*h*(e*g + 2*d*h))
)*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (-1/24*((1155*b^4*f*h^4 - 1
28*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h + 35*b*
(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h) + b^2*
(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h*(3*e
*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3*f*h
^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g*h*
(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*g
+ 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/c^2 - (35*h*(256*c^5*d*g^3 - 33*b^
5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g*(e*g + 3*d*h)) + 6
*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(10*a^2*f*h^2 + 14*a
*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 16*c^3*(2*a^2*h^
2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*b*h*(3*f*g^2 + h
*(3*e*g + d*h))))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/
(16*c^2))/(10*c))/(4*c))/(14*c*h)
```

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.186.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.89

method	result	size
risch	Expression too large to display	1757
default	Expression too large to display	2098

input
```
int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/107520*(15360*c^6*f*h^3*x^6+1280*b*c^5*f*h^3*x^5+17920*c^6*e*h^3*x^5+537
60*c^6*f*g*h^2*x^5+3072*a*c^5*f*h^3*x^4-1408*b^2*c^4*f*h^3*x^4+1792*b*c^5*
e*h^3*x^4+5376*b*c^5*f*g*h^2*x^4+21504*c^6*d*h^3*x^4+64512*c^6*e*g*h^2*x^4
+64512*c^6*f*g^2*h*x^4-5056*a*b*c^4*f*h^3*x^3+4480*a*c^5*e*h^3*x^3+13440*a
*c^5*f*g*h^2*x^3+1584*b^3*c^3*f*h^3*x^3-2016*b^2*c^4*e*h^3*x^3-6048*b^2*c^
4*f*g*h^2*x^3+2688*b*c^5*d*h^3*x^3+8064*b*c^5*e*g*h^2*x^3+8064*b*c^5*f*g^2
*h*x^3+80640*c^6*d*g*h^2*x^3+80640*c^6*e*g^2*h*x^3+26880*c^6*f*g^3*x^3-409
6*a^2*c^4*f*h^3*x^2+7776*a*b^2*c^3*f*h^3*x^2-7616*a*b*c^4*e*h^3*x^2-22848*
a*b*c^4*f*g*h^2*x^2+7168*a*c^5*d*h^3*x^2+21504*a*c^5*e*g*h^2*x^2+21504*a*c
^5*f*g^2*h*x^2-1848*b^4*c^2*f*h^3*x^2+2352*b^3*c^3*e*h^3*x^2+7056*b^3*c^3*
f*g*h^2*x^2-3136*b^2*c^4*d*h^3*x^2-9408*b^2*c^4*e*g*h^2*x^2-9408*b^2*c^4*f
*g^2*h*x^2+13440*b*c^5*d*g*h^2*x^2+13440*b*c^5*e*g^2*h*x^2+4480*b*c^5*f*g^
3*x^2+107520*c^6*d*g^2*h*x^2+35840*c^6*e*g^3*x^2+12704*a^2*b*c^3*f*h^3*x-6
720*a^2*c^4*e*h^3*x-20160*a^2*c^4*f*g*h^2*x-12096*a*b^3*c^2*f*h^3*x+12544*
a*b^2*c^3*e*h^3*x+37632*a*b^2*c^3*f*g*h^2*x-12992*a*b*c^4*d*h^3*x-38976*a*
b*c^4*e*g*h^2*x-38976*a*b*c^4*f*g^2*h*x+40320*a*c^5*d*g*h^2*x+40320*a*c^5*
e*g^2*h*x+13440*a*c^5*f*g^3*x+2310*b^5*c*f*h^3*x-2940*b^4*c^2*e*h^3*x-8820
*b^4*c^2*f*g*h^2*x+3920*b^3*c^3*d*h^3*x+11760*b^3*c^3*e*g*h^2*x+11760*b^3*
c^3*f*g^2*h*x-16800*b^2*c^4*d*g*h^2*x-16800*b^2*c^4*e*g^2*h*x-5600*b^2*c^4
*f*g^3*x+26880*b*c^5*d*g^2*h*x+8960*b*c^5*e*g^3*x+53760*c^6*d*g^3*x+819...
```

3.186.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 2817, normalized size of antiderivative = 3.03

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
 (5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*
c^5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*
c^5)*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a
*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 -
 64*a^3*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*
f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^
2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e +
b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)
*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*
c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^
5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b
^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e +
 (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c
^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1
808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 819
2*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h...
```

3.186.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4820 vs. 2(966) = 1932.

Time = 1.38 (sec) , antiderivative size = 4820, normalized size of antiderivative = 5.18

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**6/7 + x**5*(b*f*h**3/14 + c*e
*h**3 + 3*c*f*g*h**2)/(6*c) + x**4*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 -
 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*
h**2 + 3*c*f*g**2*h)/(5*c) + x**3*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3
/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**
2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*
h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*
c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + x**2*(a*d*h**3 + 3*a*
e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b
*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2
+ 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*f*g**3 - 7*b*(a*e*
h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) +
b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*
f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3
+ 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*
g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + x*(3*a*d*g*h**2 + 3*a*e*g**
2*h + a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**
3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*
f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*
g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d...
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1657, normalized size of antiderivative = 1.78

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h
^2 + 14*c^6*e*h^3 + b*c^5*f*h^3)/c^6)*x + (504*c^6*f*g^2*h + 504*c^6*e*g*h
^2 + 42*b*c^5*f*g*h^2 + 168*c^6*d*h^3 + 14*b*c^5*e*h^3 - 11*b^2*c^4*f*h^3
+ 24*a*c^5*f*h^3)/c^6)*x + (1680*c^6*f*g^3 + 5040*c^6*e*g^2*h + 504*b*c^5*
f*g^2*h + 5040*c^6*d*g*h^2 + 504*b*c^5*e*g*h^2 - 378*b^2*c^4*f*g*h^2 + 840
*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 - 126*b^2*c^4*e*h^3 + 280*a*c^5*e*h^3 + 9
9*b^3*c^3*f*h^3 - 316*a*b*c^4*f*h^3)/c^6)*x + (4480*c^6*e*g^3 + 560*b*c^5*
f*g^3 + 13440*c^6*d*g^2*h + 1680*b*c^5*e*g^2*h - 1176*b^2*c^4*f*g^2*h + 26
88*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 - 1176*b^2*c^4*e*g*h^2 + 2688*a*c^5*
e*g*h^2 + 882*b^3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 +
 896*a*c^5*d*h^3 + 294*b^3*c^3*e*h^3 - 952*a*b*c^4*e*h^3 - 231*b^4*c^2*f*h
^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3)/c^6)*x + (26880*c^6*d*g^3 +
4480*b*c^5*e*g^3 - 2800*b^2*c^4*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g
^2*h - 8400*b^2*c^4*e*g^2*h + 20160*a*c^5*e*g^2*h + 5880*b^3*c^3*f*g^2*h -
 19488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 + 5880
*b^3*c^3*e*g*h^2 - 19488*a*b*c^4*e*g*h^2 - 4410*b^4*c^2*f*g*h^2 + 18816*a*
b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c^3*d*h^3 - 6496*a*b*c^
4*d*h^3 - 1470*b^4*c^2*e*h^3 + 6272*a*b^2*c^3*e*h^3 - 3360*a^2*c^4*e*h^3 +
 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6352*a^2*b*c^3*f*h^3)/c^6)*x +
(26880*b*c^5*d*g^3 - 13440*b^2*c^4*e*g^3 + 35840*a*c^5*e*g^3 + 8400*b^3...
```

3.186.9 Mupad [B] (verification not implemented)

Time = 25.11 (sec) , antiderivative size = 3262, normalized size of antiderivative = 3.51

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*
x^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6)
+ (d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)
^(3/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x
+ c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x
)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*l
og((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c
^(5/2)) - (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1
/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e
*g^3*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
 + (33*b^7*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048
*c^(13/2)) + (f*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((lo
g((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^
(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24
*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a
 + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1...
```

3.186. $\int (g+hx)^3\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.187.1 Optimal result

Integrand size = 32, antiderivative size = 584

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(2bg(eg+2dh)+a(fg^2+2egh+dh^2))+8c^2(}{512c^5}$$
$$-\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{(105b^3fh^3+64c^3g(fg^2-2h(eg+5dh))-28bch^2(7afh+5b(2fg+eh))+8c^2h(16ah(2fg+eh)+b(}{}$$
$$-\frac{(b^2-4ac)\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(2bg(eg+2dh)+a(fg^2+2egh+\right.}{1024c^{11/2}}$$

output
```
-1/20*(3*b*f*h-4*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2/h+1/6*f*
(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^3+64*c^3*g*(f*g^2-2*h
*(5*d*h+e*g))-28*b*c*h^2*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*h*(16*a*h*(e*h+2*
f*g)+b*(7*f*g^2+25*h*(d*h+2*e*g)))-6*c*h*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*
e*h+2*b*f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4/h-1
/1024*(-4*a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2
*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+8*c^2*(2*a^2*f*
h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d*g^2+21*b^4*f*h^2-
28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*
g*h+f*g^2))+8*c^2*(2*a^2*f*h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*
g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

3.187.2 Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.12

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(315b^5fh^2-210b^4ch(4fg+2eh+fhx)+8b^3c(-210afh^2+5ch(30eg+15dh+7ehx}{}$$

input
```
Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f*h^2 - 210*b^4*c*h*(4*f*g + 2*e*h
 + f*h*x) + 8*b^3*c*(-210*a*f*h^2 + 5*c*h*(30*e*g + 15*d*h + 7*e*h*x) + c*
f*(75*g^2 + 70*g*h*x + 21*h^2*x^2)) - 16*b^2*c^2*(-(a*h*(230*f*g + 115*e*h
 + 56*f*h*x)) + c*(5*d*h*(24*g + 5*h*x) + 2*e*(30*g^2 + 25*g*h*x + 7*h^2*x
^2) + f*x*(25*g^2 + 28*g*h*x + 9*h^2*x^2))) + 16*b*c^2*(113*a^2*f*h^2 - 2*
a*c*(h*(130*e*g + 65*d*h + 29*e*h*x) + f*(65*g^2 + 58*g*h*x + 17*h^2*x^2))
 + 4*c^2*(5*d*(6*g^2 + 4*g*h*x + h^2*x^2) + x*(f*x*(5*g^2 + 6*g*h*x + 2*h^
2*x^2) + e*(10*g^2 + 10*g*h*x + 3*h^2*x^2)))) - 32*c^3*(a^2*h*(64*f*g + 32
*e*h + 15*f*h*x) - 2*a*c*(5*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x +
5*h^2*x^2) + e*(40*g^2 + 30*g*h*x + 8*h^2*x^2)) - 4*c^2*x*(5*d*(6*g^2 + 8*
g*h*x + 3*h^2*x^2) + x*(2*e*(10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2
+ 24*g*h*x + 10*h^2*x^2))))) - 15*(b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*
h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*
g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*
h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqr
t[a + x*(b + c*x)])])/(7680*c^(11/2))
```

3.187.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{3}{2}h(g+hx)^2(bfg-4cdh+2afh+(2cfg-4ceh+3bfh)x)\sqrt{cx^2+bx+a}dx}{6ch^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$\big\downarrow$ 27

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\int (g+hx)^2(bfg-4cdh+2afh+(2cfg-4ceh+3bfh)x)\sqrt{cx^2+bx+a}dx}{4ch}$$

$\big\downarrow$ 1236

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{\int -\frac{1}{2}(g+hx)\left(9fghb^2+12afh^2b-4cg(fg+3eh)b+4ch(10cdg-3afg-4aeh)+\left(-8\left(fg^2-h(2eg+5dh)\right)c^2-4h(2bfg+7beh+5afh)c+21b^2fh^2\right)x\right)\sqrt{cx}}{5c}}{4ch}$$

$\big\downarrow$ 27

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{\int (g+hx)\left(9fghb^2+12afh^2b-4cg(fg+3eh)b+4ch(10cdg-3afg-4aeh)+\left(-8\left(fg^2-h(2eg+5dh)\right)c\right.}{10c}}{4ch}$$

$\big\downarrow$ 1225

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+a\right.}{16c^2}$$

$\big\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+a\right.}{16c^2}$$

$\big\downarrow$ 1092

3.187. $\int (g+hx)^2\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+a\right.\right.}{}$$

$\Big\downarrow$ 219

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{5h\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\right.\right.}{}$$

input
```
Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/(6*c*h) - (((2*c*f*g - 4*c*e*h + 3
*b*f*h)*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (-1/24*((105*b^3*f*h^
3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 28*b*c*h^2*(7*a*f*h + 5*b*(2*f*
g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*
h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7*b*e*h + 5*a*f*h) - 8*c^2*(f
*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)^(3/2))/c^2 + (5*h*(128*c^4
*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a
*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6
*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c^2))/(10*c))/(4*c*h)
```

3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.187. $\int (g+hx)^2\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.187. $\int (g+hx)^2\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.187.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.76

method	result	size
risch	Expression too large to display	1027
default	Expression too large to display	1212

input
```
int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7680*(1280*c^5*f*h^2*x^5+128*b*c^4*f*h^2*x^4+1536*c^5*e*h^2*x^4+3072*c^5
*f*g*h*x^4+320*a*c^4*f*h^2*x^3-144*b^2*c^3*f*h^2*x^3+192*b*c^4*e*h^2*x^3+3
84*b*c^4*f*g*h*x^3+1920*c^5*d*h^2*x^3+3840*c^5*e*g*h*x^3+1920*c^5*f*g^2*x^
3-544*a*b*c^3*f*h^2*x^2+512*a*c^4*e*h^2*x^2+1024*a*c^4*f*g*h*x^2+168*b^3*c
^2*f*h^2*x^2-224*b^2*c^3*e*h^2*x^2-448*b^2*c^3*f*g*h*x^2+320*b*c^4*d*h^2*x
^2+640*b*c^4*e*g*h*x^2+320*b*c^4*f*g^2*x^2+5120*c^5*d*g*h*x^2+2560*c^5*e*g
^2*x^2-480*a^2*c^3*f*h^2*x+896*a*b^2*c^2*f*h^2*x-928*a*b*c^3*e*h^2*x-1856*
a*b*c^3*f*g*h*x+960*a*c^4*d*h^2*x+1920*a*c^4*e*g*h*x+960*a*c^4*f*g^2*x-210
*b^4*c*f*h^2*x+280*b^3*c^2*e*h^2*x+560*b^3*c^2*f*g*h*x-400*b^2*c^3*d*h^2*x
-800*b^2*c^3*e*g*h*x-400*b^2*c^3*f*g^2*x+1280*b*c^4*d*g*h*x+640*b*c^4*e*g^
2*x+3840*c^5*d*g^2*x+1808*a^2*b*c^2*f*h^2-1024*a^2*c^3*e*h^2-2048*a^2*c^3*
f*g*h-1680*a*b^3*c*f*h^2+1840*a*b^2*c^2*e*h^2+3680*a*b^2*c^2*f*g*h-2080*a*
b*c^3*d*h^2-4160*a*b*c^3*e*g*h-2080*a*b*c^3*f*g^2+5120*a*c^4*d*g*h+2560*a*
c^4*e*g^2+315*b^5*f*h^2-420*b^4*c*e*h^2-840*b^4*c*f*g*h+600*b^3*c^2*d*h^2+
1200*b^3*c^2*e*g*h+600*b^3*c^2*f*g^2-1920*b^2*c^3*d*g*h-960*b^2*c^3*e*g^2+
1920*b*c^4*d*g^2)/c^5*(c*x^2+b*x+a)^(1/2)+1/1024*(64*a^3*c^3*f*h^2-240*a^2
*b^2*c^2*f*h^2+192*a^2*b*c^3*e*h^2+384*a^2*b*c^3*f*g*h-128*a^2*c^4*d*h^2-2
56*a^2*c^4*e*g*h-128*a^2*c^4*f*g^2+140*a*b^4*c*f*h^2-160*a*b^3*c^2*e*h^2-3
20*a*b^3*c^2*f*g*h+192*a*b^2*c^3*d*h^2+384*a*b^2*c^3*e*g*h+192*a*b^2*c^3*f
*g^2-512*a*b*c^4*d*g*h-256*a*b*c^4*e*g^2+512*a*c^5*d*g^2-21*b^6*f*h^2+2...
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 1791, normalized size of antiderivative = 3.07

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4
)*d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^
2 + 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d -
4*(7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*
a^2*b^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*
x^5 + 128*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^
2 + 24*(10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 2
0*a*c^5)*f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^
3*c^3 - 52*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^
3 - 52*a*b*c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (
40*(15*b^3*c^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*
c^4)*e + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c
^6*e + b*c^5*f)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)
*g*h + (40*b*c^5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4
)*f)*h^2)*x^2 + 2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^
2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^
4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e
+ (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b...
```

3.187.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2440 vs. $2(607) = 1214$.

Time = 1.19 (sec) , antiderivative size = 2440, normalized size of antiderivative = 4.18

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**5/6 + x**4*(b*f*h**2/12 + c*e
*h**2 + 2*c*f*g*h)/(5*c) + x**3*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(
b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g*
*2)/(4*c) + x**2*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c
*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h*
*2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**
2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(3*c) + x*(a*d*h**
2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b
*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**
2)/(4*c) + 2*b*d*g*h + b*e*g**2 - 5*b*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**
2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b
*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*
g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**
2)/(6*c) + c*d*g**2)/(2*c) + (2*a*d*g*h + a*e*g**2 - 2*a*(a*e*h**2 + 2*a*f
*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g
*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12
+ c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) +
2*c*d*g*h + c*e*g**2)/(3*c) + b*d*g**2 - 3*b*(a*d*h**2 + 2*a*e*g*h + a*f*g
**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2
 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + 2*b*d*g...
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.68

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,fh^2x+\frac{24\,c^5fgh+12\,c^5eh^2+bc^4fh^2}{c^5}\right)x+\frac{120\,c^5fg^2+240\,c^5egh\right.\right.\right.\right.\right.$$
$$+\frac{(128\,b^2c^4dg^2-512\,ac^5dg^2-64\,b^3c^3eg^2+256\,abc^4eg^2+40\,b^4c^2fg^2-192\,ab^2c^3fg^2+128\,a^2c^4fg^2-128}{}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + 12*
c^5*e*h^2 + b*c^4*f*h^2)/c^5)*x + (120*c^5*f*g^2 + 240*c^5*e*g*h + 24*b*c^
4*f*g*h + 120*c^5*d*h^2 + 12*b*c^4*e*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^
2)/c^5)*x + (320*c^5*e*g^2 + 40*b*c^4*f*g^2 + 640*c^5*d*g*h + 80*b*c^4*e*g
*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g*h + 40*b*c^4*d*h^2 - 28*b^2*c^3*e*h^
2 + 64*a*c^4*e*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2)/c^5)*x + (1920*c
^5*d*g^2 + 320*b*c^4*e*g^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c
^4*d*g*h - 400*b^2*c^3*e*g*h + 960*a*c^4*e*g*h + 280*b^3*c^2*f*g*h - 928*a
*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 + 140*b^3*c^2*e*h^2 - 4
64*a*b*c^3*e*h^2 - 105*b^4*c*f*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h
^2)/c^5)*x + (1920*b*c^4*d*g^2 - 960*b^2*c^3*e*g^2 + 2560*a*c^4*e*g^2 + 60
0*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^3*d*g*h + 5120*a*c^4*d*g
*h + 1200*b^3*c^2*e*g*h - 4160*a*b*c^3*e*g*h - 840*b^4*c*f*g*h + 3680*a*b^
2*c^2*f*g*h - 2048*a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2
- 420*b^4*c*e*h^2 + 1840*a*b^2*c^2*e*h^2 - 1024*a^2*c^3*e*h^2 + 315*b^5*f*
h^2 - 1680*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2)/c^5) + 1/1024*(128*b^2*c^
4*d*g^2 - 512*a*c^5*d*g^2 - 64*b^3*c^3*e*g^2 + 256*a*b*c^4*e*g^2 + 40*b^4*
c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 128*b^3*c^3*d*g*h +
512*a*b*c^4*d*g*h + 80*b^4*c^2*e*g*h - 384*a*b^2*c^3*e*g*h + 256*a^2*c^4*e
*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h - 384*a^2*b*c^3*f*g*h + 40*...
```

3.187.9 Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.22

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^
2)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x
/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*
x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/
(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b
 + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2
)) - (2*a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
 + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) +
 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x
^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*
g^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c)
)/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1
/2))/(24*c^2)))/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*
g^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(
a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)...
```

3.188 $\int (g+hx)\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.188.1 Optimal result

Integrand size = 30, antiderivative size = 322

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh))\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^4}$$
$$+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}$$
$$+\frac{\left(35b^2fh^2-16c^2(3fg^2-5h(eg+dh))-2ch(16afh+25b(fg+eh))-6ch(6cfg-10ceh+7bfh)x\right)(a}{240c^3h}$$
$$-\frac{\left(b^2-4ac\right)\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh))\right)\operatorname{arctanh}\left(\frac{}{2\sqrt{c}}\right.}{256c^{9/2}}$$

output
```
1/5*f*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-16*c^2*(3*f*g^
2-5*h*(d*h+e*g))-2*c*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6
*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*
f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arc
tanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/128*(32*c^3*d*g-
7*b^3*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)
))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

3.188.2 Mathematica [A] (verified)

Time = 3.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^4fh+10b^3c(15fg+15eh+7fhx)-4b^2c(-115afh+c(60eg+60dh+25fgx$$

input
```
Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^4*f*h + 10*b^3*c*(15*f*g + 15*e*h +
 7*f*h*x) - 4*b^2*c*(-115*a*f*h + c*(60*e*g + 60*d*h + 25*f*g*x + 25*e*h*x
 + 14*f*h*x^2)) + 8*b*c^2*(20*c*d*(3*g + h*x) - a*(65*f*g + 65*e*h + 29*f*
h*x) + 2*c*x*(5*e*(2*g + h*x) + f*x*(5*g + 3*h*x))) + 16*c^2*(-16*a^2*f*h
+ a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g + 8*h*x)) + 2*c^2*x*(10*d*(3
*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*g + 4*h*x))))) + 15*(b^2 - 4
*a*c)*(-32*c^3*d*g + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h)
 - 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt
[a + x*(b + c*x)])])/(1920*c^(9/2))
```

3.188.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2184, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 2184}$$
$$\frac{\int -\frac{1}{2}h(g+hx)(3bfg-10cdh+4afh+(6cfg-10ceh+7bfh)x)\sqrt{cx^2+bx+a}dx}{5ch^2}+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{\int (g+hx)(3bfg-10cdh+4afh+(6cfg-10ceh+7bfh)x)\sqrt{cx^2+bx+a}dx}{10ch}$$

$\big\downarrow$ 1225

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\int \sqrt{cx^2+bx+a}dx}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-2ch(16afh+25b(eh+fg))+}{}}{10ch}$$

$\big\downarrow$ 1087

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c^2}-\frac{\left(a+bx+cx^2\right)}{}}{10ch}$$

$\big\downarrow$ 1092

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c^2}-}{10ch}$$

$\big\downarrow$ 219

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)}{16c^2}-}{10ch}$$

input
```
Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

3.188. $\int (g+hx)\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

output
```
(f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) - (-1/24*((35*b^2*f*h^2 -
16*c^2*(3*f*g^2 - 5*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) -
 6*c*h*(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/c^2 - (5
*h*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2
*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*
c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(3/2))))/(16*c^2))/(10*c*h)
```

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.188.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{\left(-384hf\,c^4x^4-48b\,c^3fh\,x^3-480c^4eh\,x^3-480c^4fg\,x^3-128a\,c^3fh\,x^2+56b^2c^2fh\,x^2-80b\,c^3eh\,x^2-80b\,c^3fg\,x^2-640c^4dh\,x^2-640c^4\cdots\right.}{\cdots}$
default	$dg\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+hf\left(\frac{x^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{5c}-\frac{7b\left(\frac{x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right.$

input
```
int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(-384*c^4*f*h*x^4-48*b*c^3*f*h*x^3-480*c^4*e*h*x^3-480*c^4*f*g*x^3
-128*a*c^3*f*h*x^2+56*b^2*c^2*f*h*x^2-80*b*c^3*e*h*x^2-80*b*c^3*f*g*x^2-64
0*c^4*d*h*x^2-640*c^4*e*g*x^2+232*a*b*c^2*f*h*x-240*a*c^3*e*h*x-240*a*c^3*
f*g*x-70*b^3*c*f*h*x+100*b^2*c^2*e*h*x+100*b^2*c^2*f*g*x-160*b*c^3*d*h*x-1
60*b*c^3*e*g*x-960*c^4*d*g*x+256*a^2*c^2*f*h-460*a*b^2*c*f*h+520*a*b*c^2*e
*h+520*a*b*c^2*f*g-640*a*c^3*d*h-640*a*c^3*e*g+105*b^4*f*h-150*b^3*c*e*h-1
50*b^3*c*f*g+240*b^2*c^2*d*h+240*b^2*c^2*e*g-480*b*c^3*d*g)/c^4*(c*x^2+b*x
+a)^(1/2)+1/256*(48*a^2*b*c^2*f*h-32*a^2*c^3*e*h-32*a^2*c^3*f*g-40*a*b^3*c
*f*h+48*a*b^2*c^2*e*h+48*a*b^2*c^2*f*g-64*a*b*c^3*d*h-64*a*b*c^3*e*g+128*a
*c^4*d*g+7*b^5*f*h-10*b^4*c*e*h-10*b^4*c*f*g+16*b^3*c^2*d*h+16*b^3*c^2*e*g
-32*b^2*c^3*d*g)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1009, normalized size of antiderivative = 3.13

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2
*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*
c^2)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10
*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*
c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 -
 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d
 - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c
^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80
*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*
x)*sqrt(c*x^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8
*(b^3*c^2 - 4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (1
6*(b^3*c^2 - 4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7
*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4
 + 48*(10*c^5*f*g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f
)*g + (80*c^5*d + 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b
*c^4*d - 8*(3*b^2*c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*
(3*b^2*c^3 - 8*a*c^4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c -...
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(332) = 664$.

Time = 1.04 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.08

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fhx^4}{5}+\frac{x^3\left(\frac{bfh}{10}+ceh+cfg\right)}{4c}+\frac{x^2\left(\frac{afh}{5}+beh+bfg-\frac{7b\left(\frac{bfh}{10}+ceh+cfg\right)}{8c}+cdh+ceg\right)}{3c}+\frac{x\left(aeh+afg-\frac{3a\left(\frac{bfh}{10}+ceh+\cdots\right)}{4c}\cdots\right)}{\cdots}\right. \\ \frac{2\left(\frac{fh(a+bx)^{\frac{9}{2}}}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}(-3afh+beh+bfg)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(3a^2fh-2abeh-2abfg+b^2dh+b^2eg\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\left(-a^3fh+a^2beh+a^2bfg-ab^2dh-ab^2eg+b^3dg\right)}{3b^3}\right)}{b} \\ \sqrt{a}\left(dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}\right) \end{cases}$$

input

```
integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

3.188. $\int (g+hx)\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**4/5 + x**3*(b*f*h/10 + c*e*h + c
*f*g)/(4*c) + x**2*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*
g)/(8*c) + c*d*h + c*e*g)/(3*c) + x*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h
 + c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*
h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(2*c) + (a*d*h
 + a*e*g - 2*a*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(
8*c) + c*d*h + c*e*g)/(3*c) + b*d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 +
 c*e*h + c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b
*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/c
) + (a*d*g - a*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*c) + b*d
*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)
/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(2*c) - b*(a*d*h + a*e*g - 2*a*(a*f
*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*
g)/(3*c) + b*d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*
c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h
+ c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/(2*c))*Piecewise((lo
g(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c)
, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), N
e(c, 0)), (2*(f*h*(a + b*x)**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-3*a*f*h +
 b*e*h + b*f*g)/(7*b**3) + (a + b*x)**(5/2)*(3*a**2*f*h - 2*a*b*e*h - 2...
```

3.188.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.48

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,fhx+\frac{10\,c^4fg+10\,c^4eh+bc^3fh}{c^4}\right)x+\frac{80\,c^4eg+10\,bc^3fg+80\,c^4dh+}{c^4}\right.\right.\right.$$
$$+\frac{(32\,b^2c^3dg-128\,ac^4dg-16\,b^3c^2eg+64\,abc^3eg+10\,b^4cfg-48\,ab^2c^2fg+32\,a^2c^3fg-16\,b^3c^2dh+64\,a}{}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + 10*c^4*e*h
+ b*c^3*f*h)/c^4)*x + (80*c^4*e*g + 10*b*c^3*f*g + 80*c^4*d*h + 10*b*c^3*e
*h - 7*b^2*c^2*f*h + 16*a*c^3*f*h)/c^4)*x + (480*c^4*d*g + 80*b*c^3*e*g -
50*b^2*c^2*f*g + 120*a*c^3*f*g + 80*b*c^3*d*h - 50*b^2*c^2*e*h + 120*a*c^3
*e*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h)/c^4)*x + (480*b*c^3*d*g - 240*b^2*c
^2*e*g + 640*a*c^3*e*g + 150*b^3*c*f*g - 520*a*b*c^2*f*g - 240*b^2*c^2*d*h
 + 640*a*c^3*d*h + 150*b^3*c*e*h - 520*a*b*c^2*e*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g -
 16*b^3*c^2*e*g + 64*a*b*c^3*e*g + 10*b^4*c*f*g - 48*a*b^2*c^2*f*g + 32*a^
2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h + 10*b^4*c*e*h - 48*a*b^2*c^2*
e*h + 32*a^2*c^3*e*h - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h)*log(
abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.72

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = d\,g\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a} - \frac{2\,a\,f\,h\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{5\,c} - \frac{5\,b\,e\,h\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} - \frac{5\,b\,f\,g\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} + \frac{d\,h\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2} + \frac{e\,g\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2} + \frac{e\,h\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c} + \frac{f\,g\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c} + \frac{7\,b\,f\,h\left(\frac{5\,b\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c}-\frac{x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}+\frac{a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right) \cdots\right.}{\cdots}\right)}{10\,c} + \frac{f\,h\,x^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{5\,c} - \frac{a\,e\,h\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} - \frac{a\,f\,g\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} + \frac{d\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} + \frac{d\,h\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} + \frac{e\,g\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}$$

input
```
int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c
^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) -
 (5*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*
a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x
^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*
b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a +
c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a
 + b*x + c*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*
b*f*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c
*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x
/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*
x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(
a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))
/(2*c^(3/2))))/(4*c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (
log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^...
```

3.188. $\int (g+hx)\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

3.189 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

3.189.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3}\\&\quad+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}\\&\quad-\frac{\left(b^2-4ac\right)\left(16c^2d+5b^2f-4c(2be+af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

output
```
1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-
1/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c
^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

3.189.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+2}{192c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*
(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e
+ a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(7
/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \ 2192$$

$$\frac{\int \frac{1}{2}(8cd-2af+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{4c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (2(4cd-af)+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{(-4acf+5b^2f-8bce+16c^2d)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{(-4acf+5b^2f-8bce+16c^2d)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)(-4acf+5b^2f-8bce+16c^2d)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((8*c*e - 5*b*f)*(a + b*x + c*x^2)^
(3/2))/(3*c) + ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(2*c))/(8*c)
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.189.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{(-48f\,c^3x^3-8b\,c^2f\,x^2-64c^3e\,x^2-24a\,c^2fx+10b^2cfx-16b\,c^2ex-96c^3dx+52abcf-64a\,c^2e-15b^3f+24b^2ce-48bd\,c^2)\sqrt{c\,x^2+bx+a}}{192c^3}$
default	$d\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+f\left(\frac{x(c\,x^2+bx+a)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)}{}\right.}{}\right)}{}\right.$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*c^3*f*x^3-8*b*c^2*f*x^2-64*c^3*e*x^2-24*a*c^2*f*x+10*b^2*c*f*x
-16*b*c^2*e*x-96*c^3*d*x+52*a*b*c*f-64*a*c^2*e-15*b^3*f+24*b^2*c*e-48*b*c^
2*d)*(c*x^2+b*x+a)^(1/2)/c^3-1/128*(16*a^2*c^2*f-24*a*b^2*c*f+32*a*b*c^2*e
-64*a*c^3*d+5*b^4*f-8*b^3*c*e+16*b^2*c^2*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.66

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$=\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{}\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 2
4*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^
3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c -
52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqr
t(c*x^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4
*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c
^4*f*x^3 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3
)*e + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 1
2*a*c^3)*f)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

3.189.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 0.54 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.19

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fx^3}{4}+\frac{x^2\left(\frac{bf}{8}+ce\right)}{3c}+\frac{x\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{2c}+\frac{ae-\frac{2a\left(\frac{bf}{8}+ce\right)}{3c}+bd-\frac{3b\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{4c}}{c}\right)+\left(\right. \\ \frac{2\left(\frac{f(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{5}{2}}(-2af+be)}{5b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(a^2f-abe+b^2d\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*x**3/4 + x**2*(b*f/8 + c*e)/(3*c) + x
*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) + (a*e - 2*a*(b*f/8 +
 c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(4*c
))/c) + (a*d - a*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) - b*(
a*e - 2*a*(b*f/8 + c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)
/(6*c) + c*d)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*(a + b*x)**(7/2)/
(7*b**2) + (a + b*x)**(5/2)*(-2*a*f + b*e)/(5*b**2) + (a + b*x)**(3/2)*(a*
*2*f - a*b*e + b**2*d)/(3*b**2))/b, Ne(b, 0)), (sqrt(a)*(d*x + e*x**2/2 +
f*x**3/3), True))
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{8\,c^3e+bc^2f}{c^3}\right)x+\frac{48\,c^3d+8\,bc^2e-5\,b^2cf+12\,ac^2f}{c^3}\right)x+\frac{48\,bc^2d-2\right.$$
$$+\frac{\left(16\,b^2c^2d-64\,ac^3d-8\,b^3ce+32\,abc^2e+5\,b^4f-24\,ab^2cf+16\,a^2c^2f\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{\right.\right.}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (8*c^3*e + b*c^2*f)/c^3)*x + (4
8*c^3*d + 8*b*c^2*e - 5*b^2*c*f + 12*a*c^2*f)/c^3)*x + (48*b*c^2*d - 24*b^
2*c*e + 64*a*c^2*e + 15*b^3*f - 52*a*b*c*f)/c^3) + 1/128*(16*b^2*c^2*d - 6
4*a*c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2
*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.83

$$\int \sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$$
$$= d\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a} - \frac{af\left(\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}}\right)}{4c} + \frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\left(ac-\frac{b^2}{4}\right)}{2c^{3/2}} + \frac{e\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)(b^3-4abc)}{16c^{5/2}} - \frac{5bf\left(\frac{\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)(b^3-4abc)}{16c^{5/2}}+\frac{(-3b^2+2cxb+8c(cx^2+a))\sqrt{cx^2+bx+a}}{24c^2}\right)}{8c} + \frac{e(-3b^2+2cxb+8c(cx^2+a))\sqrt{cx^2+bx+a}}{24c^2} + \frac{fx(cx^2+bx+a)^{3/2}}{4c}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b +
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
+ ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2) + (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.190 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$

3.190.1 Optimal result

Integrand size = 32, antiderivative size = 321

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+bfh)x)\sqrt{a+bx+cx^2}}{8c^2h^3} + \frac{f(a+bx+cx^2)^{3/2}}{3ch} + \frac{(4ch(2cg-bh)(bfg-2cdh)-(2cfg-2ceh+bfh)(8c^2g^2-b^2h^2-4ch(bg-ah)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}h^4} + \frac{\sqrt{cg^2-bgh+ah^2}(fg^2-egh+dh^2)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

output
```
1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/16*(4*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*
f*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-b^2*h^2-4*c*h*(-a*h+b*g)))*arctanh(1/2*(2*
c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^4+(d*h^2-e*g*h+f*g^2)*arctan
h(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(
1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^4-1/8*(4*c*h*(b*f*g-2*c*d*h)-(-b*h+4*c*g
)*(b*f*h-2*c*e*h+2*c*f*g)+2*c*h*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(
1/2)/c^2/h^3
```

3.190.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx} \, dx$$

$$= \frac{\frac{h\sqrt{a+x(b+cx)}(-3b^2fh^2+2ch(4afh+b(-3fg+3eh+fhx))+4c^2(3h(-2eg+2dh+ehx)+f(6g^2-3ghx+2h^2x^2)))}{c^2} + 48\sqrt{-cg^2+h(bg-}$$

input

```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output

```
((h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e
*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x +
2*h^2*x^2))))/c^2 + 48*Sqrt[-(c*g^2) + h*(b*g - a*h)]*(f*g^2 + h*(-(e*g) +
 d*h))*ArcTan[(Sqrt[-(c*g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sq
rt[a + x*(b + c*x)])] - (3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2
*a*f*h) + 16*c^3*(f*g^3 + g*h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(
e*g) + d*h) + a*h*(-(f*g) + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a
+ x*(b + c*x)])])/c^(5/2))/(24*h^4)
```

3.190.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx} \, dx$$

$$\Big\downarrow \; 2184$$

$$\frac{\int -\frac{3h(bfg-2cdh+(2cfg-2ceh+bfh)x)\sqrt{cx^2+bx+a}}{2(g+hx)}dx}{3ch^2} + \frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$\Big\downarrow \; 27$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch} - \frac{\int \frac{(bfg-2cdh+(2cfg-2ceh+bfh)x)\sqrt{cx^2+bx+a}}{g+hx}dx}{2ch}$$

$\Big\downarrow$ 1231

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\int \frac{4ch(bg-2ah)(bfg-2cdh)-g\left(-hb^2+4cgb-4ach\right)(2cfg-2ceh \cdots}{\cdots}}{\cdots}}{2ch}$$

$\Big\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\int \frac{4ch(bg-2ah)(bfg-2cdh)-g\left(-hb^2+4cgb-4ach\right)(2cfg-2ceh \cdots}{\cdots}}{\cdots}}{2ch}$$

$\Big\downarrow$ 1269

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bf \cdots\right.}{h}\cdots}{\cdots}}{2ch}$$

$\Big\downarrow$ 1092

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{2\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bf \cdots\right.}{h}\cdots}{\cdots}}{2ch}$$

$\Big\downarrow$ 219

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{16c^2\left(ah^2-bgh+cg^2\right)\left(fg^2-h(eg-dh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a \cdots}}}{h}\cdots}{\cdots}}{2ch}$$

$\Big\downarrow$ 1154

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(\cdots\right.\right.}{\sqrt{c}h}\cdots}{\cdots}}{2 \cdots}$$

$\Big\downarrow$ 219

3.190. $\displaystyle\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-\right.\right.}{\sqrt{c}h}}{}}{2ch}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + b*x + c*x^2)^(3/2))/(3*c*h) - (((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g
 - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*
Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h
) - (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + (16
*c^2*Sqrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - h*(e*g - d*h))*ArcTanh[(b*g - 2*
a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
])])/h)/(8*c*h^2))/(2*c*h)
```

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.190.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.60

method	result
risch	$\frac{\left(8f\,h^{2}c^{2}x^{2}+2bcf\,h^{2}x+12c^{2}e\,h^{2}x-12c^{2}fghx+8acf\,h^{2}-3b^{2}f\,h^{2}+6bce\,h^{2}-6bcfgh+24c^{2}d\,h^{2}-24c^{2}egh+24c^{2}f\,g^{2}\right)\sqrt{c\,x^{2}+bx+a}}{24c^{2}h^{3}}-$
default	$\frac{eh\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+fh\left(\frac{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{\cdots}\right)}{8c^{\frac{3}{2}}}\cdots\right.}{2c}\cdots\right.}{h^{2}}$

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*f*h^2*x^2+2*b*c*f*h^2*x+12*c^2*e*h^2*x-12*c^2*f*g*h*x+8*a*c*f*
h^2-3*b^2*f*h^2+6*b*c*e*h^2-6*b*c*f*g*h+24*c^2*d*h^2-24*c^2*e*g*h+24*c^2*f
*g^2)/c^2*(c*x^2+b*x+a)^(1/2)/h^3-1/16/c^2/h^3*(16*(a*d*h^4-a*e*g*h^3+a*f*
g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*h+c*f*g^4)*c^2
/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c
*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(4*a*b*c*f*h^3-
8*a*c^2*e*h^3+8*a*c^2*f*g*h^2-b^3*f*h^3+2*b^2*c*e*h^3-2*b^2*c*f*g*h^2-8*b*
c^2*d*h^3+8*b*c^2*e*g*h^2-8*b*c^2*f*g^2*h+16*c^3*d*g*h^2-16*c^3*e*g^2*h+16
*c^3*f*g^3)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

3.190.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

3.190.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

3.190.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.191 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2} dx$

3.191.1 Optimal result

Integrand size = 32, antiderivative size = 459

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2} dx = -\frac{\left(bfh^2(bg-ah)+4c^2g(3fg^2-h(2eg-dh))+ch(4ah(2fg-eh)-b(13fg^2-8egh+4dh^2))+2ch^2\left(\right.}{4ch^3(cg^2-bgh+ah^2)}$$
$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{3/2}}{h(cg^2-bgh+ah^2)(g+hx)}$$
$$-\frac{(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}h^4}$$
$$-\frac{(2cg(3fg^2-h(2eg-dh))+h(2ah(2fg-eh)-b(5fg^2-3egh+dh^2)))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+c}}\right.}{2h^4\sqrt{cg^2-bgh+ah^2}}$$

output
```
-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/
8*(b^2*f*h^2+4*c*h*(-a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*g*
(3*f*g^2-h*(-d*h+2*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(d*h^2-3*e*g*h+5*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(1/2)-1/4*(b*f*h^2*(-a*h+b*g)+4*c^2*g
*(3*f*g^2-h*(-d*h+2*e*g))+c*h*(4*a*h*(-e*h+2*f*g)-b*(4*d*h^2-8*e*g*h+13*f*
g^2))+2*c*h^2*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(
1/2)/c/h^3/(a*h^2-b*g*h+c*g^2)
```

3.191.2 Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

$$= \frac{-8(2fg-eh)\sqrt{a+x(b+cx)}+\frac{2fh(b+2cx)\sqrt{a+x(b+cx)}}{c}-\frac{8(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{g+hx}+\frac{(-b^2+4ac)fh\operatorname{arctanh}\left(\frac{}{2\sqrt{}}\right.}{c^{3/2}}}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
(-8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)] + (2*f*h*(b + 2*c*x)*Sqrt[a + x*(b
 + c*x)])/c - (8*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/(g + h*
x) + ((-b^2 + 4*a*c)*f*h*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])])/c^(3/2) + (4*(f*g^2 + h*(-(e*g) + d*h))*(2*Sqrt[c]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h +
2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c
*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)]))/h + (4*(2*f*g - e*h)*((2*c*g - b*
h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt
[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqr
t[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*h))/(8*h^3)
```

3.191.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+2cdg-3beg-2afg+bdh+2aeh-2\left(-\frac{3cfg^2}{h}+2ceg+bfg-2cdh-afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)}dx}{ah^2-bgh+cg^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{3fg^2}{h}+3eg-dh\right)-2\left(-\frac{3cfg^2}{h}+2ceg+bfg-2cdh-afh\right)x\right)\sqrt{cx^2+bx+a}}{g+hx}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1231$$

$$-\frac{\int \frac{\left(cg^2-bhg+ah^2\right)\left(fghb^2-4c\left(3fg^2-h(2eg-dh)\right)b+4ach(3fg-2eh)+\left(-8\left(3fg^2-h(2eg-dh)\right)c^2+4h(2bfg-beh-afh)c+b^2fh^2\right)x\right)}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{4ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2ch\right.}{}}{2\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\left(ah^2-bgh+cg^2\right)\int \frac{fghb^2-4c\left(3fg^2-h(2eg-dh)\right)b+4ach(3fg-2eh)+\left(-8\left(3fg^2-h(2eg-dh)\right)c^2+4h(2bfg-beh-afh)c+b^2fh^2\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{4ch^3}-\frac{\sqrt{a+bx+cx^2}\left(2ch\right.}{}}{2\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h}+\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-eh)-bh(3eg-dh\right.\right.}{h}\right.}{4ch^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

3.191. $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{2\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}+\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-\right.\right.}{}\right.}{4ch^3}$$

$$\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-eh)-bh(3eg-dh)+5bfg^2\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-\right.}{}\right.}{4ch^3}$$

$$\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1154

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{\sqrt{c}h}-\frac{8c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-eh\right.\right.}{}\right.}{4ch^3}$$

$$\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{\sqrt{c}h}+\frac{4c\,\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right.}{}\right.}{4ch^3}$$

$$\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

input

```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

3.191. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}dx$

output
```
-(((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x))) + (-1/2*((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/
h - d*h) + 4*a*c*h*(2*f*g - e*h) - b*c*(13*f*g^2 - 4*h*(2*e*g - d*h)) + 2*
c*h*(2*c*e*g + b*f*g - (3*c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x +
c*x^2])/(c*h^2) - ((c*g^2 - b*g*h + a*h^2)*(((b^2*f*h^2 + 4*c*h*(2*b*f*g -
 b*e*h - a*f*h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + (4*c*(2*c*(3*f*g^3 - g*h*
(2*e*g - d*h)) - h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*
ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2])))/(4*c*h^3))/(2*(c*g
^2 - b*g*h + a*h^2))
```

3.191.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.191. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.191.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(2cfhx+bfh+4ehc-8cfg)\sqrt{c x^2+bx+a}}{4c h^3}+\frac{\frac{\left(4acf h^2-b^2 f h^2+4bce h^2-8bcfgh+8c^2 d h^2-16c^2 egh+24c^2 f g^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{h\sqrt{c}}-}{}$
default	Expression too large to display

3.191. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*h*x+b*f*h+4*c*e*h-8*c*f*g)/c*(c*x^2+b*x+a)^(1/2)/h^3+1/8/c/h^3*
((4*a*c*f*h^2-b^2*f*h^2+4*b*c*e*h^2-8*b*c*f*g*h+8*c^2*d*h^2-16*c^2*e*g*h+2
4*c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-8*c/h^2
*(a*e*h^3-2*a*f*g*h^2+b*d*h^3-2*b*e*g*h^2+3*b*f*g^2*h-2*c*d*g*h^2+3*c*e*g^
2*h-4*c*f*g^3)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h
^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+8*c*(
a*d*h^4-a*e*g*h^3+a*f*g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-
c*e*g^3*h+c*f*g^4)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h
/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*
g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))
))
```

3.191.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.191.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

3.191.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,(f\,x^2+e\,x+d)}{{(g+h\,x)}^2}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.192 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.192.1 Optimal result

Integrand size = 32, antiderivative size = 448

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$
$$= \frac{\left(\frac{4cg^2(3fg-eh)}{h}+4ah(3fg-eh)-b(11fg^2-3egh-dh^2)-2h\left(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\right)x\right)\sqrt{a}}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$
$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}-\frac{(6cfg-2ceh-bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}h^4}$$
$$+\frac{(8c^2g^3(3fg-eh)-4ch(bg^2(10fg-3eh)-ah(9fg^2-3egh+dh^2))+h^2(8a^2fh^2-4abh(6fg-eh)+}{8h^4\left(cg^2-bgh+ah^2\right)^{3/2}}$$

output
```
-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g
)^2+1/8*(8*c^2*g^3*(-e*h+3*f*g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*
e*g*h+9*f*g^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h
+3*e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2
)/(c*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(3/2)-1/2*(-b*f*h-2*c*e*h+6
*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/c^(1/2)+1/4
*(4*c*g^2*(-e*h+3*f*g)/h+4*a*h*(-e*h+3*f*g)-b*(-d*h^2-3*e*g*h+11*f*g^2)-2*
h*(c*e*g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/(a*
h^2-b*g*h+c*g^2)/(h*x+g)
```

3.192.2 Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$= \frac{8f\sqrt{a+x(b+cx)} + \frac{8(2fg-eh)\sqrt{a+x(b+cx)}}{g+hx} + \frac{2h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2} + \frac{(-b^2+4ac)h(fg^2+h}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
(8*f*Sqrt[a + x*(b + c*x)] + (8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)])/(g +
h*x) + (2*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c
*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 + 4
*a*c)*h*(f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x)
)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-
(b*g) + a*h))^(3/2) - (4*(2*f*g - e*h)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x
+ b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/
Sqrt[c*g^2 + h*(-(b*g) + a*h)]))/h + (4*f*(((-2*c*g + b*h)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*
g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/h)/(8*h^3)
```

3.192.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2181, 27, 1230, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+4cdg-3beg-4afg-bdh+4aeh-2\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^2}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(4cdg-4afg+4aeh-b\left(-\frac{3fg^2}{h}+3eg+dh\right)-2\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^2}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1230$$

$$\frac{-\frac{\int -\frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^2}-\frac{\sqrt{a+bx+c}}{}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^2}-\frac{\sqrt{a+bx+cx^2}\,(}{}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^3}-\frac{\sqrt{a+bx+cx^2}\,(}{}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

3.192. $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

$$\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{4\left(ah^2-bgh+\right.}{}}{2h^3}}{4\left(ah^2\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1092

$$\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{8\left(ah^2-bgh+\right.}{}}{2h^3}}{4}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{4\text{arctanh}}{}}{2h^3}}{4(}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1154

$$\frac{-\frac{2\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int\frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2cg}{cx^2+bx-}}}{h}}{2h^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-\right.}{h\sqrt{ah^2-bgh+cg^2}}}{2h^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

3.192. $\int\frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + (-(((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f
*g - e*h))/h - 4*a*h*(3*f*g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h
- c*d*h - 2*a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(h^2*(g + h*x))) + ((-4*(6*c*
f*g - 2*c*e*h - b*f*h)*(c*g^2 - b*g*h + a*h^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c
*h*(b*g^2*(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2
*f*h^2 - 4*a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTan
h[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*h^3))/(4*(c*g^2 - b*g*
h + a*h^2))
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.192. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.192.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(420) = 840$.

Time = 0.96 (sec) , antiderivative size = 1238, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1238
default	Expression too large to display	2166

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
f/h^3*(c*x^2+b*x+a)^(1/2)+1/2/h^3*((b*f*h+2*c*e*h-6*c*f*g)/h*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(2*a*f*h^2+2*b*e*h^2-6*b*f*g*h+2*c*
d*h^2-6*c*e*g*h+12*c*f*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+1/h*g))+(2*a*e*h^3-4*a*f*g*h^2+2*b*d*h^3-4*b*e*g*h^2+6*b*f*g^2*h-4*c*d
*g*h^2+6*c*e*g^2*h-8*c*f*g^3)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((
x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b
*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h
^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+1/h*g)))+1/h^4*(2*a*d*h^4-2*a*e*g*h^3+2*a*f*g^2*h^2-2*b*d*g*h^3+2*b*e*
g^2*h^2-2*b*f*g^3*h+2*c*d*g^2*h^2-2*c*e*g^3*h+2*c*f*g^4)*(-1/2/(a*h^2-b*g*
h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*
h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2/((a...
```

3.192.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.192.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.192.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(420) = 840$.

Time $= 0.47$ (sec) , antiderivative size $= 2364$, normalized size of antiderivative $= 5.28$

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/4*(24*c^2*f*g^4 - 8*c^2*e*g^3*h - 40*b*c*f*g^3*h + 12*b*c*e*g^2*h^2 + 15
*b^2*f*g^2*h^2 + 36*a*c*f*g^2*h^2 - 3*b^2*e*g*h^3 - 12*a*c*e*g*h^3 - 24*a*
b*f*g*h^3 - b^2*d*h^4 + 4*a*c*d*h^4 + 4*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-(
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a
*h^2))/((c*g^2*h^4 - b*g*h^5 + a*h^6)*sqrt(-c*g^2 + b*g*h - a*h^2)) + sqrt
(c*x^2 + b*x + a)*f/h^3 + 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^
2*f*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^2 - 32*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 8*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*b*c*e*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 2
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*e*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + (sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h
^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 40*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*c^(5/2)*e*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*f*g^4*h + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 + 20
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 + 3*(sqrt(c)...
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.193 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.193.1 Optimal result

Integrand size = 32, antiderivative size = 603

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{(8c^2fg^5-2cgh(7bfg^3-6afg^2h+bdgh^2-2adh^3)+h^2(4a^2eh^3+b^2g(5fg^2+egh+dh^2)-2abh(3fg^2 \cdots}{}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3}+\frac{\sqrt{c}f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^4}$$

$$-\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+dh^2)) \cdots}{16h^4\,(cg^2 \cdots}$$

output
```
-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g
)^3-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*
a^2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^
2*(-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*ar
ctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c
*x^2+b*x+a)^(1/2))*c^(1/2)/h^4-1/8*(8*c^2*f*g^5-2*c*g*h*(-2*a*d*h^3-6*a*f*
g^2*h+b*d*g*h^2+7*b*f*g^3)+h^2*(4*a^2*e*h^3+b^2*g*(d*h^2+e*g*h+5*f*g^2)-2*
a*b*h*(d*h^2+2*e*g*h+3*f*g^2))+h*(4*c^2*(-d*g^2*h^2+3*f*g^4)+h^2*(8*a^2*f*
h^2-2*a*b*h*(-e*h+10*f*g)+b^2*(11*f*g^2-h*(d*h+e*g)))+2*c*g*h*(2*a*h*(-e*h
+6*f*g)-b*(12*f*g^2-h*(2*d*h+e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^3/(a*h^2-b*g
*h+c*g^2)^2/(h*x+g)^2
```

3.193.2 Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

$$= \frac{-\frac{48f\sqrt{a+x(b+cx)}}{g+hx} - \frac{16h^2(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(cg^2+h(-bg+ah))(g+hx)^3} + \frac{12h(-2fg+eh)\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2} - \frac{6(b^2-4ac)h(-2}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
((-48*f*Sqrt[a + x*(b + c*x)])/(g + h*x) - (16*h^2*(f*g^2 + h*(-(e*g) + d*
h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^3) + (1
2*h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) - (6*(b^2 - 4*a*c)*h*(-2*f*g + e
*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a
*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) + (3*h*(2*c
*g - b*h)*(f*g^2 + h*(-(e*g) + d*h))*((2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2
*c*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 +
 4*a*c)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g)
 + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(c*g^
2 + h*(-(b*g) + a*h)) + (24*f*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g -
h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^
2 + h*(-(b*g) + a*h)]))/h)/(48*h^3)
```

3.193.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

$\downarrow$ 2181

$$-\frac{\int -\frac{3\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^3}dx}{3\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^3}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1229

$$-\frac{\int -\frac{16cfx\left(cg^2-bhg+ah^2\right)^2+h\left(\frac{8bc^2fg^4}{h}+8a^2bfh^3+4abch\left(7fg^2-h(eg+dh)\right)+b^3h\left(5fg^2+h(eg+dh)\right)-8ac\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2b^2\left(a(6fg+eh)h^2 \cdots\right.\right.}{2h(g+hx)\sqrt{cx^2+bx+a}}}{4h^2\left(ah^2-bgh+cg^2\right)} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{16cfx\left(cg^2-bhg+ah^2\right)^2+h\left(\frac{8bc^2fg^4}{h}+8a^2bfh^3+4abch\left(7fg^2-h(eg+dh)\right)+b^3h\left(5fg^2+h(eg+dh)\right)-8ac\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2b^2\left(a(6fg+eh)h^2+c\left(\cdots\right.\right.\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{8h^3\left(ah^2-bgh+cg^2\right)} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1269

$$\frac{\frac{16cf\left(ah^2-bgh+cg^2\right)^2\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)+b^2\left(dh^2+egh+ \cdots\right.\right.\right.}{h}}{8h^3\left(ah^2-bgh+cg^2\right)} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1092

3.193. $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

$$\frac{\frac{32cf\left(ah^2-bgh+cg^2\right)^2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)+\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{16\sqrt{c}f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)^2}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{2\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)+b^2\left(dh^2+egh+5fg^2\right)\right)-8c^2gh\left(adh^3-5afg^2h+5bfg^3\right)+1\right.}{h}}{8h^3(ah^2-bgh+cg^2}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{16\sqrt{c}f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)^2}{h}-\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(\right.\right.\right.}{h\sqrt{}}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

3.193. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

output
```
-1/3*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^3) + (-1/4*(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(
3*f*g^2 + d*h^2) + b^2*g*h*(5*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2
 + 2*e*g*h + d*h^2) + c*(7*f*g^4 + d*g^2*h^2)) + (8*f*(c*g^2 - h*(b*g - a*
h))^2 + (2*c*g - b*h)*(2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d
*h) - 2*a*h*(2*f*g - e*h))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h
 + a*h^2)*(g + h*x)^2) + ((16*Sqrt[c]*f*(c*g^2 - b*g*h + a*h^2)^2*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h - ((16*c^3*f*g^5 - 8*c^2
*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b*h*(6
*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*f*g -
 e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))*Ar
cTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2]])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(8*h^3*(c*g^2 - b*g*h
 + a*h^2)))/(2*(c*g^2 - b*g*h + a*h^2))
```

3.193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.193. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.193.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3089 vs. $2(577) = 1154$.

Time = 1.15 (sec) , antiderivative size = 3090, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	3090

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
f/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g
^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2
*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-
b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)
/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+2*
c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^
2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2
-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/
h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2))))+(e*h-2*f*g)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x
+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-
2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h
*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/...
```

3.193.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas
")
```

output
```
Timed out
```

3.193.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.193.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6846 vs. $2(577) = 1154$.

Time = 4.82 (sec) , antiderivative size = 6846, normalized size of antiderivative = 11.35

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

output
```
-1/8*(16*c^3*f*g^5 - 40*b*c^2*f*g^4*h + 30*b^2*c*f*g^3*h^2 + 40*a*c^2*f*g^
3*h^2 - 5*b^3*f*g^2*h^3 - 60*a*b*c*f*g^2*h^3 + 2*b^2*c*d*g*h^4 - 8*a*c^2*d
*g*h^4 - b^3*e*g*h^4 + 4*a*b*c*e*g*h^4 + 12*a*b^2*f*g*h^4 + 32*a^2*c*f*g*h
^4 - b^3*d*h^5 + 4*a*b*c*d*h^5 + 2*a*b^2*e*h^5 - 8*a^2*c*e*h^5 - 8*a^2*b*f
*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 + b*g*h - a*h^2))/((c^2*g^4*h^4 - 2*b*c*g^3*h^5 + b^2*g^2*h^6 + 2*a*c*
g^2*h^6 - 2*a*b*g*h^7 + a^2*h^8)*sqrt(-c*g^2 + b*g*h - a*h^2)) - sqrt(c)*f
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/h^4 - 1/24*(
144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*f*g^5*h^2 - 48*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*c^3*e*g^4*h^3 - 312*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b*c^2*f*g^4*h^3 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*
e*g^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*f*g^3*h^4 + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 48*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^2*c*e*g^2*h^5 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a*c^2*e*g^2*h^5 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*f
*g^2*h^5 - 300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 - 6*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*c^2*d*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^3*e*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e*g*h^6 + 6
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*f*g*h^6 + 96*(sqrt(c)*x -...
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.194 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5} dx$

3.194.1 Optimal result

Integrand size = 32, antiderivative size = 497

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$
$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)+b^2(5fg^2+3egh+5dh^2)-4c(2bg(eg+2dh)+a(fg^2-5egh+}{64\,(cg^2-bgh+ah^2)^3\,(g+hx)^2}$$
$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{4h\,(cg^2-bgh+ah^2)\,(g+hx)^4}$$
$$+\frac{(2cg(3fg^2+h(eg-5dh))+h(8ah(2fg-eh)-b(11fg^2-3egh-5dh^2)))\,(a+bx+cx^2)^{3/2}}{24h\,(cg^2-bgh+ah^2)^2\,(g+hx)^3}$$
$$-\frac{(b^2-4ac)\,(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)+b^2(5fg^2+3egh+5dh^2)-4c(2bg(eg+2dh)+a(f}{128\,(cg^2-bgh+ah^2)^{7/2}}$$

output
```
-1/4*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g
)^4+1/24*(2*c*g*(3*f*g^2+h*(-5*d*h+e*g))+h*(8*a*h*(-e*h+2*f*g)-b*(-5*d*h^2
-3*e*g*h+11*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3
-1/128*(-4*a*c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)+b^2*(5*
d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*h+f*g^2)))*ar
ctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*
h*(e*h+2*f*g)+b^2*(5*d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^
2-5*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b
*g*h+c*g^2)^3/(h*x+g)^2
```

3.194.2 Mathematica [A] (verified)

Time = 13.35 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$$= \frac{-48h(cg^2+h(-bg+ah))^{5/2}\left(fg^2+h(-eg+dh)\right)(a+x(b+cx))^{3/2}+64h(2fg-eh)\left(cg^2+h(-bg+a\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
(-48*h*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(f*g^2 + h*(-(e*g) + d*h))*(a + x*
(b + c*x))^(3/2) + 64*h*(2*f*g - e*h)*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g
+ h*x)*(a + x*(b + c*x))^(3/2) + 48*f*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g
+ h*x)^2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) - (f*g^2 +
 h*(-(e*g) + d*h))*(g + h*x)*(40*h*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h)
)^(3/2)*(a + x*(b + c*x))^(3/2) + 3*(8*c^2*g^2 + (5*b^2*h^2)/2 - 2*c*h*(4*
b*g + a*h))*(g + h*x)*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c
*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) - (b^2 - 4*a*c)*(g + h*x)^2*ArcTanh[
(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[
a + x*(b + c*x)])])) - 24*(b^2 - 4*a*c)*f*(c*g^2 + h*(-(b*g) + a*h))^2*(g
+ h*x)^4*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*Sqrt[a + x*(b + c*x)])] + 12*(2*c*g - b*h)*(2*f*g - e*h)*(c*g^2
+ h*(-(b*g) + a*h))*(g + h*x)^2*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a
+ x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) + (b^2 - 4*a*c)*(g + h*x)^
2*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h
)]*Sqrt[a + x*(b + c*x)])]))/(192*h^2*(c*g^2 + h*(-(b*g) + a*h))^(7/2)*(g
+ h*x)^4)
```

3.194.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2181, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+8cdg-3beg-8afg-5bdh+8aeh+2\left(\frac{3cfg^2}{h}+ceg-4bfg-cdh+4afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^4}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(8cdg-8afg+8aeh-b\left(-\frac{3fg^2}{h}+3eg+5dh\right)-2\left(4bfg-4afh-c\left(\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^4}dx}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\int \frac{\sqrt{cx^2+bx+a}}{(g+hx)^3}dx}{2(ah^2-bgh+cg^2)}+\frac{\left(a+bx+cx^2\right)^{3/2}(8a\cdots}{\cdots}}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1152$$

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-\cdots\right.}{\cdots}\cdots\right)}{2(ah^2-bgh+cg^2)}\cdots}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+\cdots}{cx^2\cdots}}\cdots}{4\left(ah^2-bg\cdots\right.}\cdots\right)}{2(ah^2-bgh+cg^2)}\cdots}{8\left(a\cdots\right.}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

3.194. $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

$$\Big\downarrow 219$$

$$\frac{\frac{\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)\left(16a^2fh^2-4c(-ah(5eg-dh)+afg^2+2bg(2dh+eg))-8a\right.}{2(ah^2-bgh+cg^2)}}{8\left(ah^2-bgh\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^4) + (((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d*h) + 8*a*h^2*(2*f
*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x + c*x^2)^(3/2))/(
3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((16*c^2*d*g^2 + 16*a^2*f*h^2 -
 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(5*e*g - d*h) + 2*b*g*(e*g + 2
*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(((b*g - 2*a*h + (2*c*g - b*h)
*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2
 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(2*(c*
g^2 - b*g*h + a*h^2)))/(8*(c*g^2 - b*g*h + a*h^2))
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4939 vs. $2(475) = 950$.

Time = 1.40 (sec) , antiderivative size = 4940, normalized size of antiderivative = 9.94

method	result	size
default	Expression too large to display	4940

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h
+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^
2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+
2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1/4*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h
^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+
1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)
/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2
*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a...
```

3.194.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas
")
```

output
```
Timed out
```

3.194.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.194.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.194.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{cx^2+bx+a}(fx^2+ex+d)}{(hx+g)^5}\,dx$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.195 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.195.1 Optimal result

Integrand size = 32, antiderivative size = 824

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$= \frac{(32c^3dg^3-8c^2g(2bg(eg+3dh)+a(fg^2-6egh+3dh^2))-bh(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+\cdots}{\cdots}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5}$$

$$+\frac{(2cg(3fg^2+h(2eg-7dh))+h(10ah(2fg-eh)-b(13fg^2-3egh-7dh^2)))\,(a+bx+cx^2)^{3/2}}{40h\,(cg^2-bgh+ah^2)^2\,(g+hx)^4}$$

$$+\frac{(4c^2g^2(3fg^2+h(2eg-27dh))-5h^2(16a^2fh^2-2abh(6fg+5eh)+b^2(3fg^2+3egh+7dh^2))-2ch(bg\cdots}{240h\,(cg^2-bgh+ah^2)^3\,(g+\cdots}$$

$$-\frac{(b^2-4ac)\,(32c^3dg^3-8c^2g(2bg(eg+3dh)+a(fg^2-6egh+3dh^2))-bh(16a^2fh^2-2abh(6fg+5eh)\cdots}{\cdots}$$

output
```
-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g
)^5+1/40*(2*c*g*(3*f*g^2+h*(-7*d*h+2*e*g))+h*(10*a*h*(-e*h+2*f*g)-b*(-7*d*
h^2-3*e*g*h+13*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g
)^4+1/240*(4*c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-5*h^2*(16*a^2*f*h^2-2*a*b
*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*c*h*(b*g*(-54*d*h^2-21*e
*g*h+16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(
a*h^2-b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*(32*c^3*d*g^3-8*c^2*g*(2
*b*g*(3*d*h+e*g)+a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e
*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b
*h*(-d*h^2+3*e*g*h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*arctanh(1/2
*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
/(a*h^2-b*g*h+c*g^2)^(9/2)+1/128*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3*d*h+e*g)+
a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*
d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^2+3*e*g*
h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(
c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

3.195. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.195.2 Mathematica [A] (verified)

Time = 16.24 (sec) , antiderivative size = 1334, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{5h\,(cg^2-h(bg-ah))\,(g+hx)^5} + \frac{(2fg-eh)\,(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{4h\,(cg^2-h(bg-ah))\,(g+hx)^4} - \frac{f(a+bx+cx^2)\,\sqrt{a+x(b+cx)}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3}$$

$$- \frac{(-2fg+eh)\sqrt{a+x(b+cx)}\left(\frac{\left(cgh-\frac{1}{2}h(-8cg+5bh)\right)(a+bx+cx^2)^{3/2}}{3(cg^2-bgh+ah^2)(g+hx)^3} - \frac{\left(-2\left(ach^2+\frac{1}{2}cg(-8cg+5bh)\right)+b\left(cgh+\frac{1}{2}h(-8cg+5bh)\right)\right.}{\cdots}\right.}{4h^2\,(cg^2-bgh+ah^2)\,\sqrt{a+bx+cx^2}\cdots}$$

$$- \frac{(fg^2-egh+dh^2)\,\sqrt{a+x(b+cx)}\left(-\frac{\left(-2cgh+\frac{1}{2}h(-10cg+7bh)\right)(a+bx+cx^2)^{3/2}}{4(cg^2-bgh+ah^2)(g+hx)^4} - \frac{\left(-\frac{7}{2}cgh(2cg-bh)-\frac{1}{4}h\left(80c^2g^2+35b^2h^2-2c\cdots\right.\right.}{3\left(cg^2-bgh+ah^2\right)\cdots}\right.}{\cdots}$$

$$+ \frac{f(2cg-bh)\sqrt{a+x(b+cx)}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2} - \frac{(b^2-4ac)\,\mathrm{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+bx+cx^2}}\right)}{(cg^2-h(bg-ah))^{3/2}}\right)}{16h^2\,(cg^2-h(bg-ah))\,\sqrt{a+bx+cx^2}}$$

input

```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(h*
(c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*f*g - e*h)*(a + b*x + c*x^2)*Sq
rt[a + x*(b + c*x)])/(4*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^4) - (f*(a + b
*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
3) - ((-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(((c*g*h - (h*(-8*c*g + 5*b*h))
/2)*(a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-
2*(a*c*h^2 + (c*g*(-8*c*g + 5*b*h))/2) + b*(c*g*h + (h*(-8*c*g + 5*b*h))/2
))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*
g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*
g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(2*Sqr
t[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*
h + a*h^2))))/(4*h^2*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) - ((f*
g^2 - e*g*h + d*h^2)*Sqrt[a + x*(b + c*x)]*(-1/4*((-2*c*g*h + (h*(-10*c*g
+ 7*b*h))/2)*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4
) - ((((-7*c*g*h*(2*c*g - b*h))/2 - (h*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(4
7*b*g + 16*a*h)))/4)*(a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(
g + h*x)^3) - ((-2*((-7*a*c*h^2*(2*c*g - b*h))/2 + (c*g*(80*c^2*g^2 + 35*b
^2*h^2 - 2*c*h*(47*b*g + 16*a*h)))/4) + b*((-7*c*g*h*(2*c*g - b*h))/2 + (h
*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(47*b*g + 16*a*h)))/4))*(((b*g - 2*a*h +
 (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g ...
```

3.195.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 730, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2181, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+10cdg-3beg-10afg-7bdh+10aeh+2\left(\frac{3cfg^2}{h}+2ceg-5bfg-2cdh+5afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^5}dx}{5\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(10cdg-b\left(-\frac{3fg^2}{h}+3eg+7dh\right)-10a(fg-eh)-2\left(5bfg-5afh-c\left(\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^5}dx}{10\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1237

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}\left(10ah^2(2fg-eh)-bh\left(13fg^2-h(7dh+3eg)\right)+2cgh(2eg-7dh)+6cfg^3\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\frac{\int -\frac{\left(h\left(5\left(3fg^2+h(3eg+7dh)\right)b^2-2cg\left(-\frac{3fg^2}{h}+18eg+47dh\right)\right.}{}}{}}{10\left(ah^2-b\right.}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(h\left(5\left(3fg^2+h(3eg+7dh)\right)b^2-2cg\left(-\frac{3fg^2}{h}+18eg+47dh\right)b-10ah(6fg+5eh)b+80c^2dg^2+80a^2fh^2-16ac\left(2fg^2-h(7eg-2dh)\right)\right)+2c\left(6cfg^3+2ch(2eg-7dh)g+10a\right.}{(g+hx)^4}}{8h\left(ah^2-bgh+cg^2\right)}}{10\left(ah^2-b\right.}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1228

$$\frac{\frac{5h\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh+2eg)+5fg^3\right)\right)-bh\left(16a^2fh^2-2abh(5eh+6fg)+b^2\left(h(7dh+3eg)+3fg^2\right)\right)-8c^2g\left(-3ah(2eg-dh)+a\right.\right.}{2\left(ah^2-bgh+cg^2\right)}}{}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1152

$$\frac{\frac{5h\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh+2eg)+5fg^3\right)\right)-bh\left(16a^2fh^2-2abh(5eh+6fg)+b^2\left(h(7dh+3eg)+3fg^2\right)\right)-8c^2g\left(-3ah(2eg-dh)+a\right.\right.}{}}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1154

3.195. $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

$$5h\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh+2eg)+5fg^3\right)\right)-bh\left(16a^2fh^2-2abh(5eh+6fg)+b^2\left(h(7dh+3eg)+3fg^2\right)\right)-8c^2g\left(-3ah(2eg-dh)+a\right.$$

$$2\left(ah^2-bgh\right.$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{219}$$

$$5h\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh\right.\right.\right.$$

$$2\left(ah^2-bgh+cg^2\right)$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^5) + (((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(
2*f*g - e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2)
)/(4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + (((4*c^2*(3*f*g^4 + g^2*h*(2
*e*g - 27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f
*g^2 + 3*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) -
 2*a*h*(18*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(3*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)^3) + (5*h*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 -
3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h)
- 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*
h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3*e
*g + 7*d*h))))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4
*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*
h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])
])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(8*h*(
c*g^2 - b*g*h + a*h^2)))/(10*(c*g^2 - b*g*h + a*h^2))
```

3.195. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.195. $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.195.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7713 vs. 2(798) = 1596.

Time = 1.55 (sec) , antiderivative size = 7714, normalized size of antiderivative = 9.36

method	result	size
default	Expression too large to display	7714

input
```
int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.195.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas
")
```

output
```
Timed out
```

3.195.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$$

input
```
integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

3.195.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28577 vs. $2(798) = 1596$.

Time = 1.87 (sec) , antiderivative size = 28577, normalized size of antiderivative = 34.68

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

output
```
-1/128*(32*b^2*c^3*d*g^3 - 128*a*c^4*d*g^3 - 16*b^3*c^2*e*g^3 + 64*a*b*c^3
*e*g^3 + 10*b^4*c*f*g^3 - 48*a*b^2*c^2*f*g^3 + 32*a^2*c^3*f*g^3 - 48*b^3*c
^2*d*g^2*h + 192*a*b*c^3*d*g^2*h + 12*b^4*c*e*g^2*h - 192*a^2*c^3*e*g^2*h
- 3*b^5*f*g^2*h - 24*a*b^3*c*f*g^2*h + 144*a^2*b*c^2*f*g^2*h + 30*b^4*c*d*
g*h^2 - 144*a*b^2*c^2*d*g*h^2 + 96*a^2*c^3*d*g*h^2 - 3*b^5*e*g*h^2 - 24*a*
b^3*c*e*g*h^2 + 144*a^2*b*c^2*e*g*h^2 + 12*a*b^4*f*g*h^2 - 192*a^3*c^2*f*g
*h^2 - 7*b^5*d*h^3 + 40*a*b^3*c*d*h^3 - 48*a^2*b*c^2*d*h^3 + 10*a*b^4*e*h^
3 - 48*a^2*b^2*c*e*h^3 + 32*a^3*c^2*e*h^3 - 16*a^2*b^3*f*h^3 + 64*a^3*b*c*
f*h^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c
*g^2 + b*g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a
*c^3*g^6*h^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b
^2*c*g^4*h^4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 +
6*a^2*b^2*g^2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2
 + b*g*h - a*h^2)) + 1/1920*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2
*c^3*d*g^3*h^8 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^4*d*g^3*h^
8 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*e*g^3*h^8 + 960*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*e*g^3*h^8 + 150*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*b^4*c*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^9*a*b^2*c^2*f*g^3*h^8 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^
2*c^3*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*d*g...
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.196.1 Optimal result

Integrand size = 32, antiderivative size = 1169

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx =$$

$$-\frac{(b^2-4ac)\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)\right.}{}$$

$$+\frac{(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)+6b^2(3fg^2}{}$$

$$+\frac{(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2(5fg^2-3h(3eg+8dh)))(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{2016c^3h}$$

$$-\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$+\frac{(3003b^4fh^4-192c^4g^2(5fg^2-3h(3eg+64dh))-198b^2ch^3(38afh+21b(3fg+eh))+8c^2h^2(256a^2fh^2+}{}$$

$$+\frac{(b^2-4ac)^2\left(1536c^5dg^3-143b^5fh^3+22b^3ch^2(20afh+9b(3fg+eh))-48bc^2h(5a^2fh^2+9abh(3fg+eh)\right.}{}$$

output
```
1/12288*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*
g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g
^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h
^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e
*g))))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h^2-2*c*h*(64*a
*f*h+99*b*e*h+24*b*f*g)-12*c^2*(5*f*g^2-3*h*(8*d*h+3*e*g)))*(h*x+g)^2*(c*x
^2+b*x+a)^(5/2)/c^3/h-1/144*(13*b*f*h-18*c*e*h+10*c*f*g)*(h*x+g)^3*(c*x^2+
b*x+a)^(5/2)/c^2/h+1/9*f*(h*x+g)^4*(c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b
^4*f*h^4-192*c^4*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-198*b^2*c*h^3*(38*a*f*h+
21*b*(e*h+3*f*g))+8*c^2*h^2*(256*a^2*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553
*f*g^2+756*h*(d*h+3*e*g)))-16*c^3*h*(32*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g
*(13*f*g^2+9*h*(196*d*h+141*e*g)))-10*c*h*(429*b^3*f*h^3-22*b*c*h^2*(34*a*
f*h+27*b*e*h+29*b*f*g)+16*c^3*g*(5*f*g^2-9*h*(12*d*h+e*g))+8*c^2*h*(a*h*(6
3*e*h+61*f*g)+3*b*(f*g^2+6*h*(6*d*h+7*e*g))))*x)*(c*x^2+b*x+a)^(5/2)/c^5/h
+1/65536*(-4*a*c+b^2)^2*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f
*h+9*b*(e*h+3*f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h
^2+3*e*g*h+3*f*g^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))
+32*c^3*(3*a^2*h^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*
f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/
c^(15/2)-1/32768*(-4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^...
```

3.196.2 Mathematica [A] (verified)

Time = 13.65 (sec) , antiderivative size = 1683, normalized size of antiderivative = 1.44

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(72*h*(f*g^2 + h*(-(e*g) + d*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + 63*
h*(-2*f*g + e*h)*(g + h*x)^3*(a + x*(b + c*x))^(5/2) + 56*f*h*(g + h*x)^4*
(a + x*(b + c*x))^(5/2) - (f*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-45045*b^8*
h^5 + 2310*b^7*c*h^4*(135*g + 13*h*x) - 84*b^6*c*h^3*(-5225*a*h^2 + c*(108
00*g^2 + 2475*g*h*x + 286*h^2*x^2)) + 72*b^5*c^2*h^2*(-7*a*h^2*(5475*g + 5
17*h*x) + 2*c*(9800*g^3 + 4200*g^2*h*x + 1155*g*h^2*x^2 + 143*h^3*x^3)) -
16*b^4*c^2*h*(86499*a^2*h^4 - 9*a*c*h^2*(50400*g^2 + 11235*g*h*x + 1276*h^
2*x^2) + 2*c^2*(37800*g^4 + 29400*g^3*h*x + 15120*g^2*h^2*x^2 + 4455*g*h^3
*x^3 + 572*h^4*x^4)) - 128*b*c^4*(a^3*h^4*(41355*g + 3701*h*x) - 6*a^2*c*h
^2*(22680*g^3 + 8760*g^2*h*x + 2265*g*h^2*x^2 + 269*h^3*x^3) + 40*a*c^2*(6
30*g^5 + 434*g^4*h*x - 1036*g^3*h^2*x^2 - 2292*g^2*h^3*x^3 - 1675*g*h^4*x^
4 - 433*h^5*x^5) + 80*c^3*x^2*(378*g^5 + 1162*g^4*h*x + 1288*g^3*h^2*x^2 +
 456*g^2*h^3*x^3 - 131*g*h^4*x^4 - 91*h^5*x^5)) - 256*c^4*(1024*a^4*h^5 -
a^3*c*h^3*(23040*g^2 + 4725*g*h*x + 512*h^2*x^2) + 80*c^4*g*x^3*(126*g^4 +
 448*g^3*h*x + 616*g^2*h^2*x^2 + 384*g*h^3*x^3 + 91*h^4*x^4) - 2*a^2*c^2*h
*(-17920*g^4 - 3640*g^3*h*x + 7680*g^2*h^2*x^2 + 7385*g*h^3*x^3 + 2048*h^4
*x^4) + 40*a*c^3*x*(630*g^5 + 1792*g^4*h*x + 2044*g^3*h^2*x^2 + 960*g^2*h^
3*x^3 + 49*g*h^4*x^4 - 64*h^5*x^5)) + 32*b^3*c^3*(9*a^2*h^4*(25515*g + 235
3*h*x) - 4*a*c*h^2*(79800*g^3 + 32760*g^2*h*x + 8775*g*h^2*x^2 + 1067*h^3*
x^3) + 40*c^2*(378*g^5 + 630*g^4*h*x + 588*g^3*h^2*x^2 + 324*g^2*h^3*x^...
```

3.196.3 Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 801, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^3(5bfg-18cdh+8afh+(10cfg-18ceh+13bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{9ch^2}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\int(g+hx)^3(5bfg-18cdh+8afh+(10cfg-18ceh+13bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{18ch}$$

$\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{\int-\frac{1}{2}(g+hx)^2\left(65fghb^2+78afh^2b-30cg(fg+3eh)b+4ch(72cdg-17afg-27aeh)+\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+143\right.\right.}{8c}}{18ch}$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\int(g+hx)^2\left(65fghb^2+78afh^2b-30cg(fg+3eh)b+4ch(72cdg-17afg-27aeh)+\left(-12\left(5fg^2-3h\right.\right.\right.}{16c}}{18ch}$$

$\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\int-\frac{1}{2}(g+hx)\left(715fgh^2b^3+2\left(286afh^3-5cgh(115fg+99eh)\right)b^2-4c\left(ah^2(481fg+198eh)-30cg\left(fg^2+3h(5\right.\right.\right.}{}}{}$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{\int(g+}{}}{}}{}$$

$\downarrow$ 1225

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{-\frac{21}{}}{}}{}}{}$$

$\downarrow$ 1087

3.196. $\int(g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-$$

21

$\downarrow$ 1087

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-$$

21

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\left(cx^2+bx+a\right)^{5/2}}{9ch}-$$

$$\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(cx^2+bx+a\right)^{5/2}}{8c}-\frac{\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+143b^2fh^2\right)(g+hx)^2\left(cx^2+bx+a\right)^{5/2}}{7c}-$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\left(cx^2+bx+a\right)^{5/2}}{9ch}-$$

$$\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(cx^2+bx+a\right)^{5/2}}{8c}-\frac{\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+143b^2fh^2\right)(g+hx)^2\left(cx^2+bx+a\right)^{5/2}}{7c}-$$

input
```
Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

3.196. $\int (g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

output
```
(f*(g + h*x)^4*(a + b*x + c*x^2)^(5/2))/(9*c*h) - (((10*c*f*g - 18*c*e*h +
 13*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(8*c) - (((143*b^2*f*h^2 -
 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8
*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) - (-1/20*((3003*b^4*f*h
^4 - 192*c^4*(5*f*g^4 - 3*g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*
h + 21*b*(3*f*g + e*h)) + 8*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*
h) + b^2*(1553*f*g^2 + 756*h*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2
+ 9*h*(3*e*g + d*h)) + b*g*(13*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*
(429*b^3*f*h^3 - 22*b*c*h^2*(29*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f
*g^3 - 9*g*h*(e*g + 12*d*h)) + 8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2
 + 6*h*(7*e*g + 6*d*h))))*x)*(a + b*x + c*x^2)^(5/2))/c^2 - (21*h*(1536*c^
5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(
e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(
5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2)
)/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((
b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^
(3/2))))/(16*c)))/(8*c^2))/(14*c))/(16*c))/(18*c*h)
```

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.196.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2877 vs. 2(1135) = 2270.

Time = 1.00 (sec) , antiderivative size = 2878, normalized size of antiderivative = 2.46

method	result	size
default	Expression too large to display	2878
risch	Expression too large to display	3146

input
```
int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^3*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+
b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))+f*h^3*(1/9*x^4*(c*x^2+b*x+a)^(5/2)/c-13/18*b/c*(1/8*x
^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c
*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b
^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c
-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c
*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))))))-3/8*a/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16
*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*
(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2
)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-
4/9*a/c*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)
/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+...
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. $2(1135) = 2270$.

Time = 2.75 (sec) , antiderivative size = 4751, normalized size of antiderivative = 4.06

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
[-1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2
*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c
^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*
(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(3
2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*
c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*
a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 -
(96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b
^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*
e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a
^4*b*c^4)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*c^9*f*h^3*x^8 + 71680*(
54*c^9*f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h
+ 54*(16*c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7
 + 320*a*c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^
8*f)*g^2*h + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2
 + (1440*b*c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f
)*h^3)*x^5 + 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*
b*c^8*e + 3*(b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*...
```

3.196.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19122 vs. 2(1221) = 2442.

Time = 1.54 (sec) , antiderivative size = 19122, normalized size of antiderivative = 16.36

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**3*x**8/9 + x**7*(19*b*c*f*h**3/1
8 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + x**6*(10*a*c*f*h**3/9 + b**2*f*
h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**
3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**
2*h)/(7*c) + x**5*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*
b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**
2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c
*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h
**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*
h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*
f*g**3)/(6*c) + x**4*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*
d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h
**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3
 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2
*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h*
*2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6
*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*
c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c
*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g
*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) ...
```

3.196.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2902 vs. $2(1135) = 2270$.

Time = 0.33 (sec) , antiderivative size = 2902, normalized size of antiderivative = 2.48

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (5
4*c^9*f*g*h^2 + 18*c^9*e*h^3 + 19*b*c^8*f*h^3)/c^8)*x + (864*c^9*f*g^2*h +
 864*c^9*e*g*h^2 + 918*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 306*b*c^8*e*h^3 + 3
*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3)/c^8)*x + (1344*c^9*f*g^3 + 4032*c^9*e*g^
2*h + 4320*b*c^8*f*g^2*h + 4032*c^9*d*g*h^2 + 4320*b*c^8*e*g*h^2 + 54*b^2*
c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8*d*h^3 + 18*b^2*c^7*e*h^3 + 1
512*a*c^8*e*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3)/c^8)*x + (16128*c^9
*e*g^3 + 17472*b*c^8*f*g^3 + 48384*c^9*d*g^2*h + 52416*b*c^8*e*g^2*h + 864
*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b*c^8*d*g*h^2 + 864*b^2*c^7
*e*g*h^2 + 55296*a*c^8*e*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^
2 + 288*b^2*c^7*d*h^3 + 18432*a*c^8*d*h^3 - 198*b^3*c^6*e*h^3 + 936*a*b*c^
7*e*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*c^7*f*h^3)/c^8
)*x + (161280*c^9*d*g^3 + 177408*b*c^8*e*g^3 + 4032*b^2*c^7*f*g^3 + 188160
*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*h + 12096*b^2*c^7*e*g^2*h + 564480*a*c^8
*e*g^2*h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*
g*h^2 + 564480*a*c^8*d*g*h^2 - 7776*b^3*c^6*e*g*h^2 + 38016*a*b*c^7*e*g*h^
2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 30240*a^2*c^7*f*g*h^2
 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 + 1782*b^4*c^5*e*h^3 - 10224*a
*b^2*c^6*e*h^3 + 10080*a^2*c^7*e*h^3 - 1287*b^5*c^4*f*h^3 + 8536*a*b^3*c^5
*f*h^3 - 12912*a^2*b*c^6*f*h^3)/c^8)*x + (483840*b*c^8*d*g^3 + 16128*b^...
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \int (g+h\,x)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

input
```
int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.196. $\int (g+hx)^3 (a+bx+cx^2)^{3/2} (d+ex+fx^2)\,dx$

3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$

3.197.1 Optimal result

Integrand size = 32, antiderivative size = 753

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx =$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+\dots\right.}{16384c^6}$$

$$+\frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+dh^2))+16\dots\right.}{6144c^5}$$

$$-\frac{(10cfg-16ceh+11bfh)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2h}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{\left(693b^3fh^3+96c^3g(5fg^2-8h(eg+7dh))-36bch^2(31afh+28b(2fg+eh))+8c^2h(96ah(2fg+eh)+b\dots\right.}{}$$

$$+\frac{(b^2-4ac)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(3bg(eg+2dh)+a(fg^2+2egh+\dots\right.}{32768c^{13/2}}$$

output
```
1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-12
8*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a
*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3
/2)/c^5-1/112*(11*b*f*h-16*c*e*h+10*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c
^2/h+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(693*b^3*f*h^3+96*c^3
*g*(5*f*g^2-8*h*(7*d*h+e*g))-36*b*c*h^2*(31*a*f*h+28*b*(e*h+2*f*g))+8*c^2*
h*(96*a*h*(e*h+2*f*g)+b*(31*f*g^2+196*h*(d*h+2*e*g)))-10*c*h*(99*b^2*f*h^2
-8*c^2*(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e*h+f*g)))*x)*(c
*x^2+b*x+a)^(5/2)/c^4/h+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2+99*b^4*f*h^2
-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+
2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e
*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)-
1/16384*(-4*a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e
*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*
a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c
*x^2+b*x+a)^(1/2)/c^6
```

3.197.2 Mathematica [A] (verified)

Time = 11.44 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.16

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{430080dg^2(b+2cx)(a+x(b+cx))^{3/2}+688128g(eg+2dh)(a+x(b+cx))^{5/2}+573440(fg^2+\cdots}{\cdots}$$

input
```
Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(430080*d*g^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 688128*g*(e*g + 2*d*h)
*(a + x*(b + c*x))^(5/2) + 573440*(f*g^2 + h*(2*e*g + d*h))*x*(a + x*(b +
c*x))^(5/2) + 491520*h*(2*f*g + e*h)*x^2*(a + x*(b + c*x))^(5/2) + 430080*
f*h^2*x^3*(a + x*(b + c*x))^(5/2) + (80640*(b^2 - 4*a*c)*d*g^2*(-2*Sqrt[c]
*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (13440*b*g*(e*g + 2*d*h)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
 + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (48*h*(2*f*g + e*h)*(-256*c^(5/2
)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*
a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*
Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(f*g^2 + h*(2*e*
g + d*h))*(1792*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*
c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(
b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(7/2) - (3*f*h^2*(112640*b*c^(9/2)*x^2
*(a + x*(b + c*x))^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x
+ 280*a*c^2*x)*(a + x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*
c^2)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*...
```

3.197.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.70, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^2(5bfg-16cdh+6afh+(10cfg-16ceh+11bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{8ch^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\int (g+hx)^2(5bfg-16cdh+6afh+(10cfg-16ceh+11bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{16ch}$$

$\downarrow$ 1236

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\int -\frac{1}{2}(g+hx)\left(55fghb^2+44afh^2b-20cg(fg+4eh)b+4ch(56cdg-11afg-16aeh)+\left(-8\left(5fg^2-4h(2eg+7dh)\right)c^2-12h(7afh+2b(fg+6eh))c+99b^2fh\right.\right.$$

$$7c \qquad 16ch$$

$\downarrow$ 27

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\int (g+hx)\left(55fghb^2+44afh^2b-20cg(fg+4eh)b+4ch(56cdg-11afg-16aeh)+\left(-8\left(5fg^2-4h(2\right.\right.\right.$$

$$14c \qquad 16ch$$

$\downarrow$ 1225

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(dh\right.\right.$$

$$24c^2$$

$\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(dh\right.\right.$$

$\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(dh\right.\right.$$

$\downarrow$ 1092

3.197. $\int (g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(dh\right.\right.}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{7h\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{8c^{3/2}}\right.}{16c}\right.}{}$$

input
```
Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(8*c*h) - (((10*c*f*g - 16*c*e*h +
 11*b*f*h)*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) - (-1/60*((693*b^3*f
*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36*b*c*h^2*(31*a*f*h + 28*
b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2*e*g + d*h) + 96*a*h*(2
*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2 - 4*h*(2*e*g + 7*d*h)
) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x + c*x^2)^(5/2))/c^2
+ (7*h*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a
*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c
^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h)
)))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b +
 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24*c^2))/(14*c
))/(16*c*h)
```

3.197. $\int(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

3.197.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. 2(723) = 1446.

Time = 0.85 (sec) , antiderivative size = 1680, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	1680
risch	Expression too large to display	1933

input
```
int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+
b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))+f*h^2*(1/8*x^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/7*x
^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16
*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*
(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2
)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2
/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(
3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^
2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-3/8*a/c*(1/6*x*
(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*
c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*
x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1
/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))))))+(e*h^2+2*f*g*h)*(1/7*x^2*(c*x^2+b*x+a)^(5
/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5
/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(...
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(723) = 1446$.

Time = 1.50 (sec) , antiderivative size = 3145, normalized size of antiderivative = 4.18

$$\int (g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
[1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b
^2*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5
)*d - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*(3
*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6
*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a
*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 11
20*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(c)*log(-8*c^
2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*
c) + 4*(215040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*
f)*h^2)*x^6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*
c^8*d + 240*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^
8*e + 13*b*c^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^
7)*f)*g*h + (2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52
*a*b*c^6)*f)*h^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 14
0*a*c^7)*f)*g^2 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b
^3*c^5 - 44*a*b*c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c
^5 - 44*a*b*c^6)*e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*
x^3 - 224*(120*(3*b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5
 + 128*a^2*c^6)*e + (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g...
```

3.197.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9687 vs. 2(790) = 1580.

Time = 1.33 (sec) , antiderivative size = 9687, normalized size of antiderivative = 12.86

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**2*x**7/8 + x**6*(17*b*c*f*h**2/1
6 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + x**5*(9*a*c*f*h**2/8 + b**2*f*h**2
 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c
**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + x**4
*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2
*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2
+ 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*
e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h
)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h
 + c**2*e*g**2)/(5*c) + x**3*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2
*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h*
*2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2
*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b*
*2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*
(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*
e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 +
 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e
*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)
/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h
+ c**2*e*g**2)/(10*c) + c**2*d*g**2)/(4*c) + x**2*(a**2*e*h**2 + 2*a**2...
```

3.197.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(723) = 1446.

Time = 0.32 (sec) , antiderivative size = 1802, normalized size of antiderivative = 2.39

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^
8*f*g*h + 16*c^8*e*h^2 + 17*b*c^7*f*h^2)/c^7)*x + (224*c^8*f*g^2 + 448*c^8
*e*g*h + 480*b*c^7*f*g*h + 224*c^8*d*h^2 + 240*b*c^7*e*h^2 + 3*b^2*c^6*f*h
^2 + 252*a*c^7*f*h^2)/c^7)*x + (2688*c^8*e*g^2 + 2912*b*c^7*f*g^2 + 5376*c
^8*d*g*h + 5824*b*c^7*e*g*h + 96*b^2*c^6*f*g*h + 6144*a*c^7*f*g*h + 2912*b
*c^7*d*h^2 + 48*b^2*c^6*e*h^2 + 3072*a*c^7*e*h^2 - 33*b^3*c^5*f*h^2 + 156*
a*b*c^6*f*h^2)/c^7)*x + (26880*c^8*d*g^2 + 29568*b*c^7*e*g^2 + 672*b^2*c^6
*f*g^2 + 31360*a*c^7*f*g^2 + 59136*b*c^7*d*g*h + 1344*b^2*c^6*e*g*h + 6272
0*a*c^7*e*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*d*h^2
 + 31360*a*c^7*d*h^2 - 432*b^3*c^5*e*h^2 + 2112*a*b*c^6*e*h^2 + 297*b^4*c^
4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680*a^2*c^6*f*h^2)/c^7)*x + (80640*b*c^7
*d*g^2 + 2688*b^2*c^6*e*g^2 + 86016*a*c^7*e*g^2 - 1568*b^3*c^5*f*g^2 + 806
4*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d*g*h - 3136*b^3*c^5*e
*g*h + 16128*a*b*c^6*e*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h +
12288*a^2*c^6*f*g*h - 1568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 + 1008*b^4*c
^4*e*h^2 - 5952*a*b^2*c^5*e*h^2 + 6144*a^2*c^6*e*h^2 - 693*b^5*c^3*f*h^2 +
 4680*a*b^3*c^4*f*h^2 - 7248*a^2*b*c^5*f*h^2)/c^7)*x + (26880*b^2*c^6*d*g^
2 + 537600*a*c^7*d*g^2 - 13440*b^3*c^5*e*g^2 + 75264*a*b*c^6*e*g^2 + 7840*
b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 - 26880*b^3*c^
5*d*g*h + 150528*a*b*c^6*d*g*h + 15680*b^4*c^4*e*g*h - 96768*a*b^2*c^5*...
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \int (g+h\,x)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,dx$$

input
```
int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.197. $\int (g+hx)^2 (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx$

3.198 $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

3.198.1 Optimal result

Integrand size = 30, antiderivative size = 418

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{(b^2-4ac)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\sqrt{a+}}{1024c^5}$$

$$+\frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\left(a+bx+cx^2\right)^{3/}}{384c^4}$$

$$+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$

$$+\frac{\left(63b^2fh^2-24c^2(5fg^2-7h(eg+dh))-2ch(24afh+49b(fg+eh))-10ch(10cfg-14ceh+9bfh)x\right)(a}{840c^3h}$$

$$+\frac{(b^2-4ac)^2\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)\text{arctanh}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right.}{2048c^{11/2}}$$

output
```
1/384*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a
*f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*
x^2+b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h^2-24*c^2*(5*f*g^2-7*h*(d*h+e*g))-2*
c*h*(24*a*f*h+49*b*(e*h+f*g))-10*c*h*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2
+b*x+a)^(5/2)/c^3/h+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e
*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)-1/1024*(-4*a*c+b^2)*(48*c^3*d*g
-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g
)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

3.198.2 Mathematica [A] (verified)

Time = 7.71 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.44

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}(d+ex$$

$$+fx^2)\ dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(945b^6fh-210b^5c(7fg+7eh+3fhx)+28b^4c(-270afh+c(90eg+90dh}{}$$

input
```
Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^6*f*h - 210*b^5*c*(7*f*g + 7*e*h + 3
*f*h*x) + 28*b^4*c*(-270*a*f*h + c*(90*e*g + 90*d*h + 35*f*g*x + 35*e*h*x
+ 18*f*h*x^2)) - 16*b^3*c^2*(105*c*d*(3*g + h*x) - 7*a*(95*f*g + 95*e*h +
39*f*h*x) + c*x*(7*e*(15*g + 7*h*x) + f*x*(49*g + 27*h*x))) + 48*b^2*c^2*(
343*a^2*f*h - 2*a*c*(175*d*h + 7*e*(25*g + 9*h*x) + f*x*(63*g + 31*h*x)) +
 2*c^2*x*(7*d*(5*g + 2*h*x) + x*(7*e*(2*g + h*x) + f*x*(7*g + 4*h*x)))) +
32*b*c^3*(-3*a^2*(189*f*g + 189*e*h + 73*f*h*x) + 6*a*c*(7*d*(25*g + 7*h*x
) + x*(7*e*(7*g + 3*h*x) + f*x*(21*g + 11*h*x))) + 4*c^2*x^2*(21*d*(15*g +
 11*h*x) + x*(7*e*(33*g + 26*h*x) + 2*f*x*(91*g + 75*h*x)))) + 64*c^3*(-96
*a^3*f*h + 3*a^2*c*(112*d*h + 7*e*(16*g + 5*h*x) + f*x*(35*g + 16*h*x)) +
4*c^3*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5*h*x) + 5*f*x*(7*g + 6*h*
x))) + 2*a*c^2*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g + 35*h*x) + f*x*(245
*g + 192*h*x))))) - 105*(b^2 - 4*a*c)^2*(-48*c^3*d*g + 9*b^3*f*h + 8*c^2*(
3*b*e*g + a*f*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h + 7*b*(f*g + e*h))*Ar
cTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(107520*c^(11/2))
```

3.198.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2184, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\ dx$$

$$\Big\downarrow \ 2184$$

$$\frac{\int -\frac{1}{2}h(g+hx)(5bfg-14cdh+4afh+(10cfg-14ceh+9bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{7ch^2}+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$

$\big\downarrow$ 27

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{\int (g+hx)(5bfg-14cdh+4afh+(10cfg-14ceh+9bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{14ch}$$

$\big\downarrow$ 1225

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}(-2ch(24afh+49b(eh+fg)}{}}{14ch}$$

$\big\downarrow$ 1087

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c^2}-\frac{(a+bx+}{}}{14ch}$$

$\big\downarrow$ 1087

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)}{}\right.}{16c}\right.}{24c^2}}{14ch}$$

$\big\downarrow$ 1092

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)}{}\right.}{16c}\right.}{24c^2}}{}$$

3.198. $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$

$\big\downarrow$ 219

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6a\cdots\right.}{24c^2}}{\cdots}$$

input
```
Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) - (-1/60*((63*b^2*f*h^2 -
24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b*(f*g + e*h)) -
 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)^(5/2))/c^2 -
(7*h*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) +
 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))
/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(
3/2))))/(16*c)))/(24*c^2))/(14*c*h)
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(392) = 784$.

Time = 0.77 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.24

method	result
default	Expression too large to display
risch	$-\frac{(-15360c^6fh\,x^6-19200b\,c^5fh\,x^5-17920c^6eh\,x^5-17920c^6fg\,x^5-24576a\,c^5fh\,x^4-384b^2c^4fh\,x^4-23296b\,c^5eh\,x^4-23296b\,c^5fg\,x^4}{}$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)
/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+f*h*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x
^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+
b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
)))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(
2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/
8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x
^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))))))+(e*h+f*g)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+
b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^
2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x
+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a
*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+(d*h+e*g)*(
1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/1
6*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(392) = 784$.

Time = 0.65 (sec) , antiderivative size = 1833, normalized size of antiderivative = 4.39

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c
^3 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(
7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a
*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(153
60*c^7*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(
14*(12*c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*
a*c^6)*f)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^
6)*f)*g + (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44
*a*b*c^5)*f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*
b^3*c^4 - 36*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 -
 36*a*b*c^5)*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 -
14*(120*(3*b^3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*
a^2*c^5)*e + (105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(1
5*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c
^3 + 1296*a^2*b*c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3
- 2048*a^3*c^4)*f)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 -
 28*a*b*c^5)*e + (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5
*b^3*c^4 - 28*a*b*c^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5...
```

3.198.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3990 vs. $2(435) = 870$.

Time = 1.13 (sec) , antiderivative size = 3990, normalized size of antiderivative = 9.55

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h*x**6/7 + x**5*(15*b*c*f*h/14 + c*
*2*e*h + c**2*f*g)/(6*c) + x**4*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*
c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**
2*e*g)/(5*c) + x**3*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/1
4 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e
*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*
h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*
g)/(4*c) + x**2*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g
- 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/1
4 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(5*c) + b**2*d*h +
b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*
c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h +
2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15
*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) +
c**2*d*g)/(8*c))/(3*c) + x*(a**2*e*h + a**2*f*g + 2*a*b*d*h + 2*a*b*e*g +
2*a*c*d*g - 3*a*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 +
c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g -
 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14
 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(
4*c) + b**2*d*g - 5*b*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2...
```

3.198.7 Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(392) = 784$.

Time = 0.31 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.21

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}(d+ex+fx^2)\,dx=\frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,cfhx+\frac{14\,c^7fg+14\,c^7eh+15\,bc^6fh}{c^6}\right)x+\frac{168\,c^7e}{}\right.\right.\right.\right.\right.\right.$$
$$-\frac{(48\,b^4c^3dg-384\,ab^2c^4dg+768\,a^2c^5dg-24\,b^5c^2eg+192\,ab^3c^3eg-384\,a^2bc^4eg+14\,b^6cfg-120\,ab^4c^2fg}{}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
 14*c^7*e*h + 15*b*c^6*f*h)/c^6)*x + (168*c^7*e*g + 182*b*c^6*f*g + 168*c^
7*d*h + 182*b*c^6*e*h + 3*b^2*c^5*f*h + 192*a*c^6*f*h)/c^6)*x + (1680*c^7*
d*g + 1848*b*c^6*e*g + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h +
42*b^2*c^5*e*h + 1960*a*c^6*e*h - 27*b^3*c^4*f*h + 132*a*b*c^5*f*h)/c^6)*x
 + (5040*b*c^6*d*g + 168*b^2*c^5*e*g + 5376*a*c^6*e*g - 98*b^3*c^4*f*g + 5
04*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a*c^6*d*h - 98*b^3*c^4*e*h + 504*a
*b*c^5*e*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h)/c^6)*x
+ (1680*b^2*c^5*d*g + 33600*a*c^6*d*g - 840*b^3*c^4*e*g + 4704*a*b*c^5*e*g
 + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360*a^2*c^5*f*g - 840*b^3*c^4*d
*h + 4704*a*b*c^5*d*h + 490*b^4*c^3*e*h - 3024*a*b^2*c^4*e*h + 3360*a^2*c^
5*e*h - 315*b^5*c^2*f*h + 2184*a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h)/c^6)*x
- (5040*b^3*c^4*d*g - 33600*a*b*c^5*d*g - 2520*b^4*c^3*e*g + 16800*a*b^2*c
^4*e*g - 21504*a^2*c^5*e*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d
*h + 1470*b^5*c^2*e*h - 10640*a*b^3*c^3*e*h + 18144*a^2*b*c^4*e*h - 945*b^
6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c^4*f*h)/c
^6) - 1/2048*(48*b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g - 24*b^
5*c^2*e*g + 192*a*b^3*c^3*e*g - 384*a^2*b*c^4*e*g + 14*b^6*c*f*g - 120*a*b
^4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 1...
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.199 $\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx$

3.199.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx = &-\frac{(b^2-4ac)(24c^2d+7b^2f-4c(3be+af))(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}\\ &+\frac{(24c^2d-12bce+7b^2f-4acf)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3}\\ &+\frac{(12ce-7bf)(a+bx+cx^2)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c}\\ &+\frac{(b^2-4ac)^2(24c^2d+7b^2f-4c(3be+af))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

output
```
1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c
^3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2
)/c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d
+7*b^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

3.199.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.24

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48b^2}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*b
^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(
7*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*
e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*
e + 35*f*x)))) + 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f
))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(9/2))
```

3.199.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \ 2192$$

$$\frac{\int \frac{1}{2}(12cd-2af+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{6c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (2(6cd-af)+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce}{5c}}{12c}\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}+(a+bx+}{12c}\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-4acf+7b^2f-12bce+24c^2d\right)}{2c}+(a+bx+c}{12c}\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

3.199. $\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((12*c*e - 7*b*f)*(a + b*x + c*x^2)
^(5/2))/(5*c) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*
x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c)
```

3.199.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.199.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{(-1280c^5 f x^5-1664b c^4 f x^4-1536c^5 e x^4-2240a c^4 f x^3-48b^2c^3 f x^3-2112b c^4 e x^3-1920c^5 d x^3-288ab c^3 f x^2-3072a c^4 e x^2+56b$
default	$d\left(\frac{(2cx+b)(c x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+f\left(\frac{x(c x^2+bx+a)^{\frac{5}{2}}}{6c}\right.$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^4*(-1280*c^5*f*x^5-1664*b*c^4*f*x^4-1536*c^5*e*x^4-2240*a*c^4*f*
x^3-48*b^2*c^3*f*x^3-2112*b*c^4*e*x^3-1920*c^5*d*x^3-288*a*b*c^3*f*x^2-307
2*a*c^4*e*x^2+56*b^3*c^2*f*x^2-96*b^2*c^3*e*x^2-2880*b*c^4*d*x^2-480*a^2*c
^3*f*x+432*a*b^2*c^2*f*x-672*a*b*c^3*e*x-4800*a*c^4*d*x-70*b^4*c*f*x+120*b
^3*c^2*e*x-240*b^2*c^3*d*x+1296*a^2*b*c^2*f-1536*a^2*c^3*e-760*a*b^3*c*f+1
200*a*b^2*c^2*e-2400*a*b*c^3*d+105*b^5*f-180*b^4*c*e+360*b^3*c^2*d)*(c*x^2
+b*x+a)^(1/2)-1/1024*(64*a^3*c^3*f-144*a^2*b^2*c^2*f+192*a^2*b*c^3*e-384*a
^2*c^4*d+60*a*b^4*c*f-96*a*b^3*c^2*e+192*a*b^2*c^3*d-7*b^6*f+12*b^5*c*e-24
*b^4*c^2*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.56

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \left[-\frac{15\left(24\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d-12\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)e+\left(7\,b^6-60\,ab^4c+144\,a^2\right.\right.}{}\right.$$

$$-\frac{15\left(24\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d-12\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)e+\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^
3*c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c
^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 +
 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*
x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 12
8*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^
2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*
b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*
d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^
2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 2
0*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*
c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12
*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(230) = 460$.

Time = 0.56 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.76

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*x**5/6 + x**4*(13*b*c*f/12 + c**2*e
)/(5*c) + x**3*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/
(10*c) + c**2*d)/(4*c) + x**2*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2
*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13
*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(3*c) + x*(a**2*f + 2*a*b*e +
2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(
10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12
 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e -
9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) + (a**2*e
+ 2*a*b*d - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b*
*2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c*
*2*e)/(10*c) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*
a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2
*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(
5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/c) + (a**2*d - a*(a**
2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e -
4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**
2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c...
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\ dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{12\,c^6e+13\,bc^5f}{c^5}\right)x+\frac{120\,c^6d+132\,bc^5e+3\,b^2c^4f}{c^5}\right.\right.\right.\right.$$
$$-\frac{(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f-}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (12*c^6*e + 13*b*c^5*
f)/c^5)*x + (120*c^6*d + 132*b*c^5*e + 3*b^2*c^4*f + 140*a*c^5*f)/c^5)*x +
 (360*b*c^5*d + 12*b^2*c^4*e + 384*a*c^5*e - 7*b^3*c^3*f + 36*a*b*c^4*f)/c
^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d - 60*b^3*c^3*e + 336*a*b*c^4*e + 35*
b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f)/c^5)*x - (360*b^3*c^3*d - 240
0*a*b*c^4*d - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e + 105*b^5*
c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f)/c^5) - 1/1024*(24*b^4*c^2*d - 19
2*a*b^2*c^3*d + 384*a^2*c^4*d - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^
3*e + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\ dx = \int (c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)\ dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.200 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

3.200.1 Optimal result

Integrand size = 32, antiderivative size = 660

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)+128c^4g^2(fg^2-h(eg-dh)}{}$$

$$-\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ceh+bfh)x)(a+bx+cx^2)^{3/2}}{48c^2h^3}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$-\frac{\left(4ch(2cg-bh)(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)(2cfg-2ceh+bfh))-2\left(4c^2g^2-\right.\right.}{}$$

$$+\frac{(cg^2-bgh+ah^2)^{3/2}(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

output
```
-1/48*(8*c*h*(b*f*g-2*c*d*h)-(-3*b*h+8*c*g)*(b*f*h-2*c*e*h+2*c*f*g)+6*c*h*
(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/h^3+1/5*f*(c*x^2+b*x+a)
^(5/2)/c/h-1/256*(4*c*h*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b*g)*(b*f*g-2*c*d*h)-g
*(-4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))-2*(4*c^2*g^2-1/2*b^2*
h^2-2*c*h*(-a*h+b*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2
*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^6+(a*h^2-b*g*h+c*g^2)^(3/2)*(f*g^
2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)
^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6+1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b
*e*h+b*f*g)+128*c^4*g^2*(f*g^2-h*(-d*h+e*g))-32*c^3*h*(-4*a*h+5*b*g)*(f*g^
2-h*(-d*h+e*g))-8*b*c^2*h^2*(3*a*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c
*h*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2
-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5
```

3.200.2 Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(45b^4fh^4-30b^2ch^3(10afh+b(-3fg+3eh+fhx))+12c^2h^2\left(32a^2fh^2+2abh(\right.\right.}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
((h*Sqrt[a + x*(b + c*x)]*(45*b^4*f*h^4 - 30*b^2*c*h^3*(10*a*f*h + b*(-3*f
*g + 3*e*h + f*h*x)) + 12*c^2*h^2*(32*a^2*f*h^2 + 2*a*b*h*(-25*f*g + 25*e*
h + 7*f*h*x) + b^2*(5*h*(-4*e*g + 4*d*h + e*h*x) + f*(20*g^2 - 5*g*h*x + 2
*h^2*x^2))) + 32*c^4*(f*(60*g^4 - 30*g^3*h*x + 20*g^2*h^2*x^2 - 15*g*h^3*x
^3 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x + 2*h^2*x^2) + e*(-12*g^3 +
 6*g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3))) + 16*c^3*h*(a*h*(5*h*(-32*e*g + 32
*d*h + 15*e*h*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + b*(f*(-150*g^3 +
 70*g^2*h*x - 45*g*h^2*x^2 + 33*h^3*x^3) + 5*h*(2*d*h*(-15*g + 7*h*x) + e*
(30*g^2 - 14*g*h*x + 9*h^2*x^2))))))/c^3 + 3840*Sqrt[-(c*g^2) + h*(b*g - a
*h)]*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[-(
c*g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])]
- (15*(3*b^5*f*h^5 - 6*b^3*c*h^4*(-(b*f*g) + b*e*h + 4*a*f*h) - 384*c^4*g*
h*(b*g - a*h)*(f*g^2 + h*(-(e*g) + d*h)) + 256*c^5*(f*g^5 + g^3*h*(-(e*g)
+ d*h)) + 16*b*c^2*h^3*(3*a^2*f*h^2 + 3*a*b*h*(-(f*g) + e*h) + b^2*(f*g^2
- e*g*h + d*h^2)) + 96*c^3*h^2*(a^2*h^2*(f*g - e*h) + b^2*g*(f*g^2 - e*g*h
 + d*h^2) - 2*a*b*h*(f*g^2 - e*g*h + d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a
] + Sqrt[a + x*(b + c*x)])])/c^(7/2))/(1920*h^6)
```

3.200.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\int -\frac{5h(bfg-2cdh+(2cfg-2ceh+bfh)x)(cx^2+bx+a)^{3/2}}{2(g+hx)}dx}{5ch^2}+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f(a+bx+cx^2)^{5/2}}{5ch}-\frac{\int \frac{(bfg-2cdh+(2cfg-2ceh+bfh)x)(cx^2+bx+a)^{3/2}}{g+hx}dx}{2ch}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\int\frac{\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-3hb^2+8cgb-4ach\right)(2cf\right.}{}}{2ch}$$

$\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\int\frac{\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-3hb^2+8cgb-4ach\right)(2cf\right.}{}}{2ch}$$

$\downarrow$ 1231

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}$$

$\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}$$

$\downarrow$ 1269

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}$$

$\downarrow$ 1092

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}$$

$\downarrow$ 219

3.200. $\int\frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}}{}$$

$$\Big\downarrow \text{1154}$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg\right.\right.\right.}{}}{}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - (((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g
 - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x
)*(a + b*x + c*x^2)^(3/2))/(24*c*h^2) - (((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*
g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4*a*h)*(f*g^2 - h*(e*g - d*h)) +
128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c^2*h^2*(3*a*h*(f*g - e*h) - 2*b
*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h) -
 (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b^2*h^2 - 4*c*h*(2*b*g - 3*a*
h)))*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(2*c*g - b*h)*(8*c*h*(
b*g - 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g
- 2*c*e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c
*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g
^2 - 3*b^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - (256*c^3*(c*g^2 - b*g*h + a*h^2)^(3/
2)*(f*g^2 - h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt
[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h)/(8*c*h^2))/(16*c*h^2))
/(2*c*h)
```

3.200. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.200. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.200.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.50

method	result
default	$\dfrac{eh\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+fh\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b}{}\cdots\right.}{}\cdots\right.}{}$
risch	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/h^2*(e*h*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))))+f*h*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c
*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))))-f*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^3*(1/3*((x+1/h*g)^2*c+(b*
h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4
*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/c*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^
2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)^2*c+(
b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2
)/h^2*(((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)+1/2*(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+1/h*g))/c^(1/2)+((x+1/h*g)
^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^
2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))))
```

3.200.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

3.200.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx=\int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

3.200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

3.200.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

3.201 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.201.1 Optimal result

Integrand size = 32, antiderivative size = 754

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx =$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3g(5fg^2-h(4eg-3dh))+16c^2h(4ah(2fg-eh)-b(19fg^2 \cdots}{6 \cdots}$$

$$-\frac{\left(3bfh^2(bg-ah)+8c^2g(5fg^2-h(4eg-3dh))+ch(8ah(2fg-eh)-b(43fg^2-8h(4eg-3dh)))+6ch^2 \cdots\right.}{24ch^3\,(cg^2-bgh+ah^2)}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{h\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$+\frac{(3b^4fh^4+8b^2ch^3(2bfg-beh-3afh)+128c^4g^2(5fg^2-h(4eg-3dh))+48c^2h^2(a^2fh^2-2abh(2fg-eh \cdots}{\cdots}$$

$$-\frac{\sqrt{cg^2-bgh+ah^2}(2cg(5fg^2-h(4eg-3dh))+h(2ah(2fg-eh)-b(7fg^2-5egh+3dh^2)))\operatorname{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\cdots\right.}{2h^6}$$

output
```
-1/24*(3*b*f*h^2*(-a*h+b*g)+8*c^2*g*(5*f*g^2-h*(-3*d*h+4*e*g))+c*h*(8*a*h*
(-e*h+2*f*g)-b*(43*f*g^2-8*h*(-3*d*h+4*e*g)))+6*c*h^2*(4*c*e*g+b*f*g-5*c*f
*g^2/h-4*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/c/h^3/(a*h^2-b*g*h+c*g^2)-(f*
g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*
(3*b^4*f*h^4+8*b^2*c*h^3*(-3*a*f*h-b*e*h+2*b*f*g)+128*c^4*g^2*(5*f*g^2-h*(
-3*d*h+4*e*g))+48*c^2*h^2*(a^2*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g
*h+3*f*g^2))+192*c^3*h*(a*h*(d*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4
*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1
/2*(2*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(3*d*h^2-5*e*
g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^6-1/64*(3*b^3*f*h^3
+4*b*c*h^2*(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*g*(5*f*g^2-h*(-3*d*h+4*e*g))+
16*c^2*h*(4*a*h*(-e*h+2*f*g)-b*(9*d*h^2-14*e*g*h+19*f*g^2))+2*c*h*(3*b^2*f
*h^2+4*c*h*(-3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h*(-3*d*h+4*e*g)))*x
)*(c*x^2+b*x+a)^(1/2)/c^2/h^5
```

3.201.2 Mathematica [A] (verified)

Time = 4.83 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(-9b^3fh^3(g+hx)+6bch^2(g+hx)(10afh+b(-8fg+4eh+fhx))-16c^3(f(60g\right.}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
((h*Sqrt[a + x*(b + c*x)]*(-9*b^3*f*h^3*(g + h*x) + 6*b*c*h^2*(g + h*x)*(1
0*a*f*h + b*(-8*f*g + 4*e*h + f*h*x)) - 16*c^3*(f*(60*g^4 + 30*g^3*h*x - 1
0*g^2*h^2*x^2 + 5*g*h^3*x^3 - 3*h^4*x^4) - 2*h*(3*d*h*(-6*g^2 - 3*g*h*x +
h^2*x^2) + 2*e*(12*g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))) + 8*c^2*h*(a
*h*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*g*h*x + 15*h^2*x^2)) +
 b*(f*(114*g^3 + 62*g^2*h*x - 19*g*h^2*x^2 + 9*h^3*x^3) + 2*h*(3*d*h*(9*g
+ 5*h*x) + e*(-42*g^2 - 23*g*h*x + 7*h^2*x^2))))))/(c^2*(g + h*x)) - 192*S
qrt[-(c*g^2) + h*(b*g - a*h)]*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-
7*b*f*g^2 + b*h*(5*e*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqrt[-(c*
g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])] +
(3*(3*b^4*f*h^4 - 8*b^2*c*h^3*(-2*b*f*g + b*e*h + 3*a*f*h) + 128*c^4*(5*f*
g^4 + g^2*h*(-4*e*g + 3*d*h)) + 48*c^2*h^2*(a^2*f*h^2 + 2*a*b*h*(-2*f*g +
e*h) + b^2*(3*f*g^2 - 2*e*g*h + d*h^2)) - 192*c^3*h*(-(a*h*(3*f*g^2 - 2*e*
g*h + d*h^2)) + b*g*(4*f*g^2 - 3*e*g*h + 2*d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-
Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(5/2))/(192*h^6)
```

3.201.3 Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+2cdg-5beg-2afg+3bdh+2aeh-2\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(g+hx)}dx}{ah^2-bgh+cg^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{5fg^2}{h}+5eg-3dh\right)-2\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{g+hx}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\int \frac{\left(cg^2-bhg+ah^2\right)\left(3fghb^2-8c\left(5fg^2-h(4eg-3dh)\right)b+4ach(5fg-4eh)+\left(-16\left(5fg^2-h(4eg-3dh)\right)c^2+4h(4bfg-2beh-3afh)c+3b^2fh^2\right)x\right)\sqrt{cx^2+bx+a}}{h(g+hx)}dx}{8ch^2} - \frac{}{2\left(ah^2-\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\int \frac{\left(3fghb^2-8c\left(5fg^2-h(4eg-3dh)\right)b+4ach(5fg-4eh)+\left(-16\left(5fg^2-h(4eg-3dh)\right)c^2+4h(4bfg-2beh-3afh)c+3b^2fh^2\right)x\right)\sqrt{cx^2+bx+a}}{g+hx}dx}{8ch^3} - \frac{}{2\left(ah^2-\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch\right.}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch\right.}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{1269}$$

3.201. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1092

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 219

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1154

$$-\frac{\left(\frac{8\left(5fg^3-gh(4eg-3dh)\right)c^2}{h}-\left(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh)\right)c+6h\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)xc+3bfh(bg-ah)\right)\left(cx^2+bx+a\right)^{3/}}{12ch^2}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{h\left(cg^2-bhg+ah^2\right)(g+hx)}$$

$\Big\downarrow$ 219

3.201. $\displaystyle\int\frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4bch}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
-(((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x))) + (-1/12*((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4
*e*g - 3*d*h)))/h - c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g -
 e*h)) + 6*c*h*(4*c*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a +
 b*x + c*x^2)^(3/2))/(c*h^2) - ((c*g^2 - b*g*h + a*h^2)*(((3*b^3*f*h^3 + 4
*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f*g^3 - g*h*(4*e*g - 3*
d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) - 4*a*h*(2*f*g - e*h))
 + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h) - 16*c^2*(5*f*
g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(
2*c*g - b*h)*(3*b^2*f*g*h + 4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 - h*(
4*e*g - 3*d*h))) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(3*b^2*
f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h) - 16*c^2*(5*f*g^2 - h*(4*e*g -
 3*d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c
]*h) - (64*c^2*Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g - 3*
d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh
[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b
*x + c*x^2])])/h)/(8*c*h^2)))/(8*c*h^3))/(2*(c*g^2 - b*g*h + a*h^2))
```

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.201. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.201. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

3.201.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 1324, normalized size of antiderivative = 1.76

method	result	size
risch	Expression too large to display	1324
default	Expression too large to display	1853

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192/c^2*(48*c^3*f*h^3*x^3+72*b*c^2*f*h^3*x^2+64*c^3*e*h^3*x^2-128*c^3*f*
g*h^2*x^2+120*a*c^2*f*h^3*x+6*b^2*c*f*h^3*x+112*b*c^2*e*h^3*x-224*b*c^2*f*
g*h^2*x+96*c^3*d*h^3*x-192*c^3*e*g*h^2*x+288*c^3*f*g^2*h*x+60*a*b*c*f*h^3+
256*a*c^2*e*h^3-512*a*c^2*f*g*h^2-9*b^3*f*h^3+24*b^2*c*e*h^3-48*b^2*c*f*g*
h^2+240*b*c^2*d*h^3-480*b*c^2*e*g*h^2+720*b*c^2*f*g^2*h-384*c^3*d*g*h^2+57
6*c^3*e*g^2*h-768*c^3*f*g^3)*(c*x^2+b*x+a)^(1/2)/h^5+1/128/h^5/c^2*(-128*c
^2/h^2*(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6*a*b*f*g^2*h^3-
4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4+3*b^2*e*g^2*h^
3-4*b^2*f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g^3*h^2+10*b*c*f*g^4*h-4*c^2*d*g
^3*h^2+5*c^2*e*g^4*h-6*c^2*f*g^5)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a
*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+1/h*g))+128*c^2*(a^2*d*h^6-a^2*e*g*h^5+a^2*f*g^2*h^4-2*a*b*d*g*h^5+2
*a*b*e*g^2*h^4-2*a*b*f*g^3*h^3+2*a*c*d*g^2*h^4-2*a*c*e*g^3*h^3+2*a*c*f*g^4
*h^2+b^2*d*g^2*h^4-b^2*e*g^3*h^3+b^2*f*g^4*h^2-2*b*c*d*g^3*h^3+2*b*c*e*g^4
*h^2-2*b*c*f*g^5*h+c^2*d*g^4*h^2-c^2*e*g^5*h+c^2*f*g^6)/h^3*(-1/(a*h^2-b*g
*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a...
```

3.201.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

3.201.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

3.201.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

3.201.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^2}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

3.202 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

3.202.1 Optimal result

Integrand size = 32, antiderivative size = 824

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx =$$

$$-\frac{(b^2fh^3(bg-ah)-8c^3g^2(10fg^2-3h(2eg-dh))-2c^2h(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh\cdots}{}$$

$$-\frac{\left(4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-4ah(7fg-3eh)+b(31fg^2-3h(5eg-dh))+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3c\cdots\right.\right.}{12h^2\,(cg^2-bgh+ah^2)\,(g+hx)}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{2h\,(cg^2-bgh+ah^2)\,(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3g(10fg^2-3h(2eg-dh))+24c^2h(ah(3fg-eh)-b(6fg^2-\cdots}{16c^{3/2}h^6}$$

$$+\frac{(8c^2g^2(10fg^2-3h(2eg-dh))+4ch(ah(19fg^2-9egh+3dh^2)-bg(28fg^2-15egh+6dh^2))+h^2(8a^2f\cdots}{8h^6\sqrt{cg^2-bgh+a\cdots}}$$

output
```
-1/12*(4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-4*a*h*(-3*e*h+7*f*g)+b*(31*f*g^2-3*h
*(-d*h+5*e*g))+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*x^2
+b*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*
x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*h^2
*(-2*a*f*h-b*e*h+3*b*f*g)+16*c^3*g*(10*f*g^2-3*h*(-d*h+2*e*g))+24*c^2*h*(a
*h*(-e*h+3*f*g)-b*(d*h^2-3*e*g*h+6*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g)
)+4*c*h*(a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-b*g*(6*d*h^2-15*e*g*h+28*f*g^2))+h
^2*(8*a^2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*g))))*
arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)-1/8*(b^2*f*h^3*(-a*h+b*g)-8*c^3*
g^2*(10*f*g^2-3*h*(-d*h+2*e*g))-2*c^2*h*(2*a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-
3*b*g*(5*d*h^2-12*e*g*h+22*f*g^2))-c*h^2*(8*a^2*f*h^2-18*a*b*h*(-e*h+3*f*g
)+b^2*(53*f*g^2-6*h*(-d*h+4*e*g)))+2*c*h*(b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f
*g^2-3*h*(-d*h+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3*b*(d*h^2-3*e*g*h+6*f*g^
2)))*x)*(c*x^2+b*x+a)^(1/2)/c/h^5/(a*h^2-b*g*h+c*g^2)
```

3.202.2 Mathematica [A] (verified)

Time = 11.63 (sec) , antiderivative size = 817, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{4f(a+x(b+cx))^{3/2}-\frac{6(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(g+hx)^2}+\frac{12(2fg-eh)(a+}{g+h}}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
(4*f*(a + x*(b + c*x))^(3/2) - (6*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c
*x))^(3/2))/(g + h*x)^2 + (12*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g +
h*x) + (9*(-2*f*g + e*h)*((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(h*(-4*c*g
 + 3*b*h + 2*c*h*x)*Sqrt[a + x*(b + c*x)] + 2*(2*c*g - b*h)*Sqrt[c*g^2 + h
*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2
+ h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])))/(2*Sqrt[c]*h^3) + (9*(f*g^2
 + h*(-(e*g) + d*h))*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) -
 (Sqrt[a + x*(b + c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a
*h + b*h*x)))/h^2 + (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^
2 + 4*c*h*(-2*b*g + a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) +
 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b
+ c*x)])])/(2*h^3)))/(-(c*g^2) + h*(b*g - a*h)) - (3*f*((2*c*g - b*h)*(8*c
^2*g^2 - b^2*h^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(h*Sqrt[a + x*(b + c*x)]*(-(b^2*h^2) +
 4*c^2*g*(-2*g + h*x) - 2*c*h*(-5*b*g + 4*a*h + b*h*x)) + 8*c*(c*g^2 + h*(
-(b*g) + a*h))^(3/2)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*
g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])))/(4*c^(3/2)*h^3))/(12*h^
3)
```

3.202.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 1230, 25, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+4cdg-5beg-4afg+bdh+4aeh-2\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(g+hx)^2}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(4cdg-4afg+4aeh-b\left(-\frac{5fg^2}{h}+5eg-dh\right)-2\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^2}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1230$$

$$-\frac{\int -\frac{\left(h\left(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\right)-4\left(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\left(2ah(7fg-3eh)-3b\right.\right.\right.}{h(g+hx)}}{2h^2}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(h\left(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\right)-4\left(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\left(2ah(7fg-3eh)-3b\left(6fg\right.\right.\right.\right.}{h(g+hx)}}{2h^2}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(h\left(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\right)-4\left(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\left(2ah(7fg-3eh)-3b\left(6fg\right.\right.\right.\right.}{g+hx}}{2h^3}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1231$$

$$-\frac{\int \frac{2\left(cg^2-bhg+ah^2\right)\left(fgh^2b^3-2ch\left(26fg^2-3h(4eg-dh)\right)b^2+4c\left(20cfg^3-6ch(2eg-dh)g+ah^2(17fg-6eh)\right)b-8ach\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)+\left(16\left(10fg^3-\right.\right.\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{4ch^2}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

3.202. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

$$-\frac{\left(ah^2-bgh+cg^2\right)\int \frac{fgh^2b^3-2ch\left(26fg^2-3h(4eg-dh)\right)b^2+4c\left(20cfg^3-6ch(2eg-dh)g+ah^2(17fg-6eh)\right)b-8ach\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)+\left(16\left(10fg^3-3\right.\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{2ch^2}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1269$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\left(-24c^2h\left(-ah(3fg-eh)-bh(3eg-dh)+6bfg^2\right)+6bch^2(-2afh-beh+3bfg)+b^3fh^3+16c^3\left(10fg^3-3gh(2eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{2c\left(h^2\right.}{}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1092$$

$$-\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8a^2\right.\right.}{ch^2}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24c^2h\left(-ah(3fg-eh)-bh(3eg-dh)+6bfg^2\right)+6bch^2(-2afh-beh+3bfg)+b^3fh^3+16c^3\left(10fg^3-3gh(2eg-\right.\right.}{\sqrt{c}h}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1154$$

3.202. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

$$-\frac{\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8a^2\right.}{ch^2}}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

$\downarrow$ 219

$$-\frac{\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8a^2\right.}{ch^2}}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + (-1/3*((31*b*f*g^2 + 4*c*g*(6*e*g - (10*f*g^2)/h -
3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g - 3*e*h) + 2*h*(3*c*e*g + 2*b*
f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(h^2*
(g + h*x)) + (-(((b^2*f*h^3*(b*g - a*h) - 8*c^3*(10*f*g^4 - 3*g^2*h*(2*e*g
 - d*h)) - 2*c^2*h*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2
 - 12*e*g*h + 5*d*h^2)) - c*h^2*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^
2*(53*f*g^2 - 6*h*(4*e*g - d*h))) + 2*c*h*(b*f*h^2*(b*g - a*h) + 2*c^2*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2
- 3*e*g*h + d*h^2)))*x)*Sqrt[a + b*x + c*x^2])/(c*h^2)) - ((c*g^2 - b*g*h
+ a*h^2)*(((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) -
a*h*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[c]*h) - (2*c*(8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28
*b*f*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) +
 h^2*(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g
- d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2])))/(2*c*h^2
))/(2*h^3))/(4*(c*g^2 - b*g*h + a*h^2))
```

3.202. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.202. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.202.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(792) = 1584$.

Time = 1.05 (sec) , antiderivative size = 1781, normalized size of antiderivative = 2.16

method	result	size
risch	Expression too large to display	1781
default	Expression too large to display	3600

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*c^2*f*h^2*x^2+14*b*c*f*h^2*x+12*c^2*e*h^2*x-36*c^2*f*g*h*x+32*a*
c*f*h^2+3*b^2*f*h^2+30*b*c*e*h^2-90*b*c*f*g*h+24*c^2*d*h^2-72*c^2*e*g*h+14
4*c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/h^5+1/16/c/h^5*((12*a*b*c*f*h^3+24*a*c^2*
e*h^3-72*a*c^2*f*g*h^2-b^3*f*h^3+6*b^2*c*e*h^3-18*b^2*c*f*g*h^2+24*b*c^2*d
*h^3-72*b*c^2*e*g*h^2+144*b*c^2*f*g^2*h-48*c^3*d*g*h^2+96*c^3*e*g^2*h-160*
c^3*f*g^3)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-16*c/h^2*
(a^2*f*h^4+2*a*b*e*h^4-6*a*b*f*g*h^3+2*a*c*d*h^4-6*a*c*e*g*h^3+12*a*c*f*g^
2*h^2+b^2*d*h^4-3*b^2*e*g*h^3+6*b^2*f*g^2*h^2-6*b*c*d*g*h^3+12*b*c*e*g^2*h
^2-20*b*c*f*g^3*h+6*c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^4)/((a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+16*c/h^3*(a^2*e*h^5-2*a^2*f*g*h
^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6*a*b*f*g^2*h^3-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3
-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4+3*b^2*e*g^2*h^3-4*b^2*f*g^3*h^2+6*b*c*d*g^2
*h^3-8*b*c*e*g^3*h^2+10*b*c*f*g^4*h-4*c^2*d*g^3*h^2+5*c^2*e*g^4*h-6*c^2*f*
g^5)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x
+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^
2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*
g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g
)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+16*c*(a^2*d*h...
```

3.202.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

3.202.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.202.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. 2(792) = 1584.

Time = 0.66 (sec) , antiderivative size = 2628, normalized size of antiderivative = 3.19

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*x*(4*c*f*x/h^3 - (18*c^3*f*g*h^14 - 6*c^3*e*
h^15 - 7*b*c^2*f*h^15)/(c^2*h^18)) + (144*c^3*f*g^2*h^13 - 72*c^3*e*g*h^14
 - 90*b*c^2*f*g*h^14 + 24*c^3*d*h^15 + 30*b*c^2*e*h^15 + 3*b^2*c*f*h^15 +
32*a*c^2*f*h^15)/(c^2*h^18)) + 1/4*(80*c^2*f*g^4 - 48*c^2*e*g^3*h - 112*b*
c*f*g^3*h + 24*c^2*d*g^2*h^2 + 60*b*c*e*g^2*h^2 + 35*b^2*f*g^2*h^2 + 76*a*
c*f*g^2*h^2 - 24*b*c*d*g*h^3 - 15*b^2*e*g*h^3 - 36*a*c*e*g*h^3 - 40*a*b*f*
g*h^3 + 3*b^2*d*h^4 + 12*a*c*d*h^4 + 12*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-(
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a
*h^2))/(sqrt(-c*g^2 + b*g*h - a*h^2)*h^6) + 1/16*(160*c^3*f*g^3 - 96*c^3*e
*g^2*h - 144*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 72*b*c^2*e*g*h^2 + 18*b^2*c*
f*g*h^2 + 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 6*b^2*c*e*h^3 - 24*a*c^2*e*h
^3 + b^3*f*h^3 - 12*a*b*c*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*sqrt(c) + b))/(c^(3/2)*h^6) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*c^2*f*g^4*h - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^
2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 24*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b*c*e*g^2*h^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*
g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^2*e*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*...
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^3}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

3.203 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.203.1 Optimal result

Integrand size = 32, antiderivative size = 833

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{(8c^2g^2(10fg^2-h(4eg-dh))-2ch(3bg(18fg^2-6egh+dh^2)-2ah(23fg^2-8egh+2dh^2))+h^2(12a^2f\cdots}{}$$

$$-\frac{\left(2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-6ah(3fg-eh)+b(17fg^2-h(5eg+dh))+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\cdots\right.\right.}{12h^2(cg^2-bgh+ah^2)(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{3h(cg^2-bgh+ah^2)(g+hx)^3}$$

$$+\frac{(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{(16c^3g^3(10fg^2-h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a^2\cdots}{}$$

output
```
-1/12*(2*c*g*(4*e*g-10*f*g^2/h-d*h)-6*a*h*(-e*h+3*f*g)+b*(17*f*g^2-h*(d*h+
5*e*g))+2*h*(2*c*e*g+3*b*f*g-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)
^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b
*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/16*(16*c^3*g^3*(10*f*g^2-h*(
-d*h+4*e*g))-b*h^3*(24*a^2*f*h^2-6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h
+35*f*g^2))+6*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2
)-2*a*b*h*(d*h^2-7*e*g*h+25*f*g^2))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^
2)-a*h*(d*h^2-4*e*g*h+11*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(
a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2
)+1/8*(3*b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f*g^2-h*(-d*h+4
*e*g)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/c^(1/2)-1/8
*(8*c^2*g^2*(10*f*g^2-h*(-d*h+4*e*g))-2*c*h*(3*b*g*(d*h^2-6*e*g*h+18*f*g^2
)-2*a*h*(2*d*h^2-8*e*g*h+23*f*g^2))+h^2*(12*a^2*f*h^2-6*a*b*h*(-e*h+7*f*g)
+b^2*(29*f*g^2-h*(d*h+5*e*g)))+2*h*(3*b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2
-h*(-d*h+4*e*g))+c*h*(6*a*h*(-e*h+3*f*g)-b*(d*h^2-7*e*g*h+22*f*g^2)))*x)*(
c*x^2+b*x+a)^(1/2)/h^5/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

3.203.2 Mathematica [A] (verified)

Time = 13.34 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \frac{-\frac{4(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(g+hx)^3}+\frac{6(2fg-eh)(a+x(b+cx))^{3/2}}{(g+hx)^2}-\frac{12f(a+x(b+c}{g+hx}}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
((-4*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^3 + (6*
(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - (12*f*(a + x*(b + c*x
))^(3/2))/(g + h*x) + (9*f*(2*h*(-4*c*g + 3*b*h + 2*c*h*x)*Sqrt[a + x*(b +
 c*x)] + ((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 4*(2*c*g - b*h)*Sqrt[c*g^2
+ h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g
^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(2*h^3) + (9*(-2*f*g + e*
h)*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) - (Sqrt[a + x*(b +
c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a*h + b*h*x)))/h^2
+ (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g
+ a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x +
 b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*h^3)
))/(-(c*g^2) + h*(b*g - a*h)) + (3*(f*g^2 + h*(-(e*g) + d*h))*((4*(2*c*g -
 b*h)*(c*g^2 + h*(-(b*g) + a*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 + (2
*(-4*c^2*g^2 + b^2*h^2 + 4*c*h*(b*g - 2*a*h))*(a + x*(b + c*x))^(3/2))/(g
+ h*x) + (-2*c*h*Sqrt[a + x*(b + c*x)]*(b^3*h^3 + 4*c^3*g^2*(2*g - h*x) +
b*c*h^2*(5*b*g - 10*a*h + b*h*x) - 2*c^2*h*(b*g*(7*g - 2*h*x) + 2*a*h*(-3*
g + 2*h*x))) + 16*c^(5/2)*(c*g^2 + h*(-(b*g) + a*h))^2*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-...
```

3.203.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2181, 27, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+6cdg-5beg-6afg-bdh+6aeh-2\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^3}dx}{3(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(6cdg-6afg+6aeh-b\left(-\frac{5fg^2}{h}+5eg+dh\right)-2\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)x\right)(cx^2+bx+a)^{3/2}}{(g+hx)^3}dx}{6(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$\big\downarrow$ 1230

$$-\frac{3\int -\frac{2\left(h\left((4bg-4ah)\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)+b\left(5bfg^2-bh(5eg+dh)+6h(cdg-afg+aeh)\right)\right)-2\left(2\left(10fg^3-gh(4eg-dh)\right)c^2-h\left(22bfg^2-bh(7eg-d\right.\right.}{h(g+hx)^2}}{8h^2}$$

$$\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$\big\downarrow$ 27

$$\frac{3\int \frac{\left(h\left((4bg-4ah)\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)+b\left(5bfg^2-bh(5eg+dh)+6h(cdg-afg+aeh)\right)\right)-2\left(2\left(10fg^3-gh(4eg-dh)\right)c^2-h\left(22bfg^2-bh(7eg-dh)-6c\right.\right.}{(g+hx)^2}}{4h^3}$$

$$\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$\big\downarrow$ 1230

$$3\left(-\frac{\int -\frac{h^2\left(29fg^2-h(5eg+dh)\right)b^3-6\left(a(9fg-eh)h^3+cg\left(18fg^2-h(6eg-dh)\right)h\right)b^2+4\left(6a^2fh^4+3ac\left(15fg^2-h(5eg-dh)\right)h^2+2c^2\left(10fg^4-g^2h(4eg-dh)\right)\right)b-8ach}{(g+hx)\sqrt{cx^2+bx+a}}}{2h^2}\right.$$

$$\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$\big\downarrow$ 25

$$3\left(\frac{\int \frac{h^2\left(29fg^2-h(5eg+dh)\right)b^3-6\left(a(9fg-eh)h^3+cg\left(18fg^2-h(6eg-dh)\right)h\right)b^2+4\left(6a^2fh^4+3ac\left(15fg^2-h(5eg-dh)\right)h^2+2c^2\left(10fg^4-g^2h(4eg-dh)\right)\right)b-8ach\left(10\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{2h^2}\right.$$

$$\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$\big\downarrow$ 1269

3.203. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

$$3\left(\frac{\frac{2\left(ah^2-bgh+cg^2\right)\left(-12ch(-afh-beh+4bfg)+3b^2fh^2+8c^2\left(10fg^2-h(4eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{\left(6ch^2\left(4a^2h^2(4fg-eh)-2abh\left(dh^2-7egh+25fg^2\right)+b^2g\right.\right.}{}}{}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

$$3\left(\frac{\frac{4\left(cg^2-bhg+ah^2\right)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(14f\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

$$\Big\downarrow \; 219$$

$$3\left(\frac{\frac{2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)\left(-12ch(-afh-beh+4bfg)+3b^2fh^2+8c^2\left(10fg^2-h(4eg-dh)\right)\right)}{\sqrt{c}h}-\frac{\left(6ch^2\left(4a^2h^2(4fg-eh)-2abh\left(dh^2-7egh\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$3\left(\frac{\frac{2\left(cg^2-bhg+ah^2\right)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{\sqrt{c}h}+\frac{2\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(14f\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

$$\Big\downarrow \; 219$$

3.203. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

$$3\left(\frac{\frac{2\left(cg^2-bhg+ah^2\right)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{\sqrt{c}h}-\frac{\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(14fg\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
-1/3*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^3) + (-1/2*((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h -
d*h) - b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g -
(5*c*f*g^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(h^2*(g + h
*x)^2) + (3*(-(((8*c^2*(10*f*g^4 - g^2*h*(4*e*g - d*h)) - 2*c*h*(3*b*g*(18
*f*g^2 - 6*e*g*h + d*h^2) - 2*a*h*(23*f*g^2 - 8*e*g*h + 2*d*h^2)) + h^2*(1
2*a^2*f*h^2 - 6*a*b*h*(7*f*g - e*h) + b^2*(29*f*g^2 - h*(5*e*g + d*h))) +
2*h*(3*b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(2
2*b*f*g^2 - b*h*(7*e*g - d*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*
x^2])/(h^2*(g + h*x))) + ((2*(c*g^2 - b*g*h + a*h^2)*(3*b^2*f*h^2 - 12*c*h
*(4*b*f*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - ((16*c^3*(10*
f*g^5 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h
) + b^2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) +
 b^2*g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2
)) - 24*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*
h + d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
 + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*h^2
)))/(4*h^3))/(6*(c*g^2 - b*g*h + a*h^2))
```

3.203. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.203. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.203.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2703 vs. $2(801) = 1602$.

Time = 1.04 (sec) , antiderivative size = 2704, normalized size of antiderivative = 3.25

method	result	size
risch	Expression too large to display	2704
default	Expression too large to display	6060

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*h*x+5*b*f*h+4*c*e*h-16*c*f*g)*(c*x^2+b*x+a)^(1/2)/h^5+1/8/h^5*(
(12*a*c*f*h^2+3*b^2*f*h^2+12*b*c*e*h^2-48*b*c*f*g*h+8*c^2*d*h^2-32*c^2*e*g
*h+80*c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(16
*a*b*f*h^3+16*a*c*e*h^3-64*a*c*f*g*h^2+8*b^2*e*h^3-32*b^2*f*g*h^2+16*b*c*d
*h^3-64*b*c*e*g*h^2+160*b*c*f*g^2*h-32*c^2*d*g*h^2+80*c^2*e*g^2*h-160*c^2*
f*g^3)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b
*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(8*a^2*f*
h^4+16*a*b*e*h^4-48*a*b*f*g*h^3+16*a*c*d*h^4-48*a*c*e*g*h^3+96*a*c*f*g^2*h
^2+8*b^2*d*h^4-24*b^2*e*g*h^3+48*b^2*f*g^2*h^2-48*b*c*d*g*h^3+96*b*c*e*g^2
*h^2-160*b*c*f*g^3*h+48*c^2*d*g^2*h^2-80*c^2*e*g^3*h+120*c^2*f*g^4)/h^3*(-
1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)
+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x
+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+
1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(8*a^2*e*h^5-16*a^2*f*g
*h^4+16*a*b*d*h^5-32*a*b*e*g*h^4+48*a*b*f*g^2*h^3-32*a*c*d*g*h^4+48*a*c*e*
g^2*h^3-64*a*c*f*g^3*h^2-16*b^2*d*g*h^4+24*b^2*e*g^2*h^3-32*b^2*f*g^3*h^2+
48*b*c*d*g^2*h^3-64*b*c*e*g^3*h^2+80*b*c*f*g^4*h-32*c^2*d*g^3*h^2+40*c^2*e
*g^4*h-48*c^2*f*g^5)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+...
```

3.203.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas
")
```

output
```
Timed out
```

3.203.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7155 vs. $2(801) = 1602$.

Time = 5.10 (sec) , antiderivative size = 7155, normalized size of antiderivative = 8.59

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 4*c^2*e*h^11 -
 5*b*c*f*h^11)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 64*c^3*e*g^4*h - 336*b*c^2
*f*g^4*h + 16*c^3*d*g^3*h^2 + 120*b*c^2*e*g^3*h^2 + 210*b^2*c*f*g^3*h^2 +
264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 - 60*b^2*c*e*g^2*h^3 - 96*a*c^2*e
*g^2*h^3 - 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a
*c^2*d*g*h^4 + 5*b^3*e*g*h^4 + 84*a*b*c*e*g*h^4 + 60*a*b^2*f*g*h^4 + 96*a^
2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 6*a*b^2*e*h^5 - 24*a^2*c*e*h^5
- 24*a^2*b*f*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)
*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g
^2 + b*g*h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3
*f*g^5*h^2 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*e*g^4*h^3 - 912
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*f*g^4*h^3 + 144*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*c^3*d*g^3*h^4 + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b*c^2*e*g^3*h^4 + 522*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c
*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 2
16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d*g^2*h^5 - 252*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^5*b^2*c*e*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*a*c^2*e*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^
3*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 +
78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 120*(sqrt(c)*x...
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^4}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

3.204 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

3.204.1 Optimal result

Integrand size = 32, antiderivative size = 1097

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{(64c^3g^4(5fg-eh)-16c^2g^2h(bg(41fg-7eh)-8ah(5fg-eh))}{}$$

$$-\frac{(16c^2g^4(5fg-eh)-h^2(16a^2h^2(fg-2eh)-b^2g(35fg^2+5egh+3dh^2)+4abh(7fg^2+7egh+3dh^2))-}{}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{4h\,(cg^2-bgh+ah^2)\,(g+hx)^4}-\frac{\sqrt{c}(10cfg-2ceh-3bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2h^6}$$

$$+\frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(10fg-eh)}{}$$

output
```
-1/96*(16*c^2*g^4*(-e*h+5*f*g)-h^2*(16*a^2*h^2*(-2*e*h+f*g)-b^2*g*(3*d*h^2
+5*e*g*h+35*f*g^2)+4*a*b*h*(3*d*h^2+7*e*g*h+7*f*g^2))-4*c*g*h*(b*g*(3*d*h^
2-5*e*g*h+31*f*g^2)-a*h*(9*d*h^2-5*e*g*h+25*f*g^2))+3*h*(8*c^2*g^2*(5*f*g^
2-h*(d*h+e*g))+h^2*(16*a^2*f*h^2-8*a*b*h*(-e*h+6*f*g)+b^2*(-3*d*h^2-5*e*g*
h+29*f*g^2))-4*c*h*(2*b*g*(-d*h^2-2*e*g*h+9*f*g^2)-a*h*(d*h^2-5*e*g*h+17*f
*g^2)))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/4*(f*
g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/12
8*(128*c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*h+
5*f*g))+8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h+1
4*f*g)+3*a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e*h
+6*f*g)-5*b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4*(
48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arctanh
(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))/h^6/(a*h^2-b*g*h+c*g^2)^(5/2)-1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/h^6+1/64*(64*c^3*g^4*(
-e*h+5*f*g)-16*c^2*g^2*h*(b*g*(-7*e*h+41*f*g)-8*a*h*(-e*h+5*f*g))+4*c*h^2*
(2*b^2*g^2*(-5*e*h+46*f*g)+16*a^2*h^2*(-e*h+5*f*g)-a*b*h*(-3*d*h^2-25*e*g*
h+173*f*g^2))-b*h^3*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*
h+35*f*g^2))+2*c*h*(16*c^2*g^3*(-e*h+5*f*g)-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*
h*(35*f*g^2-h*(-3*d*h+7*e*g)))+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+...
```

3.204.2 Mathematica [A] (verified)

Time = 15.59 (sec) , antiderivative size = 1005, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{\frac{128(2fg-eh)(a+x(b+cx))^{3/2}}{(g+hx)^3} - \frac{192f(a+x(b+cx))^{3/2}}{(g+hx)^2} + \frac{48h(fg^2+h(-eg+dh))(a+x(\cdots}{(cg^2+h(-b\cdots}}{\cdots}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
((128*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*x)^3 - (192*f*(a + x*(
b + c*x))^(3/2))/(g + h*x)^2 + (48*h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b
+ c*x))^(3/2)*(-2*a*h + 2*c*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h)
)*(g + h*x)^4) + (288*f*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x
) - (Sqrt[a + x*(b + c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g +
2*a*h + b*h*x)))/h^2 + (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2
*h^2 + 4*c*h*(-2*b*g + a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g
) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*
(b + c*x)])])/(2*h^3)))/(-(c*g^2) + h*(b*g - a*h)) + (24*(-2*f*g + e*h)*((
4*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*(a + x*(b + c*x))^(3/2))/(g + h
*x)^2 + (2*(-4*c^2*g^2 + b^2*h^2 + 4*c*h*(b*g - 2*a*h))*(a + x*(b + c*x))^
(3/2))/(g + h*x) + (-2*c*h*Sqrt[a + x*(b + c*x)]*(b^3*h^3 + 4*c^3*g^2*(2*g
 - h*x) + b*c*h^2*(5*b*g - 10*a*h + b*h*x) - 2*c^2*h*(b*g*(7*g - 2*h*x) +
2*a*h*(-3*g + 2*h*x))) + 16*c^(5/2)*(c*g^2 + h*(-(b*g) + a*h))^2*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*(2*c*g - b*h)*Sqrt[c*g^2
 + h*(-(b*g) + a*h)]*(8*c^2*g^2 - b^2*h^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTan
h[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqr
t[a + x*(b + c*x)])])/(c*h^3)))/(c*g^2 + h*(-(b*g) + a*h))^2 - (9*(b^2 - 4
*a*c)*h*(f*g^2 + h*(-(e*g) + d*h))*((2*Sqrt[a + x*(b + c*x)]*(-2*a*h + ...
```

3.204.3 Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+8cdg-5beg-8afg-3bdh+8aeh-2\left(-\frac{5cfg^2}{h}+ceg+4bfg-cdh-4afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^4}dx}{4(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{4h(g+hx)^4(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(8cdg-8afg+8aeh-b\left(-\frac{5fg^2}{h}+5eg+3dh\right)-2\left(-\frac{5cfg^2}{h}+ceg+4bfg-cdh-4afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^4}dx}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1229

$$-\int -\frac{(h(h(35fg^2+h(5eg+3dh))b^3-4(31cfg^3-ch(5eg-3dh)g+2ah^2(10fg+eh))b^2+48a^2fh^3b+\frac{16c^2g^3(5fg-eh)b}{h}+4ach(61fg^2-h(17eg+3dh))b-16ac(5cfg^3}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 27

$$\int \frac{(h^2(35fg^2+h(5eg+3dh))b^3-4h(31cfg^3-ch(5eg-3dh)g+2ah^2(10fg+eh))b^2+4(12a^2fh^4+ac(61fg^2-h(17eg+3dh))h^2+4c^2g^3(5fg-eh))b-16ach(5cfg^3-ch}{8h}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1230

$$\frac{(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh(173fg^2-25ehg-3dh^2))-bh^3((35fg^2+5ehg+3dh^2)}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\big\downarrow$ 25

$$\frac{\sqrt{cx^2+bx+a}(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh(173fg^2-25ehg-3dh^2))-bh^3((35fg^2-}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\big\downarrow$ 1269

3.204. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

$$\frac{\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2-\cdots\right.\right.\right.}{\cdots}}{\cdots} - \frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 1092

$$\frac{\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2-\cdots\right.\right.\right.}{\cdots}}{\cdots} - \frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 219

$$\frac{\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2-\cdots\right.\right.\right.}{\cdots}}{\cdots} - \frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 1154

$$\frac{\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2-\cdots\right.\right.\right.}{\cdots}}{\cdots} - \frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 219

$$\frac{\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2-\cdots\right.\right.\right.}{\cdots}}{\cdots} - \frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

3.204. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^4) + (-1/12*(((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h
^2*(f*g - 2*e*h) - b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2
 + 7*e*g*h + 3*d*h^2)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(
25*f*g^2 - 5*e*g*h + 9*d*h^2)) + 3*h*(4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h))
 + (8*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))/h - 8*b*c*g*(9*f*g^2 - h*(2*e*g +
 d*h)) + h*(16*a^2*f*h^2 - 8*a*b*h*(6*f*g - e*h) + b^2*(29*f*g^2 - 5*e*g*h
 - 3*d*h^2)))*x)*(a + b*x + c*x^2)^(3/2))/(h^2*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^3) + (((64*c^3*g^4*(5*f*g - e*h) - 16*c^2*g^2*h*(b*g*(41*f*g - 7*e*
h) - 8*a*h*(5*f*g - e*h)) + 4*c*h^2*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h
^2*(5*f*g - e*h) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^3*(48*a^2
*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*
c*h*(16*c^2*g^3*(5*f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g
^2 - h*(7*e*g - 3*d*h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^
2*(61*f*g^2 - h*(5*e*g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(g + h*x
)) + ((-64*Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*(c*g^2 - b*g*h + a*h^2)^
2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h + ((128*c^4*g^
5*(5*f*g - e*h) - 64*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)
) + 8*c*h^3*(24*a^3*f*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*
g - e*h) + 3*a*b^2*h*(55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b...
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
 + 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
 + b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
 + 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.204. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.204.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4358 vs. $2(1065) = 2130$.

Time = 1.21 (sec) , antiderivative size = 4359, normalized size of antiderivative = 3.97

method	result	size
risch	Expression too large to display	4359
default	Expression too large to display	10038

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*(c*x^2+b*x+a)^(1/2)*c+1/2/h^5*(c^(1/2)*(3*b*f*h+2*c*e*h-10*c*f*g)/h*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-(4*a*c*f*h^2+2*b^2*f*h^2+4*b*c
*e*h^2-20*b*c*f*g*h+2*c^2*d*h^2-10*c^2*e*g*h+30*c^2*f*g^2)/h^2/((a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(4*a*b*f*h^3+4*a*c*e*h^3-16*a*c
*f*g*h^2+2*b^2*e*h^3-8*b^2*f*g*h^2+4*b*c*d*h^3-16*b*c*e*g*h^2+40*b*c*f*g^2
*h-8*c^2*d*g*h^2+20*c^2*e*g^2*h-40*c^2*f*g^3)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*
h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))/(x+1/h*g)))+(2*a^2*f*h^4+4*a*b*e*h^4-12*a*b*f*g*h^3+4*a*c
*d*h^4-12*a*c*e*g*h^3+24*a*c*f*g^2*h^2+2*b^2*d*h^4-6*b^2*e*g*h^3+12*b^2*f*
g^2*h^2-12*b*c*d*g*h^3+24*b*c*e*g^2*h^2-40*b*c*f*g^3*h+12*c^2*d*g^2*h^2-20
*c^2*e*g^3*h+30*c^2*f*g^4)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*(
(x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(
b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x
+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a...
```

3.204.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas
")
```

output
```
Timed out
```

3.204.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.204.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

output
```
Timed out
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

3.205 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.205.1 Optimal result

Integrand size = 32, antiderivative size = 1226

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx =$$

$$-\frac{\left(128c^4fg^7-32c^3fg^5h(11bg-10ah)+8c^2gh^2(38b^2fg^4+2a^2h^2(13fg^2+3dh^2)-abgh(65fg^2+3dh^2))-\right.}{}$$

$$-\frac{(16c^2fg^5-2cgh(13bfg^3-10afg^2h+3bdgh^2-6adh^3)-h^2(4a^2h^2(2fg-3eh)-b^2g(7fg^2+3h(eg+dh}{}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{5h(cg^2-bgh+ah^2)(g+hx)^5}+\frac{c^{3/2}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16c^2h^3(35b^3f}{}$$

output
```
-1/48*(16*c^2*f*g^5-2*c*g*h*(-6*a*d*h^3-10*a*f*g^2*h+3*b*d*g*h^2+13*b*f*g^
3)-h^2*(4*a^2*h^2*(-3*e*h+2*f*g)-b^2*g*(7*f*g^2+3*h*(d*h+e*g))+2*a*b*h*(f*
g^2+3*h*(d*h+2*e*g)))+h*(4*c^2*(-3*d*g^2*h^2+7*f*g^4)+2*c*g*h*(2*a*h*(-3*e
*h+14*f*g)-b*(-6*d*h^2-3*e*g*h+28*f*g^2))+h^2*(16*a^2*f*h^2-2*a*b*h*(-3*e*
h+22*f*g)+b^2*(25*f*g^2-3*h*(d*h+e*g))))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2
-b*g*h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(
a*h^2-b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^
2+b*x+a)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+32*c^3
*g*h^2*(35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*c^2*h^
3*(35*b^3*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*f*g^2)
-3*a*b^2*g*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g
)+b^2*(7*f*g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*(e*h+8
*f*g)-b^3*(-3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))*arcta
nh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(7/2)-1/128*(128*c^4*f*g^7-32*c^3*f*g^5*h*(
-10*a*h+11*b*g)+8*c^2*g*h^2*(38*b^2*f*g^4+2*a^2*h^2*(3*d*h^2+13*f*g^2)-a*b
*g*h*(3*d*h^2+65*f*g^2))-2*c*h^3*(8*a^3*h^3*(-3*e*h+2*f*g)-2*a*b^2*g^2*h*(
3*e*h+34*f*g)+4*a^2*b*h^2*(3*d*h^2+6*e*g*h+5*f*g^2)+b^3*(-3*d*g^2*h^2+35*f
*g^4))-b*h^4*(-2*a*h+b*g)*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*g^
2+3*h*(d*h+e*g)))+h*(128*c*f*(c*g^2-h*(-a*h+b*g))^3+(-b*h+2*c*g)*(32*c^...
```

3.205. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

3.205.2 Mathematica [A] (verified)

Time = 16.34 (sec) , antiderivative size = 1363, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{f(a+x(b+cx))^{3/2}}{3h^3(g+hx)^3}$$
$$-\frac{(2fg-eh)(bg-2ah+(2cg-bh)x)(a+x(b+cx))^{3/2}}{8h^2(cg^2-h(bg-ah))(g+hx)^4}$$
$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)(a+x(b+cx))^{3/2}}{5h(cg^2-h(bg-ah))(g+hx)^5}$$
$$+\frac{(2cg-bh)(fg^2-egh+dh^2)(a+x(b+cx))^{3/2}\left(\frac{(bg-2ah+(2cg-bh)x)(a+bx+cx^2)^{3/2}}{8(cg^2-bgh+ah^2)(g+hx)^4}-\frac{3(b^2-4ac)\left(\frac{(bg-2ah+(2cg-bh)x)}{4(cg^2-bgh+ah^2)}\right.}{}\right.}{2h^2(cg^2-bgh+ah^2)(a+bx+cx^2)^{3/2}}$$
$$+\frac{f(a+x(b+cx))^{3/2}\left(-\frac{(-2cg+bh)(a+bx+cx^2)^{3/2}}{2(cg^2-bgh+ah^2)(g+hx)^2}-\frac{\frac{\left(-cg(2cg-bh)+\frac{1}{2}h\left(2bcg+b^2h-8ach\right)\right)(a+bx+cx^2)^{3/2}}{(-cg^2+bgh-ah^2)(g+hx)}+\frac{\left(-c\left(2cg-\frac{bh}{2}\right)\left(4c^2g^2-b^2h^2-\right.\right.}{}}{}\right.}{}$$
$$+\frac{3(b^2-4ac)(2fg-eh)(a+x(b+cx))^{3/2}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+b}}\right.}{(cg^2-h(bg-ah))^{3/2}}\right.}{128h^2(cg^2-h(bg-ah))(a+bx+cx^2)^{3/2}}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/3*(f*(a + x*(b + c*x))^(3/2))/(h^3*(g + h*x)^3) - ((2*f*g - e*h)*(b*g -
 2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2 - h*(b*g
- a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(
b + c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*c*g - b*h
)*(f*g^2 - e*g*h + d*h^2)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*g
- b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4)
 - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2
])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))
/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x +
c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(-1/2*((-2*c*g + b*h)*(a + b*x
+ c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g -
b*h)) + (h*(2*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*
g^2) + b*g*h - a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b
^2*h^2 - 4*c*h*(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h
^2 + 4*b*c*(2*c*g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b
*g - 2*a*h))*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 -
h*(b*g - a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h
 - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c...
```

3.205.3 Rubi [A] (verified)

Time = 2.64 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)}{(g+hx)^6} \, dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{5\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(g+hx)^5}dx}{5\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)(cx^2+bx+a)^{3/2}}{(g+hx)^5}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1229

$$-\frac{\int -\frac{\left(h^2\left(7fg^2+3h(eg+dh)\right)b^3-2\left(a(10fg+3eh)h^3+c\left(13fhg^3+3dh^3g\right)\right)b^2+4\left(4c^2fg^4+4a^2fh^4+ach^2\left(17fg^2-3h(eg+dh)\right)\right)b-24ach\left(a(2fg-eh)h^2+c\left(fg^3-d\right.\right.\right.}{2h(g+hx)^3}}{8h^2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(h^2\left(7fg^2+3h(eg+dh)\right)b^3-2\left(a(10fg+3eh)h^3+c\left(13fhg^3+3dh^3g\right)\right)b^2+4\left(4c^2fg^4+4a^2fh^4+ach^2\left(17fg^2-3h(eg+dh)\right)\right)b-24ach\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right.\right.}{(g+hx)^3}}{16h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1229

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\Big\downarrow$ 1269

3.205. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\downarrow$ 1092

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\downarrow$ 219

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\downarrow$ 1154

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$\downarrow$ 219

3.205. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3eh\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

input

```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output

```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^5) + (-1/24*(((16*c^2*f*g^5)/h - 2*c*g*(13*b*f*g^3 - 10*
a*f*g^2*h + 3*b*d*g*h^2 - 6*a*d*h^3) - h*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*
(7*f*g^3 + 3*g*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + (16
*f*(c*g^2 - h*(b*g - a*h))^2 + 3*(2*c*g - b*h)*(2*c*(f*g^3 - d*g*h^2) - h*
(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h))))*x)*(a + b*x + c*x^2)
^(3/2))/(h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + (-1/4*((128*c^4*f*g^7
- 32*c^3*f*g^5*h*(11*b*g - 10*a*h) + 8*c^2*g*h^2*(38*b^2*f*g^4 + 2*a^2*h^2
*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - 2*c*h^3*(8*a^3*h^3
*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*f*g + 3*e*h) + 4*a^2*b*h^2*(5*f*g^2 +
 6*e*g*h + 3*d*h^2) + b^3*(35*f*g^4 - 3*d*g^2*h^2)) - b*h^4*(b*g - 2*a*h)*
(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g + d*h))
) + h*(128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^3*f*g^5 - 8
*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^2*h^2*(10*
f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3 + 3*d*g*
h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*
h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h + a*h^2)*(
g + h*x)^2) - ((-256*c^(3/2)*f*(c*g^2 - b*g*h + a*h^2)^3*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h + ((256*c^5*f*g^7 - 896*c^4*f*g^5
*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*...
```

3.205. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

3.205.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.205. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13371 vs. 2(1196) = 2392.

Time = 1.39 (sec) , antiderivative size = 13372, normalized size of antiderivative = 10.91

method	result	size
default	Expression too large to display	13372

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.205.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas
")
```

output
```
Timed out
```

3.205.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

output
```
Timed out
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.205.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

output
```
Timed out
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^6}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

3.206 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.206.1 Optimal result

Integrand size = 32, antiderivative size = 657

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx =$$

$$-\frac{(b^2-4ac)\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg\right.}{512\left(cg^2-bgh+ah^2\right)^4(g+hx)^2}$$

$$+\frac{\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg^2-7egh+\right.}{192\left(cg^2-bgh+ah^2\right)^3(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6}$$

$$+\frac{(2cg(5fg^2+h(eg-7dh))+h(12ah(2fg-eh)-b(17fg^2-5egh-7dh^2)))(a+bx+cx^2)^{5/2}}{60h(cg^2-bgh+ah^2)^2(g+hx)^5}$$

$$+\frac{(b^2-4ac)^2\left(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)+b^2(7fg^2+5egh+7dh^2)-4c(3bg(eg+2dh)+a(fg\right.}{1024\left(cg^2-bgh+ah^2\right)^{9/2}}$$

output
```
1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h
+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*
h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*(f*g^2
-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1/60*(2
*c*g*(5*f*g^2+h*(-7*d*h+e*g))+h*(12*a*h*(-e*h+2*f*g)-b*(-7*d*h^2-5*e*g*h+1
7*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/1024*(-
4*a*c+b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+
5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*arctanh(
1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/
2))/(a*h^2-b*g*h+c*g^2)^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^
2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g
)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)
/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

3.206. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.206.2 Mathematica [A] (verified)

Time = 16.26 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \frac{f(bg-2ah+(2cg-bh)x)(a+x(b+cx))^{3/2}}{8h^2(cg^2-h(bg-ah))(g+hx)^4}$$
$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)(a+x(b+cx))^{3/2}}{6h(cg^2-h(bg-ah))(g+hx)^6}$$
$$+\frac{(2fg-eh)(a+bx+cx^2)(a+x(b+cx))^{3/2}}{5h(cg^2-h(bg-ah))(g+hx)^5}$$
$$+\frac{(2cg-bh)(-2fg+eh)(a+x(b+cx))^{3/2}\left(\frac{(bg-2ah+(2cg-bh)x)(a+bx+cx^2)^{3/2}}{8(cg^2-bgh+ah^2)(g+hx)^4}-\frac{3(b^2-4ac)\left(\frac{(bg-2ah+(2cg-bh)x)\sqrt{a+bx+c}}{4(cg^2-bgh+ah^2)(g+hx)^2}\right.}{1}\right.}{2h^2(cg^2-bgh+ah^2)(a+bx+cx^2)^{3/2}}$$
$$-\frac{(fg^2-egh+dh^2)(a+x(b+cx))^{3/2}\left(\frac{(cgh-\frac{1}{2}h(-12cg+7bh))(a+bx+cx^2)^{5/2}}{5(cg^2-bgh+ah^2)(g+hx)^5}-\frac{(-2(ach^2+\frac{1}{2}cg(-12cg+7bh))+b(cgh+\frac{1}{2}h(-}{}\right.}{6h^2(cg^2-bgh+}$$
$$-\frac{3(b^2-4ac)f(a+x(b+cx))^{3/2}\left(\frac{2(bg-2ah+(2cg-bh)x)\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)(g+hx)^2}-\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-h(bg-ah)}\sqrt{a+bx+cx^2}}\right)}{(cg^2-h(bg-ah))^{3/2}}\right)}{128h^2(cg^2-h(bg-ah))(a+bx+cx^2)^{3/2}}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output
```
(f*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2
- h*(b*g - a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)
*(a + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) + ((2*
f*g - e*h)*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g
 - a*h))*(g + h*x)^5) + ((2*c*g - b*h)*(-2*f*g + e*h)*(a + x*(b + c*x))^(3
/2)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g -
b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + (
(b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b
*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c
*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2
- b*g*h + a*h^2)*(a + b*x + c*x^2)^(3/2)) - ((f*g^2 - e*g*h + d*h^2)*(a +
x*(b + c*x))^(3/2)*(((c*g*h - (h*(-12*c*g + 7*b*h))/2)*(a + b*x + c*x^2)^(
5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*c*h^2 + (c*g*(-12*
c*g + 7*b*h))/2) + b*(c*g*h + (h*(-12*c*g + 7*b*h))/2))*(((b*g - 2*a*h + (
2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h
*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x +
 c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[
(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*...
```

3.206.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2181, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+12cdg-5beg-12afg-7bdh+12aeh+2\left(\frac{5cfg^2}{h}+ceg-6bfg-cdh+6afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(g+hx)^6}dx}{6\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(12cdg-b\left(-\frac{5fg^2}{h}+5eg+7dh\right)-12a(fg-eh)-2\left(6bfg-6afh-c\left(\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^6}dx}{12\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^5}dx}{2(ah^2-bgh+cg^2)}+\frac{\left(a+bx+cx^2\right)^{5/}\cdots}{\cdots}}{12\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\cdots\right.}{2(ah^2-bgh+cg^2)}}{12\left(ah^2-bgh+cg^{2}\cdots\right.}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\cdots\right.}{2(ah^2-bgh+cg^2)}}{\cdots}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1154}$$

3.206. $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx$

$$\frac{\left(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+eg))-12abh(eh+2fg)+b^2(h(7dh+5eg)+7fg^2)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\right.}{2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 219$$

$$\frac{\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)}{16\left(ah^2-bgh+cg^2\right)}\right)(24}{2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

input

```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output

```
-1/6*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^6) + (((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^
2 - b*h*(5*e*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/
(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + ((24*c^2*d*g^2 + 24*a^2*f*h^2
- 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g +
 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(((b*g - 2*a*h + (2*c*g - b*
h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (
3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(
4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a
*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2]
)])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*
(c*g^2 - b*g*h + a*h^2)))/(12*(c*g^2 - b*g*h + a*h^2))
```

3.206. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.206. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

3.206.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20683 vs. 2(631) = 1262.

Time = 1.82 (sec) , antiderivative size = 20684, normalized size of antiderivative = 31.48

method	result	size
default	Expression too large to display	20684

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.206.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas
")
```

output
```
Timed out
```

3.206.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^7}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.206.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48343 vs. $2(631) = 1262$.

Time = 9.70 (sec) , antiderivative size = 48343, normalized size of antiderivative = 73.58

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

output
```
1/512*(24*b^4*c^2*d*g^2 - 192*a*b^2*c^3*d*g^2 + 384*a^2*c^4*d*g^2 - 12*b^5
*c*e*g^2 + 96*a*b^3*c^2*e*g^2 - 192*a^2*b*c^3*e*g^2 + 7*b^6*f*g^2 - 60*a*b
^4*c*f*g^2 + 144*a^2*b^2*c^2*f*g^2 - 64*a^3*c^3*f*g^2 - 24*b^5*c*d*g*h + 1
92*a*b^3*c^2*d*g*h - 384*a^2*b*c^3*d*g*h + 5*b^6*e*g*h - 12*a*b^4*c*e*g*h
- 144*a^2*b^2*c^2*e*g*h + 448*a^3*c^3*e*g*h - 24*a*b^5*f*g*h + 192*a^2*b^3
*c*f*g*h - 384*a^3*b*c^2*f*g*h + 7*b^6*d*h^2 - 60*a*b^4*c*d*h^2 + 144*a^2*
b^2*c^2*d*h^2 - 64*a^3*c^3*d*h^2 - 12*a*b^5*e*h^2 + 96*a^2*b^3*c*e*h^2 - 1
92*a^3*b*c^2*e*h^2 + 24*a^2*b^4*f*h^2 - 192*a^3*b^2*c*f*h^2 + 384*a^4*c^2*
f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c
*g^2 + b*g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a
*c^3*g^6*h^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b
^2*c*g^4*h^4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 +
6*a^2*b^2*g^2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2
 + b*g*h - a*h^2)) + 1/7680*(15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*
c^6*f*g^8*h^5 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^5*f*g^7*h
^6 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^4*f*g^6*h^7 + 6144
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*c^5*f*g^6*h^7 - 61440*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^11*b^3*c^3*f*g^5*h^8 - 184320*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*a*b*c^4*f*g^5*h^8 + 15360*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*b^4*c^2*f*g^4*h^9 + 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x +...
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^7}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

3.207 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

3.207.1 Optimal result

Integrand size = 32, antiderivative size = 1062

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx =$$

$$-\frac{(b^2-4ac)\,(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+}{}$$

$$+\frac{(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+}{}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{7h\,(cg^2-bgh+ah^2)\,(g+hx)^7}$$

$$+\frac{(2cg(5fg^2+h(2eg-9dh))+h(14ah(2fg-eh)-b(19fg^2-5egh-9dh^2)))\,(a+bx+cx^2)^{5/2}}{84h\,(cg^2-bgh+ah^2)^2\,(g+hx)^6}$$

$$+\frac{(4c^2g^2(5fg^2+h(2eg-51dh))-7h^2(24a^2fh^2-2abh(10fg+7eh)+b^2(5fg^2+5egh+9dh^2))-2ch(3bg}{840h\,(cg^2-bgh+ah^2)^3\,(g+hx}$$

$$+\frac{(b^2-4ac)^2\,(48c^3dg^3-8c^2g(3bg(eg+3dh)+a(fg^2-8egh+3dh^2))-bh(24a^2fh^2-2abh(10fg+7eh)+}{}$$

output
```
1/384*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-
b*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*
c*(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d
*h^2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a
*h^2-b*g*h+c*g^2)^4/(h*x+g)^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)
/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^7+1/84*(2*c*g*(5*f*g^2+h*(-9*d*h+2*e*g))+h*
(14*a*h*(-e*h+2*f*g)-b*(-9*d*h^2-5*e*g*h+19*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h
/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^6+1/840*(4*c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*
g))-7*h^2*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^
2))-2*c*h*(3*b*g*(-34*d*h^2-15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*
f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*
a*c+b^2)^2*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g
^2))-b*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2
))+2*c*(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*
(21*d*h^2+10*e*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^
2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(11/2)-1/102
4*(-4*a*c+b^2)*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h
+f*g^2))-b*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f
*g^2))+2*c*(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^
2*g*(21*d*h^2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x...
```

3.207.2 Mathematica [A] (verified)

Time = 16.32 (sec) , antiderivative size = 1636, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

output
```
-1/7*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(
h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^7) + ((2*f*g - e*h)*(a + b*x + c*x^2)*
(a + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) - (f*(a
 + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g +
 h*x)^5) + (f*(2*c*g - b*h)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*
g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x
^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b
*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))
))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x
+ c*x^2)^(3/2)) - ((-2*f*g + e*h)*(a + x*(b + c*x))^(3/2)*(((c*g*h - (h*(-
12*c*g + 7*b*h))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^5) - ((-2*(a*c*h^2 + (c*g*(-12*c*g + 7*b*h))/2) + b*(c*g*h + (h*(-
12*c*g + 7*b*h))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(
3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)
*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/
(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^...
```

3.207.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 823, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+14cdg-5beg-14afg-9bdh+14aeh+2\left(\frac{5cfg^2}{h}+2ceg-7bfg-2cdh+7afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^7}dx}{7(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{7h(g+hx)^7(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(14cdg-b\left(-\frac{5fg^2}{h}+5eg+9dh\right)-14a(fg-eh)-2\left(7bfg-7afh-c\left(\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)(cx^2+bx+a)^{3/2}}{(g+hx)^7}dx}{14\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1237

$$\frac{\frac{(a+bx+cx^2)^{5/2}(2c(gh(2eg-9dh)+5fg^3)-h(-14ah(2fg-eh)-bh(9dh+5eg)+19bfg^2))}{6h(g+hx)^6(ah^2-bgh+cg^2)}-\frac{\int -\frac{\left(h\left(7\left(5fg^2+h(5eg+9dh)\right)b^2-2cg\left(-\frac{5fg^2}{h}+40eg+93c\right.\right.\right.}{}}{}}{14\left(ah\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(h\left(7\left(5fg^2+h(5eg+9dh)\right)b^2-2cg\left(-\frac{5fg^2}{h}+40eg+93dh\right)b-14ah(10fg+7eh)b+168c^2dg^2+168a^2fh^2-24ac\left(2fg^2-h(9eg-2dh)\right)\right)+2c\left(2c\left(5fg^3+h(2eg-9dh)g\right)\right.}{(g+hx)^6}}{12h(ah^2-bgh+cg^2)}}{14\left(ah^2\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1228

$$\frac{\frac{7h\left(2c\left(4a^2h^2(8fg-eh)-2abh\left(h(13eg-3dh)+13fg^2\right)+b^2\left(gh(21dh+10eg)+7fg^3\right)\right)-bh\left(24a^2fh^2-2abh(7eh+10fg)+b^2\left(h(9dh+5eg)+5fg^2\right)\right)-8c^2g\left(-ah(8eg-3c\right.\right.}{2\left(ah^2-bgh+cg^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1152

$$\frac{\frac{7h\left(2c\left(4a^2h^2(8fg-eh)-2abh\left(h(13eg-3dh)+13fg^2\right)+b^2\left(gh(21dh+10eg)+7fg^3\right)\right)-bh\left(24a^2fh^2-2abh(7eh+10fg)+b^2\left(h(9dh+5eg)+5fg^2\right)\right)-8c^2g\left(-ah(8eg-3c\right.\right.}{2\left(ah^2-bgh+cg^2\right)}}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1152

3.207. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

$$7h\Big(2c\Big(4a^2h^2(8fg-eh)-2abh\Big(h(13eg-3dh)+13fg^2\Big)+b^2\Big(gh(21dh+10eg)+7fg^3\Big)\Big)-bh\Big(24a^2fh^2-2abh(7eh+10fg)+b^2\Big(h(9dh+5eg)+5fg^2\Big)\Big)-8c^2g\Big(-ah(8eg-3c$$

$$2\Big(c$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 1154$$

$$\frac{(2c(5fg^3+h(2eg-9dh)g)-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))(cx^2+bx+a)^{5/2}}{6h(cg^2-bhg+ah^2)(g+hx)^6}+\frac{\Big(4\Big(5fg^4+h(2eg-51dh)g^2\Big)c^2-2h\Big(3bg\Big(8fg^2-15ehg-34dh^2}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{7h\left(cg^2-bhg+ah^2\right)(g+hx)^7}$$

$$\Big\downarrow 219$$

$$\frac{(2c(5fg^3+h(2eg-9dh)g)-h(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)))(cx^2+bx+a)^{5/2}}{6h(cg^2-bhg+ah^2)(g+hx)^6}+\frac{\Big(4\Big(5fg^4+h(2eg-51dh)g^2\Big)c^2-2h\Big(3bg\Big(8fg^2-15ehg-34dh^2}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{7h\left(cg^2-bhg+ah^2\right)(g+hx)^7}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

3.207. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

output
```
-1/7*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^7) + (((2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*
g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2)
)/(6*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) + (((4*c^2*(5*f*g^4 + g^2*h*(2
*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15*e*g*h - 34*d*h^2)
 - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*x^2)^(5/2))/(5*(c
*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (7*h*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2
 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*
h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(
8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*
h*(10*e*g + 21*d*h))))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)
^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g
 - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(
2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h +
 a*h^2)^(3/2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)
))/(12*h*(c*g^2 - b*g*h + a*h^2)))/(14*(c*g^2 - b*g*h + a*h^2))
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.207.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31329 vs. $2(1032) = 2064$.

Time = 2.52 (sec) , antiderivative size = 31330, normalized size of antiderivative = 29.50

method	result	size
default	Expression too large to display	31330

3.207. $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.207.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas
")
```

output
```
Timed out
```

3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

output
```
Timed out
```

3.207.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.207.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75375 vs. $2(1032) = 2064$.

Time = 31.74 (sec) , antiderivative size = 75375, normalized size of antiderivative = 70.97

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

output
```
1/1024*(48*b^4*c^3*d*g^3 - 384*a*b^2*c^4*d*g^3 + 768*a^2*c^5*d*g^3 - 24*b^
5*c^2*e*g^3 + 192*a*b^3*c^3*e*g^3 - 384*a^2*b*c^4*e*g^3 + 14*b^6*c*f*g^3 -
 120*a*b^4*c^2*f*g^3 + 288*a^2*b^2*c^3*f*g^3 - 128*a^3*c^4*f*g^3 - 72*b^5*
c^2*d*g^2*h + 576*a*b^3*c^3*d*g^2*h - 1152*a^2*b*c^4*d*g^2*h + 20*b^6*c*e*
g^2*h - 96*a*b^4*c^2*e*g^2*h - 192*a^2*b^2*c^3*e*g^2*h + 1024*a^3*c^4*e*g^
2*h - 5*b^7*f*g^2*h - 12*a*b^5*c*f*g^2*h + 336*a^2*b^3*c^2*f*g^2*h - 832*a
^3*b*c^3*f*g^2*h + 42*b^6*c*d*g*h^2 - 360*a*b^4*c^2*d*g*h^2 + 864*a^2*b^2*
c^3*d*g*h^2 - 384*a^3*c^4*d*g*h^2 - 5*b^7*e*g*h^2 - 12*a*b^5*c*e*g*h^2 + 3
36*a^2*b^3*c^2*e*g*h^2 - 832*a^3*b*c^3*e*g*h^2 + 20*a*b^6*f*g*h^2 - 96*a^2
*b^4*c*f*g*h^2 - 192*a^3*b^2*c^2*f*g*h^2 + 1024*a^4*c^3*f*g*h^2 - 9*b^7*d*
h^3 + 84*a*b^5*c*d*h^3 - 240*a^2*b^3*c^2*d*h^3 + 192*a^3*b*c^3*d*h^3 + 14*
a*b^6*e*h^3 - 120*a^2*b^4*c*e*h^3 + 288*a^3*b^2*c^2*e*h^3 - 128*a^4*c^3*e*
h^3 - 24*a^2*b^5*f*h^3 + 192*a^3*b^3*c*f*h^3 - 384*a^4*b*c^2*f*h^3)*arctan
(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h
- a*h^2))/((c^5*g^10 - 5*b*c^4*g^9*h + 10*b^2*c^3*g^8*h^2 + 5*a*c^4*g^8*h^
2 - 10*b^3*c^2*g^7*h^3 - 20*a*b*c^3*g^7*h^3 + 5*b^4*c*g^6*h^4 + 30*a*b^2*c
^2*g^6*h^4 + 10*a^2*c^3*g^6*h^4 - b^5*g^5*h^5 - 20*a*b^3*c*g^5*h^5 - 30*a^
2*b*c^2*g^5*h^5 + 5*a*b^4*g^4*h^6 + 30*a^2*b^2*c*g^4*h^6 + 10*a^3*c^2*g^4*
h^6 - 10*a^2*b^3*g^3*h^7 - 20*a^3*b*c*g^3*h^7 + 10*a^3*b^2*g^2*h^8 + 5*a^4
*c*g^2*h^8 - 5*a^4*b*g*h^9 + a^5*h^10)*sqrt(-c*g^2 + b*g*h - a*h^2)) - ...
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx=\text{Hanged}$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

output
```
\text{Hanged}
```

3.208 $\int (1+2x)^3\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

3.208.1 Optimal result

Integrand size = 32, antiderivative size = 143

$$\int (1+2x)^3\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx = \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} - \frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040} + \frac{124039\,\mathrm{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}$$

output
```
17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21
*(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*(75295+26982*x)*(3*x^2-x+2)^(3/2)+124
039/93312*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+5393/15552*(1-6*x)*(3*x^2
-x+2)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{6\sqrt{2-x+3x^2}(-543069+1493894x+3280872x^2+5497776x^3+7491456x^4+6462720x^5+2488320x^6}{3265920}$$

input
```
Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 +
7491456*x^4 + 6462720*x^5 + 2488320*x^6) + 4341365*Sqrt[3]*Log[1 - 6*x + 2
*Sqrt[6 - 3*x + 9*x^2]])/3265920
```

3.208.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$
$$\Big\downarrow \text{2184}$$
$$\frac{1}{84}\int -4(8-67x)(2x+1)^3\sqrt{3x^2-x+2}dx+\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$
$$\Big\downarrow \text{27}$$
$$\frac{2}{21}(2x+1)^4\left(3x^2-x+2\right)^{3/2}-\frac{1}{21}\int (8-67x)(2x+1)^3\sqrt{3x^2-x+2}dx$$
$$\Big\downarrow \text{1236}$$
$$\frac{1}{21}\left(\frac{67}{18}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{18}\int \frac{3}{2}(565-612x)(2x+1)^2\sqrt{3x^2-x+2}dx\right)+\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{67}{18}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{12}\int(565-612x)(2x+1)^2\sqrt{3x^2-x+2}dx\right)+\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{204}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{15}\int 3(2x+1)(2998x+4151)\sqrt{3x^2-x+2}dx\right)+\frac{67}{18}\left(3x^2-x+2\right)^{3/}\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{204}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{5}\int(2x+1)(2998x+4151)\sqrt{3x^2-x+2}dx\right)+\frac{67}{18}\left(3x^2-x+2\right)^{3/2}\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\int\sqrt{3x^2-x+2}dx-\frac{1}{54}(26982x+75295)\left(3x^2-x+2\right)^{3/2}\right)+\frac{204}{5}\left(3x^2-x+2\right)^{3/2}(2x+\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+75295)\left(3x^2-x+2\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+75\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+75295)\left(3x^2-x+2\right)^{3/2}\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

3.208. $\int(1+2x)^3\sqrt{2-x+3x^2}(1+3x+4x^2)\ dx$

input
```
Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(3/2))/21 + ((67*(1 + 2*x)^3*(2 - x + 3*x^2
)^(3/2))/18 + ((204*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2))/5 + (-1/54*((75295
+ 26982*x)*(2 - x + 3*x^2)^(3/2)) - (188755*(-1/12*((1 - 6*x)*Sqrt[2 - x +
 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/36)/5)/12)/21
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.208.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{\left(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069\right)\sqrt{3x^2-x+2}}{544320}-\frac{124039\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right.}{93312}$
trager	$\left(\frac{32}{7}x^6+\frac{748}{63}x^5+\frac{1858}{135}x^4+\frac{38179}{3780}x^3+\frac{19529}{3240}x^2+\frac{746947}{272160}x-\frac{60341}{60480}\right)\sqrt{3x^2-x+2}+\frac{124039\,\operatorname{RootOf}\left(\_Z^2-\right.}{}$
default	$-\frac{5393(-1+6x)\sqrt{3x^2-x+2}}{15552}-\frac{124039\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{93312}-\frac{45739\left(3x^2-x+2\right)^{\frac{3}{2}}}{68040}+\frac{32x^4\left(3x^2-x+2\right)^{\frac{3}{2}}}{21}+\frac{844x^3\left(3x^2-\right.}{189}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/544320*(2488320*x^6+6462720*x^5+7491456*x^4+5497776*x^3+3280872*x^2+1493
894*x-543069)*(3*x^2-x+2)^(1/2)-124039/93312*3^(1/2)*arcsinh(6/23*23^(1/2)
*(x-1/6))
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{544320}\left(2488320\,x^6+6462720\,x^5+7491456\,x^4+5497776\,x^3+3280872\,x^2+1493894\,x-543069\right)\sqrt{3\,x^2}$$
$$+\frac{124039}{186624}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*
x^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(
4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.208.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{32x^6}{7}+\frac{748x^5}{63}+\frac{1858x^4}{135}+\frac{38179x^3}{3780}+\frac{19529x^2}{3240}+\frac{746947x}{272160}-\frac{60341}{60480}\right)-\frac{124039\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{93312}$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**6/7 + 748*x**5/63 + 1858*x**4/135 + 38179*x**3
/3780 + 19529*x**2/3240 + 746947*x/272160 - 60341/60480) - 124039*sqrt(3)*
asinh(6*sqrt(23)*(x - 1/6)/23)/93312
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{32}{21}\left(3x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{7849}{3780}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{5393}{2592}\sqrt{3x^2-x+2}x-\frac{124039}{93312}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{5393}{15552}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594
/315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739
/68040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93
312*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x +
 2)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3x^2-x+}$$
$$+\frac{124039}{93312}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x
+ 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1594\,x^2\,\left(3\,x^2-x+2\right)^{3/2}}{315}+\frac{844\,x^3\,\left(3\,x^2-x+2\right)^{3/2}}{189} + \frac{32\,x^4\,\left(3\,x^2-x+2\right)^{3/2}}{21}-\frac{137057\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{136080} - \frac{5959\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{1890}-\frac{45739\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{1632960} + \frac{7849\,x\,\left(3\,x^2-x+2\right)^{3/2}}{3780}-\frac{1051997\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{3265920}$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
 + (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)
^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x
+ 2)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/16329
60 + (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 -
 x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

3.209 $\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

3.209.1 Optimal result

Integrand size = 32, antiderivative size = 118

$$\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx = \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296} + \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2} + \frac{5405\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}$$

output
```
1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+
306*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+
235/1296*(1-6*x)*(3*x^2-x+2)^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\ dx$$
$$= \frac{6\sqrt{2-x+3x^2}(5607+14638x+22344x^2+33552x^3+35712x^4+17280x^5)+27025\sqrt{3}\log\left(1-6x+2\sqrt{}\right.}{38880}$$

input
```
Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
 + 17280*x^5) + 27025*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/3888
0
```

3.209.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\ dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{72}\int -12(1-18x)(2x+1)^2\sqrt{3x^2-x+2}dx+\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{6}\int (1-18x)(2x+1)^2\sqrt{3x^2-x+2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{15}\int 12(11-17x)(2x+1)\sqrt{3x^2-x+2}dx\right)+\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\int(11-17x)(2x+1)\sqrt{3x^2-x+2}dx\right)+$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\int\sqrt{3x^2-x+2}dx-\frac{1}{108}(306x+25)\left(3x^2-x+2\right)^{3/2}\right)\right)+$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{108}(306x+\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right.\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{108}(306x+25)\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

input
```
Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((6*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/5 - (4*(-1/108*((25 + 306*x)*(2 - x + 3*x^2)^(3/2)) + (1175*(-1/12*((
1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt
[3])))/72))/5)/6
```

3.209. $\int(1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.209. $\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.209.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607)\sqrt{3x^2-x+2}}{6480}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$
trager	$\left(\frac{8}{3}x^5+\frac{248}{45}x^4+\frac{233}{45}x^3+\frac{931}{270}x^2+\frac{7319}{3240}x+\frac{623}{720}\right)\sqrt{3x^2-x+2}-\frac{5405\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-\right.\right.}{7776}$
default	$-\frac{235(-1+6x)\sqrt{3x^2-x+2}}{1296}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{7776}+\frac{277(3x^2-x+2)^{\frac{3}{2}}}{810}+\frac{8x^3(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{32x^2(3x^2-x+2)^{\frac{3}{2}}}{15}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6480*(17280*x^5+35712*x^4+33552*x^3+22344*x^2+14638*x+5607)*(3*x^2-x+2)^
(1/2)-5405/7776*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{6480}\left(17280\,x^5+35712\,x^4+33552\,x^3+22344\,x^2+14638\,x+5607\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{5405}{15552}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sq
rt(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25)
```

3.209.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{8x^5}{3}+\frac{248x^4}{45}+\frac{233x^3}{45}+\frac{931x^2}{270}+\frac{7319x}{3240}+\frac{623}{720}\right)-\frac{5405\sqrt{3}\,\mathrm{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**5/3 + 248*x**4/45 + 233*x**3/45 + 931*x**2/270
+ 7319*x/3240 + 623/720) - 5405*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/777
6
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx = \frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\sqrt{3x^2-x+2}x-\frac{5405}{7776}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{235}{1296}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3
*x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2
 - x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/129
6*sqrt(3*x^2 - x + 2)
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{6480}\left(2\left(12\left(6\left(8\left(15\,x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{5405}{7776}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt
(3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 - x + 2)) + 1)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{32\,x^2\,{\left(3\,x^2-x+2\right)}^{3/2}}{15}+\frac{8\,x^3\,{\left(3\,x^2-x+2\right)}^{3/2}}{9} - \frac{2783\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{3240} - \frac{121\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{45} + \frac{277\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{19440} + \frac{83\,x\,{\left(3\,x^2-x+2\right)}^{3/2}}{45} + \frac{6371\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{38880}$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (278
3*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (12
1*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*
x^2 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*l
og(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

3.210 $\int (1+2x)\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

3.210.1 Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592} + \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620} + \frac{437\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

output
```
2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*(745+738*x)*(3*x^2-x+2)^(3/2)+437/
15552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+19/2592*(1-6*x)*(3*x^2-x+2)^(
1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(15471+17374x+24072x^2+31536x^3+20736x^4)+2185\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9\right.}{77760}$$

input
```
Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^
4) + 2185*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/77760
```

3.210.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2184, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$

$$\Big\downarrow \; 2184$$

$$\frac{1}{60}\int 4(2x+1)(41x+2)\sqrt{3x^2-x+2}dx+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\int (2x+1)(41x+2)\sqrt{3x^2-x+2}dx+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\int \sqrt{3x^2-x+2}dx\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+\right.\right.$$
$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

input
```
Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2))/15 + (((745 + 738*x)*(2 - x + 3*x^2)
^(3/2))/108 - (95*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1
 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/72)/15
```

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.210.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\left(20736 x^{4}+31536 x^{3}+24072 x^{2}+17374 x+15471\right) \sqrt{3 x^{2}-x+2}}{12960}-\frac{437 \sqrt{3}\, \operatorname{arcsinh}\left(\frac{6 \sqrt{23}\, \left(x-\frac{1}{6}\right)}{23}\right)}{15552}$
trager	$\left(\frac{8}{5} x^{4}+\frac{73}{30} x^{3}+\frac{1003}{540} x^{2}+\frac{8687}{6480} x+\frac{191}{160}\right) \sqrt{3 x^{2}-x+2}-\frac{437 \operatorname{RootOf}\left(\_Z^{2}-3\right) \ln\left(6 \operatorname{RootOf}\left(\_Z^{2}-3\right) x+6 \sqrt{3 x^{2}\right.}{15552}$
default	$-\frac{19(-1+6 x) \sqrt{3 x^{2}-x+2}}{2592}-\frac{437 \sqrt{3}\, \operatorname{arcsinh}\left(\frac{6 \sqrt{23}\, \left(x-\frac{1}{6}\right)}{23}\right)}{15552}+\frac{961\left(3 x^{2}-x+2\right)^{\frac{3}{2}}}{1620}+\frac{8 x^{2}\left(3 x^{2}-x+2\right)^{\frac{3}{2}}}{15}+\frac{89 x\left(3 x^{2}-x+2\right)^{\frac{3}{2}}}{90}$

input
```
int((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12960*(20736*x^4+31536*x^3+24072*x^2+17374*x+15471)*(3*x^2-x+2)^(1/2)-43
7/15552*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{437}{31104}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
 x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) -
72*x^2 + 24*x - 25)
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{8x^4}{5}+\frac{73x^3}{30}+\frac{1003x^2}{540}+\frac{8687x}{6480}+\frac{191}{160}\right) - \frac{437\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**4/5 + 73*x**3/30 + 1003*x**2/540 + 8687*x/6480
+ 191/160) - 437*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/15552
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{961}{1620}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{19}{432}\sqrt{3x^2-x+2}\,x - \frac{437}{15552}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{19}{2592}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*
(3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*a
rcsinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{12960}\,(2\,(12\,(18\,(48\,x+73)x+1003)x+8687)x+15471)\sqrt{3\,x^2-x+2} + \frac{437}{15552}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
 x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
)) + 1)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8\,x^2\,\left(3\,x^2-x+2\right)^{3/2}}{15} - \frac{253\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{810} - \frac{44\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{45} + \frac{961\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{38880} + \frac{89\,x\,\left(3\,x^2-x+2\right)^{3/2}}{90} + \frac{22103\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,(6\,x-1)}{3}\right)}{77760}$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2)
+ (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/
45 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2
- x + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)
*(6*x - 1))/3))/77760
```

3.210. $\int (1+2x)\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

3.211 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$

3.211.1 Optimal result

Integrand size = 32, antiderivative size = 101

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{43\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}} - \frac{1}{8}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/8*ar
ctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/72*(13+30*x)*(3*
x^2-x+2)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{432}\left(6\sqrt{2-x+3x^2}(45+14x+48x^2) + 108\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) - 43\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

input
```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 108*Sqrt[13]*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 43*Sqrt[3]*Log[1 - 6*
x + 2*Sqrt[6 - 3*x + 9*x^2]])/432
```

3.211.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2184, 27, 1231, 25, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{2x+1}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{36}\int \frac{12(5x+4)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(5x+4)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{1}{24}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{86x+277}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{48}\int \frac{86x+277}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{1}{48}\left(43\int \frac{1}{\sqrt{3x^2-x+2}}dx+234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{3}\left(\frac{1}{48}\left(234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{43\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{3}\left(\frac{1}{48}\left(234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(\frac{1}{48}\left(\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-468\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{1}{48}\left(\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-18\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(2*(2 - x + 3*x^2)^(3/2))/9 + (((13 + 30*x)*Sqrt[2 - x + 3*x^2])/24 + ((43
*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - 18*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*
Sqrt[13]*Sqrt[2 - x + 3*x^2])])/48)/3
```

3.211. $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.211. $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{1+2x}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.211.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

method	result
risch	$\frac{(48x^2+14x+45)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8}$
default	$\frac{5(-1+6x)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{432}+\frac{2(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{8}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-\right.}{13\sqrt{12\left(x+\right.}}\right.}{8}$
trager	$\left(\frac{2}{3}x^2+\frac{7}{36}x+\frac{5}{8}\right)\sqrt{3x^2-x+2}+\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{8}$

input
```
int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/72*(48*x^2+14*x+45)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh(6/23*23^(1/
2)*(x-1/6))-1/8*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*
x+5)^(1/2))
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$$
$$= \frac{1}{72}\left(48\,x^2+14\,x+45\right)\sqrt{3\,x^2-x+2}$$
$$+ \frac{43}{864}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$
$$+ \frac{1}{16}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

output
```
1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*lo
g(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x
^2 + 4*x + 1))
```

3.211.6 Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}} + \frac{5}{12}\sqrt{3\,x^2-x+2}x + \frac{43}{432}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{1}{8}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{13}{72}\sqrt{3\,x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

output
```
2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*ar
csinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23
)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{72}\left(2\left(24\,x+7\right)x+45\right)\sqrt{3\,x^2-x+2} - \frac{43}{432}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) + \frac{1}{8}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

output
```
1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqr
t(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*s
qrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x -
sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,(4\,x^2+3\,x+1)}{2\,x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.212 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2} dx$

3.212.1 Optimal result

Integrand size = 32, antiderivative size = 108

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{8\sqrt{13}}$$

output
```
-1/13*(3*x^2-x+2)^(3/2)/(1+2*x)-11/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/
2)+17/104*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/156*
(67-96*x)*(3*x^2-x+2)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\sqrt{2-x+3x^2}(-7-2x+12x^2)}{12+24x} - \frac{17\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4\sqrt{13}} - \frac{11\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
(Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) - (17*ArcTanh[(Sqrt[
3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(4*Sqrt[13]) - (11*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6*Sqrt[3])
```

3.212.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2181, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{(64x+15)\sqrt{3x^2-x+2}}{2(2x+1)}dx-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\int \frac{(64x+15)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{26}\left(-\frac{1}{48}\int \frac{52(7-88x)}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\left(-\frac{13}{12}\int \frac{7-88x}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-44\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{(3x^2-x+2)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{44\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(-102\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{51\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(3/2)/(1 + 2*x) + (-1/6*((67 - 96*x)*Sqrt[2 - x + 3*
x^2]) - (13*((-44*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (51*ArcTanh[(9 -
 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13]))/12)/26
```

3.212. $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

3.212.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.212. $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^2}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.212.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result
risch	$\frac{36x^4-18x^3+5x^2+3x-14}{12(1+2x)\sqrt{3x^2-x+2}}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{17\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{104}$
trager	$\frac{(12x^2-2x-7)\sqrt{3x^2-x+2}}{12+24x}+\frac{17\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{104}-\frac{11\,\mathrm{Roo}}{}$
default	$\frac{(-1+6x)\sqrt{3x^2-x+2}}{12}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{18}-\frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{26\left(x+\frac{1}{2}\right)}-\frac{17\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{104}+\frac{17\sqrt{13}\,\operatorname{arctanh}\left(\right.}{}$

input
```
int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(36*x^4-18*x^3+5*x^2+3*x-14)/(1+2*x)/(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*
arcsinh(6/23*23^(1/2)*(x-1/6))+17/104*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(
1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$
$$=\frac{572\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+153\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3}}{}\right.}{1872\,(2x+1)}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*
x^2 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

3.212.6 Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{2}\sqrt{3x^2-x+2}x \\ &\quad+\frac{11}{18}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &\quad-\frac{17}{104}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right) \\ &\quad-\frac{1}{3}\sqrt{3x^2-x+2}-\frac{\sqrt{3x^2-x+2}}{4(2x+1)}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*s
qrt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqr
t(23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2
*x + 1)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(85) = 170$.

Time = 0.49 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.52

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$
$$= \frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$
$$-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$
$$-\frac{1}{8}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$
$$+\frac{67\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^3\operatorname{sgn}\left(\frac{1}{2x+1}\right)-57\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^2\operatorname{sgn}\left(\frac{1}{2x+1}\right)}{12\left(\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^2\right.}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

output
```
17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqr
t(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*s
gn(1/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x +
 1))^3*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 1
3/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sg
n(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
+ 1))^2 - 3)^2
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.213 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} \, dx$

3.213.1 Optimal result

Integrand size = 32, antiderivative size = 115

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} \, dx = \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{208\sqrt{13}}$$

output

```
-1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)-803/2704*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+11
/104*(7+10*x)*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.213.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} \, dx = \frac{\frac{39\sqrt{2-x+3x^2}(69+268x+208x^2)}{(1+2x)^2} + 2409\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) + 1859\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9\right.}{4056}$$

input

```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((39*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 + 2409*Sqrt[1
3]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 185
9*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/4056
```

3.213.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{(2x+1)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{11(10x+3)\sqrt{3x^2-x+2}}{2(2x+1)^2}dx-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{52}\int \frac{(10x+3)\sqrt{3x^2-x+2}}{(2x+1)^2}dx-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{11}{52}\left(\frac{(10x+7)\sqrt{3x^2-x+2}}{2(2x+1)}-\frac{1}{8}\int -\frac{2(47-52x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{52}\left(\frac{1}{4}\int \frac{47-52x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-26\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{26\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 222

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1154

$$\frac{11}{52}\left(\frac{1}{4}\left(-146\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 219

$$\frac{11}{52}\left(\frac{1}{4}\left(-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{73\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(3/2)/(1 + 2*x)^2 + (11*(((7 + 10*x)*Sqrt[2 - x + 3*
x^2])/(2*(1 + 2*x)) + ((-26*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (73*Ar
cTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/4))/52
```

3.213. $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.213. $\int \frac{\sqrt{2-x+3x^2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.213.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
risch	$\frac{624x^4+596x^3+355x^2+467x+138}{104(1+2x)^2\sqrt{3x^2-x+2}}-\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2704}$
trager	$\frac{(208x^2+268x+69)\sqrt{3x^2-x+2}}{104(1+2x)^2}-\frac{11\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{24}-\frac{803\,\mathrm{Ro}}{}$
default	$\frac{803\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{2704}-\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2704}+\frac{11\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right.}{338\left(x+\frac{1}{2}\right)}$

input
```
int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/104*(624*x^4+596*x^3+355*x^2+467*x+138)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-11/2
4*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-803/2704*13^(1/2)*arctanh(2/13*(9
/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$$
$$= \frac{3718\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+2409\sqrt{13}(4x^2+4x+1}{16224\,(4x^2+4x+}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4
*x + 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x +
1)
```

3.213.6 Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx &= -\frac{11}{24}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\ &\quad+\frac{803}{2704}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)\\ &\quad+\frac{55}{104}\sqrt{3x^2-x+2}\\ &\quad-\frac{(3x^2-x+2)^{\frac{3}{2}}}{26\,(4x^2+4x+1)}+\frac{11\sqrt{3x^2-x+2}}{52\,(2x+1)}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
-11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13
)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/
104*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 1
1/52*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{11}{24}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{803}{2704}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right) + \frac{1}{2}\sqrt{3x^2-x+2} + \frac{318\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-69\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-1241\sqrt{3}x+649\sqrt{3}+1241\sqrt{3x^2-x+2}}{104\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

output
```
11/24*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 803/
2704*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 1/2*sqrt(3*x^2 - x + 2) + 1/104*(318*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
^3 - 69*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1241*sqrt(3)*x + 649
*sqrt(3) + 1241*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^
2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,(4\,x^2+3\,x+1)}{(2\,x+1)^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.214 $\int (1+2x)^3 (2-x+3x^2)^{3/2} (1+3x+4x^2) \, dx$

3.214.1 Optimal result

Integrand size = 32, antiderivative size = 158

$$\int (1+2x)^3 (2-x+3x^2)^{3/2} (1+3x+4x^2) \, dx = \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)(2-x+3x^2)^{3/2}}{559872} - \frac{11(283-5850x)(2-x+3x^2)^{5/2}}{58320} + \frac{913}{486}x^2(2-x+3x^2)^{5/2} + \frac{77}{81}x^3(2-x+3x^2)^{5/2} + \frac{2}{27}(1+2x)^4(2-x+3x^2)^{5/2} + \frac{28879697\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

output
```
54593/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(
5/2)+913/486*x^2*(3*x^2-x+2)^(5/2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x
)^4*(3*x^2-x+2)^(5/2)+28879697/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)+1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int (1+2x)^3 (2-x+3x^2)^{3/2} (1+3x +4x^2) \, dx = \frac{6\sqrt{2-x+3x^2}(12499587+84014278x+201289704x^2+421626672x^3+649452672x^4+71121}{1}$$

input
```
Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*
x^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 2388
78720*x^8) + 144398485*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/134
369280
```

3.214.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1267, 27, 2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{3/2} \left(4x^2+3x+1\right) dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{108} \int 308x(2x+1)^3 \left(3x^2-x+2\right)^{3/2} dx + \frac{2}{27} \left(3x^2-x+2\right)^{5/2} (2x+1)^4$$

$$\Big\downarrow \text{ 27}$$

$$\frac{77}{27} \int x(2x+1)^3 \left(3x^2-x+2\right)^{3/2} dx + \frac{2}{27} \left(3x^2-x+2\right)^{5/2} (2x+1)^4$$

$$\Big\downarrow \text{ 1267}$$

$$\frac{77}{27} \left(\frac{1}{24} \int 4x \left(3x^2-x+2\right)^{3/2} \left(83x^2+24x+6\right) dx + \frac{1}{3} \left(3x^2-x+2\right)^{5/2} x^3\right) + \frac{2}{27} \left(3x^2-x+2\right)^{5/2} (2x+1)^4$$

$$\Big\downarrow \text{ 27}$$

$$\frac{77}{27} \left(\frac{1}{6} \int x \left(3x^2-x+2\right)^{3/2} \left(83x^2+24x+6\right) dx + \frac{1}{3} \left(3x^2-x+2\right)^{5/2} x^3\right) + \frac{2}{27} \left(3x^2-x+2\right)^{5/2} (2x+1)^4$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{21}\int -\frac{1}{2}(412-1755x)x\left(3x^2-x+2\right)^{3/2}dx+\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+ \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 27

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}-\frac{1}{42}\int(412-1755x)x\left(3x^2-x+2\right)^{3/2}dx\right)+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+ \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 1225

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\int\left(3x^2-x+2\right)^{3/2}dx-\frac{1}{60}(283-5850x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{3}\left(\right.\right. \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 1087

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\int\sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{60}(283-5850x)\left(3x^2-x+2\right)^{5/2}\right.\right.\right. \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 1087

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\right.\right.\right. \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 1090

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(\right.\right.\right.\right.\right. \frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\Big\downarrow$ 222

3.214. $\int(1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{6}\right.\right.\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

input
```
Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (77*((x^3*(2 - x + 3*x^2)^(5/2)
)/3 + ((83*x^2*(2 - x + 3*x^2)^(5/2))/21 + (-1/60*((283 - 5850*x)*(2 - x +
 3*x^2)^(5/2)) - (4963*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/
12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24
*Sqrt[3])))/16))/24)/42)/6))/27
```

3.214.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.214. $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.214.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+201289704x^2+84014278x+12499587)\sqrt{3x}}{22394880}$
trager	$\left(\frac{32}{3}x^8+\frac{205}{9}x^7+\frac{511}{18}x^6+\frac{20579}{648}x^5+\frac{563761}{19440}x^4+\frac{2927963}{155520}x^3+\frac{8387071}{933120}x^2+\frac{42007139}{11197440}x+\frac{1388843}{2488320}\right)\sqrt{3x^2-}$
default	$-\frac{54593(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{559872}-\frac{1255639(-1+6x)\sqrt{3x^2-x+2}}{4478976}-\frac{28879697\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+\frac{1207(3x^2-x+2)^{\frac{5}{2}}}{58320}$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/22394880*(238878720*x^8+510105600*x^7+635765760*x^6+711210240*x^5+649452
672*x^4+421626672*x^3+201289704*x^2+84014278*x+12499587)*(3*x^2-x+2)^(1/2)
-28879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{22394880}\left(238878720\,x^8+510105600\,x^7+635765760\,x^6+711210240\,x^5+649452672\,x^4+42\right.$$
$$+\frac{28879697}{53747712}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5
+ 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*s
qrt(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 -
x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.214.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{32x^8}{3}+\frac{205x^7}{9}+\frac{511x^6}{18}+\frac{20579x^5}{648}+\frac{563761x^4}{19440}+\frac{2927963x^3}{155520}+\frac{8387071x^2}{933120}+\frac{42007139x}{11197440}+\frac{1388843}{2488320}\right)-\frac{28879697\sqrt{3}\,\mathrm{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

input
```
integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**8/3 + 205*x**7/9 + 511*x**6/18 + 20579*x**5/64
8 + 563761*x**4/19440 + 2927963*x**3/155520 + 8387071*x**2/933120 + 420071
39*x/11197440 + 1388843/2488320) - 28879697*sqrt(3)*asinh(6*sqrt(23)*(x -
1/6)/23)/26873856
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \frac{32}{27}\left(3x^2-x+2\right)^{\frac{5}{2}}x^4 + \frac{269}{81}\left(3x^2-x+2\right)^{\frac{5}{2}}x^3 + \frac{1777}{486}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2 + \frac{1099}{648}\left(3x^2-x+2\right)^{\frac{5}{2}}x + \frac{1207}{58320}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{54593}{93312}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{54593}{559872}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{1255639}{746496}\sqrt{3x^2-x+2}x - \frac{28879697}{26873856}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{1255639}{4478976}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/
486*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/58
320*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55
9872*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 288796
97/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqr
t(3*x^2 - x + 2)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \frac{1}{22394880}\left(2\left(12\left(6\left(8\left(30\left(36\left(2\left(96x+205\right)x+511\right)x+20579\right)x+563761\right)x+2927963\right)x+83\right.\right.\right. + \frac{28879697}{26873856}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 56
3761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x
 + 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \int (2\,x+1)^3\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right) dx$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.215 $\int (1+2x)^2 (2-x+3x^2)^{3/2} (1+3x+4x^2)\,dx$

3.215.1 Optimal result

Integrand size = 32, antiderivative size = 141

$$\int (1+2x)^2 (2-x+3x^2)^{3/2} (1+3x+4x^2)\,dx = \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)(2-x+3x^2)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2(2-x+3x^2)^{5/2} + \frac{1}{12}(1+2x)^3(2-x+3x^2)^{5/2} + \frac{13(29+50x)(2-x+3x^2)^{5/2}}{2520} + \frac{48139\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

output
```
91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/63*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1
+2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29+50*x)*(3*x^2-x+2)^(5/2)+48139/165888
*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2093/27648*(1-6*x)*(3*x^2-x+2)^(1/
2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int (1+2x)^2 (2-x+3x^2)^{3/2} (1+3x+4x^2)\,dx = \frac{6\sqrt{2-x+3x^2}(1517367+2735918x+5694024x^2+10119792x^3+12173952x^4+10656000x^5}{5806080}$$

input
```
Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
 12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) + 1684865*Sqrt[3
]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5806080
```

3.215.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{3/2} \left(4x^2+3x+1\right) \, dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96} \int 4(2x+1)^2(64x+5) \left(3x^2-x+2\right)^{3/2}dx+\frac{1}{12} \left(3x^2-x+2\right)^{5/2} (2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24} \int (2x+1)^2(64x+5) \left(3x^2-x+2\right)^{3/2}dx+\frac{1}{12} \left(3x^2-x+2\right)^{5/2} (2x+1)^3$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{24} \left(\frac{1}{21} \int -13(19-90x)(2x+1) \left(3x^2-x+2\right)^{3/2}dx+\frac{64}{21}(2x+1)^2 \left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{12} \left(3x^2-x+2\right)^{5/2} (2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24} \left(\frac{64}{21}(2x+1)^2 \left(3x^2-x+2\right)^{5/2}-\frac{13}{21} \int (19-90x)(2x+1) \left(3x^2-x+2\right)^{3/2}dx\right)+\frac{1}{12} \left(3x^2-x+2\right)^{5/2} (2x+1)^3$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{24} \left(\frac{64}{21}(2x+1)^2 \left(3x^2-x+2\right)^{5/2}-\frac{13}{21} \left(\frac{49}{2} \int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{5}(50x+29) \left(3x^2-x+2\right)^{5/2}\right)\right)+\frac{1}{12} \left(3x^2-x+2\right)^{5/2} (2x+1)^3$$

$\downarrow$ 1087

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\int\sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{5}(50x+\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$\downarrow$ 1087

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$\downarrow$ 1090

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-\right.\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$\downarrow$ 222

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

input
```
Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2))/12 + ((64*(1 + 2*x)^2*(2 - x + 3*x^2)^
(5/2))/21 - (13*(-1/5*((29 + 50*x)*(2 - x + 3*x^2)^(5/2)) + (49*(-1/24*((1
 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]
) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/16))/2))/21)/24
```

3.215. $\int(1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.215. $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.215.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result
risch	$\frac{(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517367)\sqrt{3x^2-x+2}}{967680}-\frac{48139\sqrt{3}\,\text{arcsi}}{16}$
trager	$\left(6x^7+\frac{67}{7}x^6+\frac{925}{84}x^5+\frac{4529}{360}x^4+\frac{210829}{20160}x^3+\frac{33893}{5760}x^2+\frac{1367959}{483840}x+\frac{505789}{322560}\right)\sqrt{3x^2-x+2}-\frac{48139\,\text{Root}}{}$
default	$-\frac{91(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{3456}-\frac{2093(-1+6x)\sqrt{3x^2-x+2}}{27648}-\frac{48139\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}+\frac{907(3x^2-x+2)^{\frac{5}{2}}}{2520}+\frac{2x^3(3x^2}{3}$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/967680*(5806080*x^7+9262080*x^6+10656000*x^5+12173952*x^4+10119792*x^3+5
694024*x^2+2735918*x+1517367)*(3*x^2-x+2)^(1/2)-48139/165888*3^(1/2)*arcsi
nh(6/23*23^(1/2)*(x-1/6))
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int(1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx=\frac{1}{967680}\left(5806080\,x^7+9262080\,x^6+10656000\,x^5+12173952\,x^4+10119792\,x^3+5694024\,x^2+\right.$$
$$+\frac{48139}{331776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 101197
92*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/33
1776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25)
```

3.215.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int (1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(6x^7+\frac{67x^6}{7}+\frac{925x^5}{84}+\frac{4529x^4}{360}+\frac{210829x^3}{20160}+\frac{33893x^2}{5760}+\frac{1367959x}{483840}+\frac{505789}{322560}\right) - \frac{48139\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}$$

input
```
integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(6*x**7 + 67*x**6/7 + 925*x**5/84 + 4529*x**4/360 + 2
10829*x**3/20160 + 33893*x**2/5760 + 1367959*x/483840 + 505789/322560) - 4
8139*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/165888
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int (1+2x)^2\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{2}{3}\left(3x^2-x+2\right)^{\frac{5}{2}}x^3 + \frac{95}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2 + \frac{319}{252}\left(3x^2-x+2\right)^{\frac{5}{2}}x + \frac{907}{2520}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{91}{576}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{91}{3456}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{2093}{4608}\sqrt{3x^2-x+2}\,x - \frac{48139}{165888}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{2093}{27648}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*
(3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 -
 x + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x
 + 2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/276
48*sqrt(3*x^2 - x + 2)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1}{967680} (2 (12 (2 (8 (30 (12 (42 x + 67)x + 925)x + 31703)x + 210829)x + 237251)x + 1367959) + \frac{48139}{165888} \sqrt{3} \log \left(-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3x^2-x+2}\right) + 1\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*
x + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*s
qrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \int (2x+1)^2 \left(3x^2-x+2\right)^{3/2} \left(4x^2+3x+1\right) dx$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.216 $\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

3.216.1 Optimal result

Integrand size = 30, antiderivative size = 116

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736}-\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592}+\frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2}+\frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2}-\frac{37559\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}$$

output
```
-71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*
(109+102*x)*(3*x^2-x+2)^(5/2)-37559/124416*arcsinh(1/23*(1-6*x)*23^(1/2))*
3^(1/2)-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(203337+275410x+531384x^2+744336x^3+653184x^4+518400x^5+497664x^6}{870912}$$

input
```
Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 6531
84*x^4 + 518400*x^5 + 497664*x^6) - 262913*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6
- 3*x + 9*x^2]])/870912
```

3.216.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{84}\int 4(2x+1)(51x+10)\left(3x^2-x+2\right)^{3/2}dx+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\int (2x+1)(51x+10)\left(3x^2-x+2\right)^{3/2}dx+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{21}\left(\frac{497}{36}\int \left(3x^2-x+2\right)^{3/2}dx+\frac{1}{18}(102x+109)\left(3x^2-x+2\right)^{5/2}\right)+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x+109)\left(3x^2-x+2\right)^{5/2}\right)+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x \cdots\right. \frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)\right.\right.$$
$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x+10\right.$$
$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

input
```
Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + (((109 + 102*x)*(2 - x + 3*x^2)
^(5/2))/18 + (497*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((
1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt
[3])))/16))/36)/21
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.216.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337)\sqrt{3x^2-x+2}}{145152}+\frac{37559\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{124416}$
trager	$\left(\frac{24}{7}x^6+\frac{25}{7}x^5+\frac{9}{2}x^4+\frac{1723}{336}x^3+\frac{3163}{864}x^2+\frac{137705}{72576}x+\frac{7531}{5376}\right)\sqrt{3x^2-x+2}+\frac{37559\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{R}\right.}{}$
default	$\frac{71(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{2592}+\frac{1633(-1+6x)\sqrt{3x^2-x+2}}{20736}+\frac{37559\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{124416}+\frac{145(3x^2-x+2)^{\frac{5}{2}}}{378}+\frac{8x^2(3x^2-x}{21}$

input
```
int((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/145152*(497664*x^6+518400*x^5+653184*x^4+744336*x^3+531384*x^2+275410*x+
203337)*(3*x^2-x+2)^(1/2)+37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/
6))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{3}+\frac{37559}{248832}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
 275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.216.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{24x^6}{7}+\frac{25x^5}{7}+\frac{9x^4}{2}+\frac{1723x^3}{336}+\frac{3163x^2}{864}+\frac{137705x}{72576}+\frac{7531}{5376}\right)+\frac{37559\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}$$

input
```
integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(24*x**6/7 + 25*x**5/7 + 9*x**4/2 + 1723*x**3/336 + 3
163*x**2/864 + 137705*x/72576 + 7531/5376) + 37559*sqrt(3)*asinh(6*sqrt(23
)*(x - 1/6)/23)/124416
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{8}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2 + \frac{41}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x + \frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{71}{432}\left(3x^2-x+2\right)^{\frac{3}{2}}x - \frac{71}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{1633}{3456}\sqrt{3x^2-x+2}x + \frac{37559}{124416}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{1633}{20736}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(
3*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x
 + 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsi
nh(1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{1}{145152}(2(12(18(24(2(24x+25)x+63)x+1723)x+22141)x+137705)x+203337)\sqrt{3x^2-} - \frac{37559}{124416}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 13
7705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\int (2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

3.217 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

3.217.1 Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)^{5/2} + \frac{2203\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}} - \frac{13}{32}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(
1/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-
x+2)^(1/2))*13^(1/2)+1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(7977+1058x+9624x^2-1008x^3+6912x^4)+280}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) +
28080*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqr
t[13]] + 11015*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/34560
```

3.217.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{2x+1}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{60}\int \frac{20(5x+4)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{(5x+4)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{3}\left(\frac{1}{48}(30x+7)(3x^2-x+2)^{3/2}-\frac{1}{96}\int -\frac{3(134x+223)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{32}\int \frac{(134x+223)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{48}(30x+7)(3x^2-x+2)^{3/2}\right)+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{12}(402x+869)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(9965-4406x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{48}(30x+7)(3x^2-x+2)^{3/2}\right)+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\int \frac{9965-4406x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)+\frac{1}{48}(30x+7)\left(3x^2-x+2\right)^{3/2}\right)+ \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-2203\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)+ \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2203\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+86 \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \ 222$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)+\frac{1}{48} \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(-24336\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right) \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-936\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right) \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

3.217. $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

output
```
(2*(2 - x + 3*x^2)^(5/2))/15 + (((7 + 30*x)*(2 - x + 3*x^2)^(3/2))/48 + ((
(869 + 402*x)*Sqrt[2 - x + 3*x^2])/12 + ((-2203*ArcSinh[(-1 + 6*x)/Sqrt[23
]])/Sqrt[3] - 936*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^
2])])/24)/32)/3
```

3.217.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.217. $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.217.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(6912x^4-1008x^3+9624x^2+1058x+7977)\sqrt{3x^2-x+2}}{5760}-\frac{2203\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912}-\frac{13\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x}}\right.}{32}$
trager	$\left(\frac{6}{5}x^4-\frac{7}{40}x^3+\frac{401}{240}x^2+\frac{529}{2880}x+\frac{2659}{1920}\right)\sqrt{3x^2-x+2}+\frac{13\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2}}{1+2}\right.}{32}$
default	$\frac{5(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{144}+\frac{115(-1+6x)\sqrt{3x^2-x+2}}{1152}-\frac{2203\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912}+\frac{2(3x^2-x+2)^{\frac{5}{2}}}{15}+\frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\right.}{12}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/5760*(6912*x^4-1008*x^3+9624*x^2+1058*x+7977)*(3*x^2-x+2)^(1/2)-2203/691
2*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-13/32*13^(1/2)*arctanh(2/13*(9/2-
4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}\left(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{2203}{13824}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$
$$+\frac{13}{64}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

output
```
1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2
) + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x
- 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

3.217.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{15}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{5}{24}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{7}{144}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{67}{192}\sqrt{3x^2-x+2}\,x - \frac{2203}{6912}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{13}{32}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right) + \frac{869}{1152}\sqrt{3x^2-x+2}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

output
```
2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
 x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6
/23*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/a
bs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}\left(2\left(12\left(6\left(48x-7\right)x+401\right)x+529\right)x+7977\right)\sqrt{3x^2-x+2} + \frac{2203}{6912}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right) + \frac{13}{32}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

output
```
1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2
) + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2))
+ 13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2)))
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3\,x^2-x+2)^{3/2}\,(4\,x^2+3\,x+1)}{2\,x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.218 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} \, dx$

3.218.1 Optimal result

Integrand size = 32, antiderivative size = 131

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} \, dx = -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2-x+3x^2)^{5/2}}{13(1+2x)} - \frac{2327\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}} + \frac{25}{32}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
-1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-1/13*(3*x^2-x+2)^(5/2)/(1+2*x)-2327/115
2*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+25/32*arctanh(1/26*(9-8*x)*13^(1/
2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} \, dx = \frac{\frac{6\sqrt{2-x+3x^2}(-493-332x+564x^2-96x^3+288x^4)}{1+2x} - 1800\sqrt{13}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-\cdots}{\sqrt{\cdots}}\right.\cdots}{1152}$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 +
2*x) - 1800*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2
])/Sqrt[13]] - 2327*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1152
```

3.218.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{(76x+13)(3x^2-x+2)^{3/2}}{2(2x+1)}dx-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\int \frac{(76x+13)(3x^2-x+2)^{3/2}}{2x+1}dx-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{26}\left(-\frac{1}{96}\int \frac{156(1-98x)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{1}{4}(23-38x)(3x^2-x+2)^{3/2}\right)-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\left(-\frac{13}{8}\int \frac{(1-98x)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{1}{4}(23-38x)(3x^2-x+2)^{3/2}\right)-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{1}{12}(349-294x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{26(121-358x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{1}{4}(23-38x)(3x^2-x+2)^{3/2}\right)-\frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\int \frac{121-358x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}(23-38x)\left(3x^2-x+2\right)^{3/2}\right)-$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\Big\downarrow$ 1269

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-179\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\Big\downarrow$ 1090

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{179\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\Big\downarrow$ 222

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}(2\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\Big\downarrow$ 1154

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(-600\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\Big\downarrow$ 219

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{300\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}(23\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

3.218. $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}dx$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(5/2)/(1 + 2*x) + (-1/4*((23 - 38*x)*(2 - x + 3*x^2)
^(3/2)) - (13*(((349 - 294*x)*Sqrt[2 - x + 3*x^2])/12 + (13*((-179*ArcSinh
[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (300*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
 - x + 3*x^2])])/Sqrt[13]))/24))/8)/26
```

3.218.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.218.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.74

method	result
risch	$\frac{864x^6-576x^5+2364x^4-1752x^3-19x^2-171x-986}{192(1+2x)\sqrt{3x^2-x+2}}+\frac{2327\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152}+\frac{25\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right.}{32}$
trager	$\frac{(288x^4-96x^3+564x^2-332x-493)\sqrt{3x^2-x+2}}{192+384x}+\frac{25\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x}}{1+2x}\right.}{32}$
default	$\frac{(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{24}+\frac{23(-1+6x)\sqrt{3x^2-x+2}}{192}+\frac{2327\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152}-\frac{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{5}{2}}}{26\left(x+\frac{1}{2}\right)}-\frac{25\left(3\left(x+\frac{1}{2}\right)^2\right.}{15}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*(864*x^6-576*x^5+2364*x^4-1752*x^3-19*x^2-171*x-986)/(1+2*x)/(3*x^2-
x+2)^(1/2)+2327/1152*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+25/32*13^(1/2)
*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{2327\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x\right.}{}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1
) - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^
2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288
*x^4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

3.218.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx=\int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{4}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3x^2-x+2\right)^{\frac{3}{2}} \\ &+\frac{49}{32}\sqrt{3x^2-x+2}x+\frac{2327}{1152}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &-\frac{25}{32}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)-\frac{349}{192}\sqrt{3x^2-x+2}-\frac{(3x^2-x+2)^{\frac{3}{2}}}{4(2x+1)}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
 - x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) -
 25/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x +
1)
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(104) = 208$.

Time = 0.50 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.35

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{25}{32}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{2327}{1152}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{2\sqrt{13}}{2x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{13}{32}\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}\operatorname{sgn}\left(\frac{1}{2x+1}\right) + \frac{5929\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^7\operatorname{sgn}\left(\frac{1}{2x+1}\right) - 7272\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)^6\operatorname{sgn}\left(\frac{1}{2x+1}\right.}{}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

output
```
25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*
sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1)
)/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))
)*sgn(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(
2*x + 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13
)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2
*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(
2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) -
48*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))
^4*sgn(1/(2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) +
 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqr
t(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)
) + 24624*sqrt(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^2 - 3)^4
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx=\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2} \,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.219 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

3.219.1 Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} + \frac{1519\,\mathrm{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}} - \frac{1153\,\mathrm{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{64\sqrt{13}}$$

output
```
1/312*(151+122*x)*(3*x^2-x+2)^(3/2)/(1+2*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)
^2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1153/832*arctanh(1/26*(
9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/624*(1858-771*x)*(3*x^2-x+2)
^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{\frac{156\sqrt{2-x+3x^2}(182+627x+390x^2-68x^3+96x^4)}{(1+2x)^2} + 20754\sqrt{13}\,\mathrm{arctanh}\Big(\frac{\sqrt{3}+2\sqrt{3}x\cdots}{\cdots}}{7488}$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 +
2*x)^2 + 20754*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*
x^2])/Sqrt[13]] + 19747*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/74
88
```

3.219.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2181, 27, 1230, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)}{(2x+1)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{(122x+31)\left(3x^2-x+2\right)^{3/2}}{2(2x+1)^2}dx-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{52}\int \frac{(122x+31)\left(3x^2-x+2\right)^{3/2}}{(2x+1)^2}dx-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{52}\left(\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}-\frac{1}{8}\int -\frac{2(639-1028x)\sqrt{3x^2-x+2}}{2x+1}dx\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{52}\left(\frac{1}{4}\int \frac{(639-1028x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{1}{3}(1858-771x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{104(970-1519x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\int \frac{970-1519x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\downarrow$ 1269

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519}{2}\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\downarrow$ 1090

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{2\sqrt{69}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\downarrow$ 222

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(12}{}\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\downarrow$ 1154

3.219. $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(-3459\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-\frac{3459\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(122x\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(5/2)/(1 + 2*x)^2 + (((151 + 122*x)*(2 - x + 3*x^2)^
(3/2))/(6*(1 + 2*x)) + (((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/3 + (13*((-15
19*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(2*Sqrt[3]) - (3459*ArcTanh[(9 - 8*x)/(2*
Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13])))/6)/4)/52
```

3.219.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.219. $\int\frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.219. $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3} \, dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.219.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

method	result
risch	$\frac{288x^6-300x^5+1430x^4+1355x^3+699x^2+1072x+364}{48(1+2x)^2\sqrt{3x^2-x+2}}-\frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576}-\frac{1153\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16}}\right.}{832}$
trager	$\frac{(96x^4-68x^3+390x^2+627x+182)\sqrt{3x^2-x+2}}{48(1+2x)^2}-\frac{1153\,\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\operatorname{RootOf}\left(\_Z^2-13\right)x-9\,\operatorname{RootOf}\left(\_Z^2-13\right)-26\sqrt{3}}{1+2x}\right.}{832}$
default	$\frac{1153\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{4056}-\frac{257(-1+6x)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1248}-\frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576}+\frac{1153\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}{832}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/48*(288*x^6-300*x^5+1430*x^4+1355*x^3+699*x^2+1072*x+364)/(1+2*x)^2/(3*x
^2-x+2)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1153/832*13^
(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{19747\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2\right.}{}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
 4*x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x
+ 2))/(4*x^2 + 4*x + 1)
```

3.219.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{61}{312}(3x^2-x+2)^{\frac{3}{2}} - \frac{(3x^2-x+2)^{\frac{5}{2}}}{26(4x^2+4x+1)} - \frac{257}{208}\sqrt{3x^2-x+2}x - \frac{1519}{576}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{1153}{832}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{929}{312}\sqrt{3x^2-x+2} + \frac{15(3x^2-x+2)^{\frac{3}{2}}}{52(2x+1)}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1
) - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)
*x - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x +
1) - 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*
x^2 - x + 2)^(3/2)/(2*x + 1)
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(111) = 222$.

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.89

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{96}\left(2\left(24\,x-41\right)x+265\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{1519}{576}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$
$$+\frac{1153}{832}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$
$$+\frac{446\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-85\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1993\sqrt{3}x+1009\sqrt{3}+1993\sqrt{3\,x^2-x+2}}{32\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

output
```
1/96*(2*(24*x - 41)*x + 265)*sqrt(3*x^2 - x + 2) + 1519/576*sqrt(3)*log(-2
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 1153/832*sqrt(13)*log(-1
/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*s
qrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 1/32*(446*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^3 - 85*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
))^2 - 1993*sqrt(3)*x + 1009*sqrt(3) + 1993*sqrt(3*x^2 - x + 2))/(2*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)
) - 5)^2
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\left(3\,x^2-x+2\right)^{3/2}\left(4\,x^2+3\,x+1\right)}{\left(2\,x+1\right)^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.220 $\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2)\,dx$

3.220.1 Optimal result

Integrand size = 32, antiderivative size = 189

$$\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2)\,dx = \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)(2-x+3x^2)^{3/2}}{1492992} + \frac{5089(1-6x)(2-x+3x^2)^{5/2}}{155520} - \frac{(26353-21350x)(2-x+3x^2)^{7/2}}{498960} + \frac{133(1+2x)^2(2-x+3x^2)^{7/2}}{1485} + \frac{29}{330}(1+2x)^3(2-x+3x^2)^{7/2} + \frac{2}{33}(1+2x)^4(2-x+3x^2)^{7/2} + \frac{61917863\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}$$

output
```
117047/1492992*(1-6*x)*(3*x^2-x+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(
5/2)-1/498960*(26353-21350*x)*(3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-
x+2)^(7/2)+29/330*(1+2*x)^3*(3*x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(
7/2)+61917863/71663616*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2692081/1194
3936*(1-6*x)*(3*x^2-x+2)^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(9173509857+26646633218x+72088585464x^2+161269204752x^3+26363613$$

input
```
Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 161
269204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 4
19978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10)
 + 23838377255*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/27590492160
```

3.220.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2184, 27, 1236, 27, 1236, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{132}\int 4(2x+1)^3(87x+8)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{33}\int (2x+1)^3(87x+8)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{33}\left(\frac{1}{30}\int -\frac{9}{2}(111-532x)(2x+1)^2\left(3x^2-x+2\right)^{5/2}dx+\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}\right)+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\int(111-532x)(2x+1)^2\left(3x^2-x+2\right)^{5/2}dx\right)+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1236

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\int(5391-3050x)(2x+1)\left(3x^2-x+2\right)^{5/2}dx-\frac{532}{27}(2x+1)^2\left(3x^2\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1225

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\int\left(3x^2-x+2\right)^{5/2}dx+\frac{1}{84}(26353-21350x)\left(3x^2-x+2\right)\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1087

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\int\left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1087

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\int\sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1087

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$\downarrow$ 1090

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

3.220. $\int(1+2x)^3\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

$$\Big\downarrow \text{222}$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

input
```
Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + ((29*(1 + 2*x)^3*(2 - x + 3*x^2
)^(7/2))/10 - (3*((-532*(1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + (((26353 -
 21350*x)*(2 - x + 3*x^2)^(7/2))/84 + (55979*(-1/36*((1 - 6*x)*(2 - x + 3*
x^2)^(5/2)) + (115*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*(
(1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqr
t[3])))/16))/72))/8)/27))/20)/33
```

3.220.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.220.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6+347247744768x^5+263636134272x^4+1}{4598415360}$
trager	$\left(\frac{288}{11}x^{10}+\frac{2484}{55}x^9+\frac{3694}{55}x^8+\frac{120557}{1320}x^7+\frac{557147}{6160}x^6+\frac{50238389}{665280}x^5+\frac{32692973}{570240}x^4+\frac{1119925033}{31933440}x^3+\frac{42909872}{2737152}\right.$
default	$-\frac{5089(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{155520}-\frac{117047(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{1492992}-\frac{2692081(-1+6x)\sqrt{3x^2-x+2}}{11943936}-\frac{61917863\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}}{}\right.}{71663616}$

3.220. $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\, dx$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4598415360*(120394874880*x^10+207681159168*x^9+308846297088*x^8+41997815
1936*x^7+415908006912*x^6+347247744768*x^5+263636134272*x^4+161269204752*x
^3+72088585464*x^2+26646633218*x+9173509857)*(3*x^2-x+2)^(1/2)-61917863/71
663616*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} (1+3x +4x^2) \, dx = \frac{1}{4598415360} \left(120394874880 \, x^{10} + 207681159168 \, x^{9} + 308846297088 \, x^{8} + 419978151936 \, x^{7} + 41\right. + \frac{61917863}{143327232} \sqrt{3} \log \left(4 \sqrt{3} \sqrt{3 \, x^{2} - x + 2} (6 \, x - 1) - 72 \, x^{2} + 24 \, x - 25\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 41
9978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 +
161269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^
2 - x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.220.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.55

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \sqrt{3x^2-x+2} \cdot \left(\frac{288x^{10}}{11} + \frac{2484x^9}{55} + \frac{3694x^8}{55} + \frac{120557x^7}{1320} + \frac{557147x^6}{6160} + \frac{50238389x^5}{665280} + \frac{32692973x^4}{570240} + \frac{1119925033x^3}{31933440} + \frac{429098723x^2}{27371520} + \frac{13323316609x}{2299207680} + \frac{1019278873}{510935040}\right) - \frac{61917863\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{71663616}$$

input
```
integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(288*x**10/11 + 2484*x**9/55 + 3694*x**8/55 + 120557*
x**7/1320 + 557147*x**6/6160 + 50238389*x**5/665280 + 32692973*x**4/570240
 + 1119925033*x**3/31933440 + 429098723*x**2/27371520 + 13323316609*x/2299
207680 + 1019278873/510935040) - 61917863*sqrt(3)*asinh(6*sqrt(23)*(x - 1/
6)/23)/71663616
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{32}{33} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x^4 + \frac{436}{165} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x^3 + \frac{4258}{1485} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x^2 + \frac{10073}{7128} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x + \frac{92423}{498960} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} - \frac{5089}{25920} \left(3 \, x^2 - x + 2\right)^{\frac{5}{2}} x + \frac{5089}{155520} \left(3 \, x^2 - x + 2\right)^{\frac{5}{2}} - \frac{117047}{248832} \left(3 \, x^2 - x + 2\right)^{\frac{3}{2}} x + \frac{117047}{1492992} \left(3 \, x^2 - x + 2\right)^{\frac{3}{2}} - \frac{2692081}{1990656} \sqrt{3 \, x^2 - x + 2} x - \frac{61917863}{71663616} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23} (6 \, x - 1)\right) + \frac{2692081}{11943936} \sqrt{3 \, x^2 - x + 2}$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258
/1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 924
23/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 508
9/155520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 1
17047/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*
x - 61917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1
1943936*sqrt(3*x^2 - x + 2)
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} (1+3x+4x^2)\, dx = \frac{1}{4598415360}(2(12(6(8(6(36(14(48(18(40x+69)x+1847)x+120557)x+1671441)x+502\ldots + \frac{61917863}{71663616}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 120
557)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 3003691
061)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663
616*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} (1+3x+4x^2)\, dx = \int (2x+1)^3 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right) dx$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.221 $\int (1+2x)^2 (2-x+3x^2)^{5/2} (1+3x+4x^2) \, dx$

3.221.1 Optimal result

Integrand size = 32, antiderivative size = 164

$$\int (1+2x)^2 (2-x+3x^2)^{5/2} (1+3x+4x^2) \, dx = -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976}$$
$$-\frac{6739(1-6x)(2-x+3x^2)^{3/2}}{559872}-\frac{293(1-6x)(2-x+3x^2)^{5/2}}{58320}$$
$$+\frac{37}{405}(1+2x)^2(2-x+3x^2)^{7/2}+\frac{1}{15}(1+2x)^3(2-x+3x^2)^{7/2}+\frac{(2731+3430x)(2-x+3x^2)^{7/2}}{17010}-\frac{3564931\text{arcs}}{89579}$$

output
```
-6739/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)
+37/405*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/170
10*(2731+3430*x)*(3*x^2-x+2)^(7/2)-3564931/26873856*arcsinh(1/23*(1-6*x)*2
3^(1/2))*3^(1/2)-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int (1+2x)^2 (2-x+3x^2)^{5/2} (1+3x$$
$$+4x^2) \, dx = \frac{6\sqrt{2-x+3x^2}(387182961+692659234x+1693765752x^2+3096104976x^3+4171579776x^4+}{}$$

input
```
Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 3096104
976*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^
7 + 2675441664*x^8 + 2257403904*x^9) - 124772585*Sqrt[3]*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/940584960
```

3.221.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right) \, dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{120} \int 4(2x+1)^2(74x+13) \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30} \int (2x+1)^2(74x+13) \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{30} \left(\frac{1}{27} \int 2(2x+1)(980x+9) \left(3x^2-x+2\right)^{5/2}dx+\frac{74}{27}(2x+1)^2 \left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30} \left(\frac{2}{27} \int (2x+1)(980x+9) \left(3x^2-x+2\right)^{5/2}dx+\frac{74}{27}(2x+1)^2 \left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3 \left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\int \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{42}(3430x+2731)\left(3x^2-x+2\right)^{7/2}\right)+\frac{74}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1087

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{42}(3430x+2731)\left(3x^2-x+2\right)^{7/2}\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1087

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1087

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\right.\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1090

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3\right.\right.\right.\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 222

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}\right.\right.\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

3.221. $\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/15 + ((74*(1 + 2*x)^2*(2 - x + 3*x^2)^
(7/2))/27 + (2*(((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/42 + (293*(-1/36*(
(1 - 6*x)*(2 - x + 3*x^2)^(5/2)) + (115*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^
(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*
x)/Sqrt[23]])/(24*Sqrt[3])))/16))/72))/4))/27)/30
```

3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.221. $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.221.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
risch	$\frac{(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+4171579776x^4+3096104976x^3+1693765752x^2+69}{156764160}$
trager	$\left(\frac{72}{5}x^9+\frac{256}{15}x^8+\frac{1271}{45}x^7+\frac{22793}{630}x^6+\frac{722917}{22680}x^5+\frac{517309}{19440}x^4+\frac{21500729}{1088640}x^3+\frac{10081939}{933120}x^2+\frac{346329617}{78382080}x+\frac{4}{1}\right.$
default	$\frac{293(-1+6x)\left(3x^2-x+2\right)^{\frac{5}{2}}}{58320}+\frac{6739(-1+6x)\left(3x^2-x+2\right)^{\frac{3}{2}}}{559872}+\frac{154997(-1+6x)\sqrt{3x^2-x+2}}{4478976}+\frac{3564931\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/156764160*(2257403904*x^9+2675441664*x^8+4427716608*x^7+5671627776*x^6+4
996802304*x^5+4171579776*x^4+3096104976*x^3+1693765752*x^2+692659234*x+387
182961)*(3*x^2-x+2)^(1/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} (1+3x$$
$$+4x^2) \; dx = \frac{1}{156764160} \left(2257403904\,x^9 + 2675441664\,x^8 + 4427716608\,x^7 + 5671627776\,x^6 + 4996802304\,x\right.$$
$$+ \frac{3564931}{53747712}\,\sqrt{3}\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1) - 72\,x^2 + 24\,x - 25\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776
*x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 +
 692659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*l
og(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.221.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \sqrt{3x^2-x+2} \cdot \left(\frac{72x^9}{5} + \frac{256x^8}{15} + \frac{1271x^7}{45} + \frac{22793x^6}{630} + \frac{722917x^5}{22680} + \frac{517309x^4}{19440} + \frac{21500729x^3}{1088640} + \frac{10081939x^2}{933120} + \frac{346329617x}{78382080} + \frac{43020329}{17418240}\right) + \frac{3564931\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

input
```
integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(72*x**9/5 + 256*x**8/15 + 1271*x**7/45 + 22793*x**6/
630 + 722917*x**5/22680 + 517309*x**4/19440 + 21500729*x**3/1088640 + 1008
1939*x**2/933120 + 346329617*x/78382080 + 43020329/17418240) + 3564931*sqr
t(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/26873856
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{8}{15} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x^3 + \frac{472}{405} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x^2 + \frac{235}{243} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} x + \frac{5419}{17010} \left(3 \, x^2 - x + 2\right)^{\frac{7}{2}} + \frac{293}{9720} \left(3 \, x^2 - x + 2\right)^{\frac{5}{2}} x - \frac{293}{58320} \left(3 \, x^2 - x + 2\right)^{\frac{5}{2}} + \frac{6739}{93312} \left(3 \, x^2 - x + 2\right)^{\frac{3}{2}} x - \frac{6739}{559872} \left(3 \, x^2 - x + 2\right)^{\frac{3}{2}} + \frac{154997}{746496} \sqrt{3 \, x^2 - x + 2} x + \frac{3564931}{26873856} \sqrt{3} \operatorname{arsinh}\left(\frac{1}{23} \sqrt{23} (6 \, x - 1)\right) - \frac{154997}{4478976} \sqrt{3 \, x^2 - x + 2}$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/2
43*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(
3*x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x
^2 - x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sq
rt(3*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x
- 1)) - 154997/4478976*sqrt(3*x^2 - x + 2)
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{1}{156764160} (2 (12 (6 (8 (6 (36 (14 (24 (27 \, x + 32) x + 1271) x + 22793) x + 722917) x + 3621163) x - \frac{3564931}{26873856} \sqrt{3} \log \left(-2 \sqrt{3} \left(\sqrt{3} x - \sqrt{3 \, x^2 - x + 2}\right) + 1\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
 + 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 387
182961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx=\int {\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.222 $\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

3.222.1 Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} - \frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552} + \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2} - \frac{27071575\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

output
```
-51175/746496*(1-6*x)*(3*x^2-x+2)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2
)+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-270
71575/35831808*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1177025/5971968*(1-6
*x)*(3*x^2-x+2)^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65

$$\int (1+2x) \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{6\sqrt{2-x+3x^2}(10960335 + 19860062x + 41031048x^2 + 58946544x^3 + 66969216x^4 + 80034048}{3583}$$

input
```
Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^
3 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x
^8) - 27071575*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/35831808
```

3.222.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2184, 27, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(3x^2-x+2\right)^{5/2}\left(4x^2+3x+1\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{108}\int 4(2x+1)(61x+18)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int (2x+1)(61x+18)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{27}\left(\frac{445}{16}\int \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{24}(122x+137)\left(3x^2-x+2\right)^{7/2}\right)+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{24}(122x+137)\left(3x^2-x+2\right)^{7/2}\right)+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\int\sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{24}(12\right.$$
$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$\Big\downarrow$ 1087

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(\right.\right.$$
$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$\Big\downarrow$ 1090

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-\right.\right.\right.\right.$$
$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$\Big\downarrow$ 222

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1-\right.\right.$$
$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + (((137 + 122*x)*(2 - x + 3*x^2)
^(7/2))/24 + (445*(-1/36*((1 - 6*x)*(2 - x + 3*x^2)^(5/2)) + (115*(-1/24*(
(1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^
2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/16))/72))/16)/27
```

3.222. $\int(1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

3.222.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.222. $\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

3.222.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

method	result
risch	$\frac{(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+41031048x^2+19860062x+10960335)\sqrt{3x^2-x+2}}{5971968}$
trager	$\left(8x^8+\frac{61}{12}x^7+\frac{319}{24}x^6+\frac{11579}{864}x^5+\frac{58133}{5184}x^4+\frac{409351}{41472}x^3+\frac{1709627}{248832}x^2+\frac{9930031}{2985984}x+\frac{1217815}{663552}\right)\sqrt{3x^2-x+}$
default	$\frac{445(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{15552}+\frac{51175(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{746496}+\frac{1177025(-1+6x)\sqrt{3x^2-x+2}}{5971968}+\frac{27071575\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right.}{23}\right.}{35831808}$

input
```
int((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/5971968*(47775744*x^8+30357504*x^7+79377408*x^6+80034048*x^5+66969216*x^
4+58946544*x^3+41031048*x^2+19860062*x+10960335)*(3*x^2-x+2)^(1/2)+2707157
5/35831808*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx=\frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+58946544\right.$$
$$+\frac{27071575}{71663616}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 669
69216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^
2 - x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.222.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(8x^8+\frac{61x^7}{12}+\frac{319x^6}{24}+\frac{11579x^5}{864}+\frac{58133x^4}{5184}+\frac{409351x^3}{41472}+\frac{1709627x^2}{248832}+\frac{9930031x}{2985984}+\frac{1217815}{663552}\right)+\frac{27071575\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}$$

input
```
integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**8 + 61*x**7/12 + 319*x**6/24 + 11579*x**5/864 +
 58133*x**4/5184 + 409351*x**3/41472 + 1709627*x**2/248832 + 9930031*x/298
5984 + 1217815/663552) + 27071575*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3
5831808
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\frac{8}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{157}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{445}{2592}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{445}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{51175}{124416}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{51175}{746496}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{1177025}{995328}\sqrt{3x^2-x+2}\,x+\frac{27071575}{35831808}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{1177025}{5971968}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648
*(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x
^2 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3
*x^2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/3583
1808*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2
 - x + 2)
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \frac{1}{5971968}\left(2\left(12\left(6\left(8\left(6\left(36\left(2\left(96\,x+61\right)x+319\right)x+11579\right)x+58133\right)x+409351\right)x+1709627\right.\right.\right.\right.\right.\right.\right. - \frac{27071575}{35831808}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133
)*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2)
- 27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
)) + 1)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \int \left(2\,x+1\right)\,\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

3.223 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx$

3.223.1 Optimal result

Integrand size = 32, antiderivative size = 147

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} + \frac{2}{21}(2-x+3x^2)^{7/2} + \frac{944521\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}} - \frac{169}{128}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/10368*(2449+2154*x)*(3*x^2-x+2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2
)+2/21*(3*x^2-x+2)^(7/2)+944521/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1
/2)-169/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/82
944*(221999-17850*x)*(3*x^2-x+2)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(11665053-2120998x+12466776x^2-3646512x^3}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3
+ 15700608*x^4 - 3836160*x^5 + 7464960*x^6) + 45995040*Sqrt[13]*ArcTanh[(S
qrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 33058235*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/17418240
```

3.223.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2184, 27, 1231, 25, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{2x+1}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{84}\int \frac{28(5x+4)(3x^2-x+2)^{5/2}}{2x+1}dx+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(5x+4)(3x^2-x+2)^{5/2}}{2x+1}dx+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{1}{360}(150x+29)(3x^2-x+2)^{5/2}-\frac{1}{144}\int -\frac{(718x+1061)(3x^2-x+2)^{3/2}}{2x+1}dx\right)+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{144}\int \frac{(718x+1061)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{1}{360}(150x+29)(3x^2-x+2)^{5/2}\right)+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{24}(2154x+2449)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{6(33529-5950x)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{1}{360}(150x+29)\left(3\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\int \frac{(33529-5950x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{24}(2154x+2449)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{360}(150x+29)\left(3x^2\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1231

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{12}(221999-17850x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(1902791-1889042x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{24}(2154x+2449)\left(3x\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\int \frac{1902791-1889042x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(221999-17850x)\right)+\frac{1}{24}(2154x+2449)\left(3x^2-\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1269

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-944521\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(221999\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1090

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{944521\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 222

3.223. $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(221999-\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 1154

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(-5694624\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(221\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-219024\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(2\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(2*(2 - x + 3*x^2)^(7/2))/21 + (((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/360 +
 (((2449 + 2154*x)*(2 - x + 3*x^2)^(3/2))/24 + (((221999 - 17850*x)*Sqrt[2
 - x + 3*x^2])/12 + ((-944521*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - 2190
24*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/24)/16)/1
44)/3
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.223. $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.223. $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} \, dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.223.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result
risch	$\frac{(7464960x^6-3836160x^5+15700608x^4-3646512x^3+12466776x^2-2120998x+11665053)\sqrt{3x^2-x+2}}{2903040}-\frac{944521\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(\ldots}{23}\right.}{497664}$
trager	$\left(\frac{18}{7}x^6-\frac{37}{28}x^5+\frac{649}{120}x^4-\frac{8441}{6720}x^3+\frac{74207}{17280}x^2-\frac{1060499}{1451520}x+\frac{144013}{35840}\right)\sqrt{3x^2-x+2}+\frac{169\,\mathrm{RootOf}\left(\_Z^2-13\right)\ldots}{}$
default	$\frac{5(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{216}+\frac{575(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{10368}+\frac{13225(-1+6x)\sqrt{3x^2-x+2}}{82944}-\frac{944521\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{497664}+\ldots$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2903040*(7464960*x^6-3836160*x^5+15700608*x^4-3646512*x^3+12466776*x^2-2
120998*x+11665053)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)*arcsinh(6/23*23
^(1/2)*(x-1/6))-169/128*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/
2)^2-16*x+5)^(1/2))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{2903040}\left(7464960\,x^6 - 3836160\,x^5 + 15700608\,x^4 - 3646512\,x^3 + \right.$$
$$+ \frac{944521}{995328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1) - 72\,x^2 + 24\,x - 25\right)$$
$$+ \frac{169}{256}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9) + 220\,x^2 - 196\,x + 185}{4\,x^2+4\,x+1}\right)$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

output
```
1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 124667
76*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)
*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/2
56*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196
*x + 185)/(4*x^2 + 4*x + 1))
```

3.223.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{2}{21}\left(3\,x^2-x+2\right)^{\frac{7}{2}} + \frac{5}{36}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x + \frac{29}{1080}\left(3\,x^2-x+2\right)^{\frac{5}{2}} + \frac{359}{1728}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x + \frac{2449}{10368}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{2975}{13824}\sqrt{3\,x^2-x+2}x - \frac{944521}{497664}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{169}{128}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{221999}{82944}\sqrt{3\,x^2-x+2}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

output
```
2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
 - x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x
 + 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt
(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^
2 - x + 2)
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{2903040}\left(2\left(12\left(18\left(8\left(30\left(72\,x-37\right)x+4543\right)x-8441\right)x+519449\right.\right.\right. + \frac{944521}{497664}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) + \frac{169}{128}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

output
```
1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x
- 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-
6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2
*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqr
t(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3\,x^2-x+2)^{5/2}\,(4\,x^2+3\,x+1)}{2\,x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

3.224 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} \, dx$

3.224.1 Optimal result

Integrand size = 32, antiderivative size = 154

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} \, dx = -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{315623\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}} + \frac{429}{128}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
-11/864*(67-78*x)*(3*x^2-x+2)^(3/2)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-1/
13*(3*x^2-x+2)^(7/2)/(1+2*x)-315623/41472*arcsinh(1/23*(1-6*x)*23^(1/2))*3
^(1/2)+429/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1
1/6912*(4727-3090*x)*(3*x^2-x+2)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2} \, dx = \frac{\frac{6\sqrt{2-x+3x^2}\left(-364257-322972x+310660x^2-115680x^3+251424x^4-65664x^5+103680x^6\right)}{1+2x}}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 25
1424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) - 1389960*Sqrt[13]*ArcTanh[(
Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 1578115*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/207360
```

3.224.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{11(8x+1)(3x^2-x+2)^{5/2}}{2(2x+1)}dx-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{26}\int \frac{(8x+1)(3x^2-x+2)^{5/2}}{2x+1}dx-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{11}{26}\left(-\frac{1}{144}\int \frac{52(1-52x)(3x^2-x+2)^{3/2}}{2x+1}dx-\frac{1}{90}(37-60x)(3x^2-x+2)^{5/2}\right)-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{26}\left(-\frac{13}{36}\int \frac{(1-52x)(3x^2-x+2)^{3/2}}{2x+1}dx-\frac{1}{90}(37-60x)(3x^2-x+2)^{5/2}\right)-\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{12}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{12(187-1030x)\sqrt{3x^2-x+2}}{2x+1}dx\right)-\frac{1}{90}(37-60x)\left(3x^2-x\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 27$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\int \frac{(187-1030x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{12}(67-78x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{90}(37-60x)\left(3x^2-x+2\right)\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 1231$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{12}(4727-3090x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(26063-57386x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}(67-78x)\left(3x^2-x+\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 27$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\int \frac{26063-57386x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(4727-3090x)\right)+\frac{1}{12}(67-78x)\left(3x^2-x+2\right)\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 1269$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-28693\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727-3\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 1090$$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{28693\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \; 222$$

3.224. $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727-3090\right.\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\big\downarrow$ 1154

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(-109512\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727\right.\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\big\downarrow$ 219

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-4212\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727\right.\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(7/2)/(1 + 2*x) + (11*(-1/90*((37 - 60*x)*(2 - x + 3
*x^2)^(5/2)) - (13*(((67 - 78*x)*(2 - x + 3*x^2)^(3/2))/12 + (((4727 - 309
0*x)*Sqrt[2 - x + 3*x^2])/12 + ((-28693*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt
[3] - 4212*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/2
4)/8))/36))/26
```

3.224.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.224. $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.224. $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

3.224.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
risch	$\frac{311040x^8-300672x^7+1027296x^6-729792x^5+1550508x^4-1510936x^3-148479x^2-281687x-728514}{34560(1+2x)\sqrt{3x^2-x+2}}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(}{23}\right.}{41472}$
trager	$\frac{(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)\sqrt{3x^2-x+2}}{34560+69120x}+\frac{429\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\right.}{}\right.}{}$
default	$\frac{(-1+6x)(3x^2-x+2)^{\frac{5}{2}}}{36}+\frac{115(-1+6x)(3x^2-x+2)^{\frac{3}{2}}}{1728}+\frac{2645(-1+6x)\sqrt{3x^2-x+2}}{13824}+\frac{315623\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{41472}-\frac{(3(}{}$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/34560*(311040*x^8-300672*x^7+1027296*x^6-729792*x^5+1550508*x^4-1510936*
x^3-148479*x^2-281687*x-728514)/(1+2*x)/(3*x^2-x+2)^(1/2)+315623/41472*3^(
1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+429/128*13^(1/2)*arctanh(2/13*(9/2-4*x
)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{1578115\,\sqrt{3}(2\,x+1)\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+2\right.}{}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*s
qrt(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) +
 12*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 32297
2*x - 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

3.224.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{6}\left(3x^2-x+2\right)^{\frac{5}{2}}x - \frac{7}{90}\left(3x^2-x+2\right)^{\frac{5}{2}} \\ &+ \frac{143}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}x - \frac{737}{864}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{4(2x+1)} \\ &+ \frac{5665}{1152}\sqrt{3x^2-x+2}x + \frac{315623}{41472}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{429}{128}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|} - \frac{9\sqrt{23}}{23\,|2x+1|}\right) - \frac{51997}{6912}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2
 - x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/
2)/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcs
inh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(
23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 -
 x + 2)
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(123) = 246$.

Time = 0.57 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.94

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

output
```
429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*ab
s(-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x
 + 1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
+ 1)))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*
sgn(1/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
3) + sqrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(
2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) +
 39468815*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9
*sgn(1/(2*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x +
1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084
680*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
)^6*sgn(1/(2*x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)
+ sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x
 + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 139
7032245*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*s
gn(1/(2*x + 1)) - 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538...
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3\,x^2-x+2)^{5/2}\,(4\,x^2+3\,x+1)}{(2\,x+1)^2}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

3.224. $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

3.225 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3} dx$

3.225.1 Optimal result

Integrand size = 32, antiderivative size = 161

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3} dx = \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} + \frac{118423\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}} - \frac{1631}{256}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/832*(1227-838*x)*(3*x^2-x+2)^(3/2)+1/520*(257+134*x)*(3*x^2-x+2)^(5/2)/(
1+2*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*2
3^(1/2))*3^(1/2)-1631/256*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
*13^(1/2)+1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3} dx = \frac{\frac{6\sqrt{2-x+3x^2}(142057+464446x+256564x^2-76200x^3+83616x^4-22464x^5+27648x^6)}{(1+2x)^2} + 58}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 8361
6*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 + 587160*Sqrt[13]*ArcTanh[(Sqr
t[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 592115*Sqrt[3]*Log
[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/46080
```

3.225.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2181, 27, 1230, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{(2x+1)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{(134x+29)(3x^2-x+2)^{5/2}}{2(2x+1)^2}dx-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{52}\int \frac{(134x+29)(3x^2-x+2)^{5/2}}{(2x+1)^2}dx-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{52}\left(\frac{(134x+257)(3x^2-x+2)^{5/2}}{10(2x+1)}-\frac{1}{8}\int -\frac{2(793-1676x)(3x^2-x+2)^{3/2}}{2x+1}dx\right)-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{52}\left(\frac{1}{4}\int \frac{(793-1676x)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{(134x+257)(3x^2-x+2)^{5/2}}{10(2x+1)}\right)-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{1}{4}(1227-838x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{156(1517-3490x)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{(134x+257)\left(3x^2-\right.}{10(2x+1)}\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\int \frac{(1517-3490x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{4}(1227-838x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{(134x+257)\left(3x^2-x+2\right.}{10(2x+1)}\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1231

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{12}(21317-10470x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(136013-236846x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{4}(1227-838x)\left(3x^2-x\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\int \frac{136013-236846x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(21317-10470x)\right)+\frac{1}{4}(1227-838x)\left(3x^2-x+\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1269

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-118423\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21317-\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1090

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{118423\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(2\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

3.225. $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

$$\downarrow \text{222}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21317-104\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\downarrow \text{1154}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(-508872\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21317\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\downarrow \text{219}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-19572\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(213\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(7/2)/(1 + 2*x)^2 + (((257 + 134*x)*(2 - x + 3*x^2)^
(5/2))/(10*(1 + 2*x)) + (((1227 - 838*x)*(2 - x + 3*x^2)^(3/2))/4 + (13*((
(21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/12 + ((-118423*ArcSinh[(-1 + 6*x)/S
qrt[23]])/Sqrt[3] - 19572*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 -
x + 3*x^2])])/24))/8)/4)/52
```

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.225. $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.225.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result
risch	$\frac{82944x^8-95040x^7+328608x^6-357144x^5+1013124x^4+984374x^3+474853x^2+786835x+284114}{7680(1+2x)^2\sqrt{3x^2-x+2}}-\frac{118423\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right.}{23}\right.}{9216}$
trager	$\frac{(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)\sqrt{3x^2-x+2}}{7680(1+2x)^2}-\frac{7\,\operatorname{RootOf}\left(\_Z^2-705757\right)\ln\left(-\frac{8\,\operatorname{RootOf}\left(\right.}{}\right.}{}$
default	$\frac{1631\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{5}{2}}}{6760}-\frac{419(-1+6x)\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}{2496}-\frac{1745(-1+6x)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{1536}-\frac{118423\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{}}{}\right.}{9216}$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7680*(82944*x^8-95040*x^7+328608*x^6-357144*x^5+1013124*x^4+984374*x^3+4
74853*x^2+786835*x+284114)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-118423/9216*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))-1631/256*13^(1/2)*arctanh(2/13*(9/2-4*x)*1
3^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{592115\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2\right.}{}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2
)*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
 + 4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*
x^2 + 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

3.225.6 Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx &= \frac{67}{520}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{(3x^2-x+2)^{\frac{7}{2}}}{26(4x^2+4x+1)}\\ &- \frac{419}{416}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{1227}{832}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{19(3x^2-x+2)^{\frac{5}{2}}}{52(2x+1)}\\ &- \frac{1745}{256}\sqrt{3x^2-x+2}\,x - \frac{118423}{9216}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right)\\ &+ \frac{1631}{256}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{21317}{1536}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1
) - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/
52*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 1184
23/9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(1
3)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21
317/1536*sqrt(3*x^2 - x + 2)
```

3.225.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.68

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{7680}(6(4(18(16x-29)x+1321)x-7937)x+103837)\sqrt{3x^2-x+2} + \frac{118423}{9216}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{1631}{256}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right) + \frac{13\left(574\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-101\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-2745\sqrt{3}x+1369\sqrt{3}+2745\sqrt{3x^2-x+2}\right)}{128\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

output
```
1/7680*(6*(4*(18*(16*x - 29)*x + 1321)*x - 7937)*x + 103837)*sqrt(3*x^2 -
x + 2) + 118423/9216*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x +
2)) + 1) + 1631/256*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sq
rt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(
3*x^2 - x + 2))) + 13/128*(574*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 101*s
qrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 2745*sqrt(3)*x + 1369*sqrt(3)
 + 2745*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx=\int \frac{{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

3.226 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

3.226.1 Optimal result

Integrand size = 32, antiderivative size = 693

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2(12fg^2-20h(3eg+4dh)))\,(g+hx)^2\sqrt{a+bx+cx^2}}{240c^3h}$$
$$-\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$
$$+\frac{(945b^4fh^4-64c^4g^2(3fg^2-5h(3eg+16dh))-210b^2ch^3(14afh+5b(3fg+eh))+8c^2h^2(128a^2fh^2+}{}$$
$$+\frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-80bc^2h(3a^2fh^2+3abh(3fg+eh)+b^2(3fg^2+3}{}$$

output
```
1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-80*
b*c^2*h*(3*a^2*f*h^2+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))-128*
c^4*g*(b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+96*c^3*(a^2*h^2*(e*h+3*f*g
)+b^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(63*b^2*f*h^2-2*c*
h*(32*a*f*h+35*b*e*h+24*b*f*g)-c^2*(12*f*g^2-20*h*(4*d*h+3*e*g)))*(h*x+g)^
2*(c*x^2+b*x+a)^(1/2)/c^3/h-1/40*(9*b*f*h+2*c*(-5*e*h+f*g))*(h*x+g)^3*(c*x
^2+b*x+a)^(1/2)/c^2/h+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/1920*(945*
b^4*f*h^4-64*c^4*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-210*b^2*c*h^3*(14*a*f*h+
5*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b^2*(129
*f*g^2+50*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))+b*g*
(39*f*g^2+5*h*(54*d*h+47*e*g)))-2*c*h*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f*h+
25*b*e*h+39*b*f*g)+16*c^3*g*(3*f*g^2-5*h*(10*d*h+3*e*g))+8*c^2*h*(a*h*(45*
e*h+71*f*g)+b*(50*d*h^2+80*e*g*h+21*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^5/h
```

3.226.2 Mathematica [A] (verified)

Time = 2.67 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.85

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^4fh^3-210b^2ch^2(5beh+14afh+3bf(5g+hx))+32c^4(10dh(18g^2+9ghx+2}{}$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h +
 14*a*f*h + 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*
x^2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 +
20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h
*(825*f*g + 275*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) +
 3*f*(300*g^2 + 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*
d*h + 9*e*h*x) + f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 +
50*g^2*h*x + 35*g*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54
*g^2 + 30*g*h*x + 7*h^2*x^2))))) + 15*(-256*c^5*d*g^3 + 63*b^5*f*h^3 - 70*
b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) + 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g +
d*h) + b*g*(e*g + 3*d*h)) + 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h
) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2
*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b
 + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

3.226.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(g+hx)^3(bfg-10cdh+8afh+(2cfg-10ceh+9bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{5ch^2}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\int \frac{(g+hx)^3(bfg-10cdh+8afh+(2cfg-10ceh+9bfh)x)}{\sqrt{cx^2+bx+a}}dx}{10ch}$$

$$\Big\downarrow \text{1236}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-$$
$$\frac{\frac{\int -\frac{(g+hx)^2\left(9fghb^2+54afh^2b-2cg(3fg+5eh)b+4ch(20cdg-13afg-15aeh)+\left(-4\left(3fg^2-5h(3eg+4dh)\right)c^2-2h(24bfg+35beh+32afh)c+63b^2fh^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+(g+}{10ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-$$
$$\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\int \frac{(g+hx)^2\left(9fghb^2+54afh^2b-2cg(3fg+5eh)b+4ch(20cdg-13afg-15aeh)+\left(-\left(\left(12fg^2-20h(3eg+4dh)\right)c^2\right)-\right.\right.}{\sqrt{cx^2+bx+a}}}{8c}}{10ch}$$

$$\Big\downarrow \text{1236}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-$$
$$\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\int -\frac{(g+hx)\left(63fgh^2b^3+2\left(126afh^3-cgh(51fg+35eh)\right)b^2+4c\left(6cfg^3+10ch(3eg+2dh)g-5ah^2(29fg+14eh)\right)b-\right.}{}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} - \frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c} - \int \frac{(g+hx)\left(63fgh\right.}{}}{}}{}$$

$\downarrow$ 1225

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} - \frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c} - \frac{-\frac{15h\left(96c^3\left(a^2h\right.\right.}{}}{}}{}}{}$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} - \frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c} - \frac{-\frac{15h\left(96c^3\left(a^2h\right.\right.}{}}{}}{}}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} - \frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c} - \frac{-\frac{15h\,\text{arctanh}}{}}{}}{}}{}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

3.226. $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

output
```
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) - (((2*c*f*g - 10*c*e*h + 9*
b*f*h)*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c) - (((63*b^2*f*h^2 - 2*c*h*
(24*b*f*g + 35*b*e*h + 32*a*f*h) - 4*c^2*(3*f*g^2 - 5*h*(3*e*g + 4*d*h)))*
(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4*((945*b^4*f*h^4 - 64*c^4*
(3*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*
g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(12
9*f*g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g +
 d*h)) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 -
14*b*c*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + c^3*(48*f*g^3 - 80*g*h*(3*e*
g + 10*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g
+ 45*e*h)))*x)*Sqrt[a + b*x + c*x^2])/c^2 - (15*h*(256*c^5*d*g^3 - 63*b^5*
f*h^3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*
a*h*(e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(
3*f*g + e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g +
 e*h) + b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*
h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))
/(6*c))/(8*c))/(10*c*h)
```

3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.226. $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.226.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.17

method	result
risch	$\frac{(384 f h^3 c^4 x^4 - 432 b c^3 f h^3 x^3 + 480 c^4 e h^3 x^3 + 1440 c^4 f g h^2 x^3 - 512 a c^3 f h^3 x^2 + 504 b^2 c^2 f h^3 x^2 - 560 b c^3 e h^3 x^2 - 1680 b c^3 f g h^2 x^2 + 640 c}{}$
default	Expression too large to display

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*c^4*f*h^3*x^4-432*b*c^3*f*h^3*x^3+480*c^4*e*h^3*x^3+1440*c^4*f
*g*h^2*x^3-512*a*c^3*f*h^3*x^2+504*b^2*c^2*f*h^3*x^2-560*b*c^3*e*h^3*x^2-1
680*b*c^3*f*g*h^2*x^2+640*c^4*d*h^3*x^2+1920*c^4*e*g*h^2*x^2+1920*c^4*f*g^
2*h*x^2+1288*a*b*c^2*f*h^3*x-720*a*c^3*e*h^3*x-2160*a*c^3*f*g*h^2*x-630*b^
3*c*f*h^3*x+700*b^2*c^2*e*h^3*x+2100*b^2*c^2*f*g*h^2*x-800*b*c^3*d*h^3*x-2
400*b*c^3*e*g*h^2*x-2400*b*c^3*f*g^2*h*x+2880*c^4*d*g*h^2*x+2880*c^4*e*g^2
*h*x+960*c^4*f*g^3*x+1024*a^2*c^2*f*h^3-2940*a*b^2*c*f*h^3+2200*a*b*c^2*e*
h^3+6600*a*b*c^2*f*g*h^2-1280*a*c^3*d*h^3-3840*a*c^3*e*g*h^2-3840*a*c^3*f*
g^2*h+945*b^4*f*h^3-1050*b^3*c*e*h^3-3150*b^3*c*f*g*h^2+1200*b^2*c^2*d*h^3
+3600*b^2*c^2*e*g*h^2+3600*b^2*c^2*f*g^2*h-4320*b*c^3*d*g*h^2-4320*b*c^3*e
*g^2*h-1440*b*c^3*f*g^3+5760*c^4*d*g^2*h+1920*c^4*e*g^3)*(c*x^2+b*x+a)^(1/
2)/c^5-1/256*(240*a^2*b*c^2*f*h^3-96*a^2*c^3*e*h^3-288*a^2*c^3*f*g*h^2-280
*a*b^3*c*f*h^3+240*a*b^2*c^2*e*h^3+720*a*b^2*c^2*f*g*h^2-192*a*b*c^3*d*h^3
-576*a*b*c^3*e*g*h^2-576*a*b*c^3*f*g^2*h+384*a*c^4*d*g*h^2+384*a*c^4*e*g^2
*h+128*a*c^4*f*g^3+63*b^5*f*h^3-70*b^4*c*e*h^3-210*b^4*c*f*g*h^2+80*b^3*c^
2*d*h^3+240*b^3*c^2*e*g*h^2+240*b^3*c^2*f*g^2*h-288*b^2*c^3*d*g*h^2-288*b^
2*c^3*e*g^2*h-96*b^2*c^3*f*g^3+384*b*c^4*d*g^2*h+128*b*c^4*e*g^3-256*c^5*d
*g^3)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 1435, normalized size of antiderivative = 2.07

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*
(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h
 + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*
c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d
- 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 24
0*a^2*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c
^5*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^
4)*f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^
2 - 44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2
- 44*a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*
(30*c^5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 3
0*(8*c^5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*
a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h +
30*(48*c^5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*
d - 10*(35*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*
sqrt(c*x^2 + b*x + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*
c^3 - 4*a*c^4)*f)*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3
*c^2 - 12*a*b*c^3)*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2
 - 12*a*b*c^3)*e + (35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (...
```

3.226.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. 2(721) = 1442.

Time = 1.28 (sec) , antiderivative size = 1613, normalized size of antiderivative = 2.33

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**4/(5*c) + x**3*(-9*b*f*h**3/(
10*c) + e*h**3 + 3*f*g*h**2)/(4*c) + x**2*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f
*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2
*h)/(3*c) + x*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b
*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c)
 + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g
**3)/(2*c) + (-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 +
 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3*b*(-3*a*(
-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h**3/(5*c) -
 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**
2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c) + 3*d*g**2
*h + e*g**3)/c) + (-a*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*
c) - 5*b*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**
2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2
*h + f*g**3)/(2*c) - b*(-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c)
+ e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3
*b*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h*
*3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 +
 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c)
+ 3*d*g**2*h + e*g**3)/(2*c) + d*g**3)*Piecewise((log(b + 2*sqrt(c)*sqr...
```

3.226.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.15

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,fh^3x}{c}+\frac{30\,c^4fgh^2+10\,c^4eh^3-9\,bc^3fh^3}{c^5}\right)x+\frac{240\,c^4fg^2h+240\,c^4egh^2}{}\right.\right.\right.$$
$$-\frac{(256\,c^5dg^3-128\,bc^4eg^3+96\,b^2c^3fg^3-128\,ac^4fg^3-384\,bc^4dg^2h+288\,b^2c^3eg^2h-384\,ac^4eg^2h-240\,b}{}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 + 10*
c^4*e*h^3 - 9*b*c^3*f*h^3)/c^5)*x + (240*c^4*f*g^2*h + 240*c^4*e*g*h^2 - 2
10*b*c^3*f*g*h^2 + 80*c^4*d*h^3 - 70*b*c^3*e*h^3 + 63*b^2*c^2*f*h^3 - 64*a
*c^3*f*h^3)/c^5)*x + (480*c^4*f*g^3 + 1440*c^4*e*g^2*h - 1200*b*c^3*f*g^2*
h + 1440*c^4*d*g*h^2 - 1200*b*c^3*e*g*h^2 + 1050*b^2*c^2*f*g*h^2 - 1080*a*
c^3*f*g*h^2 - 400*b*c^3*d*h^3 + 350*b^2*c^2*e*h^3 - 360*a*c^3*e*h^3 - 315*
b^3*c*f*h^3 + 644*a*b*c^2*f*h^3)/c^5)*x + (1920*c^4*e*g^3 - 1440*b*c^3*f*g
^3 + 5760*c^4*d*g^2*h - 4320*b*c^3*e*g^2*h + 3600*b^2*c^2*f*g^2*h - 3840*a
*c^3*f*g^2*h - 4320*b*c^3*d*g*h^2 + 3600*b^2*c^2*e*g*h^2 - 3840*a*c^3*e*g*
h^2 - 3150*b^3*c*f*g*h^2 + 6600*a*b*c^2*f*g*h^2 + 1200*b^2*c^2*d*h^3 - 128
0*a*c^3*d*h^3 - 1050*b^3*c*e*h^3 + 2200*a*b*c^2*e*h^3 + 945*b^4*f*h^3 - 29
40*a*b^2*c*f*h^3 + 1024*a^2*c^2*f*h^3)/c^5) - 1/256*(256*c^5*d*g^3 - 128*b
*c^4*e*g^3 + 96*b^2*c^3*f*g^3 - 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h + 288*
b^2*c^3*e*g^2*h - 384*a*c^4*e*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f*
g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 - 240*b^3*c^2*e*g*h^2 + 57
6*a*b*c^3*e*g*h^2 + 210*b^4*c*f*g*h^2 - 720*a*b^2*c^2*f*g*h^2 + 288*a^2*c^
3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*h^3 + 70*b^4*c*e*h^3 - 240*a*
b^2*c^2*e*h^3 + 96*a^2*c^3*e*h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*
a^2*b*c^2*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b
))/c^(11/2)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.227 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

3.227.1 Optimal result

Integrand size = 32, antiderivative size = 420

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$
$$- \frac{(105b^3fh^3 + 32c^3g(fg^2-4h(eg+3dh)) - 20bch^2(11afh+6b(2fg+eh)) + 8c^2h(16ah(2fg+eh)+b}{}$$
$$+ \frac{(128c^4dg^2 + 35b^4fh^2 - 40b^2ch(2bfg+beh+3afh) - 64c^3(bg(eg+2dh)+a(fg^2+2egh+dh^2)) + 48}{128c^{9/2}}$$

output
```
1/128*(128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^
3*(b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h
+2*f*g)+b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))/c^(9/2)-1/24*(7*b*f*h-8*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a
)^(1/2)/c^2/h+1/4*f*(h*x+g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^3
+32*c^3*g*(f*g^2-4*h*(3*d*h+e*g))-20*b*c*h^2*(11*a*f*h+6*b*(e*h+2*f*g))+8*
c^2*h*(16*a*h*(e*h+2*f*g)+b*(11*f*g^2+18*h*(d*h+2*e*g)))-2*c*h*(35*b^2*f*h
^2-4*c*h*(9*a*f*h+10*b*e*h+6*b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g)))*x)*(c
*x^2+b*x+a)^(1/2)/c^4/h
```

3.227.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.81

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3fh^2+10bch(22afh+b(24fg+12eh+7fhx))+16c^3(6dh(4g+hx)+4e(3}{}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*
(24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*
g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e
*g + 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20
*g*h*x + 7*h^2*x^2))) + 3*(-128*c^4*d*g^2 - 35*b^4*f*h^2 + 40*b^2*c*h*(2*b
*f*g + b*e*h + 3*a*f*h) + 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g +
 2*d*h)) - 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e
*g + d*h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2
))
```

3.227.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2184, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(g+hx)^2(bfg-8cdh+6afh+(2cfg-8ceh+7bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{4ch^2}+\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\int \frac{(g+hx)^2(bfg-8cdh+6afh+(2cfg-8ceh+7bfh)x)}{\sqrt{cx^2+bx+a}}dx}{8ch}$$

$$\Big\downarrow \ 1236$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\frac{\int -\frac{(g+hx)\left(7fghb^2+4\left(7afh^2-cg(fg+2eh)\right)b+4ch(12cdg-7afg-8aeh)+\left(-8\left(fg^2-2h(2eg+3dh)\right)c^2-4h(6bfg+10beh+9afh)c+35b^2fh^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c} + \frac{(g+hx)^2\sqrt{}}{}}{8ch}$$

$$\Big\downarrow \ 27$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c} - \frac{\int \frac{(g+hx)\left(7fghb^2+28afh^2b-4cg(fg+2eh)b+4ch(12cdg-7afg-8aeh)+\left(-8\left(fg^2-2h(2eg+3dh)\right)c^2-4h(6bfg+\right.\right.}{\sqrt{cx^2+bx+a}}}{6c}}{8ch}$$

$$\Big\downarrow \ 1225$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c} - \frac{\frac{3h\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(ah(dh+2eg)+afg^2+\right.\right.}{8c^2}}{}}{}$$

$$\Big\downarrow \ 1092$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c} - \frac{\frac{3h\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(ah(dh+2eg)+afg^2+\right.\right.}{4c^2}}{}}{}$$

$$\Big\downarrow \ 219$$

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c} - \frac{\frac{3h\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2\right.}{8c^{5/2}}}{}}{}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

3.227. $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

output
```
(f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - (((2*c*f*g - 8*c*e*h + 7*b
*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4*((105*b^3*f*h^3 + 3
2*c^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g +
e*h)) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h))
 - 2*c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g
^2 - 2*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/c^2 + (3*h*(128*c^4*d
*g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f
*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*
h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c*h)
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.227. $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.227.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.04

method	result
risch	$\frac{(48 f h^2 c^3 x^3 - 56 b c^2 f h^2 x^2 + 64 c^3 e h^2 x^2 + 128 c^3 f g h x^2 - 72 a c^2 f h^2 x + 70 b^2 c f h^2 x - 80 b c^2 e h^2 x - 160 b c^2 f g h x + 96 c^3 d h^2 x + 192 c^3 e g h x \cdots}{\cdots}$
default	$\frac{d g^2 \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^2+b x+a}\right)}{\sqrt{c}} + f h^2 \left(\frac{x^3 \sqrt{c x^2+b x+a}}{4 c} - \frac{7 b \left(\frac{x^2 \sqrt{c x^2+b x+a}}{3 c} - \frac{5 b \left(\frac{x \sqrt{c x^2+b x+a}}{2 c} - \frac{3 b \left(\frac{\sqrt{c x^2+b x+a}}{c} - \frac{b \ln\left(\frac{\frac{b}{2}+c \cdots}{\sqrt{c}} \cdots\right)}{\cdots} \right)}{4 c} \cdots \right)}{6 \cdots} \cdots \right)}{\cdots} \cdots \right.$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*c^3*f*h^2*x^3-56*b*c^2*f*h^2*x^2+64*c^3*e*h^2*x^2+128*c^3*f*g*h*
x^2-72*a*c^2*f*h^2*x+70*b^2*c*f*h^2*x-80*b*c^2*e*h^2*x-160*b*c^2*f*g*h*x+9
6*c^3*d*h^2*x+192*c^3*e*g*h*x+96*c^3*f*g^2*x+220*a*b*c*f*h^2-128*a*c^2*e*h
^2-256*a*c^2*f*g*h-105*b^3*f*h^2+120*b^2*c*e*h^2+240*b^2*c*f*g*h-144*b*c^2
*d*h^2-288*b*c^2*e*g*h-144*b*c^2*f*g^2+384*c^3*d*g*h+192*c^3*e*g^2)*(c*x^2
+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*f*h^2-120*a*b^2*c*f*h^2+96*a*b*c^2*e*h
^2+192*a*b*c^2*f*g*h-64*a*c^3*d*h^2-128*a*c^3*e*g*h-64*a*c^3*f*g^2+35*b^4*
f*h^2-40*b^3*c*e*h^2-80*b^3*c*f*g*h+48*b^2*c^2*d*h^2+96*b^2*c^2*e*g*h+48*b
^2*c^2*f*g^2-128*b*c^3*d*g*h-64*b*c^3*e*g^2+128*c^4*d*g^2)/c^(9/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.227. $\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}} \, dx$

3.227.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.05

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}} \, dx$$
$$= \left[\frac{3\,(16\,(8\,c^4d-4\,bc^3e+(3\,b^2c^2-4\,ac^3)f)g^2-16\,(8\,bc^3d-2\,(3\,b^2c^2-4\,ac^3)e+(5\,b^3c-12\,abc^2)f)gh+(\right.$$
$$-\frac{3\,(16\,(8\,c^4d-4\,bc^3e+(3\,b^2c^2-4\,ac^3)f)g^2-16\,(8\,bc^3d-2\,(3\,b^2c^2-4\,ac^3)e+(5\,b^3c-12\,abc^2)f)gh+}{}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*
b*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*
(3*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4
*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c
^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44
*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(4
8*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*
b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(
8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*
b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*
a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2
)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g
^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c
^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*
(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c
^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f
)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.227.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(432) = 864$.

Time = 1.10 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.17

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fh^2x^3}{4c} + \frac{x^2\left(-\frac{7bfh^2}{8c}+eh^2+2fgh\right)}{3c} + \frac{x\left(-\frac{3afh^2}{4c}-\frac{5b\left(-\frac{7bfh^2}{8c}+eh^2+2fgh\right)}{6c}+dh^2+2egh+fg^2\right)}{2c} + \frac{-\frac{2a\left(-\frac{7bfh^2}{8c}+\right.}{3c}}{} \right. \\ \frac{2\left(\frac{fh^2(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{(a+bx)^{\frac{7}{2}}\left(-4afh^2+beh^2+2bfgh\right)}{7b^4}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2fh^2-3abeh^2-6abfgh+b^2dh^2+2b^2egh+b^2fg^2\right)}{5b^4}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3fh^2+3a^2beh^2+6a^2b\right.}{}\right.}{b} \\ \frac{dg^2x+\frac{fh^2x^5}{5}+\frac{x^4\left(eh^2+2fgh\right)}{4}+\frac{x^3\left(dh^2+2egh+fg^2\right)}{3}+\frac{x^2\cdot\left(2dgh+eg^2\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**3/(4*c) + x**2*(-7*b*f*h**2/(
8*c) + e*h**2 + 2*f*g*h)/(3*c) + x*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2/(
8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) + (-2*a*
(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c) - 5*
b*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2
)/(4*c) + 2*d*g*h + e*g**2)/c) + (-a*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2
/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) - b*(-
2*a*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c)
- 5*b*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*
g**2)/(4*c) + 2*d*g*h + e*g**2)/(2*c) + d*g**2)*Piecewise((log(b + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*
(f*h**2*(a + b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a*f*h**2 + b*e*h*
*2 + 2*b*f*g*h)/(7*b**4) + (a + b*x)**(5/2)*(6*a**2*f*h**2 - 3*a*b*e*h**2
- 6*a*b*f*g*h + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2)/(5*b**4) + (a +
b*x)**(3/2)*(-4*a**3*f*h**2 + 3*a**2*b*e*h**2 + 6*a**2*b*f*g*h - 2*a*b**2*
d*h**2 - 4*a*b**2*e*g*h - 2*a*b**2*f*g**2 + 2*b**3*d*g*h + b**3*e*g**2)/(3
*b**4) + sqrt(a + b*x)*(a**4*f*h**2 - a**3*b*e*h**2 - 2*a**3*b*f*g*h + a**
2*b**2*d*h**2 + 2*a**2*b**2*e*g*h + a**2*b**2*f*g**2 - 2*a*b**3*d*g*h - a*
b**3*e*g**2 + b**4*d*g**2)/b**4)/b, Ne(b, 0)), ((d*g**2*x + f*h**2*x**5...
```

3.227.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.05

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,fh^2x}{c}+\frac{16\,c^3fgh+8\,c^3eh^2-7\,bc^2fh^2}{c^4}\right)x+\frac{48\,c^3fg^2+96\,c^3egh-80\,bc^2fg\right.\right.$$
$$-\frac{(128\,c^4dg^2-64\,bc^3eg^2+48\,b^2c^2fg^2-64\,ac^3fg^2-128\,bc^3dgh+96\,b^2c^2egh-128\,ac^3egh-80\,b^3cfgh-$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h + 8*c^3*e*h
^2 - 7*b*c^2*f*h^2)/c^4)*x + (48*c^3*f*g^2 + 96*c^3*e*g*h - 80*b*c^2*f*g*h
 + 48*c^3*d*h^2 - 40*b*c^2*e*h^2 + 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2)/c^4)*x
 + (192*c^3*e*g^2 - 144*b*c^2*f*g^2 + 384*c^3*d*g*h - 288*b*c^2*e*g*h + 24
0*b^2*c*f*g*h - 256*a*c^2*f*g*h - 144*b*c^2*d*h^2 + 120*b^2*c*e*h^2 - 128*
a*c^2*e*h^2 - 105*b^3*f*h^2 + 220*a*b*c*f*h^2)/c^4) - 1/128*(128*c^4*d*g^2
 - 64*b*c^3*e*g^2 + 48*b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h +
96*b^2*c^2*e*g*h - 128*a*c^3*e*g*h - 80*b^3*c*f*g*h + 192*a*b*c^2*f*g*h +
48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 - 40*b^3*c*e*h^2 + 96*a*b*c^2*e*h^2 + 35
*b^4*f*h^2 - 120*a*b^2*c*f*h^2 + 48*a^2*c^2*f*h^2)*log(abs(2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(g+h\,x)^2\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.228 $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx$

3.228.1 Optimal result

Integrand size = 30, antiderivative size = 223

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{(15b^2fh^2-8c^2(fg^2-3h(eg+dh))-2ch(8afh+9b(fg+eh))-2ch(2cfg-6ceh+5bfh)x)\sqrt{a+b}}{24c^3h} + \frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}$$

output
```
1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*h+b*e*g)+6*b*c*(2*a*f*h+
b*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1
/3*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h^2-8*c^2*(f*g^2-3*h
*(d*h+e*g))-2*c*h*(8*a*f*h+9*b*(e*h+f*g))-2*c*h*(5*b*f*h-6*c*e*h+2*c*f*g)*
x)*(c*x^2+b*x+a)^(1/2)/c^3/h
```

3.228.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}} dx = \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2fh+4c^2(6eg+6dh+3fgx+3ehx+2fhx^2)-2c(8afh+b(9fg+9eh+5fh}{}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*f*h + 4*c^2*(6*e*g + 6*d*h + 3*f*
g*x + 3*e*h*x + 2*f*h*x^2) - 2*c*(8*a*f*h + b*(9*f*g + 9*e*h + 5*f*h*x)))
+ 3*(-16*c^3*d*g + 5*b^3*f*h + 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b
*c*(b*f*g + b*e*h + 2*a*f*h))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/(48*c^(7/2))
```

3.228.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2184, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(g+hx)(bfg-6cdh+4afh+(2cfg-6ceh+5bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{3ch^2}+\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-\frac{\int \frac{(g+hx)(bfg-6cdh+4afh+(2cfg-6ceh+5bfh)x)}{\sqrt{cx^2+bx+a}}dx}{6ch}$$

$$\Big\downarrow \text{1225}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-$$
$$\frac{-\frac{3h\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-\right.}{4c}}{6ch}$$

$$\Big\downarrow \text{1092}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-$$
$$\frac{-\frac{3h\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg)\right.}{6ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} - \frac{-\frac{3h\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg)}{8c^{5/2}} - \frac{\sqrt{a+bx+cx^2}(-2ch(8afh+9b(eh+fg))}{}}{6ch}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) - (-1/4*((15*b^2*f*h^2 - 8*c
^2*(f*g^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(
2*c*f*g - 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/c^2 - (3*h*(16*c^3*
d*g - 5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b
*e*h + 2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)))/(6*c*h)
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.228. $\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.228.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(-8fh\,c^{2}x^{2}+10bcfhx-12c^{2}ehx-12c^{2}fgx+16acfh-15b^{2}fh+18bceh+18bcfg-24c^{2}dh-24c^{2}eg\right)\sqrt{c\,x^{2}+bx+a}}{24c^{3}}+\frac{(12abcfh-8a\,c\ldots}{\ldots}$
default	$\frac{dg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+fh\left(\frac{x^{2}\sqrt{c\,x^{2}+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\ldots}{\ldots}\right)}{6c}\ldots\right.$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*c^2*f*h*x^2+10*b*c*f*h*x-12*c^2*e*h*x-12*c^2*f*g*x+16*a*c*f*h-15
*b^2*f*h+18*b*c*e*h+18*b*c*f*g-24*c^2*d*h-24*c^2*e*g)*(c*x^2+b*x+a)^(1/2)/
c^3+1/16*(12*a*b*c*f*h-8*a*c^2*e*h-8*a*c^2*f*g-5*b^3*f*h+6*b^2*c*e*h+6*b^2
*c*f*g-8*b*c^2*d*h-8*b*c^2*e*g+16*c^3*d*g)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.07

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[\frac{3\,(2\,(8\,c^3d-4\,bc^2e+(3\,b^2c-4\,ac^2)f)g-(8\,bc^2d-2\,(3\,b^2c-4\,ac^2)e+(5\,b^3-12\,abc)f)h)\sqrt{c}\log\left(-8\,c^2\right.}{}\right.$$

$$-\frac{3\,(2\,(8\,c^3d-4\,bc^2e+(3\,b^2c-4\,ac^2)f)g-(8\,bc^2d-2\,(3\,b^2c-4\,ac^2)e+(5\,b^3-12\,abc)f)h)\sqrt{-c}\arctan}{}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2
 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) +
4*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (1
5*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt
(c*x^2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*
c^2)*f)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h
)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d
 - 18*b*c^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b
*c^2*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

3.228.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.96

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fhx^2}{3c}+\frac{x\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}+\frac{-\frac{2afh}{3c}-\frac{3b\left(-\frac{5bfh}{6c}+eh+fg\right)}{4c}+dh+eg}{c}\right)+\left(-\frac{a\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}-\frac{b\left(-\frac{2afh}{3c}-\right.}{} \right. \\ \frac{2\left(\frac{fh(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}(-3afh+beh+bfg)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(3a^2fh-2abeh-2abfg+b^2dh+b^2eg\right)}{3b^3}+\frac{\sqrt{a+bx}\left(-a^3fh+a^2beh+a^2bfg-ab^2dh-ab^2eg+b^3dg\right)}{b^3}\right)}{b} \\ \frac{dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**2/(3*c) + x*(-5*b*f*h/(6*c) + e*
h + f*g)/(2*c) + (-2*a*f*h/(3*c) - 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c)
+ d*h + e*g)/c) + (-a*(-5*b*f*h/(6*c) + e*h + f*g)/(2*c) - b*(-2*a*f*h/(3*
c) - 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c) + d*h + e*g)/(2*c) + d*g)*Piec
ewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b
**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2),
True)), Ne(c, 0)), (2*(f*h*(a + b*x)**(7/2)/(7*b**3) + (a + b*x)**(5/2)*(-
3*a*f*h + b*e*h + b*f*g)/(5*b**3) + (a + b*x)**(3/2)*(3*a**2*f*h - 2*a*b*e
*h - 2*a*b*f*g + b**2*d*h + b**2*e*g)/(3*b**3) + sqrt(a + b*x)*(-a**3*f*h
+ a**2*b*e*h + a**2*b*f*g - a*b**2*d*h - a*b**2*e*g + b**3*d*g)/b**3)/b, N
e(b, 0)), ((d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2)/
sqrt(a), True))
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.90

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,fhx}{c}+\frac{6\,c^2fg+6\,c^2eh-5\,bcfh}{c^3}\right)x+\frac{24\,c^2eg-18\,bcfg+24\,c^2dh-18\,bceh+1}{c^3}\right.$$
$$-\frac{(16\,c^3dg-8\,bc^2eg+6\,b^2cfg-8\,ac^2fg-8\,bc^2dh+6\,b^2ceh-8\,ac^2eh-5\,b^3fh+12\,abcfh)\log\left(\left|2\left(\sqrt{cx}\right.\right.\right.}{16\,c^{\frac{7}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g + 6*c^2*e*h - 5*b*c*
f*h)/c^3)*x + (24*c^2*e*g - 18*b*c*f*g + 24*c^2*d*h - 18*b*c*e*h + 15*b^2*
f*h - 16*a*c*f*h)/c^3) - 1/16*(16*c^3*d*g - 8*b*c^2*e*g + 6*b^2*c*f*g - 8*
a*c^2*f*g - 8*b*c^2*d*h + 6*b^2*c*e*h - 8*a*c^2*e*h - 5*b^3*f*h + 12*a*b*c
*f*h)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

3.229.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{(8c^2d+3b^2f-4c(be+af))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c
*x^2+b*x+a)^(1/2)/c
```

3.229.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{c}(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4c(be+af))\,\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^
2*f - 4*c*(b*e + a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x
)])])/(4*c^(5/2))
```

3.229.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \ 2192$$

$$\frac{\int \frac{4cd-2af+(4ce-3bf)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2(2cd-af)+(4ce-3bf)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

3.229. $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}dx$

output
```
(f*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2
])/c + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c)
```

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.229. $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

3.229.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^2+bx+a}}{4c^2}-\frac{(4acf-3b^2f+4bce-8c^2d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx-}\right.}{2c^{\frac{3}{2}}}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*c*f*x+3*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2-1/8*(4*a*c*f-3*b^2*f+4
*b*c*e-8*c^2*d)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-}{16\,c^3}\right.$$
$$-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx+4\,c^2e-3\,bcf)\sqrt{cx^2+b}}{8\,c^3}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2
*c^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d -
 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b
*c*f)*sqrt(c*x^2 + b*x + a))/c^3]
```

3.229.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(\frac{fx}{2c}+\frac{-\frac{3bf}{4c}+e}{c}\right)\sqrt{a+bx+cx^2}+\left(-\frac{af}{2c}-\frac{b\left(-\frac{3bf}{4c}+e\right)}{2c}+d\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((((f*x/(2*c) + (-3*b*f/(4*c) + e)/c)*sqrt(a + b*x + c*x**2) + (-a
*f/(2*c) - b*(-3*b*f/(4*c) + e)/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) +
 x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*d*sq
rt(a + b*x) + 2*e*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sq
rt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b, Ne(b,
0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

3.229.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}+\frac{4\,ce-3\,bf}{c^2}\right) - \frac{(8\,c^2d-4\,bce+3\,b^2f-4\,acf)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c + (4*c*e - 3*b*f)/c^2) - 1/8*(8*c^2*d -
 4*b*c*e + 3*b^2*f - 4*a*c*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*sqrt(c) + b))/c^(5/2)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

3.230.1 Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} + \frac{(fg^2-h(eg-dh))\,\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^2\sqrt{cg^2-bgh+ah^2}}$$

output
```
-1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)
*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)
^(1/2)+f*(c*x^2+b*x+a)^(1/2)/c/h
```

3.230.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{2fh\sqrt{a+x(b+cx)}}{c} + \frac{4\sqrt{-cg^2+bgh-ah^2}\,(fg^2+h(-eg+dh))\,\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{cg^2+h(-bg+ah)} - \frac{(2cfg-2ceh+bfh)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b\cdots}}\right)}{c^{3/2}}}{2h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*f*h*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^
2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])
/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(b*g) + a*h)) - ((2*c*f*g -
 2*c*e*h + b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
c^(3/2))/(2*h^2)
```

3.230.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(bfg-2cdh+(2cfg-2ceh+bfh)x)}{2(g+hx)\sqrt{cx^2+bx+a}}dx}{ch^2}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\int \frac{bfg-2cdh+(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2ch}$$

$$\Big\downarrow \text{1269}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{(bfh-2ceh+2cfg)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \text{1092}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{2(bfh-2ceh+2cfg)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \; 1154$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{4c(dh^2-egh+fg^2)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{h}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}}{2ch}$$

$$\Big\downarrow \; 219$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}-\frac{2c(dh^2-egh+fg^2)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h\sqrt{ah^2-bgh+cg^2}}}{2ch}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(f*Sqrt[a + b*x + c*x^2])/(c*h) - (((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - (2*c*(f*g^2 -
e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g
*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*c
*h)
```

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.230.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.40

method	result
risch	$\frac{f\sqrt{c\,x^{2}+bx+a}}{ch}-\frac{\frac{(bfh-2ehc+2cfg)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{h\sqrt{c}}+\frac{2\left(d\,h^{2}-egh+f\,g^{2}\right)c\ln\left(\frac{\frac{2a\,h^{2}-2bgh+2c\,g^{2}}{h^{2}}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^{2}-bg\cdots}{h^{2}}}\cdots}{\cdots}\right)}{h^{2}\sqrt{\frac{a\,h^{2}-bgh+c\,g\cdots}{h^{2}}}}}{2hc}$
default	$\frac{\frac{eh\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+fh\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{fg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}}{h^{2}}-\frac{\left(d\,h^{2}-egh+f\,g^{2}\right)\ln\left(\cdots\right)}{\cdots}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h/c*((b*f*h-2*c*e*h+2*c*f*g)/h*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*(d*h^2-e*g*h+f*g^2)*c/h^2/((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/
h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h
*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.230.5 Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.230.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

3.230.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

3.230.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

3.231.1 Optimal result

Integrand size = 32, antiderivative size = 241

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h(cg^2-bgh+ah^2)(g+hx)}+\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} - \frac{(2c(fg^3-dgh^2)+h(2ah(2fg-eh)-b(3fg^2-egh-dh^2)))\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2h^2(cg^2-bgh+ah^2)^{3/2}}$$

output
```
-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-e*g*h+3*f*g^2))
)*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+
b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/h^2/c^(1/2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2
)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

3.231.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{\sqrt{-cg^2+bgh-ah^2}\,(2c(fg^3-dgh^2)+h(-3bfg^2+bh(eg+dh)-2ah(-2fg+eh)))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{\cdots}}{\sqrt{-cg^2+h(\cdots}}\right.}{(cg^2+h(-bg+ah))^2}\cdots}{h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g
) + a*h))*(g + h*x)) + (Sqrt[-(c*g^2) + b*g*h - a*h^2]*(2*c*(f*g^3 - d*g*h
^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqr
t[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])
/(c*g^2 + h*(-(b*g) + a*h))^2 + (f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)]])/Sqrt[c])/h^2)
```

3.231.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2181, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\frac{bfg^2}{h}+2cdg-beg-2afg-bdh+2aeh-2f\left(-\frac{cg^2}{h}+bg-ah\right)x}{2(g+hx)\sqrt{cx^2+bx+a}}dx}{ah^2-bgh+cg^2}-\frac{\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{h(g+hx)\,(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2\,(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{h(g+hx)\,(ah^2-bgh+cg^2)}$$

$\Big\downarrow$ 1269

$$\frac{\frac{2f(ah^2-bgh+cg^2)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h^2}-\frac{(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2))\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

$\Big\downarrow$ 1092

$$\frac{\frac{4f(ah^2-bgh+cg^2)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h^2}-\frac{(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2))\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

$\Big\downarrow$ 219

$$\frac{\frac{2f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(ah^2-bgh+cg^2)}{\sqrt{c}h^2}-\frac{(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2))\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{2(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2))\int \frac{1}{4(cg^2-bhg+ah^2)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{h^2}+\frac{2f\text{arctanh}\big(}{}}{2(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

$\Big\downarrow$ 219

$$\frac{\frac{2f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(ah^2-bgh+cg^2)}{\sqrt{c}h^2}-\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+3}{h^2\sqrt{ah^2-bgh+cg^2}}}{2(ah^2-bgh+cg^2)}-\frac{\sqrt{a+bx+cx^2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

3.231. $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

output
```
-(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x))) + ((2*f*(c*g^2 - b*g*h + a*h^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(
3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h +
 (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(h^2*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*(c*g^2 - b*g*h + a*h^2))
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.231. $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.231.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(221) = 442$.

Time = 0.84 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.01

method	result
default	$\frac{f \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{h^2\sqrt{c}} - \frac{(eh-2fg)\ln\left(\frac{\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+}}{x+\frac{g}{h}}\right.}{h^3\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}}$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g
)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+1/h^4*(d*h^2-e*g*h
+f*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*
c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.231.5 Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.231.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

3.231.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

3.231.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

3.232.1 Optimal result

Integrand size = 32, antiderivative size = 336

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{(2cg(fg^2+h(eg-3dh))+h(4ah(2fg-eh)-b(5fg^2-egh-3dh^2)))\sqrt{a+bx+cx^2}}{4h\left(cg^2-bgh+ah^2\right)^2(g+hx)} + \frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)+b^2(3fg^2+egh+3dh^2)-4c(bg(eg+2dh)+a(fg^2-3egh+dh}{8\left(cg^2-bgh+ah^2\right)^{5/2}}$$

output
```
1/8*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)+b^2*(3*d*h^2+e*g*h+3*f*g^
2)-4*c*(b*g*(2*d*h+e*g)+a*(d*h^2-3*e*g*h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(
-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h
+c*g^2)^(5/2)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+
c*g^2)/(h*x+g)^2+1/4*(2*c*g*(f*g^2+h*(-3*d*h+e*g))+h*(4*a*h*(-e*h+2*f*g)-b
*(-3*d*h^2-e*g*h+5*f*g^2)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h
*x+g)
```

3.232.2 Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.40

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{4h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)^2}+\frac{8h(-2fg+eh)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{4(2cg-bh)(2fg-eh)\text{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}-\cdots}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/8*((4*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(
-(b*g) + a*h))*(g + h*x)^2) + (8*h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)])/(
(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (4*(2*c*g - b*h)*(2*f*g - e*h)*Arc
Tanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sq
rt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) - (8*f*ArcTanh[(-2
*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)] + ((f*g^2 + h*(-(e*g) + d*h)
)*(6*h*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]
- (8*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(2*b*g + a*h))*(g + h*x)*ArcTanh[(-2*a*h
+ 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
 c*x)])]))/((c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g + h*x)))/h^2
```

3.232.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{\frac{bfg^2}{h}+4cdg-beg-4afg-3bdh+4aeh+2\left(\frac{cfg^2}{h}+ceg-2bfg-cdh+2afh\right)x}{2(g+hx)^2\sqrt{cx^2+bx+a}}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4cdg-4afg+4aeh-b\left(-\frac{fg^2}{h}+eg+3dh\right)-2\left(2bfg-2afh-c\left(\frac{fg^2}{h}+eg-dh\right)\right)x}{(g+hx)^2\sqrt{cx^2+bx+a}}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2(ah^2-bgh+cg^2)}+\frac{\sqrt{a+bx+cx^2}\left(2c(g\right.}{}}{4\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2c\left(gh(eg-3dh)+fg^3\right)-h\left(-4ah(2fg-eh)-bh(3dh+eg)+5bfg^2\right)\right)}{h(g+hx)(ah^2-bgh+cg^2)}-\frac{\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2\right.}{}}{4\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)}{2(ah^2-bgh+cg^2)^{3/2}}+}{4\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

3.232. $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a
*h^2)*(g + h*x)^2) + (((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b
*h*(e*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2
- b*g*h + a*h^2)*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g
 + e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f
*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c
*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^
(3/2)))/(4*(c*g^2 - b*g*h + a*h^2))
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.232.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(318) = 636$.

Time = 0.99 (sec) , antiderivative size = 1013, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1013

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-
2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2
*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g))+(e*h-2*f*g)/h
^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1
/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/
h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(d*h^2-e*g*h+f*g^2
)/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)^2*((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h
+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)*((x+1/h*g)^2*c+(b*h-2*c*g)/h
*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*
c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/2*c/(a*h^2-
b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)
/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(318) = 636.

Time = 34.05 (sec) , antiderivative size = 2034, normalized size of antiderivative = 6.05

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f -
 (b^2 + 12*a*c)*e)*g^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2
 + ((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f -
 (b^2 + 12*a*c)*e)*g*h^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^
2 + 2*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f
- (b^2 + 12*a*c)*e)*g^2*h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^
3)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*
c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 -
b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(
4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2))
 - 4*(2*a^2*d*h^5 - (4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2
+ 2*a*c)*f)*g^4*h - (13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*
e + 6*a^2*f - 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*
c^2*f*g^5 + (2*c^2*e - 7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)
*f)*g^3*h^2 + (9*b*c*d - 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e +
8*a^2*f - 3*(b^2 + 2*a*c)*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^
2 + b*x + a))/(c^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3
*(b^2*c + a*c^2)*g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4
*h^4 + (c^3*g^6*h^2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c
 + a*c^2)*g^4*h^4 - (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6...
```

3.232.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)^3\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

3.232.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(318) = 636$.

Time = 0.33 (sec) , antiderivative size = 2279, normalized size of antiderivative = 6.78

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*c^2*d*g^2 - 4*b*c*e*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h +
 b^2*e*g*h + 12*a*c*e*g*h - 8*a*b*f*g*h + 3*b^2*d*h^2 - 4*a*c*d*h^2 - 4*a*
b*e*h^2 + 8*a^2*f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sq
rt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h
^2 + 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2))
+ 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*c^2*d*g^2*h^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*g^
2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*b*c*d*g*h^4 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*g*h
^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*c^(5/2)*f*g^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5
/2)*e*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h -
 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x ...
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.233 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

3.233.1 Optimal result

Integrand size = 32, antiderivative size = 504

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$+\frac{\left(12c^2d-6bce+7b^2f-16acf\right)h(g+hx)^2\sqrt{a+bx+cx^2}}{3c^2\left(b^2-4ac\right)}$$

$$+\frac{h(192c^4dg^2+105b^4fh^2-10b^2ch(46afh+9b(3fg+eh))-16c^3(3bg(2eg+3dh)+4a(7fg^2+9egh+3dh}{}$$

$$-\frac{(35b^3fh^3-30bch^2(3bfg+beh+2afh)-16c^3g(fg^2+3h(eg+dh))+24c^2h(ah(3fg+eh)+b(3fg^2+3}{16c^{9/2}}$$

output
```
-1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*g*(f*g^2+3*h
*(d*h+e*g))+24*c^2*h*(a*h*(e*h+3*f*g)+b*(d*h^2+3*e*g*h+3*f*g^2)))*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+2*(c*(2*a*e-b*(d+a*f/c)
)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^3/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
^(1/2)+1/3*(-16*a*c*f+7*b^2*f-6*b*c*e+12*c^2*d)*h*(h*x+g)^2*(c*x^2+b*x+a)^
(1/2)/c^2/(-4*a*c+b^2)+1/24*h*(192*c^4*d*g^2+105*b^4*f*h^2-10*b^2*c*h*(46*
a*f*h+9*b*(e*h+3*f*g))-16*c^3*(3*b*g*(3*d*h+2*e*g)+4*a*(3*d*h^2+9*e*g*h+7*
f*g^2))+8*c^2*(32*a^2*f*h^2+39*a*b*h*(e*h+3*f*g)+b^2*(20*f*g^2+9*h*(d*h+3*
e*g)))+2*c*h*(48*c^3*d*g-35*b^3*f*h-8*c^2*(9*a*e*h+11*a*f*g+3*b*d*h+3*b*e*
g)+2*b*c*(58*a*f*h+15*b*e*h+17*b*f*g))*x)*(c*x^2+b*x+a)^(1/2)/c^4/(-4*a*c+
b^2)
```

3.233.2 Mathematica [A] (verified)

Time = 4.72 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.38

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} \, dx =$$
$$-\frac{-105b^5fh^3x-5b^4h^2(21afh+cx(-54fg-18eh+7fhx))+2b^3ch(5ah(27fg+9eh+53fhx)+cx(3h(-}{}$$
$$+\frac{(-35b^3fh^3+30bch^2(3bfg+beh+2afh)+16c^3(fg^3+3gh(eg+dh))-24c^2h(3bfg^2+bh(3eg+dh)+a}{8c^{9/2}}$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
-1/24*(-105*b^5*f*h^3*x - 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h + 7*
f*h*x)) + 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*e*g
 - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) + 16*c^2*(-16
*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) - 3
*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
 3*h*(4*d*h + 3*e*(4*g + h*x)))) + 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3
*d*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h
*x - 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*
g*h*x - 5*h^2*x^2)))) + 4*b^2*c*(115*a^2*f*h^3 - a*c*h*(3*h*(18*e*g + 6*d*
h + 31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) - c^2*x*(f*(-12*g^3 +
 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*
g^2 + 6*g*h*x + h^2*x^2)))))/(c^4*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (
(-35*b^3*f*h^3 + 30*b*c*h^2*(3*b*f*g + b*e*h + 2*a*f*h) + 16*c^3*(f*g^3 +
3*g*h*(e*g + d*h)) - 24*c^2*h*(3*b*f*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g
+ e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(8*c^(9/
2))
```

3.233.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2175, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(g+hx)^2\left(fgb^2+6(cd+af)hb-4ac(fg+3eh)+c\left(\frac{7fb^2}{c}-6eb+12cd-16af\right)hx\right)}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(g+hx)^2\left(fgb^2+6(cd+af)hb-4ac(fg+3eh)+\left(7fb^2-6ceb+12c^2d-16acf\right)hx\right)}{\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}+\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1236}$$

$$\frac{\frac{\int -\frac{(g+hx)\left(7fghb^3+\left(28afh^2-6cg(fg+eh)\right)b^2-4ch(6cdg+13afg+6aeh)b-8ac\left(8afh^2-3c\left(fg^2+3ehg+2dh^2\right)\right)-h\left(-35fhb^3+2c(17bfg+15beh+58afh)b+48c^3dg\right)\right)}{2\sqrt{cx^2+bx+a}}}{3c}}{c\left(b^2-4ac\right)}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\int \frac{(g+hx)\left(7fghb^3+\left(28afh^2-6cg(fg+eh)\right)b^2-4ch(6cdg+13afg+6aeh)b-8ac\left(8afh^2-3c\left(fg^2+3e\right)\right)\right)}{\sqrt{cx^2}}}{}}{c\left(b^2-4ac\right)}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1225}$$

$$\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2afh+beh+3bfg)+35b^3fh^3-16c^3\right)}{8c^2}}{}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

3.233. $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}dx$

$$\frac{h(g+hx)^2\sqrt{a+bx+cx^2}(-16acf+7b^2f-6bce+12c^2d)}{3c}-\frac{\frac{3(b^2-4ac)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2afh+beh+3bfg)+35b^3fh^3-16c^3}{4c^2}}{}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{h(g+hx)^2\sqrt{a+bx+cx^2}(-16acf+7b^2f-6bce+12c^2d)}{3c}-\frac{\frac{3(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2a}{8c^{5/2}}}{}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((12*c^2*d - 6*b*c*
e + 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4
*((192*c^4*d*g^2*h + 105*b^4*f*h^3 - 10*b^2*c*h^2*(46*a*f*h + 9*b*(3*f*g +
 e*h)) - 16*c^3*h*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*h^
2)) + 8*c^2*h*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h
*(3*e*g + d*h))) + 2*c*h^2*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*
a*f*g + 3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sq
rt[a + b*x + c*x^2])/c^2 + (3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*
b*f*g + b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(
3*b*f*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(c*(b^2 - 4*a*c))
```

3.233. $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

3.233. $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.233.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(484) = 968$.

Time = 1.28 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{h\left(-8f\,h^{2}c^{2}x^{2}+22bcf\,h^{2}x-12c^{2}e\,h^{2}x-36c^{2}fghx+40acf\,h^{2}-57b^{2}f\,h^{2}+42bce\,h^{2}+126bcfgh-24c^{2}d\,h^{2}-72c^{2}egh-72c^{2}f\,g^{2}\right)\sqrt{c\,x}}{24c^{4}}$
default	Expression too large to display

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*h*(-8*c^2*f*h^2*x^2+22*b*c*f*h^2*x-12*c^2*e*h^2*x-36*c^2*f*g*h*x+40*
a*c*f*h^2-57*b^2*f*h^2+42*b*c*e*h^2+126*b*c*f*g*h-24*c^2*d*h^2-72*c^2*e*g*
h-72*c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/c^4+1/16/c^4*(32*c^4*d*g^3*(2*c*x+b)/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-16*a^2*c^2*e*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(1/2)-38*a*b^3*f*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1
6*a*b*c^2*d*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+56*a^2*b*c*f*h^3
*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a^2*c^2*f*g*h^2*(2*c*x+b)/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+28*a*b^2*c*e*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)+84*a*b^2*c*f*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
)-48*a*b*c^2*e*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a*b*c^2*
f*g^2*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(60*a*b*c^2*f*h^3-24*a*c
^3*e*h^3-72*a*c^3*f*g*h^2-35*b^3*c*f*h^3+30*b^2*c^2*e*h^3+90*b^2*c^2*f*g*h
^2-24*b*c^3*d*h^3-72*b*c^3*e*g*h^2-72*b*c^3*f*g^2*h+48*c^4*d*g*h^2+48*c^4*
e*g^2*h+16*c^4*f*g^3)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a
)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(16*a^2*c^2*f*h^3+12*a*b^2*c*f*h^3+8*a
*b*c^2*e*h^3+24*a*b*c^2*f*g*h^2-16*a*c^3*d*h^3-48*a*c^3*e*g*h^2-48*a*c^3*f
*g^2*h-19*b^4*f*h^3+14*b^3*c*e*h^3+42*b^3*c*f*g*h^2-8*b^2*c^2*d*h^3-24*b^2
*c^2*e*g*h^2-24*b^2*c^2*f*g^2*h+48*c^4*d*g^2*h+16*c^4*e*g^3)*(-1/c/(c*x^2+
b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(482) = 964$.

Time = 7.72 (sec) , antiderivative size = 2937, normalized size of antiderivative = 5.83

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)
*e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d
 - 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3
*c^3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b
^2*c^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3
+ (16*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3
 - 4*a*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*
c^4)*e + 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3
 - 4*a*b*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*
g^3 + 24*(2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h +
 6*(8*(b^3*c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c
- 24*a*b^3*c^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d -
6*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a
^2*b^2*c^2)*f)*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^
3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c
^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c
^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*
c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c -...
```

3.233.6 Sympy [F]

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.233.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(482) = 964$.

Time = 0.30 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.04

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\left(b^2c^3fh^3-4\,ac^4fh^3\right)x}{b^2c^4-4\,ac^5} + \frac{18\,b^2c^3fgh^2-72\,ac^4fgh^2+6\,b^2c^3eh^3-24\,ac^4eh^3-7\,b^3c^2fh^3+28}{b^2c^4-4\,ac^5}\right.\right.\right.}{}$$
$$-\frac{\left(16\,c^3fg^3+48\,c^3eg^2h-72\,bc^2fg^2h+48\,c^3dgh^2-72\,bc^2egh^2+90\,b^2cfgh^2-72\,ac^2fgh^2-24\,bc^2dh^3+30\right.}{16\,c^{\frac{9}{2}}}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b
^2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 + 6*b^2*c^3*e*h^3 - 24*a*c^4*e*h^3 - 7*b
^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^
2*h - 288*a*c^4*f*g^2*h + 72*b^2*c^3*e*g*h^2 - 288*a*c^4*e*g*h^2 - 90*b^3*
c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^3*d*h^3 - 96*a*c^4*d*h^3 - 30
*b^3*c^2*e*h^3 + 120*a*b*c^3*e*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3
+ 128*a^2*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 - 48*b*c^4*e*g
^3 + 48*b^2*c^3*f*g^3 - 96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h + 144*b^2*c^3*e
*g^2*h - 288*a*c^4*e*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^2*h + 1
44*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 - 216*b^3*c^2*e*g*h^2 + 720*a*b*c^3
*e*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^
2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^3 + 90*b^4*c*e*h^3 - 372*a*b^2*c^2*
e*h^3 + 144*a^2*c^3*e*h^3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*
c^2*f*h^3)/(b^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 - 96*a*c^4*e*g^3 + 48*
a*b*c^3*f*g^3 - 288*a*c^4*d*g^2*h + 144*a*b*c^3*e*g^2*h - 216*a*b^2*c^2*f*
g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 - 216*a*b^2*c^2*e*g*h^2
+ 576*a^2*c^3*e*g*h^2 + 270*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a
*b^2*c^2*d*h^3 + 192*a^2*c^3*d*h^3 + 90*a*b^3*c*e*h^3 - 312*a^2*b*c^2*e*h^
3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3)/(b^2*c^4 -
4*a*c^5))/sqrt(c*x^2 + b*x + a) - 1/16*(16*c^3*f*g^3 + 48*c^3*e*g^2*h -...
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.234 $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

3.234.1 Optimal result

Integrand size = 32, antiderivative size = 289

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)(g+hx)^2}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}$$
$$+\frac{h(32c^3dg-15b^3fh-8c^2(2beg+8afg+bdh+4aeh)+4bc(6bfg+3beh+13afh)+2c(8c^2d-4bce+5b}{4c^3\,(b^2-4ac)}$$
$$+\frac{(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

output
```
1/8*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g))
)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(c*(2*a*e-b
*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^2/c/(-4*a*c+b^2)/(c*
x^2+b*x+a)^(1/2)+1/4*h*(32*c^3*d*g-15*b^3*f*h-8*c^2*(4*a*e*h+8*a*f*g+b*d*h
+2*b*e*g)+4*b*c*(13*a*f*h+3*b*e*h+6*b*f*g)+2*c*(-12*a*c*f+5*b^2*f-4*b*c*e+
8*c^2*d)*h*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)
```

3.234.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.35

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{\sqrt{c}(15b^4fh^2x+b^3h(15afh+cx(-24fg-12eh+5fhx))+4bc(-13a^2fh^2+2c^2g(-egx+d(g-2hx))+\cdots}{\cdots}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-((Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*
f*h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*
(2*h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*
c*(a*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f
*(-4*g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*
e*h + 3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) +
 f*x*(-2*g^2 + 4*g*h*x + h^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]
)) + (15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(
2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4
*c^(7/2))
```

3.234.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2175, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\,(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(g+hx)\left(fgb^2+4(cd+af)hb-4ac(fg+2eh)+c\left(\frac{5fb^2}{c}+8cd-4(be+3af)\right)hx\right)}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(g+hx)(fgb^2+4(cd+af)hb-4ac(fg+2eh)+(5fb^2-4ceb+8c^2d-12acf)hx)}{\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}+\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\big\downarrow$ 1225

$$\frac{\frac{(b^2-4ac)(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2(h(dh+2eg)+fg^2))\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}+\frac{h\sqrt{a+bx+cx^2}(2chx(-12acf+5b^2f-4bce+8c^2d)-8c^2}{}}{c(b^2-4ac)}$$

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\big\downarrow$ 1092

$$\frac{\frac{(b^2-4ac)(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2(h(dh+2eg)+fg^2))\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}+\frac{h\sqrt{a+bx+cx^2}(2chx(-12acf+5b^2f-4bce}{}}{c(b^2-4ac)}$$

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\big\downarrow$ 219

$$\frac{\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2(h(dh+2eg)+fg^2))}{8c^{5/2}}+\frac{h\sqrt{a+bx+cx^2}(2chx(-12acf+5b^2f-4bce}{}}{c(b^2-4ac)}$$

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x(-2acf+b^2f-bce+2c^2d)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((h*(32*c^3*d*g - 15
*b^3*f*h - 8*c^2*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*c*(6*b*f*g +
3*b*e*h + 13*a*f*h) + 2*c*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sq
rt[a + b*x + c*x^2])/(4*c^2) + ((b^2 - 4*a*c)*(15*b^2*f*h^2 - 12*c*h*(2*b*
f*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(c*(b^2 - 4*a*c))
```

3.234. $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.234. $\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

3.234.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(273) = 546.

Time = 0.96 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{h(-2cfhx+7bfh-4ehc-8cfg)\sqrt{c\,x^2+bx+a}}{4c^3}-\frac{-\frac{16c^3d\,g^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{8a^2cf\,h^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}-\frac{14a\,b^2f\,h^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{8abce}{(4ac-b^2}\cdots}{\cdots}$
default	$\frac{2d\,g^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f\,h^2\left(\frac{x^3}{2c\sqrt{c\,x^2+bx+a}}-\frac{5b\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+}\cdots}\right)}{2c}\right)}{2c}\right)}{\cdots}\cdots\right.$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*h*(-2*c*f*h*x+7*b*f*h-4*c*e*h-8*c*f*g)*(c*x^2+b*x+a)^(1/2)/c^3-1/8/c^
3*(-16*c^3*d*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*a^2*c*f*h^2*(
2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-14*a*b^2*f*h^2*(2*c*x+b)/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)+8*a*b*c*e*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)+16*a*b*c*f*g*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(12*a*c^2*f
*h^2-15*b^2*c*f*h^2+12*b*c^2*e*h^2+24*b*c^2*f*g*h-8*c^3*d*h^2-16*c^3*e*g*h
-8*c^3*f*g^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2)))+(-4*a*b*c*f*h^2+8*a*c^2*e*h^2+16*a*c^2*f*g*h-7
*b^3*f*h^2+4*b^2*c*e*h^2+8*b^2*c*f*g*h-16*c^3*d*g*h-8*c^3*e*g^2)*(-1/c/(c*
x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(271) = 542$.

Time = 5.98 (sec) , antiderivative size = 1769, normalized size of antiderivative = 6.12

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e
- 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*
b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 +
(8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4
*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e +
 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b
*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*
h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 2
4*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^
2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 -
 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4
*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*
a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^
3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)
*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^
4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*
c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2
 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 +
(b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 -
4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*...
```

3.234.6 Sympy [F]

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx = \int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}} dx$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.234.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(271) = 542$.

Time = 0.29 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.96

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(b^2c^2fh^2-4\,ac^3fh^2)x}{b^2c^3-4\,ac^4} + \frac{8\,b^2c^2fgh-32\,ac^3fgh+4\,b^2c^2eh^2-16\,ac^3eh^2-5\,b^3cfh^2+20\,abc^2fh^2}{b^2c^3-4\,ac^4}\right.\right.}{}$$
$$- \frac{(8\,c^2fg^2+16\,c^2egh-24\,bcfgh+8\,c^2dh^2-12\,bceh^2+15\,b^2fh^2-12\,acfh^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+}\right.\right.\right.}{8\,c^{\frac{7}{2}}}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^
2*f*g*h - 32*a*c^3*f*g*h + 4*b^2*c^2*e*h^2 - 16*a*c^3*e*h^2 - 5*b^3*c*f*h^
2 + 20*a*b*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 - 8*b*c^3*e*g
^2 + 8*b^2*c^2*f*g^2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h + 16*b^2*c^2*e*g*h
- 32*a*c^3*e*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8*b^2*c^2*d*h^2 - 1
6*a*c^3*d*h^2 - 12*b^3*c*e*h^2 + 40*a*b*c^2*e*h^2 + 15*b^4*f*h^2 - 62*a*b^
2*c*f*h^2 + 24*a^2*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 - 16
*a*c^3*e*g^2 + 8*a*b*c^2*f*g^2 - 32*a*c^3*d*g*h + 16*a*b*c^2*e*g*h - 24*a*
b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d*h^2 - 12*a*b^2*c*e*h^2 + 32*a
^2*c^2*e*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2)/(b^2*c^3 - 4*a*c^4))/sqr
t(c*x^2 + b*x + a) - 1/8*(8*c^2*f*g^2 + 16*c^2*e*g*h - 24*b*c*f*g*h + 8*c^
2*d*h^2 - 12*b*c*e*h^2 + 15*b^2*f*h^2 - 12*a*c*f*h^2)*log(abs(2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.235 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

3.235.1 Optimal result

Integrand size = 30, antiderivative size = 186

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d-2bce+3b^2f-8acf\right)h\sqrt{a+bx+cx^2}}{c^2\left(b^2-4ac\right)} - \frac{(3bfh-2c(fg+eh))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}}$$

output
```
-1/2*(3*b*f*h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)+2*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(
h*x+g)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e)
)*h*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{-3b^3fhx+2bc(aeh-cegx+cd(g-hx)+af(g+5hx))+b^2(-3afh+c}{c^2\left(-b^2+4a\right.} + \frac{(-3bfh+2c(fg+eh))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g + 5*h*x))
+ b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h + c^2*d*g*
x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x))))/(c^2*(-b^2 + 4*a*c)*Sqrt[a +
 x*(b + c*x)]) + ((-3*b*f*h + 2*c*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[
a] + Sqrt[a + x*(b + c*x)]])])/c^(5/2)
```

3.235.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2175, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{fgb^2+2(cd+af)hb-4ac(fg+eh)+c\left(\frac{3fb^2}{c}-2eb+4cd-8af\right)hx}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{fgb^2+2(cd+af)hb-4ac(fg+eh)+\left(3fb^2-2ceb+4c^2d-8acf\right)hx}{\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c}-\frac{\left(b^2-4ac\right)(3bfh-2c(eh+fg))\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}}{c\left(b^2-4ac\right)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c}-\frac{(b^2-4ac)(3bfh-2c(eh+fg))\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}}{c\left(b^2-4ac\right)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(3bfh-2c(eh+fg))}{2c^{3/2}}}{c\left(b^2-4ac\right)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((4*c^2*d - 2*b*c*e +
 3*b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/c - ((b^2 - 4*a*c)*(3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(2*c^(3/2)))/(c*(b^2 - 4*a*c))
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.235. $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.235.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.67

method	result
risch	$\frac{fh\sqrt{cx^2+bx+a}}{c^2} - \frac{\frac{2abfh(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}} - \frac{4c^2dg(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}} + (3bcfh-2c^2eh-2c^2fg)\left(-\frac{x}{c\sqrt{cx^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{cx^2+bx+a}} - \frac{c(\dots}{\dots}\right)}{2\dots}\dots\right)}{\dots}$
default	$\frac{2dg(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}} + fh\left(\frac{x^2}{c\sqrt{cx^2+bx+a}} - \frac{3b\left(-\frac{x}{c\sqrt{cx^2+bx+a}} - \frac{b\left(-\frac{1}{c\sqrt{cx^2+bx+a}} - \frac{b(2cx+b)}{c(4ac-b^2)\sqrt{cx^2+bx+a}}\right)}{2c} + \frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\dots\right)}{c\dots}\right)}{2c}\dots\right)$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
f*h/c^2*(c*x^2+b*x+a)^(1/2)-1/2/c^2*(2*a*b*f*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)-4*c^2*d*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(3*b*c*
f*h-2*c^2*e*h-2*c^2*f*g)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*
x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(2*a*c*f*h+b^2*f*h-2*c^2*d*h-2*c^2*
e*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)))
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(171) = 342$.

Time = 3.87 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.87

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \left[-\frac{(2\,(ab^2c-4\,a^2c^2)fg+(2\,(b^2c^2-4\,ac^3)fg+(2\,(b^2c^2-4\,ac^3)e-3\,(b^3c\right.$$

$$-\frac{(2\,(ab^2c-4\,a^2c^2)fg+(2\,(b^2c^2-4\,ac^3)fg+(2\,(b^2c^2-4\,ac^3)e-3\,(b^3c-4\,abc^2)f)h)x^2+(2\,(ab^2c-4\,a^2c^2}{}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2
*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*
c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3
*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(
(b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a
*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*
e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b
^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4
+ (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c -
4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3
*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4
*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3
*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2
 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a
*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*
g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)
*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 +
 (b^3*c^3 - 4*a*b*c^4)*x)]
```

3.235.6 Sympy [F]

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.235.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.41

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\frac{(b^2cfh-4\,ac^2fh)x}{b^2c^2-4\,ac^3} - \frac{4\,c^3dg-2\,bc^2eg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh+2\,b^2ceh-4\,ac^2eh-3\,b^3fh+10\,ab}{b^2c^2-4\,ac^3}\right.}{\sqrt{cx^2+bx+a}} - \frac{(2\,cfg+2\,ceh-3\,bfh)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
(((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g - 2*b*c^2*e
*g + 2*b^2*c*f*g - 4*a*c^2*f*g - 2*b*c^2*d*h + 2*b^2*c*e*h - 4*a*c^2*e*h -
 3*b^3*f*h + 10*a*b*c*f*h)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g - 4*a*c^2
*e*g + 2*a*b*c*f*g - 4*a*c^2*d*h + 2*a*b*c*e*h - 3*a*b^2*f*h + 8*a^2*c*f*h
)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g + 2*c*e*h - 3*
b*f*h)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{(g+h\,x)\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

3.236 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$

3.236.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b
*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{c\,(-b^2+4ac)\,\sqrt{a+x(b+cx)}} + \frac{2f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b
^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (2*f*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
Sqrt[a + x*(b + c*x)])])/c^(3/2)
```

3.236.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2f\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.236.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.81

method	result
default	$\frac{2d(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*(-x/c/(c*x^2+b*x+a)^(1/2)-
1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(99) = 198$.

Time = 0.47 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.86

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b\right.}{2\,(ab^2c^2}\right.$$

$$-\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(bc^2d-2\,ac^2e}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc^3)}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f
)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^
2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^
2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b
*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x
)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
+ (b^3*c^2 - 4*a*b*c^3)*x)]
```

3.236.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.236.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{d + ex + fx^2}{(a + bx + cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2\,c^2d - bce + b^2f - 2\,acf)x}{b^2c - 4\,ac^2} + \frac{bcd - 2\,ace + abf}{b^2c - 4\,ac^2}\right)}{\sqrt{cx^2 + bx + a}} - \frac{f\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x/(b^2*c - 4*a*c^2) + (b*c*d - 2*a
*c*e + a*b*f)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 13.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c -
 b^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c
 - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.237 $\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx$

3.237.1 Optimal result

Integrand size = 32, antiderivative size = 225

$$\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-(2c^2dg+bf(bg\cdots}{(b^2-4ac)(cg^2-bgh+ah^2)\sqrt{a+bx\cdots}} + \frac{(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{(cg^2-bgh+ah^2)^{3/2}}$$

output
```
(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c
*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)+2*(b^2*d*h-b*(a
*e*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)-(2*c^2*d*g+b*f*(-a*h+b*g)-c*(-2*
a*e*h+2*a*f*g+b*d*h+b*e*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^2+b*x
+a)^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx = 2\left(\frac{-2a^2fh+2c^2dgx+b^2(-dh+fgx)+2ac(-eg+dh-fgx+ehx)\cdots}{(b^2-4ac)(-cg^2+h(bg-ah))\sqrt{a\cdots}} + \frac{\sqrt{-cg^2+bgh-ah^2}(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{(cg^2+h(-bg+ah))^2}\right)$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
2*((-2*a^2*f*h + 2*c^2*d*g*x + b^2*(-(d*h) + f*g*x) + 2*a*c*(-(e*g) + d*h
- f*g*x + e*h*x) + b*c*(-(e*g*x) + d*(g - h*x)) + a*b*(e*h + f*(g - h*x)))
/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) + (Sqrt[
-(c*g^2) + b*g*h - a*h^2]*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g +
h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 +
h*(-(b*g) + a*h))^2)
```

3.237.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 2177$$

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$
$$\frac{2\int -\frac{\left(b^2-4ac\right)\left(fg^2-h(eg-dh)\right)}{2\left(cg^2-bhg+ah^2\right)(g+hx)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(fg^2-h(eg-dh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{ah^2-bgh+cg^2}+$$
$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1154$$

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$
$$\frac{2\left(fg^2-h(eg-dh)\right)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{ah^2-bgh+cg^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(fg^2 - h(eg - dh))\,\text{arctanh}\left(\frac{-2ah + x(2cg - bh) + bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{(ah^2 - bgh + cg^2)^{3/2}} + \frac{2(-x(-c(-2aeh + 2afg + bdh + beg) + bf(bg - ah) + 2c^2dg) - b(aeh + afg + cdg) + 2a(afh - cdh + ceg) + b^2}{(b^2 - 4ac)\sqrt{a + bx + cx^2}\,(ah^2 - bgh + cg^2)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2
*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((
b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*
(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
 + a*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(215) = 430$.

Time = 0.76 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.43

method	result
default	$\frac{\frac{2eh(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}}+fh\left(-\frac{1}{c\sqrt{cx^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{cx^2+bx+a}}\right)-\frac{2fg(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}}}{h^2}+\frac{(dh^2-egh+fg^2)\Bigg(\frac{}{(ah^2-bgh+cg^2}}{}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/h^2*(2*e*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*h*(-1/c/(c*x^2+b*
x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-2*f*g*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2-b*g*h+c
*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c
*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(
x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+1/h
*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*
g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(215) = 430$.

Time = 9.79 (sec) , antiderivative size = 1905, normalized size of antiderivative = 8.47

$$\int \frac{d + ex + fx^2}{(g + hx)\left(a + bx + cx^2\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*
c)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4
*a*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3
 - 4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h
^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 -
 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g
 - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 +
 2*g*h*x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2
*(b^2*c - a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*
c)*d - (a*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)
*d)*h^3 + ((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b
^2*c + 2*a*c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2
)*d - 2*(a*b^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)
*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g^3*h + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*
a^3*b*c)*g*h^3 + (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b
^3*c^2 - 4*a*b*c^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*
(a*b^3*c - 4*a^2*b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*g^4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*g^2*h^2 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*...
```

3.237.6 Sympy [F]

$$\int \frac{d + ex + fx^2}{(g + hx)\left(a + bx + cx^2\right)^{3/2}}\, dx = \int \frac{d + ex + fx^2}{(g + hx)\left(a + bx + cx^2\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(215) = 430$.

Time = 0.31 (sec) , antiderivative size = 708, normalized size of antiderivative = 3.15

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(\frac{(2c^3dg^3-bc^2eg^3+b^2cfg^3-2ac^2fg^3-3bc^2dg^2h+b^2ceg^2h+2ac^2eg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2ac^2dgh^2-3abcegh^2+2ab^2fgh^2-2a^2c}{b^2c^2g^4-4ac^3g^4-2b^3cg^3h+8abc^2g^3h+b^4g^2h^2-2ab^2cg^2h^2-8a^2c^2g^2h^2-2ab^3gh^3+8a^2bcgh^3+a^2b^2h^4-4a^3}\right.}{} + \frac{2\left(fg^2-egh+dh^2\right)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2+bgh-ah^2}}\right)}{\left(cg^2-bgh+ah^2\right)\sqrt{-cg^2+bgh-ah^2}}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^3*d*g^3 - b*c^2*e*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g
^2*h + b^2*c*e*g^2*h + 2*a*c^2*e*g^2*h - b^3*f*g^2*h + a*b*c*f*g^2*h + b^2
*c*d*g*h^2 + 2*a*c^2*d*g*h^2 - 3*a*b*c*e*g*h^2 + 2*a*b^2*f*g*h^2 - 2*a^2*c
*f*g*h^2 - a*b*c*d*h^3 + 2*a^2*c*e*h^3 - a^2*b*f*h^3)*x/(b^2*c^2*g^4 - 4*a
*c^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h
^2 - 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4
*a^3*c*h^4) + (b*c^2*d*g^3 - 2*a*c^2*e*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h
 + 2*a*c^2*d*g^2*h + 3*a*b*c*e*g^2*h - a*b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b
^3*d*g*h^2 - a*b*c*d*g*h^2 - a*b^2*e*g*h^2 - 2*a^2*c*e*g*h^2 + 3*a^2*b*f*g
*h^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 + a^2*b*e*h^3 - 2*a^3*f*h^3)/(b^2*c^2*g
^4 - 4*a*c^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2
*c*g^2*h^2 - 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2
*h^4 - 4*a^3*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 - e*g*h + d*h^2)*arc
tan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g
*h - a*h^2))/((c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\,(a+bx+cx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}}\,dx$

3.238.1 Optimal result

Integrand size = 32, antiderivative size = 421

$$\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b(c^2dg^2+a^2fh^2+ac(fg^2+2egh-3}{(b^2-4ac)(cg^2-bgh}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{(cg^2-bgh+ah^2)^2(g+hx)}$$

$$+\frac{(2cg(fg^2-h(2eg-3dh))-h(2ah(2fg-eh)-b(fg^2+egh-3dh^2)))\,\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2(cg^2-bgh+ah^2)^{5/2}}$$

output
```
1/2*(2*c*g*(f*g^2-h*(-3*d*h+2*e*g))-h*(2*a*h*(-e*h+2*f*g)-b*(-3*d*h^2+e*g*
h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2
)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)-2*(b^3*d*h^2-b^2*h*(a*e*h
+2*c*d*g)-2*a*c*(c*g*(-2*d*h+e*g)+a*h*(-e*h+2*f*g))+b*(c^2*d*g^2+a^2*f*h^2
+a*c*(-3*d*h^2+2*e*g*h+f*g^2))+c*(2*c^2*d*g^2+2*a^2*f*h^2-a*b*h*(e*h+2*f*g
)+b^2*(d*h^2+f*g^2)-c*(b*g*(2*d*h+e*g)+2*a*(d*h^2-2*e*g*h+f*g^2)))*x)/(-4*
a*c+b^2)/(a*h^2-b*g*h+c*g^2)^2/(c*x^2+b*x+a)^(1/2)-h*(f*g^2-h*(-d*h+e*g))*
(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

3.238.2 Mathematica [A] (verified)

Time = 10.97 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.23

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2f(b+2cx)}{(b^2-4ac)\,h^2\sqrt{a+x(b+cx)}}$$
$$+\frac{2(fg^2+h(-eg+dh))\left(-b^2h+2c(ah+cgx)+bc(g-hx)\right)}{(b^2-4ac)\,h^2\left(-cg^2+h(bg-ah)\right)(g+hx)\sqrt{a+x(b+cx)}}$$
$$-\frac{2(-2fg+eh)\left(b^2h-2c(ah+cgx)+bc(-g+hx)\right)}{(b^2-4ac)\,h^2\left(-cg^2+h(bg-ah)\right)\sqrt{a+x(b+cx)}}$$
$$-\frac{(fg^2+h(-eg+dh))\left(\frac{2\left(4c^2g^2+3b^2h^2-4ch(bg+2ah)\right)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}-\frac{3(b^2-4ac)h(-2cg+bh)\operatorname{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b}}\right.}{(cg^2+h(-bg+ah))^{3/2}}\right.}{2\left(b^2-4ac\right)h\left(cg^2+h(-bg+ah)\right)}$$
$$-\frac{(2fg-eh)\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{\left(cg^2+h(-bg+ah)\right)^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*f*(b + 2*c*x))/((b^2 - 4*a*c)*h^2*Sqrt[a + x*(b + c*x)]) + (2*(f*g^2 +
 h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x)))/((b^2 -
 4*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) -
(2*(-2*f*g + e*h)*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*
a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - ((f*g^2 + h*(
-(e*g) + d*h))*((2*(4*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a +
x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*
(-2*c*g + b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 +
h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2
)))/(2*(b^2 - 4*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))) - ((2*f*g - e*h)*ArcTan
h[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[
a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)
```

3.238.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$-\frac{2\int -\frac{(b^2-4ac)\left(-\left(\left(afg^2-b(eg-2dh)g-adh^2\right)h^2\right)-\left(cg(eg-2dh)+ah(2fg-eh)-b\left(fg^2-dh^2\right)\right)xh^2+c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{2(cg^2-bhg+ah^2)^2(g+hx)^2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(\ldots}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left(\left(afg^2-b(eg-2dh)g-adh^2\right)h^2\right)-\left(cg(eg-2dh)+ah(2fg-eh)-b\left(fg^2-dh^2\right)\right)xh^2+c\left(fg^4-g^2h(2eg-3dh)\right)}{(g+hx)^2\sqrt{cx^2+bx+a}}dx}{(ah^2-bgh+cg^2)^2}-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(\ldots}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{1}{2}\left(h\left(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2\right)+2c\left(fg^3-gh(2eg-3dh)\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx-\frac{h\sqrt{a+bx+cx^2}(dh\ldots}{g+h\ldots}}{(ah^2-bgh+cg^2)^2}-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(\ldots}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\left(h\left(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2\right)+2c\left(fg^3-gh(2eg-3dh)\right)\right)\int \frac{1}{4(cg^2-bhg+ah^2)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\Big(\ldots}{(ah^2-bgh+cg^2)^2}-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(\ldots}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h\left(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2\right)+2c\left(fg^3-gh(2eg-3dh)\right)\right)}{2\sqrt{ah^2-bgh+cg^2}}-\frac{h\sqrt{a+bx+cx^2}\left(dh^2-egh+fg^2\right)}{g+hx}}{\left(ah^2-bgh+cg^2\right)^2} -$$

$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh^2\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,(}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*
(2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2
)) + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h
^2) - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4
*a*c)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) + (-((h*(f*g^2 - e*
g*h + d*h^2)*Sqrt[a + b*x + c*x^2])/(g + h*x)) + ((2*c*(f*g^3 - g*h*(2*e*g
 - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTan
h[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]))/(c*g^2 - b*g*h + a*h^2)^2
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.238.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(407) = 814$.

Time = 0.76 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	1115

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*
h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h*g)+(b*h-2*c
*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h
-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*
g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*c+(b*h-2*c*g
)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+1/h^4*(d*h^2-e*g
*h+f*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*g)
/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h
+c*g^2)*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1
/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1
/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-
b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)
/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^
2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-4*
c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+
c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(407) = 814$.

Time = 35.52 (sec) , antiderivative size = 5098, normalized size of antiderivative = 12.11

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

3.238.8 Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \int \frac{fx^2+ex+d}{\left(cx^2+bx+a\right)^{\frac{3}{2}}(hx+g)^2}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx$

3.239.1 Optimal result

Integrand size = 32, antiderivative size = 713

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h(3c^2dg^2+a^2fh^2+ach(3eg-4dh))}{}$$

$$-\frac{h(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2\left(cg^2-bgh+ah^2\right)^2(g+hx)^2}$$

$$-\frac{h(2cg(3fg^2-h(5eg-7dh))-h(4ah(2fg-eh)-b(fg^2+3egh-7dh^2)))\sqrt{a+bx+cx^2}}{4\left(cg^2-bgh+ah^2\right)^3(g+hx)}$$

$$+\frac{(8c^2g^2(fg^2-3egh+6dh^2)+h^2(8a^2fh^2+4abh(2fg-3eh)-b^2(fg^2+3h(eg-5dh)))-4ch(ah(11fg^2}{8\left(cg^2-bgh+ah^2\right)^{7/2}}$$

output
```
1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+h^2*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*
f*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g)))-4*c*h*(a*h*(3*d*h^2-9*e*g*h+11*f*g^2)-b
*g*(2*f*g^2+3*h*(-4*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*
h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+2*(b
^4*d*h^3-b^3*h^2*(a*e*h+3*c*d*g)+b^2*h*(3*c^2*d*g^2+a^2*f*h^2+a*c*h*(-4*d*
h+3*e*g))-b*c*(c^2*d*g^3+3*a^2*h^2*(-e*h+f*g)+a*c*g*(-9*d*h^2+3*e*g*h+f*g^
2))-2*a*c*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(d*h^2-3*e*g*h+3*f*g^2))-c
*(2*c^3*d*g^3-b*(a^2*f-a*b*e+b^2*d)*h^3-c^2*g*(b*g*(3*d*h+e*g)+2*a*(3*d*h^
2-3*e*g*h+f*g^2))+c*(2*a^2*h^2*(-e*h+3*f*g)-3*a*b*h*(-d*h^2+e*g*h+f*g^2)+b
^2*(3*d*g*h^2+f*g^3)))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^3/(c*x^2+b*x+a)
^(1/2)-1/2*h*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^
2/(h*x+g)^2-1/4*h*(2*c*g*(3*f*g^2-h*(-7*d*h+5*e*g))-h*(4*a*h*(-e*h+2*f*g)-
b*(-7*d*h^2+3*e*g*h+f*g^2)))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*
x+g)
```

3.239.2 Mathematica [A] (verified)

Time = 11.87 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.19

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx = \frac{2(fg^2+h(-eg+dh))(-b^2h+2c(ah+cgx)+bc(g-hx))}{(b^2-4ac)h^2(-cg^2+h(bg-ah))(g+hx)^2\sqrt{a+x(b+cx)}} - \frac{2f(b^2h-2c(ah+cgx)+bc(-g+hx))}{(b^2-4ac)h^2(-cg^2+h(bg-ah))\sqrt{a+x(b+cx)}} - \frac{2(-2fg+eh)(b^2h-2c(ah+cgx)+bc(-g+hx))}{(b^2-4ac)h^2(-cg^2+h(bg-ah))(g+hx)\sqrt{a+x(b+cx)}} - \frac{(-2fg+eh)\left(\frac{2(4c^2g^2+3b^2h^2-4ch(bg+2ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)} - \frac{3(b^2-4ac)h(-2cg+bh)\operatorname{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}\right)}{2(b^2-4ac)h(cg^2+h(-bg+ah))} - \frac{(fg^2+h(-eg+dh))\left(\frac{4(8c^2g^2+5b^2h^2-4ch(2bg+3ah))\sqrt{a+x(b+cx)}}{(g+hx)^2} + \frac{2(2cg-bh)(8c^2g^2+15b^2h^2-4ch(2bg+13ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)} -}{8(b^2-4ac)h(cg^2+h(-bg+ah))^2} + \frac{f\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(f*g^2 + h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x
)))/((b^2 - 4*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)^2*Sqrt[a + x*(
b + c*x)]) - (2*f*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*
a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (2*(-2*f*g +
e*h)*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*a*c)*h^2*(-(c
*g^2) + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) - ((-2*f*g + e*h)*
((2*(4*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a + x*(b + c*x)])/(
(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*(-2*c*g + b*h)*
ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h
)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(2*(b^2 - 4
*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))) - ((f*g^2 + h*(-(e*g) + d*h))*((4*(8*c
^2*g^2 + 5*b^2*h^2 - 4*c*h*(2*b*g + 3*a*h))*Sqrt[a + x*(b + c*x)])/(g + h*
x)^2 + (2*(2*c*g - b*h)*(8*c^2*g^2 + 15*b^2*h^2 - 4*c*h*(2*b*g + 13*a*h))*
Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (3*(b^2 -
4*a*c)*h*(16*c^2*g^2 + 5*b^2*h^2 - 4*c*h*(4*b*g + a*h))*ArcTanh[(-(b*g) +
2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
 c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(8*(b^2 - 4*a*c)*h*(c*g^2 + h
*(-(b*g) + a*h))^2) + (f*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[
c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*
h))^(3/2)
```

3.239.3 Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^3 \left(a+bx+cx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right.\right.\right.}{}$$

$$2\int -\frac{\frac{\left(b^2-4ac\right)\left(\left(fa^2-bea+b^2d\right)h^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)+bc\left(fg^3-3dgh^2\right)\right)x^2h^3}{\left(cg^2-bhg+ah^2\right)^3}-\frac{\left(b^2-4ac\right)\left(c^2(3eg-8dh)g^3-c\left(3bfg^2+bh(eg-9dh)-2a\right.\right.}{}}{}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\frac{\left(b^2-4ac\right)\left(\left(fa^2-bea+b^2d\right)h^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)+bc\left(fg^3-3dgh^2\right)\right)x^2h^3}{\left(cg^2-bhg+ah^2\right)^3}-\frac{\left(b^2-4ac\right)\left(c^2(3eg-8dh)g^3-c\left(3bfg^2+bh(eg-9dh)-2ah(4f\ldots\right.\right.}{\left(cg\ldots\right.}}{}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\ldots\right.\right.\right.$$

$\downarrow$ 2181

$$-\frac{\int -\frac{\left(b^2-4ac\right)\left(4c^2\left(fg^2-3ehg+6dh^2\right)g^3+ch\left(bg\left(fg^2+11ehg-31dh^2\right)-8ah\left(2fg^2-ehg-dh^2\right)\right)g-h^2\left(g\left(fg^2+3ehg-11dh^2\right)b^2-ah\left(9fg^2-5ehg-7dh^2\right)b+4a^2h\ldots\right.\right.}{2\left(cg^2-bhg+ah^2\right)^2(g+hx)^2\sqrt{cx^2+bx\ldots}}}{2(ah^2-bgh+cg^2)}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\ldots\right.\right.\right.$$

$\downarrow$ 27

$$\frac{(b^2-4ac)\int \frac{4c^2\left(fg^2-3ehg+6dh^2\right)g^3+ch\left(bg\left(fg^2+11ehg-31dh^2\right)-8ah\left(2fg^2-ehg-dh^2\right)\right)g-h^2\left(g\left(fg^2+3ehg-11dh^2\right)b^2-ah\left(9fg^2-5ehg-7dh^2\right)b+4a^2h^2(fg\ldots\right.}{(g+hx)^2\sqrt{cx^2+bx+a}}}{4(ah^2-bgh+cg^2)^3}$$

$$b^2\ldots$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\ldots\right.\right.\right.$$

$\downarrow$ 1228

$$\frac{(b^2-4ac)\left(\frac{1}{2}\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg-4dh)+2bfg^3\right)+8c^2g^2\left(6dh^2-3eg\ldots\right.\right.\right.}{4(ah^2-bgh+cg^2)^3}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\ldots\right.\right.\right.$$

$\downarrow$ 1154

$$\frac{(b^2-4ac)\left(-\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg-4dh)+2bfg^3\right)+8c^2g^2\left(6dh^2-3e\ldots\right.\right.\right.}{4(ah\ldots}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\ldots\right.\right.\right.$$

$\downarrow$ 219

3.239. $\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx$

$$\frac{(b^2-4ac)\left(\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg-4\right.\right.}{2\sqrt{ah^2-bgh+cg^2}}\right.}{4(ah^2-bgh+cg^2)^3}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right.\right.\right.$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
 + a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g
*(f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) -
 a*c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e +
a^2*f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c
*(2*a^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(
e*g - d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x +
c*x^2]) + (-1/2*((b^2 - 4*a*c)*h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*
x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) + ((b^2 - 4*a*c)*(-((h*(6*c*
f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*f*g - e*h) + b*h*(f*g^2 + h*(
3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(g + h*x)) + ((8*c^2*g^2*(f*g^2 -
3*e*g*h + 6*d*h^2) + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*
g^2 - 9*e*g*h + 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b
^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(
2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g
*h + a*h^2])))/(4*(c*g^2 - b*g*h + a*h^2)^3))/(b^2 - 4*a*c)
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.239. $\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx$

3.239.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. 2(693) = 1386.

Time = 0.90 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	2268

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^3*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+1/h
*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h
*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*
g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h
^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g)^2*
c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))+(e*h
-2*f*g)/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+1/h*g)/((x+1/h*g)^2*c+(b*h-2*c*
g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g
*h+c*g^2)*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x
+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x
+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^
2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+1/h*g)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+1/h*g
)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+1/h*g)))-
4*c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+1/h*g)+(b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*
h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g*h+c*
g^2)*h^2/(x+1/h*g)^2/((x+1/h*g)^2*c+(b*h-2*c*g)/h*(x+1/h*g)+(a*h^2-b*g*...
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5149 vs. 2(693) = 1386.

Time = 162.17 (sec) , antiderivative size = 10340, normalized size of antiderivative = 14.50

$$\int \frac{d+ex+fx^2}{(g+hx)^3\,(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

3.239.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5562 vs. 2(693) = 1386.

Time = 0.41 (sec) , antiderivative size = 5562, normalized size of antiderivative = 7.80

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^7*d*g^9 - b*c^6*e*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d
*g^8*h + 3*b^2*c^5*e*g^8*h + 6*a*c^6*e*g^8*h - 3*b^3*c^4*f*g^8*h + 3*a*b*c
^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 - 3*b^3*c^4*e*g^7*h^2 - 24*a*b*c^5*e*g^7
*h^2 + 3*b^4*c^3*f*g^7*h^2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3
+ b^4*c^3*e*g^6*h^3 + 34*a*b^2*c^4*e*g^6*h^3 + 16*a^2*c^5*e*g^6*h^3 - b^5*
c^2*f*g^6*h^3 - 13*a*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c
^3*d*g^5*h^4 + 6*a*b^2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 - 21*a*b^3*c^3
*e*g^5*h^4 - 42*a^2*b*c^4*e*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2*c
^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*d
*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 + 6*a*b^4*c^2*e*g^4*h^5 + 36*a^2*b^2*c^3
*e*g^4*h^5 + 12*a^3*c^4*e*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^
3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*
g^3*h^6 - 16*a^3*c^4*d*g^3*h^6 - a*b^5*c*e*g^3*h^6 - 13*a^2*b^3*c^2*e*g^3*
h^6 - 16*a^3*b*c^3*e*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*
h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7
 + 24*a^3*b*c^3*d*g^2*h^7 + 3*a^2*b^4*c*e*g^2*h^7 + 6*a^3*b^2*c^2*e*g^2*h^
7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 + 3*a^2*b^4*c*d*g*h^8 -
 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 - 3*a^3*b^3*c*e*g*h^8 + 3*a^4*b
*c^2*e*g*h^8 + 3*a^4*b^2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 +
 3*a^4*b*c^2*d*h^9 + a^4*b^2*c*e*h^9 - 2*a^5*c^2*e*h^9 - a^5*b*c*f*h^9)...
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)^3\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

3.239. $\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx$

3.240 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

3.240.1 Optimal result

Integrand size = 32, antiderivative size = 120

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - \frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240}+\frac{9211\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}$$

output
```
9211/3888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+44/135*(1+2*x)^2*(3*x^2-x
+2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2-x+2)^(1/
2)-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{6\sqrt{2-x+3x^2}(-22383+7538x+26904x^2+22032x^3+6912x^4)+46055\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+\right.}{19440}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4
) + 46055*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/19440
```

3.240.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2184, 27, 1236, 27, 1236, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}} \, dx$$

$$\Big\downarrow \; 2184$$

$$\frac{1}{60} \int -\frac{4(16-57x)(2x+1)^3}{\sqrt{3x^2-x+2}}dx + \frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \; 27$$

$$\frac{2}{15}(2x+1)^4\sqrt{3x^2-x+2} - \frac{1}{15}\int \frac{(16-57x)(2x+1)^3}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \; 1236$$

$$\frac{1}{15}\left(\frac{19}{4}(2x+1)^3\sqrt{3x^2-x+2} - \frac{1}{12}\int \frac{3(565-352x)(2x+1)^2}{2\sqrt{3x^2-x+2}}dx\right) + \frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \; 27$$

$$\frac{1}{15}\left(\frac{19}{4}(2x+1)^3\sqrt{3x^2-x+2} - \frac{1}{8}\int \frac{(565-352x)(2x+1)^2}{\sqrt{3x^2-x+2}}dx\right) + \frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \; 1236$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{352}{9}(2x+1)^2\sqrt{3x^2-x+2} - \frac{1}{9}\int \frac{(2x+1)(6298x+7725)}{\sqrt{3x^2-x+2}}dx\right) + \frac{19}{4}\sqrt{3x^2-x+2}(2x+1)^3\right) + \frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055}{6}\int \frac{1}{\sqrt{3x^2-x+2}}dx - \frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right) + \frac{352}{9}\sqrt{3x^2-x+2}(2x+1)^2\right) + \frac{19}{4}\right. \frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{6\sqrt{69}}-\frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right)+\frac{352}{9}\sqrt{3x^2-x+2}(2x+1)^2\right.\right.$$

$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right)+\frac{352}{9}\sqrt{3x^2-x+2}(2x+1)^2\right)+\frac{19}{4}\sqrt{}\right.$$

$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 + ((19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/4 + ((352*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + (-1/3*((24897 + 6298*x
)*Sqrt[2 - x + 3*x^2]) - (46055*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]))
/9)/8)/15
```

3.240.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.240. $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.240.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(6912x^4+22032x^3+26904x^2+7538x-22383)\sqrt{3x^2-x+2}}{3240}-\frac{9211\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$
trager	$\left(\frac{32}{15}x^4+\frac{34}{5}x^3+\frac{1121}{135}x^2+\frac{3769}{1620}x-\frac{829}{120}\right)\sqrt{3x^2-x+2}-\frac{9211\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3}\right.}{3888}$
default	$-\frac{9211\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}-\frac{829\sqrt{3x^2-x+2}}{120}+\frac{32x^4\sqrt{3x^2-x+2}}{15}+\frac{34x^3\sqrt{3x^2-x+2}}{5}+\frac{1121x^2\sqrt{3x^2-x+2}}{135}+\frac{3769x}{}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3240*(6912*x^4+22032*x^3+26904*x^2+7538*x-22383)*(3*x^2-x+2)^(1/2)-9211/
3888*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{3240}\left(6912\,x^4+22032\,x^3+26904\,x^2+7538\,x-22383\right)\sqrt{3\,x^2-x+2} + \frac{9211}{7776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x
+ 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*
x^2 + 24*x - 25)
```

3.240.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{32x^4}{15}+\frac{34x^3}{5}+\frac{1121x^2}{135}+\frac{3769x}{1620}-\frac{829}{120}\right) - \frac{9211\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**4/15 + 34*x**3/5 + 1121*x**2/135 + 3769*x/1620
 - 829/120) - 9211*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3888
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{32}{15}\sqrt{3\,x^2-x+2}x^4+\frac{34}{5}\sqrt{3\,x^2-x+2}x^3 + \frac{1121}{135}\sqrt{3\,x^2-x+2}x^2+\frac{3769}{1620}\sqrt{3\,x^2-x+2}x - \frac{9211}{3888}\sqrt{3}\,\text{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{829}{120}\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sq
rt(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{3240}(2\,(12\,(18\,(16\,x+51)x+1121)x+3769)x-22383)\sqrt{3\,x^2-x+2} + \frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 -
x + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)
) + 1)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{{\left(2\,x+1\right)}^3\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.241 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

3.241.1 Optimal result

Integrand size = 32, antiderivative size = 95

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx=-\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{4147\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

output
```
4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-143/324*(3-2*x)*(3*x^2-x+
2)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(1+2*x)^3*(3*x^2-x+2)^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx =\frac{6\sqrt{2-x+3x^2}(-243+1138x+1176x^2+432x^3)+4147\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{1944}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) + 4147*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1944
```

3.241.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2184, 27, 1236, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{\sqrt{3x^2-x+2}} dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{48} \int -\frac{44(1-4x)(2x+1)^2}{\sqrt{3x^2-x+2}} dx + \frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2} - \frac{11}{12}\int \frac{(1-4x)(2x+1)^2}{\sqrt{3x^2-x+2}} dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2} - \frac{11}{12}\left(\frac{1}{9}\int \frac{13(3-2x)(2x+1)}{\sqrt{3x^2-x+2}} dx - \frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2} - \frac{11}{12}\left(\frac{13}{9}\int \frac{(3-2x)(2x+1)}{\sqrt{3x^2-x+2}} dx - \frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2} - \frac{11}{12}\left(\frac{13}{9}\left(\frac{29}{6}\int \frac{1}{\sqrt{3x^2-x+2}} dx + \frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right) - \frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2} - \frac{11}{12}\left(\frac{13}{9}\left(\frac{29\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}} d(6x-1)}{6\sqrt{69}} + \frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right) - \frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\left(\frac{13}{9}\left(\frac{29\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right)-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 - (11*((-4*(1 + 2*x)^2*Sqrt[2 - x + 3*
x^2])/9 + (13*(((3 - 2*x)*Sqrt[2 - x + 3*x^2])/3 + (29*ArcSinh[(-1 + 6*x)/
Sqrt[23]])/(6*Sqrt[3])))/9))/12
```

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

3.241. $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.241.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(432x^3+1176x^2+1138x-243)\sqrt{3x^2-x+2}}{324}-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{1944}$
trager	$\left(\frac{4}{3}x^3+\frac{98}{27}x^2+\frac{569}{162}x-\frac{3}{4}\right)\sqrt{3x^2-x+2}-\frac{4147\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\text{RootOf}\right.}{1944}$
default	$-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{1944}-\frac{3\sqrt{3x^2-x+2}}{4}+\frac{4x^3\sqrt{3x^2-x+2}}{3}+\frac{98x^2\sqrt{3x^2-x+2}}{27}+\frac{569x\sqrt{3x^2-x+2}}{162}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/324*(432*x^3+1176*x^2+1138*x-243)*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*ar
csinh(6/23*23^(1/2)*(x-1/6))
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(432\,x^3+1176\,x^2+1138\,x-243\right)\sqrt{3\,x^2-x+2} + \frac{4147}{3888}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2 + 24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*
sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.241.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{4x^3}{3}+\frac{98x^2}{27}+\frac{569x}{162}-\frac{3}{4}\right) - \frac{4147\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(4*x**3/3 + 98*x**2/27 + 569*x/162 - 3/4) - 4147*sqrt
(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/1944
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{4}{3}\,\sqrt{3\,x^2-x+2}x^3+\frac{98}{27}\,\sqrt{3\,x^2-x+2}x^2 + \frac{569}{162}\,\sqrt{3\,x^2-x+2}x - \frac{4147}{1944}\,\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{3}{4}\,\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt
(3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3
/4*sqrt(3*x^2 - x + 2)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\,(2\,(12\,(18\,x+49)x+569)x-243)\sqrt{3\,x^2-x+2} + \frac{4147}{1944}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{(2\,x+1)^2\,(4\,x^2+3\,x+1)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.242 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

3.242.1 Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} + \frac{251\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

output
```
251/324*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2-x+2)^(
1/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(6\sqrt{2-x+3x^2}(81+110x+48x^2) + 251\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) + 251*Sqrt[3]*Log[1 - 6*x + 2
*Sqrt[6 - 3*x + 9*x^2]])/324
```

3.242.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2184, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{36}\int -\frac{4(6-31x)(2x+1)}{\sqrt{3x^2-x+2}}dx+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2-x+2}-\frac{1}{9}\int \frac{(6-31x)(2x+1)}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251}{12}\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{12\sqrt{69}}\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{12\sqrt{3}}\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + (((69 + 62*x)*Sqrt[2 - x + 3*x^2])
/6 - (251*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(12*Sqrt[3]))/9
```

3.242.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.242. $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

3.242.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result
risch	$\frac{(48x^2+110x+81)\sqrt{3x^2-x+2}}{54}-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}$
default	$-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}+\frac{3\sqrt{3x^2-x+2}}{2}+\frac{8x^2\sqrt{3x^2-x+2}}{9}+\frac{55x\sqrt{3x^2-x+2}}{27}$
trager	$\left(\frac{8}{9}x^2+\frac{55}{27}x+\frac{3}{2}\right)\sqrt{3x^2-x+2}+\frac{251\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+}\right.}{324}$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(48*x^2+110*x+81)*(3*x^2-x+2)^(1/2)-251/324*3^(1/2)*arcsinh(6/23*23^(
1/2)*(x-1/6))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{54}\left(48\,x^2+110\,x+81\right)\sqrt{3\,x^2-x+2} + \frac{251}{648}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqr
t(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

3.242.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \left(\frac{8x^2}{9}+\frac{55x}{27}+\frac{3}{2}\right)\sqrt{3x^2-x+2}-\frac{251\sqrt{3}\,\mathrm{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
(8*x**2/9 + 55*x/27 + 3/2)*sqrt(3*x**2 - x + 2) - 251*sqrt(3)*asinh(6*sqrt
(23)*(x - 1/6)/23)/324
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{8}{9}\sqrt{3\,x^2-x+2}x^2+\frac{55}{27}\sqrt{3\,x^2-x+2}x - \frac{251}{324}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)+\frac{3}{2}\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{54}\left(2\,(24\,x+55)x+81\right)\sqrt{3\,x^2-x+2} + \frac{251}{324}\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*s
qrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

3.243.1 Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}$$

output
```
-5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*arctanh(1/26*(9-8*x)*13^
(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/3*(3*x^2-x+2)^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{\sqrt{13}} - \frac{5\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2])/3 + ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x
+ 3*x^2])/Sqrt[13]]/Sqrt[13] - (5*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/
(6*Sqrt[3])
```

3.243.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{12}\int \frac{4(5x+4)}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{5x+4}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{1}{\sqrt{3x^2-x+2}}dx+\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{5\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{2\sqrt{69}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{3}\left(\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-3\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-\frac{3\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2])/3 + ((5*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(2*Sqrt[3])
- (3*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13]))/3
```

3.243.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.243.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

method	result
default	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
risch	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{26}$
trager	$\frac{2\sqrt{3x^2-x+2}}{3}+\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\operatorname{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{26}+\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\right.}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+2/3*(3*x^2-x+2)^(1/2)-1/26*13^
(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.35

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)$$
$$+ \frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)$$
$$+ \frac{2}{3}\sqrt{3x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x
- 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220
*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

3.243.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{5}{18}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)$$
$$+ \frac{1}{26}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)$$
$$+ \frac{2}{3}\sqrt{3x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcs
inh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(
3*x^2 - x + 2)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) + \frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2}{3}\sqrt{3\,x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
-5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*s
qrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2
- x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/
3*sqrt(3*x^2 - x + 2)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

3.244.1 Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=-\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

output
```
-1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+9/338*arctanh(1/26*(9-8*x)*13^
(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.244.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=-\frac{\sqrt{2-x+3x^2}}{13+26x}-\frac{9\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}-\frac{\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-(Sqrt[2 - x + 3*x^2]/(13 + 26*x)) - (9*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2
*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13]) - Log[1 - 6*x + 2*Sqrt[6 -
3*x + 9*x^2]]/Sqrt[3]
```

3.244.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{13}\int -\frac{52x+17}{2(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{26}\int \frac{52x+17}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{26}\left(26\int \frac{1}{\sqrt{3x^2-x+2}}dx-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{26}\left(\frac{26\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{26}\left(\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{26}\left(18\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}+\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{26}\left(\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) + ((26*ArcSinh[(-1 + 6*x)/Sqrt[23]])/S
qrt[3] + (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])
/26
```

3.244.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.244.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^{2}-4x+\frac{5}{4}}}{26\left(x+\frac{1}{2}\right)}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^{2}-16x+5}}\right)}{338}$
risch	$-\frac{\sqrt{3x^{2}-x+2}}{13(1+2x)}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^{2}-16x+5}}\right)}{338}$
trager	$-\frac{\sqrt{3x^{2}-x+2}}{13(1+2x)}+\frac{9\operatorname{RootOf}\left(\_Z^{2}-13\right)\ln\left(-\frac{8\operatorname{RootOf}\left(\_Z^{2}-13\right)x-9\operatorname{RootOf}\left(\_Z^{2}-13\right)-26\sqrt{3x^{2}-x+2}}{1+2x}\right)}{338}+\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\,\mathrm{l}}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5
/4)^(1/2)+9/338*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*
x+5)^(1/2))
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$
$$=\frac{338\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2}}{}\right.}{2028\,(2x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(
3*x^2 - x + 2))/(2*x + 1)
```

3.244.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=\frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)-\frac{9}{338}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{13\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcs
inh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt
(3*x^2 - x + 2)/(2*x + 1)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(66) = 132$.

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{9\,\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)}{338\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\,\sqrt{3}+2\,\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{2\,\sqrt{13}}{2\,x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)}\right)}{3\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}}{26\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
9/338*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3
) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))/sgn(
1/(2*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1
))
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)^2\,\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

3.245.1 Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=-\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}$$

output
```
-581/8788*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/26*(
3*x^2-x+2)^(1/2)/(1+2*x)^2+7/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.245.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=\frac{\frac{13(1+28x)\sqrt{2-x+3x^2}}{(1+2x)^2}+581\sqrt{13}\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4394}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

output
```
((13*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 + 581*Sqrt[13]*ArcTanh[(S
qrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/4394
```

3.245.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{7(14x+5)}{2(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{52}\int \frac{14x+5}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1228}$$

$$\frac{7}{52}\left(\frac{83}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{7}{52}\left(\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}-\frac{166}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{52}\left(\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}-\frac{83\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-1/26*Sqrt[2 - x + 3*x^2]/(1 + 2*x)^2 + (7*((4*Sqrt[2 - x + 3*x^2])/(13*(1
 + 2*x)) - (83*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(13*Sq
rt[13])))/52
```

3.245.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.245. $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

3.245.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{84x^3-25x^2+55x+2}{338(1+2x)^2\sqrt{3x^2-x+2}}-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}$	68
default	$-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{8788}+\frac{7\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{338\left(x+\frac{1}{2}\right)}-\frac{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}{104\left(x+\frac{1}{2}\right)^2}$	74
trager	$\frac{(28x+1)\sqrt{3x^2-x+2}}{338(1+2x)^2}-\frac{581\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{8788}$	78

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/338*(84*x^3-25*x^2+55*x+2)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-581/8788*13^(1/2)
*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581\sqrt{13}(4x^2+4x+1)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+52\sqrt{3x^2-x+2}(28x+1)}{17576\,(4x^2+4x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x +
2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 -
 x + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

3.245.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581}{8788}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2-x+2}}{26\,(4\,x^2+4\,x+1)} + \frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2
 - x + 2)/(2*x + 1)
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(71) = 142.

Time = 0.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581}{8788}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{190\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-53\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-489\sqrt{3}x+289\sqrt{3}+489\sqrt{3\,x^2-x+}}{338\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
 2))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x
^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

3.246 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

3.246.1 Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{353\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

output
```
353/243*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/1863*(12839-3871*x)/(3*x^
2-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*(3*x^2-x+2)^(1/2)+32/27*x^2
*(3*x^2-x+2)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{\frac{6(29997-2974x+23207x^2+13110x^3+3312x^4)}{\sqrt{2-x+3x^2}} + 8119\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{5589}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
((6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4))/Sqrt[2 - x + 3*x^
2] + 8119*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5589
```

3.246.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2191, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23\left(432x^3+1116x^2+1002x+49\right)}{81\sqrt{3x^2-x+2}}dx+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\int \frac{432x^3+1116x^2+1002x+49}{\sqrt{3x^2-x+2}}dx+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2}{81}\left(\frac{1}{9}\int \frac{9\left(1236x^2+810x+49\right)}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\left(\int \frac{1236x^2+810x+49}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2}{81}\left(\frac{1}{6}\int -\frac{18(121-373x)}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\left(-3\int \frac{121-373x}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2}{81}\left(-3\left(\frac{353}{6}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2}{81}\left(-3\left(\frac{353\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{6\sqrt{69}}-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{81}\left(-3\left(\frac{353\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (2*(206*x*Sqrt[2 - x + 3
*x^2] + 48*x^2*Sqrt[2 - x + 3*x^2] - 3*((-373*Sqrt[2 - x + 3*x^2])/3 + (35
3*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]))))/81
```

3.246.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.246. $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.246.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}$
trager	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{243}$
default	$-\frac{521(-1+6x)}{414\sqrt{3x^2-x+2}}+\frac{557}{18\sqrt{3x^2-x+2}}+\frac{32x^4}{9\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\right.}{243}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1863*(3312*x^4+13110*x^3+23207*x^2-2974*x+29997)/(3*x^2-x+2)^(1/2)-353/2
43*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{8119\,\sqrt{3}(3\,x^2-x+2)\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-2\right.}{11178\,(3\,x}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6
*x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 297
4*x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

3.246.6 Sympy [F]

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{(2x+1)^3\cdot\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx&=\frac{32\,x^4}{9\,\sqrt{3\,x^2-x+2}}+\frac{380\,x^3}{27\,\sqrt{3\,x^2-x+2}}\\&\quad+\frac{2018\,x^2}{81\,\sqrt{3\,x^2-x+2}}-\frac{353}{243}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right)-\frac{5948\,x}{1863\,\sqrt{3\,x^2-x+2}}\\&\quad+\frac{2222}{69\,\sqrt{3\,x^2-x+2}}\end{aligned}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^
2/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) -
 5948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\,((23\,(6\,(24\,x+95)x+1009)x-2974)x+29997)}{1863\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/
1863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2
)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{3/2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.247 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

3.247.1 Optimal result

Integrand size = 32, antiderivative size = 82

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} - \frac{64\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

output
```
-64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/621*(1249-2273*x)/(3*x^2-x
+2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^2-x+2)^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(1275-1003x+1196x^2+276x^3)}{207\sqrt{2-x+3x^2}} - \frac{64\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(1275 - 1003*x + 1196*x^2 + 276*x^3))/(207*Sqrt[2 - x + 3*x^2]) - (64*L
og[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

3.247.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{46\left(36x^2+75x+46\right)}{27\sqrt{3x^2-x+2}}dx+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27}\int \frac{36x^2+75x+46}{\sqrt{3x^2-x+2}}dx+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{4}{27}\left(\frac{1}{6}\int \frac{12(42x+17)}{\sqrt{3x^2-x+2}}dx+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27}\left(2\int \frac{42x+17}{\sqrt{3x^2-x+2}}dx+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{4}{27}\left(2\left(24\int \frac{1}{\sqrt{3x^2-x+2}}dx+14\sqrt{3x^2-x+2}\right)+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{4}{27}\left(2\left(8\sqrt{\frac{3}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)+14\sqrt{3x^2-x+2}\right)+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{4}{27}\left(2\left(8\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)+14\sqrt{3x^2-x+2}\right)+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (4*(6*x*Sqrt[2 - x + 3*x^2
] + 2*(14*Sqrt[2 - x + 3*x^2] + 8*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])))/
27
```

3.247.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.247. $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.247.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	size
risch	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}$	45
trager	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}-\frac{64\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}\right)}{27}$	70
default	$-\frac{89(-1+6x)}{207\sqrt{3x^2-x+2}}+\frac{107}{9\sqrt{3x^2-x+2}}+\frac{8x^3}{3\sqrt{3x^2-x+2}}+\frac{104x^2}{9\sqrt{3x^2-x+2}}-\frac{64x}{9\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}$	98

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/207*(276*x^3+1196*x^2-1003*x+1275)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*arcsi
nh(6/23*23^(1/2)*(x-1/6))
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx=\frac{2\left(368\sqrt{3}(3x^2-x+2)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x\right.\right.}{621\,(3x^2-x+}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(
3*x^2 - x + 2))/(3*x^2 - x + 2)
```

3.247.6 Sympy [F]

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{8\,x^3}{3\,\sqrt{3\,x^2-x+2}} + \frac{104\,x^2}{9\,\sqrt{3\,x^2-x+2}} + \frac{64}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{2006\,x}{207\,\sqrt{3\,x^2-x+2}} + \frac{850}{69\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/
69/sqrt(3*x^2 - x + 2)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\,((92\,(3\,x+13)x-1003)x+1275)}{207\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/2
07*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{{\left(2\,x+1\right)}^2\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.248 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

3.248.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}} + \frac{8}{9}\sqrt{2-x+3x^2} - \frac{14\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

output
```
-14/9*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-2/207*(73+367*x)/(3*x^2-x+2)^
(1/2)+8/9*(3*x^2-x+2)^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(37-153x+92x^2)}{69\sqrt{2-x+3x^2}} - \frac{14\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) - (14*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/(3*Sqrt[3])
```

3.248.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23(12x+19)}{9\sqrt{3x^2-x+2}}dx-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int \frac{12x+19}{\sqrt{3x^2-x+2}}dx-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2}{9}\left(21\int \frac{1}{\sqrt{3x^2-x+2}}dx+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2}{9}\left(7\sqrt{\frac{3}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{9}\left(7\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (2*(4*Sqrt[2 - x + 3*x^2] +
7*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/9
```

3.248. $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}dx$

3.248.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.248. $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

3.248.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{9}$	40
trager	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\text{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\text{RootOf}\left(\_Z^2-3\right)\right)}{9}$	67
default	$\frac{-\frac{8}{207}+\frac{16x}{69}}{\sqrt{3x^2-x+2}}+\frac{10}{9\sqrt{3x^2-x+2}}+\frac{8x^2}{3\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{9}$	81

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/69*(92*x^2-153*x+37)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2
)*(x-1/6))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{161\,\sqrt{3}(3\,x^2-x+2)\log\left(-4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-2\right.}{207\,(3\,x^2-x+2)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))
/(3*x^2 - x + 2)
```

3.248.6 Sympy [F]

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{8\,x^2}{3\,\sqrt{3\,x^2-x+2}} + \frac{14}{9}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right) - \frac{102\,x}{23\,\sqrt{3\,x^2-x+2}} + \frac{74}{69\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)
) - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{14}{9}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\,((92\,x-153)x+37)}{69\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69
*((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

3.249 $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}}\,dx$

3.249.1 Optimal result

Integrand size = 32, antiderivative size = 62

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}$$

output
```
-2/169*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/299*(10
1-77*x)/(3*x^2-x+2)^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(-101+77x)}{299\sqrt{2-x+3x^2}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(-101 + 77*x))/(299*Sqrt[2 - x + 3*x^2]) + (4*ArcTanh[(Sqrt[3] + 2*Sqrt
[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13])
```

3.249.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)(3x^2-x+2)^{3/2}}\,dx$$
$$\Big\downarrow \text{2177}$$
$$\frac{2}{23}\int \frac{23}{13(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{2}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$
$$\Big\downarrow \text{1154}$$
$$-\frac{4}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt
[13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.249.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

method	result	siz
risch	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{169}$	51
trager	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x-9\,\mathrm{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{169}$	71
default	$\frac{-\frac{5}{69}+\frac{10x}{23}}{\sqrt{3x^2-x+2}}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{-\frac{4}{299}+\frac{24x}{299}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{169}$	10

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/299*(-101+77*x)/(3*x^2-x+2)^(1/2)-2/169*13^(1/2)*arctanh(2/13*(9/2-4*x)*
13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{23\sqrt{13}(3\,x^2-x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+26\sqrt{3}}{3887\,(3\,x^2-x+2)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(
8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x +
 2)*(77*x - 101))/(3*x^2 - x + 2)
```

3.249.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2}{169}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{154\,x}{299\sqrt{3\,x^2-x+2}} - \frac{202}{299\sqrt{3\,x^2-x+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(77\,x-101)}{299\sqrt{3\,x^2-x+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)
)) + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx$

3.250.1 Optimal result

Integrand size = 32, antiderivative size = 87

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx=-\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

output
```
2/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/3887*(1
97-837*x)/(3*x^2-x+2)^(1/2)-4/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.250.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx=\frac{2\sqrt{2-x+3x^2}(-289+489x+1536x^2)}{3887(2+3x+x^2+6x^3)}-\frac{4\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2]*(-289 + 489*x + 1536*x^2))/(3887*(2 + 3*x + x^2 + 6
*x^3)) - (4*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[1
3]])/(169*Sqrt[13])
```

3.250.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{23}\int \frac{46(4-5x)}{169(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{169}\int \frac{4-5x}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{4}{169}\left(-\frac{1}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{4}{169}\left(\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4}{169}\left(\frac{\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) + (4*(-(Sqrt[2 - x + 3*x^2]/
(1 + 2*x)) + ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[1
3])))/169
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.250. $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx$

3.250.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\frac{3072}{3887}x^2+\frac{978}{3887}x-\frac{578}{3887}}{(1+2x)\sqrt{3x^2-x+2}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{2197}$
trager	$\frac{2(1536x^2+489x-289)\sqrt{3x^2-x+2}}{3887(6x^3+x^2+3x+2)}+\frac{2\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(-\frac{8\operatorname{RootOf}\left(\_Z^2-13\right)x-9\operatorname{RootOf}\left(\_Z^2-13\right)-26\sqrt{3x^2-x+2}}{1+2x}\right)}{2197}$
default	$\frac{-\frac{2}{23}+\frac{12x}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{26\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{1}{169\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{82(-1+6x)}{3887\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}\right.}{13\sqrt{12\left(x\right.}}\right.}{2197}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3887*(1536*x^2+489*x-289)/(1+2*x)/(3*x^2-x+2)^(1/2)+2/2197*13^(1/2)*arct
anh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{23\sqrt{13}(6x^3+x^2+3x+2)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+2}{50531\left(6x^3+x^2+3x+2\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^
2 + 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

3.250.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2 (3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{3/2}}\,dx = -\frac{2}{2197}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{1536\,x}{3887\sqrt{3\,x^2-x+2}} - \frac{279}{3887\sqrt{3\,x^2-x+2}} - \frac{1}{13\left(2\sqrt{3\,x^2-x+2}x + \sqrt{3\,x^2-x+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
-2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2)
 - 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(69) = 138.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.93

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx =$$
$$-\frac{2}{50531}\sqrt{13}\left(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)$$
$$-\frac{2\left(\frac{\frac{1047}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}$$
$$+\frac{2\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{2197\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sg
n(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*
x + 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x +
1)^2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)^2\,(3\,x^2-x+2)^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx$

3.251.1 Optimal result

Integrand size = 32, antiderivative size = 112

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}}-\frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2}-\frac{4\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

output
```
-487/28561*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/505
31*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-4/219
7*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.251.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(1673+13306x+23281x^2+14496x^3)}{50531(1+2x)^2\sqrt{2-x+3x^2}}+\frac{974\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
 + 3*x^2]) + (974*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/
Sqrt[13]])/(2197*Sqrt[13])
```

3.251.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2-x+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23} \int \frac{23(1036x^2+906x+363)}{2197(2x+1)^3\sqrt{3x^2-x+2}}dx+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1036x^2+906x+363}{(2x+1)^3\sqrt{3x^2-x+2}}dx}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{2\left(-\frac{1}{26}\int -\frac{13(958x+505)}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{2}\int \frac{958x+505}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{2\left(\frac{1}{2}\left(487\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(\frac{1}{2}\left(-974\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\downarrow 219$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) + (2*((-13*Sqrt[2 - x + 3*
x^2])/(1 + 2*x)^2 + ((-4*Sqrt[2 - x + 3*x^2])/(1 + 2*x) - (487*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/2))/2197
```

3.251.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.251. $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx$

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.251.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}+\frac{487\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{28561}$
default	$\frac{487}{4394\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{-\frac{1208}{50531}+\frac{7248x}{50531}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}+\frac{3}{338\left(x+\frac{1}{2}\right)\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/50531*(14496*x^3+23281*x^2+13306*x+1673)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-487
/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2
))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{11201\sqrt{13}(12\,x^4+8\,x^3+7\,x^2+7\,x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x}{4\,x^2+4\,x+1}\right.}{1313806\,(12\,x^4+8\,x}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(
13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x +
1)) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12
*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

3.251.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx &= \frac{487}{28561}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) \\
&+ \frac{7248\,x}{50531\sqrt{3\,x^2-x+2}} + \frac{8785}{101062\sqrt{3\,x^2-x+2}} \\
&- \frac{1}{26\left(4\sqrt{3\,x^2-x+2}x^2+4\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)} \\
&+ \frac{3}{169\left(2\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)}
\end{aligned}
$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/ab
s(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 -
x + 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(
3*x^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(90) = 180$.

Time = 0.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.99

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx &= \frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) \\
&+ \frac{2\,(3693\,x+2363)}{50531\sqrt{3\,x^2-x+2}} \\
&+ \frac{2\left(62\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-37\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+263\sqrt{3}x-71\sqrt{3}-263\sqrt{3\,x^2-x+2}\right.}{2197\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}
\end{aligned}
$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*
sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)
*x - sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
^2 + 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x -
 sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)
^2
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

3.252 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

3.252.1 Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} - \frac{296\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

output
```
2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/
2))*3^(1/2)-28/128547*(35809+42240*x)/(3*x^2-x+2)^(1/2)+32/27*(3*x^2-x+2)^
(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(-44739-119459x+8630x^2-247904x^3+76176x^4)}{14283(2-x+3x^2)^{3/2}} - \frac{296\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{27\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-44739 - 119459*x + 8630*x^2 - 247904*x^3 + 76176*x^4))/(14283*(2 - x
+ 3*x^2)^(3/2)) - (296*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(27*Sqrt[3]
)
```

3.252.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2191, 27, 2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69} \int -\frac{-29808x^3-77004x^2-69138x+4361}{81 (3x^2-x+2)^{3/2}} dx + \frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\int \frac{-29808x^3-77004x^2-69138x+4361}{(3x^2-x+2)^{3/2}}dx}{5589}$$

$$\Big\downarrow \text{2191}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\left(\frac{2}{23}\int -\frac{9522(12x+35)}{\sqrt{3x^2-x+2}}dx + \frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}\right)}{5589}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}} - 828\int \frac{12x+35}{\sqrt{3x^2-x+2}}dx\right)}{5589}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}} - 828\left(37\int \frac{1}{\sqrt{3x^2-x+2}}dx + 4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}-\frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}-828\left(\frac{37\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}+4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

$$\Big\downarrow \; 222$$

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}-\frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}-828\left(\frac{37\operatorname{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}+4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (2*((14*(35809 + 42240
*x))/(23*Sqrt[2 - x + 3*x^2]) - 828*(4*Sqrt[2 - x + 3*x^2] + (37*ArcSinh[(
-1 + 6*x)/Sqrt[23]])/Sqrt[3])))/5589
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.252. $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.252.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{296\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{81}$
trager	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}-\frac{296\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x-}\right.}{81}$
default	$\frac{-\frac{13763}{33534}+\frac{13763x}{5589}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{65264}{128547}+\frac{130528x}{42849}}{\sqrt{3x^2-x+2}}-\frac{1727}{1458(3x^2-x+2)^{\frac{3}{2}}}+\frac{32x^4}{3(3x^2-x+2)^{\frac{3}{2}}}-\frac{296x^3}{27(3x^2-x+2)^{\frac{3}{2}}}+\frac{8x^2}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{}{81(3}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/14283*(76176*x^4-247904*x^3+8630*x^2-119459*x-44739)/(3*x^2-x+2)^(3/2)+2
96/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\left(39146\sqrt{3}(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x\right.\right.}{42849}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*s
qrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904
*x^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 +
13*x^2 - 4*x + 4)
```

3.252.6 Sympy [F]

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{(2x+1)^3\cdot\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.252.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(69) = 138.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx&=\frac{32\,x^4}{3\,(3\,x^2-x+2)^{\frac{3}{2}}}\\&+\frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{2162}{(3\,x^2-x+2)^{\frac{3}{2}}}\right)\\&+\frac{296}{81}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)-\frac{42032}{42849}\sqrt{3\,x^2-x+2}-\frac{47072\,x}{42849\,\sqrt{3\,x^2-x+2}}\\&+\frac{52\,x^2}{9\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{23104}{14283\,\sqrt{3\,x^2-x+2}}-\frac{7742\,x}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}+\frac{1666}{1863\,(3\,x^2-x+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) -
4761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x
 + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sq
rt(23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3
*x^2 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 -
x + 2) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/
2)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\left(\left(2\left(8\left(4761\,x-15494\right)x+4315\right)x-119459\right)x-44739\right)}{14283\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/
14283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x
+ 2)^(3/2)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{5/2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.253 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

3.253.1 Optimal result

Integrand size = 32, antiderivative size = 68

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(1249-2273x)}{1863(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} - \frac{16\,\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

output
```
2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2)
)*3^(1/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(-17481+5837x-31664x^2+1964x^3)}{4761(2-x+3x^2)^{3/2}} - \frac{16\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-17481 + 5837*x - 31664*x^2 + 1964*x^3))/(4761*(2 - x + 3*x^2)^(3/2))
- (16*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

3.253.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{5/2}} \, dx$$

$$\Big\downarrow \ 2191$$

$$\frac{2}{69} \int \frac{2\left(2484x^2+5175x+901\right)}{27\left(3x^2-x+2\right)^{3/2}}dx+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{4\int \frac{2484x^2+5175x+901}{\left(3x^2-x+2\right)^{3/2}}dx}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \ 2191$$

$$\frac{4\left(\frac{2}{23}\int \frac{9522}{\sqrt{3x^2-x+2}}dx-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{4\left(828\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1090$$

$$\frac{4\left(12\sqrt{69}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \ 222$$

$$\frac{4\left(276\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) + (4*((-2*(23257 - 1473*x
))/(23*Sqrt[2 - x + 3*x^2]) + 276*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/1
863
```

3.253.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.253. $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

3.253.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}$
trager	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}-\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{27}$
default	$\frac{-\frac{4585}{11178}+\frac{4585x}{1863}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{18892}{42849}+\frac{37784x}{14283}}{\sqrt{3x^2-x+2}}-\frac{2653}{486(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{92x^2}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{67x}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x}{9\sqrt{3x^2-x}}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/4761*(1964*x^3-31664*x^2+5837*x-17481)/(3*x^2-x+2)^(3/2)+16/27*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))
```

3.253.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\left(2116\sqrt{3}(9x^4-6x^3+13x^2-4x+4)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-\right.\right.}{14283\,(9x^4-}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^
2 + 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4
)
```

3.253.6 Sympy [F]

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.253.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{16}{14283}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}} - \frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{71}{\sqrt{3\,x^2-x+2}} + \frac{805\,x}{(3\,x^2-x}\right.$$
$$+\frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{2272}{14283}\sqrt{3\,x^2-x+2} + \frac{28184\,x}{14283\sqrt{3\,x^2-x+2}}$$
$$-\frac{28\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2956}{4761\sqrt{3\,x^2-x+2}} - \frac{106\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{3394}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 7
1/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)
^(3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt
(3*x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x
+ 2)^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/
2) - 3394/621/(3*x^2 - x + 2)^(3/2)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{2\left((4(491x-7916)x+5837)x-17481\right)}{4761(3x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/4
761*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2(4x^2+3x+1)}{(3x^2-x+2)^{5/2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

3.254 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

3.254.1 Optimal result

Integrand size = 30, antiderivative size = 47

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}} - \frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

output
```
-2/621*(73+367*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/2
)
```

3.254.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(-1915+1833x-3546x^2+2860x^3)}{1587(2-x+3x^2)^{3/2}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

3.254.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2191, 27, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{828x+577}{9(3x^2-x+2)^{3/2}}dx-\frac{2(367x+73)}{621(3x^2-x+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{621}\int \frac{828x+577}{(3x^2-x+2)^{3/2}}dx-\frac{2(367x+73)}{621(3x^2-x+2)^{3/2}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621(3x^2-x+2)^{3/2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283
*Sqrt[2 - x + 3*x^2])
```

3.254.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.254.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}$	30
default	$\frac{-\frac{715}{3726}+\frac{715x}{621}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{2860}{14283}+\frac{5720x}{4761}}{\sqrt{3x^2-x+2}}-\frac{295}{162(3x^2-x+2)^{\frac{3}{2}}}-\frac{8x^2}{3(3x^2-x+2)^{\frac{3}{2}}}-\frac{13x}{9(3x^2-x+2)^{\frac{3}{2}}}$	86

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\,\left(2860\,x^3-3546\,x^2+1833\,x-1915\right)\sqrt{3\,x^2-x+2}}{1587\,\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 -
6*x^3 + 13*x^2 - 4*x + 4)
```

3.254.6 Sympy [F]

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}} - \frac{8\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2860}{14283\,\sqrt{3\,x^2-x+2}} - \frac{182\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1250}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/142
83/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2
 - x + 2)^(3/2)
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\,((2\,(1430\,x-1773)x+1833)x-1915)}{1587\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{442\,x-5720\,x\,(3\,x^2-x+2)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,(14283\,x^2-4761\,x+9522)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
-(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/
2)*(14283*x^2 - 4761*x + 9522))
```

3.255 $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}}\,dx$

3.255.1 Optimal result

Integrand size = 32, antiderivative size = 85

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=-\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}}-\frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}}-\frac{8\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

output
```
-2/897*(101-77*x)/(3*x^2-x+2)^(3/2)-8/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(
3*x^2-x+2)^(1/2))*13^(1/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-32963+79077x-31482x^2+82008x^3)}{268203\left(2-x+3x^2\right)^{3/2}}+\frac{16\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/
2)) + (16*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]
])/(169*Sqrt[13])
```

3.255.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2177, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int \frac{308x+223}{13(2x+1)\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{897}\int \frac{308x+223}{(2x+1)\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2}{897}\left(\frac{2}{299}\int \frac{3174}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{897}\left(\frac{276}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{2}{897}\left(-\frac{552}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2}{897}\left(-\frac{276\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) + (2*((-2*(691 - 13668*x))/(
299*Sqrt[2 - x + 3*x^2]) - (276*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/(13*Sqrt[13])))/897
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.255.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
trager	$\frac{\frac{54672}{89401}x^3-\frac{20988}{89401}x^2+\frac{52718}{89401}x-\frac{65926}{268203}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{2197}$
default	$\frac{-\frac{5}{207}+\frac{10x}{69}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{40}{1587}+\frac{80x}{529}}{\sqrt{3x^2-x+2}}-\frac{2}{9(3x^2-x+2)^{\frac{3}{2}}}+\frac{1}{39\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{4}{897}+\frac{8x}{299}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{784}{89401}+\frac{4704x}{89401}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/268203*(82008*x^3-31482*x^2+79077*x-32963)/(3*x^2-x+2)^(3/2)+8/2197*Root
Of(_Z^2-13)*ln((8*RootOf(_Z^2-13)*x+26*(3*x^2-x+2)^(1/2)-9*RootOf(_Z^2-13)
)/(1+2*x))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(3174\sqrt{13}(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220}{4\,x^2+4\,x+1}\right.\right.}{3486639\,(9\,x^4-6\,}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1
)) + 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*
x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

3.255.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{8}{2197}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)\\ &+\frac{18224\,x}{89401\sqrt{3\,x^2-x+2}}-\frac{2764}{268203\sqrt{3\,x^2-x+2}}\\ &+\frac{154\,x}{897\,(3\,x^2-x+2)^{\frac{3}{2}}}-\frac{202}{897\,(3\,x^2-x+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x
+ 2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\left(3\left(6\left(4556\,x-1749\right)x+26359\right)x-32963\right)}{268203\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqr
t(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)
^(3/2)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx$

3.256.1 Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx = -\frac{2(197-837x)}{11661(2-x+3x^2)^{3/2}} - \frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}} - \frac{16\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{56\,\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

output
```
-2/11661*(197-837*x)/(3*x^2-x+2)^(3/2)-56/28561*arctanh(1/26*(9-8*x)*13^(1
/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-24/1162213*(841-6633*x)/(3*x^2-x+2)^(1/2)-
16/2197*(3*x^2-x+2)^(1/2)/(1+2*x)
```

3.256.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx = \frac{2(-170239+569989x+1021566x^2+133308x^3+1318464x^4)}{3486639(1+2x)(2-x+3x^2)^{3/2}} + \frac{112\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4))/(3486639
*(1 + 2*x)*(2 - x + 3*x^2)^(3/2)) + (112*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x -
2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(2197*Sqrt[13])
```

3.256.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2177, 27, 2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int \frac{6\left(1116x^2+1001x+371\right)}{169(2x+1)^2\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\int \frac{1116x^2+1001x+371}{(2x+1)^2(3x^2-x+2)^{3/2}}dx}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{4\left(\frac{2}{23}\int \frac{1058(3x+8)}{13(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\left(\frac{92}{13}\int \frac{3x+8}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{4\left(\frac{92}{13}\left(\frac{7}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{4\left(\frac{92}{13}\left(-7\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\,(3x^2-x+2)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{4\left(\frac{92}{13}\left(-\frac{7\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\,(3x^2-x+2)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) + (4*((-6*(841 - 6633*x))
/(299*Sqrt[2 - x + 3*x^2]) + (92*(-(Sqrt[2 - x + 3*x^2]/(1 + 2*x)) - (7*Ar
cTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13])))/13))/388
7
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.256. $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{5/2}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.256.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{878976}{1162213}x^4+\frac{168}{2197}x^3+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}}{(1+2x)(3x^2-x+2)^{\frac{3}{2}}}-\frac{56\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{28561}$
trager	$\frac{\frac{878976}{1162213}x^4+\frac{168}{2197}x^3+\frac{52388}{89401}x^2+\frac{1139978}{3486639}x-\frac{340478}{3486639}}{(1+2x)(3x^2-x+2)^{\frac{3}{2}}}+\frac{56\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_\right.}{1+2x}\right.}{28561}$
default	$\frac{-\frac{2}{69}+\frac{4x}{23}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{-\frac{16}{529}+\frac{96x}{529}}{\sqrt{3x^2-x+2}}-\frac{1}{26\left(x+\frac{1}{2}\right)\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{7}{507\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{128(-1+6x)}{11661\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}-$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3486639*(1318464*x^4+133308*x^3+1021566*x^2+569989*x-170239)/(3*x^2-x+2)
^(3/2)/(1+2*x)-56/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/
2)^2-16*x+5)^(1/2))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(22218\sqrt{13}(18x^5-3x^4+20x^3+5x^2+4x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}}{}\right.\right.}{4532630}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log
(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^
2 + 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17
0239)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

3.256.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{\left(2x+1\right)^2\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{56}{28561}\,\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{146496\,x}{1162213\,\sqrt{3\,x^2-x+2}} - \frac{9604}{1162213\,\sqrt{3\,x^2-x+2}} + \frac{420\,x}{3887\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1}{13\,\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)} - \frac{49}{11661\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^
2 - x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3
/2)*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{56}{15108769}\,\sqrt{13}\left(872\,\sqrt{13}\sqrt{3}-529\,\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right) - \frac{56\,\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)}{28561\,\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)} + \frac{8\left(\frac{\frac{\frac{13\left(\frac{77756}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}+\frac{20631}{(2\,x+1)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}\right)}{2\,x+1}-\frac{1399650}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}}{2\,x+1}+\frac{625905}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}}{2\,x+1}-\frac{164808}{\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)}\right)}{3486639\left(\frac{8}{2\,x+1}-\frac{13}{(2\,x+1)^2}-3\right)\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4
))*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) +
13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/348663
9*(((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x
 + 1) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*
x + 1) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt
(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3))
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{5/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx$

3.257.1 Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593(2-x+3x^2)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2-x+3x^2}}{28561(1+2x)} - \frac{2084\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{28561\sqrt{13}}$$

output
```
2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)-2084/371293*arctanh(1/26*(9-8*x)*
13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+12/15108769*(25771+103526*x)/(3*x^2-x
+2)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144/28561*(3*x^2-x+2)^(1/2)/(
1+2*x)
```

3.257.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2\sqrt{2-x+3x^2}(847141+10777477x+21890266x^2+19381992x^3+200}{45326307(2+3x+x^2+6x^3)^2} + \frac{4168\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{28561\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2]*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3
+ 20074356*x^4 + 20304864*x^5))/(45326307*(2 + 3*x + x^2 + 6*x^3)^2) + (41
68*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(285
61*Sqrt[13])
```

3.257.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2177, 27, 2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int \frac{3\left(19696x^3+53372x^2+35610x+10811\right)}{2197(2x+1)^3\left(3x^2-x+2\right)^{3/2}}dx+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{19696x^3+53372x^2+35610x+10811}{(2x+1)^3\left(3x^2-x+2\right)^{3/2}}dx}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2177}$$

$$\frac{2\left(\frac{2}{23}\int \frac{2116\left(488x^2+527x+226\right)}{13(2x+1)^3\sqrt{3x^2-x+2}}dx+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{184}{13}\int \frac{488x^2+527x+226}{(2x+1)^3\sqrt{3x^2-x+2}}dx+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{2\left(\frac{184}{13}\left(-\frac{1}{26}\int -\frac{13(898x+683)}{2(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\int \frac{898x+683}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(521\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(-1042\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(-\frac{521\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (2*((6*(25771 + 10352
6*x))/(299*Sqrt[2 - x + 3*x^2]) + (184*((-13*Sqrt[2 - x + 3*x^2])/(2*(1 +
2*x)^2) + ((-36*Sqrt[2 - x + 3*x^2])/(1 + 2*x) - (521*ArcTanh[(9 - 8*x)/(2
*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/4))/13))/50531
```

3.257. $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx$

3.257.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.257. $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.257.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{13536576}{15108769}x^5+\frac{13382904}{15108769}x^4+\frac{12921328}{15108769}x^3+\frac{43780532}{45326307}x^2+\frac{21554954}{45326307}x+\frac{1694282}{45326307}}{(1+2x)^2(3x^2-x+2)^{\frac{3}{2}}}-\frac{2084\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(x+\frac{1}{2}\right)^2-16x+5}}\right)}{371293}$
trager	$\frac{2(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141)\sqrt{3x^2-x+2}}{45326307(6x^3+x^2+3x+2)^2}+\frac{2084\,\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\operatorname{RootO}}{}\right.}{}$
default	$\frac{521}{13182\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{886}{151593}+\frac{1772x}{50531}}{\left(3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{-\frac{188008}{15108769}+\frac{1128048x}{15108769}}{\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}+\frac{1042}{28561\sqrt{3\left(x+\frac{1}{2}\right)^2-4x+\frac{5}{4}}}-\frac{2084\sqrt{13}\,\operatorname{arctanl}}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45326307*(20304864*x^5+20074356*x^4+19381992*x^3+21890266*x^2+10777477*x
+847141)/(3*x^2-x+2)^(3/2)/(1+2*x)^2-2084/371293*13^(1/2)*arctanh(2/13*(9/
2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(826827\sqrt{13}(36\,x^6+12\,x^5+37\,x^4+30\,x^3+13\,x^2+12\,x+4)\log\left(-\right.\right.}{}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
 12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*
x + 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x
^3 + 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12
*x^5 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

3.257.6 Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{\left(2x+1\right)^3\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{2084}{371293}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)\\ &+\frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}}+\frac{363210}{15108769\sqrt{3\,x^2-x+2}}+\frac{1772\,x}{50531\,(3\,x^2-x+2)^{\frac{3}{2}}}\\ &-\frac{1}{26\left(4\,(3\,x^2-x+2)^{\frac{3}{2}}x^2+4\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)}\\ &-\frac{1}{169\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)}+\frac{10211}{303186\,(3\,x^2-x+2)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/
abs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/s
qrt(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 -
 x + 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1
/169/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3
*x^2 - x + 2)^(3/2)
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.73

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$
$$+\frac{2\left(3\left(6\left(310578x-26213\right)x+1455755\right)x+1634293\right)}{45326307\left(3x^2-x+2\right)^{\frac{3}{2}}}$$
$$-\frac{8\left(66\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3+21\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-1015\sqrt{3}x+431\sqrt{3}+1015\sqrt{3x^2-x}\right.}{28561\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) +
4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 -
x + 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(
3*x^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 2
1*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(
3) + 1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^3\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

3.258.1 Optimal result

Integrand size = 47, antiderivative size = 208

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2dh)))\,(b+2cx)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{2(fg^2-egh+dh^2)}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

output
```
-f/c/h^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+1/3*(6*b*c*e*h^2-3*b^2*f*
h^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*(2*c*x+b)/c/h^2/(-b*h+2*c*g)^3/(-g*(-b*h+
c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*
x+g)/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2b^2h^2(-h(2eg+dh+3ehx)+f(8g^2+12ghx+3h^2x^2))}{}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2
*x^2)^(3/2)),x]
```

output
```
(2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2
)) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*
x^2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x)
 + h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g
 + b*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

3.258.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.085$, Rules used = {2169, 27, 1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(bgh+bh^2x-cg^2+ch^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 2169$$

$$-\frac{\int \frac{h^3(bfg-2cdh+(2cfg-2ceh+bfh)x)}{2(g+hx)(cx^2h^2+bxh^2-g(cg-bh))^{3/2}}dx}{ch^4}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{bfg-2cdh+(2cfg-2ceh+bfh)x}{(g+hx)(cx^2h^2+bxh^2-g(cg-bh))^{3/2}}dx}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$$\Big\downarrow \ 1220$$

$$-\frac{\frac{\left(-3b^2fh^2+6bceh^2+c^2\left(4fg^2-4h(2dh+eg)\right)\right)\int \frac{1}{\left(cx^2h^2+bxh^2-g(cg-bh)\right)^{3/2}}dx}{3h(2cg-bh)}-\frac{4c\left(dh^2-egh+fg^2\right)}{3h^2(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$\big\downarrow$ 1088

$$-\frac{-\frac{2(b+2cx)\left(-3b^2fh^2+6bceh^2+c^2\left(4fg^2-4h(2dh+eg)\right)\right)}{3h(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{4c\left(dh^2-egh+fg^2\right)}{3h^2(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^
(3/2)),x]
```

output
```
-(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) - ((-2*(6*b*c*e*
h^2 - 3*b^2*f*h^2 + c^2*(4*f*g^2 - 4*h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*h*(
2*c*g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) - (4*c*(f*g^2
 - e*g*h + d*h^2))/(3*h^2*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) +
b*h^2*x + c*h^2*x^2]))/(2*c*h)
```

3.258.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

3.258. $\int \frac{d+ex+fx^2}{(g+hx)(-cg^2+bgh+bh^2x+ch^2x^2)^{3/2}}\,dx$

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 2169

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

3.258.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.56

method	result
gosper	$-\frac{2(chx+bh-cg)\left(-3b^2 f h^4x^2+6bce h^4x^2-8c^2d h^4x^2-4c^2eg h^3x^2+4c^2f g^2h^2x^2+3b^2e h^4x-12b^2fg h^3x-4bcd h^4x+4bceg h^3x+20b\cdots\right.}{3(b^3h^3-6b^2cg h^2+12b c^2g^2\cdots}$
trager	$-\frac{2\left(-3b^2 f h^4x^2+6bce h^4x^2-8c^2d h^4x^2-4c^2eg h^3x^2+4c^2f g^2h^2x^2+3b^2e h^4x-12b^2fg h^3x-4bcd h^4x+4bceg h^3x+20bcf g^2h^2x-8c^2\cdots\right.}{3h^3(b^2h^2-4bcg\cdots}$
default	$\frac{\frac{2eh\left(2c h^2x+b h^2\right)}{\left(4c h^2\left(bgh-c g^2\right)-b^2h^4\right)\sqrt{c x^2h^2+bx h^2+bgh-c g^2}}+fh\left(-\frac{1}{c h^2\sqrt{c x^2h^2+bx h^2+bgh-c g^2}}-\frac{b\left(2c h^2x+b h^2\right)}{c\left(4c h^2\left(bgh-c g^2\right)-b^2h^4\right)\sqrt{c x^2h^2+bx h^2+bgh-\cdots}}\right.\cdots}{h^2}$

input

```
int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
-2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c
^2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*
h^4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x
-8*c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b
*c*e*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^
3*h^3-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h
-c*g^2)^(3/2)
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(198) = 396$.

Time = 13.84 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.24

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2\left(8\,c^2fg^4-b^2dh^4+4\,(c^2e-4\,bcf)g^3h-2\,(2\,c^2d+bce-\right.}{3\left(8\,c^4g^6h^3-20\,bc^3g^5h^4+18\,b^2c^2g^4h^5-7\,b^3\right.}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="fricas")
```

output
```
2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*
c*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*
c^2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*
(c^2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d
 - 3*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h
^3 - 20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7
 - (8*c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 -
(8*c^4*g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4
*h^9)*x^2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^
3*c*g^2*h^7 + 2*b^4*g*h^8)*x)
```

3.258.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{\left((g+hx)\,(bh-cg+chx)\right)^{\frac{3}{2}}(g+hx)}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2
),x)
```

output
```
Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h
*x)), x)
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

3.258.8 Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{fx^2+ex+d}{\left(ch^2x^2+bh^2x-cg^2+bgh\right)^{\frac{3}{2}}(hx+g)}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(
h*x + g)), x)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.24

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{16\,c^2\,f\,g^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-2\,b^2\,d\,h^4\,\sqrt{-}}{}$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/
2)),x)
```

output
```
(16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b
*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g
^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*
x^2 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h
)^(1/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*
b^2*e*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b
*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
 b*g*h)^(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 4*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b
*c*e*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*
x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c
*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
 b*g*h)^(1/2) + 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 16*b*c*d*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*
f*g^3*h*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h
^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*
g^2 + c*h^2*x^2 + b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^
9*x^2 - 60*b*c^3*g^5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*...
```

3.259 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}(A+Bx+Cx^2)\,dx$

3.259.1 Optimal result

Integrand size = 34, antiderivative size = 906

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{2\sqrt{d+ex}(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3d(8Cd^2-3e(4Bd-7Ae))+3c^2e(ae(Cd-5Be)-b}{}$$
$$-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{21c^2e}+\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2\left(4c^2d^2-b^2e^2-\frac{3}{2}ce(bd-2ae)\right)\left(8b^2Ce^2-ce(bCd+12bBe+7aCe)-c^2(2Cd^2-3e(E\right.\right.}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-4bBe\right.}{}$$

output
```
2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-2/21*(-3*B*c*e+2*C*b*e+2*C*c*d
)*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c^2/e+2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*
B*b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e
+C*d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e
+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2
)/c^3/e^3+1/315*(2*(4*c^2*d^2-b^2*e^2-3/2*c*e*(-2*a*e+b*d))*(8*b^2*C*e^2-c
*e*(12*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))-5*c*e*(-b*e+2*c
*d)*(6*b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C*e^2-c*d*(9*B*e+C*
d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
)*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)/c^4/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2)-2/315*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e^2*(-7*A*b*e-10*
B*a*e+B*b*d+2*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*c^3*d*(8*C*d^2-3
*e*(-7*A*e+4*B*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^4/(e*x+d)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.259.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.80 (sec) , antiderivative size = 15669, normalized size of antiderivative = 17.29

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

output
```
Result too large to show
```

3.259.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2184, 27, 1236, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \; 2184$$

$$\frac{2\int -\frac{3}{2}e\sqrt{d+ex}(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cx^2+bx+a}dx}{9ce^2}+\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\int \sqrt{d+ex}(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cx^2+bx+a}dx}{3ce}$$

$$\Big\downarrow \; 1236$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\frac{2\int -\frac{\left(6Cdeb^2+2aCe^2b-cd(Cd+9Be)b+ce(21Acd-5aCd-3aBe)+\left(-\left(\left(2Cd^2-3e(Bd+7Ae)\right)c^2\right)-e(bCd+12bBe+7aCe)c+8b^2Ce^2\right)x\right)\sqrt{cx^2+bx+a}}{2\sqrt{d+ex}}dx}{7c}+2\sqrt{d+}}{3ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{\int \frac{\left(6Cdeb^2+2aCe^2b-cd(Cd+9Be)b+ce(21Acd-5aCd-3aBe)+\left(-\left(\left(2Cd^2-3e(Bd+7Ae)\right)c^2\right)-e(bCd+12b\right.\right.}{\sqrt{d+ex}}}{7c}}{3ce}$$

$$\Big\downarrow \; 1231$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(3cex\left(-ce(7aCe+12bBe+bCd)-\left(c^2\left(2Cd^2-3e(7Ae+Bd)\right)\right)+8b^2Ce^2\right)-3c^2e\left(-ae(C\right.\right.}{15ce^2}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(3cex\left(-ce(7aCe+12bBe+bCd)-\left(c^2\left(2Cd^2-3e(7Ae+Bd)\right)\right)+8b^2Ce^2\right)-3c^2e\left(-ae(C\right.\right.}{15ce^2}}{}$$

$$\Big\downarrow \; 1269$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(3cex\left(-ce(7aCe+12bBe+bCd)-\left(c^2\left(2Cd^2-3e(7Ae+Bd)\right)\right)+8b^2Ce^2\right)-3c^2e\left(-ae(C\right.\right.}{15ce^2}}{}$$

$$\Big\downarrow \; 1172$$

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-$$

$$\frac{\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c}-\frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd-7Ae)\right)c^3-3e\left(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be)\right)c^2-3be^2(bCd+4bBe-aCe)c\right.}{15ce^2}}{}$$

$$\Big\downarrow \; 321$$

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-$$

$$\frac{\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c}-\frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd-7Ae)\right)c^3-3e\left(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be)\right)c^2-3be^2(bCd+4bBe-aCe)c\right.}{15ce^2}}{}$$

$$\Big\downarrow \; 327$$

3.259. $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-$$

$$\frac{\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c}-\frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd-7Ae)\right)c^3-3e\left(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be)\right)c^2-3be^2(bCd+4bBe-aCe)c\cdots}{15ce^2}}{\cdots}$$

input
```
Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

output
```
(2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - ((2*(2*c*C*d - 3*B
*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7*c) - ((2*Sqrt[d
+ e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(8*C*d^3 -
 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) - a*e*(C*
d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e) - c^2*
(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*c*e^2) - ((Sq
rt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 -
b*c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 -
 b^2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e +
 7*a*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c
*d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e
- 10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d
*e*(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a...
```

3.259. $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}(A+Bx+Cx^2)\,dx$

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.259. $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}(A+Bx+Cx^2)\,dx$

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.259.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(834) = 1668.

Time = 3.46 (sec) , antiderivative size = 1736, normalized size of antiderivative = 1.92

method	result	size
elliptic	Expression too large to display	1736
risch	Expression too large to display	7113
default	Expression too large to display	19955

input
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*C*x^3
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/7*(B*c*e+C*b*e+C*c*d-2/
9*(4*b*e+4*c*d)*C)/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)
+2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b
*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*
d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B
*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b
*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4
*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+
c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(d*A*a-2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d
-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(
3*b*e+3*c*d))/c/e*a*d-2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B*c*e+C*
b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b*e+B*c*
d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*
c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c...
```

3.259.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.13

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/945*((16*C*c^5*d^5 - 8*(2*C*b*c^4 + 3*B*c^5)*d^4*e - (5*C*b^2*c^3 - 42*A
*c^5 - 3*(10*C*a + 9*B*b)*c^4)*d^3*e^2 - (5*C*b^3*c^2 + 3*(22*B*a + 21*A*b
)*c^4 - 3*(7*C*a*b + 4*B*b^2)*c^3)*d^2*e^3 - (16*C*b^4*c - 378*A*a*c^4 + 3
*(22*C*a^2 + 41*B*a*b + 21*A*b^2)*c^3 - 3*(28*C*a*b^2 + 9*B*b^3)*c^2)*d*e^
4 + (16*C*b^5 - 9*(10*B*a^2 + 21*A*a*b)*c^3 + 3*(41*C*a^2*b + 41*B*a*b^2 +
 14*A*b^3)*c^2 - 24*(4*C*a*b^3 + B*b^4)*c)*e^5)*sqrt(c*e)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^
3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^
3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^5*d^4*e - 8*(C*b*c^4
+ 3*B*c^5)*d^3*e^2 - 3*(2*C*b^2*c^3 - 14*A*c^5 - (6*C*a + 5*B*b)*c^4)*d^2*
e^3 - (8*C*b^3*c^2 + 6*(8*B*a + 7*A*b)*c^4 - 15*(2*C*a*b + B*b^2)*c^3)*d*e
^4 + (16*C*b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 -
24*(3*C*a*b^2 + B*b^3)*c^2)*e^5)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3
)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(35*C*c^5*e^5*x^3 + 8*C
*c^5*d^3*e^2 - 3*(C*b*c^4 + 4*B*c^5)*d^2*e^3 - (3*C*b^2*c^3 - 21*A*c^5 - 2
*(4*C*a + 3*B*b)*c^4)*d*e^4 + (8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3...
```

3.259.6 Sympy [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{d+ex}\left(A+Bx+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

3.259.7 Maxima [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (Cx^2+Bx+A)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

3.259.8 Giac [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (Cx^2+Bx+A)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

3.260 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$

3.260.1 Optimal result

Integrand size = 34, antiderivative size = 668

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx =$$

$$-\frac{2\sqrt{d+ex}(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe)+3ce(6cCd-7Bce+4bCe)}{105c^2e^3}$$

$$+\frac{2C\sqrt{d+ex}(a+bx+cx^2)^{3/2}}{7ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(5ce(2cd-be)(3bCd-7Ace+aCe)-(6cCd-7Bce+4bCe)\left(8c^2d^2-2b^2e^2-3ce(bd-\right.}{105c^3e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd-5Ae)\right)}{105c^3e^4\sqrt{d+ex}\sqrt{a}}$$

output
```
2/7*C*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c/e-2/105*(5*c*e*(-7*A*c*e+C*a*e+3
*C*b*d)-(-b*e+4*c*d)*(-7*B*c*e+4*C*b*e+6*C*c*d)+3*c*e*(-7*B*c*e+4*C*b*e+6*
C*c*d)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/105*(5*c*e*(-b*e+2*c
*d)*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e+6*C*c*d)*(8*c^2*d^2-2*b^2*e
^2-3*c*e*(-2*a*e+b*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*
x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/105*(a*e^2-b*d*e+c*d^2)*(4*b^2*C*e^2+c*e*(
-7*B*b*e-10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d)))*EllipticF(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.260.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.38 (sec) , antiderivative size = 9965, normalized size of antiderivative = 14.92

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

output
```
Result too large to show
```

3.260.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2184, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2\int -\frac{e(3bCd-7Ace+aCe+(6cCd-7Bce+4bCe)x)\sqrt{cx^2+bx+a}}{2\sqrt{d+ex}}dx}{7ce^2}+\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-\frac{\int \frac{(3bCd-7Ace+aCe+(6cCd-7Bce+4bCe)x)\sqrt{cx^2+bx+a}}{\sqrt{d+ex}}dx}{7ce}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{2\int \frac{5ce(bd-2ae)(3bCd-7Ace+aCe)-2(}{}}{}}{7ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\int \frac{5ce(bd-2ae)(3bCd-7Ace+aCe)-(6c}{}}{}}{7ce}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\left(ae^2-bde+cd^2\right)\left(ce(-10aCe-7bBe+8b\right.}{}$$

$$\Big\downarrow \text{1172}$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-\right.}{}}{}$$

$$\Big\downarrow \text{321}$$

3.260. $\displaystyle\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} -$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(}{}$$

$$\Big\downarrow \; 327$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} -$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-\right.}{}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

output
```
(2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7*c*e) - ((2*Sqrt[d + e*x]*(5
*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C*d - 7*B*c*e + 4*b*
C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b*x + c*x^2])/(15*c
*e^2) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*
e + a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*
(b*d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x +
 c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*
A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))
/(15*c*e^2))/(7*c*e)
```

3.260. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.260. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.260.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 1202, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1202
risch	Expression too large to display	4505
default	Expression too large to display	12761

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/e*x
^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*c+b*C-2/7*C/e*(3
*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*
c+B*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/
e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(a*
A-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*a*d-2/3*(A*c+B*b+C*a-2/7*C/e*(5/
2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*
(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*
x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))+2*(A*b+B*a-4/7*a*d/e*C-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(3/
2*a*e+3/2*b*d)-2/3*(A*c+B*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7
*C/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+
b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/
(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*...
```

3.260.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$$

$$= \frac{2\Big((48\,Cc^4d^4 - 8\,(5\,Cbc^3 + 7\,Bc^4)d^3e - (10\,Cb^2c^2 - 70\,Ac^4 - (62\,Ca + 49\,Bb)c^3)d^2e^2 - (5\,Cb^3c + 14\,(6\,B}{}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

output
```
2/315*((48*C*c^4*d^4 - 8*(5*C*b*c^3 + 7*B*c^4)*d^3*e - (10*C*b^2*c^2 - 70*
A*c^4 - (62*C*a + 49*B*b)*c^3)*d^2*e^2 - (5*C*b^3*c + 14*(6*B*a + 5*A*b)*c
^3 - 2*(11*C*a*b + 7*B*b^2)*c^2)*d*e^3 - (8*C*b^4 - 210*A*a*c^3 + (30*C*a^
2 + 63*B*a*b + 35*A*b^2)*c^2 - (41*C*a*b^2 + 14*B*b^3)*c)*e^4)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^4*d^3*
e - 8*(2*C*b*c^3 + 7*B*c^4)*d^2*e^2 - (9*C*b^2*c^2 - 70*A*c^4 - (26*C*a +
21*B*b)*c^3)*d*e^3 - (8*C*b^3*c + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14*B
*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c
^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2
*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*
d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*
(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 24*C*c^4*d^2*e^2 - (
5*C*b*c^3 + 28*B*c^4)*d*e^3 - (4*C*b^2*c^2 - 35*A*c^4 - (10*C*a + 7*B*b)*c
^3)*e^4 - 3*(6*C*c^4*d*e^3 - (C*b*c^3 + 7*B*c^4)*e^4)*x)*sqrt(c*x^2 + b*x
+ a)*sqrt(e*x + d))/(c^4*e^5)
```

3.260.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

3.260.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

3.260.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

3.261 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

3.261.1 Optimal result

Integrand size = 34, antiderivative size = 749

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2d(24Cd^2-5e(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4Bde+3A\right.}{15ce^3\left(cd^2-bde+ae^2\right)}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx}{b^2-4ac}}}{15c^2e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))-ce(2ae(9Cd-5Be)-b(32Cd^2-5e(5}{15c^2e^4\sqrt{d+ex}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^
(1/2)-2/15*(b*C*e^2*(-a*e+b*d)+c^2*d*(24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(a*
e*(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+C*b*d-6*c
*C*d^2/e-5*A*c*e-C*a*e)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-
b*d*e+c*d^2)-1/15*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*C*a*e+8*C*b*d)-c^2*(48*C*d^
2-10*e*(-3*A*e+4*B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*
x+a)/(-4*a*c+b^2))^(1/2)/c^2/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(24*C*d^2-5
*e*(-3*A*e+4*B*d))-c*e*(2*a*e*(-5*B*e+9*C*d)-b*(32*C*d^2-5*e*(-3*A*e+5*B*d
))))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+
d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*
x+a)^(1/2)
```

3.261.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.14 (sec) , antiderivative size = 1276, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \sqrt{d+ex}\sqrt{a+x(b+cx)}\left(\frac{2(-9cCd+5Bce+bCe)}{15ce^3} + \frac{2Cx}{5e^2} - \frac{2(Cd^2-Bde+Ae^2)}{e^3(d+ex)}\right)$$

$$+ \frac{(d+ex)^{3/2}\sqrt{a+x(b+cx)}\left(-4\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)+c^2(-48Cd^2+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

output
```
Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]*((2*(-9*c*C*d + 5*B*c*e + b*C*e))/(15*
c*e^3) + (2*C*x)/(5*e^2) - (2*(C*d^2 - B*d*e + A*e^2))/(e^3*(d + e*x)) +
((d + e*x)^(3/2)*Sqrt[a + x*(b + c*x)]*(-4*Sqrt[(c*d^2 + e*(-(b*d) + a*e))
/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5
*b*B*e - 6*a*C*e) + c^2*(-48*C*d^2 + 10*e*(4*B*d - 3*A*e)))*(c*(-1 + d/(d
+ e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) + (I*Sq
rt[2]*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(2*b^2*C*e^2 + c*e*(8*b*C*d
- 5*b*B*e - 6*a*C*e) + c^2*(-48*C*d^2 + 10*e*(4*B*d - 3*A*e)))*Sqrt[(Sqrt[
(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*
(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqr
t[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*
(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipti
cE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - (I*Sqrt[2]
*(-2*b^3*C*e^3 + b^2*e^2*(-6*c*C*d + 5*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]
) + b*(8*a*c*C*e^3 + c*e*Sqrt[(b^2 - 4*a*c)*e^2]*(8*C*d - 5*B*e)) - 2*c*(a
*e^2*(-12*c*C*d + 10*B*c*e + 3*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*Sqrt[(b^2 -
4*a*c)*e^2]*(24*C*d^2 + 5*e*(-4*B*d + 3*A*e))))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e
^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/...
```

3.261.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2181, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+5Bcd+bCd-5Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e\sqrt{d+ex}}dx}{ae^2-bde+cd^2}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+5Bcd+bCd-5Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{\sqrt{d+ex}}dx}{e\left(ae^2-bde+cd^2\right)} - \frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{2\int -\frac{\left(cd^2-bed+ae^2\right)\left(-Cdeb^2+24cCd^2b-aCe^2b-5ce(4Bd-3Ae)b-2ace(6Cd-5Be)-\left(-\left(\left(48Cd^2-10e(4Bd-3Ae)\right)c^2\right)+e(8bCd-5bBe-6aCe)c+2b^2Ce^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{15ce^2} \quad e(\ldots$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(ae^2-bde+cd^2\right)\int \frac{-Cdeb^2+24cCd^2b-aCe^2b-5ce(4Bd-3Ae)b-2ace(6Cd-5Be)-\left(-\left(\left(48Cd^2-10e(4Bd-3Ae)\right)c^2\right)+e(8bCd-5bBe-6aCe)c+2b^2Ce^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{15ce^2} - \quad e(ae\ldots$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\left(ae^2-bde+cd^2\right)\left(\frac{\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-ae)-2c^2d\left(24Cd^2-5e(4Bd-3Ae)\right)\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e} - \frac{\left(ce(-6aCe-5bBe\ldots\right.}{\ldots}\right.}{15ce^2}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\left(cd^2-bed+ae^2\right)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(-2d\left(24Cd^2-5e(4Bd-3Ae)\right)c^2+e\left(32bCd^2-5be(5Bd-3Ae)-2ae(9Cd-5Be)\right)c+bCe^2(bd-ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(c\ldots}{\ldots}}}{ce\sqrt{d+ex}\sqrt{cx^2+bx+a}}\right.$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{e\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

3.261. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx$

$$\Big\downarrow \; 321$$

$$\frac{(cd^2-bed+ae^2)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(-2d\left(24Cd^2-5e(4Bd-3Ae)\right)c^2+e\left(32bCd^2-5be(5Bd-3Ae)-2ae(9Cd-5Be)\right)c+bCe^2(bd-ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(c\cdots}{\cdots}}}{ce\sqrt{d+ex}\sqrt{cx^2+bx+a}}\cdots\right.}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{e\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\Big\downarrow \; 327$$

$$\frac{(ae^2-bde+cd^2)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-ae)-2c^2d\left(24Cd^2-5e(4B\cdots\right.\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\cdots\right.}{\cdots}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

3.261. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a
*e^2)*Sqrt[d + e*x]) + ((-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C
*d^3 - 5*d*e*(4*B*d - 3*A*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 -
4*B*d*e + 3*A*e^2)) + 3*c*e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e -
 a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(15*c*e^2) + ((c*d^2 - b*d*e + a*e^2)*(-
((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*
e) - c^2*(48*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*
e^2*(b*d - a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d
^2 - 5*b*e*(5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])))/(15*c*e^2))/(e*(c*d^2 - b*
d*e + a*e^2))
```

3.261.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.261. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.261. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}} \, dx$

3.261.4 Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.62

method	result	size
elliptic	Expression too large to display	1215
risch	Expression too large to display	1864
default	Expression too large to display	8221

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)*(A*e^2-B*d*e+C*d^2)/e^4/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+
2/5*C/e^2*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(1/e^2*(B*
c*e+C*b*e-C*c*d)-2/5/e^2*(2*b*e+2*c*d)*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e
*x+b*d*x+a*d)^(1/2)+2*((A*b*e^3-A*c*d*e^2+B*a*e^3-B*b*d*e^2+B*c*d^2*e-C*a*
d*e^2+C*b*d^2*e-C*c*d^3)/e^4-(A*e^2-B*d*e+C*d^2)/e^4*(b*e-c*d)+b/e^3*(A*e^
2-B*d*e+C*d^2)-2/5*a*d/e^2*C-2/3*(1/e^2*(B*c*e+C*b*e-C*c*d)-2/5/e^2*(2*b*e
+2*c*d)*C)/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d
*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+2*(1/e^3*(A*c*e^2+B*b*e^2-B*c*d*e+C*a*e^2-C*b*d*e+C
*c*d^2)+(A*e^2-B*d*e+C*d^2)/e^3*c-2/5*C/e^2*(3/2*a*e+3/2*b*d)-2/3*(1/e^2*(
B*c*e+C*b*e-C*c*d)-2/5/e^2*(2*b*e+2*c*d)*C)/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(...
```

3.261.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\left(\left(48\,Cc^3d^4-8\,(4\,Cbc^2+5\,Bc^3)d^3e-(7\,Cb^2c-30\,Ac^3-(42\,Ca+25\,Bb)c^2)d^2e^2-(2\,Cb^3+15\,(2\,Ba+\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fr
icas")
```

output
```
-2/45*((48*C*c^3*d^4 - 8*(4*C*b*c^2 + 5*B*c^3)*d^3*e - (7*C*b^2*c - 30*A*c
^3 - (42*C*a + 25*B*b)*c^2)*d^2*e^2 - (2*C*b^3 + 15*(2*B*a + A*b)*c^2 - (9
*C*a*b + 5*B*b^2)*c)*d*e^3 + (48*C*c^3*d^3*e - 8*(4*C*b*c^2 + 5*B*c^3)*d^2
*e^2 - (7*C*b^2*c - 30*A*c^3 - (42*C*a + 25*B*b)*c^2)*d*e^3 - (2*C*b^3 + 1
5*(2*B*a + A*b)*c^2 - (9*C*a*b + 5*B*b^2)*c)*e^4)*x)*sqrt(c*e)*weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3
)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^3*d^3*e - 8*(C*b
*c^2 + 5*B*c^3)*d^2*e^2 - (2*C*b^2*c - 30*A*c^3 - (6*C*a + 5*B*b)*c^2)*d*e
^3 + (48*C*c^3*d^2*e^2 - 8*(C*b*c^2 + 5*B*c^3)*d*e^3 - (2*C*b^2*c - 30*A*c
^3 - (6*C*a + 5*B*b)*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
 b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e
- 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstras
sPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*
c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^
3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(3*C*c^3*e^4*x^2 - 24*
C*c^3*d^2*e^2 - 15*A*c^3*e^4 + (C*b*c^2 + 20*B*c^3)*d*e^3 - (6*C*c^3*d*e^3
 - (C*b*c^2 + 5*B*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*e
^6*x + c^3*d*e^5)
```

3.261.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

3.261.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

3.261.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

3.262 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx$

3.262.1 Optimal result

Integrand size = 34, antiderivative size = 712

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\left(e(bd-ae)(7Cd-3Be)-cd(8Cd^2-e(4Bd-Ae))+e^2\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)x\right)\sqrt{a+}}{3e^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2\left(4cd-\frac{be}{2}\right)\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)+6c(bd(Cd-Be)+e(Acd-aCd+aBe\right.}{3ce^3\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{E}}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(3/2)-2/3*(e*(-a*e+b*d)*(-3*B*e+7*C*d)-c*d*(8*C*d^2-e*(-A*e+4*B*d))+e^2*
(B*c*d+C*b*d-2*c*C*d^2/e-A*c*e-C*a*e)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*
d*e+c*d^2)/(e*x+d)^(1/2)+1/3*(2*(4*c*d-1/2*b*e)*(B*c*d+C*b*d-2*c*C*d^2/e-A
*c*e-C*a*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d)))*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+
b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^3/(a*e^2-b*d*
e+c*d^2)/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)-2/3*(e*(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))*Elli
pticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.262.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.08 (sec) , antiderivative size = 8456, normalized size of antiderivative = 11.88

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

output
```
Result too large to show
```

3.262.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2181, 27, 1230, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{3\left(bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{3/2}}dx}{3\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{3/2}}dx}{e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1230}$$

$$-\frac{\frac{2\int \frac{2(2bd-ae)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-3be(bd(Cd-Be)+e(Acd-aCd+aBe))+\left((8cd-be)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-6ce(bd(Cd-Be)+e(Ac\ldots\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3e^2}}{e\left(ae^2-bde+cd\ldots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{2(2bd-ae)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-3be(bd(Cd-Be)+e(Acd-aCd+aBe))+\left((8cd-be)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-6ce(bd(Cd-Be)+e(Acd-\ldots\right.}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3e^2}}{e\left(ae^2-bde+cd^2\ldots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{\frac{\frac{\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{\left(ae^2-bde+cd^2\right)\left(e(-2aCe-3bBe+8bCd)-2c\left(\ldots\right.\right.}{\ldots}}{3e^2}}{e\left(ae^2-\ldots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

3.262. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\sqrt{cx^2+bx+a}\left(\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)xe^2+(bd-ae)(7Cd-3Be)e-c\left(8Cd^3-de(4Bd-Ae)\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\left(e(8bCd-\right.}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{3e\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$\Big\downarrow$ 321

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 327

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

3.262. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(3/2)) + ((-2*(e*(b*d - a*e)*(7*C*d - 3*B*e) - c*(8*C*d^
3 - d*e*(4*B*d - A*e)) + e^2*(B*c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*
e)*x)*Sqrt[a + b*x + c*x^2])/(3*e^2*Sqrt[d + e*x]) - ((Sqrt[2]*Sqrt[b^2 -
4*a*c]*(b*C*e^2*(b*d - a*e) + 2*c^2*(8*C*d^3 - d*e*(4*B*d - A*e)) - c*e*(1
6*b*C*d^2 - b*e*(7*B*d - A*e) - 2*a*e*(7*C*d - 3*B*e)))*Sqrt[d + e*x]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(c*d^2 - b*d*e + a*e^2)*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) - 2*c*(8*C*
d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a
 + b*x + c*x^2]))/(3*e^2))/(e*(c*d^2 - b*d*e + a*e^2))
```

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

3.262. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.262.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(646) = 1292$.

Time = 4.86 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.88

method	result	size
elliptic	Expression too large to display	1340
risch	Expression too large to display	2764
default	Expression too large to display	21038

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3*(A*e
^2-B*d*e+C*d^2)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^2-2/3*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3
*B*a*e^3-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/((x+d/
e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2/3*C/e^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)+2*((A*c*e^2+B*b*e^2-2*B*c*d*e+C*a*e^2-2*C*b*d*e+3*C*c*d^2)/e
^4-1/3*(A*e^2-B*d*e+C*d^2)/e^4*c-1/3/e^4*(b*e-c*d)*(A*b*e^3-2*A*c*d*e^2+3*
B*a*e^3-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-
b*d*e+c*d^2)+1/3*b/e^3/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3-
4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)-2/3*C/e^3*(1/2*
a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d
)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
))+2*(1/e^3*(B*c*e+C*b*e-2*C*c*d)+1/3*c/e^3*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3
-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-b*d*e+c
*d^2)-2/3*C/e^3*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)...
```

3.262.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1385, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fr
icas")
```

output
```
2/9*((16*C*c^3*d^6 - 8*(3*C*b*c^2 + B*c^3)*d^5*e + (6*C*b^2*c + 2*A*c^3 +
(26*C*a + 11*B*b)*c^2)*d^4*e^2 + (C*b^3 - 2*(6*B*a + A*b)*c^2 - 2*(7*C*a*b
 + B*b^2)*c)*d^3*e^3 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A*b^2)*
c)*d^2*e^4 + (16*C*c^3*d^4*e^2 - 8*(3*C*b*c^2 + B*c^3)*d^3*e^3 + (6*C*b^2*
c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^2*e^4 + (C*b^3 - 2*(6*B*a + A*b)*c^
2 - 2*(7*C*a*b + B*b^2)*c)*d*e^5 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a
*b - A*b^2)*c)*e^6)*x^2 + 2*(16*C*c^3*d^5*e - 8*(3*C*b*c^2 + B*c^3)*d^4*e^
2 + (6*C*b^2*c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^3*e^3 + (C*b^3 - 2*(6*
B*a + A*b)*c^2 - 2*(7*C*a*b + B*b^2)*c)*d^2*e^4 - (C*a*b^2 - 6*A*a*c^2 - (
6*C*a^2 + 3*B*a*b - A*b^2)*c)*d*e^5)*x)*sqrt(c*e)*weierstrassPInverse(4/3*
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^3*d^5*e - 8*(2*C*b*c^2 + B*c^
3)*d^4*e^2 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^3*e^3 - (C*a*b*c
+ (6*B*a + A*b)*c^2)*d^2*e^4 + (16*C*c^3*d^3*e^3 - 8*(2*C*b*c^2 + B*c^3)*d
^2*e^4 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d*e^5 - (C*a*b*c + (6*B
*a + A*b)*c^2)*e^6)*x^2 + 2*(16*C*c^3*d^4*e^2 - 8*(2*C*b*c^2 + B*c^3)*d^3*
e^3 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^2*e^4 - (C*a*b*c + (6*B*
a + A*b)*c^2)*d*e^5)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
 (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^...
```

3.262.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx=\int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

3.262.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

3.262.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

3.263 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

3.263.1 Optimal result

Integrand size = 34, antiderivative size = 992

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx =$$

$$-\frac{2(c^2d^3(24Cd^2-e(4Bd+Ae))+e^2(15b^2Cd^3+5a^2e^2(Cd+Be)-abe(22Cd^2+3Bde+2Ae^2))-cde(bd\cdots}{\cdots}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^2d^2(24Cd^2-e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3Bde-2A\cdots}{15e^4\left(cd^2\cdots\right.}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(15bCe^2(bd-ae)+2c^2d(24Cd^2-e(4Bd+Ae))+ce(10ae(5Cd-Be)-b(64Cd^2-9Bd\cdots}{15ce^4\left(cd^2-bde+ae^2\right)\sqrt{\cdots}}$$

output
```
-2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(5/2)-2/15*(c^2*d^3*(24*C*d^2-e*(A*e+4*B*d))+e^2*(15*b^2*C*d^3+5*a^2*e^2
*(B*e+C*d)-a*b*e*(2*A*e^2+3*B*d*e+22*C*d^2))-c*d*e*(b*d*(A*e^2-6*B*d*e+41*
C*d^2)-a*e*(7*A*e^2-7*B*d*e+37*C*d^2))+e*(5*c^2*d^2*(6*C*d^2-e*(A*e+B*d))+
e^2*(15*a^2*C*e^2-5*a*b*e*(-B*e+8*C*d)+b^2*(-2*A*e^2-3*B*d*e+23*C*d^2))-c*
e*(5*b*d*(-A*e^2-2*B*d*e+11*C*d^2)-a*e*(3*A*e^2-13*B*d*e+53*C*d^2)))*x)*(c
*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/2)+1/15*(2*c^2*d^2*
(24*C*d^2-e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A
*e^2-3*B*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e
^2-8*B*d*e+43*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x
+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(
e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/15*(15*b*C*e^2*(-a*e+b*d)
+2*c^2*d*(24*C*d^2-e*(A*e+4*B*d))+c*e*(10*a*e*(-B*e+5*C*d)-b*(-A*e^2-9*B*d
*e+64*C*d^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(a*e^2-b*d*e+c*
d^2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.263.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.86 (sec) , antiderivative size = 12997, normalized size of antiderivative = 13.10

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

output
```
Result too large to show
```

3.263.3 Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2181, 27, 1229, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \ \mathbf{2181}$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+Bcd+5bCd-Ace-5aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{5/2}}dx}{5\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ \mathbf{27}$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+Bcd+5bCd-Ace-5aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{5/2}}dx}{5e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ \mathbf{1229}$$

$$-\frac{2\int -\frac{15Cd^2e^2b^3-de\left(41cCd^2+30aCe^2-ce(6Bd-Ae)\right)b^2+\left(15a^2Ce^4+ac\left(59Cd^2-e(14Bd+Ae)\right)e^2+c^2\left(24Cd^4-d^2e(4Bd+Ae)\right)\right)b-2ace(6cCd^3-ce(Bd+4Ae)c}{}}{3}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ \mathbf{27}$$

$$\frac{\int \frac{15Cd^2e^2b^3-de\left(41cCd^2+30aCe^2-ce(6Bd-Ae)\right)b^2+\left(15a^2Ce^4+ac\left(59Cd^2-e(14Bd+Ae)\right)e^2+c^2\left(24Cd^4-d^2e(4Bd+Ae)\right)\right)b-2ace(6cCd^3-ce(Bd+4Ae)d+5ae^2}{\sqrt{d+e}}}{3e^2(d}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ \mathbf{1269}$$

$$\frac{\frac{c\left(e^2\left(30a^2Ce^2-5abe(14Cd-Be)+b^2\left(-2Ae^2-3Bde+38Cd^2\right)\right)-ce\left(bd\left(-2Ae^2-13Bde+88Cd^2\right)-2ae\left(3Ae^2-8Bde+43Cd^2\right)\right)+c^2\left(48Cd^4-2d^2e(Ae+4Bd)\right)\right)\int \frac{}{\sqrt{}}}{e}}{3e^2(ae^2-bde}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2Ce^2\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$\downarrow$ 321

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2Ce^2\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$\downarrow$ 327

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2Ce^2\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

3.263. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx$

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(5/2)) + ((-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^
2*(15*b^2*C*d^3 + 5*a^2*e^2*(C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*
e^2)) - c*d*e*(b*d*(41*C*d^2 - 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e
+ 7*A*e^2)) - e*(3*e*(B*c*d + 5*b*C*d - (6*c*C*d^2)/e - A*c*e - 5*a*C*e)*(
c*d^2 - e*(b*d - a*e)) - (2*c*d - b*e)*(6*c*C*d^3 - c*d*e*(B*d + 4*A*e) +
5*a*e^2*(2*C*d - B*e) - b*e*(8*C*d^2 - e*(3*B*d + 2*A*e))))*x)*Sqrt[a + b*
x + c*x^2])/(3*e^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + ((Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C*e^
2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(b*
d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2)))
*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*Sqrt[(c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(15*b*C*e^2*(b*d - a*e) + c
^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e)
- 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b...
```

3.263.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.263. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.263. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

3.263.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 1766, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1766
default	Expression too large to display	48427

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5*(A*e
^2-B*d*e+C*d^2)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^3-2/15*(A*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e
^2+11*C*b*d^2*e-12*C*c*d^3)/e^5/(a*e^2-b*d*e+c*d^2)*(c*e*x^3+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2-2/15*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*
d*e+c*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*
b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2
*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*
c^2*d^4)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*((B*c*e+C*b*e-3*C*c*d)/e^4-
1/15*c*(A*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e^2
+11*C*b*d^2*e-12*C*c*d^3)/e^4/(a*e^2-b*d*e+c*d^2)-1/15/e^4*(b*e-c*d)*(6*A*
a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e
^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^
3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4)/(a*e^2-b*
d*e+c*d^2)^2+1/15*b/e^3/(a*e^2-b*d*e+c*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A
*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b
*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C
*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+...
```

3.263.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 2430, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fr
icas")
```

output
```
-2/45*((48*C*c^3*d^8 - 8*(14*C*b*c^2 + B*c^3)*d^7*e + (73*C*b^2*c - 2*A*c^
3 + (122*C*a + 17*B*b)*c^2)*d^6*e^2 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (1
61*C*a*b + 8*B*b^2)*c)*d^5*e^3 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*
C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3
 + 3*(10*B*a^2 - 3*A*a*b)*c)*d^3*e^5 + (48*C*c^3*d^5*e^3 - 8*(14*C*b*c^2 +
 B*c^3)*d^4*e^4 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^3*e^5
- (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^2*e^6 + (20
*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d*e^7
 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*e^8)*x^3
+ 3*(48*C*c^3*d^6*e^2 - 8*(14*C*b*c^2 + B*c^3)*d^5*e^3 + (73*C*b^2*c - 2*A
*c^3 + (122*C*a + 17*B*b)*c^2)*d^4*e^4 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 +
 (161*C*a*b + 8*B*b^2)*c)*d^3*e^5 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (
90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*
b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*d*e^7)*x^2 + 3*(48*C*c^3*d^7*e - 8*(14*C*b
*c^2 + B*c^3)*d^6*e^2 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^
5*e^3 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^4*e^4
 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)
*d^3*e^5 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*d
^2*e^6)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a...
```

3.263.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

3.263.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

3.263.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

3.264 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx$

3.264.1 Optimal result

Integrand size = 34, antiderivative size = 1363

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \frac{2(2c^3d^3(24Cd^2+e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+B}{}$$

$$-\frac{2(c^2d^3(24Cd^2+e(4Bd+3Ae))-e^2(7a^2e^2(Cd-3Be)-b^2d(15Cd^2+6Bde+8Ae^2)+abe(12Cd^2+23I}{}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2c^3d^3(24Cd^2+e(4Bd+3Ae))-be^3(35a^2Ce^2-14abe(3Cd+Be)+b^2(15Cd^2+6Bde+8}{}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2c^2d^2(24Cd^2+e(4Bd+3Ae))+ce(2ae(51Cd^2+e(12Bd-5Ae))-bd(104Cd^2+3e(5Bd}{}$$

output
```
-2/7*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(7/2)-2/105*(c^2*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-e^2*(7*a^2*e^2*(-3*B*e+C
*d)-b^2*d*(8*A*e^2+6*B*d*e+15*C*d^2)+a*b*e*(12*A*e^2+23*B*d*e+12*C*d^2))-c
*d*e*(b*d*(15*A*e^2+6*B*d*e+43*C*d^2)-a*e*(19*A*e^2+9*B*d*e+33*C*d^2))+e*(
7*c^2*d^2*(6*C*d^2+e*(-3*A*e+B*d))+e^2*(35*a^2*C*e^2-7*a*b*e*(-B*e+12*C*d)
+b^2*(-4*A*e^2-3*B*d*e+45*C*d^2))+c*e*(a*e*(-5*A*e^2-9*B*d*e+93*C*d^2)-b*(
-21*A*d*e^2+91*C*d^3)))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(
e*x+d)^(5/2)+2/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e
^2-14*a*b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69
*C*d^2+e*(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2
*e^2*(-3*B*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*
(19*A*e+9*B*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1
/2)-1/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*b
*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*
(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*
B*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9
*B*d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^
2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c...
```

3.264.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.64 (sec) , antiderivative size = 19853, normalized size of antiderivative = 14.57

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}} \, dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

output
```
Result too large to show
```

3.264.3 Rubi [A] (verified)

Time = 3.27 (sec) , antiderivative size = 1413, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2181, 27, 1229, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)+e\left(\frac{6cCd^2}{e}+Bcd-7bCd-Ace+7aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{7/2}}dx}{7\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)+e\left(\frac{6cCd^2}{e}+Bcd-7bCd-Ace+7aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{7/2}}dx}{7e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1229$$

$$-\frac{2\int -\frac{e^2\left(15Cd^2+6Bed+8Ae^2\right)b^3-\left(14a(3Cd+Be)e^3+cd\left(43Cd^2+3e(2Bd+5Ae)\right)e\right)b^2+\left(35a^2Ce^4+ac\left(111Cd^2-e(6Bd+29Ae)\right)e^2+c^2\left(24Cd^4+e(4Bd+3Ae)d^2\right.\right.}{}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{e^2\left(15Cd^2+6Bed+8Ae^2\right)b^3-\left(14a(3Cd+Be)e^3+cd\left(43Cd^2+3e(2Bd+5Ae)\right)e\right)b^2+\left(35a^2Ce^4+ac\left(111Cd^2-e(6Bd+29Ae)\right)e^2+c^2\left(24Cd^4+e(4Bd+3Ae)d^2\right)\right)b-6}{}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1237$$

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\downarrow$ 27

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\downarrow$ 1269

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\downarrow$ 1172

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\downarrow$ 321

3.264. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\Big\downarrow$ 327

$$\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right)b\right.\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

3.264. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(7*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(7/2)) + ((-2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) -
e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2) + a*b*
e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*e + 15*
A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(5*e*(B*c*d - 7*b*C*d +
(6*c*C*d^2)/e - A*c*e + 7*a*C*e)*(c*d^2 - e*(b*d - a*e)) + (2*c*d - b*e)*(
6*c*C*d^3 + c*d*e*(B*d - 8*A*e) + 7*a*e^2*(2*C*d - B*e) - b*e*(10*C*d^2 -
e*(3*B*d + 4*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(15*e^2*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + ((2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*
e^3*(35*a^2*C*e^2 - 14*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A
*e^2)) + c^2*d*e*(2*a*e*(69*C*d^2 + 15*B*d*e - 29*A*e^2) - b*d*(128*C*d^2
+ 19*B*d*e + 9*A*e^2)) + c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C
*d^2 + B*d*e - 29*A*e^2) + b^2*d*(103*C*d^2 + 9*B*d*e + 19*A*e^2)))*Sqrt[a
 + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (c*((Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*
C*e^2 - 14*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2
*d*e*(2*a*e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d
+ 9*A*e))) + c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*
d - 29*A*e)) + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqr...
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.264. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.264.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 2484, normalized size of antiderivative = 1.82

method	result	size
elliptic	Expression too large to display	2484
default	Expression too large to display	88790

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/7*(A*e
^2-B*d*e+C*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^4-2/35*(A*b*e^3-2*A*c*d*e^2+7*B*a*e^3-8*B*b*d*e^2+9*B*c*d^2*e-14*C*a*d*e
^2+15*C*b*d^2*e-16*C*c*d^3)/(a*e^2-b*d*e+c*d^2)/e^6*(c*e*x^3+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^3-2/105*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*
c*d*e^3-6*A*c^2*d^2*e^2+7*B*a*b*e^4-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*
d^2*e^2-8*B*c^2*d^3*e+35*C*a^2*e^4-84*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b
^2*d^2*e^2-106*C*b*c*d^3*e+57*C*c^2*d^4)/e^5/(a*e^2-b*d*e+c*d^2)^2*(c*e*x^
3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/105*(c*e*x^2+b*e*x+a*
e)/e^4/(a*e^2-b*d*e+c*d^2)^3*(29*A*a*b*c*e^5-58*A*a*c^2*d*e^4-8*A*b^3*e^5+
19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B*a^2*c*e^5+14*B*a*b
^2*e^5-B*a*b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*B*b^2*c*d^2*e^3-19
*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c*d*e^4+42*C*a*b^2
*d*e^4-237*C*a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^2*e^3+103*C*b^2*
c*d^3*e^2-128*C*b*c^2*d^4*e+48*C*c^3*d^5)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1
/2)+2*(c*C/e^4-1/105*c*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*c*d*e^3-6*A*c^2*d^2
*e^2+7*B*a*b*e^4-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*d^2*e^2-8*B*c^2*d^3
*e+35*C*a^2*e^4-84*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b^2*d^2*e^2-106*C*b*
c*d^3*e+57*C*c^2*d^4)/e^4/(a*e^2-b*d*e+c*d^2)^2+1/105/e^4*(b*e-c*d)*(29*A*
a*b*c*e^5-58*A*a*c^2*d*e^4-8*A*b^3*e^5+19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e...
```

3.264.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 4543, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fr
icas")
```

output
```
2/315*((48*C*c^4*d^10 - 8*(19*C*b*c^3 - B*c^4)*d^9*e + (158*C*b^2*c^2 + 6*
A*c^4 + (174*C*a - 23*B*b)*c^3)*d^8*e^2 - (47*C*b^3*c - 12*(3*B*a - A*b)*c
^3 + (384*C*a*b - 17*B*b^2)*c^2)*d^7*e^3 - (15*C*b^4 - 104*A*a*c^3 - (208*
C*a^2 - 106*B*a*b - 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^6*e^4 +
(42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^
2 - 23*A*b^3)*c)*d^5*e^5 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2
*c^2 - (210*C*a^3 - 63*B*a^2*b + 41*A*a*b^2)*c)*d^4*e^6 + (48*C*c^4*d^6*e^
4 - 8*(19*C*b*c^3 - B*c^4)*d^5*e^5 + (158*C*b^2*c^2 + 6*A*c^4 + (174*C*a -
 23*B*b)*c^3)*d^4*e^6 - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b -
17*B*b^2)*c^2)*d^3*e^7 - (15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b
- 17*A*b^2)*c^2 - 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^2*e^8 + (42*C*a*b^3 - 6*B*
b^4 + 52*(3*B*a^2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d
*e^9 - (35*C*a^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 -
63*B*a^2*b + 41*A*a*b^2)*c)*e^10)*x^4 + 4*(48*C*c^4*d^7*e^3 - 8*(19*C*b*c^
3 - B*c^4)*d^6*e^4 + (158*C*b^2*c^2 + 6*A*c^4 + (174*C*a - 23*B*b)*c^3)*d^
5*e^5 - (47*C*b^3*c - 12*(3*B*a - A*b)*c^3 + (384*C*a*b - 17*B*b^2)*c^2)*d
^4*e^6 - (15*C*b^4 - 104*A*a*c^3 - (208*C*a^2 - 106*B*a*b - 17*A*b^2)*c^2
- 3*(79*C*a*b^2 + 4*B*b^3)*c)*d^3*e^7 + (42*C*a*b^3 - 6*B*b^4 + 52*(3*B*a^
2 - 2*A*a*b)*c^2 - (364*C*a^2*b - B*a*b^2 - 23*A*b^3)*c)*d^2*e^8 - (35*C*a
^2*b^2 - 14*B*a*b^3 + 8*A*b^4 + 30*A*a^2*c^2 - (210*C*a^3 - 63*B*a^2*b ...
```

3.264.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx=\int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

3.264.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

3.264.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

3.265 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx$

3.265.1 Optimal result

Integrand size = 34, antiderivative size = 1904

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

output
```
-2/9*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(9/2)+2/315*(2*c^3*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*
e^2+7*B*d*e+9*C*d^2)-b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-
5*B*e+17*C*d)-a*b*e*(-9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*
C*d^2))-b*e^3*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C
*d^2)))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(3/2)-2/105*
(c^2*d^3*(8*C*d^2+e*(5*A*e+4*B*d))-e^2*(3*a^2*e^2*(-5*B*e+3*C*d)-a*b*e*(-1
0*A*e^2-17*B*d*e+2*C*d^2)-b^2*d*(8*A*e^2+4*B*d*e+5*C*d^2))-c*d*e*(3*b*d*(5
*A*e^2+2*B*d*e+5*C*d^2)-a*e*(13*A*e^2+11*B*d*e+7*C*d^2))+e*(3*c^2*d^2*(6*C
*d^2+e*(-5*A*e+3*B*d))+c*e*(a*e*(-7*A*e^2+B*d*e+47*C*d^2)-3*b*d*(-5*A*e^2+
2*B*d*e+15*C*d^2))+e^2*(21*a^2*C*e^2-3*a*b*e*(-B*e+16*C*d)+b^2*(25*C*d^2-e
*(2*A*e+B*d))))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(
7/2)+2/315*(2*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*
a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34
*A*e^2-5*B*d*e+30*C*d^2)-a^2*e^2*(7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A
*e^2+3*B*d*e+11*C*d^2))-c*e^3*(126*a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6
*a*b^2*e*(-12*A*e^2+7*B*d*e+5*C*d^2)+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c
^3*d^2*e*(6*a*e*(-34*A*e^2+8*B*d*e+11*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*
d)))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^(1/2)-1/315*(2
*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B...
```

3.265.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 38.31 (sec) , antiderivative size = 29140, normalized size of antiderivative = 15.30

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

output
```
Result too large to show
```

3.265.3 Rubi [A] (verified)

Time = 4.55 (sec) , antiderivative size = 1966, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2181, 27, 1229, 27, 1237, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \ 2181$$

$$-\frac{2\int -\frac{3\left(bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{9/2}}dx}{9\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{9e(d+ex)^{9/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{9/2}}dx}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{9e(d+ex)^{9/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \ 1229$$

$$-\frac{2\int -\frac{e^2\left(5Cd^2+4e(Bd+2Ae)\right)b^3-3\left(2a(3Cd+2Be)e^3+cd\left(5Cd^2+e(2Bd+5Ae)\right)e\right)b^2+\left(21a^2Ce^4+3ac\left(19Cd^2+e(2Bd-9Ae)\right)e^2+c^2\left(8Cd^4+e(4Bd+5Ae)d^2\right)\right)b-\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{9e(d+ex)^{9/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\int \frac{e^2\left(5Cd^2+4e(Bd+2Ae)\right)b^3-3\left(2a(3Cd+2Be)e^3+cd\left(5Cd^2+e(2Bd+5Ae)\right)e\right)b^2+\left(21a^2Ce^4+3ac\left(19Cd^2+e(2Bd-9Ae)\right)e^2+c^2\left(8Cd^4+e(4Bd+5Ae)d^2\right)\right)b-10ace\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{9e(d+ex)^{9/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1237

$$\frac{\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\cdots\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}\cdots}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 27

$$\frac{\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\cdots\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}\cdots}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 1237

$$\frac{\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\cdots\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}\cdots}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 27

3.265. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

$$\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 1269

$$\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 1172

$$\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 321

3.265. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

$$\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

$\downarrow$ 327

$$\frac{2\left(2\left(8Cd^5+e(4Bd+5Ae)d^3\right)c^3+3de\left(2ae\left(9Cd^2+7Bed-9Ae^2\right)-bd\left(16Cd^2+7Bed+5Ae^2\right)\right)c^2+3e^2\left(d\left(15Cd^2+3Bed+7Ae^2\right)b^2-ae\left(41Cd^2+5Bed-9Ae^2\right)b+2a^2e^2\right.\right.}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

3.265. $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(9/2)) + ((-2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e
^2*(3*a^2*e^2*(3*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^
2*d*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A
*e^2) - a*e*(7*C*d^2 + 11*B*d*e + 13*A*e^2)) + e*(7*e*(B*c*d - 3*b*C*d + (
2*c*C*d^2)/e - A*c*e + 3*a*C*e)*(c*d^2 - e*(b*d - a*e)) + (2*c*d - b*e)*(2
*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*
(B*d + 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(35*e^2*(c*d^2 - b*d*e + a*e^2)
*(d + e*x)^(7/2)) + ((2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d
*e*(2*a*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^
2)) + 3*c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*
A*e^2) + b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a
*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x +
c*x^2])/(3*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) - ((-2*(2*c^4*(8*C*d^6
 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e
) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a
*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b
*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*
e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3
*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)...
```

3.265.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.265. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx$

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.265. $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.265.4 Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 3498, normalized size of antiderivative = 1.84

method	result	size
elliptic	Expression too large to display	3498
default	Expression too large to display	153623

input
```
int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x,method=_RETURNVERBO
SE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/9*(A*e
^2-B*d*e+C*d^2)/e^8*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^5-2/63*(A*b*e^3-2*A*c*d*e^2+9*B*a*e^3-10*B*b*d*e^2+11*B*c*d^2*e-18*C*a*d
*e^2+19*C*b*d^2*e-20*C*c*d^3)/(a*e^2-b*d*e+c*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d
*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^4-2/315*(14*A*a*c*e^4-6*A*b^2*e^4+10*A
*b*c*d*e^3-10*A*c^2*d^2*e^2+9*B*a*b*e^4-32*B*a*c*d*e^3-3*B*b^2*d*e^3+17*B*
b*c*d^2*e^2-8*B*c^2*d^3*e+63*C*a^2*e^4-144*C*a*b*d*e^3+176*C*a*c*d^2*e^2+7
5*C*b^2*d^2*e^2-170*C*b*c*d^3*e+89*C*c^2*d^4)/e^6/(a*e^2-b*d*e+c*d^2)^2*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^3+2/315*(27*A*a*b*c*
e^5-54*A*a*c^2*d*e^4-8*A*b^3*e^5+21*A*b^2*c*d*e^4-15*A*b*c^2*d^2*e^3+10*A*
c^3*d^3*e^2-30*B*a^2*c*e^5+12*B*a*b^2*e^5-15*B*a*b*c*d*e^4+42*B*a*c^2*d^2*
e^3-4*B*b^3*d*e^4+9*B*b^2*c*d^2*e^3-21*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-21*C*
a^2*b*e^5+102*C*a^2*c*d*e^4+18*C*a*b^2*d*e^4-123*C*a*b*c*d^2*e^3+54*C*a*c^
2*d^3*e^2-5*C*b^3*d^2*e^3+45*C*b^2*c*d^3*e^2-48*C*b*c^2*d^4*e+16*C*c^3*d^5
)/e^5/(a*e^2-b*d*e+c*d^2)^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2
)/(x+d/e)^2+2/315*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c*d^2)^4*(42*A*a^2*
c^2*e^6-72*A*a*b^2*c*e^6+204*A*a*b*c^2*d*e^5-204*A*a*c^3*d^2*e^4+16*A*b^4*
e^6-56*A*b^3*c*d*e^5+66*A*b^2*c^2*d^2*e^4-20*A*b*c^3*d^3*e^3+10*A*c^4*d^4*
e^2+87*B*a^2*b*c*e^6-216*B*a^2*c^2*d*e^5-24*B*a*b^3*e^6+42*B*a*b^2*c*d*e^5
+30*B*a*b*c^2*d^2*e^4+48*B*a*c^3*d^3*e^3+8*B*b^4*d*e^5-25*B*b^3*c*d^2*e...
```

3.265.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.28 (sec) , antiderivative size = 7780, normalized size of antiderivative = 4.09

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="f
ricas")
```

output
```
Too large to include
```

3.265.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(11/2), x)
```

3.265.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

3.265.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{11/2}}\,dx=\int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{11/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2), x)
```

3.266 $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

3.266.1 Optimal result

Integrand size = 34, antiderivative size = 724

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5A}{105c^3e}$$

$$-\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3d(6Cd^2-7e(3Bd+20Ae))+c^2e(ae(82Cd}{105c^4e^2\sqrt{\frac{c(}{2cd-(b}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae)))}{105c^4e^2\sqrt{d+ex}\sqrt{a+}}$$

output
```
-2/35*(-7*B*c*e+6*C*b*e+2*C*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2
/7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/105*(24*b^2*C*e^2-c*e*(28*B*b
*e+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(e*x+d)^(1/2)*(c*x^
2+b*x+a)^(1/2)/c^3/e-1/105*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+13*C*a*e+9*C*b
*d)+c^3*d*(6*C*d^2-7*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*C*d)+b*(70*A*
e^2+91*B*d*e+12*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b
*x+a)/(-4*a*c+b^2))^(1/2)/c^4/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e
*(28*B*b*e+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*EllipticF(1
/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c
+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.266.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.48 (sec) , antiderivative size = 1314, normalized size of antiderivative = 1.81

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d+ex}\left(a+bx+cx^2\right)\left(\frac{2(3c^2Cd^2+42Bc^2de-33bcCde-28bBce^2+35Ac^2e^2+24b^2Ce^2}{105c^3e}\right.}{\sqrt{a+x(b+cx)}}$$

$$+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}\left(-\left((48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3(6Cd^3-7de(3Bd+20Ae\right.\right.}{}$$

input
```
Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[d + e*x]*(a + b*x + c*x^2)*((2*(3*c^2*C*d^2 + 42*B*c^2*d*e - 33*b*c*
C*d*e - 28*b*B*c*e^2 + 35*A*c^2*e^2 + 24*b^2*C*e^2 - 25*a*c*C*e^2))/(105*c
^3*e) + (2*(8*c*C*d + 7*B*c*e - 6*b*C*e)*x)/(35*c^2) + (2*C*e*x^2)/(7*c)))
/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-((48*b
^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*(6*C*d^3 - 7*d*e
*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C*d^2 + 91*B*d*e
 + 70*A*e^2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(
d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((
2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*
(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2
*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d +
 7*b*B*e + 13*a*C*e) + c^3*(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e
*(82*C*d + 63*B*e) + b*(12*C*d^2 + 91*B*d*e + 70*A*e^2)))*EllipticE[I*ArcS
inh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c
*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (-48*b^4*C*e^4 + 8*b^3*e^3*(15*c*C
*d + 7*B*c*e + 6*C*Sqrt[(b^2 - 4*a*c)*e^2]) - b^2*c*e^2*(78*c*C*d^2 - 152*
a*C*e^2 + 7*c*e*(21*B*d + 10*A*e) + 8*Sqrt[(b^2 - 4*a*c)*e^2]*(9*C*d + 7*B
*e)) + c^2*(-50*a^2*C*e^4 - 3*c*d^2*Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 7*B*
e) - 70*A*c*e^2*(3*c*d^2 - a*e^2 + 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2...
```

3.266.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{2\int -\frac{e(d+ex)^{3/2}(bCd-7Ace+5aCe+(2cCd-7Bce+6bCe)x)}{2\sqrt{cx^2+bx+a}}dx}{7ce^2}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-\frac{\int \frac{(d+ex)^{3/2}(bCd-7Ace+5aCe+(2cCd-7Bce+6bCe)x)}{\sqrt{cx^2+bx+a}}dx}{7ce}$$

$$\Big\downarrow \; 1236$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} - \frac{\frac{2\int -\frac{\sqrt{d+ex}\left(6Cdeb^2+18aCe^2b-cd(3Cd+7Be)b+ce(35Acd-19aCd-21aBe)+\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right)-e(15bCd+28bBe+25aCe)c+24b^2Ce^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{5c} + \frac{2 \cdots}{}}{7ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} - \frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c} - \frac{\int \frac{\sqrt{d+ex}\left(6Cdeb^2+18aCe^2b-cd(3Cd+7Be)b+ce(35Acd-19aCd-21aBe)+\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right) \cdots\right.\right.}{\sqrt{cx^2+bx+a}} \cdots}{5c}}{7ce}$$

$$\Big\downarrow \; 1236$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} - \frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c} - \frac{2\int -\frac{24Cde^2b^3+\left(24aCe^3-cde(33Cd+28Be)\right)b^2+c\left(3cCd^3+7ce(6Bd+5Ae)d-2ae^2(47Cd+14Be)\right)b-ce\left(35 \cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} - \frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c} - \frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c} - \frac{\int \frac{24Cde^2b^3 \cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 1269$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} - \frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c} - \frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c} - \frac{\left(c^2e\left(ae(63 \cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 1172$$

$$\frac{2C(d+ex)^{5/2}\sqrt{cx^2+bx+a}}{7ce} - \frac{\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}{5c} - \frac{\frac{2\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right)-e(15bCd+28bBe+25aCe)c+24b^2Ce^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c} - \frac{2\sqrt{2}\sqrt{b^2-4 \cdots}}{\cdots}}{\cdots}}{\cdots}$$

3.266. $\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

$\big\downarrow$ 321

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{\sqrt{2}\sqrt{b^2-4a}}{}}{}}{}$$

$\big\downarrow$ 327

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{2\sqrt{2}\sqrt{b^2-4}}{}}{}}{}$$

input
```
Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - ((2*(2*c*C*d - 7*B*c
*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - ((2*(24*b^2*C
*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*e*(3*B*d +
5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - ((Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*(6*
C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C*d
^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2
)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b
*x + c*x^2]))/(3*c))/(5*c))/(7*c*e)
```

3.266. $\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.266. $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.266.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.74

method	result	size
elliptic	Expression too large to display	1261
risch	Expression too large to display	4796
default	Expression too large to display	14084

input
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/c*e
*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*e^2+2*d*e*C-2/
7*C/c*e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)+2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+2*d*e*C
-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+
a*e*x+b*d*x+a*d)^(1/2)+2*(A*d^2-2/5*(B*e^2+2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d)
)/c/e*a*d-2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+
2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))
*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ellip
ticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2*A*d*e
+B*d^2-4/7*a/c*d*e*C-2/5*(B*e^2+2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(3/2*
a*e+3/2*b*d)-2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C/c*e*(5/2*a*e+5/2*b*d)-2/5*(B*e
^2+2*d*e*C-2/7*C/c*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2...
```

3.266.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^{3/2} (A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}} \, dx = \frac{2\left(\left(6\,Cc^4d^4+3\left(3\,Cbc^3-7\,Bc^4\right)d^3e+\left(39\,Cb^2c^2+175\,Ac^4-\left(71\,Ca\right.\right.\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/315*((6*C*c^4*d^4 + 3*(3*C*b*c^3 - 7*B*c^4)*d^3*e + (39*C*b^2*c^2 + 175*
A*c^4 - (71*C*a + 56*B*b)*c^3)*d^2*e^2 - (96*C*b^3*c + 7*(27*B*a + 25*A*b)
*c^3 - (260*C*a*b + 119*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*
C*a^2 + 147*B*a*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*sqrt(
c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*
e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^
3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(6*C*c^4
*d^3*e + 3*(4*C*b*c^3 - 7*B*c^4)*d^2*e^2 - (72*C*b^2*c^2 + 140*A*c^4 - (82
*C*a + 91*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13*C
*a*b + 7*B*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 3*C*c^4*d^
2*e^2 - 3*(11*C*b*c^3 - 14*B*c^4)*d*e^3 + (24*C*b^2*c^2 + 35*A*c^4 - (25*C
*a + 28*B*b)*c^3)*e^4 + 3*(8*C*c^4*d*e^3 - (6*C*b*c^3 - 7*B*c^4)*e^4)*x)*s
qrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^5*e^3)
```

3.266.6 Sympy [F]

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.266.7 Maxima [F]

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

3.266.8 Giac [F]

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

3.267.1 Optimal result

Integrand size = 34, antiderivative size = 557

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2Ce^2-ce(3bCd+10bBe+9aCe)-c^2(2Cd^2-5e(Bd+3Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{15c^3e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\text{a}\right.}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/15*(-5*B*c*e+4*C*b*e+2*C*c*d
)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+1/15*(8*b^2*C*e^2-c*e*(10*B*b*e+
9*C*a*e+3*C*b*d)-c^2*(2*C*d^2-5*e*(3*A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*
x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^2/(c*x^2+b*x+a)^(1/
2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-5*B*c*e+4*C*b
*e+2*C*c*d)*(a*e^2-b*d*e+c*d^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(
-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/
e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.267.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.14 (sec) , antiderivative size = 862, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{d+ex}\left(\frac{2(a+x(b+cx))(5Bce-4bCe+cC(d+3ex))}{c^2e} - \frac{2(d+ex)\left(\frac{e^2\left(-8b^2Ce^2+ce(3bCd+10bBe+9aCe)+c^2\left(2Cd^2-5e(Bd+3Ae)\right)\right)(a+x(b+cx))}{(d+ex)^2} + \cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[d + e*x]*((2*(a + x*(b + c*x))*(5*B*c*e - 4*b*C*e + c*C*(d + 3*e*x))
)/(c^2*e) - (2*(d + e*x)*((e^2*(-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9
*a*C*e) + c^2*(2*C*d^2 - 5*e*(B*d + 3*A*e)))*(a + x*(b + c*x)))/(d + e*x)^
2 + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b
^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(
-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 -
b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]],
-((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c*e + 8*C*Sqrt[(b^2 -
 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 5*B*e) - 15*A*c*e^
2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d + 10*B*c*e + 9*C*Sqr
t[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 + 3*C*d*Sqrt[(b^2
- 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*
ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/
(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*
d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(c^3
*e^3)))/(15*Sqrt[a + x*(b + c*x)])
```

3.267.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2184, 27, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2\int -\frac{e\sqrt{d+ex}(bCd-5Ace+3aCe+(2cCd-5Bce+4bCe)x)}{2\sqrt{cx^2+bx+a}}dx}{5ce^2}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\int \frac{\sqrt{d+ex}(bCd-5Ace+3aCe+(2cCd-5Bce+4bCe)x)}{\sqrt{cx^2+bx+a}}dx}{5ce}$$

$\downarrow$ 1236

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\int -\frac{4Cdeb^2+4aCe^2b-cd(Cd+5Be)b+ce(15Acd-7aCd-5aBe)+\left(-\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2\right)-e(3bCd+10bBe+9aCe)c+8b^2Ce^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3c}+2\sqrt{d+ex}\sqrt{a+bx+}}{5ce}$$

$\downarrow$ 27

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\int \frac{4Cdeb^2+4aCe^2b-cd(Cd+5Be)b+ce(15Acd-7aCd-5aBe)+\left(-\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2\right)-e(3bCd+10bBe-}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3c}}{5ce}$$

$\downarrow$ 1269

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\frac{\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{\left(ae^2-bde+cd^2\right)(4bCe}{}}{3c}}{5ce}$$

$\downarrow$ 1172

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}$$

$\downarrow$ 321

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}$$

$\downarrow$ 327

3.267. $\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} - \frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\text{a}\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}$$

input
```
Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) - ((2*(2*c*C*d - 5*B*c
*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - ((Sqrt[2]*Sqrt[
b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C*
d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e + 4*b
*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*
Sqrt[a + b*x + c*x^2]))/(3*c))/(5*c*e)
```

3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.267. $\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

3.267.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2Cx\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{5c}+\frac{2\left(Be+Cd-\frac{2(2be+2cd)C}{5c}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ce}+\frac{2\left(dA-\frac{2a}{5}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input

```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output

```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*C/c*x
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(B*e+C*d-2/5/c*(2*b*e
+2*c*d)*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(d*A-2/5*
a/c*d*C-2/3*(B*e+C*d-2/5/c*(2*b*e+2*c*d)*C)/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x
+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A*e+B*d-2/5*C/c
*(3/2*a*e+3/2*b*d)-2/3*(B*e+C*d-2/5/c*(2*b*e+2*c*d)*C)/c/e*(b*e+c*d))*(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

3.267.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\left(2\,Cc^3d^3+(2\,Cbc^2-5\,Bc^3)d^2e+(7\,Cb^2c+30\,Ac^3-2\,(6\,Ca+5\,Bb)c^2)de^2-(8\,Cb^3+15\,(Ba+Ab)c\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/45*((2*C*c^3*d^3 + (2*C*b*c^2 - 5*B*c^3)*d^2*e + (7*C*b^2*c + 30*A*c^3 -
 2*(6*C*a + 5*B*b)*c^2)*d*e^2 - (8*C*b^3 + 15*(B*a + A*b)*c^2 - (21*C*a*b
+ 10*B*b^2)*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e +
 (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c
 - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d +
 b*e)/(c*e)) + 3*(2*C*c^3*d^2*e + (3*C*b*c^2 - 5*B*c^3)*d*e^2 - (8*C*b^2*c
 + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*sqrt(c*e)*weierstrassZeta(4/3*(c^
2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2
*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), wei
erstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4
/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*
b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*C*c^3*e^3*x
+ C*c^3*d*e^2 - (4*C*b*c^2 - 5*B*c^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(e*x
+ d))/(c^4*e^3)
```

3.267.6 Sympy [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

3.267.7 Maxima [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

3.267.8 Giac [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{d+e\,x}\,(C\,x^2+B\,x+A)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

3.268.1 Optimal result

Integrand size = 34, antiderivative size = 471

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2cd-\left(b+\sqrt{b^2-\cdots}\right.}\cdots}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)+c(2Cd^2-3e(Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\text{EllipticF}\left(\text{ar}\cdots\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)
*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2
)/c^2/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2)+2/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*EllipticF(1/2*((b+2
*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b
^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.268.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.18 (sec) , antiderivative size = 980, normalized size of antiderivative = 2.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\sqrt{d+ex}\left(4cCe^2(a+x(b+cx))+\frac{(d+ex)\left(-\frac{4e^2(2cCd-3Bce+2bCe)\sqrt{\frac{cd^2+e(-bd+ae)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}(a+x(b+cx))}{(d+ex)^2}+\frac{i\sqrt{2}(2cCd-3Bce+2bCe)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(4*c*C*e^2*(a + x*(b + c*x)) + ((d + e*x)*((-4*e^2*(2*c*C*d
 - 3*B*c*e + 2*b*C*e)*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt
[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 + (I*Sqrt[2]*(2*c*C*d
 - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e
^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x +
 b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[
(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^
2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x)
)]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] +
(I*Sqrt[2]*(2*b^2*C*e^2 - b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*
(6*A*c*e^2 - 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(-2*C*d + 3*B*e)))*Sqrt[(
-2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*c*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^
2]*x + b*e*(d - e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*Sqrt[(2*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*c*d*e*x + e*Sqrt[(b^2 - 4*a
*c)*e^2]*x + b*e*(-d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d
+ e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt
[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d ...
```

3.268.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2\int -\frac{e(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{(2bCe-3Bce+2cCd)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}$$

$\downarrow$ 1172

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\ldots}}{\ldots}}{3ce}$$

$\downarrow$ 321

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\ldots}}{\ldots}}{3ce}$$

$\downarrow$ 327

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx-\ldots\right)}{b^2-4\ldots}}}{\ldots}}{3ce}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

3.268. $\int\frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

output
```
(2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C
*e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e)
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.268.4 Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2C\sqrt{ce\,x^3+be\,x^2+cd\,x^2+aex+bdx+ad}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\cdots}{\cdots}}{-\frac{d}{e}-\frac{-b\cdots}{\cdots}}}\cdots}{\sqrt{ce\,x^3+be\cdots}}\cdots\right)}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*C/c/e
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(B-2/3*C/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

3.268.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}Cc^2e^2 + (2\,Cc^2d^2 + (Cbc-3\,Bc^2)de + (2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2)\sqrt{c}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b
*c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d*e +
 (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

3.268.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

3.268.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

3.268.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

3.269.1 Optimal result

Integrand size = 34, antiderivative size = 508

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)-c(2Cd^2-e(Bd-Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ce^2\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{}}{2cd-(\ldots}\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^
(1/2)-(C*e*(-a*e+b*d)-c*(2*C*d^2-e*(-A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*
x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*
(-B*e+2*C*d)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
)^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*
(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(e*x+d)^(1/2)/(c*
x^2+b*x+a)^(1/2)
```

3.269.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.43 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(-e^2(Cd^2+e(-Bd+Ae))\,(a+x(b+cx))+\frac{e^2(2cCd^2+Ce(-bd+ae)+ce(-B}{c}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2
+ C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(
d + e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/
((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) +
 2*a*c*C*d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e
*Sqrt[(b^2 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*
d + Sqrt[(b^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sq
rt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e +
 a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]
))]))/(Sqrt[2]*c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a +
 x*(b + c*x)])
```

3.269.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2181, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{2e\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c}{\cdots}}\cdots}{\cdots}}{e\left(ae^2-bde+c\cdots\right.}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 321

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c}{}}}{}}{e\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 327

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-}}{}}{e\left(ae^2-bde+\right.}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e
^2)*Sqrt[d + e*x]) + (-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(B*c*d + b*C*d - (2*c*C
*d^2)/e - A*c*e - a*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
+ b*x + c*x^2])) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d - B*e)*(c*d^2 - b*d
*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x
^2]))/(e*(c*d^2 - b*d*e + a*e^2))
```

3.269. $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.269. $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(456) = 912$.

Time = 4.17 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(-\frac{2\left(ce\,x^2+bex+ae\right)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(e^2a-bde+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex+ae\right)}}+\frac{2\left(\frac{Be-Cd}{e^2}-\frac{(be-cd)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(e^2a-bde+c\,d^2\right)}+\frac{b\left(A\,e^2-Bde+C\,d^2\right)}{e\left(e^2a-bde+c\,d^2\right)}\right)\left(\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/((x+d/e)*(c*e*x^
2+b*e*x+a*e))^(1/2)+2*((B*e-C*d)/e^2-(b*e-c*d)/e^2*(A*e^2-B*d*e+C*d^2)/(a*
e^2-b*d*e+c*d^2)+b/e/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2))*(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e+c/e*(A*e^2-B*d*e
+C*d^2)/(a*e^2-b*d*e+c*d^2))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+
a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x
+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+
b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(
(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2))))
```

3.269.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left((2\,Cc^2d^4-(2\,Cbc+Bc^2)d^3e-(Cb^2+2\,Ac^2-2\,(2\,Ca+Bb)c)d^2e^2+(Cab-(3\,Ba-Ab)c)de^3+(2\,C\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2/3*((2*C*c^2*d^4 - (2*C*b*c + B*c^2)*d^3*e - (C*b^2 + 2*A*c^2 - 2*(2*C*a
 + B*b)*c)*d^2*e^2 + (C*a*b - (3*B*a - A*b)*c)*d*e^3 + (2*C*c^2*d^3*e - (2
*C*b*c + B*c^2)*d^2*e^2 - (C*b^2 + 2*A*c^2 - 2*(2*C*a + B*b)*c)*d*e^3 + (C
*a*b - (3*B*a - A*b)*c)*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d^3*e - (C*b*c + B*c^2)*d^2*e^2 + (C
*a*c + A*c^2)*d*e^3 + (2*C*c^2*d^2*e^2 - (C*b*c + B*c^2)*d*e^3 + (C*a*c +
A*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(
3*c*e*x + c*d + b*e)/(c*e))) + 3*(C*c^2*d^2*e^2 - B*c^2*d*e^3 + A*c^2*e^4)
*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2
*d*e^5 + (c^3*d^2*e^4 - b*c^2*d*e^5 + a*c^2*e^6)*x)
```

3.269.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

3.269.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

3.269.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

3.270.1 Optimal result

Integrand size = 34, antiderivative size = 684

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$
$$+\frac{2(cd(2Cd^2+e(Bd-4Ae))+e(3ae(2Cd-Be)-b(4Cd^2-Bde-2Ae^2)))\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(cd(2Cd^2+e(Bd-4Ae))+e(3ae(2Cd-Be)-b(4Cd^2-Bde-2Ae^2)))\sqrt{d+ex}\sqrt{-\frac{c(a+}{b^2}}}{3e^2\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(3Ce(bd-ae)-c(2Cd^2+e(Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\text{arcs}\right.}{3ce^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(3/2)+2/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^
2-B*d*e+4*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/
2)-1/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d
*e+4*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a
*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*(3*C*e*(-a*e+b*d)-c*(2*C*d^2+e*(
-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(
c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2
)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.270.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.72 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{d+ex}(a+bx+cx^2)\left(-\frac{2(Cd^2-Bde+Ae^2)}{3e(cd^2-bde+ae^2)(d+ex)^2}-\frac{2(-2cCd^3-Bcd^2e+4bCd^2e-b}{3e(cd^2-}\right.}{\sqrt{a+x(b+cx)}}$$

$$+\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}\left(-\left((2cCd^3+cde(Bd-4Ae)-3ae^2(-2Cd+Be)+be(-4Cd^2+e(Bd+2A\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
 - b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e
- b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(
3/2)*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(
-2*C*d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2
+ (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1
- (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*
(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[
(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(
c*d*(2*C*d^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*
e*(-2*C*d + B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^
2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] -
 (2*a*c*C*d^2*e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*
a*c)*e^2] + B*c*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*
a*c)*e^2] - 3*a*B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^
2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b
*e*(2*A*e^2*(3*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[
(b^2 - 4*a*c)*e^2]) + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2...
```

3.270.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2181, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x}{2e(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{3\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1237

$$\frac{\frac{2\sqrt{a+bx+cx^2}\left(3ae^2(2Cd-Be)-be\left(4Cd^2-e(2Ae+Bd)\right)+cde(Bd-4Ae)+2cCd^3\right)}{\sqrt{d+ex}(ae^2-bde+cd^2)}-\frac{2\int -\frac{3b^2Ced^2-6abCe^2d-bc\left(Cd^2+e(2Bd+Ae)\right)d+Ace\left(3cd^2-ae^2\right.}{}}{}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{3b^2Ced^2-b\left(cCd^2+6aCe^2+ce(2Bd+Ae)\right)d+e\left(Ac\left(3cd^2-ae^2\right)+a\left(3aCe^2-cd(Cd-4Be)\right)\right)-c\left(2cCd^3+ce(Bd-4Ae)d+3ae^2(2Cd-Be)-be\left(4Cd^2-e(Bd+2Ae)\right)\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{ae^2-bde+cd^2}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1269

$$\frac{\frac{\frac{\left(ae^2-bde+cd^2\right)\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{c\left(3ae^2(2Cd-Be)-be\left(4Cd^2-e(2Ae+Bd)\right)+cde(Bd-4Ae)+2cCd^3\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}}{e}}{ae^2-bde+cd^2}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$\frac{\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(Bd-4Ae)d+3ae^2(2Cd-Be)-be\left(4Cd^2-e(Bd+2Ae)\right)\right)}{(cd^2-bed+ae^2)\sqrt{d+ex}}+\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\left(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae)\right)}{}}{}}{}-\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{3e\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$\downarrow$ 321

3.270. $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{\cdots}}{2cd-(\cdots}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 327

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{\cdots}}{2cd-(\cdots}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input

```
Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output

```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(3/2)) + ((2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2
*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/((c*
d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) + (-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C
*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d
 + 2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2
])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(2*c*C*d^2 - 3*
C*e*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
e*x]*Sqrt[a + b*x + c*x^2]))/(c*d^2 - b*d*e + a*e^2))/(3*e*(c*d^2 - b*d*e
+ a*e^2))
```

3.270. $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.270. $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.270.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(620) = 1240$.

Time = 3.61 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.82

method	result	size
elliptic	Expression too large to display	1248
default	Expression too large to display	20481

input
```
int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/e^3/
(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)/(x+d/e)^2+2/3*(c*e*x^2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)^2*
(2*A*b*e^3-4*A*c*d*e^2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2
*e+2*C*c*d^3)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(C/e^2-1/3*c/e^2*(A*e^
2-B*d*e+C*d^2)/(a*e^2-b*d*e+c*d^2)+1/3*(b*e-c*d)/e^2*(2*A*b*e^3-4*A*c*d*e^
2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*e^2-
b*d*e+c*d^2)^2-1/3*b/e/(a*e^2-b*d*e+c*d^2)^2*(2*A*b*e^3-4*A*c*d*e^2-3*B*a*
e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3))*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/3*c/e*(2*A*b*e^3-4*A*c*
d*e^2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*
e^2-b*d*e+c*d^2)^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b...
```

3.270.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1305, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/9*((2*C*c^2*d^6 - (5*C*b*c - B*c^2)*d^5*e + (5*C*b^2 + 5*A*c^2 + (3*C*a
- 4*B*b)*c)*d^4*e^2 - (12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d^3*e^3 + (9*
C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d^2*e^4 + (2*C*c^2*d^4*e^2 - (5*C*b*c
 - B*c^2)*d^3*e^3 + (5*C*b^2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^2*e^4 - (12*
C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d*e^5 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 -
3*A*a*c)*e^6)*x^2 + 2*(2*C*c^2*d^5*e - (5*C*b*c - B*c^2)*d^4*e^2 + (5*C*b^
2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^3*e^3 - (12*C*a*b - B*b^2 - (9*B*a - 5*
A*b)*c)*d^2*e^4 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d*e^5)*x)*sqrt(c
*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e
^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3
 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*
d^5*e - (3*B*a - 2*A*b)*c*d^2*e^4 - (4*C*b*c - B*c^2)*d^4*e^2 - (4*A*c^2 -
 (6*C*a + B*b)*c)*d^3*e^3 + (2*C*c^2*d^3*e^3 - (3*B*a - 2*A*b)*c*e^6 - (4*
C*b*c - B*c^2)*d^2*e^4 - (4*A*c^2 - (6*C*a + B*b)*c)*d*e^5)*x^2 + 2*(2*C*c
^2*d^4*e^2 - (3*B*a - 2*A*b)*c*d*e^5 - (4*C*b*c - B*c^2)*d^3*e^3 - (4*A*c^
2 - (6*C*a + B*b)*c)*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*...
```

3.270.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

3.270.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

3.270.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{5/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

3.271.1 Optimal result

Integrand size = 34, antiderivative size = 944

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$
$$+\frac{2(cd(2Cd^2+e(3Bd-8Ae))+e(5ae(2Cd-Be)-b(6Cd^2-Bde-4Ae^2)))\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}$$
$$+\frac{2(c^2d^2(2Cd^2+e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae^2))-ce(bd(7C\ldots}{15e\left(cd^2-bde+ae^2\right)^3\sqrt{d+ex}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(c^2d^2(2Cd^2+e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae^2\ldots}{15e^2\left(cd^2-\ldots\right.}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd(2Cd^2+e(3Bd-8Ae))+e(5ae(2Cd-Be)-b(6Cd^2-Bde-4Ae^2)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4\ldots\right.}}}}{15e^2\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}\sqrt{a+\ldots}}$$

output
```
-2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(5/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A
*e^2-B*d*e+6*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(3/2)+2/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e
*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*
d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+
c*d^2)^3/(e*x+d)^(1/2)-1/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a
^2*C*e^2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A
*e^2-7*B*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*EllipticE(1/2*((b+
2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+
b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+
c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e
^2-B*d*e+6*C*d^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(a*e^2-b*d*e
+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

3.271.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.76 (sec) , antiderivative size = 1746, normalized size of antiderivative = 1.85

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(5*e*(c*d^2
 - b*d*e + a*e^2)*(d + e*x)^3) - (2*(-2*c*C*d^3 - 3*B*c*d^2*e + 6*b*C*d^2*
e - b*B*d*e^2 + 8*A*c*d*e^2 - 10*a*C*d*e^2 - 4*A*b*e^3 + 5*a*B*e^3))/(15*e
*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) - (2*(-2*c^2*C*d^4 - 3*B*c^2*d^3*e
 + 7*b*c*C*d^3*e - 7*b*B*c*d^2*e^2 + 23*A*c^2*d^2*e^2 + 3*b^2*C*d^2*e^2 -
19*a*c*C*d^2*e^2 + 2*b^2*B*d*e^3 - 23*A*b*c*d*e^3 + 29*a*B*c*d*e^3 - 10*a*
b*C*d*e^3 + 8*A*b^2*e^4 - 10*a*b*B*e^4 - 9*a*A*c*e^4 + 15*a^2*C*e^4))/(15*
e*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e
*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-((c^2*(2*C*d^4 + d^2*e*(3*B*d - 23*A*e))
 - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e + 8*A
*e^2)) + c*e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 + 7*B*d*
e + 23*A*e^2)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/
(d + e*x)))/(d + e*x))) - ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/(
(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e
*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((
2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c^2*(-2*C*d^4 + d^2*e*(-3*B*d + 23
*A*e)) + e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*B*d*e
 + 8*A*e^2)) - c*e*(a*e*(19*C*d^2 - 29*B*d*e + 9*A*e^2) + b*d*(-7*C*d^2 +
7*B*d*e + 23*A*e^2)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a
*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*...
```

3.271.3 Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 986, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2181, 27, 1237, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+3Bcd-5bCd-3Ace+5aCe\right)x}{2e(d+ex)^{5/2}\sqrt{cx^2+bx+a}}dx}{5\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+3Bcd-5bCd-3Ace+5aCe\right)x}{(d+ex)^{5/2}\sqrt{cx^2+bx+a}}dx}{5e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1237

$$\frac{\frac{2\sqrt{a+bx+cx^2}\left(5ae^2(2Cd-Be)-be\left(6Cd^2-e(4Ae+Bd)\right)+cde(3Bd-8Ae)+2cCd^3\right)}{3(d+ex)^{3/2}(ae^2-bde+cd^2)}-\frac{2\int -\frac{e\left(3Cd^2+2e(Bd+4Ae)\right)b^2+\left(cCd^3-ce(6Bd+19Ae)d-10ae^2\right.}{}}{}}{5e\left(ae^2-bde+cd^2\right.}$$
$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{e\left(3Cd^2+2e(Bd+4Ae)\right)b^2+\left(cCd^3-ce(6Bd+19Ae)d-10ae^2(Cd+Be)\right)b+3e\left(Ac\left(5cd^2-3ae^2\right)+a\left(5aCe^2-cd(3Cd-8Be)\right)\right)+c\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd\right.}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}}{3(ae^2-bde+cd^2)}}{5e\left(ae^2-bde+cd^2\right)}$$
$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1237

$$\frac{2\sqrt{a+bx+cx^2}\left(-e^2\left(15a^2Ce^2-10abe(Be+Cd)+b^2\left(8Ae^2+2Bde+3Cd^2\right)\right)-ce\left(bd\left(-23Ae^2-7Bde+7Cd^2\right)-ae\left(9Ae^2-29Bde+19Cd^2\right)\right)+c^2\left(d^2e(3Bd-23Ae)+2Cd\right.\right.}{\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$
$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\frac{c\int \frac{de\left(9Cd^2+e(Bd+4Ae)\right)b^2-\left(cCd^4+26aCe^2d^2+ce(9Bd+11Ae)d^2+4ae^3(Bd-Ae)\right)b+e\left(Acd\left(15cd^2-17ae^2\right)-a\left(cd^2(7Cd-27Be)-5ae^2(5Cd-Be)\right)\right)-\left(\left(2Cd^4+\right.\right.}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{ae^2-bde+cd^2}$$
$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1269

3.271. $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

$$c\left(\frac{\left(ae^2-bde+cd^2\right)\left(5ae^2(2Cd-Be)-be\left(6Cd^2-e(4Ae+Bd)\right)+cde(3Bd-8Ae)+2cCd^3\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(-e^2\left(15a^2Ce^2-10abe(Be+Cd)+b^2\left(8Ae^2+2B\right.\right.\right.}{ae^2-bde+cd^2}\right.$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be\left(6Cd^2-e(Bd+4Ae)\right)\right)}{3(cd^2-bed+ae^2)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-\right.\right.\right.}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$\downarrow$ 321

$$\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be\left(6Cd^2-e(Bd+4Ae)\right)\right)}{3(cd^2-bed+ae^2)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-\right.\right.\right.}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$\downarrow$ 327

3.271. $\int\frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\frac{2\sqrt{cx^2+bx+a}(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))}{3(cd^2-bed+ae^2)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-\right.\right.\right.}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(5/2)) + ((2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*
(2*C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(3
*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + ((2*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 +
2*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*
d^2 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2
)*Sqrt[d + e*x]) + (c*(-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*
(3*B*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^
2 + 2*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(1
9*C*d^2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6
*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*...
```

3.271. $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.271.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. 2(874) = 1748.

Time = 4.39 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1757
default	Expression too large to display	46695

input
```
int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5/e^4/
(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)/(x+d/e)^3+2/15*(4*A*b*e^3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*
B*c*d^2*e+10*C*a*d*e^2-6*C*b*d^2*e+2*C*c*d^3)/e^3/(a*e^2-b*d*e+c*d^2)^2*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/15*(c*e*x^2+b*e*
x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^3-2
3*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e^2+
3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*e^2
-7*C*b*c*d^3*e+2*C*c^2*d^4)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(1/15*c*
(4*A*b*e^3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*B*c*d^2*e+10*C*a*d*e^2-6*C*b*
d^2*e+2*C*c*d^3)/e^2/(a*e^2-b*d*e+c*d^2)^2+1/15*(b*e-c*d)/e^2*(9*A*a*c*e^4
-8*A*b^2*e^4+23*A*b*c*d*e^3-23*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2
*B*b^2*d*e^3+7*B*b*c*d^2*e^2+3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*
C*a*c*d^2*e^2-3*C*b^2*d^2*e^2-7*C*b*c*d^3*e+2*C*c^2*d^4)/(a*e^2-b*d*e+c*d^
2)^3-1/15*b/e/(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^
3-23*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e
^2+3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*
e^2-7*C*b*c*d^3*e+2*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2...
```

3.271.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 2656, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/45*((2*C*c^3*d^8 - (8*C*b*c^2 - 3*B*c^3)*d^7*e + (17*C*b^2*c + 22*A*c^3
- (2*C*a + 17*B*b)*c^2)*d^6*e^2 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C
*a*b - 8*B*b^2)*c)*d^5*e^3 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^
2 - 31*B*a*b + 27*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 +
 3*(5*B*a^2 - 7*A*a*b)*c)*d^3*e^5 + (2*C*c^3*d^5*e^3 - (8*C*b*c^2 - 3*B*c^
3)*d^4*e^4 + (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^3*e^5 - (3*C
*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^2*e^6 + (10*C*a*b
^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d*e^7 - (1
5*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*e^8)*x^3 + 3*(
2*C*c^3*d^6*e^2 - (8*C*b*c^2 - 3*B*c^3)*d^5*e^3 + (17*C*b^2*c + 22*A*c^3 -
 (2*C*a + 17*B*b)*c^2)*d^4*e^4 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*
a*b - 8*B*b^2)*c)*d^3*e^5 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2
 - 31*B*a*b + 27*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 +
3*(5*B*a^2 - 7*A*a*b)*c)*d*e^7)*x^2 + 3*(2*C*c^3*d^7*e - (8*C*b*c^2 - 3*B*
c^3)*d^6*e^2 + (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^5*e^3 - (3
*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^4*e^4 + (10*C*a
*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d^3*e^5
- (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*d^2*e^6)*x
)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e...
```

3.271.6 Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

3.271.7 Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

3.271.8 Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{7/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

3.272.1 Optimal result

Integrand size = 30, antiderivative size = 510

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)}$$
$$+ \frac{(fh(bg-ah)(1+m)+c(2fg^2(1+p)-h(eg-dh)(3+m+2p)))(g+hx)^{1+m}\left(a+bx+cx^2\right)^p \left(1-\right.}{ch^3(1}$$
$$- \frac{(bfh(2+m+p)+c(2fg(1+p)-eh(3+m+2p)))(g+hx)^{2+m}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)}\right.}{ch^3(2+m)(3}$$

output
```
f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+c*
(2*f*g^2*(p+1)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appe
llF1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g
)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/
(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b
^2)^(1/2))))^p)-(b*f*h*(2+m+p)+c*(2*f*g*(p+1)-e*h*(3+m+2*p)))*(h*x+g)^(2+m
)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b
^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(2+m)/(3+m
+2*p)/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)
/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p)
```

3.272.2 Mathematica [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$$
$$= \int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$$

input
```
Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

3.272.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2184, 25, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right) (g+hx)^m \left(a+bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -h(g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p+}{ch^2(m+2p+3)}$$
$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-$$
$$\frac{\int h(g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p+3)}{ch^2(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\int (g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p+3))x}{ch(m+2p+3)}$$

$\big\downarrow$ 1269

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\int (g+hx)^{m+1}\left(cx^2+bx+a\right)^p dx}{h}-\frac{(fh(m+1)(bg-ah)-ch(m+2p+3)(eg-dh)+2cfg^2(p+1))\int (g+hx}{h}}{ch(m+2p+3)}$$

$\big\downarrow$ 1179

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{(a+bx+cx^2)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\int (g+hx)^{m+1}\left(1-\frac{2c(g}{2cg-(b-\sqrt{}}\right.}{h^2}}{}$$

$\big\downarrow$ 150

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{(g+hx)^{m+2}(a+bx+cx^2)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\operatorname{AppellF1}\left(m+2\right.}{h^2(m+2)}}{}$$

input
```
Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - (-((
(f*h*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*
p))*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (
2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g
 - (b + Sqrt[b^2 - 4*a*c])*h)])/(h^2*(1 + m)*(1 - (2*c*(g + h*x))/(2*c*g -
 (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h))^p)) + ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3 + m
+ 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 +
m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2
*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(h^2*(2 + m)*(1 - (2*c*(g + h*x))/(2*c
*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt
[b^2 - 4*a*c])*h))^p))/(c*h*(3 + m + 2*p))
```

3.272. $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c)))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.272.4 Maple [F]

$$\int (hx+g)^m \left(c\,x^2+bx+a\right)^p \left(f\,x^2+ex+d\right) dx$$

input
```
int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

3.272.5 Fricas [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m \, dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \text{Timed out}$$

input
```
integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

3.272.8 Giac [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int (g+h\,x)^m \left(c\,x^2+b\,x+a\right)^p \left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

3.273 $\int (g+hx)^m \sqrt{a+bx+cx^2} (d+ex+fx^2) \, dx$

3.273.1 Optimal result

Integrand size = 32, antiderivative size = 496

$$\int (g+hx)^m \sqrt{a+bx+cx^2} (d+ex+fx^2) \, dx = \frac{f(g+hx)^{1+m} (a+bx+cx^2)^{3/2}}{ch(4+m)}$$
$$+ \frac{(fh(bg-ah)(1+m)+c(3fg^2-h(eg-dh)(4+m)))(g+hx)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,-\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+h}{2cg-\left(b+\sqrt{b^2}\right.}}}$$
$$- \frac{(bfh(5+2m)+c(6fg-2eh(4+m)))(g+hx)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\frac{}{2c}\right.}{2ch^3(2+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}}$$

output
```
f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(f*h*(-a*h+b*g)*(1+m)+c*(3*f
*g^2-h*(-d*h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(h*
x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)
^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-h*(b
-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)-1/2*(b*f*h*(5+2*m)+c*(6*f*g-2*e*h*(4+m)))*(h*x+g)^(2+m)*AppellF1
(2+m,-1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g
)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(2+m)/(4+m)/
(1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c
*g-h*(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

3.273.2 Mathematica [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

3.273.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 492, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2184, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)(g+hx)^m\,dx$$

$$\Big\downarrow \ 2184$$

$$\frac{\int -\frac{1}{2}h(g+hx)^m(3bfg+2afh(m+1)-2cdh(m+4)+(6cfg-2ceh(m+4)+bfh(2m+5))x)\sqrt{cx^2+bx+a}dx}{ch^2(m+4)}$$
$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

$$\Big\downarrow \ 27$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\int (g+hx)^m(3bfg+2afh(m+1)-2cdh(m+4)+(6cfg-2ceh(m+4)+bfh(2m+5))x)\sqrt{cx^2+bx+a}dx}{2ch(m+4)}$$

$$\Big\downarrow \ 1269$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{(bfh(2m+5)-2ceh(m+4)+6cfg)\int (g+hx)^{m+1}\sqrt{cx^2+bx+a}dx}{h}-\frac{2\left(fh(m+1)(bg-ah)-ch(m+4)(eg-dh)+3cfg^2\right)\int (g+hx)^m\sqrt{cx^2+bx+a}dx}{h}}{2ch(m+4)}$$

$$\Big\downarrow \text{1179}$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{\sqrt{a+bx+cx^2}(bfh(2m+5)-2ceh(m+4)+6cfg)\int (g+hx)^{m+1}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}d(g+hx)}{h^2\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+bx+cx^2}(fh(m+}{}}{2ch(m+4)}$$

$$\Big\downarrow \text{150}$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+2}(bfh(2m+5)-2ceh(m+4)+6cfg)\operatorname{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)}{h^2(m+2)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+bx}}{}}{2ch(m+}$$

input
```
Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) - ((-2*(3*c*f*
g^2 + f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)
*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^
2 - 4*a*c])*h)])/(h^2*(1 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) + ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*
Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(
2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h)])/(h^2*(2 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b
^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])
*h)]))/(2*c*h*(4 + m))
```

3.273. $\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

3.273.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.273.4 Maple [F]

$$\int (hx+g)^m \left(f\,x^2+ex+d\right)\sqrt{c\,x^2+bx+a}dx$$

input
```
int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

3.273.5 Fricas [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.273.6 Sympy [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int (g+hx)^m\,\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

input
```
integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

3.273.7 Maxima [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.273.8 Giac [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+h\,x)^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

3.274 $\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$

3.274.1 Optimal result

Integrand size = 34, antiderivative size = 590

$$\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx$$
$$= -\frac{(fg^2-h(eg-dh))(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}$$
$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)^{-p}\text{AppellF1}\left(-2p,-p\right.}{2h^3p}$$
$$-\frac{(2c(fg^3-dgh^2)+h(2ah(2fg-eh)-b(3fg^2-egh-dh^2)))\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cg-\left(b-\sqrt{b^2-4ac}\right.\right.}{\left(2cg-\left(b+\sqrt{b^2-4ac}\right.\right.}\right.}{2h^2\left(2cg-\left(b-\right.\right.}$$

output
```
-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(p+1)/h/(a*h^2-b*g*h+c*g^2)/(p+1)/
((h*x+g)^(2+2*p))-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^
2-e*g*h+3*f*g^2)))*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p]
,[-2*p],-4*c*(h*x+g)*(-4*a*c+b^2)^(1/2)/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*
g-h*(b+(-4*a*c+b^2)^(1/2))))*(b+2*c*x-(-4*a*c+b^2)^(1/2))/h^2/(a*h^2-b*g*h
+c*g^2)/(1+2*p)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2)))/(((2*c*g-h*(b-(-4*a*c+b^2
)^(1/2)))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g
-h*(b+(-4*a*c+b^2)^(1/2))))^p)-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1
-2*p,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+
(-4*a*c+b^2)^(1/2))))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-
4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p
)
```

3.274.2 Mathematica [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

3.274.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2186, 25, 1179, 150, 1228, 1155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)(g+hx)^{-2p-3}\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{2186}$$

$$\frac{\int -(g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}+\frac{f\int (g+hx)^{-2p-1}\left(cx^2+bx+a\right)^p dx}{h^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{f\int (g+hx)^{-2p-1}\left(cx^2+bx+a\right)^p dx}{h^2}-\frac{\int (g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}$$

$$\Big\downarrow \text{1179}$$

$$\frac{f\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\int(g+hx)^{-2p-1}\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}\right.}{h^3}$$

$$\frac{\int(g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}$$

$\downarrow$ 150

$$-\frac{\int(g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}-$$

$$\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-\right.}{2h^3p}$$

$\downarrow$ 1228

$$-\frac{\frac{\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int(g+hx)^{-2(p+1)}\left(cx^2+bx+a\right)^p dx}{2\left(ah^2-bgh+cg^2\right)}+\frac{h(g+hx)^{-2(p+1)}\left(a+bx+cx^2\right)^{p+1}\left(fg^2-h(eg-\right.}{2(p+1)\left(ah^2-bgh+cg^2\right)}}{h^2}$$

$$\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1-\right.}{2h^3p}$$

$\downarrow$ 1155

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,\right.}{2h^3p}$$

$$\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(g+hx)^{-2p-1}\left(a+bx+cx^2\right)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg\right.\right.}{2(2p+1)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

3.274. $\int(g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

output
```
-1/2*(f*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*(g + h*x)
)/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[
b^2 - 4*a*c])*h)])/(h^3*p*(g + h*x)^(2*p)*(1 - (2*c*(g + h*x))/(2*c*g - (b
 - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h))^p) - ((h*(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2
*(c*g^2 - b*g*h + a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) + ((2*c*(f*g^3 - d
*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt
[b^2 - 4*a*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeome
tric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (
b + Sqrt[b^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*g - (
b - Sqrt[b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sq
rt[b^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p))/h^2
```

3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1155
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(b - q + 2*c*x))*(d + e*x)^
(m + 1)*((a + b*x + c*x^2)^p/((m + 1)*(2*c*d - b*e + e*q)*((2*c*d - b*e + e
*q)*((b + q + 2*c*x)/((2*c*d - b*e - e*q)*(b - q + 2*c*x))))^p))*Hypergeome
tric2F1[m + 1, -p, m + 2, -4*c*q*((d + e*x)/((2*c*d - b*e - e*q)*(b - q + 2
*c*x)))], x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[m + 2*p + 2, 0]
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

3.274. $\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2186
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Simp[Coeff[Pq, x, q]/e^q   Int[(
d + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Simp[1/e^q   Int[(d + e*x)^m
*(a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x],
x], x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b,
 c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.274.4 Maple [F]

$$\int (hx+g)^{-3-2p} \left(c\,x^2+bx+a\right)^p \left(f\,x^2+ex+d\right) dx$$

input
```
int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

3.274.5 Fricas [F]

$$\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx$$
$$= \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fri
cas")
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.274.7 Maxima [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.274.8 Giac [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx=\int \frac{\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^{2\,p+3}}\,dx$$

input
```
int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

output
```
int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

3.275 $\int (d+fx^2)^p (2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x$

3.275.1 Optimal result

Integrand size = 42, antiderivative size = 41

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = \frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p}+2cfx\left(d+fx^2\right)^{1+p}$$

output
```
b*f*(3+2*p)*(f*x^2+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+d)^(p+1)
```

3.275.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.90

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = \frac{f(d+fx^2)^p\left(1+\frac{fx^2}{d}\right)^{-p}\left(6cd(1+p)x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{fx^2}{d}\right)+(3+2p)\left(3b(d+fx^2)\left(1\right.\right.\right.}{3(1+p)}$$

input
```
Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)
*x^2),x]
```

output
```
(f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2
)/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*H
ypergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)
^p)
```

3.275.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2346, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+fx^2\right)^p \left(2bf^2(2p+3)x+2cdf+2cf^2(2p+3)x^2\right) \, dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int 2bf^3(2p+3)^2x\left(fx^2+d\right)^p dx}{f(2p+3)}+2cfx\left(d+fx^2\right)^{p+1}$$

$$\Big\downarrow \text{27}$$

$$2bf^2(2p+3)\int x\left(fx^2+d\right)^p dx+2cfx\left(d+fx^2\right)^{p+1}$$

$$\Big\downarrow \text{241}$$

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

input
```
Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),
x]
```

output
```
(b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.275.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{f\left(f\,x^{2}+d\right)^{1+p}\left(2pcx+2bp+2cx+3b\right)}{1+p}$
risch	$\frac{f\left(2cfp\,x^{3}+2bfp\,x^{2}+2cf\,x^{3}+3bf\,x^{2}+2cdpx+2bdp+2cdx+3bd\right)\left(f\,x^{2}+d\right)^{p}}{1+p}$
norman	$\frac{bdf(3+2p)\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}}{1+p}+\frac{b\,f^{2}(3+2p)x^{2}\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}}{1+p}+2c\,f^{2}x^{3}\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}+2cdfx\,\mathrm{e}^{p\ln\left(f\,x^{2}+d\right)}$
parallelrisch	$\frac{2x^{3}\left(f\,x^{2}+d\right)^{p}c\,f^{3}p+2x^{3}\left(f\,x^{2}+d\right)^{p}c\,f^{3}+2x^{2}\left(f\,x^{2}+d\right)^{p}b\,f^{3}p+3x^{2}\left(f\,x^{2}+d\right)^{p}b\,f^{3}+2x\left(f\,x^{2}+d\right)^{p}cd\,f^{2}p+2x\left(f\,x^{2}+d\right)^{p}cd\,f^{2}+2}{f(1+p)}$

input
```
int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_
RETURNVERBOSE)
```

output
```
f/(1+p)*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{(2\,bdfp+2\,(cf^2p+cf^2)x^3+3\,bdf+(2\,bf^2p+3\,bf^2)x^2+2\,(cdfp+cdf)x)(fx^2+d)^p}{p+1}$$

input
```
integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, a
lgorithm="fricas")
```

output
```
(2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
 + 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(37) = 74$.

Time = 3.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 5.15

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\begin{cases}\frac{2bdfp(d+fx^2)^p}{p+1}+\frac{3bdf(d+fx^2)^p}{p+1}+\frac{2bf^2px^2(d+fx^2)^p}{p+1}+\frac{3bf^2x^2(d+fx^2)^p}{p+1}+\frac{2cdfpx(d+fx^2)^p}{p+1}+\frac{2cdfx(d+fx^2)^p}{p+1}+\frac{2cf^2px^3(d+f}{p+1} \\ bf\log\left(x-\sqrt{-\frac{d}{f}}\right)+bf\log\left(x+\sqrt{-\frac{d}{f}}\right)+2cfx\end{cases}$$

input
```
integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2)
,x)
```

output
```
Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p
+ 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2
)**p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2
)**p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d
+ f*x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(x - sqrt(-d/f)) + b*f*log(x + s
qrt(-d/f)) + 2*c*f*x, True))
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) \, dx = \frac{(2\,cf^2(p+1)x^3+bf^2(2\,p+3)x^2+2\,cdf(p+1)x+bdf(2\,p+3))(fx^2+d)^p}{p+1}$$

input
```
integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, a
lgorithm="maxima")
```

output
```
(2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*
p + 3))*(f*x^2 + d)^p/(p + 1)
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(41) = 82.

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.44

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) \, dx = \frac{2\,(fx^2+d)^p cf^2px^3+2\,(fx^2+d)^p bf^2px^2+2\,(fx^2+d)^p cf^2x^3+2\,(fx^2+d)^p cdfpx+3\,(fx^2+d)^p bf^2x^2 -}{p+1}$$

input
```
integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, a
lgorithm="giac")
```

output
```
(2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)
^p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(
f*x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p
 + 1)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int (d+fx^2)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\left(f\,x^2+d\right)^p\left(2\,c\,f^2\,x^3+2\,c\,d\,f\,x+\frac{b\,f^2\,x^2\,\left(2\,p+3\right)}{p+1}+\frac{b\,d\,f\,\left(2\,p+3\right)}{p+1}\right)$$

input
```
int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),
x)
```

output
```
(d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (
b*d*f*(2*p + 3))/(p + 1))
```

3.276 $\int (d+ex+fx^2)^p (-2ce^2+2cdf-ce^2p+2cf^2(3+$

3.276.1 Optimal result

Integrand size = 46, antiderivative size = 46

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right) dx = -\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p} + 2cfx\left(d+ex+fx^2\right)^{1+p}$$

output
```
-c*e*(2+p)*(f*x^2+e*x+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+e*x+d)^(p+1)
```

3.276.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right) dx = \frac{c(-e(2+p)+2f(1+p)x)(d+x(e+fx))^{1+p}}{1+p}$$

input
```
Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
 2*p)*x^2),x]
```

output
```
(c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)
```

3.276.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2192, 25, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)^p \left(2cdf-ce^2p-2ce^2+2cf^2(2p+3)x^2\right) \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{\int -cef(p+2)(2p+3)(e+2fx)\left(fx^2+ex+d\right)^p dx}{f(2p+3)}+2cfx\left(d+ex+fx^2\right)^{p+1}$$

$$\Big\downarrow \text{ 25}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{\int cef(p+2)(2p+3)(e+2fx)\left(fx^2+ex+d\right)^p dx}{f(2p+3)}$$

$$\Big\downarrow \text{ 27}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-ce(p+2)\int (e+2fx)\left(fx^2+ex+d\right)^p dx$$

$$\Big\downarrow \text{ 1104}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

input
```
Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

output
```
-((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*
x^2)^(1 + p)
```

3.276.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.276.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{c\left(f\,x^{2}+ex+d\right)^{1+p}\left(-2fpx+ep-2fx+2e\right)}{1+p}$
risch	$-\frac{c\left(-2p\,f^{2}x^{3}-efp\,x^{2}-2f^{2}x^{3}-2dfpx+e^{2}px+dep-2dfx+2e^{2}x+2de\right)\left(f\,x^{2}+ex+d\right)^{p}}{1+p}$
norman	$\frac{c\left(2dfp-e^{2}p+2df-2e^{2}\right)x\,\mathrm{e}^{p\ln\left(f\,x^{2}+ex+d\right)}}{1+p}+\frac{cefp\,x^{2}\mathrm{e}^{p\ln\left(f\,x^{2}+ex+d\right)}}{1+p}+2c\,f^{2}x^{3}\mathrm{e}^{p\ln\left(f\,x^{2}+ex+d\right)}-\frac{cde(2+p)\mathrm{e}^{p\ln\left(f\,x \cdots\right.}}{1+p}$
parallelrisch	$\frac{2x^{3}\left(f\,x^{2}+ex+d\right)^{p}cd\,f^{2}p+2x^{3}\left(f\,x^{2}+ex+d\right)^{p}cd\,f^{2}+x^{2}\left(f\,x^{2}+ex+d\right)^{p}cdefp+2x\left(f\,x^{2}+ex+d\right)^{p}c\,d^{2}fp-x\left(f\,x^{2}+ex+d\right)^{p}cd\,e^{2}p \cdots}{(1+p)d}$

input
```
int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,metho
d=_RETURNVERBOSE)
```

output
```
-c/(1+p)*(f*x^2+e*x+d)^(1+p)*(-2*f*p*x+e*p-2*f*x+2*e)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int (d+ex+fx^2)^p\left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{(cefpx^2-cdep+2\,(cf^2p+cf^2)x^3-2\,cde-(2\,ce^2-2\,cdf+(ce^2-2\,cdf)p)x)(fx^2+ex+d)^p}{p+1}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x
, algorithm="fricas")
```

output
```
(c*e*f*p*x^2 - c*d*e*p + 2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e - (2*c*e^2 - 2*
c*d*f + (c*e^2 - 2*c*d*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

3.276.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(42) = 84.

Time = 53.17 (sec) , antiderivative size = 280, normalized size of antiderivative = 6.09

$$\int (d+ex+fx^2)^p\left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx$$
$$=\begin{cases}-\frac{cdep(d+ex+fx^2)^p}{p+1}-\frac{2cde(d+ex+fx^2)^p}{p+1}+\frac{2cdfpx(d+ex+fx^2)^p}{p+1}+\frac{2cdfx(d+ex+fx^2)^p}{p+1}-\frac{ce^2px(d+ex+fx^2)^p}{p+1}-\frac{2ce^2x(d+ex+f}{p+1}\\ -ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)+2cfx\end{cases}$$

input
```
integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x
**2),x)
```

output
```
Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d
 + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2
*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)*
*p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3
*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-
4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) +
2*c*f*x, True))
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \frac{(2\,cf^2(p+1)x^3+cefpx^2-cde(p+2)-(e^2(p+2)-2\,df(p+1))cx)(fx^2+ex+d)^p}{p+1}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x
, algorithm="maxima")
```

output
```
(2*c*f^2*(p + 1)*x^3 + c*e*f*p*x^2 - c*d*e*(p + 2) - (e^2*(p + 2) - 2*d*f*
(p + 1))*c*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

3.276.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(46) = 92.

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.93

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = \frac{2\,(fx^2+ex+d)^p cf^2px^3+(fx^2+ex+d)^p cefpx^2+2\,(fx^2+ex+d)^p cf^2x^3-(fx^2+ex+d)^p ce^2px+2}{}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x
, algorithm="giac")
```

output
```
(2*(f*x^2 + e*x + d)^p*c*f^2*p*x^3 + (f*x^2 + e*x + d)^p*c*e*f*p*x^2 + 2*(
f*x^2 + e*x + d)^p*c*f^2*x^3 - (f*x^2 + e*x + d)^p*c*e^2*p*x + 2*(f*x^2 +
e*x + d)^p*c*d*f*p*x - (f*x^2 + e*x + d)^p*c*d*e*p - 2*(f*x^2 + e*x + d)^p
*c*e^2*x + 2*(f*x^2 + e*x + d)^p*c*d*f*x - 2*(f*x^2 + e*x + d)^p*c*d*e)/(p
 + 1)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 13.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.70

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right) dx$$
$$= \left(f\,x^2+e\,x+d\right)^p \left(2\,c\,f^2\,x^3+\frac{c\,x\,\left(2\,d\,f-e^2\,p-2\,e^2+2\,d\,f\,p\right)}{p+1}-\frac{c\,d\,e\,(p+2)}{p+1}+\frac{c\,e\,f\,p\,x^2}{p+1}\right)$$

input
```
int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p +
 3)),x)
```

output
```
(d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p))
/(p + 1) - (c*d*e*(p + 2))/(p + 1) + (c*e*f*p*x^2)/(p + 1))
```

3.277 $\int (d+ex+fx^2)^p (-2ce^2+2cdf+3bef-ce^2p+2$

3.277.1 Optimal result

Integrand size = 69, antiderivative size = 57

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = -\frac{(ce(2+p)-bf(3+2p))\,(d+ex+fx^2)^{1+p}}{1+p}+2cfx\left(d+ex+fx^2\right)^{1+p}$$

output
```
-(c*e*(2+p)-b*f*(3+2*p))*(f*x^2+e*x+d)^(p+1)/(p+1)+2*c*f*x*(f*x^2+e*x+d)^(
p+1)
```

3.277.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int (d+ex+fx^2)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = \frac{(-ce(2+p)+bf(3+2p)+2cf(1+p)x)(d+x(e+fx))^{1+p}}{1+p}$$

input
```
Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*
b*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

output
```
((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
 p))/(1 + p)
```

3.277.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {2192, 25, 27, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex+fx^2)^p \left(2befp+3bef+2bf^2(2p+3)x+2cdf-ce^2p-2ce^2+2cf^2(2p+3)x^2\right)\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int -f(2p+3)(ce(p+2)-bf(2p+3))(e+2fx)\left(fx^2+ex+d\right)^p dx}{f(2p+3)}+2cfx\left(d+ex+fx^2\right)^{p+1}$$

$$\Big\downarrow \text{25}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{\int f(2p+3)(ce(p+2)-bf(2p+3))(e+2fx)\left(fx^2+ex+d\right)^p dx}{f(2p+3)}$$

$$\Big\downarrow \text{27}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-(ce(p+2)-bf(2p+3))\int (e+2fx)\left(fx^2+ex+d\right)^p dx$$

$$\Big\downarrow \text{1104}$$

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

input
```
Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*
p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

output
```
-(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c
*f*x*(d + e*x + f*x^2)^(1 + p)
```

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.277.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{(f\,x^2+ex+d)^{1+p}(2cfxp+2bfp-cep+2cfx+3bf-2ce)}{1+p}$
risch	$\frac{(2pc\,f^2x^3+2b\,f^2p\,x^2+cefp\,x^2+2c\,f^2x^3+2befpx+3b\,f^2x^2+2cdfpx-c\,e^2px+2bdfp+3befx-cdep+2cdfx-2c\,e^2x+3bdf-2cde)}{1+p}$
norman	$\frac{d(2bfp-cep+3bf-2ce)e^{p\ln\left(f\,x^2+ex+d\right)}}{1+p}+\frac{(2befp+2cdfp-c\,e^2p+3bef+2cdf-2c\,e^2)x\,e^{p\ln\left(f\,x^2+ex+d\right)}}{1+p}+\frac{f(2bfp+cep+3b}{}$
parallelrisch	$2x^3(f\,x^2+ex+d)^p cd\,f^2p+2x^3(f\,x^2+ex+d)^p cd\,f^2+2x^2(f\,x^2+ex+d)^p bd\,f^2p+x^2(f\,x^2+ex+d)^p cdefp+3x^2(f\,x^2+ex+d)^p bd$

input
```
int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3
+2*p)*x+2*c*f^2*(3+2*p)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(1+p)*(f*x^2+e*x+d)^(1+p)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)
```

3.277.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.16

$$\int (d+ex+fx^2)^p\,(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2)\ dx$$
$$= \frac{(2\,(cf^2p+cf^2)x^3-2\,cde+3\,bdf+(3\,bf^2+(cef+2\,bf^2)p)x^2-(cde-2\,bdf)p-(2\,ce^2-(2\,cd+3\,be)f}{p+1}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*
f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

output
```
(2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e + 3*b*d*f + (3*b*f^2 + (c*e*f + 2*b*f^2
)*p)*x^2 - (c*d*e - 2*b*d*f)*p - (2*c*e^2 - (2*c*d + 3*b*e)*f + (c*e^2 - 2
*(c*d + b*e)*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(51) = 102$.

Time = 58.82 (sec) , antiderivative size = 483, normalized size of antiderivative = 8.47

$$\int (d+ex+fx^2)^p\,(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2)\ dx$$
$$= \begin{cases} \frac{2bdfp(d+ex+fx^2)^p}{p+1} + \frac{3bdf(d+ex+fx^2)^p}{p+1} + \frac{2befpx(d+ex+fx^2)^p}{p+1} + \frac{3befx(d+ex+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+ex+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+ex-}{p+1} \\ bf\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)+bf\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+\right. \end{cases}$$

input
```
integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+
2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

output
```
Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*
x**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(
d + e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p +
1) + 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x*
*2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d +
e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**
2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p
+ 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e
*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(
p, -1)), (b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*
f) + x + sqrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f +
e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x,
 True))
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72

$$\int (d+ex+fx^2)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{(2\,cf^2(p+1)x^3+bdf(2\,p+3)-cde(p+2)+(bf^2(2\,p+3)+cefp)x^2+(bef(2\,p+3)-(e^2(p+2)-2\,d}{p+1}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*
f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

output
```
(2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3)
+ c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f
*x^2 + e*x + d)^p/(p + 1)
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(56) = 112.

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.19

$$\int (d+ex+fx^2)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx$$
$$=\frac{2\,(fx^2+ex+d)^pcf^2px^3+(fx^2+ex+d)^pcefpx^2+2\,(fx^2+ex+d)^pbf^2px^2+2\,(fx^2+ex+d)^pcf^2x^3}{}$$

input
```
integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*
f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

output
```
(2*(f*x^2 + e*x + d)^p*c*f^2*p*x^3 + (f*x^2 + e*x + d)^p*c*e*f*p*x^2 + 2*(
f*x^2 + e*x + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + e*x + d)^p*c*f^2*x^3 - (f*x^2
+ e*x + d)^p*c*e^2*p*x + 2*(f*x^2 + e*x + d)^p*c*d*f*p*x + 2*(f*x^2 + e*x
+ d)^p*b*e*f*p*x + 3*(f*x^2 + e*x + d)^p*b*f^2*x^2 - (f*x^2 + e*x + d)^p*c
*d*e*p + 2*(f*x^2 + e*x + d)^p*b*d*f*p - 2*(f*x^2 + e*x + d)^p*c*e^2*x + 2
*(f*x^2 + e*x + d)^p*c*d*f*x + 3*(f*x^2 + e*x + d)^p*b*e*f*x - 2*(f*x^2 +
e*x + d)^p*c*d*e + 3*(f*x^2 + e*x + d)^p*b*d*f)/(p + 1)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.11

$$\int (d+ex+fx^2)^p\left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)\,dx=\left(f\,x^2+e\,x+d\right)^p\left(\frac{x^2\,(3\,b\,f^2+2\,b\,f^2\,p+c\,e\,f\,p)}{p+1}+2\,c\,f^2\,x^3+\frac{d\,(3\,b\,f-2\,c\,e+2\,b\,f\,p-c\,e\,p)}{p+1}+\frac{x\,(3\,b\,e\,f-2\,c\,e^2+2\,c\,d\,f-c\,e^2\,p+2\,b\,e\,f\,p+2\,c\,d\,f\,p)}{p+1}\right)$$

input
```
int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x
*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

output
```
(d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f
^2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c
*e^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

3.278 $\int (d+ex)^3 (a+bx+cx^2)^5 (d(6bd+5ae)+(12cd^2+$

3.278.1 Optimal result

Integrand size = 75, antiderivative size = 20

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = (d+ex)^5 \left(a+bx+cx^2\right)^6$$

output
```
(e*x+d)^5*(c*x^2+b*x+a)^6
```

3.278.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2 +17ce^2x^3\right) \, dx = x\left(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+cx)^3(d+ex)^5 + 15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5+x^5(b+cx)^6(d+ex)^5 + a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)\right)$$

input
```
Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
 17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

output
```
x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3
*x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^
4*(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 1
0*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

3.278.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2155, 2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(x\left(5ae^2+17bde+12cd^2\right)+d(5ae+6bd)+ex^2(11be+29cd)+17ce^2x^3\right) dx$$

$$\Big\downarrow \textbf{2155}$$

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(5ae+x(11be+12cd)+6bd+17cex^2\right) dx$$

$$\Big\downarrow \textbf{2023}$$

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

input
```
Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*
d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

output
```
(d + e*x)^5*(a + b*x + c*x^2)^6
```

3.278.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

rule 2155
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

3.278.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. $2(20) = 40$.

Time = 0.91 (sec) , antiderivative size = 2052, normalized size of antiderivative = 102.60

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
parallelrisch	Expression too large to display	2468
default	Expression too large to display	8419

input
```
int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)
*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x,method=_RETURNVERBOSE)
```

output
```
(5*a^6*d^4*e+6*a^5*b*d^5)*x+(10*a^6*d^3*e^2+30*a^5*b*d^4*e+6*a^5*c*d^5+15*
a^4*b^2*d^5)*x^2+(10*a^6*d^2*e^3+60*a^5*b*d^3*e^2+30*a^5*c*d^4*e+75*a^4*b^
2*d^4*e+30*a^4*b*c*d^5+20*a^3*b^3*d^5)*x^3+(5*a^6*d*e^4+60*a^5*b*d^2*e^3+6
0*a^5*c*d^3*e^2+150*a^4*b^2*d^3*e^2+150*a^4*b*c*d^4*e+15*a^4*c^2*d^5+100*a
^3*b^3*d^4*e+60*a^3*b^2*c*d^5+15*a^2*b^4*d^5)*x^4+(a^6*e^5+30*a^5*b*d*e^4+
60*a^5*c*d^2*e^3+150*a^4*b^2*d^2*e^3+300*a^4*b*c*d^3*e^2+75*a^4*c^2*d^4*e+
200*a^3*b^3*d^3*e^2+300*a^3*b^2*c*d^4*e+60*a^3*b*c^2*d^5+75*a^2*b^4*d^4*e+
60*a^2*b^3*c*d^5+6*a*b^5*d^5)*x^5+(6*a^5*b*e^5+30*a^5*c*d*e^4+75*a^4*b^2*d
*e^4+300*a^4*b*c*d^2*e^3+150*a^4*c^2*d^3*e^2+200*a^3*b^3*d^2*e^3+600*a^3*b
^2*c*d^3*e^2+300*a^3*b*c^2*d^4*e+20*a^3*c^3*d^5+150*a^2*b^4*d^3*e^2+300*a^
2*b^3*c*d^4*e+90*a^2*b^2*c^2*d^5+30*a*b^5*d^4*e+30*a*b^4*c*d^5+b^6*d^5)*x^
6+(6*a^5*c*e^5+15*a^4*b^2*e^5+150*a^4*b*c*d*e^4+150*a^4*c^2*d^2*e^3+100*a^
3*b^3*d*e^4+600*a^3*b^2*c*d^2*e^3+600*a^3*b*c^2*d^3*e^2+100*a^3*c^3*d^4*e+
150*a^2*b^4*d^2*e^3+600*a^2*b^3*c*d^3*e^2+450*a^2*b^2*c^2*d^4*e+60*a^2*b*c
^3*d^5+60*a*b^5*d^3*e^2+150*a*b^4*c*d^4*e+60*a*b^3*c^2*d^5+5*b^6*d^4*e+6*b
^5*c*d^5)*x^7+(30*a^4*b*c*e^5+75*a^4*c^2*d*e^4+20*a^3*b^3*e^5+300*a^3*b^2*
c*d*e^4+600*a^3*b*c^2*d^2*e^3+200*a^3*c^3*d^3*e^2+75*a^2*b^4*d*e^4+600*a^2
*b^3*c*d^2*e^3+900*a^2*b^2*c^2*d^3*e^2+300*a^2*b*c^3*d^4*e+15*a^2*c^4*d^5+
60*a*b^5*d^2*e^3+300*a*b^4*c*d^3*e^2+300*a*b^3*c^2*d^4*e+60*a*b^2*c^3*d^5+
10*b^6*d^3*e^2+30*b^5*c*d^4*e+15*b^4*c^2*d^5)*x^8+(15*a^4*c^2*e^5+60*a^...
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3\left(a+bx+cx^2\right)^5\left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

3.278.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.

Time = 0.16 (sec) , antiderivative size = 2281, normalized size of antiderivative = 114.05

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e
+12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

output
```
c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*
e**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(
30*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e
**4 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5
+ 60*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**
4*c**2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d
**3*e**2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e*
*4 + 60*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3
 + 60*a*c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c*
*3*d**2*e**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x
**11*(20*a**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4
+ 150*a**2*c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 60
0*a*b**2*c**3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6
*e**5 + 30*b**5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e*
*2 + 75*b**2*c**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 1
00*a**3*c**3*d*e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 60
0*a**2*b*c**3*d**2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*
b**4*c*d*e**4 + 600*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 15
0*a*b*c**4*d**4*e + 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 +
150*b**4*c**2*d**3*e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x...
```

3.278.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 2467, normalized size of antiderivative = 123.35

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

output
```
c^6*e^5*x^17 + 5*c^6*d*e^4*x^16 + 6*b*c^5*e^5*x^16 + 10*c^6*d^2*e^3*x^15 +
 30*b*c^5*d*e^4*x^15 + 15*b^2*c^4*e^5*x^15 + 6*a*c^5*e^5*x^15 + 10*c^6*d^3
*e^2*x^14 + 60*b*c^5*d^2*e^3*x^14 + 75*b^2*c^4*d*e^4*x^14 + 30*a*c^5*d*e^4
*x^14 + 20*b^3*c^3*e^5*x^14 + 30*a*b*c^4*e^5*x^14 + 5*c^6*d^4*e*x^13 + 60*
b*c^5*d^3*e^2*x^13 + 150*b^2*c^4*d^2*e^3*x^13 + 60*a*c^5*d^2*e^3*x^13 + 10
0*b^3*c^3*d*e^4*x^13 + 150*a*b*c^4*d*e^4*x^13 + 15*b^4*c^2*e^5*x^13 + 60*a
*b^2*c^3*e^5*x^13 + 15*a^2*c^4*e^5*x^13 + c^6*d^5*x^12 + 30*b*c^5*d^4*e*x^
12 + 150*b^2*c^4*d^3*e^2*x^12 + 60*a*c^5*d^3*e^2*x^12 + 200*b^3*c^3*d^2*e^
3*x^12 + 300*a*b*c^4*d^2*e^3*x^12 + 75*b^4*c^2*d*e^4*x^12 + 300*a*b^2*c^3*
d*e^4*x^12 + 75*a^2*c^4*d*e^4*x^12 + 6*b^5*c*e^5*x^12 + 60*a*b^3*c^2*e^5*x
^12 + 60*a^2*b*c^3*e^5*x^12 + 6*b*c^5*d^5*x^11 + 75*b^2*c^4*d^4*e*x^11 + 3
0*a*c^5*d^4*e*x^11 + 200*b^3*c^3*d^3*e^2*x^11 + 300*a*b*c^4*d^3*e^2*x^11 +
 150*b^4*c^2*d^2*e^3*x^11 + 600*a*b^2*c^3*d^2*e^3*x^11 + 150*a^2*c^4*d^2*e
^3*x^11 + 30*b^5*c*d*e^4*x^11 + 300*a*b^3*c^2*d*e^4*x^11 + 300*a^2*b*c^3*d
*e^4*x^11 + b^6*e^5*x^11 + 30*a*b^4*c*e^5*x^11 + 90*a^2*b^2*c^2*e^5*x^11 +
 20*a^3*c^3*e^5*x^11 + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 100*b^3*c^
3*d^4*e*x^10 + 150*a*b*c^4*d^4*e*x^10 + 150*b^4*c^2*d^3*e^2*x^10 + 600*a*b
^2*c^3*d^3*e^2*x^10 + 150*a^2*c^4*d^3*e^2*x^10 + 60*b^5*c*d^2*e^3*x^10 + 6
00*a*b^3*c^2*d^2*e^3*x^10 + 600*a^2*b*c^3*d^2*e^3*x^10 + 5*b^6*d*e^4*x^10
+ 150*a*b^4*c*d*e^4*x^10 + 450*a^2*b^2*c^2*d*e^4*x^10 + 100*a^3*c^3*d*e...
```

3.278.9 Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 2026, normalized size of antiderivative = 101.30

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c
*d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

output
```
x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^
2*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^
2 + 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e
 + 300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^
11*(b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2
*c^2*e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3
 + 30*a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2
+ 300*a*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5
*(a^6*e^5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4
*d^4*e + 75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a
^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^
2) + x^3*(20*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^
3*e^2 + 30*a^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 6
0*a*b^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 +
 75*b^4*c^2*d*e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d
^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b
^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*
d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b
^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 +
 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 ...
```

3.278. $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right)\,dx$

3.279 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

3.279.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)$$

output
```
1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

3.279.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)$$

input
```
Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
```

output
```
x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

3.279.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2027, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2}{x^2+x-2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(x+1)}{x^2+x-2}dx$$

$$\Big\downarrow \text{1200}$$

$$\int \left(\frac{2x}{x^2+x-2}+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

input
```
Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

output
```
x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

3.279.3.1 Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.279.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x \right)}{3}+\frac{4 \ln \left(2+x \right)}{3}$	19
norman	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x \right)}{3}+\frac{4 \ln \left(2+x \right)}{3}$	19
risch	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x \right)}{3}+\frac{4 \ln \left(2+x \right)}{3}$	19
parallelrisch	$\frac{x^{2}}{2}+\frac{2 \ln \left(-1+x \right)}{3}+\frac{4 \ln \left(2+x \right)}{3}$	19

input
```
int((x^3+x^2)/(x^2+x-2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+2/3*ln(-1+x)+4/3*ln(2+x)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{1}{2}\,x^{2}+\frac{4}{3}\,\log\left(x+2\right)+\frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
```

output
```
1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

3.279.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{x^2}{2} + \frac{2\log(x-1)}{3} + \frac{4\log(x+2)}{3}$$

input
```
integrate((x**3+x**2)/(x**2+x-2),x)
```

output
```
x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{4}{3}\,\log(x+2) + \frac{2}{3}\,\log(x-1)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
```

output
```
1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{1}{2}\,x^2 + \frac{4}{3}\,\log(|x+2|) + \frac{2}{3}\,\log(|x-1|)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
```

output
```
1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

3.279.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx = \frac{2\ln(x-1)}{3}+\frac{4\ln(x+2)}{3}+\frac{x^2}{2}$$

input
```
int((x^2 + x^3)/(x + x^2 - 2),x)
```

output
```
(2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

3.280 $\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

3.280.1 Optimal result

Integrand size = 33, antiderivative size = 346

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(80c^2e-70bcf+63b^2g-64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} - \frac{(1050b^3cf+40bc^2(36cd-55af)-945b^4g-60b^2c(20ce-49ag)+256ac^2(5ce-4ag)-2c(480c^3d-4}{1920c^5} + \frac{(70b^4cf+48b^2c^2(2cd-5af)-32ac^3(4cd-3af)-63b^5g-40b^3c(2ce-7ag)+48abc^2(4ce-5ag))\,\mathrm{ar}}{256c^{11/2}}$$

output

```
1/256*(70*b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5
*g-40*b^3*c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(-64*a*c*g+63*b^2*g-70*b*c*f
+80*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x
+a)^(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-1/1920*(1050*b^3*c*f+40*b*c^
2*(-55*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5
*c*e)-2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*e)-315*b^3*g+14*b*c*(46*a*g+25*b*f
))*x)*(c*x^2+b*x+a)^(1/2)/c^5
```

3.280.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^4g-210b^3c(5f+3gx)+4b^2c(300ce-735ag+7cx(25f+18gx))-8bc^2(-a(2}{}$$

input
```
Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*
b^2*c*(300*c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f +
161*g*x)) + 2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g
- a*c*(80*e + x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*
g*x))))) + 15*(-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d
- 3*a*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*
g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

3.280.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2184, 27, 2184, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int \frac{x^2\left((10cf-9bg)x^2+2(5ce-4ag)x+10cd\right)}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2\left((10cf-9bg)x^2+2(5ce-4ag)x+10cd\right)}{\sqrt{cx^2+bx+a}}dx}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{\int \frac{x^2\left(80dc^2-60afc+54abg+\left(63gb^2-70cfb+80c^2e-64acg\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{x^3\sqrt{a+bx+cx^2}(10cf-9bg)}{4c}}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{x^2\left(2\left(40dc^2-30afc+27abg\right)+\left(63gb^2-70cfb+80c^2e-64acg\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{x^3\sqrt{a+bx+cx^2}(10cf-9bg)}{4c}}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 1236

$$\frac{\frac{\frac{\int -\frac{x\left(4a\left(63gb^2-70cfb+80c^2e-64acg\right)-\left(-315gb^3+14c(25bf+46ag)b+480c^3d-40c^2(10be+9af)\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}}{8c}+\frac{x^3}{\ldots}}{10c}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 27

$$\frac{\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\int \frac{x\left(4a\left(63gb^2-70cfb+80c^2e-64acg\right)-\left(-315gb^3+14c(25bf+46ag)b+480c^3d-40c^2(10be+9af)\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}}{8c}+\frac{x^3\sqrt{\ldots}}{\ldots}}{10c}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 1225

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd-5\ldots\right.}{4c^2}}{8\ldots}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 1092

$$\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd-5\ldots\right.}{4c^2}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\downarrow$ 219

3.280. $\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd-5\right.}{4c^2}}{}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

input
```
Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^4*Sqrt[a + b*x + c*x^2])/(5*c) + (((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + (((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a
 + b*x + c*x^2])/(3*c) - (((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 94
5*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*
x)*Sqrt[a + b*x + c*x^2])/(4*c^2) - (15*(70*b^4*c*f + 48*b^2*c^2*(2*c*d -
5*a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) +
48*a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c))/(10*c)
```

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.280.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\left(384g\,c^{4}x^{4}-432b\,c^{3}g\,x^{3}+480c^{4}f\,x^{3}-512a\,c^{3}g\,x^{2}+504b^{2}c^{2}g\,x^{2}-560b\,c^{3}f\,x^{2}+640c^{4}e\,x^{2}+1288ab\,c^{2}gx-720a\,c^{3}fx-630gc\,b^{3}x+700f\right.}{192}$
default	Expression too large to display

input
```
int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*c^4*g*x^4-432*b*c^3*g*x^3+480*c^4*f*x^3-512*a*c^3*g*x^2+504*b^
2*c^2*g*x^2-560*b*c^3*f*x^2+640*c^4*e*x^2+1288*a*b*c^2*g*x-720*a*c^3*f*x-6
30*b^3*c*g*x+700*b^2*c^2*f*x-800*b*c^3*e*x+960*c^4*d*x+1024*a^2*c^2*g-2940
*a*b^2*c*g+2200*a*b*c^2*f-1280*a*c^3*e+945*b^4*g-1050*b^3*c*f+1200*b^2*c^2
*e-1440*b*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^5-1/256*(240*a^2*b*c^2*g-96*a^2*c^3
*f-280*a*b^3*c*g+240*a*b^2*c^2*f-192*a*b*c^3*e+128*a*c^4*d+63*b^5*g-70*b^4
*c*f+80*b^3*c^2*e-96*b^2*c^3*d)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.03

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \left[-\frac{15\,(32\,(3\,b^2c^3-4\,ac^4)d-16\,(5\,b^3c^2-12\,abc^3)e+2\,(35\,b^4c-120\,ab^2c^2+48\,a^2c^3)f-(63\,b^5-280\,ab^3c}{}\right.$$

$$-\frac{15\,(32\,(3\,b^2c^3-4\,ac^4)d-16\,(5\,b^3c^2-12\,abc^3)e+2\,(35\,b^4c-120\,ab^2c^2+48\,a^2c^3)f-(63\,b^5-280\,ab^3c}{}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240
*a^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*
(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a
*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)
*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*
c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*s
qrt(c*x^2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5
*b^3*c^2 - 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (
63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4
- 1440*b*c^4*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f
+ (63*b^2*c^3 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b
^3*c^2 - 44*a*b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2
*(480*c^5*d - 400*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 -
 92*a*b*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

3.280.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 700, normalized size of antiderivative = 2.02

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \left(-\frac{a\left(-\frac{3a\left(-\frac{9bg}{10c}+f\right)}{4c}-\frac{5b\left(-\frac{4ag}{5c}-\frac{7b\left(-\frac{9bg}{10c}+f\right)}{8c}+e\right)}{6c}+d\right)}{2c} - \frac{b\left(-\frac{2a\left(-\frac{4ag}{5c}-\frac{7b\left(-\frac{9bg}{10c}+f\right)}{8c}+e\right)}{3c}-\frac{3b\left(-\frac{3a\left(-\frac{9bg}{10c}+f\right)}{4c}-\frac{5b\left(-\frac{4ag}{5c}-\frac{7b\left(-\frac{9bg}{10c}+f\cdots\right)}{8c}\cdots\right)}{6c}\cdots\right)}{4c}\cdots\right)}{2c}\cdots \right. \\ \frac{2\left(\frac{g(a+bx)^{\frac{11}{2}}}{11b^3}+\frac{(a+bx)^{\frac{9}{2}}(-5ag+bf)}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}\cdot\left(10a^2g-4abf+b^2e\right)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\left(-10a^3g+6a^2bf-3ab^2e+b^3d\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(5a^4g-4a^3bf+3a^2b^2e-2ab^3d\right)}{3b^3}\cdots\right)}{b^3} \\ \frac{\frac{dx^3}{3}+\frac{ex^4}{4}+\frac{fx^5}{5}+\frac{gx^6}{6}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(
-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) - b*(-2*a*(-4*a*g/(5*c) - 7
*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4
*c) - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4
*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/s
qrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/
(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**4/(5*c) + x**3*(-9*b
*g/(10*c) + f)/(4*c) + x**2*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c)
+ e)/(3*c) + x*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(
-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) + (-2*a*(-4*a*g/(5*c) - 7*b
*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4*c
) - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4*c
))/c), Ne(c, 0)), (2*(g*(a + b*x)**(11/2)/(11*b**3) + (a + b*x)**(9/2)*(-5
*a*g + b*f)/(9*b**3) + (a + b*x)**(7/2)*(10*a**2*g - 4*a*b*f + b**2*e)/(7*
b**3) + (a + b*x)**(5/2)*(-10*a**3*g + 6*a**2*b*f - 3*a*b**2*e + b**3*d)/(
5*b**3) + (a + b*x)**(3/2)*(5*a**4*g - 4*a**3*b*f + 3*a**2*b**2*e - 2*a*b*
*3*d)/(3*b**3) + sqrt(a + b*x)*(-a**5*g + a**4*b*f - a**3*b**2*e + a**2*b*
*3*d)/b**3)/b**3, Ne(b, 0)), ((d*x**3/3 + e*x**4/4 + f*x**5/5 + g*x**6/6)/
sqrt(a), True))
```

3.280.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.93

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,gx}{c}+\frac{10\,c^4f-9\,bc^3g}{c^5}\right)x+\frac{80\,c^4e-70\,bc^3f+63\,b^2c^2g-64\,ac^3g}{c^5}\right)x+\right.\right.$$
$$-\frac{(96\,b^2c^3d-128\,ac^4d-80\,b^3c^2e+192\,abc^3e+70\,b^4cf-240\,ab^2c^2f+96\,a^2c^3f-63\,b^5g+280\,ab^3cg-2}{256\,c^{\frac{11}{2}}}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^
5)*x + (80*c^4*e - 70*b*c^3*f + 63*b^2*c^2*g - 64*a*c^3*g)/c^5)*x + (480*c
^4*d - 400*b*c^3*e + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c
^2*g)/c^5)*x - (1440*b*c^3*d - 1200*b^2*c^2*e + 1280*a*c^3*e + 1050*b^3*c*
f - 2200*a*b*c^2*f - 945*b^4*g + 2940*a*b^2*c*g - 1024*a^2*c^2*g)/c^5) - 1
/256*(96*b^2*c^3*d - 128*a*c^4*d - 80*b^3*c^2*e + 192*a*b*c^3*e + 70*b^4*c
*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f - 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b
*c^2*g)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/
2)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.281 $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

3.281.1 Optimal result

Integrand size = 31, antiderivative size = 245

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$
$$+\frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)\,x)\,\sqrt{a-}}{192c^4}$$
$$-\frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\,\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}$$

output
```
-1/128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e
)+16*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/
2))/c^(9/2)+1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x
^2+b*x+a)^(1/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(
11*a*g+6*b*f)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^
(1/2)/c^4
```

3.281.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.81

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3g+10bc(12bf+22ag+7bgx)-8c^2(18be+16af+10bfx+9agx+7bgx^2)}{}$$

input
```
Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7
*b*g*x) - 8*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^
3*(12*d + x*(6*e + 4*f*x + 3*g*x^2))) + 3*(40*b^3*c*f + 32*b*c^2*(2*c*d -
3*a*f) - 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

3.281.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2184, 27, 2184, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int \frac{x\left((8cf-7bg)x^2+2(4ce-3ag)x+8cd\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x\left((8cf-7bg)x^2+2(4ce-3ag)x+8cd\right)}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{\int \frac{x\left(4\left(12dc^2-8afc+7abg\right)+\left(35gb^2-40cfb+48c^2e-36acg\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{3c}}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x\left(4\left(12dc^2-8afc+7abg\right)+\left(35gb^2-40cfb+48c^2e-36acg\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{3c}}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \; 1225$$

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\left(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3ag)\right.}{8c^2}}{6c}}{8c}$$

$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\left(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3ag)\right.}{4c^2}}{6c}}{8c}$$

$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24b^2c(2ce-5ag)+32\right.}{8c^{5/2}}}{6c}}{8c}$$

$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

input
```
Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + (((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x +
 c*x^2])/(3*c) + (((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*
c*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sq
rt[a + b*x + c*x^2])/(4*c^2) - (3*(40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) -
 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c)
```

3.281. $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.281. $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}} dx$

3.281.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\left(48g\,c^3x^3-56b\,c^2g\,x^2+64c^3f\,x^2-72a\,c^2gx+70b^2cgx-80b\,c^2f\,x+96c^3ex+220abcg-128a\,c^2f-105b^3g+120b^2cf-144b\,c^2e+192c^3d\right)\sqrt{\ }}{192c^4}$
default	$g\left(\frac{x^3\sqrt{c\,x^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{2c^{\frac{3}{2}}}\right)}{6c}}{8c}\right.$

input
```
int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*c^3*g*x^3-56*b*c^2*g*x^2+64*c^3*f*x^2-72*a*c^2*g*x+70*b^2*c*g*x-
80*b*c^2*f*x+96*c^3*e*x+220*a*b*c*g-128*a*c^2*f-105*b^3*g+120*b^2*c*f-144*
b*c^2*e+192*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*g-120*a*b^2*c
*g+96*a*b*c^2*f-64*a*c^3*e+35*b^4*g-40*b^3*c*f+48*b^2*c^2*e-64*b*c^3*d)/c^
(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.04

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{3\,(64\,bc^3d-16\,(3\,b^2c^2-4\,ac^3)e+8\,(5\,b^3c-12\,abc^2)f-(35\,b^4-120\,ab^2c+48\,a^2c^2)g)\sqrt{c}\log\left(-8\,c^2x^2\right.}{}\right.$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*
c^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(4
8*c^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(1
5*b^2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b
*c^3*f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(
3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f -
(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 1
92*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*
a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^
2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.96

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{a\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{2c}-\frac{b\left(-\frac{2a\left(-\frac{7bg}{8c}+f\right)}{3c}-\frac{3b\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{4c}+d\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{othe}\end{cases}\right. \\ \frac{2\left(\frac{g(a+bx)^{\frac{9}{2}}}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}(-4ag+bf)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2g-3abf+b^2e\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3g+3a^2bf-2ab^2e+b^3d\right)}{3b^3}+\frac{\sqrt{a+bx}\left(a^4g-a^3bf+a^2b^2e-ab^3d\right)}{b^3}\right)}{b^2} \\ \frac{\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}+\frac{gx^5}{5}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(2*c) - b
*(-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) +
f)/(6*c) + e)/(4*c) + d)/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(
2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**
3/(4*c) + x**2*(-7*b*g/(8*c) + f)/(3*c) + x*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8
*c) + f)/(6*c) + e)/(2*c) + (-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(
4*c) - 5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(4*c) + d)/c), Ne(c, 0)), (2*(g*(
a + b*x)**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-4*a*g + b*f)/(7*b**3) + (a +
 b*x)**(5/2)*(6*a**2*g - 3*a*b*f + b**2*e)/(5*b**3) + (a + b*x)**(3/2)*(-4
*a**3*g + 3*a**2*b*f - 2*a*b**2*e + b**3*d)/(3*b**3) + sqrt(a + b*x)*(a**4
*g - a**3*b*f + a**2*b**2*e - a*b**3*d)/b**3)/b**2, Ne(b, 0)), ((d*x**2/2
+ e*x**3/3 + f*x**4/4 + g*x**5/5)/sqrt(a), True))
```

3.281.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x+\frac{48\,c^3e-40\,bc^2f+35\,b^2cg-36\,ac^2g}{c^4}\right)x+\frac{192\,c^3d\right.$$
$$+\frac{(64\,bc^3d-48\,b^2c^2e+64\,ac^3e+40\,b^3cf-96\,abc^2f-35\,b^4g+120\,ab^2cg-48\,a^2c^2g)\log\left(\left|2\left(\sqrt{c}x-\sqrt{c}\right.\right.\right.}{128\,c^{\frac{9}{2}}}$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x
+ (48*c^3*e - 40*b*c^2*f + 35*b^2*c*g - 36*a*c^2*g)/c^4)*x + (192*c^3*d -
144*b*c^2*e + 120*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g)/c^4) +
1/128*(64*b*c^3*d - 48*b^2*c^2*e + 64*a*c^3*e + 40*b^3*c*f - 96*a*b*c^2*f
- 35*b^4*g + 120*a*b^2*c*g - 48*a^2*c^2*g)*log(abs(2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\,(g\,x^3+f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

3.282.1 Optimal result

Integrand size = 30, antiderivative size = 177

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx &= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} \\ &+ \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\ &+ \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}\end{aligned}$$

output
```
1/16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/24*(-16*a*c*g+15*b^2*g-18*b*c
*f+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(
1/2)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c
```

3.282.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2g-2c(9bf+8ag+5bgx)+4c^2(6e+x(3f+2gx)))+3(-16c^3d+8c^2(be+af}{48c^{7/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
 + 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(-16*c^3*d + 8*c^2*(b*e + a*f) + 5*b
^3*g - 6*b*c*(b*f + 2*a*g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]])/(48*c^(7/2))
```

3.282.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2192}$$
$$\frac{\int \frac{(6cf-5bg)x^2+2(3ce-2ag)x+6cd}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(6cf-5bg)x^2+2(3ce-2ag)x+6cd}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$
$$\Big\downarrow \text{2192}$$
$$\frac{\frac{\int \frac{24dc^2-12afc+10abg+\left(15gb^2-18cfb+24c^2e-16acg\right)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{2\left(12dc^2-6afc+5abg\right)+\left(15gb^2-18cfb+24c^2e-16acg\right)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\downarrow 1160$$

$$\frac{\frac{\frac{3\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\downarrow 1092$$

$$\frac{\frac{\frac{3\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\downarrow 219$$

$$\frac{\frac{\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^2*Sqrt[a + b*x + c*x^2])/(3*c) + (((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c
*x^2])/(2*c) + (((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x
+ c*x^2])/c + (3*(16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f + 2*
a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))
/(4*c))/(6*c)
```

3.282. $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.282.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(-8g\,c^2x^2+10bcgx-12c^2fx+16acg-15b^2g+18bcf-24c^2e)\sqrt{c\,x^2+bx+a}}{24c^3}+\frac{(12abcg-8a\,c^2f-5b^3g+6b^2cf-8b\,c^2e+16c^3d)\ln\left(\frac{b}{2}\right.}{16c^{\frac{7}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+g\left(\frac{x^2\sqrt{c\,x^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{b}{2}\right.}{}\right)}{6c}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*c^2*g*x^2+10*b*c*g*x-12*c^2*f*x+16*a*c*g-15*b^2*g+18*b*c*f-24*c^
2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/16*(12*a*b*c*g-8*a*c^2*f-5*b^3*g+6*b^2*c*f-
8*b*c^2*e+16*c^3*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+c}\right.}{}\right.$$
$$-\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(8\,c^3gx}{48\,c^4}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b
*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*
b^2*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^
4, -1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*
a*b*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(172) = 344$.

Time = 0.48 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{gx^2}{3c}+\frac{x\left(-\frac{5bg}{6c}+f\right)}{2c}+\frac{-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e}{c}\right)+\left(-\frac{a\left(-\frac{5bg}{6c}+f\right)}{2c}-\frac{b\left(-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e\right)}{2c}+d\right)\Bigg(\\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2g\left(-a^3\sqrt{a+bx}+a^2(a+bx)^{\frac{3}{2}}-\frac{3a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}+\frac{gx^4}{4}}{\sqrt{a}} \end{cases}$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(g*x**2/(3*c) + x*(-5*b*g/(6*c) + f)/(2*
c) + (-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/c) + (-a*(-5*b*g/(6
*c) + f)/(2*c) - b*(-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/(2*c)
 + d)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c)
, Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
+ x)**2), True)), Ne(c, 0)), ((2*d*sqrt(a + b*x) + 2*e*(-a*sqrt(a + b*x) +
 (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3
+ (a + b*x)**(5/2)/5)/b**2 + 2*g*(-a**3*sqrt(a + b*x) + a**2*(a + b*x)**(3
/2) - 3*a*(a + b*x)**(5/2)/5 + (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((d
*x + e*x**2/2 + f*x**3/3 + g*x**4/4)/sqrt(a), True))
```

3.282.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,gx}{c}+\frac{6\,c^2f-5\,bcg}{c^3}\right)x+\frac{24\,c^2e-18\,bcf+15\,b^2g-16\,acg}{c^3}\right) - \frac{(16\,c^3d-8\,bc^2e+6\,b^2cf-8\,ac^2f-5\,b^3g+12\,abcg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x + (24*
c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)/c^3) - 1/16*(16*c^3*d - 8*b*c^2*e
+ 6*b^2*c*f - 8*a*c^2*f - 5*b^3*g + 12*a*b*c*g)*log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

3.283.1 Optimal result

Integrand size = 33, antiderivative size = 155

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{(8c^2e+3b^2g-4c(bf+ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*c^2*e+3*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))/c^(5/2)-d*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/
a^(1/2)+1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(
1/2)/c
```

3.283.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{1}{8}\left(\frac{2(4cf-3bg+2cgx)\sqrt{a+x(b+cx)}}{c^2}+\frac{16d\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-8c^2e-3b^2g+4c(bf+ag))\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*(4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/c^2 + (16*d*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + ((-8*c^2*e - 3*b^2*
g + 4*c*(b*f + a*g))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])
])/c^(5/2))/8
```

3.283.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2184, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\int \frac{(4cf-3bg)x^2+2(2ce-ag)x+4cd}{2x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(4cf-3bg)x^2+2(2ce-ag)x+4cd}{x\sqrt{cx^2+bx+a}}dx}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\frac{\int \frac{8dc^2+\left(3gb^2+8c^2e-4c(bf+ag)\right)x}{2x\sqrt{cx^2+bx+a}}dx}{c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{8dc^2+\left(3gb^2+8c^2e-4c(bf+ag)\right)x}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{(-4c(ag+bf)+3b^2g+8c^2e)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{2(-4c(ag+bf)+3b^2g+8c^2e)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}-16c^2d\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}-\frac{8c^2d\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

3.283. $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

output
```
(g*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2
])/c + ((-8*c^2*d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
Sqrt[a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/(2*c))/(4*c)
```

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.283. $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.283.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.44

method	result
default	$\frac{e\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+g\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+\right.}{2c^{\frac{3}{2}}}$

input
```
int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+g*(1/2*x/c*(c*x^2+b*
x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))+f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2)))-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1
/2))/x)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.73

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{8\sqrt{a}c^3d\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)-(8\,ac^2e-4\,abcf+(3\,ab^2-4\,a^2c)g)\sqrt{c}\log\left(-\right.}{16}\right.$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a
*b^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*
b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x
+ (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)
/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*
(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16
*(16*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f +
 (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b
*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3)]
```

3.283.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

3.283.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.283.8 Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

3.284.1 Optimal result

Integrand size = 33, antiderivative size = 139

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\sqrt{a+bx+cx^2}}{c}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{(bd-2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

output
```
1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2
)+1/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3
/2)+g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x
```

3.284.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{(-cd+agx)\sqrt{a+x(b+cx)}}{acx}+\frac{(2cf-bg)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+\frac{(bd-2ae)\log(x)}{2a^{3/2}}+\frac{(-bd+2ae)\log\left(a\left(2a+bx-2\sqrt{a}\sqrt{a+x(b+cx)}\right)\right)}{2a^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((2*c*f - b*g)*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(3/2) + ((b*d - 2*a*e)*
Log[x])/(2*a^(3/2)) + ((-(b*d) + 2*a*e)*Log[a*(2*a + b*x - 2*Sqrt[a]*Sqrt[
a + x*(b + c*x)])])/(2*a^(3/2))
```

3.284.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2181, 27, 2184, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{-2agx^2-2afx+bd-2ae}{2x\sqrt{cx^2+bx+a}}dx}{a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2agx^2-2afx+bd-2ae}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{2184}$$

$$-\frac{\frac{\int \frac{c(bd-2ae)-a(2cf-bg)x}{x\sqrt{cx^2+bx+a}}dx}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-a(2cf-bg)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2a(2cf-bg)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{ 1154}$$

$$-\frac{\frac{-2c(bd-2ae)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{-\frac{c(bd-2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((d*Sqrt[a + b*x + c*x^2])/(a*x)) - ((-2*a*g*Sqrt[a + b*x + c*x^2])/c + (
-((c*(b*d - 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])
/Sqrt[a]) - (a*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/Sqrt[c])/c)/(2*a)
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.284. $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

3.284.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{d\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{\frac{2fa\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+2ag\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+}}{x}\right.}{\sqrt{a}}}{2a}$
default	$\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+g\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{e\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+d\Big(-$

input
```
int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-d*(c*x^2+b*x+a)^(1/2)/a/x+1/2/a*(2*f*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))/c^(1/2)+2*a*g*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-(2*a*e-b*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.06

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[-\frac{(2\,a^2cf-a^2bg)\sqrt{c}x\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)+(bc^2d-2\,ac^2e}{4\,a^2c^2x}\right.$$
$$-\frac{2\,(2\,a^2cf-a^2bg)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+(bc^2d-2\,ac^2e)\sqrt{a}x\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx}}{x}\right.}{4\,a^2c^2x}$$
$$-\frac{2\,(bc^2d-2\,ac^2e)\sqrt{-a}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+(2\,a^2cf-a^2bg)\sqrt{c}x\log\left(-8\,c^2x^2-8\,bcx-b^2\right.}{4\,a^2c^2x}$$
$$-\frac{(bc^2d-2\,ac^2e)\sqrt{-a}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+(2\,a^2cf-a^2bg)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-}}{2\,(c^2x^2+bcx+ac)}\right.}{2\,a^2c^2x}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)
*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x
+ a))/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*
d - 2*a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2
+ b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqr
t(c*x^2 + b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*
arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a
^2)) + (2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*
d)*sqrt(c*x^2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a
)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x
 + a^2)) + (2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)
*sqrt(c*x^2 + b*x + a))/(a^2*c^2*x)]
```

3.284.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

3.284.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx+a}\,g}{c} - \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{(2\,cf-bg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bd+2\,a\sqrt{c}d}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
 - a)*a)
```

3.284.9 Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\sqrt{c\,x^2+b\,x+a}}{c} - \frac{e\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}} + \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}} - \frac{b\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\,c^{3/2}} - \frac{d\sqrt{c\,x^2+b\,x+a}}{a\,x} + \frac{b\,d\,\mathrm{atanh}\left(\frac{a+\frac{b\,x}{2}}{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}\right)}{2\,a^{3/2}}$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
(g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x
^2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1
/2)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(
2*c^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/
(a^(1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

3.284. $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

3.285.1 Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{(3b^2d-4acd-4abe+8a^2f)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}} + \frac{g\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
-1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^
2+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a
)^(1/2)/a^2/x
```

3.285.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(3bdx-2a(d+2ex))}{x^2}+(3b^2d+8a^2f)\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+4a(cd+be)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(}}{\sqrt{a}}\right.}{4a^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/x^2 + (3*b^2*
d + 8*a^2*f)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 4*a*(c
*d + b*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]] - (4*a^(
5/2)*g*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/(4*a^(5/
2))
```

3.285.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2181, 27, 2181, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{-4agx^2+2(cd-2af)x+3bd-4ae}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-4agx^2+2(cd-2af)x+3bd-4ae}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{2181}$$

$$-\frac{-\frac{\int \frac{8fa^2+8gxa^2-4cda-4bea+3b^2d}{2x\sqrt{cx^2+bx+a}}dx}{a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{8fa^2+8gxa^2-4cda-4bea+3b^2d}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{-\frac{(8a^2f-4abe-4acd+3b^2d)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+8a^2g\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \; 1092$$

$$-\frac{-\frac{\left(8a^2f-4abe-4acd+3b^2d\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+16a^2g\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\left(8a^2f-4abe-4acd+3b^2d\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\left(8a^2f-4abe-4acd+3b^2d\right)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{\sqrt{a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(d*Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((3*b*d - 4*a*e)*Sqrt[a + b*x
+ c*x^2])/(a*x)) - (-(((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + (8*a^2*g*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/(2*a))/(4*a)
```

3.285. $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

3.285.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.285. $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

3.285.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,(4 a e x-3 b d x+2 a d)}{4 a^{2} x^{2}}+\frac{\frac{8 a^{2} g \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-\frac{\left(8 a^{2} f-4 a b e-4 a c d+3 b^{2} d\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{a}}}{8 a^{2}}$
default	$\frac{g \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-\frac{f \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{a}}+d\left(-\frac{\sqrt{c\,x^{2}+b x+a}}{2 a\,x^{2}}-\frac{3 b\left(-\frac{\sqrt{c\,x^{2}+b x+a}}{a x}+\frac{b \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{\ldots}}{x}\right)}{2 a^{\frac{3}{2}}}\ldots\right)}{4 a}\ldots\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(4*a*e*x-3*b*d*x+2*a*d)/a^2/x^2+1/8/a^2*(8*a^2*g*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(8*a^2*f-4*a*b*e-4*a*c
*d+3*b^2*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))
```

3.285.5 Fricas [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 783, normalized size of antiderivative = 4.92

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[\frac{8\,a^3\sqrt{c}gx^2\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-\left(4\,abce-8\,a^2cf-(3\,b^2c\ldots\right.}{\ldots}\right.$$
$$-\frac{16\,a^3\sqrt{-c}gx^2\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+\left(4\,abce-8\,a^2cf-(3\,b^2c-4\,ac^2)d\right)\sqrt{a}x^2\log\left(-\frac{8\,abx+(b^2+\ldots}{\ldots}\right.}{16\,a^3cx^2}$$
$$-\frac{8\,a^3\sqrt{-c}gx^2\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+\left(4\,abce-8\,a^2cf-(3\,b^2c-4\,ac^2)d\right)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{cx^2+b\ldots}}{2\,(a\ldots}\right.}{8\,a^3cx^2}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c
 - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^
2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d
 - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c
)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b
*c*x + a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2
*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*s
qrt(a) + 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c
*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c
*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d -
 (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^
3*sqrt(-c)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sq
rt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2
+ a*b*x + a^2)) + 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^3*c*x^2)]
```

3.285.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

3.285.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.285.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(133) = 266$.

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.18

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}} + \frac{(3\,b^2d-4\,acd-4\,abe+8\,a^2f)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} - \frac{3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe-8\,(\sqrt{}}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) +
1/4*(3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2 - a)^2*a^2)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

3.286.1 Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}\\ &\quad-\frac{(15b^2d-16acd-18abe+24a^2f)\sqrt{a+bx+cx^2}}{24a^3x}\\ &\quad+\frac{(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{7/2}}\end{aligned}$$

output
```
1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh(1
/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)-1/3*d*(c*x^2+b*x+a)^(1/2
)/a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(24*a^2*f-18*
a*b*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x
```

3.286.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(-15b^2dx^2+2ax(5bd+8cdx+9bex)-4a^2(2d+3x(e+2fx)))}{x^3}+3(-5b^3d+16a^3g)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+}{24a^{7/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-15*b^2*d*x^2 + 2*a*x*(5*b*d + 8*c*d*x +
9*b*e*x) - 4*a^2*(2*d + 3*x*(e + 2*f*x))))/x^3 + 3*(-5*b^3*d + 16*a^3*g)*A
rcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 6*a*(-6*b*c*d - 3*b^
2*e + 4*a*c*e + 4*a*b*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sq
rt[a]])/(24*a^(7/2))
```

3.286.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{\int \frac{-6agx^2+2(2cd-3af)x+5bd-6ae}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{-6agx^2+2(2cd-3af)x+5bd-6ae}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{-\frac{\int \frac{15db^2-18aeb-8a(2cd-3af)+2\left(12ga^2-6cea+5bcd\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{24fa^2-16cda-18bea+15b^2d+2\left(12ga^2-6cea+5bcd\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{3\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\frac{3\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\frac{3\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{2a^{3/2}}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/3*(d*Sqrt[a + b*x + c*x^2])/(a*x^3) - (-1/2*((5*b*d - 6*a*e)*Sqrt[a + b
*x + c*x^2])/(a*x^2) - (-(((15*b^2*d - 18*a*b*e - 8*a*(2*c*d - 3*a*f))*Sqr
t[a + b*x + c*x^2])/(a*x)) + (3*(5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*
f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(2*a^(3/2)))/(4*a))/(6*a)
```

3.286. $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.286. $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

3.286.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(24a^{2}f\,x^{2}-18abe\,x^{2}-16acd\,x^{2}+15b^{2}d\,x^{2}+12a^{2}ex-10abdx+8a^{2}d)}{24a^{3}x^{3}}-\frac{(16a^{3}g-8a^{2}bf-8a^{2}ce+6a\,b^{2}e+12abcd-5b^{3}}{16a^{\frac{7}{2}}}$
default	$-\frac{g\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+e\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{2a\,x^{2}}-\frac{3b\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx}{}\right.}{}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(24*a^2*f*x^2-18*a*b*e*x^2-16*a*c*d*x^2+15*b^2*d
*x^2+12*a^2*e*x-10*a*b*d*x+8*a^2*d)/a^3/x^3-1/16*(16*a^3*g-8*a^2*b*f-8*a^2
*c*e+6*a*b^2*e+12*a*b*c*d-5*b^3*d)/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{a}x^3\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a}{x^2}\right.}{96\,a}\right.$$
$$-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(8\,a}{48\,a^4x^3}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^
2*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^
3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2
 + b*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c
)*d - 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*
e - 24*a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*s
qrt(c*x^2 + b*x + a))/(a^4*x^3)]
```

3.286.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

3.286.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.286.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(164) = 328$.

Time = 0.31 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.66

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{\left(5\,b^3d-12\,abcd-6\,ab^2e+8\,a^2ce+8\,a^2bf-16\,a^3g\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^3}$$
$$+\frac{15\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5b^3d-36\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5abcd-18\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5ab^2e}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(5*b^3*d - 12*a*b*c*d - 6*a*b^2*e + 8*a^2*c*e + 8*a^2*b*f - 16*a^3*g)
*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/
24*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*b*c*d - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b
^2*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*e + 24*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a^2*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^3*sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d + 48*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^2*b^2*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*
f + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d + 48*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*e - 96*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^4*sqrt(c)*f + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
3*d + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d - 30*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^4*c*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*f + 48*a^3*b^2*sqrt(
c)*d - 32*a^4*c^(3/2)*d - 48*a^4*b*sqrt(c)*e + 48*a^5*sqrt(c)*f)/(((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3*a^3)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

3.287.1 Optimal result

Integrand size = 33, antiderivative size = 270

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx=-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3}-\frac{(35b^2d-36acd-40abe+48a^2f)\sqrt{a+bx+cx^2}}{96a^3x^2}+\frac{(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag))\sqrt{a+bx+cx^2}}{192a^4x}-\frac{(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{9/2}}$$

output
```
-1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f+3*c*d)-24*a*b^2*(-2*a*f+5*c*d
)+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/
2))/a^(9/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x
+a)^(1/2)/a^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)
^(1/2)/a^3/x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*
(-3*a*g+2*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
```

3.287.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(105b^3dx^3-10abx^2(7bd+22cdx+12bex)+8a^2x(7bd+cx(9d+16ex)+2bx(5e+9fx))-16a^3(3d+4ex+6x^2(f+2gx)))}{x^4}+105b^4d}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*
d*x + 12*b*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x
)) - 16*a^3*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/x^4 + 105*b^4*d*ArcTanh[(S
qrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 24*a*(5*b^3*e + 3*b^2*(5*c*d
- 2*a*f) + 2*a*c*(-3*c*d + 4*a*f) + 4*a*b*(-3*c*e + 2*a*g))*ArcTanh[(-(Sqr
t[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(9/2))
```

3.287.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2181, 27, 2181, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{-8agx^2+2(3cd-4af)x+7bd-8ae}{2x^4\sqrt{cx^2+bx+a}}dx}{4a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-8agx^2+2(3cd-4af)x+7bd-8ae}{x^4\sqrt{cx^2+bx+a}}dx}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$$\Big\downarrow \text{2181}$$

$$-\frac{-\frac{\int \frac{35db^2-40aeb-12a(3cd-4af)+4\left(12ga^2-8cea+7bcd\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{3ax^3}}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{48fa^2-36cda-40bea+35b^2d+4\left(12ga^2-8cea+7bcd\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{3ax^3}}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 1237

$$-\frac{-\frac{-\frac{\int \frac{105db^3-120aeb^2-4a(55cd-36af)b+64a^2(2ce-3ag)+2c\left(35db^2-40aeb-12a(3cd-4af)\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}}{6a}-\sqrt{a+bx+\cdots}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{-192ga^3+128cea^2+144bfa^2-220bcda-120b^2ea+105b^3d+2c\left(35db^2-40aeb-12a(3cd-4af)\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}}{6a}-\sqrt{a\cdots}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{3\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-192a^3g+144a^2bf+128a^2ce-120ab^2e-220c\cdots\right.}{ax}}{4a}}{6a}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-\frac{\frac{3\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(-192a^3g+144a^2bf+128a^2ce-12\cdots\right.}{ax}}{4a}}{6a}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\Big\downarrow$ 219

3.287. $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{-\frac{-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}-\frac{\frac{3\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)}{2a^{3/2}}-\cdots}{4a}}{6a}\cdots}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/4*(d*Sqrt[a + b*x + c*x^2])/(a*x^4) - (-1/3*((7*b*d - 8*a*e)*Sqrt[a + b
*x + c*x^2])/(a*x^3) - (-1/2*((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*
Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((105*b^3*d - 220*a*b*c*d - 120*a*b^2*
e + 128*a^2*c*e + 144*a^2*b*f - 192*a^3*g)*Sqrt[a + b*x + c*x^2])/(a*x)) +
 (3*(35*b^4*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d -
2*a*f) + 32*a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*a^(3/2)))/(4*a))/(6*a))/(8*a)
```

3.287.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.287.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(192a^3g\,x^3-144a^2bf\,x^3-128x^3a^2ce+120x^3a\,b^2e+220x^3abcd-105x^3b^3d+96a^3f\,x^2-80x^2a^2be-72x^2a^2cd+70x^2b^2d}{192a^4x^4}$
default	$d\left(-\frac{\sqrt{c\,x^2+bx+a}}{4a\,x^4}-\frac{7b\left(-\frac{\sqrt{c\,x^2+bx+a}}{3a\,x^3}-\frac{5b\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2}{}\right.}{}\right)}{6a}\right)}{8a}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(192*a^3*g*x^3-144*a^2*b*f*x^3-128*a^2*c*e*x^3+
120*a*b^2*e*x^3+220*a*b*c*d*x^3-105*b^3*d*x^3+96*a^3*f*x^2-80*a^2*b*e*x^2-
72*a^2*c*d*x^2+70*a*b^2*d*x^2+64*a^3*e*x-56*a^2*b*d*x+48*a^3*d)/a^4/x^4+1/
128*(64*a^3*b*g+64*a^3*c*f-48*a^2*b^2*f-96*a^2*b*c*e-48*a^2*c^2*d+40*a*b^3
*e+120*a*b^2*c*d-35*b^4*d)/a^(9/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)
```

3.287. $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

3.287.5 Fricas [A] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.94

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\left(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f\right)\sqrt{a}x^4\log\left(-\frac{8\,abx}{\ldots}\right.}{\ldots}\right.$$
$$-\frac{3\left(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f\right)\sqrt{-a}x^4\arctan\left(\ldots\right.}{\ldots}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x +
 (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/
x^2) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*
d - 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*
b^2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))
/(a^5*x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d
+ 8*(5*a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2
)) + 2*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d
- 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^
2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(
a^5*x^4)]
```

3.287.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

3.287.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.287.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. 2(244) = 488.

Time = 0.33 (sec) , antiderivative size = 1433, normalized size of antiderivative = 5.31

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d - 40*a*b^3*e + 96*a^2*b*c*e
+ 48*a^2*b^2*f - 64*a^3*c*f - 64*a^3*b*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d
+ 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d - 120*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a^2*b*c*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*f - 192*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^3*b*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt
(c)*g - 385*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^3*c^2*d + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e
 - 1056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 528*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*g - 768*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2)*e - 384*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*sqrt(c)*g + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d
 - 1752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^4*c^2*d - 584*(sqrt(c)*x - sqrt(c*x^2 + b...
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

3.288.1 Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}$$
$$-\frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3}$$
$$+\frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2}$$
$$-\frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx+cx^2}}{1920a^5x}$$
$$+\frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag))\,\mathrm{ar}}{256a^{11/2}}$$

output
```
1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c*d)-40*a*b^3*(-2*a*f+7*c*
d)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c*e))*arctanh(1/2*(b*x+2*a
)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/
40*(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64
*a*c*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x^3+1/960*(315*b^3*d-350*a*b^2*e-
4*a*b*(-100*a*f+161*c*d)+120*a^2*(-4*a*g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
^2-1/1920*(945*b^4*d-1050*a*b^3*e-60*a*b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*
a*f+4*c*d)+40*a^2*b*(-36*a*g+55*c*e))*(c*x^2+b*x+a)^(1/2)/a^5/x
```

3.288.2 Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\left(-945b^4dx^4+210ab^2x^3(3bd+14cdx+5bex)-32a^4\left(12d+5x\left(3e+4fx+6gx^2\right)\right)-4a^2x^2\left(256c^2dx^2+2bcx(161d+275ex)+b^2(126d+\right.\right.}{x^5}}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-945*b^4*d*x^4 + 210*a*b^2*x^3*(3*b*d + 1
4*c*d*x + 5*b*e*x) - 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) - 4*a^2*x
^2*(256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 1
2*f*x))) + 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e
+ 2*x*(5*f + 9*g*x))))))/x^5 - 15*(63*b^5*d + 128*a^4*c*g)*ArcTanh[(Sqrt[c
]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 30*a*(35*b^4*e + 48*a^2*c^2*e + 20
*b^3*(7*c*d - 2*a*f) + 24*a*b*c*(-5*c*d + 4*a*f) + 24*a*b^2*(-5*c*e + 2*a*
g))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(1920*a^(11/2
))
```

3.288.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2181, 27, 2181, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2181}$$
$$-\frac{\int \frac{-10agx^2+2(4cd-5af)x+9bd-10ae}{2x^5\sqrt{cx^2+bx+a}}dx}{5a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{-10agx^2+2(4cd-5af)x+9bd-10ae}{x^5\sqrt{cx^2+bx+a}}dx}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{2181}$$

$$-\frac{-\frac{\int \frac{63db^2-70aeb-16a(4cd-5af)+2\left(40ga^2-30cea+27bcd\right)x}{2x^4\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{4ax^4}}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{80fa^2-64cda-70bea+63b^2d+2\left(40ga^2-30cea+27bcd\right)x}{x^4\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{4ax^4}}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{-\frac{\int \frac{315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)+4c\left(63db^2-70aeb-16a(4cd-5af)\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}}{8a}-\frac{\sqrt{a+\cdots}}{\cdots}}{10a}$$
$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{\int \frac{-480ga^3+360cea^2+400bfa^2-644bcda-350b^2ea+315b^3d+4c\left(63db^2-70aeb-16a(4cd-5af)\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}}{8a}-\frac{\sqrt{a-\cdots}}{\cdots}}{10a}$$
$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{945db^4-1050aeb^3-60a(49cd-20af)b^2+40a^2(55ce-36ag)b+256a^2c(4cd-5af)+2c\left(315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+\cdots}}{\cdots}}{6a}}{8a}}{10a}$$
$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{945db^4-1050aeb^3-60a(49cd-20af)b^2+40a^2(55ce-36ag)b+256a^2c(4cd-5af)+2c\left(315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+\cdots}}{\cdots}}{6a}}{8a}}{10a}$$
$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \text{1228}$$

3.288. $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

$$-\frac{-\frac{-\frac{-\frac{15\left(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab^3(7cd-2af)+63b^5d\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(55ce-36ag)\cdots\right.}{\cdots}}{4a}\cdots}{6a}\cdots}{\cdots}\cdots$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$\big\downarrow$ 1154

$$-\frac{-\frac{-\frac{-\frac{15\left(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab^3(7cd-2af)+63b^5d\right)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(\cdots\right.}{\cdots}}{4a}\cdots}{6a}\cdots}{\cdots}\cdots$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$\big\downarrow$ 219

$$-\frac{-\frac{-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}-\frac{-\frac{\sqrt{a+bx+cx^2}\left(120a^2(3ce-4ag)-350ab^2e-4ab(161cd-100af)+315b^3d\right)}{2ax^2}-\frac{15\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\cdots}{\cdots}}{\cdots}}{\cdots}\cdots}{\cdots}\cdots$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/5*(d*Sqrt[a + b*x + c*x^2])/(a*x^5) - (-1/4*((9*b*d - 10*a*e)*Sqrt[a +
b*x + c*x^2])/(a*x^4) - (-1/3*((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)
*Sqrt[a + b*x + c*x^2])/(a*x^3) - (-1/2*((315*b^3*d - 350*a*b^2*e - 4*a*b*
(161*c*d - 100*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(a*x
^2) - (-(((945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2
*c*(4*c*d - 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(a
*x)) + (15*(63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*
(7*c*d - 2*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a))
/(6*a))/(8*a))/(10*a)
```

3.288. $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

3.288.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.288. $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

3.288.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,\left(-1440 a^{3} b g\,x^{4}-1280 a^{3} c f\,x^{4}+1200 a^{2} b^{2} f\,x^{4}+2200 x^{4} a^{2} b c e+1024 x^{4} a^{2} c^{2} d-1050 x^{4} a\,b^{3} e-2940 x^{4} a\,b^{2} c d+945 x^{4} b^{4} d+\right.}{}$
default	Expression too large to display

input

```
int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/1920*(c*x^2+b*x+a)^(1/2)*(-1440*a^3*b*g*x^4-1280*a^3*c*f*x^4+1200*a^2*b
^2*f*x^4+2200*a^2*b*c*e*x^4+1024*a^2*c^2*d*x^4-1050*a*b^3*e*x^4-2940*a*b^2
*c*d*x^4+945*b^4*d*x^4+960*a^4*g*x^3-800*a^3*b*f*x^3-720*a^3*c*e*x^3+700*a
^2*b^2*e*x^3+1288*a^2*b*c*d*x^3-630*a*b^3*d*x^3+640*a^4*f*x^2-560*a^3*b*e*
x^2-512*a^3*c*d*x^2+504*a^2*b^2*d*x^2+480*a^4*e*x-432*a^3*b*d*x+384*a^4*d)
/a^5/x^5+1/256*(128*a^4*c*g-96*a^3*b^2*g-192*a^3*b*c*f-96*a^3*c^2*e+80*a^2
*b^3*f+240*a^2*b^2*c*e+240*a^2*b*c^2*d-70*a*b^4*e-280*a*b^3*c*d+63*b^5*d)/
a^(11/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 11.94 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[\frac{15\left((63\,b^5-280\,ab^3c+240\,a^2bc^2)d-2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)e+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\,a\right.}{}\right.$$
$$-\frac{15\left((63\,b^5-280\,ab^3c+240\,a^2bc^2)d-2\,(35\,ab^4-120\,a^2b^2c+48\,a^3c^2)e+16\,(5\,a^2b^3-12\,a^3bc)f-32\,(3\right.}{}$$

input

```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*
a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2
- 4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b
*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a
^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g
 + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(
70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 1
0*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a
*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e +
16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^
2)) + 2*(384*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^
3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*
x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*
a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a
^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^
5)]
```

3.288.6 Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

3.288.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.288.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2155 vs. $2(341) = 682$.

Time = 0.31 (sec) , antiderivative size = 2155, normalized size of antiderivative = 5.81

$$\int \frac{d + ex + fx^2 + gx^3}{x^6\sqrt{a + bx + cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d - 70*a*b^4*e + 240*a^2*
b^2*c*e - 96*a^3*c^2*e + 80*a^2*b^3*f - 192*a^3*b*c*f - 96*a^3*b^2*g + 128
*a^4*c*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*
a^5) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b*c^2*d - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*
b^4*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^2*e + 1200*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b^3*f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*
b*c*f - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*c*g - 4410*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c
*d - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d + 4900*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^3*b^2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4
*c^2*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a^4*b^2*g - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*
c*g + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - s...
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

3.289 $\int (d+ex)^3 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

3.289.1 Optimal result

Integrand size = 36, antiderivative size = 258

$$\begin{aligned}\int (d+ex)^3 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx &= \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^7}\\ &\quad-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^5}{5e^7}\\ &\quad+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^6}{6e^7}\\ &\quad-\frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^7}{7e^7}\\ &\quad+\frac{(300d^2+85de+17e^2)(d+ex)^8}{8e^7}-\frac{(120d+17e)(d+ex)^9}{9e^7}+\frac{2(d+ex)^{10}}{e^7}\end{aligned}$$

output
```
1/4*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e^
7-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^5/e^
7+1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^6/e^7-2/7*(2
00*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+85*d*e+17*e^2)*
(e*x+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^7
```

3.289.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 6d^3x+\frac{1}{2}d^2(7d+18e)x^2+d\left(7d^2+7de+6e^2\right)x^3$$
$$+\frac{1}{4}\left(-4d^3+63d^2e+21de^2+6e^3\right)x^4+\frac{1}{5}\left(17d^3-12d^2e+63de^2+7e^3\right)x^5$$
$$+\frac{1}{6}\left(-17d^3+51d^2e-12de^2+21e^3\right)x^6+\frac{1}{7}\left(20d^3-51d^2e+51de^2-4e^3\right)x^7$$
$$+\frac{1}{8}e\left(60d^2-51de+17e^2\right)x^8+\frac{1}{9}(60d-17e)e^2x^9+2e^3x^{10}$$

input
```
Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*
d^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2
 + 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20
*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)
*x^8)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

3.289.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^3\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^7}{e^6}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^6}{e^6}+\frac{\left(300d^4+170d^3e+102d^2e^2\right.}{e}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(300d^2+85de+17e^2)(d+ex)^8}{8e^7}-\frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^7}{7e^7}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^6}{6e^7}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^7}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^5}{5e^7}+\frac{2(d+ex)^{10}}{e^7}-\frac{(120d+17e)(d+ex)^9}{9e^7}$$

input
```
Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
+ e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*
e^4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 +
12*d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^
2 + 2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8
)/(8*e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

3.289.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.289.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.78

method	result
norman	$2e^3x^{10}+\left(\frac{20}{3}d\,e^2-\frac{17}{9}e^3\right)x^9+\left(\frac{15}{2}d^2e-\frac{51}{8}d\,e^2+\frac{17}{8}e^3\right)x^8+\left(\frac{20}{7}d^3-\frac{51}{7}d^2e+\frac{51}{7}d\,e^2-\frac{4}{7}e^3\right)x$
default	$2e^3x^{10}+\frac{(60d\,e^2-17e^3)x^9}{9}+\frac{(60d^2e-51d\,e^2+17e^3)x^8}{8}+\frac{(20d^3-51d^2e+51d\,e^2-4e^3)x^7}{7}+\frac{(-17d^3+51d^2e-12d\,e^2+2}{6}$
gosper	$7d^2e\,x^3+\frac{17}{2}x^6d^2e-2x^6d\,e^2-\frac{12}{5}x^5d^2e+\frac{63}{5}x^5d\,e^2+\frac{63}{4}x^4d^2e+\frac{21}{4}x^4d\,e^2-\frac{51}{7}x^7d^2e+\frac{51}{7}x^7d$
risch	$7d^2e\,x^3+\frac{17}{2}x^6d^2e-2x^6d\,e^2-\frac{12}{5}x^5d^2e+\frac{63}{5}x^5d\,e^2+\frac{63}{4}x^4d^2e+\frac{21}{4}x^4d\,e^2-\frac{51}{7}x^7d^2e+\frac{51}{7}x^7d$
parallelrisch	$7d^2e\,x^3+\frac{17}{2}x^6d^2e-2x^6d\,e^2-\frac{12}{5}x^5d^2e+\frac{63}{5}x^5d\,e^2+\frac{63}{4}x^4d^2e+\frac{21}{4}x^4d\,e^2-\frac{51}{7}x^7d^2e+\frac{51}{7}x^7d$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
2*e^3*x^10+(20/3*d*e^2-17/9*e^3)*x^9+(15/2*d^2*e-51/8*d*e^2+17/8*e^3)*x^8+
(20/7*d^3-51/7*d^2*e+51/7*d*e^2-4/7*e^3)*x^7+(-17/6*d^3+17/2*d^2*e-2*d*e^2
+7/2*e^3)*x^6+(17/5*d^3-12/5*d^2*e+63/5*d*e^2+7/5*e^3)*x^5+(-d^3+63/4*d^2*
e+21/4*d*e^2+3/2*e^3)*x^4+(7*d^3+7*d^2*e+6*d*e^2)*x^3+(7/2*d^3+9*d^2*e)*x^
2+6*x*d^3
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

$$\begin{aligned}\int (d+ex)^3 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx &= 2e^3x^{10}+\frac{1}{9}(60de^2-17e^3)x^9+\frac{1}{8}(60d^2e-51de^2+17e^3)x^8\\ &+\frac{1}{7}(20d^3-51d^2e+51de^2-4e^3)x^7-\frac{1}{6}(17d^3-51d^2e+12de^2-21e^3)x^6\\ &+\frac{1}{5}(17d^3-12d^2e+63de^2+7e^3)x^5-\frac{1}{4}(4d^3-63d^2e-21de^2-6e^3)x^4\\ &+6d^3x+(7d^3+7d^2e+6de^2)x^3+\frac{1}{2}(7d^3+18d^2e)x^2\end{aligned}$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e
^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 5
1*d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e
^3)*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3
 + 7*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

3.289.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.89

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = 6d^3x + 2e^3x^{10} + x^9 \cdot \left(\frac{20de^2}{3} - \frac{17e^3}{9}\right) + x^8 \cdot \left(\frac{15d^2e}{2} - \frac{51de^2}{8} + \frac{17e^3}{8}\right) + x^7 \cdot \left(\frac{20d^3}{7} - \frac{51d^2e}{7} + \frac{51de^2}{7} - \frac{4e^3}{7}\right) + x^6\left(-\frac{17d^3}{6} + \frac{17d^2e}{2} - 2de^2 + \frac{7e^3}{2}\right) + x^5 \cdot \left(\frac{17d^3}{5} - \frac{12d^2e}{5} + \frac{63de^2}{5} + \frac{7e^3}{5}\right) + x^4\left(-d^3 + \frac{63d^2e}{4} + \frac{21de^2}{4} + \frac{3e^3}{2}\right) + x^3 \cdot \left(7d^3 + 7d^2e + 6de^2\right) + x^2 \cdot \left(\frac{7d^3}{2} + 9d^2e\right)$$

input
```
integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e
/2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/
7 - 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x*
*5*(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d
**2*e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) +
x**2*(7*d**3/2 + 9*d**2*e)
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = 2\,e^3x^{10} + \frac{1}{9}\left(60\,de^2 - 17\,e^3\right)x^9 + \frac{1}{8}\left(60\,d^2e - 51\,de^2 + 17\,e^3\right)x^8 + \frac{1}{7}\left(20\,d^3 - 51\,d^2e + 51\,de^2 - 4\,e^3\right)x^7 - \frac{1}{6}\left(17\,d^3 - 51\,d^2e + 12\,de^2 - 21\,e^3\right)x^6 + \frac{1}{5}\left(17\,d^3 - 12\,d^2e + 63\,de^2 + 7\,e^3\right)x^5 - \frac{1}{4}\left(4\,d^3 - 63\,d^2e - 21\,de^2 - 6\,e^3\right)x^4 + 6\,d^3x + \left(7\,d^3 + 7\,d^2e + 6\,de^2\right)x^3 + \frac{1}{2}\left(7\,d^3 + 18\,d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e
^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 5
1*d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e
^3)*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3
 + 7*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.92

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 2\,e^3x^{10} + \frac{20}{3}\,de^2x^9 - \frac{17}{9}\,e^3x^9 + \frac{15}{2}\,d^2ex^8 - \frac{51}{8}\,de^2x^8 + \frac{17}{8}\,e^3x^8 + \frac{20}{7}\,d^3x^7 - \frac{51}{7}\,d^2ex^7 + \frac{51}{7}\,de^2x^7 - \frac{4}{7}\,e^3x^7 - \frac{17}{6}\,d^3x^6 + \frac{17}{2}\,d^2ex^6 - 2\,de^2x^6 + \frac{7}{2}\,e^3x^6 + \frac{17}{5}\,d^3x^5 - \frac{12}{5}\,d^2ex^5 + \frac{63}{5}\,de^2x^5 + \frac{7}{5}\,e^3x^5 - d^3x^4 + \frac{63}{4}\,d^2ex^4 + \frac{21}{4}\,de^2x^4 + \frac{3}{2}\,e^3x^4 + 7\,d^3x^3 + 7\,d^2ex^3 + 6\,de^2x^3 + \frac{7}{2}\,d^3x^2 + 9\,d^2ex^2 + 6\,d^3x$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
2*e^3*x^10 + 20/3*d*e^2*x^9 - 17/9*e^3*x^9 + 15/2*d^2*e*x^8 - 51/8*d*e^2*x
^8 + 17/8*e^3*x^8 + 20/7*d^3*x^7 - 51/7*d^2*e*x^7 + 51/7*d*e^2*x^7 - 4/7*e
^3*x^7 - 17/6*d^3*x^6 + 17/2*d^2*e*x^6 - 2*d*e^2*x^6 + 7/2*e^3*x^6 + 17/5*
d^3*x^5 - 12/5*d^2*e*x^5 + 63/5*d*e^2*x^5 + 7/5*e^3*x^5 - d^3*x^4 + 63/4*d
^2*e*x^4 + 21/4*d*e^2*x^4 + 3/2*e^3*x^4 + 7*d^3*x^3 + 7*d^2*e*x^3 + 6*d*e^
2*x^3 + 7/2*d^3*x^2 + 9*d^2*e*x^2 + 6*d^3*x
```

3.289.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.76

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 6\,d^3\,x + x^8\left(\frac{15\,d^2\,e}{2} - \frac{51\,d\,e^2}{8} + \frac{17\,e^3}{8}\right) - x^6\left(\frac{17\,d^3}{6} - \frac{17\,d^2\,e}{2} + 2\,d\,e^2 - \frac{7\,e^3}{2}\right) + x^4\left(-d^3 + \frac{63\,d^2\,e}{4} + \frac{21\,d\,e^2}{4} + \frac{3\,e^3}{2}\right) + x^5\left(\frac{17\,d^3}{5} - \frac{12\,d^2\,e}{5} + \frac{63\,d\,e^2}{5} + \frac{7\,e^3}{5}\right) + x^7\left(\frac{20\,d^3}{7} - \frac{51\,d^2\,e}{7} + \frac{51\,d\,e^2}{7} - \frac{4\,e^3}{7}\right) + 2\,e^3\,x^{10} + d\,x^3\left(7\,d^2 + 7\,d\,e + 6\,e^2\right) + \frac{d^2\,x^2\,(7\,d + 18\,e)}{2} + \frac{e^2\,x^9\,(60\,d - 17\,e)}{9}$$

input
```
int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 -
(17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4
- d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^
3)/5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3
*x^10 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^
9*(60*d - 17*e))/9
```

3.290 $\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

3.290.1 Optimal result

Integrand size = 36, antiderivative size = 157

$$\begin{aligned}\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx &= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}(21d^2+14de+6e^2)x^3-\frac{1}{4}(4d^2-42de-7e^2)x^4\\ &+\frac{1}{5}(17d^2-8de+21e^2)x^5-\frac{1}{6}(17d^2-34de+4e^2)x^6\\ &+\frac{1}{7}(20d^2-34de+17e^2)x^7+\frac{1}{8}(40d-17e)ex^8+\frac{20e^2x^9}{9}\end{aligned}$$

output
```
6*d^2*x+1/2*d*(7*d+12*e)*x^2+1/3*(21*d^2+14*d*e+6*e^2)*x^3-1/4*(4*d^2-42*d
*e-7*e^2)*x^4+1/5*(17*d^2-8*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+
1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9
```

3.290.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\begin{aligned}\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx &= \frac{e^2x^3(5040+4410x+10584x^2-1680x^3+6120x^4-5355x^5+5600x^6)}{2520}\\ &+d^2\left(6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\right)\\ &+de\left(6x^2+\frac{14x^3}{3}+\frac{21x^4}{2}-\frac{8x^5}{5}+\frac{17x^6}{3}-\frac{34x^7}{7}+5x^8\right)\end{aligned}$$

input
```
Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
(e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 560
0*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/
6 + (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x
^6)/3 - (34*x^7)/7 + 5*x^8)
```

3.290.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)(d+ex)^2\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^6(20d^2-34de+17e^2)-x^5(17d^2-34de+4e^2)+x^4(17d^2-8de+21e^2)-x^3(4d^2-42de-7e^2)+x^2(21d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(20d^2-34de+17e^2)-\frac{1}{6}x^6(17d^2-34de+4e^2)+\frac{1}{5}x^5(17d^2-8de+21e^2)-\frac{1}{4}x^4(4d^2-42de-7e^2)+\frac{1}{3}x^3(21d^2+14de+6e^2)+6d^2x+\frac{1}{8}ex^8(40d-17e)+\frac{1}{2}dx^2(7d+12e)+\frac{20e^2x^9}{9}$$

input
```
Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4
*d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d
^2 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d
- 17*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

3.290.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.290.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result
norman	$\frac{20e^2x^9}{9}+\left(5de-\frac{17}{8}e^2\right)x^8+\left(\frac{20}{7}d^2-\frac{34}{7}de+\frac{17}{7}e^2\right)x^7+\left(-\frac{17}{6}d^2+\frac{17}{3}de-\frac{2}{3}e^2\right)x^6+\left(\frac{17}{5}d^2-\frac{8}{5}\right.$
default	$\frac{20e^2x^9}{9}+\frac{(40de-17e^2)x^8}{8}+\frac{(20d^2-34de+17e^2)x^7}{7}+\frac{(-17d^2+34de-4e^2)x^6}{6}+\frac{(17d^2-8de+21e^2)x^5}{5}+\frac{(-4d^2+42de}{4}$
gosper	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2$
risch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2$
parallelrisch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d^2$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
20/9*e^2*x^9+(5*d*e-17/8*e^2)*x^8+(20/7*d^2-34/7*d*e+17/7*e^2)*x^7+(-17/6*
d^2+17/3*d*e-2/3*e^2)*x^6+(17/5*d^2-8/5*d*e+21/5*e^2)*x^5+(-d^2+21/2*d*e+7
/4*e^2)*x^4+(7*d^2+14/3*d*e+2*e^2)*x^3+(7/2*d^2+6*d*e)*x^2+6*x*d^2
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int(d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{20}{9}\,e^2x^9+\frac{1}{8}\left(40\,de-17\,e^2\right)x^8+\frac{1}{7}\left(20\,d^2-34\,de+17\,e^2\right)x^7\\&\quad-\frac{1}{6}\left(17\,d^2-34\,de+4\,e^2\right)x^6+\frac{1}{5}\left(17\,d^2-8\,de+21\,e^2\right)x^5-\frac{1}{4}\left(4\,d^2-42\,de-7\,e^2\right)x^4\\&\quad+\frac{1}{3}\left(21\,d^2+14\,de+6\,e^2\right)x^3+6\,d^2x+\frac{1}{2}\left(7\,d^2+12\,de\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*
x^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^
5 - 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 +
 6*d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

3.290.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int (d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= 6d^2x+\frac{20e^2x^9}{9}+x^8\cdot\left(5de-\frac{17e^2}{8}\right)+x^7\cdot\left(\frac{20d^2}{7}-\frac{34de}{7}+\frac{17e^2}{7}\right)+x^6\left(-\frac{17d^2}{6}+\frac{17de}{3}-\frac{2e^2}{3}\right)+x^5\cdot\left(\frac{17d^2}{5}-\frac{8de}{5}+\frac{21e^2}{5}\right)+x^4\left(-d^2+\frac{21de}{2}+\frac{7e^2}{4}\right)+x^3\cdot\left(7d^2+\frac{14de}{3}+2e^2\right)+x^2\cdot\left(\frac{7d^2}{2}+6de\right)$$

input
```
integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 3
4*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*
d**2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*
(7*d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int (d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{20}{9}\,e^2x^9+\frac{1}{8}\left(40\,de-17\,e^2\right)x^8+\frac{1}{7}\left(20\,d^2-34\,de+17\,e^2\right)x^7-\frac{1}{6}\left(17\,d^2-34\,de+4\,e^2\right)x^6+\frac{1}{5}\left(17\,d^2-8\,de+21\,e^2\right)x^5-\frac{1}{4}\left(4\,d^2-42\,de-7\,e^2\right)x^4+\frac{1}{3}\left(21\,d^2+14\,de+6\,e^2\right)x^3+6\,d^2x+\frac{1}{2}\left(7\,d^2+12\,de\right)x^2$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*
x^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^
5 - 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 +
 6*d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = \frac{20}{9} e^2x^9 + 5\,dex^8 - \frac{17}{8} e^2x^8 + \frac{20}{7} d^2x^7 - \frac{34}{7} dex^7 + \frac{17}{7} e^2x^7 - \frac{17}{6} d^2x^6 + \frac{17}{3} dex^6 - \frac{2}{3} e^2x^6 + \frac{17}{5} d^2x^5 - \frac{8}{5} dex^5 + \frac{21}{5} e^2x^5 - d^2x^4 + \frac{21}{2} dex^4 + \frac{7}{4} e^2x^4 + 7\,d^2x^3 + \frac{14}{3} dex^3 + 2\,e^2x^3 + \frac{7}{2} d^2x^2 + 6\,dex^2 + 6\,d^2x$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
20/9*e^2*x^9 + 5*d*e*x^8 - 17/8*e^2*x^8 + 20/7*d^2*x^7 - 34/7*d*e*x^7 + 17
/7*e^2*x^7 - 17/6*d^2*x^6 + 17/3*d*e*x^6 - 2/3*e^2*x^6 + 17/5*d^2*x^5 - 8/
5*d*e*x^5 + 21/5*e^2*x^5 - d^2*x^4 + 21/2*d*e*x^4 + 7/4*e^2*x^4 + 7*d^2*x^
3 + 14/3*d*e*x^3 + 2*e^2*x^3 + 7/2*d^2*x^2 + 6*d*e*x^2 + 6*d^2*x
```

3.290.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = x^3 \left(7\,d^2 + \frac{14\,d\,e}{3} + 2\,e^2\right) + x^4 \left(-d^2 + \frac{21\,d\,e}{2} + \frac{7\,e^2}{4}\right) - x^6 \left(\frac{17\,d^2}{6} - \frac{17\,d\,e}{3} + \frac{2\,e^2}{3}\right) + x^5 \left(\frac{17\,d^2}{5} - \frac{8\,d\,e}{5} + \frac{21\,e^2}{5}\right) + x^7 \left(\frac{20\,d^2}{7} - \frac{34\,d\,e}{7} + \frac{17\,e^2}{7}\right) + 6\,d^2\,x + \frac{20\,e^2\,x^9}{9} + \frac{d\,x^2\,(7\,d + 12\,e)}{2} + \frac{e\,x^8\,(40\,d - 17\,e)}{8}$$

input
```
int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^
6*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (2
1*e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2
*x^9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

3.290. $\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx$

3.291 $\int (d+ex)(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,d$

3.291.1 Optimal result

Integrand size = 34, antiderivative size = 93

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4\\ &\quad+\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}\end{aligned}$$

output
```
6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*
x^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

3.291.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\ &= 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3+\frac{1}{4}(-4d+21e)x^4\\ &\quad+\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}\end{aligned}$$

input
```
Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4
+ ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e
*x^8)/2
```

3.291.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^6(20d-17e)-17x^5(d-e)+x^4(17d-4e)-x^3(4d-21e)+7x^2(3d+e)+x(7d+6e)+6d+20ex^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

input
```
Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*
x^8)/2
```

3.291.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.291.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

method	result
norman	$\frac{5e\,x^8}{2}+\left(\frac{20d}{7}-\frac{17e}{7}\right)x^7+\left(-\frac{17d}{6}+\frac{17e}{6}\right)x^6+\left(\frac{17d}{5}-\frac{4e}{5}\right)x^5+\left(-d+\frac{21e}{4}\right)x^4+\left(7d+\frac{7e}{3}\right)x^3+($
gosper	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3-$
default	$\frac{5e\,x^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+\frac{(7d+6e)x^2}{2}+6dx$
risch	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3-$
parallelrisch	$\frac{5}{2}e\,x^8+\frac{20}{7}d\,x^7-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3-$

input
```
int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*e*x^8+(20/7*d-17/7*e)*x^7+(-17/6*d+17/6*e)*x^6+(17/5*d-4/5*e)*x^5+(-d+
21/4*e)*x^4+(7*d+7/3*e)*x^3+(7/2*d+3*e)*x^2+6*d*x
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5$$
$$-\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fric
as")
```

output
```
5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^
5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

3.291.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 6dx+\frac{5ex^8}{2}+x^7\cdot\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\cdot\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\cdot\left(7d+\frac{7e}{3}\right)+x^2\cdot\left(\frac{7d}{2}+3e\right)$$

input
```
integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x*
*5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/
2 + 3*e)
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5-\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxi
ma")
```

output
```
5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^
5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{5}{2}\,ex^8+\frac{20}{7}\,dx^7-\frac{17}{7}\,ex^7-\frac{17}{6}\,dx^6+\frac{17}{6}\,ex^6+\frac{17}{5}\,dx^5-\frac{4}{5}\,ex^5-dx^4+\frac{21}{4}\,ex^4+7\,dx^3+\frac{7}{3}\,ex^3+\frac{7}{2}\,dx^2+3\,ex^2+6\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac
")
```

output
```
5/2*e*x^8 + 20/7*d*x^7 - 17/7*e*x^7 - 17/6*d*x^6 + 17/6*e*x^6 + 17/5*d*x^5
 - 4/5*e*x^5 - d*x^4 + 21/4*e*x^4 + 7*d*x^3 + 7/3*e*x^3 + 7/2*d*x^2 + 3*e*
x^2 + 6*d*x
```

3.291.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=\frac{5\,e\,x^8}{2}+\left(\frac{20\,d}{7}-\frac{17\,e}{7}\right)\,x^7+\left(\frac{17\,e}{6}-\frac{17\,d}{6}\right)\,x^6+\left(\frac{17\,d}{5}-\frac{4\,e}{5}\right)\,x^5+\left(\frac{21\,e}{4}-d\right)\,x^4+\left(7\,d+\frac{7\,e}{3}\right)\,x^3+\left(\frac{7\,d}{2}+3\,e\right)\,x^2+6\,d\,x$$

input
```
int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6
*((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 -
 x^4*(d - (21*e)/4)
```

3.292 $\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx$

3.292.1 Optimal result

Integrand size = 29, antiderivative size = 42

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

output
```
6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

3.292.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

input
```
Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

3.292.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+2x+3)\left(4x^4-5x^3+3x^2+x+2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(20x^6-17x^5+17x^4-4x^3+21x^2+7x+6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

input
```
Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

3.292.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.292.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
gosper	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
default	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
norman	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
risch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
parallelrisch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

3.292.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int (3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

3.292.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

input
```
int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.293.1 Optimal result

Integrand size = 36, antiderivative size = 228

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx \\
&= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6} \\
&\quad+\frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x^2}{2e^5}-\frac{(20d^3+17d^2e+17de^2+4e^3)\,x^3}{3e^4} \\
&\quad+\frac{(20d^2+17de+17e^2)\,x^4}{4e^3}-\frac{(20d+17e)x^5}{5e^2}+\frac{10x^6}{3e} \\
&\quad+\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}
\end{aligned}$$

output
```
-(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+1
7*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4
*e^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+1
0/3*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x
+d)/e^7
```

3.293.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{ex(-1200d^5+60d^4e(-17+10x)-10d^3e^2(102-51x+40x^2)+10d^2e^3(-24+51x-34x^2+30x^3)-5d}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
(e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
 + 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^
2 - 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200
*x^5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d
*e^5 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

3.293.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)}{d+ex}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x^3(20d^2+17de+17e^2)}{e^3}-\frac{x^2(20d^3+17d^2e+17de^2+4e^3)}{e^4}+\frac{x(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{e^5}+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}+\frac{x^2(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{2e^5}-\frac{x(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)}{e^6}-\frac{x^5(20d+17e)}{5e^2}+\frac{10x^6}{3e}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
-(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
 + ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((2
0*d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17
*e^2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^
2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e
*x])/e^7
```

3.293.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.293.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96

method	result
norman	$\frac{10x^6}{3e}-\frac{(20d+17e)x^5}{5e^2}+\frac{(20d^2+17de+17e^2)x^4}{4e^3}-\frac{(20d^3+17d^2e+17d\,e^2+4e^3)x^3}{3e^4}+\frac{(20d^4+17d^3e+17d^2e^2+4d\,e^3+21e^4)}{2e^5}$
default	$-\frac{-\frac{10}{3}e^5x^6+4d\,e^4x^5+\frac{17}{5}e^5x^5-5d^2e^3x^4-\frac{17}{4}d\,e^4x^4-\frac{17}{4}e^5x^4+\frac{20}{3}d^3e^2x^3+\frac{17}{3}d^2e^3x^3+\frac{17}{3}d\,e^4x^3+\frac{4}{3}e^5x^3-10d^4e\,x^2-\frac{17}{2}d^3e^2x^2}{e^6}$
parallelrisch	$\frac{-204e^6x^5+200e^6x^6-400d^3x^3e^3-1200d^5ex+600d^4e^2x^2+300d^2e^4x^4-240d\,e^5x^5+255x^4e^6-80x^3e^6+630x^2e^6+420x\,e^6+1200}{}$
risch	$-\frac{17x^5}{5e}+\frac{17\ln(ex+d)d^4}{e^5}+\frac{4\ln(ex+d)d^3}{e^4}+\frac{21\ln(ex+d)d^2}{e^3}-\frac{7\ln(ex+d)d}{e^2}+\frac{20\ln(ex+d)d^6}{e^7}+\frac{17\ln(ex+d)d^5}{e^6}+$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
10/3*x^6/e-1/5*(20*d+17*e)*x^5/e^2+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/3*
(20*d^3+17*d^2*e+17*d*e^2+4*e^3)*x^3/e^4+1/2*(20*d^4+17*d^3*e+17*d^2*e^2+4
*d*e^3+21*e^4)*x^2/e^5-(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^
5)*x/e^6+(20*d^6+17*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e
^7*ln(e*x+d)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^6x^6 - 12\,(20\,de^5 + 17\,e^6)x^5 + 15\,(20\,d^2e^4 + 17\,de^5 + 17\,e^6)x^4 - 20\,(20\,d^3e^3 + 17\,d^2e^4 + 17\,de^5 + 4\,e^6)}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fric
as")
```

output
```
1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
 + 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*
(20*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5
*e + 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^
6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(
e*x + d))/e^7
```

3.293.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\cdot\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2\cdot\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+x\left(-\frac{20d^5}{e^6}-\frac{17d^4}{e^5}-\frac{17d^3}{e^4}-\frac{4d^2}{e^3}-\frac{21d}{e^2}+\frac{7}{e}\right)+\frac{10x^6}{3e}+\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

output
```
x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e)
) + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e))
+ x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21
/(2*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 2
1*d/e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d*
*3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^5x^6 - 12\,(20\,de^4 + 17\,e^5)x^5 + 15\,(20\,d^2e^3 + 17\,de^4 + 17\,e^5)x^4 - 20\,(20\,d^3e^2 + 17\,d^2e^3 + 17\,de^4 + 4\,e^5)}{}$$
$$+ \frac{(20\,d^6 + 17\,d^5e + 17\,d^4e^2 + 4\,d^3e^3 + 21\,d^2e^4 - 7\,de^5 + 6\,e^6)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxi
ma")
```

output
```
1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
 + 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*
(20*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 +
 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x
+ d)/e^7
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^5x^6 - 240\,de^4x^5 - 204\,e^5x^5 + 300\,d^2e^3x^4 + 255\,de^4x^4 + 255\,e^5x^4 - 400\,d^3e^2x^3 - 340\,d^2e^3x^3 - 340\,de}{}$$
$$+ \frac{(20\,d^6 + 17\,d^5e + 17\,d^4e^2 + 4\,d^3e^3 + 21\,d^2e^4 - 7\,de^5 + 6\,e^6)\log(|ex+d|)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac
")
```

output
```
1/60*(200*e^5*x^6 - 240*d*e^4*x^5 - 204*e^5*x^5 + 300*d^2*e^3*x^4 + 255*d*
e^4*x^4 + 255*e^5*x^4 - 400*d^3*e^2*x^3 - 340*d^2*e^3*x^3 - 340*d*e^4*x^3
- 80*e^5*x^3 + 600*d^4*e*x^2 + 510*d^3*e^2*x^2 + 510*d^2*e^3*x^2 + 120*d*e
^4*x^2 + 630*e^5*x^2 - 1200*d^5*x - 1020*d^4*e*x - 1020*d^3*e^2*x - 240*d^
2*e^3*x - 1260*d*e^4*x + 420*e^5*x)/e^6 + (20*d^6 + 17*d^5*e + 17*d^4*e^2
+ 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(abs(e*x + d))/e^7
```

3.293. $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.293.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)$$
$$-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right)+x^2\left(\frac{21}{2\,e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{2\,e}\right)$$
$$+\frac{10\,x^6}{3\,e}+\frac{\ln(d+e\,x)\,(20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6)}{e^7}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

output
```
x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e
))/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e
))/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) +
 x^2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e
)) + (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 +
 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

3.294.1 Optimal result

Integrand size = 36, antiderivative size = 228

$$\begin{aligned}&\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx\\&=\frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6}-\frac{(80d^3+51d^2e+34de^2+4e^3)\,x^2}{2e^5}\\&\quad+\frac{(60d^2+34de+17e^2)\,x^3}{3e^4}-\frac{(40d+17e)x^4}{4e^3}+\frac{4x^5}{e^2}\\&\quad-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}\\&\quad-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}\end{aligned}$$

output
```
(100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34
*d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x
^4/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)
/e^7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*
x+d)/e^7
```

3.294.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{12e(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x-6e^2(80d^3+51d^2e+34de^2+4e^3)\,x^2+4e^3(60d^2+34de+}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,
x]
```

output
```
(12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d
^3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x
^3 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^
4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5
 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(1
2*e^7)
```

3.294.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)\,(4x^4-5x^3+3x^2+x+2)}{(d+ex)^2}\,dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{x^2(60d^2+34de+17e^2)}{e^4}-\frac{x(80d^3+51d^2e+34de^2+4e^3)}{e^5}+\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6}+\frac{-12\cdots}{}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(80d^3+51d^2e+34de^2+4e^3)}{2e^5}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}+\frac{x(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)}{e^6}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}-\frac{x^4(40d+17e)}{4e^3}+\frac{4x^5}{e^2}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

output
```
((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 +
51*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^
3)/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e +
 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - (
(120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d +
e*x])/e^7
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.294.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\frac{(120d^6+85d^5e+68d^4e^2+12d^3e^3+42d^2e^4-7de^5+6e^6)x}{e^6 d}+\frac{4x^6}{e}-\frac{(24d+17e)x^5}{4e^2}+\frac{(120d^2+85de+68e^2)x^4}{12e^3}-\frac{(120d^3+85d^2e+68de^2+12e^3)x^3}{6e^4}}{ex+d}$
default	$\frac{4e^4x^5-10de^3x^4-\frac{17}{4}e^4x^4+20d^2e^2x^3+\frac{34}{3}de^3x^3+\frac{17}{3}e^4x^3-40d^3ex^2-\frac{51}{2}d^2e^2x^2-17de^3x^2-2e^4x^2+100d^4x+68d^3ex+51d^2e^2x}{e^6}$
risch	$\frac{7\ln(ex+d)}{e^2}-\frac{17x^4}{4e^2}-\frac{120\ln(ex+d)d^5}{e^7}-\frac{85\ln(ex+d)d^4}{e^6}-\frac{68\ln(ex+d)d^3}{e^5}-\frac{12\ln(ex+d)d^2}{e^4}-\frac{42\ln(ex+d)d}{e^3}-\frac{10dx}{e^3}$
parallelrisch	$-\frac{51e^6x^5+72e^6-48e^6x^6+1020d^5e+504\ln(ex+d)xde^5+1440\ln(ex+d)xd^5e+1020\ln(ex+d)xd^4e^2+816\ln(ex+d)xd^3e^3+144}{}$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBOS
E)
```

output
```
((120*d^6+85*d^5*e+68*d^4*e^2+12*d^3*e^3+42*d^2*e^4-7*d*e^5+6*e^6)/e^6/d*x
+4*x^6/e-1/4*(24*d+17*e)/e^2*x^5+1/12*(120*d^2+85*d*e+68*e^2)/e^3*x^4-1/6*
(120*d^3+85*d^2*e+68*d*e^2+12*e^3)/e^4*x^3+1/2*(120*d^4+85*d^3*e+68*d^2*e^
2+12*d*e^3+42*e^4)/e^5*x^2)/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^
3+42*d*e^4-7*e^5)*ln(e*x+d)/e^7
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.40

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{48\,e^6x^6 - 240\,d^6 - 204\,d^5e - 204\,d^4e^2 - 48\,d^3e^3 - 252\,d^2e^4 + 84\,de^5 - 72\,e^6 - 3\,(24\,de^5 + 17\,e^6)x^5 + (120}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fr
icas")
```

output
```
1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^
2*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*
d*e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3
 + 6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*
(100*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d
^6 + 85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*
e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x +
d))/(e^8*x + d*e^7)
```

3.294.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= x^4\left(-\frac{10d}{e^3}-\frac{17}{4e^2}\right)+x^3\cdot\left(\frac{20d^2}{e^4}+\frac{34d}{3e^3}+\frac{17}{3e^2}\right)$$
$$+x^2\left(-\frac{40d^3}{e^5}-\frac{51d^2}{2e^4}-\frac{17d}{e^3}-\frac{2}{e^2}\right)+x\left(\frac{100d^4}{e^6}+\frac{68d^3}{e^5}+\frac{51d^2}{e^4}+\frac{8d}{e^3}+\frac{21}{e^2}\right)$$
$$+\frac{-20d^6-17d^5e-17d^4e^2-4d^3e^3-21d^2e^4+7de^5-6e^6}{de^7+e^8x}+\frac{4x^5}{e^2}$$
$$-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

output
```
x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/
(3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) +
 x*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-
20*d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5
 - 6*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d*
*3*e**2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= -\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}$$
$$+\frac{48\,e^4x^5-3\,(40\,de^3+17\,e^4)x^4+4\,(60\,d^2e^2+34\,de^3+17\,e^4)x^3-6\,(80\,d^3e+51\,d^2e^2+34\,de^3+4\,e^4)x^2+}{12\,e^6}$$
$$-\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="ma
xima")
```

output
```
-(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^
6)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*
d^2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4
*e^4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6
 - (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e
*x + d)/e^7
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.43

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$

$$-\frac{(ex+d)^5\left(\frac{3\,(120\,de+17\,e^2)}{(ex+d)e}-\frac{4\,(300\,d^2e^2+85\,de^3+17\,e^4)}{(ex+d)^2e^2}+\frac{12\,(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\,e^6)}{(ex+d)^3e^3}-\frac{12\,(300\,d^4e^4+170\,d^3e^5+102\,d^2e}{(ex+d)^4e^4}\right.}{12\,e^7}$$

$$+\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$

$$-\frac{\frac{20\,d^6e^5}{ex+d}+\frac{17\,d^5e^6}{ex+d}+\frac{17\,d^4e^7}{ex+d}+\frac{4\,d^3e^8}{ex+d}+\frac{21\,d^2e^9}{ex+d}-\frac{7\,de^{10}}{ex+d}+\frac{6\,e^{11}}{ex+d}}{e^{12}}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gi
ac")
```

output
```
-1/12*(e*x + d)^5*(3*(120*d*e + 17*e^2)/((e*x + d)*e) - 4*(300*d^2*e^2 + 8
5*d*e^3 + 17*e^4)/((e*x + d)^2*e^2) + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*d*
e^5 + 2*e^6)/((e*x + d)^3*e^3) - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2*e
^6 + 12*d*e^7 + 21*e^8)/((e*x + d)^4*e^4) - 48)/e^7 + (120*d^5 + 85*d^4*e
+ 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(abs(e*x + d)/((e*x + d)^
2*abs(e)))/e^7 - (20*d^6*e^5/(e*x + d) + 17*d^5*e^6/(e*x + d) + 17*d^4*e^7
/(e*x + d) + 4*d^3*e^8/(e*x + d) + 21*d^2*e^9/(e*x + d) - 7*d*e^10/(e*x +
d) + 6*e^11/(e*x + d))/e^12
```

3.294.9 Mupad [B] (verification not implemented)

Time = 13.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.59

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= x^3\left(\frac{17}{3e^2}-\frac{20d^2}{3e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{3e}\right) - x^2\left(\frac{2}{e^2}+\frac{d\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{2e^2}\right) - x^4\left(\frac{10d}{e^3}+\frac{17}{4e^2}\right) + x\left(\frac{21}{e^2} + \frac{2d\left(\frac{4}{e^2}+\frac{2d\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e^2}\right)}{e} - \frac{d^2\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e^2}\right)$$
$$+\frac{4x^5}{e^2}-\frac{\ln(d+ex)\,(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)}{e^7} - \frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{e\,(xe^7+de^6)}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

output
```
x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) -
x^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e
 - (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) +
x*(21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 +
 17/e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20
*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d +
e*x)*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^
7 - (17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4
*e^2)/(e*(d*e^6 + e^7*x))
```

3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

3.295.1 Optimal result

Integrand size = 36, antiderivative size = 231

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)\,x^2}{2e^5} -\frac{(60d+17e)x^3}{3e^4}+\frac{5x^4}{e^3}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2} +\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)} +\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}$$

output
```
-(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/
e^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3
*e+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^
2*e^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+
21*e^4)*ln(e*x+d)/e^7
```

3.295.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{660d^6+d^5e(459-480x)-51d^4e^2(-7+2x+40x^2)-3d^3e^3(-20-34x+357x^2+200x^3)+d^2e^4(189+}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,
x]
```

output
```
(660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^
3*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x
^3 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) +
e^6*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^
3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d
 + e*x)^2)
```

3.295.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x(120d^2+51de+17e^2)}{e^5}+\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{300d^4+170d^3e+102d^2e^2+12de^3+21e^4}{e^6(d+ex)}+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}+\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)}-\frac{x^3(60d+17e)}{3e^4}+\frac{5x^4}{e^3}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

output
```
-(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
 17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^
2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(
d + e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7
*e^5)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 2
1*e^4)*Log[d + e*x])/e^7
```

3.295.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.295.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{(600d^5+340d^4e+204d^3e^2+24d^2e^3+42de^4-7e^5)x}{e^6}+\frac{5x^6}{e}+\frac{900d^6+510d^5e+306d^4e^2+36d^3e^3+63d^2e^4-7de^5-6e^6}{2e^7}-\frac{(30d+17e)x^5}{3e^2}+\frac{(150d^2}{}}{(ex+d)^2}$
default	$-\frac{-5e^3x^4+20de^2x^3+\frac{17}{3}e^3x^3-60d^2ex^2-\frac{51}{2}de^2x^2-\frac{17}{2}e^3x^2+200xd^3+102d^2ex+51de^2x+4e^3x}{e^6}-\frac{-120d^5-85d^4e-68d^3e^2}{e^7(ex}$
risch	$\frac{5x^4}{e^3}-\frac{20dx^3}{e^4}-\frac{17x^3}{3e^3}+\frac{60d^2x^2}{e^5}+\frac{51dx^2}{2e^4}+\frac{17x^2}{2e^3}-\frac{200xd^3}{e^6}-\frac{102d^2x}{e^5}-\frac{51dx}{e^4}-\frac{4x}{e^3}+\frac{(120d^5+85d^4e+68d^3e^2}{}$
parallelrisch	$\frac{-34e^6x^5-18e^6+30e^6x^6+1530d^5e+1800\ln(ex+d)x^2d^4e^2+612\ln(ex+d)x^2d^2e^4+72\ln(ex+d)x^2de^5+1020\ln(ex+d)x^2d^3e^3+2}{}$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBOS
E)
```

output
```
((600*d^5+340*d^4*e+204*d^3*e^2+24*d^2*e^3+42*d*e^4-7*e^5)/e^6*x+5*x^6/e+1
/2*(900*d^6+510*d^5*e+306*d^4*e^2+36*d^3*e^3+63*d^2*e^4-7*d*e^5-6*e^6)/e^7
-1/3*(30*d+17*e)/e^2*x^5+1/6*(150*d^2+85*d*e+51*e^2)/e^3*x^4-2/3*(150*d^3+
85*d^2*e+51*d*e^2+6*e^3)/e^4*x^3)/(e*x+d)^2+(300*d^4+170*d^3*e+102*d^2*e^2
+12*d*e^3+21*e^4)*ln(e*x+d)/e^7
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.56

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{30\,e^6x^6+660\,d^6+459\,d^5e+357\,d^4e^2+60\,d^3e^3+189\,d^2e^4-21\,de^5-18\,e^6-2\,(30\,de^5+17\,e^6)x^5+(150}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fr
icas")
```

output
```
1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2
*e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d
*e^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 -
 3*(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e
+ 17*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 +
 170*d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^
3*e^3 + 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2
+ 102*d^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8
*x + d^2*e^7)
```

3.295.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\cdot\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)$$
$$+\frac{220d^6+153d^5e+119d^4e^2+20d^3e^3+63d^2e^4-7de^5-6e^6+x(240d^5e+170d^4e^2+136d^3e^3+24d^2e^4-}{2d^2e^7+4de^8x+2e^9x^2}$$
$$+\frac{5x^4}{e^3}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

output
```
x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/
(2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220
*d**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**
5 - 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4
+ 84*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/
e**3 + (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d
 + e*x)/e**7
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$=\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4-}{2\,(e^9x^2+2\,de^8x+d^2e^7)}$$
$$+\frac{30\,e^3x^4-2\,(60\,de^2+17\,e^3)x^3+3\,(120\,d^2e+51\,de^2+17\,e^3)x^2-6\,(200\,d^3+102\,d^2e+51\,de^2+4\,e^3)x}{6\,e^6}$$
$$+\frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="ma
xima")
```

output
```
1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
 - 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5
- 7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^
2 + 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102
*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 1
2*d*e^3 + 21*e^4)*log(e*x + d)/e^7
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log(|ex+d|)}{e^7}$$
$$+ \frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e}{2\,(ex+d)^2e^7}$$
$$+ \frac{30\,e^9x^4-120\,de^8x^3-34\,e^9x^3+360\,d^2e^7x^2+153\,de^8x^2+51\,e^9x^2-1200\,d^3e^6x-612\,d^2e^7x-306\,de^8x}{6\,e^{12}}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gi
ac")
```

output
```
(300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*log(abs(e*x + d))/
e^7 + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7
*d*e^5 - 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*
d*e^5 - 7*e^6)*x)/((e*x + d)^2*e^7) + 1/6*(30*e^9*x^4 - 120*d*e^8*x^3 - 34
*e^9*x^3 + 360*d^2*e^7*x^2 + 153*d*e^8*x^2 + 51*e^9*x^2 - 1200*d^3*e^6*x -
 612*d^2*e^7*x - 306*d*e^8*x - 24*e^9*x)/e^12
```

3.295.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.29

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^2\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right)$$
$$+\frac{x\,(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5)+\frac{220\,d^6+153\,d^5\,e+119\,d^4\,e^2+20\,d^3\,e^3+63\,d^2\,e^4-7\,d\,e^5-6\,e^6}{2\,e}}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2}$$
$$-x\left(\frac{4}{e^3}+\frac{20\,d^3}{e^6}+\frac{3\,d\,\left(\frac{17}{e^3}-\frac{60\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e^2}\right)$$
$$+\frac{5\,x^4}{e^3}+\frac{\ln\left(d+e\,x\right)\,\left(300\,d^4+170\,d^3\,e+102\,d^2\,e^2+12\,d\,e^3+21\,e^4\right)}{e^7}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

output
```
x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*
((20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12
*d^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e
^4 + 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*
(4/e^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 1
7/e^3))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d
+ e*x)*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

3.296.1 Optimal result

Integrand size = 38, antiderivative size = 391

$$\begin{aligned}&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx\\
&=\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^4}{4e^9}\\
&\quad-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^5}{5e^9}\\
&\quad+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^6}{6e^9}\\
&\quad-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^7}{7e^9}\\
&\quad+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^8}{8e^9}\\
&\quad-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9}\\
&\quad+\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{5(160d+9e)(d+ex)^{11}}{11e^9}+\frac{25(d+ex)^{12}}{3e^9}\end{aligned}$$

output
```
1/4*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/
e^9-1/5*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e
^4-11*e^5)*(e*x+d)^5/e^9+1/6*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+
888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^6/e^9-1/7*(5600*d^5+1575*d^4*e+2220
*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^7/e^9+1/8*(7000*d^4+1575*d^
3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^8/e^9-1/9*(5600*d^3+945*d^2*e+
666*d*e^2+37*e^3)*(e*x+d)^9/e^9+1/10*(2800*d^2+315*d*e+111*e^2)*(e*x+d)^10
/e^9-5/11*(160*d+9*e)*(e*x+d)^11/e^9+25/3*(e*x+d)^12/e^9
```

3.296.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.71

$$\begin{aligned}&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx\\&=18d^3x+\frac{3}{2}d^2(11d+18e)x^2+\frac{1}{3}d\left(107d^2+99de+54e^2\right)x^3\\&+\frac{1}{4}\left(65d^3+321d^2e+99de^2+18e^3\right)x^4+\frac{1}{5}\left(148d^3+195d^2e+321de^2+33e^3\right)x^5\\&+\frac{1}{6}\left(-37d^3+444d^2e+195de^2+107e^3\right)x^6+\frac{1}{7}\left(111d^3-111d^2e+444de^2+65e^3\right)x^7\\&+\frac{1}{8}\left(-45d^3+333d^2e-111de^2+148e^3\right)x^8+\frac{1}{9}\left(100d^3-135d^2e+333de^2-37e^3\right)x^9\\&+\frac{3}{10}e\left(100d^2-45de+37e^2\right)x^{10}+\frac{15}{11}(20d-3e)e^2x^{11}+\frac{25e^3x^{12}}{3}\end{aligned}$$

input
```
Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^
3)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*
d^2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 1
07*e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-4
5*d^3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e +
333*d*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (
15*(20*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

3.296.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^3 \, dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(\frac{(2800d^2+315de+111e^2)(d+ex)^9}{e^8} + \frac{(-5600d^3-945d^2e-666de^2-37e^3)(d+ex)^8}{e^8} + \frac{(7000d^4+1575d^3e\right.$$

$$\downarrow 2009$$

$$\frac{(2800d^2+315de+111e^2)(d+ex)^{10}}{10e^9} - \frac{(5600d^3+945d^2e+666de^2+37e^3)(d+ex)^9}{9e^9} + \frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)(d+ex)^8}{8e^9} + \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^9} - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)(d+ex)^7}{7e^9} - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)^5}{5e^9} + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)(d+ex)^6}{6e^9} + \frac{25(d+ex)^{12}}{3e^9} - \frac{5(160d+9e)(d+ex)^{11}}{11e^9}$$

input
```
Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d
^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6
+ 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e
^6)*(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^
2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 +
 945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d
*e + 111*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*
e^9) + (25*(d + e*x)^12)/(3*e^9)
```

3.296.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.296.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65

method	result
norman	$\frac{25e^3x^{12}}{3}+\left(\frac{300}{11}d\,e^2-\frac{45}{11}e^3\right)x^{11}+\left(30d^2e-\frac{27}{2}d\,e^2+\frac{111}{10}e^3\right)x^{10}+\left(\frac{100}{9}d^3-15d^2e+37d\,e^2-\frac{37}{9}\right.$
default	$\frac{25e^3x^{12}}{3}+\frac{(300d\,e^2-45e^3)x^{11}}{11}+\frac{(300d^2e-135d\,e^2+111e^3)x^{10}}{10}+\frac{(100d^3-135d^2e+333d\,e^2-37e^3)x^9}{9}+\frac{(-45d^3+333d^2}{}$
gosper	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{4}{}$
risch	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{4}{}$
parallelrisch	$33d^2e\,x^3+74x^6d^2e+\frac{65}{2}x^6d\,e^2+39x^5d^2e+\frac{321}{5}x^5d\,e^2+\frac{321}{4}x^4d^2e+\frac{99}{4}x^4d\,e^2-\frac{111}{7}x^7d^2e+\frac{4}{}$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
25/3*e^3*x^12+(300/11*d*e^2-45/11*e^3)*x^11+(30*d^2*e-27/2*d*e^2+111/10*e^
3)*x^10+(100/9*d^3-15*d^2*e+37*d*e^2-37/9*e^3)*x^9+(-45/8*d^3+333/8*d^2*e-
111/8*d*e^2+37/2*e^3)*x^8+(111/7*d^3-111/7*d^2*e+444/7*d*e^2+65/7*e^3)*x^7
+(-37/6*d^3+74*d^2*e+65/2*d*e^2+107/6*e^3)*x^6+(148/5*d^3+39*d^2*e+321/5*d
*e^2+33/5*e^3)*x^5+(65/4*d^3+321/4*d^2*e+99/4*d*e^2+9/2*e^3)*x^4+(107/3*d^
3+33*d^2*e+18*d*e^2)*x^3+(33/2*d^3+27*d^2*e)*x^2+18*x*d^3
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.67

$$
\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{25}{3}e^3x^{12}+\frac{15}{11}\left(20\,de^2-3\,e^3\right)x^{11}+\frac{3}{10}\left(100\,d^2e-45\,de^2+37\,e^3\right)x^{10} \\
&\quad+\frac{1}{9}\left(100\,d^3-135\,d^2e+333\,de^2-37\,e^3\right)x^9-\frac{1}{8}\left(45\,d^3-333\,d^2e+111\,de^2-148\,e^3\right)x^8 \\
&\quad+\frac{1}{7}\left(111\,d^3-111\,d^2e+444\,de^2+65\,e^3\right)x^7-\frac{1}{6}\left(37\,d^3-444\,d^2e-195\,de^2-107\,e^3\right)x^6 \\
&\quad+\frac{1}{5}\left(148\,d^3+195\,d^2e+321\,de^2+33\,e^3\right)x^5+\frac{1}{4}\left(65\,d^3+321\,d^2e+99\,de^2+18\,e^3\right)x^4 \\
&\quad+18\,d^3x+\frac{1}{3}\left(107\,d^3+99\,d^2e+54\,de^2\right)x^3+\frac{3}{2}\left(11\,d^3+18\,d^2e\right)x^2
\end{aligned}
$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
 + 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8
*(45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e
 + 444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3
)*x^6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 +
 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e +
 54*d*e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

3.296.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.76

$$\begin{aligned}
&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx \\
&= 18d^3x + \frac{25e^3x^{12}}{3} + x^{11} \cdot \left(\frac{300de^2}{11} - \frac{45e^3}{11}\right) + x^{10} \cdot \left(30d^2e - \frac{27de^2}{2} + \frac{111e^3}{10}\right) + x^9 \\
&\cdot \left(\frac{100d^3}{9} - 15d^2e + 37de^2 - \frac{37e^3}{9}\right) + x^8\left(-\frac{45d^3}{8} + \frac{333d^2e}{8} - \frac{111de^2}{8} + \frac{37e^3}{2}\right) + x^7 \\
&\cdot \left(\frac{111d^3}{7} - \frac{111d^2e}{7} + \frac{444de^2}{7} + \frac{65e^3}{7}\right) + x^6\left(-\frac{37d^3}{6} + 74d^2e + \frac{65de^2}{2} + \frac{107e^3}{6}\right) \\
&+ x^5 \cdot \left(\frac{148d^3}{5} + 39d^2e + \frac{321de^2}{5} + \frac{33e^3}{5}\right) + x^4 \cdot \left(\frac{65d^3}{4} + \frac{321d^2e}{4} + \frac{99de^2}{4} + \frac{9e^3}{2}\right) \\
&+ x^3 \cdot \left(\frac{107d^3}{3} + 33d^2e + 18de^2\right) + x^2 \cdot \left(\frac{33d^3}{2} + 27d^2e\right)
\end{aligned}$$

input
```
integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(
30*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37
*d*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37
*e**3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x
**6*(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5
 + 39*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4
+ 99*d*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x*
*2*(33*d**3/2 + 27*d**2*e)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.67

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{25}{3}e^3x^{12} + \frac{15}{11}\left(20\,de^2 - 3\,e^3\right)x^{11} + \frac{3}{10}\left(100\,d^2e - 45\,de^2 + 37\,e^3\right)x^{10} + \frac{1}{9}\left(100\,d^3 - 135\,d^2e + 333\,de^2 - 37\,e^3\right)x^9 - \frac{1}{8}\left(45\,d^3 - 333\,d^2e + 111\,de^2 - 148\,e^3\right)x^8 + \frac{1}{7}\left(111\,d^3 - 111\,d^2e + 444\,de^2 + 65\,e^3\right)x^7 - \frac{1}{6}\left(37\,d^3 - 444\,d^2e - 195\,de^2 - 107\,e^3\right)x^6 + \frac{1}{5}\left(148\,d^3 + 195\,d^2e + 321\,de^2 + 33\,e^3\right)x^5 + \frac{1}{4}\left(65\,d^3 + 321\,d^2e + 99\,de^2 + 18\,e^3\right)x^4 + 18\,d^3x + \frac{1}{3}\left(107\,d^3 + 99\,d^2e + 54\,de^2\right)x^3 + \frac{3}{2}\left(11\,d^3 + 18\,d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
 + 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8
*(45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e
 + 444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3
)*x^6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 +
 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e +
 54*d*e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.78

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{25}{3}e^3x^{12} + \frac{300}{11}de^2x^{11} - \frac{45}{11}e^3x^{11} + 30\,d^2ex^{10} - \frac{27}{2}de^2x^{10} + \frac{111}{10}e^3x^{10} + \frac{100}{9}d^3x^9 - 15\,d^2ex^9 + 37\,de^2x^9 - \frac{37}{9}e^3x^9 - \frac{45}{8}d^3x^8 + \frac{333}{8}d^2ex^8 - \frac{111}{8}de^2x^8 + \frac{37}{2}e^3x^8 + \frac{111}{7}d^3x^7 - \frac{111}{7}d^2ex^7 + \frac{444}{7}de^2x^7 + \frac{65}{7}e^3x^7 - \frac{37}{6}d^3x^6 + 74\,d^2ex^6 + \frac{65}{2}de^2x^6 + \frac{107}{6}e^3x^6 + \frac{148}{5}d^3x^5 + 39\,d^2ex^5 + \frac{321}{5}de^2x^5 + \frac{33}{5}e^3x^5 + \frac{65}{4}d^3x^4 + \frac{321}{4}d^2ex^4 + \frac{99}{4}de^2x^4 + \frac{9}{2}e^3x^4 + \frac{107}{3}d^3x^3 + 33\,d^2ex^3 + 18\,de^2x^3 + \frac{33}{2}d^3x^2 + 27\,d^2ex^2 + 18\,d^3x$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
25/3*e^3*x^12 + 300/11*d*e^2*x^11 - 45/11*e^3*x^11 + 30*d^2*e*x^10 - 27/2*
d*e^2*x^10 + 111/10*e^3*x^10 + 100/9*d^3*x^9 - 15*d^2*e*x^9 + 37*d*e^2*x^9
 - 37/9*e^3*x^9 - 45/8*d^3*x^8 + 333/8*d^2*e*x^8 - 111/8*d*e^2*x^8 + 37/2*
e^3*x^8 + 111/7*d^3*x^7 - 111/7*d^2*e*x^7 + 444/7*d*e^2*x^7 + 65/7*e^3*x^7
 - 37/6*d^3*x^6 + 74*d^2*e*x^6 + 65/2*d*e^2*x^6 + 107/6*e^3*x^6 + 148/5*d^
3*x^5 + 39*d^2*e*x^5 + 321/5*d*e^2*x^5 + 33/5*e^3*x^5 + 65/4*d^3*x^4 + 321
/4*d^2*e*x^4 + 99/4*d*e^2*x^4 + 9/2*e^3*x^4 + 107/3*d^3*x^3 + 33*d^2*e*x^3
 + 18*d*e^2*x^3 + 33/2*d^3*x^2 + 27*d^2*e*x^2 + 18*d^3*x
```

3.296.9 Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.64

$$\begin{aligned}\int (d+ex)^3\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= 18\,d^3\,x+x^3\left(\frac{107\,d^3}{3}+33\,d^2\,e+18\,d\,e^2\right)+x^9\left(\frac{100\,d^3}{9}-15\,d^2\,e+37\,d\,e^2-\frac{37\,e^3}{9}\right)\\ &\quad+x^6\left(-\frac{37\,d^3}{6}+74\,d^2\,e+\frac{65\,d\,e^2}{2}+\frac{107\,e^3}{6}\right)+x^4\left(\frac{65\,d^3}{4}+\frac{321\,d^2\,e}{4}+\frac{99\,d\,e^2}{4}+\frac{9\,e^3}{2}\right)\\ &\quad-x^8\left(\frac{45\,d^3}{8}-\frac{333\,d^2\,e}{8}+\frac{111\,d\,e^2}{8}-\frac{37\,e^3}{2}\right)+x^5\left(\frac{148\,d^3}{5}+39\,d^2\,e+\frac{321\,d\,e^2}{5}+\frac{33\,e^3}{5}\right)\\ &\quad+x^7\left(\frac{111\,d^3}{7}-\frac{111\,d^2\,e}{7}+\frac{444\,d\,e^2}{7}+\frac{65\,e^3}{7}\right)+\frac{25\,e^3\,x^{12}}{3}\\ &\quad+\frac{3\,e\,x^{10}\left(100\,d^2-45\,d\,e+37\,e^2\right)}{10}+\frac{3\,d^2\,x^2\left(11\,d+18\,e\right)}{2}+\frac{15\,e^2\,x^{11}\left(20\,d-3\,e\right)}{11}\end{aligned}$$

input
```
int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^
2*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/
6 + (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3
)/2) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5
*((321*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/
7 - (111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^1
0*(100*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*
x^11*(20*d - 3*e))/11
```

3.297 $\int (d+ex)^2 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)$

3.297.1 Optimal result

Integrand size = 38, antiderivative size = 201

$$\begin{aligned}\int (d+ex)^2 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\,dx &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}(107d^2+66de+18e^2)x^3+\frac{1}{4}(65d^2+214de+33e^2)x^4\\ &+\frac{1}{5}(148d^2+130de+107e^2)x^5-\frac{1}{6}(37d^2-296de-65e^2)x^6+\frac{37}{7}(3d^2-2de+4e^2)x^7\\ &-\frac{1}{8}(45d^2-222de+37e^2)x^8+\frac{1}{9}(100d^2-90de+111e^2)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}\end{aligned}$$

output
```
18*d^2*x+3/2*d*(11*d+12*e)*x^2+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/4*(65*d^2
+214*d*e+33*e^2)*x^4+1/5*(148*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e
-65*e^2)*x^6+37/7*(3*d^2-2*d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+
1/9*(100*d^2-90*d*e+111*e^2)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11
```

3.297.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex)^2 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\,dx &= 18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}(107d^2+66de+18e^2)x^3+\frac{1}{4}(65d^2+214de+33e^2)x^4\\ &+\frac{1}{5}(148d^2+130de+107e^2)x^5+\frac{1}{6}(-37d^2+296de+65e^2)x^6+\frac{37}{7}(3d^2-2de+4e^2)x^7\\ &+\frac{1}{8}(-45d^2+222de-37e^2)x^8+\frac{1}{9}(100d^2-90de+111e^2)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}\end{aligned}$$

input
```
Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
 + ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^
5)/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*
x^7)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e
^2)*x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

3.297.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^2 \, dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(x^8 \left(100d^2-90de+111e^2\right)-x^7 \left(45d^2-222de+37e^2\right)+37x^6 \left(3d^2-2de+4e^2\right)-x^5 \left(37d^2-296de-65e^2\right)+\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(148d^2+130de+107e^2\right)+\frac{1}{4}x^4\left(65d^2+214de+33e^2\right)+\frac{1}{3}x^3\left(107d^2+66de+18e^2\right)+18d^2x+\frac{1}{2}ex^{10}(40d-9e)+\frac{3}{2}dx^2(11d+12e)+\frac{100e^2x^{11}}{11}$$

input
```
Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
 + ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^
5)/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x
^7)/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2
)*x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

3.297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.297.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

method	result
norman	$\frac{100e^2x^{11}}{11} + \left(20de - \frac{9}{2}e^2\right)x^{10} + \left(\frac{100}{9}d^2 - 10de + \frac{37}{3}e^2\right)x^9 + \left(-\frac{45}{8}d^2 + \frac{111}{4}de - \frac{37}{8}e^2\right)x^8 + \left(\frac{111}{7}\right.$
default	$\frac{100e^2x^{11}}{11} + \frac{(200de-45e^2)x^{10}}{10} + \frac{(100d^2-90de+111e^2)x^9}{9} + \frac{(-45d^2+222de-37e^2)x^8}{8} + \frac{(111d^2-74de+148e^2)x^7}{7} +$
gosper	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$
risch	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$
parallelrisch	$\frac{111}{4}x^8de - \frac{74}{7}x^7de + \frac{148}{3}x^6de + \frac{107}{2}x^4de + 22x^3de + 26x^5de - 10x^9de + 20x^{10}de - \frac{45}{8}x^8d^2 +$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
100/11*e^2*x^11+(20*d*e-9/2*e^2)*x^10+(100/9*d^2-10*d*e+37/3*e^2)*x^9+(-45
/8*d^2+111/4*d*e-37/8*e^2)*x^8+(111/7*d^2-74/7*d*e+148/7*e^2)*x^7+(-37/6*d
^2+148/3*d*e+65/6*e^2)*x^6+(148/5*d^2+26*d*e+107/5*e^2)*x^5+(65/4*d^2+107/
2*d*e+33/4*e^2)*x^4+(107/3*d^2+22*d*e+6*e^2)*x^3+(33/2*d^2+18*d*e)*x^2+18*
x*d^2
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{100}{11} e^2x^{11} + \frac{1}{2} (40 \, de - 9 \, e^2)x^{10} + \frac{1}{9} (100 \, d^2 - 90 \, de + 111 \, e^2)x^9 - \frac{1}{8} (45 \, d^2 - 222 \, de + 37 \, e^2)x^8 + \frac{37}{7} (3 \, d^2 - 2 \, de + 4 \, e^2)x^7 - \frac{1}{6} (37 \, d^2 - 296 \, de - 65 \, e^2)x^6 + \frac{1}{5} (148 \, d^2 + 130 \, de + 107 \, e^2)x^5 + \frac{1}{4} (65 \, d^2 + 214 \, de + 33 \, e^2)x^4 + \frac{1}{3} (107 \, d^2 + 66 \, de + 18 \, e^2)x^3 + 18 \, d^2x + \frac{3}{2} (11 \, d^2 + 12 \, de)x^2$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*
e^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e
^2)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 1
07*e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e
+ 18*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

3.297.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 18d^2x + \frac{100e^2x^{11}}{11} + x^{10} \cdot \left(20de - \frac{9e^2}{2}\right) + x^9 \cdot \left(\frac{100d^2}{9} - 10de + \frac{37e^2}{3}\right) + x^8\left(-\frac{45d^2}{8} + \frac{111de}{4} - \frac{37e^2}{8}\right) + x^7 \cdot \left(\frac{111d^2}{7} - \frac{74de}{7} + \frac{148e^2}{7}\right) + x^6\left(-\frac{37d^2}{6} + \frac{148de}{3} + \frac{65e^2}{6}\right) + x^5 \cdot \left(\frac{148d^2}{5} + 26de + \frac{107e^2}{5}\right) + x^4 \cdot \left(\frac{65d^2}{4} + \frac{107de}{2} + \frac{33e^2}{4}\right) + x^3 \cdot \left(\frac{107d^2}{3} + 22de + 6e^2\right) + x^2 \cdot \left(\frac{33d^2}{2} + 18de\right)$$

input
```
integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2
/9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**
7*(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65
*e**2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107
*d*e/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/
2 + 18*d*e)
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{100}{11}\,e^2x^{11} + \frac{1}{2}\,(40\,de-9\,e^2)x^{10} + \frac{1}{9}\,(100\,d^2-90\,de+111\,e^2)x^9 \\
&\quad - \frac{1}{8}\,(45\,d^2-222\,de+37\,e^2)x^8 + \frac{37}{7}\,(3\,d^2-2\,de+4\,e^2)x^7 - \frac{1}{6}\,(37\,d^2-296\,de-65\,e^2)x^6 \\
&\quad + \frac{1}{5}\,(148\,d^2+130\,de+107\,e^2)x^5 + \frac{1}{4}\,(65\,d^2+214\,de+33\,e^2)x^4 \\
&\quad + \frac{1}{3}\,(107\,d^2+66\,de+18\,e^2)x^3 + 18\,d^2x + \frac{3}{2}\,(11\,d^2+12\,de)x^2
\end{aligned}$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*
e^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e
^2)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 1
07*e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e
+ 18*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \frac{100}{11} e^2x^{11} + 20\, dex^{10} - \frac{9}{2} e^2x^{10} + \frac{100}{9} d^2x^9 - 10\, dex^9 + \frac{37}{3} e^2x^9 - \frac{45}{8} d^2x^8 + \frac{111}{4} dex^8 - \frac{37}{8} e^2x^8 + \frac{111}{7} d^2x^7 - \frac{74}{7} dex^7 + \frac{148}{7} e^2x^7 - \frac{37}{6} d^2x^6 + \frac{148}{3} dex^6 + \frac{65}{6} e^2x^6 + \frac{148}{5} d^2x^5 + 26\, dex^5 + \frac{107}{5} e^2x^5 + \frac{65}{4} d^2x^4 + \frac{107}{2} dex^4 + \frac{33}{4} e^2x^4 + \frac{107}{3} d^2x^3 + 22\, dex^3 + 6\, e^2x^3 + \frac{33}{2} d^2x^2 + 18\, dex^2 + 18\, d^2x$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
100/11*e^2*x^11 + 20*d*e*x^10 - 9/2*e^2*x^10 + 100/9*d^2*x^9 - 10*d*e*x^9
+ 37/3*e^2*x^9 - 45/8*d^2*x^8 + 111/4*d*e*x^8 - 37/8*e^2*x^8 + 111/7*d^2*x
^7 - 74/7*d*e*x^7 + 148/7*e^2*x^7 - 37/6*d^2*x^6 + 148/3*d*e*x^6 + 65/6*e^
2*x^6 + 148/5*d^2*x^5 + 26*d*e*x^5 + 107/5*e^2*x^5 + 65/4*d^2*x^4 + 107/2*
d*e*x^4 + 33/4*e^2*x^4 + 107/3*d^2*x^3 + 22*d*e*x^3 + 6*e^2*x^3 + 33/2*d^2
*x^2 + 18*d*e*x^2 + 18*d^2*x
```

3.297.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = x^3 \left(\frac{107\, d^2}{3} + 22\, d\, e + 6\, e^2\right) + x^9 \left(\frac{100\, d^2}{9} - 10\, d\, e + \frac{37\, e^2}{3}\right) + x^4 \left(\frac{65\, d^2}{4} + \frac{107\, d\, e}{2} + \frac{33\, e^2}{4}\right) - x^8 \left(\frac{45\, d^2}{8} - \frac{111\, d\, e}{4} + \frac{37\, e^2}{8}\right) + x^6 \left(-\frac{37\, d^2}{6} + \frac{148\, d\, e}{3} + \frac{65\, e^2}{6}\right) + x^5 \left(\frac{148\, d^2}{5} + 26\, d\, e + \frac{107\, e^2}{5}\right) + x^7 \left(\frac{111\, d^2}{7} - \frac{74\, d\, e}{7} + \frac{148\, e^2}{7}\right) + 18\, d^2\, x + \frac{100\, e^2\, x^{11}}{11} + \frac{3\, d\, x^2\, (11\, d + 12\, e)}{2} + \frac{e\, x^{10}\, (40\, d - 9\, e)}{2}$$

input
```
int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/
3) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*
d*e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(
26*d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148
*e^2)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x
^10*(40*d - 9*e))/2
```

3.297. $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.298 $\int (d+ex)(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)$

3.298.1 Optimal result

Integrand size = 36, antiderivative size = 121

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}$$

output
```
18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(1
48*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(
20*d-9*e)*x^9+10*e*x^10
```

3.298.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5-\frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}$$

input
```
Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)
*x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^
7)/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

3.298.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex) \, dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(5x^8(20d-9e)-3x^7(15d-37e)+37x^6(3d-e)-37x^5(d-4e)+x^4(148d+65e)+x^3(65d+107e)+x^2(107\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2(11d+6e)+18dx+10ex^{10}$$

input
```
Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)
*x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^
7)/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

3.298.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.298.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

method	result
norman	$10e\,x^{10}+\left(\frac{100d}{9}-5e\right)x^{9}+\left(-\frac{45d}{8}+\frac{111e}{8}\right)x^{8}+\left(\frac{111d}{7}-\frac{37e}{7}\right)x^{7}+\left(-\frac{37d}{6}+\frac{74e}{3}\right)x^{6}+\left(\frac{148d}{5}+1\right.$
gosper	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$
default	$10e\,x^{10}+\frac{(100d-45e)x^{9}}{9}+\frac{(-45d+111e)x^{8}}{8}+\frac{(111d-37e)x^{7}}{7}+\frac{(-37d+148e)x^{6}}{6}+\frac{(148d+65e)x^{5}}{5}+\frac{(65d+107e)x}{4}$
risch	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$
parallelrisch	$10e\,x^{10}+\frac{100}{9}d\,x^{9}-5e\,x^{9}-\frac{45}{8}d\,x^{8}+\frac{111}{8}e\,x^{8}+\frac{111}{7}d\,x^{7}-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}$

input
```
int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
10*e*x^10+(100/9*d-5*e)*x^9+(-45/8*d+111/8*e)*x^8+(111/7*d-37/7*e)*x^7+(-3
7/6*d+74/3*e)*x^6+(148/5*d+13*e)*x^5+(65/4*d+107/4*e)*x^4+(107/3*d+11*e)*x
^3+(33/2*d+9*e)*x^2+18*d*x
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=10\,ex^{10}+\frac{5}{9}(20\,d-9\,e)x^9-\frac{3}{8}(15\,d-37\,e)x^8+\frac{37}{7}(3\,d-e)x^7-\frac{37}{6}(d-4\,e)x^6$$
$$+\frac{1}{5}(148\,d+65\,e)x^5+\frac{1}{4}(65\,d+107\,e)x^4+\frac{1}{3}(107\,d+33\,e)x^3+\frac{3}{2}(11\,d+6\,e)x^2+18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*
x^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4
 + 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

3.298.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 18dx+10ex^{10}+x^9\cdot\left(\frac{100d}{9}-5e\right)+x^8\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^7\cdot\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^6\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^5\cdot\left(\frac{148d}{5}+13e\right)+x^4\cdot\left(\frac{65d}{4}+\frac{107e}{4}\right)+x^3\cdot\left(\frac{107d}{3}+11e\right)+x^2\cdot\left(\frac{33d}{2}+9e\right)$$

input
```
integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x*
*7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) +
x**4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*
x^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4
 + 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int (d+ex) (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 10\,ex^{10} + \frac{100}{9}\,dx^9 - 5\,ex^9 - \frac{45}{8}\,dx^8 + \frac{111}{8}\,ex^8 + \frac{111}{7}\,dx^7 - \frac{37}{7}\,ex^7 - \frac{37}{6}\,dx^6 + \frac{74}{3}\,ex^6 + \frac{148}{5}\,dx^5 + 13\,ex^5 + \frac{65}{4}\,dx^4 + \frac{107}{4}\,ex^4 + \frac{107}{3}\,dx^3 + 11\,ex^3 + \frac{33}{2}\,dx^2 + 9\,ex^2 + 18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
10*e*x^10 + 100/9*d*x^9 - 5*e*x^9 - 45/8*d*x^8 + 111/8*e*x^8 + 111/7*d*x^7
 - 37/7*e*x^7 - 37/6*d*x^6 + 74/3*e*x^6 + 148/5*d*x^5 + 13*e*x^5 + 65/4*d*
x^4 + 107/4*e*x^4 + 107/3*d*x^3 + 11*e*x^3 + 33/2*d*x^2 + 9*e*x^2 + 18*d*x
```

3.298.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int (d+ex) (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 10\,e\,x^{10} + \left(\frac{100\,d}{9} - 5\,e\right)\,x^9 + \left(\frac{111\,e}{8} - \frac{45\,d}{8}\right)\,x^8 + \left(\frac{111\,d}{7} - \frac{37\,e}{7}\right)\,x^7 + \left(\frac{74\,e}{3} - \frac{37\,d}{6}\right)\,x^6 + \left(\frac{148\,d}{5} + 13\,e\right)\,x^5 + \left(\frac{65\,d}{4} + \frac{107\,e}{4}\right)\,x^4 + \left(\frac{107\,d}{3} + 11\,e\right)\,x^3 + \left(\frac{33\,d}{2} + 9\,e\right)\,x^2 + 18\,d\,x$$

input
```
int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^
6*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13
*e) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 1
0*e*x^10
```

3.298. $\int (d+ex) \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.299 $\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx$

3.299.1 Optimal result

Integrand size = 31, antiderivative size = 60

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

output
```
18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100
/9*x^9
```

3.299.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

input
```
Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

3.299.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(100x^8-45x^7+111x^6-37x^5+148x^4+65x^3+107x^2+33x+18\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

input
```
Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

3.299.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.299.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result	size
gosper	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
default	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
norman	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
risch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
parallelrisch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100
/9*x^9
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\,dx = \frac{100}{9}x^9-\frac{45}{8}x^8+\frac{111}{7}x^7-\frac{37}{6}x^6+\frac{148}{5}x^5+\frac{65}{4}x^4+\frac{107}{3}x^3+\frac{33}{2}x^2+18x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

3.299.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\, dx = \frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
 107*x**3/3 + 33*x**2/2 + 18*x
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\, dx = \frac{100}{9}\,x^9 - \frac{45}{8}\,x^8 + \frac{111}{7}\,x^7 - \frac{37}{6}\,x^6 + \frac{148}{5}\,x^5 + \frac{65}{4}\,x^4 + \frac{107}{3}\,x^3 + \frac{33}{2}\,x^2 + 18\,x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)\, dx = \frac{100}{9}\,x^9 - \frac{45}{8}\,x^8 + \frac{111}{7}\,x^7 - \frac{37}{6}\,x^6 + \frac{148}{5}\,x^5 + \frac{65}{4}\,x^4 + \frac{107}{3}\,x^3 + \frac{33}{2}\,x^2 + 18\,x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

3.299.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx$$
$$= \frac{100\,x^9}{9} - \frac{45\,x^8}{8} + \frac{111\,x^7}{7} - \frac{37\,x^6}{6} + \frac{148\,x^5}{5} + \frac{65\,x^4}{4} + \frac{107\,x^3}{3} + \frac{33\,x^2}{2} + 18\,x$$

input
```
int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

3.300 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.300.1 Optimal result

Integrand size = 38, antiderivative size = 352

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)\,x}{e^8} + \frac{(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)\,x^2}{2e^7} - \frac{(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)\,x^3}{3e^6} + \frac{(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)\,x^4}{4e^5} - \frac{(100d^3+45d^2e+111de^2+37e^3)\,x^5}{5e^4} + \frac{(100d^2+45de+111e^2)\,x^6}{6e^3} - \frac{5(20d+9e)x^7}{7e^2} + \frac{25x^8}{2e} + \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

output
```
-(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6
-33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65
*d*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d
*e^4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x
^4/e^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e
+111*e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*
(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

3.300.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{x(-42000d^7+2100d^6e(-9+10x)-70d^5e^2(666-135x+200x^2)+210d^4e^3(-74+111x-30x^2+50x^3}{}$$
$$+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),
x]
```

output
```
(x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^
2) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x
 + 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^
3 - 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9
324*x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x
^3 - 1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*
d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])
/e^9
```

3.300.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2(4x^4-5x^3+3x^2+x+2)}{d+ex}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x^5(100d^2+45de+111e^2)}{e^3}-\frac{x^4(100d^3+45d^2e+111de^2+37e^3)}{e^4}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2}{e^8(d+ex)}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}+\frac{x^4(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)}{4e^5}-\frac{x^3(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)}{3e^6}+\frac{x^2(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)}{2e^7}-\frac{x(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)}{e^8}-\frac{5x^7(20d+9e)}{7e^2}+\frac{25x^8}{2e}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
-(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e
^5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37
*d^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45
*d^4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((1
00*d^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100
*d^3 + 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e +
111*e^2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((
5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log
[d + e*x])/e^9
```

3.300.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.300. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.300.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.01

method	result
norman	$\frac{25x^8}{2e} - \frac{5(20d+9e)x^7}{7e^2} + \frac{(100d^2+45de+111e^2)x^6}{6e^3} - \frac{(100d^3+45d^2e+111d\,e^2+37e^3)x^5}{5e^4} + \frac{(100d^4+45d^3e+111d^2e^2+37d}{4e^5}$
default	$-\frac{-\frac{37}{2}e^7x^6+\frac{45}{7}e^7x^7+37d^3x^3e^4+45d^6ex-\frac{45}{2}d^5e^2x^2+15d^4e^3x^3-\frac{45}{4}d^3e^4x^4+9d^2e^5x^5-\frac{15}{2}d\,e^6x^6-\frac{25}{2}e^7x^8+107d\,e^6x-65d^2e^5}{}$
parallelrisch	$\frac{42000\ln(ex+d)d^8+7560\ln(ex+d)e^8+7770x^6e^8-3108x^5e^8+15540x^4e^8+9100x^3e^8+22470x^2e^8+13860x\,e^8+11655x^4d^2e^6+21}{}$
risch	$\frac{18\ln(ex+d)}{e} - \frac{37d^3x^3}{e^4} - \frac{45d^6x}{e^7} + \frac{45d^5x^2}{2e^6} - \frac{15d^4x^3}{e^5} + \frac{45d^3x^4}{4e^4} - \frac{9d^2x^5}{e^3} + \frac{15d\,x^6}{2e^2} - \frac{107dx}{e^2} + \frac{65d^2x}{e^3} - \frac{148d^3x}{e^4}$

input

```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output

```
25/2*x^8/e-5/7*(20*d+9*e)*x^7/e^2+1/6*(100*d^2+45*d*e+111*e^2)*x^6/e^3-1/5
*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/4*(100*d^4+45*d^3*e+111*d^2
*e^2+37*d*e^3+148*e^4)*x^4/e^5-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^
3+148*d*e^4-65*e^5)*x^3/e^6+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+1
48*d^2*e^4-65*d*e^5+107*e^6)*x^2/e^7-(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*
e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-33*e^7)*x/e^8+(100*d^8+45*d^7*e+111*d
^6*e^2+37*d^5*e^3+148*d^4*e^4-65*d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9*
ln(e*x+d)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{5250\,e^8x^8 - 300\,(20\,de^7+9\,e^8)x^7 + 70\,(100\,d^2e^6+45\,de^7+111\,e^8)x^6 - 84\,(100\,d^3e^5+45\,d^2e^6+111\,de^7}{}$$

input

```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*
e^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^
5 + 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4
- 140*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 6
5*e^8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 14
8*d^2*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^
5*e^3 + 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 42
0*(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^
5 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

3.300.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\cdot\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4\cdot\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\frac{37}{e}\right)+x^3\left(-\frac{100d^5}{3e^6}-\frac{15d^4}{e^5}-\frac{37d^3}{e^4}-\frac{37d^2}{3e^3}-\frac{148d}{3e^2}+\frac{65}{3e}\right)+x^2\cdot\left(\frac{50d^6}{e^7}+\frac{45d^5}{2e^6}+\frac{111d^4}{2e^5}+\frac{37d^3}{2e^4}+\frac{74d^2}{e^3}-\frac{65d}{2e^2}+\frac{107}{2e}\right)+x\left(-\frac{100d^7}{e^8}-\frac{45d^6}{e^7}-\frac{111d^5}{e^6}-\frac{37d^4}{e^5}-\frac{148d^3}{e^4}+\frac{65d^2}{e^3}-\frac{107d}{e^2}+\frac{33}{e}\right)+\frac{25x^8}{2e}+\frac{(5d^2-2de+3e^2)^2\cdot(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

output
```
x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
 + 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*
e)) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*
e**2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 3
7*d**2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**
5/(2*e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2
*e**2) + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 3
7*d**4/e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/
(2*e) + (5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*
e**3 + 2*e**4)*log(d + e*x)/e**9
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \frac{5250\,e^7x^8-300\,(20\,de^6+9\,e^7)x^7+70\,(100\,d^2e^5+45\,de^6+111\,e^7)x^6-84\,(100\,d^3e^4+45\,d^2e^5+111\,de^6+\cdots}{\cdots} + \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*
e^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^
5 + 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4
- 140*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 6
5*e^7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*
d^2*e^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2
+ 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (10
0*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 1
07*d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.18

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \frac{5250\,e^7x^8-6000\,de^6x^7-2700\,e^7x^7+7000\,d^2e^5x^6+3150\,de^6x^6+7770\,e^7x^6-8400\,d^3e^4x^5-3780\,d^2e^5x^5\cdots}{\cdots} + \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log(|ex+d|)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gi
ac")
```

output
```
1/420*(5250*e^7*x^8 - 6000*d*e^6*x^7 - 2700*e^7*x^7 + 7000*d^2*e^5*x^6 + 3
150*d*e^6*x^6 + 7770*e^7*x^6 - 8400*d^3*e^4*x^5 - 3780*d^2*e^5*x^5 - 9324*
d*e^6*x^5 - 3108*e^7*x^5 + 10500*d^4*e^3*x^4 + 4725*d^3*e^4*x^4 + 11655*d^
2*e^5*x^4 + 3885*d*e^6*x^4 + 15540*e^7*x^4 - 14000*d^5*e^2*x^3 - 6300*d^4*
e^3*x^3 - 15540*d^3*e^4*x^3 - 5180*d^2*e^5*x^3 - 20720*d*e^6*x^3 + 9100*e^
7*x^3 + 21000*d^6*e*x^2 + 9450*d^5*e^2*x^2 + 23310*d^4*e^3*x^2 + 7770*d^3*
e^4*x^2 + 31080*d^2*e^5*x^2 - 13650*d*e^6*x^2 + 22470*e^7*x^2 - 42000*d^7*
x - 18900*d^6*e*x - 46620*d^5*e^2*x - 15540*d^4*e^3*x - 62160*d^3*e^4*x +
27300*d^2*e^5*x - 44940*d*e^6*x + 13860*e^7*x)/e^8 + (100*d^8 + 45*d^7*e +
 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e^6 - 33*d*
e^7 + 18*e^8)*log(abs(e*x + d))/e^9
```

3.300. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.300.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.23

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

output
```
x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((1
00*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e
)) + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*
(111/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(
111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/
e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x
^2*(107/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/
e^2 + 45/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d
^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^
4 + 37*d^5*e^3 + 111*d^6*e^2))/e^9
```

3.300. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx$

3.301.1 Optimal result

Integrand size = 38, antiderivative size = 353

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2} dx \\
&= \frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)x}{e^8} \\
&\quad - \frac{(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)x^2}{2e^7} \\
&\quad + \frac{(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)x^3}{3e^6} \\
&\quad - \frac{(400d^3+135d^2e+222de^2+37e^3)x^4}{4e^5} + \frac{3(100d^2+30de+37e^2)x^5}{5e^4} \\
&\quad - \frac{5(40d+9e)x^6}{6e^3} + \frac{100x^7}{7e^2} - \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)} \\
&\quad - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
(700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*
x/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2
/e^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400
*d^3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e
^4-5/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3
*e+3*d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4
*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

3.301.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.97

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{420e(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)\,x-210e^2(600d^5+225d^4e+444}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
2,x]
```

output
```
(420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 13
0*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^
2*e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e
^2 + 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 +
37*e^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*
e)*x^6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e +
3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5
*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d
 + e*x])/(420*e^9)
```

3.301.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2(4x^4-5x^3+3x^2+x+2)}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{3x^4(100d^2+30de+37e^2)}{e^4} - \frac{x^3(400d^3+135d^2e+222de^2+37e^3)}{e^5} + \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e}{e^8(d+ex)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)}+\frac{x^3(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)}{3e^6}-\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}-\frac{x^2(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)}{2e^7}+\frac{x(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)}{e^8}-\frac{5x^6(40d+9e)}{6e^3}+\frac{100x^7}{7e^2}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

output
```
((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^
5 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 +
296*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 7
4*d*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d
+ 9*e)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^
4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*
e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e
^5)*Log[d + e*x])/e^9
```

3.301.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.301.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\left(800d^8+315d^7e+666d^6e^2+185d^5e^3+592d^4e^4-195d^3e^5+214d^2e^6-33d\,e^7+18e^8\right)x}{e^8d}+\frac{100x^8}{7e}-\frac{5(160d+63e)x^7}{42e^2}+\frac{\left(800d^2+315de+666e^2\right)x^6}{30e^3}$
default	$\frac{111}{5}e^6x^5-\frac{15}{2}e^6x^6+60d^3x^3e^3+270d^5ex-\frac{225}{2}d^4e^2x^2-\frac{135}{4}d^2e^4x^4+18d\,e^5x^5+\frac{100}{7}e^6x^7-\frac{37}{4}x^4e^6+\frac{148}{3}x^3e^6+\frac{65}{2}x^2e^6+107x\,e^6+$
risch	$-\frac{111d^6}{e^7(ex+d)}-\frac{37d^5}{e^6(ex+d)}-\frac{148d^4}{e^5(ex+d)}+\frac{65d^3}{e^4(ex+d)}-\frac{107d^2}{e^3(ex+d)}+\frac{33d}{e^2(ex+d)}-\frac{15x^6}{2e^2}+\frac{60d^3x^3}{e^5}+\frac{270d^5x}{e^7}-\frac{225}{2}$
parallelrisch	$-\frac{336000\ln(ex+d)d^8-9324x^6e^8+3885x^5e^8-20720x^4e^8-13650x^3e^8-44940x^2e^8-13860d\,e^7-81900d^3e^5+89880d^2e^6+77700}{}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
((800*d^8+315*d^7*e+666*d^6*e^2+185*d^5*e^3+592*d^4*e^4-195*d^3*e^5+214*d^
2*e^6-33*d*e^7+18*e^8)/e^8/d*x+100/7*x^8/e-5/42*(160*d+63*e)/e^2*x^7+1/30*
(800*d^2+315*d*e+666*e^2)/e^3*x^6-1/20*(800*d^3+315*d^2*e+666*d*e^2+185*e^
3)/e^4*x^5+1/12*(800*d^4+315*d^3*e+666*d^2*e^2+185*d*e^3+592*e^4)/e^5*x^4-
1/6*(800*d^5+315*d^4*e+666*d^3*e^2+185*d^2*e^3+592*d*e^4-195*e^5)/e^6*x^3+
1/2*(800*d^6+315*d^5*e+666*d^4*e^2+185*d^3*e^3+592*d^2*e^4-195*d*e^5+214*e
^6)/e^7*x^2)/(e*x+d)-(800*d^7+315*d^6*e+666*d^5*e^2+185*d^4*e^3+592*d^3*e^
4-195*d^2*e^5+214*d*e^6-33*e^7)/e^9*ln(e*x+d)
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$=\frac{6000\,e^8x^8-42000\,d^8-18900\,d^7e-46620\,d^6e^2-15540\,d^5e^3-62160\,d^4e^4+27300\,d^3e^5-44940\,d^2e^6+1}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*
e^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e
^8 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*
x^6 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d
^4*e^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^
5*e^3 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3
 + 210*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^
6 - 195*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3
+ 148*d^4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 +
315*d^7*e + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^
2*e^6 - 33*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 +
592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x +
 d*e^9)
```

3.301.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= x^6\left(-\frac{100d}{3e^3}-\frac{15}{2e^2}\right)+x^5\cdot\left(\frac{60d^2}{e^4}+\frac{18d}{e^3}+\frac{111}{5e^2}\right) \\
&\quad+x^4\left(-\frac{100d^3}{e^5}-\frac{135d^2}{4e^4}-\frac{111d}{2e^3}-\frac{37}{4e^2}\right)+x^3\cdot\left(\frac{500d^4}{3e^6}+\frac{60d^3}{e^5}+\frac{111d^2}{e^4}+\frac{74d}{3e^3}+\frac{148}{3e^2}\right) \\
&\quad+x^2\left(-\frac{300d^5}{e^7}-\frac{225d^4}{2e^6}-\frac{222d^3}{e^5}-\frac{111d^2}{2e^4}-\frac{148d}{e^3}+\frac{65}{2e^2}\right) \\
&\quad+x\left(\frac{700d^6}{e^8}+\frac{270d^5}{e^7}+\frac{555d^4}{e^6}+\frac{148d^3}{e^5}+\frac{444d^2}{e^4}-\frac{130d}{e^3}+\frac{107}{e^2}\right) \\
&\quad+\frac{-100d^8-45d^7e-111d^6e^2-37d^5e^3-148d^4e^4+65d^3e^5-107d^2e^6+33de^7-18e^8}{de^9+e^{10}x} \\
&\quad+\frac{100x^7}{7e^2} \\
&\quad-\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}
\end{aligned}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

output
```
x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 11
1/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) -
37/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74
*d/(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 2
22*d**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6
/e**8 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 13
0*d/e**3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e*
*3 - 148*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(
d*e**9 + e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**
5 + 127*d**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d +
 e*x)/e**9
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= -\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}$$
$$+\frac{6000\,e^6x^7-350\,(40\,de^5+9\,e^6)x^6+252\,(100\,d^2e^4+30\,de^5+37\,e^6)x^5-105\,(400\,d^3e^3+135\,d^2e^4+222}{}$$
$$-\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
-(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7
 - 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5
- 105*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*
e^2 + 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e
 + 225*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420
*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^
5 + 107*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 5
92*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$

$$-\frac{(ex+d)^7\left(\frac{350\,(160\,de+9\,e^2)}{(ex+d)e}-\frac{84\,(2800\,d^2e^2+315\,de^3+111\,e^4)}{(ex+d)^2e^2}+\frac{105\,(5600\,d^3e^3+945\,d^2e^4+666\,de^5+37\,e^6)}{(ex+d)^3e^3}-\frac{140\,(7000\,d^4e^4+157}{}\right.}{}$$

$$+\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9}$$

$$-\frac{\frac{100\,d^8e^7}{ex+d}+\frac{45\,d^7e^8}{ex+d}+\frac{111\,d^6e^9}{ex+d}+\frac{37\,d^5e^{10}}{ex+d}+\frac{148\,d^4e^{11}}{ex+d}-\frac{65\,d^3e^{12}}{ex+d}+\frac{107\,d^2e^{13}}{ex+d}-\frac{33\,de^{14}}{ex+d}+\frac{18\,e^{15}}{ex+d}}{e^{16}}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-1/420*(e*x + d)^7*(350*(160*d*e + 9*e^2)/((e*x + d)*e) - 84*(2800*d^2*e^2
 + 315*d*e^3 + 111*e^4)/((e*x + d)^2*e^2) + 105*(5600*d^3*e^3 + 945*d^2*e^
4 + 666*d*e^5 + 37*e^6)/((e*x + d)^3*e^3) - 140*(7000*d^4*e^4 + 1575*d^3*e
^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)/((e*x + d)^4*e^4) + 210*(5600*d^5
*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)/((
e*x + d)^5*e^5) - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^3
*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)/((e*x + d)^6*e^6) - 6000)/e^9
 + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^
2*e^5 + 214*d*e^6 - 33*e^7)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^9 - (
100*d^8*e^7/(e*x + d) + 45*d^7*e^8/(e*x + d) + 111*d^6*e^9/(e*x + d) + 37*
d^5*e^10/(e*x + d) + 148*d^4*e^11/(e*x + d) - 65*d^3*e^12/(e*x + d) + 107*
d^2*e^13/(e*x + d) - 33*d*e^14/(e*x + d) + 18*e^15/(e*x + d))/e^16
```

3.301.9 Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.66

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = x^2\left(\frac{65}{2\,e^2} - \frac{d\left(\frac{148}{e^2}+\frac{2\,d\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e^2}\right)}{e} + \frac{d^2\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)}{2\,e^2}\right)$$

3.301. $\left(\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \right.$ $2\,d\left(\frac{37}{e^2}+\frac{2\,d\left(\frac{111}{e^2}-\frac{100\,d^2}{e^4}+\frac{2\,d\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{200\,d}{e^3}+\frac{45}{e^2}\right)}{e^2}\right)$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

output
```
x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e
^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2
))/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^
2))/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 +
 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(
3*e^2) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3
+ 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^
2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4
*e^2) + (d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*
e) - (d^2*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e
^4 + (2*d*((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^
2)) - x*((2*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e
^2))/e^2))/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2)
)/e))/e^2))/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*
d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107
/e^2 + (d^2*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d
*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d
^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2)
+ (100*x^7)/(7*e^2) - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2...
```

3.301. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

3.302 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

3.302.1 Optimal result

Integrand size = 38, antiderivative size = 354

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx \\
&= -\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)\,x}{e^8} \\
&\quad+\frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)\,x^2}{2e^7} \\
&\quad-\frac{(1000d^3+270d^2e+333de^2+37e^3)\,x^3}{3e^6}+\frac{3(200d^2+45de+37e^2)\,x^4}{4e^5} \\
&\quad-\frac{3(20d+3e)x^5}{e^4}+\frac{50x^6}{3e^3}-\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2} \\
&\quad+\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)} \\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
-(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*
(1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+2
70*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(
20*d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+
88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665
*d^4*e^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

3.302.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{9000d^8-390d^7e(-9+40x)-18d^6e^2(-407+240x+2300x^2)-2d^5e^3(-999+2664x+6750x^2+5600x$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
3,x]
```

output
```
(9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
 2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x
 - 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 666
0*x^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3
 + 1665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*
x^3 + 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3
 + 888*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d
+ e*x)^2*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

3.302.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2(4x^4-5x^3+3x^2+x+2)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{3x^3(200d^2+45de+37e^2)}{e^5}-\frac{x^2(1000d^3+270d^2e+333de^2+37e^3)}{e^6}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2}{e^8(d+ex)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^4(200d^2+45de+37e^2)}{4e^5}-\frac{x^3(1000d^3+270d^2e+333de^2+37e^3)}{3e^6}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2}+\frac{x^2(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)}{2e^7}+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)}-\frac{x(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)}{e^8}+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}-\frac{3x^5(20d+3e)}{e^4}+\frac{50x^6}{3e^3}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

output
```
-(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5
)*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^
2)/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (
3*(200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (5
0*x^6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 +
 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x))
+ ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*
d*e^5 + 107*e^6)*Log[d + e*x])/e^9
```

3.302.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.302. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

3.302.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\left(5600d^7+1890d^6e+3330d^5e^2+740d^4e^3+1776d^3e^4-390d^2e^5+214d\,e^6-33e^7\right)x}{e^8}+\frac{50x^8}{3e}+\frac{8400d^8+2835d^7e+4995d^6e^2+1110d^5e^3+2664d^4e^4}{2e^9}$
default	$-\frac{-\frac{50}{3}e^5x^6+60d\,e^4x^5+9e^5x^5-150d^2e^3x^4-\frac{135}{4}d\,e^4x^4-\frac{111}{4}e^5x^4+\frac{1000}{3}d^3e^2x^3+90d^2e^3x^3+111d\,e^4x^3+\frac{37}{3}e^5x^3-750d^4e\,x^2-2}{e^8}$
risch	$\frac{2800\ln(ex+d)d^6}{e^9}+\frac{945\ln(ex+d)d^5}{e^8}-\frac{37x^3}{3e^3}+\frac{\left(800d^7+315d^6e+666d^5e^2+185d^4e^3+592d^3e^4-195d^2e^5+214d\,e^6-33e^7\right)x}{e^8(e}$
parallelrisch	$33600\ln(ex+d)d^8+333x^6e^8-148x^5e^8+888x^4e^8+780x^3e^8-396x\,e^8-198d\,e^7-3510d^3e^5+1926d^2e^6+6660d^5e^3+15984d^4e^4+2$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
((5600*d^7+1890*d^6*e+3330*d^5*e^2+740*d^4*e^3+1776*d^3*e^4-390*d^2*e^5+21
4*d*e^6-33*e^7)/e^8*x+50/3*x^8/e+1/2*(8400*d^8+2835*d^7*e+4995*d^6*e^2+111
0*d^5*e^3+2664*d^4*e^4-585*d^3*e^5+321*d^2*e^6-33*d*e^7-18*e^8)/e^9-1/3*(8
0*d+27*e)/e^2*x^7+1/12*(560*d^2+189*d*e+333*e^2)/e^3*x^6-1/6*(560*d^3+189*
d^2*e+333*d*e^2+74*e^3)/e^4*x^5+1/12*(2800*d^4+945*d^3*e+1665*d^2*e^2+370*
d*e^3+888*e^4)/e^5*x^4-1/3*(2800*d^5+945*d^4*e+1665*d^3*e^2+370*d^2*e^3+88
8*d*e^4-195*e^5)/e^6*x^3)/(e*x+d)^2+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d
^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{200\,e^8x^8+9000\,d^8+3510\,d^7e+7326\,d^6e^2+1998\,d^5e^3+6216\,d^4e^4-1950\,d^3e^5+1926\,d^2e^6-198\,de^7-1}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 +
6216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d
*e^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*
e^5 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5
+ 1665*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4
+ 1665*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2
+ 2250*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^
2 - 12*(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5
+ 130*d^2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^
6*e^2 + 370*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*
e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 +
 107*e^8)*x^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 +
 888*d^3*e^5 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e
^10*x + d^2*e^9)
```

3.302.6 Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\cdot\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right)$$
$$+x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\cdot\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d^2}{e^5}+\frac{111d}{2e^4}+\frac{74}{e^3}\right)$$
$$+x\left(-\frac{2100d^5}{e^8}-\frac{675d^4}{e^7}-\frac{1110d^3}{e^6}-\frac{222d^2}{e^5}-\frac{444d}{e^4}+\frac{65}{e^3}\right)$$
$$+\frac{1500d^8+585d^7e+1221d^6e^2+333d^5e^3+1036d^4e^4-325d^3e^5+321d^2e^6-33de^7-18e^8+x(1600d^7e - }{2d^2e^9+4de^{10}x+2e^{11}x^2}$$
$$+\frac{50x^6}{3e^3}$$
$$+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

output
```
x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4
*e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e*
*3)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4
) + 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d
**2/e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**
2 + 333*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*
e**7 - 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**
4*e**4 + 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e
**9 + 4*d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d*
*5*e + 1665*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e
**6)*log(d + e*x)/e**9
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\,(800\,d^7e}{2\,(e^{11}x^2+2\,de^{10}x+d^2e^9)}$$
$$+\frac{200\,e^5x^6-36\,(20\,de^4+3\,e^5)x^5+9\,(200\,d^2e^3+45\,de^4+37\,e^5)x^4-4\,(1000\,d^3e^2+270\,d^2e^3+333\,de^4+}{}$$
$$+\frac{(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 32
5*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 +
 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^
8)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4
 + 3*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2
+ 270*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 66
6*d^2*e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3
*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x +
d)/e^9
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6)\log\left(|ex+d|\right)}{e^9}$$
$$+ \frac{1500\,d^8+585\,d^7e+1221\,d^6e^2+333\,d^5e^3+1036\,d^4e^4-325\,d^3e^5+321\,d^2e^6-33\,de^7-18\,e^8+2\,(800\,d}{2\,(ex+d)^2e^9}$$
$$+ \frac{200\,e^{15}x^6-720\,de^{14}x^5-108\,e^{15}x^5+1800\,d^2e^{13}x^4+405\,de^{14}x^4+333\,e^{15}x^4-4000\,d^3e^{12}x^3-1080\,d^2e^{1}}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
(2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e
^5 + 107*e^6)*log(abs(e*x + d))/e^9 + 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6
*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 -
 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3
*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)/((e*x + d)^2*e^9) + 1/12*(200*
e^15*x^6 - 720*d*e^14*x^5 - 108*e^15*x^5 + 1800*d^2*e^13*x^4 + 405*d*e^14*
x^4 + 333*e^15*x^4 - 4000*d^3*e^12*x^3 - 1080*d^2*e^13*x^3 - 1332*d*e^14*x
^3 - 148*e^15*x^3 + 9000*d^4*e^11*x^2 + 2700*d^3*e^12*x^2 + 3996*d^2*e^13*
x^2 + 666*d*e^14*x^2 + 888*e^15*x^2 - 25200*d^5*e^10*x - 8100*d^4*e^11*x -
 13320*d^3*e^12*x - 2664*d^2*e^13*x - 5328*d*e^14*x + 780*e^15*x)/e^18
```

3.302.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.18

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = x^4\left(\frac{111}{4\,e^3}-\frac{75\,d^2}{e^5}+\frac{3\,d\,\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{4\,e}\right) - x^3\left(\frac{37}{3\,e^3}+\frac{100\,d^3}{3\,e^6}+\frac{d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\,\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right) - x^5\left(\frac{60\,d}{e^4}+\frac{9}{e^3}\right) + x\left(\frac{65}{e^3} - \frac{3\,d\left(\frac{148}{e^3}+\frac{3\,d\left(\frac{37}{e^3}+\frac{100\,d^3}{e^6}+\frac{3\,d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)}{e}-\frac{3\,d^2\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e^2}+\frac{d^3\,(\cdots}{\cdots}\right)}{e} + \frac{3\,d^2\left(\frac{37}{e^3}+\frac{100\,d^3}{e^6}+\frac{3\,d\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)}{e^2} \cdots\right.$$

$$d^3\left(\frac{111}{e^3}-\frac{300\,d^2}{e^5}+\frac{3\,d\left(\frac{300\,d}{e^4}+\frac{45}{e^3}\right)}{e}\right) \cdots$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

output
```
x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^
3*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((3
00*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*
d)/e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6
 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3
*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (
3*d*((300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e
 + (3*d^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*(
(300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 -
(d^3*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x
*(214*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 1
85*d^4*e^3 + 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 32
1*d^2*e^6 - 325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e)
)/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d
*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e
^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*
(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3
*((300*d)/e^4 + 45/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 +
 2800*d^6 + 107*e^6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

3.302. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

3.303 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$

3.303.1 Optimal result

Integrand size = 38, antiderivative size = 360

$$\begin{aligned}\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx &= \frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8}\\ &-\frac{(2000d^3+450d^2e+444de^2+37e^3)\,x^2}{2e^7}+\frac{(1000d^2+180de+111e^2)\,x^3}{3e^6}\\ &-\frac{5(80d+9e)x^4}{4e^5}+\frac{20x^5}{e^4}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3}\\ &+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2}\\ &-\frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)}\\ &-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}\end{aligned}$$

output
```
2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450
*d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/
4*(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3
*d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d
^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2+(-2800*d^6-945*d^
5*e-1665*d^4*e^2-370*d^3*e^3-888*d^2*e^4+195*d*e^5-107*e^6)/e^9/(e*x+d)-(5
600*d^5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^
9
```

3.303.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$

$$= \frac{24e(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x-6e^2(2000d^3+450d^2e+444de^2+37e^3)\,x^2+4e^3(100}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
4,x]
```

output
```
(24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(
2000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e
 + 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*
e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 +
(6*(800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^
2*e^5 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 166
5*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) -
12*(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^
5)*Log[d + e*x])/(12*e^9)
```

3.303.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2(4x^4-5x^3+3x^2+x+2)}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{x^2(1000d^2+180de+111e^2)}{e^6}-\frac{x(2000d^3+450d^2e+444de^2+37e^3)}{e^7}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3c}{e^8(d+ex)^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(1000d^2+180de+111e^2)}{3e^6}-\frac{x^2(2000d^3+450d^2e+444de^2+37e^3)}{2e^7}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3}+\frac{2x(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8}+\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2}-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}-\frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)}-\frac{5x^4(80d+9e)}{4e^5}+\frac{20x^5}{e^4}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

output
```
(2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((200
0*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*
e + 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 -
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/
(3*e^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d
^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 +
 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^
6)/(e^9*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
+ 592*d*e^4 - 65*e^5)*Log[d + e*x])/e^9
```

3.303.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.303.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00

method	result
norman	$\frac{\frac{20x^8}{e} - \frac{61600d^8+17325d^7e+24420d^6e^2+4070d^5e^3+6512d^4e^4-715d^3e^5+214d^2e^6+33d\,e^7+36e^8}{6e^9} - \frac{5(32d+9e)x^7}{4e^2} + \frac{(1120d^2+315de+444e^2)x}{12e^3}}{}$
default	$\frac{20e^4x^5-100d\,e^3x^4-\frac{45}{4}e^4x^4+\frac{1000}{3}d^2e^2x^3+60d\,e^3x^3+37e^4x^3-1000d^3e\,x^2-225d^2e^2x^2-222d\,e^3x^2-\frac{37}{2}e^4x^2+3500d^4x+900d^3}{e^8}$
risch	$\frac{20x^5}{e^4} - \frac{100d\,x^4}{e^5} - \frac{45x^4}{4e^4} + \frac{1000d^2x^3}{3e^6} + \frac{60d\,x^3}{e^5} + \frac{37x^3}{e^4} - \frac{1000d^3x^2}{e^7} - \frac{225d^2x^2}{e^6} - \frac{222d\,x^2}{e^5} - \frac{37x^2}{2e^4} + \frac{3500d^4x}{e^8}$
parallelrisch	$-\frac{67200\ln(ex+d)d^8-444x^6e^8+222x^5e^8-1776x^4e^8+1284x^2e^8+198x\,e^8+66d\,e^7-1430d^3e^5+428d^2e^6+8140d^5e^3+13024d^4e^4}{}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
(20*x^8/e-1/6*(61600*d^8+17325*d^7*e+24420*d^6*e^2+4070*d^5*e^3+6512*d^4*e
^4-715*d^3*e^5+214*d^2*e^6+33*d*e^7+36*e^8)/e^9-5/4*(32*d+9*e)/e^2*x^7+1/1
2*(1120*d^2+315*d*e+444*e^2)/e^3*x^6-1/4*(1120*d^3+315*d^2*e+444*d*e^2+74*
e^3)/e^4*x^5+1/4*(5600*d^4+1575*d^3*e+2220*d^2*e^2+370*d*e^3+592*e^4)/e^5*
x^4-(16800*d^6+4725*d^5*e+6660*d^4*e^2+1110*d^3*e^3+1776*d^2*e^4-195*d*e^5
+107*e^6)/e^7*x^2-1/2*(50400*d^7+14175*d^6*e+19980*d^5*e^2+3330*d^4*e^3+53
28*d^3*e^4-585*d^2*e^5+214*d*e^6+33*e^7)/e^8*x)/(e*x+d)^3-(5600*d^5+1575*d
^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.63

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$
$$= \frac{240\,e^8x^8 - 29200\,d^8 - 9630\,d^7e - 16428\,d^6e^2 - 3478\,d^5e^3 - 7696\,d^4e^4 + 1430\,d^3e^5 - 428\,d^2e^6 - 66\,de^7 - }{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3
- 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d
*e^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3
*e^5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*
e^5 + 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d
^4*e^4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2
 + 3060*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 2
14*e^8)*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 +
2664*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d
^7*e + 2220*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e
^3 + 1575*d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 +
 3*(5600*d^6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6
 - 65*d*e^7)*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e
^4 + 592*d^3*e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 +
 3*d^2*e^10*x + d^3*e^9)
```

3.303.6 Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$
$$= x^4\left(-\frac{100d}{e^5}-\frac{45}{4e^4}\right)+x^3\cdot\left(\frac{1000d^2}{3e^6}+\frac{60d}{e^5}+\frac{37}{e^4}\right)$$
$$+x^2\left(-\frac{1000d^3}{e^7}-\frac{225d^2}{e^6}-\frac{222d}{e^5}-\frac{37}{2e^4}\right)+x\left(\frac{3500d^4}{e^8}+\frac{900d^3}{e^7}+\frac{1110d^2}{e^6}+\frac{148d}{e^5}+\frac{148}{e^4}\right)$$
$$+\frac{-14600d^8-4815d^7e-8214d^6e^2-1739d^5e^3-3848d^4e^4+715d^3e^5-214d^2e^6-33de^7-36e^8+x^2(-1}{}$$
$$+\frac{20x^5}{e^4}-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

output
```
x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 +
37/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4
)) + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148
/e**4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 38
48*d**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*
(-16800*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 532
8*d**2*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2
 - 17982*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 64
2*d*e**7 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*
e**12*x**3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 3
70*d**2*e**3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.08

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx =$$

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(28\cdots}{\cdots}$$

$$+\frac{240\,e^4x^5-15\,(80\,de^3+9\,e^4)x^4+4\,(1000\,d^2e^2+180\,de^3+111\,e^4)x^3-6\,(2000\,d^3e+450\,d^2e^2+444\,de^3\cdots}{12\,e^8}$$

$$-\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
-1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
- 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^
5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^
2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d
^2*e^10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(
1000*d^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 44
4*d*e^3 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3
+ 74*e^4)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 5
92*d*e^4 - 65*e^5)*log(e*x + d)/e^9
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.06

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$
$$= -\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(|ex+d|)}{e^9}$$
$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(28}{}$$
$$+\frac{240\,e^{16}x^5-1200\,de^{15}x^4-135\,e^{16}x^4+4000\,d^2e^{14}x^3+720\,de^{15}x^3+444\,e^{16}x^3-12000\,d^3e^{13}x^2-2700\,d^2}{12}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
-(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)
*log(abs(e*x + d))/e^9 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739
*d^5*e^3 + 3848*d^4*e^4 - 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 +
6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 -
 195*d*e^7 + 107*e^8)*x^2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 +
 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/((e*x
+ d)^3*e^9) + 1/12*(240*e^16*x^5 - 1200*d*e^15*x^4 - 135*e^16*x^4 + 4000*d
^2*e^14*x^3 + 720*d*e^15*x^3 + 444*e^16*x^3 - 12000*d^3*e^13*x^2 - 2700*d^
2*e^14*x^2 - 2664*d*e^15*x^2 - 222*e^16*x^2 + 42000*d^4*e^12*x + 10800*d^3
*e^13*x + 13320*d^2*e^14*x + 1776*d*e^15*x + 1776*e^16*x)/e^20
```

3.303.9 Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.56

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx = x^3\left(\frac{37}{e^4}-\frac{200\,d^2}{e^6}+\frac{4\,d\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{3\,e}\right)$$
$$-x^2\left(\frac{37}{2\,e^4}+\frac{200\,d^3}{e^7}+\frac{2\,d\,\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{3\,d^2\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)$$
$$-\frac{x\,\left(5200\,d^7+\frac{3465\,d^6\,e}{2}+2997\,d^5\,e^2+\frac{1295\,d^4\,e^3}{2}+1480\,d^3\,e^4-\frac{585\,d^2\,e^5}{2}+107\,d\,e^6+\frac{33\,e^7}{2}\right)+\frac{14600\,d^8+4815\,d^7\,e\cdots}{\cdots}}{d\cdots}$$
$$-x^4\left(\frac{100\,d}{e^5}+\frac{45}{4\,e^4}\right)$$
$$+x\left(\frac{148}{e^4}-\frac{100\,d^4}{e^8}+\frac{4\,d\,\left(\frac{37}{e^4}+\frac{400\,d^3}{e^7}+\frac{4\,d\,\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{6\,d^2\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)}{e}\right.$$
$$\left.-\frac{6\,d^2\,\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e^2}+\frac{4\,d^3\,\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^3}\right)$$
$$+\frac{20\,x^5}{e^4}-\frac{\ln\left(d+e\,x\right)\,\left(5600\,d^5+1575\,d^4\,e+2220\,d^3\,e^2+370\,d^2\,e^3+592\,d\,e^4-65\,e^5\right)}{e^9}$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

output
```
x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(3
7/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/
e^5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6
 + (3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4
 + (1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 +
 36*e^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d
^6*e^2)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*
d^3*e^4 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x +
 3*d*e^10*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e
^8 + (4*d*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*(
(400*d)/e^5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6
*d^2*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*
d^3*((400*d)/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4
 + 1575*d^4*e + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

3.303. $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$

3.304 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

3.304.1 Optimal result

Integrand size = 38, antiderivative size = 221

$$\begin{aligned}\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625}\\ &\quad-\frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250}+\frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875}\\ &\quad+\frac{3}{500}e\left(100d^2-165de+27e^2\right)x^4+\frac{3}{125}(20d-11e)e^2x^5+\frac{2e^3x^6}{15}\\ &\quad-\frac{(52875d^3+449175d^2e-274845de^2-53189e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{78125\sqrt{14}}\\ &\quad+\frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\log\left(3+2x+5x^2\right)}{156250}\end{aligned}$$

output
```
1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-60
75*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458
*e^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2
/15*e^3*x^6+1/156250*(57250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^
2+2*x+3)-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arctan(
1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{35x(250d^3(486-495x+200x^2)+450d^2e(916+405x-550x^2+250x^3)+45de^2(-3524+4580x+2700}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
(35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2
+ 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) +
e^3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) -
6*Sqrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1
 + 5*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)
*Log[3 + 2*x + 5*x^2])/6562500
```

3.304.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^3}{5x^2+2x+3}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{3}{125}ex^3(100d^2-165de+27e^2)+\frac{1}{625}x^2(500d^3-2475d^2e+1215de^2+458e^3)+\frac{875d^3-103050d^2e+x(57}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(52875d^3+449175d^2e-274845de^2-53189e^3\right)}{78125\sqrt{14}}+\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(500d^3-2475d^2e+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(4125d^3-6075d^2e-6870de^2+881e^3\right)}{6250}+\frac{\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(5x^2+2x+3\right)}{156250}+\frac{x\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)}{15625}+\frac{3}{125}e^2x^5(20d-11e)+\frac{2e^3x^6}{15}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3
- 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e +
1215*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/5
00 + (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175
*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt
[14]) + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x +
 5*x^2])/156250
```

3.304.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.304.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

method	result
default	$\frac{2e^3x^6}{15}+\frac{12x^5d\,e^2}{25}-\frac{33x^5e^3}{125}+\frac{3x^4d^2e}{5}-\frac{99x^4d\,e^2}{100}+\frac{81e^3x^4}{500}+\frac{4d^3x^3}{15}-\frac{33d^2e\,x^3}{25}+\frac{81d\,e^2x^3}{125}+\frac{458e^3x^3}{1875}-\frac{33x^2d^3}{50}$
risch	$-\frac{33d^2e\,x^3}{25}+\frac{12x^5d\,e^2}{25}+\frac{3x^4d^2e}{5}-\frac{99x^4d\,e^2}{100}+\frac{458e^3x^3}{1875}-\frac{17967\sqrt{14}\,d^2e\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{43750}+\frac{54969\sqrt{14}\,d\,e^2\arctan(}{218750}$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
2/15*e^3*x^6+12/25*x^5*d*e^2-33/125*x^5*e^3+3/5*x^4*d^2*e-99/100*x^4*d*e^2
+81/500*e^3*x^4+4/15*d^3*x^3-33/25*d^2*e*x^3+81/125*d*e^2*x^3+458/1875*e^3
*x^3-33/50*x^2*d^3+243/250*d^2*e*x^2+687/625*d*e^2*x^2-881/6250*e^3*x^2+81
/125*x*d^3+1374/625*d^2*e*x-2643/3125*d*e^2*x-5108/15625*e^3*x+1/156250*(5
7250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*x+3)+1/218750*(-105
75*d^3-89835*d^2*e+54969*d*e^2+53189/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*
14^(1/2))
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= \frac{2}{15}\,e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5 \\ &+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right)x^3 \\ &-\frac{1}{6250}\left(4125\,d^3-6075\,d^2e-6870\,de^2+881\,e^3\right)x^2 \\ &-\frac{1}{1093750}\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) \\ &+\frac{1}{15625}\left(10125\,d^3+34350\,d^2e-13215\,de^2-5108\,e^3\right)x \\ &+\frac{1}{156250}\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^
2 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3
 - 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*s
qrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*
sqrt(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 510
8*e^3)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*lo
g(5*x^2 + 2*x + 3)
```

3.304.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{2e^3x^6}{15} + x^5\cdot\left(\frac{12de^2}{25} - \frac{33e^3}{125}\right) + x^4\cdot\left(\frac{3d^2e}{5} - \frac{99de^2}{100} + \frac{81e^3}{500}\right)$$
$$+ x^3\cdot\left(\frac{4d^3}{15} - \frac{33d^2e}{25} + \frac{81de^2}{125} + \frac{458e^3}{1875}\right) + x^2\left(-\frac{33d^3}{50} + \frac{243d^2e}{250} + \frac{687de^2}{625} - \frac{881e^3}{6250}\right)$$
$$+ x\left(\frac{81d^3}{125} + \frac{1374d^2e}{625} - \frac{2643de^2}{3125} - \frac{5108e^3}{15625}\right) + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250}\right.$$
$$\left. - \frac{\sqrt{14}i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \frac{53}{}}{52875d^3 + 449175d^{}}\right.$$
$$+ \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250}\right.$$
$$\left. + \frac{\sqrt{14}i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \frac{53}{}}{52875d^3 + 449175d^{}}\right.$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99
*d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/12
5 + 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 -
 881*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 51
08*e**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23
431*e**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 -
 53189*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 -
53189*e**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53
189*e**3)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) +
(229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 +
 sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187
500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sq
rt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(528
75*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{2}{15}\,e^3x^6+\frac{3}{125}\,\left(20\,de^2-11\,e^3\right)x^5 + \frac{3}{500}\,\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\,\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right)x^3 - \frac{1}{6250}\,\left(4125\,d^3-6075\,d^2e-6870\,de^2+881\,e^3\right)x^2 - \frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{15625}\,\left(10125\,d^3+34350\,d^2e-13215\,de^2-5108\,e^3\right)x + \frac{1}{156250}\,\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^
2 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3
 - 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*s
qrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*
sqrt(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 510
8*e^3)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*lo
g(5*x^2 + 2*x + 3)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{2}{15}\,e^3x^6+\frac{12}{25}\,de^2x^5-\frac{33}{125}\,e^3x^5+\frac{3}{5}\,d^2ex^4 - \frac{99}{100}\,de^2x^4+\frac{81}{500}\,e^3x^4+\frac{4}{15}\,d^3x^3-\frac{33}{25}\,d^2ex^3+\frac{81}{125}\,de^2x^3+\frac{458}{1875}\,e^3x^3-\frac{33}{50}\,d^3x^2 + \frac{243}{250}\,d^2ex^2+\frac{687}{625}\,de^2x^2-\frac{881}{6250}\,e^3x^2+\frac{81}{125}\,d^3x+\frac{1374}{625}\,d^2ex-\frac{2643}{3125}\,de^2x-\frac{5108}{15625}\,e^3x - \frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{156250}\,\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
2/15*e^3*x^6 + 12/25*d*e^2*x^5 - 33/125*e^3*x^5 + 3/5*d^2*e*x^4 - 99/100*d
*e^2*x^4 + 81/500*e^3*x^4 + 4/15*d^3*x^3 - 33/25*d^2*e*x^3 + 81/125*d*e^2*
x^3 + 458/1875*e^3*x^3 - 33/50*d^3*x^2 + 243/250*d^2*e*x^2 + 687/625*d*e^2
*x^2 - 881/6250*e^3*x^2 + 81/125*d^3*x + 1374/625*d^2*e*x - 2643/3125*d*e^
2*x - 5108/15625*e^3*x - 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 27
4845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/156250*(57250*
d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

3.304.9 Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= x^2\left(\frac{26\,e^2\,(12\,d-5\,e)}{625}-\frac{33\,e\,(4\,d^2-5\,d\,e+e^2)}{250}-\frac{3\,d\,e^2}{50}+\frac{3\,d^2\,e}{2}-\frac{33\,d^3}{50}+\frac{622\,e^3}{3125}\right)$$
$$-x^3\left(\frac{11\,e^2\,(12\,d-5\,e)}{375}+\frac{2\,e\,(4\,d^2-5\,d\,e+e^2)}{25}-\frac{3\,d\,e^2}{5}+d^2\,e-\frac{4\,d^3}{15}-\frac{111\,e^3}{625}\right)$$
$$+x^5\left(\frac{e^2\,(12\,d-5\,e)}{25}-\frac{8\,e^3}{125}\right)$$
$$-\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{229\,d^3}{625}+\frac{2643\,d^2\,e}{6250}+\frac{7662\,d\,e^2}{15625}-\frac{23431\,e^3}{156250}\right)$$
$$-x^4\left(\frac{e^2\,(12\,d-5\,e)}{50}-\frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{20}+\frac{11\,e^3}{125}\right)+\frac{2\,e^3\,x^6}{15}+x\left(\frac{61\,e^2\,(12\,d-5\,e)}{3125}\right.$$
$$\left.+\frac{3\,d\,(d^2+d\,e+2\,e^2)}{5}+\frac{156\,e\,(4\,d^2-5\,d\,e+e^2)}{625}-\frac{129\,d\,e^2}{125}+\frac{3\,d^2\,e}{5}+\frac{6\,d^3}{125}-\frac{7483\,e^3}{15625}\right)$$
$$+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{1093750}+\frac{\sqrt{14}\,x\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{218750}}{-\frac{423\,d^3}{625}-\frac{17967\,d^2\,e}{3125}+\frac{54969\,d\,e^2}{15625}+\frac{53189\,e^3}{78125}}\right)\,\left(-52875\,d^3-4\right.}{1093750}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e
^2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d
- 5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^
3)/15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2
*x + 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 -
(23431*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e
^2))/20 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 +
 (3*d*(d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*
e^2)/125 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(
((14^(1/2)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
+ (14^(1/2)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/21875
0)/((54969*d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)
/78125))*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

3.304. $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

3.305.1 Optimal result

Integrand size = 38, antiderivative size = 156

$$\begin{aligned}\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx &= \frac{(2025d^2+4580de-881e^2)\,x}{3125}-\frac{(825d^2-810de-458e^2)\,x^2}{1250}\\&\quad+\frac{1}{375}(100d^2-330de+81e^2)\,x^3+\frac{1}{100}(40d-33e)ex^4\\&\quad+\frac{4e^2x^5}{25}-\frac{(10575d^2+59890de-18323e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{15625\sqrt{14}}\\&\quad+\frac{(5725d^2-4405de-2554e^2)\log(3+2x+5x^2)}{15625}\end{aligned}$$

output
```
1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+
1/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5+1/
15625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)-1/218750*(10575*d^2+598
90*d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} \, dx$$
$$= \frac{35x(50d^2(486-495x+200x^2)+60de(916+405x-550x^2+250x^3)+3e^2(-3524+4580x+2700x^2-}{}$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
(35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 25
0*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt
[14]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(
5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

3.305.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^2}{5x^2+2x+3} \, dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{1}{125}x^2(100d^2-330de+81e^2)+\frac{2x(5725d^2-4405de-2554e^2)+175d^2-13740de+2643e^2}{3125(5x^2+2x+3)}-\frac{1}{625}x(825d^2\right.$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(10575d^2+59890de-18323e^2)}{15625\sqrt{14}}+\frac{1}{375}x^3(100d^2-330de+81e^2)-\frac{x^2(825d^2-810de-458e^2)}{1250}+\frac{(5725d^2-4405de-2554e^2)\log(5x^2+2x+3)}{15625}+\frac{x(2025d^2+4580de-881e^2)}{3125}+\frac{1}{100}ex^4(40d-33e)+\frac{4e^2x^5}{25}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*
x^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)
/100 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5
*x)/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3
+ 2*x + 5*x^2])/15625
```

3.305.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.305.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

method	result
default	$\frac{4e^2x^5}{25}+\frac{2x^4de}{5}-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}-\frac{22x^3de}{25}+\frac{27x^3e^2}{125}-\frac{33d^2x^2}{50}+\frac{81de\,x^2}{125}+\frac{229e^2x^2}{625}+\frac{81x\,d^2}{125}+\frac{916dex}{625}-$ 881e… / 312…
risch	$\frac{2x^4de}{5}-\frac{22x^3de}{25}-\frac{5989\sqrt{14}\,de\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21875}+\frac{229e^2x^2}{625}-\frac{881e^2x}{3125}-\frac{33d^2x^2}{50}-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}-$ 881de ln(3…

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*
x^3*e^2-33/50*d^2*x^2+81/125*d*e*x^2+229/625*e^2*x^2+81/125*x*d^2+916/625*
d*e*x-881/3125*e^2*x+1/31250*(11450*d^2-8810*d*e-5108*e^2)*ln(5*x^2+2*x+3)
+1/43750*(-2115*d^2-11978*d*e+18323/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*1
4^(1/2))
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx \\
&= \frac{4}{25}\,e^2x^5+\frac{1}{100}\,(40\,de-33\,e^2)x^4+\frac{1}{375}\,(100\,d^2-330\,de+81\,e^2)x^3 \\
&\quad -\frac{1}{1250}\,(825\,d^2-810\,de-458\,e^2)x^2 \\
&\quad -\frac{1}{218750}\,\sqrt{14}(10575\,d^2+59890\,de-18323\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad +\frac{1}{3125}\,(2025\,d^2+4580\,de-881\,e^2)x \\
&\quad +\frac{1}{15625}\,(5725\,d^2-4405\,de-2554\,e^2)\log\left(5\,x^2+2\,x+3\right)
\end{aligned}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81
*e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(
10575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/312
5*(2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*
e^2)*log(5*x^2 + 2*x + 3)
```

3.305.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{4e^2x^5}{25} + x^4\cdot\left(\frac{2de}{5}-\frac{33e^2}{100}\right) + x^3\cdot\left(\frac{4d^2}{15}-\frac{22de}{25}+\frac{27e^2}{125}\right) + x^2\left(-\frac{33d^2}{50}+\frac{81de}{125}+\frac{229e^2}{625}\right) + x\left(\frac{81d^2}{125}+\frac{916de}{625}-\frac{881e^2}{3125}\right) + \left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625} - \frac{\sqrt{14}i(10575d^2+59890de-18323e^2)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}+\frac{\sqrt{14}i(10575d^2+59890de-1832}{5}}{10575d^2+59890de-18323e^2}\right. + \left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625} + \frac{\sqrt{14}i(10575d^2+59890de-18323e^2)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}-\frac{\sqrt{14}i(10575d^2+59890de-1832}{5}}{10575d^2+59890de-18323e^2}\right.$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/2
5 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*
d**2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2
554*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*
log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 5
9890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d*
*2/625 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d
*e - 18323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - s
qrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e
 - 18323*e**2))
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$$
$$= \frac{4}{25}\,e^2x^5 + \frac{1}{100}\,\left(40\,de - 33\,e^2\right)x^4 + \frac{1}{375}\,\left(100\,d^2 - 330\,de + 81\,e^2\right)x^3$$
$$- \frac{1}{1250}\,\left(825\,d^2 - 810\,de - 458\,e^2\right)x^2$$
$$- \frac{1}{218750}\,\sqrt{14}\left(10575\,d^2 + 59890\,de - 18323\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)$$
$$+ \frac{1}{3125}\,\left(2025\,d^2 + 4580\,de - 881\,e^2\right)x$$
$$+ \frac{1}{15625}\,\left(5725\,d^2 - 4405\,de - 2554\,e^2\right)\log\left(5\,x^2 + 2\,x + 3\right)$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81
*e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(
10575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/312
5*(2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*
e^2)*log(5*x^2 + 2*x + 3)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$$
$$= \frac{4}{25}\,e^2x^5 + \frac{2}{5}\,dex^4 - \frac{33}{100}\,e^2x^4 + \frac{4}{15}\,d^2x^3 - \frac{22}{25}\,dex^3 + \frac{27}{125}\,e^2x^3$$
$$- \frac{33}{50}\,d^2x^2 + \frac{81}{125}\,dex^2 + \frac{229}{625}\,e^2x^2 + \frac{81}{125}\,d^2x + \frac{916}{625}\,dex - \frac{881}{3125}\,e^2x$$
$$- \frac{1}{218750}\,\sqrt{14}\left(10575\,d^2 + 59890\,de - 18323\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)$$
$$+ \frac{1}{15625}\,\left(5725\,d^2 - 4405\,de - 2554\,e^2\right)\log\left(5\,x^2 + 2\,x + 3\right)$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
4/25*e^2*x^5 + 2/5*d*e*x^4 - 33/100*e^2*x^4 + 4/15*d^2*x^3 - 22/25*d*e*x^3
 + 27/125*e^2*x^3 - 33/50*d^2*x^2 + 81/125*d*e*x^2 + 229/625*e^2*x^2 + 81/
125*d^2*x + 916/625*d*e*x - 881/3125*e^2*x - 1/218750*sqrt(14)*(10575*d^2
+ 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/15625*(5725*d
^2 - 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

3.305.9 Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = x\left(\frac{4\,d\,e}{5}+\frac{52\,e\,(8\,d-5\,e)}{625}+\frac{81\,d^2}{125}+\frac{419\,e^2}{3125}\right) - \ln\left(5\,x^2+2\,x+3\right)\left(-\frac{229\,d^2}{625}+\frac{881\,d\,e}{3125}+\frac{2554\,e^2}{15625}\right) + x^4\left(\frac{e\,(8\,d-5\,e)}{20}-\frac{2\,e^2}{25}\right) - x^3\left(\frac{2\,d\,e}{3}+\frac{2\,e\,(8\,d-5\,e)}{75}-\frac{4\,d^2}{15}-\frac{31\,e^2}{375}\right) + x^2\left(d\,e-\frac{11\,e\,(8\,d-5\,e)}{250}-\frac{33\,d^2}{50}+\frac{183\,e^2}{1250}\right) + \frac{4\,e^2\,x^5}{25} - \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}+\frac{\sqrt{14}\,x\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{43750}}{\frac{423\,d^2}{625}+\frac{11978\,d\,e}{3125}-\frac{18323\,e^2}{15625}}\right)\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - l
og(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) +
x^4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))
/75 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33
*d^2)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(5
9890*d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575
*d^2 - 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/
15625))*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

3.306 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

3.306.1 Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}-\frac{(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3125\sqrt{14}}+\frac{(2290d-881e)\log(3+2x+5x^2)}{6250}$$

output
```
1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4
+1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)-1/43750*(2115*d+5989*e)*arctan(1/14
*(1+5*x)*14^(1/2))*14^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{35x(5d(486-495x+200x^2)+3e(916+405x-550x^2+250x^3))-3\sqrt{14}(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{131250}$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
(35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3)
) - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d -
 881*e)*Log[3 + 2*x + 5*x^2])/131250
```

3.306.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{5x^2+2x+3}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{1}{25}x^2(20d-33e)+\frac{x(2290d-881e)+35d-1374e}{625\left(5x^2+2x+3\right)}-\frac{3}{125}x(55d-27e)+\frac{1}{625}(405d+458e)+\frac{4ex^3}{5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(2115d+5989e)}{3125\sqrt{14}}+\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)+\frac{ex^4}{5}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/
75 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt
[14]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.306.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
default	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{(2290d-881e)\ln(5x^2+2x+3)}{6250}+\frac{\left(-423d-\frac{5989e}{5}\right)\sqrt{14}\,\text{arct}}{8750}$
risch	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{229d\ln(350x^2+140x+210)}{625}-\frac{881e\ln(350x^2+140x+210)}{6250}$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/5*e*x^4+4/15*d*x^3-11/25*e*x^3-33/50*d*x^2+81/250*e*x^2+81/125*d*x+458/6
25*e*x+1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)+1/8750*(-423*d-5989/5*e)*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int \frac{(d+ex)\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx\\&=\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2\\&\quad-\frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\&\quad+\frac{1}{625}\,(405\,d+458\,e)x+\frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fric
as")
```

output
```
1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqr
t(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 4
58*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

3.306.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.65

$$\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{ex^4}{5} + x^3\cdot\left(\frac{4d}{15}-\frac{11e}{25}\right) + x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right) + x\left(\frac{81d}{125}+\frac{458e}{625}\right) \\
&\quad + \left(\frac{229d}{625}-\frac{881e}{6250}-\frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}+\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right) \\
&\quad + \left(\frac{229d}{625}-\frac{881e}{6250}+\frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}-\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right)
\end{aligned}$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/
125 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/
87500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d
 + 5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500
)*log(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 59
89*e))
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx \\
&= \frac{1}{5}\,ex^4 + \frac{1}{75}\,(20\,d-33\,e)x^3 - \frac{3}{250}\,(55\,d-27\,e)x^2 \\
&\quad - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) \\
&\quad + \frac{1}{625}\,(405\,d+458\,e)x + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)
\end{aligned}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxi
ma")
```

output
```
1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqr
t(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 4
58*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx\\&=\frac{1}{5}\,ex^4+\frac{4}{15}\,dx^3-\frac{11}{25}\,ex^3-\frac{33}{50}\,dx^2+\frac{81}{250}\,ex^2\\&\quad-\frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)\\&\quad+\frac{81}{125}\,dx+\frac{458}{625}\,ex+\frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)\end{aligned}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac
")
```

output
```
1/5*e*x^4 + 4/15*d*x^3 - 11/25*e*x^3 - 33/50*d*x^2 + 81/250*e*x^2 - 1/4375
0*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x
+ 458/625*e*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

3.306.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx\\&=x^3\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5}\\&\quad+x\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(2115\,d+5989\,e)}{43750}+\frac{\sqrt{14}\,x\,(2115\,d+5989\,e)}{8750}}{\frac{423\,d}{625}+\frac{5989\,e}{3125}}\right)(2115\,d+5989\,e)}{43750}\end{aligned}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^
2 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/
625) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2
115*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/4
3750
```

3.306. $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$

3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx$

3.307.1 Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx = \frac{81x}{125} - \frac{33x^2}{50} + \frac{4x^3}{15} - \frac{423 \arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}} + \frac{229}{625}\log\left(3+2x+5x^2\right)$$

output
```
81/125*x-33/50*x^2+4/15*x^3+229/625*ln(5*x^2+2*x+3)-423/8750*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)
```

3.307.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx = \frac{35x(486-495x+200x^2) - 1269\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right) + 9618\log\left(3+2x+5x^2\right)}{26250}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

output
```
(35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
 9618*Log[3 + 2*x + 5*x^2])/26250
```

3.307.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{5x^2+2x+3}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{4x^2}{5}+\frac{458x+7}{125(5x^2+2x+3)}-\frac{33x}{25}+\frac{81}{125}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{423\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

output
```
(81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(
625*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

3.307.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.307.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(5x^2+2x+3)}{625}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}$	44
risch	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(25x^2+10x+15)}{625}-\frac{423\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{8750}$	44

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

3.307.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*
sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{33}{50}\,x^2 - \frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x + \frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{33}{50}\,x^2 - \frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x + \frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} dx = \frac{81\,x}{125} + \frac{229\,\ln\left(5\,x^2+2\,x+3\right)}{625} - \frac{423\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750} - \frac{33\,x^2}{50} + \frac{4\,x^3}{15}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

output
```
(81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/
2)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

3.308.1 Optimal result

Integrand size = 38, antiderivative size = 168

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\,(5d^2-2de+3e^2)} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)} + \frac{(458d-7e)\log(3+2x+5x^2)}{250\,(5d^2-2de+3e^2)}$$

output
```
-1/25*(20*d+33*e)*x/e^2+2/5*x^2/e+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln
(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/250*(458*d-7*e)*ln(5*x^2+2*x+3)/(5*d^2-2
*d*e+3*e^2)-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d
*e+3*e^2)*14^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = \frac{70e(5d^2-2de+3e^2)\,x(-20d+e(-33+10x))-\sqrt{14}(423d-1367e)e^3\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+1750(4d^4+5d^3e \cdots}{1750e^3\,(5d^2-2de+3e^2)}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

output
```
(70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
 - 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^
2])/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

3.308.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)(d+ex)}dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x(458d-7e)+7d+272e}{25(5x^2+2x+3)(5d^2-2de+3e^2)}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)}+\frac{-20d-33e}{25e^2}+\frac{4x}{5e}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d-1367e)}{125\sqrt{14}(5d^2-2de+3e^2)}+\frac{(458d-7e)\log(5x^2+2x+3)}{250(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3(5d^2-2de+3e^2)}-\frac{x(20d+33e)}{25e^2}+\frac{2x^2}{5e}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

output
```
-1/25*((20*d + 33*e)*x)/e^2 + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3
*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)
) + ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

3.308.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.308.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-10e\,x^2+20dx+33ex}{25e^2}+\frac{\frac{(458d-7e)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423d}{5}+\frac{1367e}{5}\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{125d^2-50de+75e^2}+\frac{(4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4)\,\mathrm{l}}{e^3(5d^2-2de+3e^2)}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/25/e^2*(-10*e*x^2+20*d*x+33*e*x)+1/(125*d^2-50*d*e+75*e^2)*(1/10*(458*d
-7*e)*ln(5*x^2+2*x+3)+1/14*(-423/5*d+1367/5*e)*14^(1/2)*arctan(1/28*(10*x+
2)*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^3/(5*d^2-2
*d*e+3*e^2)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = \frac{700\,(5\,d^2e^2-2\,de^3+3\,e^4)x^2-\sqrt{14}(423\,de^3-1367\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-70\,(100\,d^3e+125\,d^2e^2}{1750\,(5\,d^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="fric
as")
```

output
```
1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367
*e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*
e^3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e
*x + d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4
 + 3*e^5)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(ex+d)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{(458\,d-7\,e)\log(5\,x^2+2\,x+3)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{10\,ex^2-(20\,d+33\,e)x}{25\,e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxi
ma")
```

output
```
-1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x +
d)/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3
)/(5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(|ex+d|\right)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{10\,ex^2-20\,dx-33\,ex}{25\,e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac
")
```

output
```
-1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
 2*d*e + 3*e^2) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e
+ 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(abs(e*x + d))
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/25*(10*e*x^2 - 20*d*x - 33*e*x)/e^2
```

3.308.9 Mupad [B] (verification not implemented)

Time = 15.97 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$$
$$= \frac{2\,x^2}{5\,e} - \ln(d+e\,x)\left(\frac{\frac{458\,d}{125}-\frac{7\,e}{125}}{5\,d^2-2\,d\,e+3\,e^2} - \frac{100\,d^2+165\,d\,e+81\,e^2}{125\,e^3}\right) - x\left(\frac{4\,(5\,d+2\,e)}{25\,e^2}+\frac{1}{e}\right)$$
$$- \frac{\ln\left(\frac{-28\,d^3+1053\,d^2\,e+1791\,d\,e^2+916\,e^3}{25\,e^2} - \frac{x\,(1832\,d^3+2318\,d^2\,e+321\,d\,e^2-2249\,e^3)}{25\,e^2} + \frac{\left(d\left(\frac{423\,\sqrt{14}}{3500}-\frac{229}{125}\mathrm{i}\right)-e\left(\frac{1367\,\sqrt{14}}{3500}-\frac{7}{250}\mathrm{i}\right)\right)\left(-10\cdots\right.}{\cdots}\right)}{\cdots}$$
$$+ \frac{\ln\left(\frac{-28\,d^3+1053\,d^2\,e+1791\,d\,e^2+916\,e^3}{25\,e^2} - \frac{x\,(1832\,d^3+2318\,d^2\,e+321\,d\,e^2-2249\,e^3)}{25\,e^2} - \frac{\left(d\left(\frac{423\,\sqrt{14}}{3500}+\frac{229}{125}\mathrm{i}\right)-e\left(\frac{1367\,\sqrt{14}}{3500}+\frac{7}{250}\mathrm{i}\right)\right)\left(-10\cdots\right.}{\cdots}\right)}{\cdots}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

output
```
(2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3
*e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e
^2) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) -
(x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14
^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3
 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e
^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e
^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6
250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^
(1/2))/3500 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^
2*3i))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/
250)))/(d^2*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3
 + 916*e^3)/(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/
(25*e^2) - ((d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500
+ 7i/250))*((4751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/
(25*e^2) + (x*(8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^
2))/(25*e^2) + (((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(250
0*d*e^4 + 10250*e^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 2
29i/125) - e*((1367*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i))
)/(d^2*5i - d*e*2i + e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*(...
```

3.308. $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

3.309.1 Optimal result

Integrand size = 38, antiderivative size = 233

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} - \frac{(423d^2-2734de+293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2} - \frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2} + \frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

output
```
4/5*x/e^2+(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(
e*x+d)-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)*ln(e*x+d)/e^3/(5*d
^2-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2+2*x+3)/(5*d^2-2*d*
e+3*e^2)^2-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))/
(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$$
$$= \frac{4x}{5e^2} + \frac{-4d^4-5d^3e-3d^2e^2+de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)} + \frac{(-423d^2+2734de-293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2}$$
$$+ \frac{(-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}$$
$$+ \frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),
x]
```

output
```
(4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
 - 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^
4*e - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 -
 2*d*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(2
5*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.309.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)(d+ex)^2}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{2x(229d^2-7de-136e^2)+7d^2+544de-113e^2}{5(5x^2+2x+3)(5d^2-2de+3e^2)^2} + \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2} + \frac{-40d^5-d^4e-28d^3e^2}{e^2(5d^2-2de}\right.$$

$$\downarrow 2009$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d^2-2734de+293e^2)}{25\sqrt{14}\,(5d^2-2de+3e^2)^2}+\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25\,(5d^2-2de+3e^2)^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}+\frac{4x}{5e^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

output
```
(4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e
 + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*
d*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

3.309.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.309.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.91

method	result
default	$\frac{4x}{5e^2}+\frac{\frac{(458d^2-14de-272e^2)\ln(5x^2+2x+3)}{10}+\frac{\left(-\frac{423}{5}d^2+\frac{2734}{5}de-\frac{293}{5}e^2\right)\sqrt{14}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{5(5d^2-2de+3e^2)^2}-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{e^3(5d^2-2de+3e^2)(ex+d)}+$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
4/5*x/e^2+1/5/(5*d^2-2*d*e+3*e^2)^2*(1/10*(458*d^2-14*d*e-272*e^2)*ln(5*x^
2+2*x+3)+1/14*(-423/5*d^2+2734/5*d*e-293/5*e^2)*14^(1/2)*arctan(1/28*(10*x
+2)*14^(1/2)))-1/e^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*
e^2)/(e*x+d)+(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^5)/e^3/(5*d^2-
2*d*e+3*e^2)^2*ln(e*x+d)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.79

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx =$$
$$-\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\,(25\,d^4e^2-20\,d^3e^3}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
-1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
 - 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*
d*e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (4
23*d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 28
0*(25*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*
d^6 + d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^
4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229
*d^3*e^3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*lo
g(5*x^2 + 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9
*d*e^7 + (25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.26

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^2-2734\,de+293\,e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5)\log\left(ex+d\right)}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7} + \frac{(229\,d^2-7\,de-136\,e^2)\log\left(5\,x^2+2\,x+3\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4}{5\,d^3e^3-2\,d^2e^4+3\,de^5+(5\,d^2e^4-2\,de^5+3\,e^6)x} + \frac{4\,x}{5\,e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
-1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e
 + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*
d^3*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2
)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
 - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 +
3*d*e^5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.53

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$$
$$= \frac{(229\,d^2-7\,de-136\,e^2)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}$$
$$-\frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{5\,d^2e^6-2\,de^7+3\,e^8}$$
$$-\frac{\sqrt{14}(423\,d^2e^2-2734\,de^3+293\,e^4)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)e^2}$$
$$+\frac{(40\,d+33\,e)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{25\,e^3}+\frac{4\,(ex+d)}{5\,e^3}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
1/25*(229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 +
 2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d)^2 + 5)/(25*d^4 - 20*d
^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(
e*x + d) + 3*d^2*e^5/(e*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(5*d^2
*e^6 - 2*d*e^7 + 3*e^8) - 1/350*sqrt(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e
^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) + 2*d*e/(e*x + d) - e - 3*
e^2/(e*x + d))/e)/((25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*e^2
) + 1/25*(40*d + 33*e)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + 4/5*(e
*x + d)/e^3
```

3.309.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.34

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \frac{4\,x}{5\,e^2} - \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\sqrt{14}}{700}-\frac{229}{25}\mathrm{i}\right)d^2+\left(-\frac{1367\sqrt{14}}{350}+\frac{7}{25}\mathrm{i}\right)d\,e+\left(\frac{293\sqrt{14}}{700}+\frac{136}{25}\mathrm{i}\right)e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} + \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\sqrt{14}}{700}+\frac{229}{25}\mathrm{i}\right)d^2+\left(-\frac{1367\sqrt{14}}{350}-\frac{7}{25}\mathrm{i}\right)d\,e+\left(\frac{293\sqrt{14}}{700}-\frac{136}{25}\mathrm{i}\right)e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} - \frac{5\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\left(5\,x\,e^3+5\,d\,e^2\right)\left(5\,d^2-2\,d\,e+3\,e^2\right)} - \frac{\ln\left(d+e\,x\right)\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^3\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

output
```
(4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
 229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350
- 7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log
(x + (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((29
3*14^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i
- d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*
d^4 + 2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2))
- (log(d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2)
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$

3.310.1 Optimal result

Integrand size = 38, antiderivative size = 317

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx &= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3(5d^2-2de+3e^2)^2(d+ex)} \\ &\quad -\frac{(423d^3-4101d^2e+879de^2+703e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{5\sqrt{14}(5d^2-2de+3e^2)^3} \\ &\quad +\frac{(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6)\log(d+ex)}{e^3(5d^2-2de+3e^2)^3} \\ &\quad +\frac{(458d^3-21d^2e-816de^2+113e^3)\log(3+2x+5x^2)}{10(5d^2-2de+3e^2)^3}\end{aligned}$$

output
```
1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)
^2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2
)^2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e
^5-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/10*(458*d^3-21*d^2*e-816*d*e
^2+113*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3-1/70*(423*d^3-4101*d^2*e
+879*d*e^2+703*e^3)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14
^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx =$$
$$-\frac{\frac{35(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^3(d+ex)^2} - \frac{70(5d^2-2de+3e^2)(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)}{e^3(d+ex)} + \sqrt{14}(423d^3-41\ldots}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),
x]
```

output
```
-1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e
+ 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(42
3*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70
*(-100*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e
^5 + e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

3.310.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)(d+ex)^3}\,dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{7d^3+816d^2e+x(458d^3-21d^2e-816de^2+113e^3)-339de^2-118e^3}{(5x^2+2x+3)(5d^2-2de+3e^2)^3} + \frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^3} + \ldots\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(423d^3-4101d^2e+879de^2+703e^3\right)}{5\sqrt{14}\left(5d^2-2de+3e^2\right)^3}+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

output
```
-1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 - 2*d*e + 3
*e^2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 +
 e^5)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e +
 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d
*e + 3*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d
^2*e^4 + 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) +
((458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d
^2 - 2*d*e + 3*e^2)^3)
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.310.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(458d^3-21d^2e-816d\,e^2+113e^3\right)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423}{5}d^3+\frac{4101}{5}d^2e-\frac{879}{5}d\,e^2-\frac{703}{5}e^3\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-d}{2e^3\left(5d^2-2de+3e^2\right)(}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^3*(1/10*(458*d^3-21*d^2*e-816*d*e^2+113*e^3)*ln(5*x^
2+2*x+3)+1/14*(-423/5*d^3+4101/5*d^2*e-879/5*d*e^2-703/5*e^3)*14^(1/2)*arc
tan(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3
/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2-(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^
4-e^5)/e^3/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-24
2*d^3*e^3+141*d^2*e^4+120*d*e^5-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(308) = 616$.

Time = 0.40 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$$
$$= \frac{10500\,d^8 - 6825\,d^7e + 14175\,d^6e^2 + 10395\,d^5e^3 - 6160\,d^4e^4 + 12145\,d^3e^5 - 4305\,d^2e^6 + 1365\,de^7 - 630\,e^8}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^
4 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^
5*e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2
*e^6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*
e^6 + 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d
^6*e^2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 +
3*e^8)*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e
^4 + 120*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 24
2*d^3*e^5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^
2 + 228*d^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(
e*x + d) + 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*
d^3*e^5 - 21*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*
e^5 - 816*d^2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150
*d^7*e^4 + 285*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e
^9 + (125*d^6*e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9
- 54*d*e^10 + 27*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 -
188*d^4*e^7 + 171*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log(ex+d)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}$$
$$+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log(5\,x^2+2\,x+3)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{60\,d^6-15\,d^5e+39\,d^4e^2+84\,d^3e^3-25\,d^2e^4+9\,de^5-6\,e^6+2\,(40\,d^5e+d^4e^2+28\,d}{2\,(25\,d^6e^3-20\,d^5e^4+34\,d^4e^5-12\,d^3e^6+9\,d^2e^7+(25\,d^4e^5-20\,d^3e^6+34\,d^2e^7-12\,de^8+9\,e^9)x^2+2\,(}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
-1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sq
rt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d
^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3
*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*
e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*
(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 -
 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
+ 1/2*(60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5
- 6*e^6 + 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)
*x)/(25*d^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d
^4*e^5 - 20*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 -
 20*d^4*e^5 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.38

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log\left(5\,x^2+2\,x+3\right)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log\left(|ex+d|\right)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}$$
$$+\frac{2\,(200\,d^7-75\,d^6e+258\,d^5e^2+167\,d^4e^3-14\,d^3e^4+141\,d^2e^5-8\,de^6+3\,e^7)x+\frac{300\,d^8-195\,d^7e+405\,d^6e^2+297}{}}{2\,(5\,d^2-2\,de+3\,e^2)^3(ex+d)^2e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
-1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sq
rt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d
^2*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e
^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3
 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 -
242*d^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(e*x + d))/(125*d^6*e^
3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*
e^9) + 1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4
 + 141*d^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 +
 297*d^5*e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8
)/e)/((5*d^2 - 2*d*e + 3*e^2)^3*(e*x + d)^2*e^2)
```

3.310.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx = \frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x\,(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5)}{e^2\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{d^2+2\,d\,e\,x+e^2\,x^2} - \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\sqrt{14}}{140}-\frac{229}{5}\mathrm{i}\right)d^3+\left(-\frac{4101\sqrt{14}}{140}+\frac{21}{10}\mathrm{i}\right)d^2\,e+\left(\frac{879\sqrt{14}}{140}+\frac{408}{5}\mathrm{i}\right)d\,e^2+\left(\frac{703\sqrt{14}}{140}-\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}} + \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\sqrt{14}}{140}+\frac{229}{5}\mathrm{i}\right)d^3+\left(-\frac{4101\sqrt{14}}{140}-\frac{21}{10}\mathrm{i}\right)d^2\,e+\left(\frac{879\sqrt{14}}{140}-\frac{408}{5}\mathrm{i}\right)d\,e^2+\left(\frac{703\sqrt{14}}{140}+\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}} + \frac{\ln(d+e\,x)\,(100\,d^6-120\,d^5\,e+228\,d^4\,e^2-242\,d^3\,e^3+141\,d^2\,e^4+120\,d\,e^5-e^6)}{e^3\,(5\,d^2-2\,d\,e+3\,e^2)^3}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

output
```
((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e
^2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*
e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d
^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x
 - (14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*1
4^(1/2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((41
01*14^(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i
+ d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 +
 1/5)*(d^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/
10) + d*e^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 2
1i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3
*e^3*188i + d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6
 - e^6 + 141*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3
*e^2)^3)
```

3.311 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.311.1 Optimal result

Integrand size = 38, antiderivative size = 189

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{(2800d^3-17220d^2e+9921de^2+6053e^3)\,x}{17500}+\frac{e(840d^2-1722de+373e^2)\,x^2}{3500} + \frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}-\frac{(1367+423x)(d+ex)^3}{3500\,(3+2x+5x^2)} + \frac{(32825d^3+317565d^2e-221643de^2-67499e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}} - \frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log\left(3+2x+5x^2\right)}{6250}$$

output
```
1/17500*(2800*d^3-17220*d^2*e+9921*d*e^2+6053*e^3)*x+1/3500*e*(840*d^2-172
2*d*e+373*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-1/3500*(1367+423
*x)*(e*x+d)^3/(5*x^2+2*x+3)-1/6250*(1025*d^3-1545*d^2*e-2601*d*e^2+832*e^3
)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^3+317565*d^2*e-221643*d*e^2-67499*e^3
)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\, dx$$
$$= \frac{5880(500d^3-3075d^2e+1545de^2+867e^3)\, x+14700e(300d^2-615de+103e^2)\, x^2+49000(60d-41e)e^2x}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
(5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
 615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (
42*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 598
9*x) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825
*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]]
 + 2940*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^
2])/18375000
```

3.311.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)\,(d+ex)^3}{(5x^2+2x+3)^2}dx$$

$$\Big\downarrow \ 2175$$

$$\frac{1}{56}\int \frac{2(d+ex)^2\,(2800ex^3+140(20d-33e)x^2-6(770d-519e)x+3(615d+1367e))}{125\,(5x^2+2x+3)}dx - \frac{(423x+1367)(d+ex)^3}{3500\,(5x^2+2x+3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(d+ex)^2(2800ex^3+140(20d-33e)x^2-6(770d-519e)x+3(615d+1367e))}{5x^2+2x+3}dx}{3500} - \frac{(423x+1367)(d+ex)^3}{3500(5x^2+2x+3)}$$

$$\Big\downarrow \; 2159$$

$$\frac{\int \left(560e^3x^3+28(60d-41e)e^2x^2+2e(840d^2-1722ed+373e^2)x+\frac{1}{5}(2800d^3-17220ed^2+9921e^2d+6053e^3)\right.}{3500}$$

$$\frac{(423x+1367)(d+ex)^3}{3500(5x^2+2x+3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(32825d^3+317565d^2e-221643de^2-67499e^3)}{25\sqrt{14}} + ex^2(840d^2-1722de+373e^2) - \frac{14}{25}(1025d^3-1545d^2e-2601de}{3500}$$

$$\frac{(423x+1367)(d+ex)^3}{3500(5x^2+2x+3)}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x)^3)/(3 + 2*x + 5*x^2) + (((2800*d^3 - 172
20*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/5 + e*(840*d^2 - 1722*d*e + 373*e^2)*
x^2 + (28*(60*d - 41*e)*e^2*x^3)/3 + 140*e^3*x^4 + ((32825*d^3 + 317565*d^
2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[14])
- (14*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x^2])
/25)/3500
```

3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.311.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.13

method	result
default	$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6d^2e\,x^2}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4x\,d^3}{25}-\frac{123d^2ex}{125}+\frac{309d\,e^2x}{625}+\frac{867e^3x}{3125}-\frac{\left(\frac{2115}{28}d^3+\right.}{}$
risch	$\frac{309d^2e\ln\left(350x^2+140x+210\right)}{1250}+\frac{2601d\,e^2\ln\left(350x^2+140x+210\right)}{6250}+\frac{1313\sqrt{14}\,d^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}-\frac{41e^3x^3}{375}+\frac{63513\sqrt{14}}{}$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*d^2*e*x^2-123/250*d*e^2*x^
2+103/1250*e^3*x^2+4/25*x*d^3-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3
*x-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+68
35/28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-1/1
75000*(28700*d^3-43260*d^2*e-72828*d*e^2+23296*e^3)*ln(5*x^2+2*x+3)-1/2450
00*(-6565*d^3-63513*d^2*e+221643/5*d*e^2+67499/5*e^3)*14^(1/2)*arctan(1/28
*(10*x+2)*14^(1/2))
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(172) = 344$.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{3675000\,e^3x^6 + 1225000\,(12\,de^2 - 7\,e^3)x^5 + 122500\,(180\,d^2e - 321\,de^2 + 47\,e^3)x^4 + 147000\,(100\,d^3 - 555}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180
*d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2
 + 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e
^3 + 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14
)*(98475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 3
17565*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e
- 221643*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(15712
5*d^3 - 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 46
35*d^2*e - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 +
 832*e^3)*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*
x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

3.311.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx$$
$$= \frac{e^3x^4}{25} + x^3 \cdot \left(\frac{4de^2}{25} - \frac{41e^3}{375}\right) + x^2 \cdot \left(\frac{6d^2e}{25} - \frac{123de^2}{250} + \frac{103e^3}{1250}\right)$$
$$+ x\left(\frac{4d^3}{25} - \frac{123d^2e}{125} + \frac{309de^2}{625} + \frac{867e^3}{3125}\right) + \left(-\frac{41d^3}{250} + \frac{309d^2e}{1250} + \frac{2601de^2}{6250} - \frac{416e^3}{3125}\right.$$
$$\left. - \frac{\sqrt{14}i(32825d^3 + 317565d^2e - 221643de^2 - 67499e^3)}{2450000}\right) \log\left(x + \frac{6565d^3 + 63513d^2e - \frac{221643de^2}{5} - \frac{67499}{5}}{32825d^3 + 317565d^2e}\right.$$
$$+ \left(-\frac{41d^3}{250} + \frac{309d^2e}{1250} + \frac{2601de^2}{6250} - \frac{416e^3}{3125}\right.$$
$$\left. + \frac{\sqrt{14}i(32825d^3 + 317565d^2e - 221643de^2 - 67499e^3)}{2450000}\right) \log\left(x + \frac{6565d^3 + 63513d^2e - \frac{221643de^2}{5} - \frac{67499}{5}}{32825d^3 + 317565d^2e}\right.$$
$$+ \frac{-170875d^3 + 95175d^2e + 269505de^2 - 54969e^3 + x(-52875d^3 - 449175d^2e + 274845de^2 + 53189e^3)}{2187500x^2 + 875000x + 1312500}$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*
d*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/6
25 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 -
 416*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 -
67499*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 -
 67499*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6
7499*e**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) +
 (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt
(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*
log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(
14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*
d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 9517
5*d**2*e + 269505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 2
74845*d*e**2 + 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

3.311. $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx$

3.311.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx$$
$$= \frac{1}{25} e^3 x^4 + \frac{1}{375} \left(60 \, d e^2 - 41 \, e^3\right) x^3 + \frac{1}{1250} \left(300 \, d^2 e - 615 \, d e^2 + 103 \, e^3\right) x^2$$
$$+ \frac{1}{1225000} \sqrt{14} \left(32825 \, d^3 + 317565 \, d^2 e - 221643 \, d e^2 - 67499 \, e^3\right) \arctan\left(\frac{1}{14} \sqrt{14} (5 \, x + 1)\right)$$
$$+ \frac{1}{3125} \left(500 \, d^3 - 3075 \, d^2 e + 1545 \, d e^2 + 867 \, e^3\right) x$$
$$- \frac{1}{6250} \left(1025 \, d^3 - 1545 \, d^2 e - 2601 \, d e^2 + 832 \, e^3\right) \log\left(5 \, x^2 + 2 \, x + 3\right)$$
$$- \frac{170875 \, d^3 - 95175 \, d^2 e - 269505 \, d e^2 + 54969 \, e^3 + \left(52875 \, d^3 + 449175 \, d^2 e - 274845 \, d e^2 - 53189 \, e^3\right) x}{437500 \, \left(5 \, x^2 + 2 \, x + 3\right)}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e
^2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*
d*e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 307
5*d^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d
*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e
- 269505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53
189*e^3)*x)/(5*x^2 + 2*x + 3)
```

3.311.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx = \frac{1}{25} e^3 x^4 + \frac{4}{25} d e^2 x^3 - \frac{41}{375} e^3 x^3$$
$$+ \frac{6}{25} d^2 e x^2 - \frac{123}{250} d e^2 x^2 + \frac{103}{1250} e^3 x^2 + \frac{4}{25} d^3 x - \frac{123}{125} d^2 e x + \frac{309}{625} d e^2 x + \frac{867}{3125} e^3 x$$
$$+ \frac{1}{1225000} \sqrt{14} \left(32825 \, d^3 + 317565 \, d^2 e - 221643 \, d e^2 - 67499 \, e^3\right) \arctan\left(\frac{1}{14} \sqrt{14} (5 \, x + 1)\right)$$
$$- \frac{1}{6250} \left(1025 \, d^3 - 1545 \, d^2 e - 2601 \, d e^2 + 832 \, e^3\right) \log\left(5 \, x^2 + 2 \, x + 3\right)$$
$$- \frac{170875 \, d^3 - 95175 \, d^2 e - 269505 \, d e^2 + 54969 \, e^3 + \left(52875 \, d^3 + 449175 \, d^2 e - 274845 \, d e^2 - 53189 \, e^3\right) x}{437500 \, \left(5 \, x^2 + 2 \, x + 3\right)}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
1/25*e^3*x^4 + 4/25*d*e^2*x^3 - 41/375*e^3*x^3 + 6/25*d^2*e*x^2 - 123/250*
d*e^2*x^2 + 103/1250*e^3*x^2 + 4/25*d^3*x - 123/125*d^2*e*x + 309/625*d*e^
2*x + 867/3125*e^3*x + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^3 -
 1545*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(17087
5*d^3 - 95175*d^2*e - 269505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e
 - 274845*d*e^2 - 53189*e^3)*x)/(5*x^2 + 2*x + 3)
```

3.311.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{\frac{53901\,d\,e^2}{28}+\frac{19035\,d^2\,e}{28}+x\left(-\frac{10575\,d^3}{28}-\frac{89835\,d^2\,e}{28}+\frac{54969\,d\,e^2}{28}+\frac{53189\,e^3}{140}\right)-\frac{34175\,d^3}{28}-\frac{54969\,e^3}{140}}{15625\,x^2+6250\,x+9375}$$
$$+x^3\left(\frac{e^2\,(12\,d-5\,e)}{75}-\frac{16\,e^3}{375}\right)$$
$$-x\left(\frac{18\,e^2\,(12\,d-5\,e)}{625}+\frac{12\,e\,(4\,d^2-5\,d\,e+e^2)}{125}-\frac{9\,d\,e^2}{25}+\frac{3\,d^2\,e}{5}-\frac{4\,d^3}{25}-\frac{717\,e^3}{3125}\right)$$
$$+\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{41\,d^3}{250}+\frac{309\,d^2\,e}{1250}+\frac{2601\,d\,e^2}{6250}-\frac{416\,e^3}{3125}\right)$$
$$-x^2\left(\frac{2\,e^2\,(12\,d-5\,e)}{125}-\frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{50}+\frac{36\,e^3}{625}\right)+\frac{e^3\,x^4}{25}$$
$$-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{245000}}{-\frac{1313\,d^3}{3500}-\frac{63513\,d^2\,e}{17500}+\frac{221643\,d\,e^2}{87500}+\frac{67499\,e^3}{87500}}\right)\,(-32825\,d^3-3}{1225000}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)
/28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140
)/(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375)
 - x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*
e^2)/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3
)*((2601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) -
 x^2*((2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)
/625) + (e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^
2*e - 32825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565
*d^2*e - 32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^
2*e)/17500 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*
d^2*e - 32825*d^3 + 67499*e^3))/1225000
```

3.311. $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.312 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.312.1 Optimal result

Integrand size = 38, antiderivative size = 140

$$\begin{aligned}\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx &= \frac{(2800d^2-11480de+3307e^2)\,x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}\\ &\quad-\frac{(1367+423x)(d+ex)^2}{3500\,(3+2x+5x^2)}+\frac{(32825d^2+211710de-73881e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}}\\ &\quad-\frac{(1025d^2-1030de-867e^2)\log(3+2x+5x^2)}{6250}\end{aligned}$$

output
```
1/17500*(2800*d^2-11480*d*e+3307*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x
^3-1/3500*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)-1/6250*(1025*d^2-1030*d*e-8
67*e^2)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(
1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$
$$= \frac{5880(100d^2-410de+103e^2)x+14700(40d-41e)ex^2+196000e^2x^3-\frac{42(25d^2(1367+423x)+10de(-1269+5989x)}{3+2x+5x^2}}{}$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
(5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000
*e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(179
67 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7
3881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2
)*Log[3 + 2*x + 5*x^2])/3675000
```

3.312.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^2}{(5x^2+2x+3)^2} dx$$
$$\Big\downarrow \text{ 2175}$$
$$\frac{1}{56}\int \frac{2(d+ex)\left(2800ex^3+140(20d-33e)x^2-3(1540d-897e)x+1845d+2734e\right)}{125\left(5x^2+2x+3\right)}dx - \frac{(423x+1367)(d+ex)^2}{3500\left(5x^2+2x+3\right)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{(d+ex)\left(2800ex^3+140(20d-33e)x^2-3(1540d-897e)x+1845d+2734e\right)}{5x^2+2x+3}dx}{3500} - \frac{(423x+1367)(d+ex)^2}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{\int \left(560e^2x^2+28(40d-41e)ex+\frac{1}{5}\left(2800d^2-11480ed+3307e^2\right)+\frac{825d^2+48110ed-9921e^2-28(1025d^2-1030ed-867e^2)x}{5(5x^2+2x+3)}\right)}{3500}$$
$$\frac{(423x+1367)(d+ex)^2}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(32825d^2+211710de-73881e^2\right)}{25\sqrt{14}}-\frac{14}{25}\left(1025d^2-1030de-867e^2\right)\log\left(5x^2+2x+3\right)+\frac{1}{5}x\left(2800d^2-11480de\right.}{3500}$$
$$\frac{(423x+1367)(d+ex)^2}{3500\left(5x^2+2x+3\right)}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x)^2)/(3 + 2*x + 5*x^2) + (((2800*d^2 - 114
80*d*e + 3307*e^2)*x)/5 + 14*(40*d - 41*e)*e*x^2 + (560*e^2*x^3)/3 + ((328
25*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[14])
 - (14*(1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/25)/3500
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.312. $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.312.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04

method	result
default	$\frac{4x^3e^2}{75}+\frac{4de\,x^2}{25}-\frac{41e^2x^2}{250}+\frac{4x\,d^2}{25}-\frac{82dex}{125}+\frac{103e^2x}{625}-\frac{\left(\frac{423}{28}d^2+\frac{5989}{70}de-\frac{18323}{700}e^2\right)x+\frac{1367d^2}{28}-\frac{1269de}{70}-\frac{17967e^2}{700}}{625\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{(2870}{}$
risch	$\frac{103de\ln\left(350x^2+140x+210\right)}{625}+\frac{21171\sqrt{14}\,de\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{122500}-\frac{41e^2x^2}{250}+\frac{103e^2x}{625}-\frac{82dex}{125}+\frac{\frac{\left(-\frac{423}{28}d^2-\frac{5989}{70}de+\frac{18323}{700}\right.}{625}}{x}$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
4/75*x^3*e^2+4/25*d*e*x^2-41/250*e^2*x^2+4/25*x*d^2-82/125*d*e*x+103/625*e
^2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d
*e-17967/700*e^2)/(x^2+2/5*x+3/5)-1/175000*(28700*d^2-28840*d*e-24276*e^2)
*ln(5*x^2+2*x+3)-1/245000*(-6565*d^2-42342*d*e+73881/5*e^2)*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{980000\,e^2x^5 + 24500\,(120\,de - 107\,e^2)x^4 + 58800\,(50\,d^2 - 185\,de + 41\,e^2)x^3 + 2940\,(400\,d^2 - 1040\,de - 2}{}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2
- 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt
(14)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e
- 221643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(
14)*(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 2
32090*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 30
75*d^2 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*
x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

3.312.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.13

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{4e^2x^3}{75} + x^2\cdot\left(\frac{4de}{25} - \frac{41e^2}{250}\right) + x\left(\frac{4d^2}{25} - \frac{82de}{125} + \frac{103e^2}{625}\right) + \left(-\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} - \frac{\sqrt{14}i(32825d^2 + 211710de - 73881e^2)}{2450000}\right)\log\left(x + \frac{6565d^2 + 42342de - \frac{73881e^2}{5} - \frac{\sqrt{14}i(32825d^2+211710de-73}{5}}{32825d^2 + 211710de - 73881e^2}\right.$$
$$+ \left(-\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} + \frac{\sqrt{14}i(32825d^2 + 211710de - 73881e^2)}{2450000}\right)\log\left(x + \frac{6565d^2 + 42342de - \frac{73881e^2}{5} + \frac{\sqrt{14}i(32825d^2+211710de-73}{5}}{32825d^2 + 211710de - 73881e^2}\right.$$
$$+ \frac{-34175d^2 + 12690de + 17967e^2 + x(-10575d^2 - 59890de + 18323e^2)}{437500x^2 + 175000x + 262500}$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
 + 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*
I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 423
42*d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/
5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 +
 867*e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000
)*log(x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 +
 211710*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-3
4175*d**2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e*
*2))/(437500*x**2 + 175000*x + 262500)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{4}{75}\,e^2x^3 + \frac{1}{250}\,(40\,de - 41\,e^2)x^2 + \frac{1}{1225000}\,\sqrt{14}(32825\,d^2 + 211710\,de - 73881\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\,(100\,d^2 - 410\,de + 103\,e^2)x - \frac{1}{6250}\,(1025\,d^2 - 1030\,de - 867\,e^2)\log\left(5\,x^2 + 2\,x + 3\right) - \frac{34175\,d^2 - 12690\,de - 17967\,e^2 + (10575\,d^2 + 59890\,de - 18323\,e^2)x}{87500\,(5\,x^2 + 2\,x + 3)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^
2 - 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^
2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 5
9890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{4}{75}e^2x^3 + \frac{4}{25}dex^2 - \frac{41}{250}e^2x^2 + \frac{4}{25}d^2x - \frac{82}{125}dex + \frac{103}{625}e^2x + \frac{1}{1225000}\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{1}{6250}\left(1025\,d^2 - 1030\,de - 867\,e^2\right)\log\left(5\,x^2+2\,x+3\right) - \frac{34175\,d^2 - 12690\,de - 17967\,e^2 + (10575\,d^2 + 59890\,de - 18323\,e^2)x}{87500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
4/75*e^2*x^3 + 4/25*d*e*x^2 - 41/250*e^2*x^2 + 4/25*d^2*x - 82/125*d*e*x +
 103/625*e^2*x + 1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*a
rctan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*lo
g(5*x^2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d
^2 + 59890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \ln\left(5\,x^2+2\,x+3\right)\left(-\frac{41\,d^2}{250} + \frac{103\,d\,e}{625} + \frac{867\,e^2}{6250}\right) - x\left(\frac{2\,d\,e}{5} + \frac{4\,e\,(8\,d-5\,e)}{125} - \frac{4\,d^2}{25} - \frac{3\,e^2}{625}\right) + x^2\left(\frac{e\,(8\,d-5\,e)}{50} - \frac{8\,e^2}{125}\right) + \frac{\frac{1269\,d\,e}{14} - x\left(\frac{2115\,d^2}{28} + \frac{5989\,d\,e}{14} - \frac{18323\,e^2}{140}\right) - \frac{6835\,d^2}{28} + \frac{17967\,e^2}{140}}{3125\,x^2 + 1250\,x + 1875} + \frac{4\,e^2\,x^3}{75} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000} + \frac{\sqrt{14}\,x\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{245000}}{\frac{1313\,d^2}{3500} + \frac{21171\,d\,e}{8750} - \frac{73881\,e^2}{87500}}\right)\left(32825\,d^2 + 211710\,d\,e - 73881\,e^2\right)}{1225000}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*(
(2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8
*d - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d
^2)/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 312
5*x^2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 3
2825*d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 738
81*e^2))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))
*(211710*d*e + 32825*d^2 - 73881*e^2))/1225000
```

3.312. $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.313 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$

3.313.1 Optimal result

Integrand size = 36, antiderivative size = 97

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx = \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500(3+2x+5x^2)} + \frac{(6565d+21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{17500\sqrt{14}} - \frac{(205d-103e)\log(3+2x+5x^2)}{1250}$$

output

```
1/125*(20*d-41*e)*x+2/25*e*x^2-1/3500*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)-1
/1250*(205*d-103*e)*ln(5*x^2+2*x+3)+1/245000*(6565*d+21171*e)*arctan(1/14*
(1+5*x)*14^(1/2))*14^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{1960(20d-41e)x+19600ex^2-\frac{14(5d(1367+423x)+e(-1269+5989x))}{3+2x+5x^2}+\sqrt{14}(6565d+21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+1}{245000}$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,
x]
```

output
```
(1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
 5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x
)/Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

3.313.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2175, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{\left(5x^2+2x+3\right)^2}dx$$

$$\Big\downarrow \text{ 2175}$$

$$\frac{1}{56}\int \frac{2\left(2800ex^3+140(20d-33e)x^2-84(55d-27e)x+1845d+1367e\right)}{125\left(5x^2+2x+3\right)}dx-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2800ex^3+140(20d-33e)x^2-84(55d-27e)x+1845d+1367e}{5x^2+2x+3}dx}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{\int \left(28(20d-41e)+560ex+\frac{165d+4811e-28(205d-103e)x}{5x^2+2x+3}\right)dx}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d+21171e)}{5\sqrt{14}}-\frac{14}{5}(205d-103e)\log\left(5x^2+2x+3\right)+28x(20d-41e)+280ex^2}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2) + (28*(20*d - 41*e)*x
 + 280*e*x^2 + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]
) - (14*(205*d - 103*e)*Log[3 + 2*x + 5*x^2])/5)/3500
```

3.313.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.313. $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.313.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

method	result
default	$\frac{2e\,x^2}{25}+\frac{4dx}{25}-\frac{41ex}{125}-\frac{\left(\frac{423d}{140}+\frac{5989e}{700}\right)x+\frac{1367d}{140}-\frac{1269e}{700}}{125\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{(5740d-2884e)\ln\left(5x^2+2x+3\right)}{35000}-\frac{\left(-1313d-\frac{21171e}{5}\right)\sqrt{14}\,\arctan\left(\right.}{49000}$
risch	$\frac{2e\,x^2}{25}+\frac{4dx}{25}-\frac{41ex}{125}+\frac{\frac{\left(-\frac{423d}{140}-\frac{5989e}{700}\right)x}{125}-\frac{1367d}{17500}+\frac{1269e}{87500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{41d\ln\left(350x^2+140x+210\right)}{250}+\frac{103e\ln\left(350x^2+140x+210\right)}{1250}+$ 131

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOS
E)
```

output
```
2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-
1269/700*e)/(x^2+2/5*x+3/5)-1/35000*(5740*d-2884*e)*ln(5*x^2+2*x+3)-1/4900
0*(-1313*d-21171/5*e)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$=\frac{98000\,ex^4+9800\,(20\,d-37\,e)x^3+7840\,(10\,d-13\,e)x^2+\sqrt{14}(5\,(6565\,d+21171\,e)x^2+2\,(6565\,d+2117}{}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fr
icas")
```

output
```
1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 +
sqrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 63
513*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*
(205*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x
 + 3) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

3.313.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.70

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{2ex^2}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x(-2115d-5989e)}{87500x^2+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}-\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)+\left(-\frac{41d}{250}+\frac{103e}{1250}+\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}+\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 598
9*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*
I*(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(656
5*d + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)
*I*(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(65
65*d + 21171*e)/5)/(6565*d + 21171*e))
```

3.313.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{2}{25}\,ex^2+\frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{1}{125}\,(20\,d-41\,e)x-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right)-\frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="ma
xima")
```

output
```
2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*
x + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x +
 3) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{2}{25}\,ex^2 + \frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{4}{25}\,dx - \frac{41}{125}\,ex - \frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right) - \frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gi
ac")
```

output
```
2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*
x + 1)) + 4/25*d*x - 41/125*e*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x +
 3) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex)\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{2\,e\,x^2}{25} - \ln\left(5\,x^2+2\,x+3\right)\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right) + x\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right) - \frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(6565\,d+21171\,e)}{245000}+\frac{\sqrt{14}\,x\,(6565\,d+21171\,e)}{49000}}{\frac{1313\,d}{3500}+\frac{21171\,e}{17500}}\right)(6565\,d+21171\,e)}{245000}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
(2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)
/25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)
/140))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171
*e))/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (211
71*e)/17500))*(6565*d + 21171*e))/245000
```

3.313. $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx$

3.314.1 Optimal result

Integrand size = 31, antiderivative size = 63

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4x}{25}-\frac{1367+423x}{3500(3+2x+5x^2)}+\frac{1313\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)$$

output
```
4/25*x+1/3500*(-1367-423*x)/(5*x^2+2*x+3)-41/250*ln(5*x^2+2*x+3)+1313/4900
0*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{7840x-\frac{14(1367+423x)}{3+2x+5x^2}+1313\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)-8036\log\left(3+2x+5x^2\right)}{49000}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

output
```
(7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1
+ 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

3.314.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{\left(5x^2+2x+3\right)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{56}\int \frac{2\left(560x^2-924x+369\right)}{25\left(5x^2+2x+3\right)}dx-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{700}\int \frac{560x^2-924x+369}{5x^2+2x+3}dx-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{700}\int \left(\frac{33-1148x}{5x^2+2x+3}+112\right)dx-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{700}\left(\frac{1313\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}}-\frac{574}{5}\log\left(5x^2+2x+3\right)+112x\right)-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*(1367 + 423*x)/(3 + 2*x + 5*x^2) + (112*x + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(5*Sqrt[14]) - (574*Log[3 + 2*x + 5*x^2])/5)/700
```

3.314.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.314.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{4x}{25}+\frac{-\frac{423x}{17500}-\frac{1367}{17500}}{x^{2}+\frac{2}{5}x+\frac{3}{5}}-\frac{41\ln\left(25x^{2}+10x+15\right)}{250}+\frac{1313\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{49000}$	50
default	$\frac{4x}{25}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^{2}+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{41\ln\left(5x^{2}+2x+3\right)}{250}+\frac{1313\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}$	51

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
4/25*x+(-423/17500*x-1367/17500)/(x^2+2/5*x+3/5)-41/250*ln(25*x^2+10*x+15)
+1313/49000*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{39200\,x^3+1313\,\sqrt{14}(5\,x^2+2\,x+3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+15680\,x^2-8036\,(5\,x^2+2\,x+3)\log\left(5\,x\right.}{49000\,(5\,x^2+2\,x+3)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

output
```
1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*
(5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 175
98*x - 19138)/(5*x^2 + 2*x + 3)
```

3.314.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4x}{25} + \frac{-423x-1367}{17500x^2+7000x+10500} - \frac{41\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{250} + \frac{1313\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x
/5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

3.314.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{1313}{49000}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{4}{25}\,x - \frac{423\,x+1367}{3500\,(5\,x^2+2\,x+3)} - \frac{41}{250}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

output
```
1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423
*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{4}{25}\,x - \frac{423\,x+1367}{3500\,(5\,x^2+2\,x+3)}-\frac{41}{250}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

output
```
1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423
*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4\,x}{25}-\frac{41\,\ln\left(5\,x^2+2\,x+3\right)}{250}-\frac{\frac{423\,x}{17500}+\frac{1367}{17500}}{x^2+\frac{2\,x}{5}+\frac{3}{5}} + \frac{1313\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

output
```
(4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((
2*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14)
)/49000
```

3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx$

3.315.1 Optimal result

Integrand size = 38, antiderivative size = 224

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = -\frac{1367d-293e+(423d-1367e)x}{700(5d^2-2de+3e^2)(3+2x+5x^2)} + \frac{(6565d^3-26423d^2e+11089de^2-6623e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{700\sqrt{14}(5d^2-2de+3e^2)^2} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e(5d^2-2de+3e^2)^2} - \frac{(205d^3-61d^2e+23de^2+14e^3)\log(3+2x+5x^2)}{50(5d^2-2de+3e^2)^2}$$

output
```
1/700*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+(
4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/5
0*(205*d^3-61*d^2*e+23*d*e^2+14*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2
+1/9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14
^(1/2))/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

3.315.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{\frac{14(5d^2-2de+3e^2)(-d(1367+423x)+e(293+1367x))}{3+2x+5x^2}+\sqrt{14}(6565d^3-26423d^2e+11089de^2-6623e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+}{9800\left(5d^2-2de+\right.}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),
x]
```

output
```
((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3
+ 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3
)*ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Lo
g[3 + 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.315.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{\left(5x^2+2x+3\right)^2(d+ex)}\,dx$$
$$\Big\downarrow \ 2177$$
$$\frac{1}{56}\int \frac{2\left(112x^2-\frac{(924d^2-285ed+281e^2)x}{5d^2-2ed+3e^2}+\frac{369d^2-421ed+280e^2}{5d^2-2ed+3e^2}\right)}{5(d+ex)\left(5x^2+2x+3\right)}dx-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}$$
$$\Big\downarrow \ 27$$
$$\frac{1}{140}\int \frac{112x^2-\frac{(924d^2-285ed+281e^2)x}{5d^2-2ed+3e^2}+\frac{369d^2-421ed+280e^2}{5d^2-2ed+3e^2}}{(d+ex)\left(5x^2+2x+3\right)}dx-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{140}\int\left(\frac{140(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)}+\frac{165d^3-4943ed^2+2089e^2d-1403e^3-28(205d^3-61ed^2+23e}{(5d^2-2ed+3e^2)^2(5x^2+2x+3)}\right.$$
$$\frac{x(423d-1367e)+1367d-293e}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{140}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d^3-26423d^2e+11089de^2-6623e^3)}{5\sqrt{14}(5d^2-2de+3e^2)^2}-\frac{14(205d^3-61d^2e+23de^2+14e^3)\log(5x^2+}{5(5d^2-2de+3e^2)^2}\right.$$
$$\frac{x(423d-1367e)+1367d-293e}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/700*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
 2*x + 5*x^2)) + (((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTa
n[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + (140*(4*d^
4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e +
 3*e^2)^2) - (14*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*
x^2])/(5*(5*d^2 - 2*d*e + 3*e^2)^2))/140
```

3.315.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.315. $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx$

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.315.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

method	result
default	$-\frac{\frac{\left(\frac{423}{700}d^3-\frac{7681}{3500}d^2e+\frac{4003}{3500}d\,e^2-\frac{4101}{3500}e^3\right)x+\frac{1367d^3}{700}-\frac{4199d^2e}{3500}+\frac{4687d\,e^2}{3500}-\frac{879e^3}{3500}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740d^3-1708d^2e+644d\,e^2+392e^3\right)\ln\left(5x^2+2x+3\right)}{1400}+\frac{(-13}{}}{\left(5d^2-2de+3e^2\right)^2}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^2*(((423/700*d^3-7681/3500*d^2*e+4003/3500*d*e^2-41
01/3500*e^3)*x+1367/700*d^3-4199/3500*d^2*e+4687/3500*d*e^2-879/3500*e^3)/
(x^2+2/5*x+3/5)+1/1400*(5740*d^3-1708*d^2*e+644*d*e^2+392*e^3)*ln(5*x^2+2*
x+3)+1/1960*(-1313*d^3+26423/5*d^2*e-11089/5*d*e^2+6623/5*e^3)*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x
+d)/e/(5*d^2-2*d*e+3*e^2)^2
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(215) = 430.

Time = 0.32 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.14

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{95690\,d^3e-58786\,d^2e^2+65618\,de^3-12306\,e^4-\sqrt{14}(19695\,d^3e-79269\,d^2e^2+33267\,de^3-19869\,e^4}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fr
icas")
```

output
```
-1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*
(19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 2
6423*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2
 + 11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d
^3*e - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e +
 9*d^2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*
e^4)*x^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d)
 + 196*(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^
2*e^2 + 23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14
*e^4)*x)*log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d
*e^4 + 27*e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*
x^2 + 2*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

3.315.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.29

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = \frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(ex+d)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5} - \frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log(5\,x^2+2\,x+3)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{(423\,d-1367\,e)x+1367\,d-293\,e}{700\,(5\,(5\,d^2-2\,de+3\,e^2)x^2+15\,d^2-6\,de+9\,e^2+2\,(5\,d^2-2\,de+3\,e^2)x)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="ma
xima")
```

output
```
1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1
/14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4
) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e -
 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e +
23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2
- 2*d*e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*
x)
```

3.315.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.32

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = \frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log(5\,x^2+2\,x+3)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(|ex+d|)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5} - \frac{6835\,d^3-4199\,d^2e+4687\,de^2-879\,e^3+(2115\,d^3-7681\,d^2e+4003\,de^2-4101\,e^3)x}{700\,(5\,d^2-2\,de+3\,e^2)^2(5\,x^2+2\,x+3)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gi
ac")
```

output
```
1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1
/14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4
) - 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25
*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^
2*e^2 - d*e^3 + 2*e^4)*log(abs(e*x + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e
^3 - 12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + 4687*d*e^2 - 879*e
^3 + (2115*d^3 - 7681*d^2*e + 4003*d*e^2 - 4101*e^3)*x)/((5*d^2 - 2*d*e +
3*e^2)^2*(5*x^2 + 2*x + 3))
```

3.315.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.47

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = \frac{\ln(d+e\,x)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}$$
$$+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}-\frac{23}{50}\mathrm{i}\right)\,d\,e^2+\left(-\frac{6623\,\sqrt{1}}{19600}\right.\right.}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}$$
$$-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}+\frac{41}{10}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}-\frac{61}{50}\mathrm{i}\right)\,d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}+\frac{23}{50}\mathrm{i}\right)\,d\,e^2+\left(-\frac{6623\,\sqrt{1}}{19600}\right.\right.}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}$$
$$-\frac{\frac{1367\,d-293\,e}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}+\frac{x\,\left(423\,d-1367\,e\right)}{700\,\left(5\,d^2-2\,d\,e+3\,e^2\right)}}{5\,x^2+2\,x+3}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

output
```
(log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3
*((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50)
 - d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*
12i + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^
2)) + (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3
) - (log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) -
 e^3*((6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i
/50) - d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*
e^3*12i + e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 +
 2*e^4 + 3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$

3.316.1 Optimal result

Integrand size = 38, antiderivative size = 313

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx &= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} \\ &\quad -\frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \\ &\quad +\frac{(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}(5d^2-2de+3e^2)^3} \\ &\quad +\frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex)}{(5d^2-2de+3e^2)^3} \\ &\quad -\frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^3}\end{aligned}$$

output
```
(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+1/1
40*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3
*e^2)^2/(5*x^2+2*x+3)+(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)
/(5*d^2-2*d*e+3*e^2)^3-1/2*(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5
*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2
+156*d*e^3-271*e^4)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14
^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$$
$$= \frac{-\frac{1960(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e(d+ex)} - \frac{14(5d^2-2de+3e^2)(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x))}{3+2x+5x^2} + 5\sqrt{14}(}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2
),x]
```

output
```
((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2
*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1
313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*
x)/Sqrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log
[d + e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3
+ 2*x + 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

3.316.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^2(d+ex)^2}dx$$
$$\Big\downarrow \text{2177}$$
$$\frac{1}{56}\int \frac{2\left(\frac{(560d^4-448ed^3+677e^2d^2+278e^3d+143e^4)x^2}{(5d^2-2ed+3e^2)^2} - \frac{2(462d^4-285ed^3+338e^2d^2-171e^3d+14e^4)x}{(5d^2-2ed+3e^2)^2} + \frac{369d^4-842ed^3+787e^2d^2-224e^3d+}{(5d^2-2ed+3e^2)^2}\right.}{(d+ex)^2(5x^2+2x+3)}$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\int \frac{\frac{(560d^4-448ed^3+677e^2d^2+278e^3d+143e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(462d^4-285ed^3+338e^2d^2-171e^3d+14e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{369d^4-842ed^3+787e^2d^2-224e^3d+168}{(5d^2-2ed+3e^2)^2}}{(d+ex)^2(5x^2+2x+3)}$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \; 2159$$

$$\frac{1}{28}\int \left(\frac{28(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)^2}-\frac{28e(-41d^4+8ed^3+60e^2d^2-24e^3d+5e^4)}{(5d^2-2ed+3e^2)^3(d+ex)}+\frac{165d^4-9820ed^3+}{}\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)}{\sqrt{14}(5d^2-2de+3e^2)^3}-\frac{14(41d^4-8d^3e-60d^2e^2+24de^3-5e}{(5d^2-2de+3e^2)^3}\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/140*(1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e^2)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((-28*(4*d^4 + 5*d^3*e + 3*
d^2*e^2 - d*e^3 + 2*e^4))/(e*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) + ((1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/
Sqrt[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (28*(41*d^4 - 8*d^3*e -
60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (
14*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2]
)/(5*d^2 - 2*d*e + 3*e^2)^3)/28
```

3.316. $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$

3.316.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.316.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\frac{\left(\frac{423}{140}d^4-\frac{3629}{175}d^3e+\frac{4101}{350}d^2e^2-\frac{2197}{175}de^3+\frac{879}{700}e^4\right)x+\frac{1367d^4}{140}-\frac{1416d^3e}{175}+\frac{879d^2e^2}{350}-\frac{88de^3}{175}-\frac{2109e^4}{700}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740d^4-1120d^3e-8400d^2e^2+3360de^3\cdots\right.}{280}\cdots}{\left(5d^2-2de+3e^2\right)^3}\cdots$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^3*(((423/140*d^4-3629/175*d^3*e+4101/350*d^2*e^2-21
97/175*d*e^3+879/700*e^4)*x+1367/140*d^4-1416/175*d^3*e+879/350*d^2*e^2-88
/175*d*e^3-2109/700*e^4)/(x^2+2/5*x+3/5)+1/280*(5740*d^4-1120*d^3*e-8400*d
^2*e^2+3360*d*e^3-700*e^4)*ln(5*x^2+2*x+3)+1/392*(-1313*d^4+10044*d^3*e-42
90*d^2*e^2-156*d*e^3+271*e^4)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-(4*
d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)+(41*d^4
-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(304) = 608$.

Time = 0.35 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.91

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^2 (3 + 2x + 5x^2)^2} dx =$$
$$-\frac{117600 d^6 + 195650 d^5 e + 20664 d^4 e^2 + 48132 d^3 e^3 + 118552 d^2 e^4 - 70686 de^5 + 35280 e^6 + 14 (14000 d}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
-1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 11855
2*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*
d^4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*
sqrt(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d
*e^5 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^
6)*x^3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*
d*e^5 - 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182
*d^2*e^4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(56
00*d^6 + 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*
e^5 - 429*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4
 - 15*d*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x
^3 + (205*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*
d^5*e + 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*lo
g(e*x + d) + 980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d
*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (2
05*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e +
 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2
 + 2*x + 3))/(375*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^
3*e^5 - 162*d^2*e^6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^
4 - 188*d^3*e^5 + 171*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 5...
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

3.316.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.75

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{\sqrt{14}(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+ \frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(ex+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}$$
$$- \frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$- \frac{1680\,d^4+3467\,d^3e+674\,d^2e^2-1123\,de^3+840\,e^4+(2800\,d^4+3500\,d^3e+}{140\,(75\,d^5e-60\,d^4e^2+102\,d^3e^3-36\,d^2e^4+27\,de^5+5\,(25\,d^4e^2-20\,d^3e^3+34\,d^2e^4-12\,de^5+9\,e^6)x^3+}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^
4)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 18
8*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 -
188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 6
0*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e +
285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680
*d^4 + 3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*
d^3*e + 2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e
 - 527*d^2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*
e^3 - 36*d^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12
*d*e^5 + 9*e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 +
21*d*e^5 + 18*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 -
 18*d*e^5 + 27*e^6)*x)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx =$$
$$-\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$-\frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{25\,d^4e^4-20\,d^3e^5+34\,d^2e^6-12\,de^7+9\,e^8}$$
$$+\frac{\sqrt{14}(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)e^2}$$
$$+\frac{\frac{423\,d^3e-4101\,d^2e^2+879\,de^3+703\,e^4}{5\,d^2-2\,de+3\,e^2}-\frac{423\,d^4e^2-5468\,d^3e^3+1758\,d^2e^4+2812\,de^5-457\,e^6}{(5\,d^2-2\,de+3\,e^2)(ex+d)e}}{28\,(5\,d^2-2\,de+3\,e^2)^2\left(\frac{10\,d}{ex+d}-\frac{5\,d^2}{(ex+d)^2}-\frac{2\,e}{ex+d}+\frac{2\,de}{(ex+d)^2}-\frac{3\,e^2}{(ex+d)^2}-5\right)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
-1/2*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(e*x + d
) + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d
)^2 + 5)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 -
54*d*e^5 + 27*e^6) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(e*x + d) + 3*d^2*e^
5/(e*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(25*d^4*e^4 - 20*d^3*e^5
+ 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^
3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*
d^2/(e*x + d) + 2*d*e/(e*x + d) - e - 3*e^2/(e*x + d))/e)/((125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*e^2)
+ 1/28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e +
3*e^2) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6
)/((5*d^2 - 2*d*e + 3*e^2)*(e*x + d)*e))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/
(e*x + d) - 5*d^2/(e*x + d)^2 - 2*e/(e*x + d) + 2*d*e/(e*x + d)^2 - 3*e^2/
(e*x + d)^2 - 5))
```

3.316.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.92

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = \ln(d+e\,x)\left(\frac{41}{25\,(5\,d^2-2\,d\,e+3\,e^2)} - \frac{4\,e^3\,(423\,d-1367\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3} + \frac{2\,e\,(310\,d-1323\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^2}\right) - \frac{\frac{1680\,d^4+3467\,d^3\,e+674\,d^2\,e^2-1123\,d\,e^3+840\,e^4}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} - \frac{x\,(-1120\,d^4-1823\,d^3\,e+527\,d^2\,e^2+573\,d\,e^3+143\,e^4)}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} + \frac{x^2\,(2800\,d^4+3500\,d^3\,e+2523\,d^2 \cdots}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2 \cdots}}{5\,e\,x^3+(5\,d+2\,e)\,x^2+(2\,d+3\,e)\,x+3\,d} + \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}-\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}+4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}+30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}-1 \cdots\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27 \cdots} - \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}+\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}-4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}-30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}+1 \cdots\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27 \cdots}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

output
```
log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*
e + 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*
e^2)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573
*d*e^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 -
20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3
 + 2800*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e
^3 + 9*e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e
)) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) -
e^4*((271*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*
e^3*((39*14^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*12
5i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*
e^2*285i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41
i/2) - e^4*((271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30
i) + d*e^3*((39*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/
(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i
 + d^4*e^2*285i)
```

3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

3.317.1 Optimal result

Integrand size = 38, antiderivative size = 412

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx &= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3(d+ex)} \\ &\quad - \frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+703e^3)x}{28(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\ &\quad + \frac{(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}(5d^2-2de+3e^2)^4} \\ &\quad + \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4} \\ &\quad - \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^4}\end{aligned}$$

output
```
1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)
^2+(-41*d^4+8*d^3*e+60*d^2*e^2-24*d*e^3+5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+
d)+1/28*(-1367*d^3+879*d^2*e+2109*d*e^2-457*e^3-(423*d^3-4101*d^2*e+879*d*
e^2+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+(205*d^5-19*d^4*e-846*
d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(
205*d^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/
(5*d^2-2*d*e+3*e^2)^4+1/392*(6565*d^5-74017*d^4*e+35022*d^3*e^2+42858*d^2*
e^3-17247*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)
^4*14^(1/2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$$
$$= \frac{-\frac{196(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e(d+ex)^2}+\frac{392(5d^2-2de+3e^2)(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)}{d+ex}-\frac{14(5d^2-2de+3e^2)(3de^2}{}}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2
),x]
```

output
```
((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*
e^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e +
60*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3
*d*e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(
293 + 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 350
22*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[
14]] + 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21
*e^5)*Log[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3
- 57*d*e^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

3.317.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^2(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2177}$$
$$\frac{1}{56}\int \frac{2\left(-\frac{e^3(423d^3-4101ed^2+879e^2d+703e^3)x^3}{(5d^2-2ed+3e^2)^3}+\frac{(2800d^6-3360ed^5+5115e^2d^4+5527e^3d^3+1311e^4d^2+1251e^5d-28e^6)x^2}{(5d^2-2ed+3e^2)^3}-\frac{(4620d^6-42}{}}{(d+ex)^3(5x^2}$$
$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$\Big\downarrow$ 27

$$\frac{1}{28}\int \frac{-\frac{e^3(423d^3-4101ed^2+879e^2d+703e^3)x^3}{(5d^2-2ed+3e^2)^3}+\frac{(2800d^6-3360ed^5+5115e^2d^4+5527e^3d^3+1311e^4d^2+1251e^5d-28e^6)x^2}{(5d^2-2ed+3e^2)^3}-\frac{(4620d^6-4275ed}{}}{(d+ex)^3(5x^2+}$$

$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$\Big\downarrow$ 2159

$$\frac{1}{28}\int\left(\frac{28(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)^3}-\frac{28e(-205d^5+19ed^4+846e^2d^3-396e^3d^2-57e^4d+21e^5)}{(5d^2-2ed+3e^2)^4(d+ex)}+\frac{3(2}{}\right.$$

$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$\Big\downarrow$ 2009

$$\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5)}{\sqrt{14}(5d^2-2de+3e^2)^4}-\frac{28(41d^4-8d^3e-60d}{(5d^2-2de+3e}\right.$$

$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/28*(1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*e^3 + (423*d^3 - 4101*d^2*e
 + 879*d*e^2 + 703*e^3)*x)/((5*d^2 - 2*d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) +
 ((-14*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e*(5*d^2 - 2*d*e +
3*e^2)^2*(d + e*x)^2) - (28*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*
e^4))/((5*d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) + ((6565*d^5 - 74017*d^4*e + 3
5022*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqr
t[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (28*(205*d^5 - 19*d^4*e - 8
46*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e
 + 3*e^2)^4 - (14*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e
^4 - 21*e^5)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^4)/28
```

3.317. $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.317.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\frac{\left(\frac{423}{28} d^5-\frac{21351}{140} d^4 e+\frac{6933}{70} d^3 e^2-\frac{5273}{70} d^2 e^3+\frac{1231}{140} d e^4+\frac{2109}{140} e^5\right) x+\frac{1367 d^5}{28}-\frac{7129 d^4 e}{140}-\frac{2343 d^3 e^2}{70}+\frac{1933 d^2 e^3}{70}-\frac{7241 d e^4}{140}+\frac{1371 e^5}{140}}{x^2+\frac{2}{5} x+\frac{3}{5}}+\frac{\left(28700 d^5\right.}{}}{}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^4*(((423/28*d^5-21351/140*d^4*e+6933/70*d^3*e^2-527
3/70*d^2*e^3+1231/140*d*e^4+2109/140*e^5)*x+1367/28*d^5-7129/140*d^4*e-234
3/70*d^3*e^2+1933/70*d^2*e^3-7241/140*d*e^4+1371/140*e^5)/(x^2+2/5*x+3/5)+
1/280*(28700*d^5-2660*d^4*e-118440*d^3*e^2+55440*d^2*e^3+7980*d*e^4-2940*e
^5)*ln(5*x^2+2*x+3)+1/392*(-6565*d^5+74017*d^4*e-35022*d^3*e^2-42858*d^2*e
^3+17247*d*e^4-579*e^5)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^
4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)^2-(41*d^4
-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+(205*d^5
-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+
3*e^2)^4
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. $2(401) = 802$.

Time = 0.45 (sec) , antiderivative size = 1499, normalized size of antiderivative = 3.64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
-1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 39516
4*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14
*(28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^
2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e
^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*
d*e^7 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^
5*e^3 + 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 -
74017*d^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4
 + 2*(32825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 4
3377*d^2*e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2
- 101263*d^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425
*d*e^7 + 1737*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 +
147924*d^4*e^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/
14*sqrt(14)*(5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620
*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 75
6*e^8)*x - 392*(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171
*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^
2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*
e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e +
725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 ...
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

3.317.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(401) = 802$.

Time = 0.30 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.07

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$$
$$= \frac{\sqrt{14}(6565\,d^5 - 74017\,d^4e + 35022\,d^3e^2 + 42858\,d^2e^3 - 17247\,de^4 + 579\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(625\,d^8 - 1000\,d^7e + 2100\,d^6e^2 - 1960\,d^5e^3 + 2086\,d^4e^4 - 1176\,d^3e^5 + 756\,d^2e^6 - 216\,de^7 + 81\,e^8)}$$
$$+ \frac{(205\,d^5 - 19\,d^4e - 846\,d^3e^2 + 396\,d^2e^3 + 57\,de^4 - 21\,e^5)\log(ex+d)}{625\,d^8 - 1000\,d^7e + 2100\,d^6e^2 - 1960\,d^5e^3 + 2086\,d^4e^4 - 1176\,d^3e^5 + 756\,d^2e^6 - 216\,de^7 + 81\,e^8}$$
$$- \frac{(205\,d^5 - 19\,d^4e - 846\,d^3e^2 + 396\,d^2e^3 + 57\,de^4 - 21\,e^5)\log(5\,x^2+2\,x+3)}{2\,(625\,d^8 - 1000\,d^7e + 2100\,d^6e^2 - 1960\,d^5e^3 + 2086\,d^4e^4 - 1176\,d^3e^5 + 756\,d^2e^6 - 216\,de^7 + 81\,e^8)}$$
$$- \frac{840\,d^6 + 5525\,d^5e - 837\,d^4e^2 - 69}{28\,(375\,d^8e - 450\,d^7e^2 + 855\,d^6e^3 - 564\,d^5e^4 + 513\,d^4e^5 - 162\,d^3e^6 + 81\,d^2e^7 + 5\,(125\,d^6e^3 - 150\,d^5e^4 +}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 1
7247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*
e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^
6 - 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1
960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e
^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*
e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*
e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/
28*(840*d^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714
*d*e^5 + 252*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^
5 + 3*e^6)*x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2
349*d^2*e^4 - 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*
e^2 - 4799*d^3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e -
450*d^7*e^2 + 855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d
^2*e^7 + 5*(125*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^
2*e^7 - 54*d*e^8 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e
^4 - 655*d^4*e^5 + 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (62
5*d^8*e - 250*d^7*e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3
*e^6 + 432*d^2*e^7 - 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^...
```

3.317.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$$
$$= \frac{\sqrt{14}(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$-\frac{(205\,d^5-19\,d^4e-846\,d^3e^2+396\,d^2e^3+57\,de^4-21\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$+\frac{(205\,d^5e-19\,d^4e^2-846\,d^3e^3+396\,d^2e^4+57\,de^5-21\,e^6)\log\left(|ex+d|\right)}{625\,d^8e-1000\,d^7e^2+2100\,d^6e^3-1960\,d^5e^4+2086\,d^4e^5-1176\,d^3e^6+756\,d^2e^7-216\,de^8+81\,e^9}$$
$$-\frac{4200\,d^8+25945\,d^7e-12715\,d^6e^2-16656\,d^5e^3+28226\,d^4e^4-31223\,d^3e^5+12753\,d^2e^6-2646\,de^7+7}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 1
7247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*
e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^
6 - 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*
e^3 + 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100
*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*
d*e^7 + 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57
*d*e^5 - 21*e^6)*log(abs(e*x + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^
3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 +
 81*e^9) - 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 +
28226*d^4*e^4 - 31223*d^3*e^5 + 12753*d^2*e^6 - 2646*d*e^7 + 756*e^8 + (28
700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e^
6 + 12711*d*e^7 + 9*e^8)*x^3 + (7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 - 62
649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7 +
 1791*e^8)*x^2 + (2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 -
17202*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^4*(e*x + d)^2*(5*x^2 + 2*x + 3)*e)
```

3.317.9 Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.15

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$$
$$= \ln(d+e\,x)\left(\frac{\frac{41\,d}{5}+\frac{29\,e}{5}}{(5\,d^2-2\,d\,e+3\,e^2)^2}+\frac{168\,e^4\,(458\,d-7\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^4}-\frac{2\,e^2\,(12610\,d+1329\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3}\right)$$
$$-\frac{\frac{840\,d^6+5525\,d^5\,e-837\,d^4\,e^2-6981\,d^3\,e^3+3355\,d^2\,e^4-714\,d\,e^5+252\,e^6}{28\,e\,(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6)}+\frac{x^3\,(5740\,d^4\,e-697\,d^3\,e^2-12501\,d^2\,e^3+4239\,d\,e^4+3\,e^5)}{28\,(125\,d^6-150\,d^5\,e+285\,d^4\,e^2-188\,d^3\,e^3+171\,d^2\,e^4-54\,d\,e^5+27\,e^6}}{x^2\,(5\,d^2+4\,d\,e+3\,e^2)+x\,(2}$$
$$+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\left(\frac{6565\,\sqrt{14}}{784}-\frac{205}{2}\mathrm{i}\right)d^5+\left(-\frac{74017\,\sqrt{14}}{784}+\frac{19}{2}\mathrm{i}\right)d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}+423\mathrm{i}\right)d^3\,e^2+\left(\frac{21429}{39}\right.\right.\right.}{d^8\,625\mathrm{i}-d^7\,e\,1000\mathrm{i}+d^6\,e^2\,2100\mathrm{i}-d^5\,e^3\,1960\mathrm{i}+d^4\,e^4\,2086\mathrm{i}-d^3\,e^5\,117}$$
$$-\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\left(\frac{6565\,\sqrt{14}}{784}+\frac{205}{2}\mathrm{i}\right)d^5+\left(-\frac{74017\,\sqrt{14}}{784}-\frac{19}{2}\mathrm{i}\right)d^4\,e+\left(\frac{17511\,\sqrt{14}}{392}-423\mathrm{i}\right)d^3\,e^2+\left(\frac{21429}{39}\right.\right.\right.}{d^8\,625\mathrm{i}-d^7\,e\,1000\mathrm{i}+d^6\,e^2\,2100\mathrm{i}-d^5\,e^3\,1960\mathrm{i}+d^4\,e^4\,2086\mathrm{i}-d^3\,e^5\,117}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

output
```
log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(
458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))
/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 2
52*e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d
^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^
3*(4239*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(12
5*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^
4*e^2)) + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4
 - 15403*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 +
27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195
*d^5*e + 560*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/
(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2
 + x^3*(10*d*e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^
5*((6565*14^(1/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*
e^2*((17511*14^(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i)
- d*e^4*((17247*14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*
e^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^
(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(...
```

3.318 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

3.318.1 Optimal result

Integrand size = 38, antiderivative size = 171

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{(83065d-126009e)e^2x}{980000}+\frac{2e^3x^2}{125}-\frac{(1367+423x)(d+ex)^3}{7000(3+2x+5x^2)^2}+\frac{(d+ex)^2(3(11449d-2105e)+(11015d+49177e)x)}{196000(3+2x+5x^2)}+\frac{3(353125d^3-855175d^2e+74085de^2+556349e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{4900000\sqrt{14}}+\frac{3e(100d^2-245de+47e^2)\log(3+2x+5x^2)}{6250}$$

output
```
1/980000*(83065*d-126009*e)*e^2*x+2/125*e^3*x^2-1/7000*(1367+423*x)*(e*x+d
)^3/(5*x^2+2*x+3)^2+1/196000*(e*x+d)^2*(34347*d-6315*e+(11015*d+49177*e)*x
)/(5*x^2+2*x+3)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(5*x^2+2*x+3)+3/686000
00*(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*arctan(1/14*(1+5*x)*14
^(1/2))*14^(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{548800(60d-49e)e^2x+5488000e^3x^2-\frac{392(e^3(54969-53189x)+125d^3(1367+423x)+75d^2e(-1269+5989x)-15de^2(17967+183}{(3+2x+5x^2)^2}}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
3,x]
```

output
```
(548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x
) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 +
 18323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3
*(34347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647
195*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 7408
5*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245
*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

3.318.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2175, 27, 2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^3}{(5x^2+2x+3)^3}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{1}{112}\int \frac{2(d+ex)^2(5600ex^3+280(20d-33e)x^2-168(55d-27e)x+3(1089d+1367e))}{125(5x^2+2x+3)^2}dx-\frac{(423x+1367)(d+ex)^3}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^2(5600ex^3+280(20d-33e)x^2-168(55d-27e)x+3(1089d+1367e))}{(5x^2+2x+3)^2}dx}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}$$

$\big\downarrow$ 2175

$$\frac{\frac{1}{56}\int \frac{10(d+ex)(6272e^2x^2+(10341d-22693e)ex+3(2825d^2-5587ed+842e^2))}{5x^2+2x+3}dx+\frac{(x(11015d+49177e)+3(11449d-2105e))(d+ex)^2}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{5}{28}\int \frac{(d+ex)(6272e^2x^2+(10341d-22693e)ex+3(2825d^2-5587ed+842e^2))}{5x^2+2x+3}dx+\frac{(x(11015d+49177e)+3(11449d-2105e))(d+ex)^2}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}$$

$\big\downarrow$ 2159

$$\frac{\frac{5}{28}\int \left(\frac{6272xe^3}{5}+\frac{1}{25}(83065d-126009e)e^2+\frac{3(70625d^3-139675ed^2-62015e^2d+126009e^3+1568e(100d^2-245ed+47e^2)x)}{25(5x^2+2x+3)}\right)dx+\frac{(x(}{}}{7000}$$

$$\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}$$

$\big\downarrow$ 2009

$$\frac{\frac{5}{28}\left(\frac{3\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(353125d^3-855175d^2e+74085de^2+556349e^3)}{125\sqrt{14}}+\frac{2352}{125}e(100d^2-245de+47e^2)\log\left(5x^2+2x+3\right)+\frac{1}{25}e^2x(\right.}{7000}$$

$$\frac{(423x+1367)(d+ex)^3}{7000\,(5x^2+2x+3)^2}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x)^3)/(3 + 2*x + 5*x^2)^2 + (((d + e*x)^2*(
3*(11449*d - 2105*e) + (11015*d + 49177*e)*x))/(28*(3 + 2*x + 5*x^2)) + (5
*(((83065*d - 126009*e)*e^2*x)/25 + (3136*e^3*x^2)/5 + (3*(353125*d^3 - 85
5175*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(125*Sq
rt[14]) + (2352*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/125))
/28)/7000
```

3.318. $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.318.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

method	result
default	$\frac{2e^3x^2}{125}+\frac{12d\,e^2x}{125}-\frac{49e^3x}{625}+\frac{\left(\frac{11015}{1568}d^3+\frac{109059}{1568}d^2e-\frac{388317}{7840}d\,e^2-\frac{621801}{39200}e^3\right)x^3+\left(\frac{38753}{1568}d^3+\frac{84921}{7840}d^2e-\frac{640827}{7840}d\,e^2+\frac{1396037}{196000}e^3\right)x^2}{25(5x^2+2x}$
risch	$\frac{44451\sqrt{14}\,d\,e^2\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{13720000}+\frac{1669047\sqrt{14}\,e^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{68600000}+\frac{6d^2e\ln\left(350x^2+140x+210\right)}{125}-\frac{147d\,e^2\ln(350x}{12}$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/156
8*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840
*d^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/78
40*d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d
^2*e-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+3/9800000*(1568
00*d^2*e-384160*d*e^2+73696*e^3)*ln(5*x^2+2*x+3)+3/13720000*(70625*d^3-171
035*d^2*e+14817*d*e^2+556349/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)
)
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(153) = 306$.

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{27440000\,e^3x^6 + 2744000\,(60\,de^2 - 41\,e^3)x^5 + 8780800\,(15\,de^2 - 8\,e^3)x^4 + 70\,(275375\,d^3 + 2726475\,d^2e +}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(
15*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 3
045929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 1786339
8*e^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^
2 + 3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*
x^4 + 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178
125*d^3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 85
5175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e
 + 74085*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(7491
25*d^3 + 1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d
^2*e - 245*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 +
 900*d^2*e - 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^
2 + 12*(100*d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

3.318.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.74

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{2e^3x^2}{125} + x\left(\frac{12de^2}{125} - \frac{49e^3}{625}\right) + \left(\frac{3e(100d^2-245de+47e^2)}{6250} - \frac{3\sqrt{14}\,i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+3271395\ldots}{105\ldots}\right.$$
$$+ \left(\frac{3e(100d^2-245de+47e^2)}{6250} + \frac{3\sqrt{14}\,i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+3271395\ldots}{105\ldots}\right.$$
$$+ \frac{1619125d^3-1464975d^2e-5773275de^2+1275957e^3+x^3\cdot(1376875d^3+13632375d^2e-9707925de^2-\ldots}{122500000x^4+9800\ldots}$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d
*e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e
**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271
395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sq
rt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(10
59375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d
**2 - 245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e
+ 74085*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d*
*2*e + 3271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**
2)/5 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e
**3)/5)/(1059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) +
(1619125*d**3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(137
6875*d**3 + 13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(48441
25*d**3 + 2123025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d*
*3 + 4332075*d**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 980
00000*x**3 + 166600000*x**2 + 58800000*x + 44100000)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2}{125}\,e^3x^2 + \frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3 - 855175\,d^2e + 74085\,de^2 + 556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\,(60\,de^2 - 49\,e^3)x + \frac{3}{6250}\,(100\,d^2e - 245\,de^2 + 47\,e^3)\log\left(5\,x^2+2\,x+3\right) + \frac{5\,(275375\,d^3 + 2726475\,d^2e - 1941585\,de^2 - 621801\,e^3)x^3 + 1619125\,d^3 - 1464975\,d^2e - 5773275\,de^2 +}{49000}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^
3)*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49
00000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 1
619125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 +
2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 144402
5*d^2*e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x
+ 9)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2}{125}\,e^3x^2 + \frac{12}{125}\,de^2x - \frac{49}{625}\,e^3x + \frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3 - 855175\,d^2e + 74085\,de^2 + 556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{3}{6250}\,(100\,d^2e - 245\,de^2 + 47\,e^3)\log\left(5\,x^2+2\,x+3\right) + \frac{5\,(275375\,d^3 + 2726475\,d^2e - 1941585\,de^2 - 621801\,e^3)x^3 + 1619125\,d^3 - 1464975\,d^2e - 5773275\,de^2 +}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
2/125*e^3*x^2 + 12/125*d*e^2*x - 49/625*e^3*x + 3/68600000*sqrt(14)*(35312
5*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/
4900000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 +
 1619125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3
+ 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444
025*d^2*e - 3046875*d*e^2 - 170563*e^3)*x)/(5*x^2 + 2*x + 3)^2
```

3.318.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = x\left(\frac{e^2\,(12\,d-5\,e)}{125}-\frac{24\,e^3}{625}\right)$$
$$-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}-\frac{1275957\,e^3}{7840}+x^3\left(-\right.}{15625\,x^4+12500\,x^3+2125}$$
$$+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{6\,d^2\,e}{125}-\frac{147\,d\,e^2}{1250}+\frac{141\,e^3}{6250}\right)+\frac{2\,e^3\,x^2}{125}$$
$$+\frac{3\,\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{3\,\sqrt{14}\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{68600000}+\frac{3\,\sqrt{14}\,x\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{13720000}}{\frac{339\,d^3}{1568}-\frac{102621\,d^2\,e}{196000}+\frac{44451\,d\,e^2}{980000}+\frac{1669047\,e^3}{4900000}}\right)\,(353125\,d^3-}{68600000}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (29299
5*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^
3)/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^
3*((1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (6218
01*e^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*
d^3)/1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x
^4 + 5625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141
*e^3)/6250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2
 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(7408
5*d*e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2
)/980000 - (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000)
)*(74085*d*e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

3.319 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$

3.319.1 Optimal result

Integrand size = 38, antiderivative size = 134

$$\begin{aligned}\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx &= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000(3+2x+5x^2)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+8553e)x)}{196000(3+2x+5x^2)} \\ &+ \frac{(211875d^2-342070de+14817e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{980000\sqrt{14}} + \frac{(40d-49e)e\log(3+2x+5x^2)}{1250}\end{aligned}$$

output
```
4/125*x*e^2-1/7000*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)^2+1/196000*(e*x+d)
*(34347*d-6413*e+5*(2203*d+8553*e)*x)/(5*x^2+2*x+3)+1/1250*(40*d-49*e)*e*l
n(5*x^2+2*x+3)+1/13720000*(211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$$

$$= \frac{5\sqrt{14}(211875d^2-342070de+14817e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right) + 70\left(\frac{5(5d^2(12953+17979x+38753x^2+11015x^3)+2de(-19533+5}{}\right.}{68}$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
3,x]
```

output
```
(5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14
]] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-195
33 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^
2 + 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d
 - 49*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

3.319.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2175, 27, 2175, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^2}{\left(5x^2+2x+3\right)^3}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{1}{112}\int \frac{2(d+ex)\left(5600ex^3+280(20d-33e)x^2-3(3080d-1371e)x+3267d+2734e\right)}{125\left(5x^2+2x+3\right)^2}dx-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)\left(5600ex^3+280(20d-33e)x^2-3(3080d-1371e)x+3267d+2734e\right)}{\left(5x^2+2x+3\right)^2}dx}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{2175}$$

$$\frac{\frac{1}{56}\int \frac{2\left(42375d^2-55870ed+6413e^2+31360e^2x^2+1568(40d-41e)ex\right)}{5x^2+2x+3}dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28\left(5x^2+2x+3\right)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{28}\int \frac{42375d^2-55870ed+6413e^2+31360e^2x^2+1568(40d-41e)ex}{5x^2+2x+3}dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{1}{28}\int \left(6272e^2+\frac{42375d^2-55870ed-12403e^2+1568(40d-49e)ex}{5x^2+2x+3}\right)dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^2-342070de+14817e^2)}{5\sqrt{14}}+\frac{784}{5}e(40d-49e)\log\left(5x^2+2x+3\right)+6272e^2x\right)+\frac{(d+ex)(5x(2203d+855}{28(5x^2+}}{7000}$$

$$\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x)^2)/(3 + 2*x + 5*x^2)^2 + (((d + e*x)*(34
347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(28*(3 + 2*x + 5*x^2)) + (6272*e^
2*x + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(
5*Sqrt[14]) + (784*(40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/5)/28)/7000
```

3.319.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.319.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

method	result
default	$\frac{4e^{2}x}{125}+\frac{\left(\frac{2203}{1568}d^{2}+\frac{36353}{3920}de-\frac{129439}{39200}e^{2}\right)x^{3}+\left(\frac{38753}{7840}d^{2}+\frac{28307}{19600}de-\frac{213609}{39200}e^{2}\right)x^{2}+\left(\frac{17979}{7840}d^{2}+\frac{57761}{19600}de-\frac{4875}{1568}e^{2}\right)x+\frac{12953d^{2}}{7840}-\frac{19533de}{19600}-}{5\left(5x^{2}+2x+3\right)^{2}}$
risch	$\frac{4e^{2}x}{125}+\frac{\frac{\left(\frac{2203}{1568}d^{2}+\frac{36353}{3920}de-\frac{129439}{39200}e^{2}\right)x^{3}}{5}+\frac{\left(\frac{38753}{7840}d^{2}+\frac{28307}{19600}de-\frac{213609}{39200}e^{2}\right)x^{2}}{5}+\frac{\left(\frac{17979}{7840}d^{2}+\frac{57761}{19600}de-\frac{4875}{1568}e^{2}\right)x}{5}+\frac{12953d^{2}}{39200}-\frac{19533de}{98000}-}{\left(5x^{2}+2x+3\right)^{2}}$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(3875
3/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/196
00*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5
*x^2+2*x+3)^2+1/1960000*(62720*d*e-76832*e^2)*ln(5*x^2+2*x+3)+1/2744000*(4
2375*d^2-68414*d*e+14817/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(121) = 242$.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.25

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{10976000\,e^2x^5 + 8780800\,e^2x^4 + 70\,(55075\,d^2 + 363530\,de + 83809\,e^2)x^3 + 70\,(193765\,d^2 + 56614\,de - 13}{}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*
e + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(1
4)*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070
*d*e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 190
6875*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*
e^2)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388
390*e^2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e -
49*e^2)*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*
e - 441*e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x
^3 + 34*x^2 + 12*x + 9)
```

3.319.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{4e^2x}{125} + \left(\frac{e(40d-49e)}{1250}\right.$$
$$\left.-\frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x + \frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5} -}{211875d^2-342070de+14}\right.$$
$$+\left(\frac{e(40d-49e)}{1250}\right.$$
$$\left.+\frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x + \frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5} +}{211875d^2-342070de+14}\right.$$
$$+\frac{64765d^2-39066de-76977e^2+x^3\cdot(55075d^2+363530de-129439e^2)+x^2\cdot(193765d^2+56614de-}{4900000x^4+3920000x^3+6664000x^2+2352000x+1764}$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*
e + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 +
 21952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e*
*2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 +
sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (4237
5*d**2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(
211875*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 1481
7*e**2)) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 36353
0*d*e - 129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(8
9895*d**2 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664
000*x**2 + 2352000*x + 1764000)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{4}{125}\,e^2x + \frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2 - 342070\,de + 14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)$$
$$+ \frac{1}{1250}\left(40\,de - 49\,e^2\right)\log\left(5\,x^2+2\,x+3\right)$$
$$+ \frac{(55075\,d^2 + 363530\,de - 129439\,e^2)x^3 + (193765\,d^2 + 56614\,de - 213609\,e^2)x^2 + 64765\,d^2 - 39066\,de -}{196000\left(25\,x^4 + 20\,x^3 + 34\,x^2 + 12\,x + 9\right)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*ar
ctan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x +
 3) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 +
56614*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d
^2 + 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} \, dx$$
$$= \frac{4}{125} e^2 x + \frac{1}{13720000} \sqrt{14} (211875 d^2 - 342070 de + 14817 e^2) \arctan \left(\frac{1}{14} \sqrt{14} (5x+1) \right)$$
$$+ \frac{1}{1250} (40 de - 49 e^2) \log (5x^2 + 2x + 3)$$
$$+ \frac{(55075 d^2 + 363530 de - 129439 e^2) x^3 + (193765 d^2 + 56614 de - 213609 e^2) x^2 + 64765 d^2 - 39066 de -}{196000 (5x^2 + 2x + 3)^2}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*ar
ctan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x +
 3) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 +
56614*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d
^2 + 115522*d*e - 121875*e^2)*x)/(5*x^2 + 2*x + 3)^2
```

3.319.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} \, dx$$
$$= \frac{x^3 \left(\frac{55075 d^2}{1568} + \frac{181765 d e}{784} - \frac{129439 e^2}{1568} \right) + x^2 \left(\frac{193765 d^2}{1568} + \frac{28307 d e}{784} - \frac{213609 e^2}{1568} \right) - \frac{19533 d e}{784} + x \left(\frac{89895 d^2}{1568} + \frac{57761 d e}{784} - \right.}{3125 x^4 + 2500 x^3 + 4250 x^2 + 1500 x + 1125}$$
$$+ \frac{4 e^2 x}{125} + \ln (5x^2 + 2x + 3) \left(\frac{4 d e}{125} - \frac{49 e^2}{1250} \right)$$
$$+ \frac{\sqrt{14} \operatorname{atan} \left(\frac{\frac{\sqrt{14} (211875 d^2 - 342070 d e + 14817 e^2)}{13720000} + \frac{\sqrt{14} x (211875 d^2 - 342070 d e + 14817 e^2)}{2744000}}{\frac{339 d^2}{1568} - \frac{34207 d e}{98000} + \frac{14817 e^2}{980000}} \right) (211875 d^2 - 342070 d e + 14817 e^2}{13720000}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
(x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((283
07*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x
*((57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/15
68 - (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) +
(4*e^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(
1/2)*atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14
^(1/2)*x*(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 -
 (34207*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817
*e^2))/13720000
```

3.319. $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

3.320 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

3.320.1 Optimal result

Integrand size = 36, antiderivative size = 103

$$\begin{aligned}\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = &-\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}\\ &+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}\\ &+\frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}\\ &+\frac{2}{125}e\log\left(3+2x+5x^2\right)\end{aligned}$$

output
```
-1/7000*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)^2+1/196000*(34347*d-6511*e+(110
15*d+36353*e)*x)/(5*x^2+2*x+3)+2/125*e*ln(5*x^2+2*x+3)+1/2744000*(42375*d-
34207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = \frac{-6835d+1269e-2115dx-5989ex}{35000\left(3+2x+5x^2\right)^2} + \frac{171735d-44399e+55075dx+181765ex}{980000\left(3+2x+5x^2\right)} + \frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e\log\left(3+2x+5x^2\right)$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,
x]
```

output
```
(-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (17
1735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((
42375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Lo
g[3 + 2*x + 5*x^2])/125
```

3.320.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2175, 27, 2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{\left(5x^2+2x+3\right)^3}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{1}{112}\int \frac{2\left(5600ex^3+280(20d-33e)x^2-30(308d-123e)x+3267d+1367e\right)}{125\left(5x^2+2x+3\right)^2}dx - \frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5600ex^3+280(20d-33e)x^2-30(308d-123e)x+3267d+1367e}{(5x^2+2x+3)^2}dx}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{2191}$$

$$\frac{\frac{1}{56}\int \frac{10(8475d-5587e+6272ex)}{5x^2+2x+3}dx+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{5}{28}\int \frac{8475d-5587e+6272ex}{5x^2+2x+3}dx+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{\frac{5}{28}\left(\frac{1}{5}(42375d-34207e)\int \frac{1}{5x^2+2x+3}dx+\frac{3136}{5}e\int \frac{2(5x+1)}{5x^2+2x+3}dx\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{5}{28}\left(\frac{1}{5}(42375d-34207e)\int \frac{1}{5x^2+2x+3}dx+\frac{6272}{5}e\int \frac{5x+1}{5x^2+2x+3}dx\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{1083}$$

$$\frac{\frac{5}{28}\left(\frac{6272}{5}e\int \frac{5x+1}{5x^2+2x+3}dx-\frac{2}{5}(42375d-34207e)\int \frac{1}{-(10x+2)^2-56}d(10x+2)\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{\frac{5}{28}\left(\frac{6272}{5}e\int \frac{5x+1}{5x^2+2x+3}dx+\frac{\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)(42375d-34207e)}{5\sqrt{14}}\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{1103}$$

3.320. $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

$$\frac{\frac{5}{28}\left(\frac{\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)(42375d-34207e)}{5\sqrt{14}}+\frac{3136}{5}e\log\left(5x^2+2x+3\right)\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2)^2 + ((34347*d - 6511*
e + (11015*d + 36353*e)*x)/(28*(3 + 2*x + 5*x^2)) + (5*(((42375*d - 34207*
e)*ArcTan[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[14]) + (3136*e*Log[3 + 2*x + 5
*x^2])/5))/28)/7000
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.320. $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.320.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
default	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{(5x^2+2x+3)^2}+\frac{2e\ln(5x^2+2x+3)}{125}+\frac{(8475d-\frac{34207}{5}$
risch	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{(5x^2+2x+3)^2}+\frac{2e\ln(350x^2+140x+210)}{125}+\frac{339\sqrt{1}}{}$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOS
E)
```

3.320. $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

output
```
25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^
2+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^
2+2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+1/548800*(8475*d-34207/5*e)*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2))
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{70\,(11015\,d+36353\,e)x^3+14\,(193765\,d+28307\,e)x^2+\sqrt{14}(25\,(42375\,d-34207\,e)x^4+20\,(42375\,d-34\ldots}{\ldots}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fr
icas")
```

output
```
1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt
(14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*
d - 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(
1/14*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20
*e*x^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462
*e)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

3.320.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \left(\frac{2e}{125}-\frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+\left(\frac{2e}{125}+\frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}+\frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+\frac{64765d-19533e+x^3\cdot(55075d+181765e)+x^2\cdot(193765d+28307e)+x(89895d+57761e)}{4900000x^4+3920000x^3+6664000x^2+2352000x+1764000}$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
(2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207
*e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 +
 sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqr
t(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e +
 x**3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 5776
1*e))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)$$
$$+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="ma
xima")
```

output
```
1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2
/125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (19376
5*d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 +
20*x^3 + 34*x^2 + 12*x + 9)
```

3.320.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)$$
$$+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(5\,x^2+2\,x+3)^2}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gi
ac")
```

output
```
1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2
/125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (19376
5*d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2
*x + 3)^2
```

3.320.9 Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9} + \frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(42375\,d-34207\,e)}{2744000}+\frac{\sqrt{14}\,x\,(42375\,d-34207\,e)}{548800}}{\frac{339\,d}{1568}-\frac{34207\,e}{196000}}\right)(42375\,d-34207\,e)}{2744000}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200
) + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761
*e)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2
+ 3))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(
1/2)*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42
375*d - 34207*e))/2744000
```

3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$

3.321.1 Optimal result

Integrand size = 31, antiderivative size = 64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = -\frac{1367+423x}{7000\,(3+2x+5x^2)^2} + \frac{34347+11015x}{196000\,(3+2x+5x^2)} + \frac{339\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

output
```
1/7000*(-1367-423*x)/(5*x^2+2*x+3)^2+1/196000*(34347+11015*x)/(5*x^2+2*x+3
)+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{\frac{14(12953+17979x+38753x^2+11015x^3)}{(3+2x+5x^2)^2}+8475\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{548800}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

output
```
((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475
*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

3.321.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{112}\int \frac{2(5600x^2-9240x+3267)}{125(5x^2+2x+3)^2}dx-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5600x^2-9240x+3267}{(5x^2+2x+3)^2}dx}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{56}\int \frac{84750}{5x^2+2x+3}dx+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{42375}{28}\int \frac{1}{5x^2+2x+3}dx+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{11015x+34347}{28(5x^2+2x+3)}-\frac{42375}{14}\int \frac{1}{-(10x+2)^2-56}d(10x+2)}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{42375\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)}{28\sqrt{14}}+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*(1367 + 423*x)/(3 + 2*x + 5*x^2)^2 + ((34347 + 11015*x)/(28*(3 + 2
*x + 5*x^2)) + (42375*ArcTan[(2 + 10*x)/(2*Sqrt[14])])/(28*Sqrt[14]))/7000
```

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.321.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}$	47
risch	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{21952}$	47

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2
*x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{154210\,x^3 + 8475\,\sqrt{14}(25\,x^4 + 20\,x^3 + 34\,x^2 + 12\,x + 9)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x + 1)\right) + 542542\,x^2 + 251706}{548800\,(25\,x^4 + 20\,x^3 + 34\,x^2 + 12\,x + 9)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

output
```
1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
*arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4
 + 20*x^3 + 34*x^2 + 12*x + 9)
```

3.321.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx = \frac{11015x^3 + 38753x^2 + 17979x + 12953}{980000x^4 + 784000x^3 + 1332800x^2 + 470400x + 352800} + \frac{339\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14} + \frac{\sqrt{14}}{14}\right)}{21952}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
(11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 1
332800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqr
t(14)/14)/21952
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

output
```
339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 +
38753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

3.321.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\,(5\,x^2+2\,x+3)^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

output
```
339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 +
38753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

3.321.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952} + \frac{\frac{2203\,x^3}{196000}+\frac{38753\,x^2}{980000}+\frac{17979\,x}{980000}+\frac{12953}{980000}}{x^4+\frac{4\,x^3}{5}+\frac{34\,x^2}{25}+\frac{12\,x}{25}+\frac{9}{25}}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

output
```
(339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/98
0000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 +
 (34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

3.321. $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$

3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx$

3.322.1 Optimal result

Integrand size = 38, antiderivative size = 329

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx = &-\frac{1367d-293e+(423d-1367e)x}{1400\,(5d^2-2de+3e^2)\,(3+2x+5x^2)^2}\\&+\frac{171735d^3-92989d^2e+36207de^2+1831e^3+25(2203d^3-9033d^2e+3635de^2-1829e^3)\,x}{39200\,(5d^2-2de+3e^2)^2\,(3+2x+5x^2)}\\&+\frac{(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}\,(5d^2-2de+3e^2)^3}\\&+\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{(5d^2-2de+3e^2)^3}\\&-\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(3+2x+5x^2)}{2\,(5d^2-2de+3e^2)^3}\end{aligned}$$

output
```
1/1400*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^
2+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d
^2*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+e*(4*d^4+
5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^
4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/2
1952*(42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e
^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

3.322.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{\frac{392\left(5d^2-2de+3e^2\right)^2(-d(1367+423x)+e(293+1367x))}{\left(3+2x+5x^2\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(e^3(1831-45725x)+5d^3(34347+11015x)+de^2(36207+90875x)-d\right.}{3+2x+5x^2}}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),
x]
```

output
```
((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/
(3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) +
5*d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*
x)))/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*
e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 548800*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274
400*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/
(548800*(5*d^2 - 2*d*e + 3*e^2)^3)
```

3.322.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2177, 27, 2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{\left(5x^2+2x+3\right)^3(d+ex)}dx$$
$$\Big\downarrow \text{2177}$$
$$\frac{1}{112}\int \frac{2\left(1120x^2-\frac{3\left(3080d^2-809ed+481e^2\right)x}{5d^2-2ed+3e^2}+\frac{3267d^2-2843ed+2800e^2}{5d^2-2ed+3e^2}\right)}{25(d+ex)\left(5x^2+2x+3\right)^2}dx-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1120x^2-\frac{3\left(3080d^2-809ed+481e^2\right)x}{5d^2-2ed+3e^2}+\frac{3267d^2-2843ed+2800e^2}{5d^2-2ed+3e^2}}{(d+ex)(5x^2+2x+3)^2}dx}{1400}-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$\downarrow$ 2177

$$\frac{\frac{1}{56}\int \frac{50\left(8475d^4-1193ed^3+8339e^2d^2-3397e^3d+3136e^4+e\left(2203d^3-9033ed^2+3635e^2d-1829e^3\right)x\right)}{(5d^2-2ed+3e^2)^2(d+ex)(5x^2+2x+3)}dx+\frac{171735d^3-92989d^2e+25x(2203d^3-90\ldots}{28(5x^2+2x+\ldots}}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{25\int \frac{8475d^4-1193ed^3+8339e^2d^2-3397e^3d+3136e^4+e\left(2203d^3-9033ed^2+3635e^2d-1829e^3\right)x}{(d+ex)\left(5x^2+2x+3\right)}dx}{28(5d^2-2de+3e^2)^2}+\frac{171735d^3-92989d^2e+25x(2203d^3-9033d^2e+3635de^2\ldots}{28(5x^2+2x+3)(5d^2-2de+3\ldots}}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$\downarrow$ 1200

$$\frac{\frac{25\int \left(\frac{1568\left(4d^4+5ed^3+3e^2d^2-e^3d+2e^4\right)e^2}{\left(5d^2-2ed+3e^2\right)(d+ex)}+\frac{42375d^5-22915ed^4+50690e^2d^3-60762e^3d^2+33379e^4d-11759e^5-7840e\left(4d^4+5ed^3+3e^2d^2-e^3d+2e^4\right)x}{\left(5d^2-2ed+3e^2\right)\left(5x^2+2x+3\right)}\right)dx}{28(5d^2-2de+3e^2)^2}+\ldots}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$\downarrow$ 2009

$$\frac{\frac{25\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)}{\sqrt{14}\left(5d^2-2de+3e^2\right)}-\frac{784e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(5x^2+2x+3\right)}{5d^2-2de+3e^2}+\frac{1568e\left(4d^4+5d^3e+\ldots\right.}{5d\ldots}\right.}{28(5d^2-2de+3e^2)^2}}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

input

```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

3.322. $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx$

output
```
-1/1400*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3
+ 2*x + 5*x^2)^2) + ((171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 +
25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(28*(5*d^2 - 2*d*e +
 3*e^2)^2*(3 + 2*x + 5*x^2)) + (25*(((42375*d^5 - 16643*d^4*e + 58530*d^3*
e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + (1568*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2
- d*e^3 + 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2) - (784*e*(4*d^4 + 5
*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e +
 3*e^2)))/(28*(5*d^2 - 2*d*e + 3*e^2)^2))/1400
```

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.322.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.09

method	result
default	$\frac{25\left(\frac{2203}{1568}d^5-\frac{49571}{7840}d^4e+\frac{4285}{784}d^3e^2-\frac{21757}{3920}d^2e^3+\frac{14563}{7840}de^4-\frac{5487}{7840}e^5\right)x^3+25\left(\frac{38753}{7840}d^5-\frac{10433}{1568}d^4e+\frac{655359}{98000}d^3e^2-\frac{388683}{98000}d^2e^3+\frac{250589}{196000}de^4-\frac{493}{196}\right.}{}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOS
E)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^3*(25*((2203/1568*d^5-49571/7840*d^4*e+4285/784*d^3*
e^2-21757/3920*d^2*e^3+14563/7840*d*e^4-5487/7840*e^5)*x^3+(38753/7840*d^5
-10433/1568*d^4*e+655359/98000*d^3*e^2-388683/98000*d^2*e^3+250589/196000*
d*e^4-49377/196000*e^5)*x^2+(17979/7840*d^5-33127/7840*d^4*e+380997/98000*
d^3*e^2-250449/98000*d^2*e^3+147247/196000*d*e^4-11211/196000*e^5)*x+12953
/7840*d^5-11637/7840*d^4*e+118119/98000*d^3*e^2-28843/98000*d^2*e^3-25611/
196000*d*e^4+18063/196000*e^5)/(5*x^2+2*x+3)^2+1/15680*(-31360*d^4*e-39200
*d^3*e^2-23520*d^2*e^3+7840*d*e^4-15680*e^5)*ln(5*x^2+2*x+3)+1/21952*(4237
5*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*14^(1/
2)*arctan(1/28*(10*x+2)*14^(1/2)))+e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)
*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(318) = 636$.

Time = 0.39 (sec) , antiderivative size = 1052, normalized size of antiderivative = 3.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fr
icas")
```

output
```
1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
 358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2
- 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d
^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 +
5*sqrt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 +
286299*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 5
6058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e +
 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d
^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)
*x^2 + 12*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811
*d*e^4 - 8623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 8
28175*d^4*e + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*
x + 109760*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e
^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
- d*e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x)*log(e*x + d) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 +
18*e^5 + 25*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e
^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e...
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

output
```
Timed out
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.74

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(ex+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}$$
$$-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{25\,(2203\,d^3-9033\,d^2e+3635\,de^2-1829\,e^3)x^3+64765\,d^3-32279\,d^2e-452}{7840\,(25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^4+225\,d^4-180\,d^3e+306\,d^2e^2-108\,de^3+81\,e^4+}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="ma
xima")
```

output
```
1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3
+ 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2
*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(1
25*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 +
27*e^6) + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 +
 64765*d^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^
2*e + 72557*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e
^2 - 3737*e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*
x^4 + 225*d^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e +
34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.50

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(4\,d^4e^2+5\,d^3e^3+3\,d^2e^4-de^5+2\,e^6)\log\left(|ex+d|\right)}{125\,d^6e-150\,d^5e^2+285\,d^4e^3-188\,d^3e^4+171\,d^2e^5-54\,de^6+27\,e^7}$$
$$+\frac{323825\,d^5-290925\,d^4e+236238\,d^3e^2-57686\,d^2e^3-25611\,de^4+18063\,e^5+25\,(11015\,d^5-49571\,d^4e}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gi
ac")
```

output
```
1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3
+ 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*
(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(12
5*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 2
7*e^6) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(e*x +
 d))/(125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 -
54*d*e^6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 236238*d^3*e^2 -
57686*d^2*e^3 - 25611*d*e^4 + 18063*e^5 + 25*(11015*d^5 - 49571*d^4*e + 42
850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + (968825*d^5 -
1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^
5)*x^2 + (449475*d^5 - 828175*d^4*e + 761994*d^3*e^2 - 500898*d^2*e^3 + 14
7247*d*e^4 - 11211*e^5)*x)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$

$$= \frac{\frac{x\,(89895\,d^3-129677\,d^2\,e+46591\,d\,e^2-3737\,e^3)}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} - \frac{-64765\,d^3+32279\,d^2\,e+4523\,d\,e^2-6021\,e^3}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} + \frac{5\,x^3\,(2203\,d^3-9033\,d^2\,e+3635\,d\,e^2-1829\,e^3)}{1568\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}$$

$$+\ln(d+e\,x)\left(\frac{4\,e}{25\,(5\,d^2-2\,d\,e+3\,e^2)}+\frac{e^2\,(205\,d+21\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^2} - \frac{e^4\,(458\,d-7\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3}\right)$$

$$-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}+2\mathrm{i}\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}+\frac{5}{2}\mathrm{i}\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}+\frac{3}{2}\mathrm{i}\right)e\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,5}$$

$$+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(-\frac{42375\,\sqrt{14}\,d^5}{43904}+\left(\frac{16643\,\sqrt{14}}{43904}-2\mathrm{i}\right)d^4\,e+\left(-\frac{29265\,\sqrt{14}}{21952}-\frac{5}{2}\mathrm{i}\right)d^3\,e^2+\left(\frac{28029\,\sqrt{14}}{21952}-\frac{3}{2}\mathrm{i}\right)e\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,5}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

output
```
((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 2
0*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 64
765*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e
^2)) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d
^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 1833
19*d^2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 +
9*e^4 + 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)
*((4*e)/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 -
2*d*e + 3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) -
 (log(x - (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (4237
5*14^(1/2)*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*
((29265*14^(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*
e^4*((31811*14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i +
 e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2
)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43
904 + d^2*e^3*((28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/
21952 + 5i/2) + d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/
2))/43904 - 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4
*171i - d^3*e^3*188i + d^4*e^2*285i)
```

3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx$

3.323.1 Optimal result

Integrand size = 38, antiderivative size = 443

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx &= -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{(5d^2-2de+3e^2)^3(d+ex)} \\ &\quad - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{280(5d^2-2de+3e^2)^2(3+2x+5x^2)^2} \\ &\quad + \frac{171735d^4-117284d^3e-200502d^2e^2+104428de^3-23189e^4+5(11015d^4-85924d^3e+34698d^2e^2+1}{7840(5d^2-2de+3e^2)^3(3+2x+5x^2)} \\ &\quad + \frac{(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}(5d^2-2de+3e^2)^4} \\ &\quad + \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4} \\ &\quad - \frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^4}\end{aligned}$$

output
```
-e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+1/2
80*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3
*e^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+104
428*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-358
9*e^4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+e*(40*d^5+83*d^4*e+12*d^3*e^
2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*e*(40*d^5
+83*d^4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d
*e+3*e^2)^4+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+3
80621*d^2*e^4-49586*d*e^5-43695*e^6)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-
2*d*e+3*e^2)^4*14^(1/2)
```

3.323.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{-\frac{109760e(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{d+ex} - \frac{392(5d^2-2de+3e^2)^2(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x))}{(3+2x+5x^2)^2} + 14}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3
),x]
```

output
```
((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) +
d^2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d
^2 - 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) -
 e^4*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 1
07405*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 20903
9*d^4*e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*Arc
Tan[(1 + 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^
2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d
^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d
^2 - 2*d*e + 3*e^2)^4)
```

3.323.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2177, 27, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^3(d+ex)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{112}\int \frac{2\left(\frac{(5600d^4-4480ed^3+6347e^2d^2+5514e^3d+1137e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(4620d^4-2427ed^3+646e^2d^2-1417e^3d+140e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{3267d^4-5686ed^3+7577}{(5d^2-2e}\right.}{5(d+ex)^2(5x^2+2x+3)^2}$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\int \frac{\frac{(5600d^4-4480ed^3+6347e^2d^2+5514e^3d+1137e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(4620d^4-2427ed^3+646e^2d^2-1417e^3d+140e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{3267d^4-5686ed^3+7577e^2d}{(5d^2-2ed+}}{(d+ex)^2(5x^2+2x+3)^2}$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{280}\left(\frac{1}{56}\int \frac{10\left(\frac{e^2(11015d^4-85924ed^3+34698e^2d^2+10348e^3d-3589e^4)x^2}{(5d^2-2ed+3e^2)^3}+\frac{2e(11015d^5-53780ed^4+28426e^2d^3-36692e^3d^2+15227e^4d-39}{(5d^2-2ed+3e^2)^3}\right.}{(d+ex)^2(5x^2+2x+3)}\right.$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{5}{28}\int \frac{\frac{e^2(11015d^4-85924ed^3+34698e^2d^2+10348e^3d-3589e^4)x^2}{(5d^2-2ed+3e^2)^3}+\frac{2e(11015d^5-53780ed^4+28426e^2d^3-36692e^3d^2+15227e^4d-3920e^5}{(5d^2-2ed+3e^2)^3}}{(d+ex)^2(5x^2+2x+3)}\right.$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{280}\left(\frac{5}{28}\int\left(-\frac{1568\left(-40d^5-83ed^4-12e^2d^3+76e^3d^2-46e^4d+9e^5\right)e^2}{\left(5d^2-2ed+3e^2\right)^4(d+ex)}+\frac{1568\left(4d^4+5ed^3+3e^2d^2-e^3d+2e\right.}{\left(5d^2-2ed+3e^2\right)^3(d+ex)^2}\right.\right.$$
$$\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{280\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{280}\left(\frac{5}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6\right)}{\sqrt{14}\left(5d^2-2de+3e^2\right)^4}\right.\right.$$
$$\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{280\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

output
```
-1/280*(1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e^2)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + ((171735*d^4 - 117284*d^3
*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 - 85924*d^3*
e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(28*(5*d^2 - 2*d*e + 3*e^2)
^3*(3 + 2*x + 5*x^2)) + (5*((-1568*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4))/((5*d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) + ((211875*d^6 + 3070*d^5*
e + 209039*d^4*e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695
*e^6)*ArcTan[(1 + 5*x)/Sqrt[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (
1568*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Lo
g[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - (784*e*(40*d^5 + 83*d^4*e + 12*d^3
*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e
 + 3*e^2)^4))/28)/280
```

3.323.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.323.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.07

method	result
default	$\frac{25\left(\frac{11015}{1568}d^6-\frac{45165}{784}d^5e+\frac{378383}{7840}d^4e^2-\frac{68857}{1960}d^3e^3+\frac{65453}{7840}d^2e^4+\frac{19111}{3920}d\,e^5-\frac{10767}{7840}e^6\right)x^3+25\left(\frac{38753}{1568}d^6-\frac{183319}{3920}d^5e+\frac{504029}{39200}d^4e^2+\frac{5109}{9800}d^3e^3-\right.}{}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^4*(25*((11015/1568*d^6-45165/784*d^5*e+378383/7840*d
^4*e^2-68857/1960*d^3*e^3+65453/7840*d^2*e^4+19111/3920*d*e^5-10767/7840*e
^6)*x^3+(38753/1568*d^6-183319/3920*d^5*e+504029/39200*d^4*e^2+5109/9800*d
^3*e^3-795401/39200*d^2*e^4+218053/19600*d*e^5-91101/39200*e^6)*x^2+(17979
/1568*d^6-129677/3920*d^5*e+606287/39200*d^4*e^2-3993/9800*d^3*e^3-86999/7
840*d^2*e^4+208007/19600*d*e^5-14979/7840*e^6)*x+12953/1568*d^6-32279/3920
*d^5*e-379131/39200*d^4*e^2+116869/9800*d^3*e^3-530209/39200*d^2*e^4+19809
/3920*d*e^5-6309/39200*e^6)/(5*x^2+2*x+3)^2+1/15680*(-313600*d^5*e-650720*
d^4*e^2-94080*d^3*e^3+595840*d^2*e^4-360640*d*e^5+70560*e^6)*ln(5*x^2+2*x+
3)+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^2
*e^4-49586*d*e^5-43695*e^6)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))+e*(40
*d^5+83*d^4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e
+3*e^2)^4-e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e
*x+d)
```

3.323.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. $2(432) = 864$.

Time = 0.52 (sec) , antiderivative size = 1734, normalized size of antiderivative = 3.91

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
 8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(101
725*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e
^5 - 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 47439
5*d^5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*
e^6 - 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 -
 279581*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 39476
7*e^7)*x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 82929
96*d^4*e^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*
d^6*e + 3070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 -
49586*d*e^6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5
*e^2 - 3771064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6
- 174780*e^7)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 566
0777*d^4*e^3 - 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815
*e^7)*x^3 + 2*(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^
4*e^3 + 941893*d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2
 + 3*(847500*d^7 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 12418
48*d^3*e^4 + 943519*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sq
rt(14)*(5*x + 1)) + 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325
*d^4*e^3 - 14261*d^3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x ...
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

output
```
Timed out
```

3.323.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(432) = 864$.

Time = 0.31 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.07

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx$$
$$= \frac{\sqrt{14}(211875\,d^6+3070\,d^5e+209039\,d^4e^2-921444\,d^3e^3+380621\,d^2e^4-49586\,de^5-43695\,e^6)\arctan\left(\frac{1}{14}\right.}{21952\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+}$$
$$+\frac{(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6)\log(ex+d)}{625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8}$$
$$-\frac{(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6)\log(5\,x^2+2\,x+3)}{2\,(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8)}$$
$$+\frac{64765\,d^5-95100\,d^4e-200706\,d^3e^2+}{1568\,(1125\,d^7-1350\,d^6e+2565\,d^5e^2-1692\,d^4e^3+1539\,d^3e^4-486\,d^2e^5+243\,de^6+25\,(125\,d^6e-150}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^
3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x +
1))/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1
176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 +
 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*
d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^
2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76
*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e +
2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
216*d*e^7 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 2
2292*d^2*e^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 -
 11178*d^2*e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e
- 272442*d^3*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (19376
5*d^5 - 412485*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131
589*e^5)*x^2 + 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3
+ 19493*d*e^4 - 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692
*d^4*e^3 + 1539*d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^
5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 +
 5*(625*d^7 - 250*d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^
2*e^5 - 81*d*e^6 + 108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2...
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.82

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx =$$

$$-\frac{\left(40\,d^5e+83\,d^4e^2+12\,d^3e^3-76\,d^2e^4+46\,de^5-9\,e^6\right)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+\right.}{2\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}$$

$$-\frac{\frac{4\,d^4e^7}{ex+d}+\frac{5\,d^3e^8}{ex+d}+\frac{3\,d^2e^9}{ex+d}-\frac{de^{10}}{ex+d}+\frac{2\,e^{11}}{ex+d}}{125\,d^6e^6-150\,d^5e^7+285\,d^4e^8-188\,d^3e^9+171\,d^2e^{10}-54\,de^{11}+27\,e^{12}}$$

$$+\frac{\sqrt{14}\left(211875\,d^6e^2+3070\,d^5e^3+209039\,d^4e^4-921444\,d^3e^5+380621\,d^2e^6-49586\,de^7-43695\,e^8\right)\text{arcta}}{21952\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-2\right.}$$

$$+\frac{275375\,d^5e-3006775\,d^4e^2+1394650\,d^3e^3+1835350\,d^2e^4-734925\,de^5+17525\,e^6-\frac{5\left(165225\,d^6e^2-19978\right.}{}}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
-1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*
log(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^
2 + 3*e^2/(e*x + d)^2 + 5)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5
*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (
4*d^4*e^7/(e*x + d) + 5*d^3*e^8/(e*x + d) + 3*d^2*e^9/(e*x + d) - d*e^10/(
e*x + d) + 2*e^11/(e*x + d))/(125*d^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 18
8*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11 + 27*e^12) + 1/21952*sqrt(14)*(211875
*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d^3*e^5 + 380621*d^2*e^6
 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) +
2*d*e/(e*x + d) - e - 3*e^2/(e*x + d))/e)/((625*d^8 - 1000*d^7*e + 2100*d^
6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e
^7 + 81*e^8)*e^2) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e
^3 + 1835350*d^2*e^4 - 734925*d*e^5 + 17525*e^6 - 5*(165225*d^6*e^2 - 1997
830*d^5*e^3 + 1218421*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d
*e^7 + 7963*e^8)/((e*x + d)*e) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449
735*d^5*e^5 + 8211175*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947
*d*e^9 - 67267*e^10)/((e*x + d)^2*e^2) - (275375*d^8*e^4 - 3975600*d^7*e^5
 + 3752280*d^6*e^6 + 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 -
 1453312*d^2*e^10 - 197784*d*e^11 + 66483*e^12)/((e*x + d)^3*e^3))/((5*d^2
 - 2*d*e + 3*e^2)^4*(10*d/(e*x + d) - 5*d^2/(e*x + d)^2 - 2*e/(e*x + d)...
```

3.323.9 Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 965, normalized size of antiderivative = 2.18

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

output
```
log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (
6*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))
/(5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965
*d^5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^
5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (120
09*d*e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^
3*e^2)/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*
d^3*e^3 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 -
11178*d^2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 2
7*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 1
38539*d*e^4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1
568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 +
 285*d^4*e^2)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e
^5 + 56850*d^2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^
5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d +
 12*e) + x^4*(25*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)
) + (log(x - (14^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((4
3695*14^(1/2))/43904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^
4*e^2*((209039*14^(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904
 + 38i) + d^5*e*((1535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))...
```

3.324 $\int (5+2x)\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

3.324.1 Optimal result

Integrand size = 38, antiderivative size = 143

$$\begin{aligned}
&\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx \\
&= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480} \\
&\quad -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2} \\
&\quad -\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}-\frac{1183005\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}
\end{aligned}$$

output
```
11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2
)+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)-1/71680*(1005757+295276*x)*(2*x^2-x+3)
^(3/2)-1183005/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-51435/32768*(
1-4*x)*(2*x^2-x+3)^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{4\sqrt{3-x+2x^2}(6231117+14742332x+11357024x^2+20304768x^3+1390592x^4+12984320x^5+491520\ldots}{13762560}$$

input
```
Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
 + 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/13762560
```

3.324.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2184, 25, 2184, 27, 2184, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+5)\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$
$$\Big\downarrow \text{2184}$$
$$\frac{1}{224}\int -\left((2x+5)\sqrt{2x^2-x+3}\left(6584x^3+10788x^2+7826x+3677\right)\right)dx+\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$
$$\Big\downarrow \text{25}$$
$$\frac{5}{112}(2x+5)^4\left(2x^2-x+3\right)^{3/2}-\frac{1}{224}\int (2x+5)\sqrt{2x^2-x+3}\left(6584x^3+10788x^2+7826x+3677\right)dx$$
$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\left(-\frac{1}{96}\int -24(2x+5)\sqrt{2x^2-x+3}\left(45732x^2+37828x+14097\right)dx-\frac{823}{6}\left(2x^2-x+3\right)^{3/2}(2x+5)^3\right)+\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(\frac{1}{4}\int (2x+5)\sqrt{2x^2-x+3}\left(45732x^2+37828x+14097\right)dx-\frac{823}{6}(2x+5)^3\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{40}\int 4(38073-147638x)(2x+5)\sqrt{2x^2-x+3}dx+\frac{11433}{5}\left(2x^2-x+3\right)^{3/2}(2x+5)^2\right)-\frac{823}{6}(2x+5)^3\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\int (38073-147638x)(2x+5)\sqrt{2x^2-x+3}dx+\frac{11433}{5}\left(2x^2-x+3\right)^{3/2}(2x+5)^2\right)-\frac{823}{6}(2x+5)^3\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\int \sqrt{2x^2-x+3}dx-\frac{1}{8}(295276x+1005757)\left(2x^2-x+3\right)^{3/2}\right)+\frac{11433}{5}\left(2x^2-x+3\right)^{3/2}\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+1005757)\left(2x^2-x+\right.\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+10\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

3.324. $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

$$\Big\downarrow \; 222$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+1005757)\left(2x^2-x+3\right)^3\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

input
```
Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*(5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 + ((-823*(5 + 2*x)^3*(3 - x + 2*
x^2)^(3/2))/6 + ((11433*(5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))/5 + (-1/8*((100
5757 + 295276*x)*(3 - x + 2*x^2)^(3/2)) + (1800225*(-1/8*((1 - 4*x)*Sqrt[3
 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/16)/10)/
4)/224
```

3.324.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.324. $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.324.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117)\sqrt{2x^2-x+3}}{3440640}+\frac{1183005\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}}{\cdots}\right.}{131072}$
trager	$\left(\frac{10}{7}x^6+\frac{317}{84}x^5+\frac{97}{240}x^4+\frac{52877}{8960}x^3+\frac{50701}{15360}x^2+\frac{3685583}{860160}x+\frac{2077039}{1146880}\right)\sqrt{2x^2-x+3}-\frac{1183005\,\mathrm{RootOf}\left(\_\cdots\right.}{}$
default	$\frac{51435(-1+4x)\sqrt{2x^2-x+3}}{32768}+\frac{1183005\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}+\frac{283x^2(2x^2-x+3)^{\frac{3}{2}}}{1120}-\frac{5179x(2x^2-x+3)^{\frac{3}{2}}}{17920}+\frac{242329(2x\cdots}{2150\cdots}$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3440640*(4915200*x^6+12984320*x^5+1390592*x^4+20304768*x^3+11357024*x^2+
14742332*x+6231117)*(2*x^2-x+3)^(1/2)+1183005/131072*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{3440640}\left(4915200\,x^6+12984320\,x^5+1390592\,x^4+20304768\,x^3+11357024\,x^2+14742332\,x+6231117\right.$$
$$+\frac{1183005}{262144}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fr
icas")
```

output
```
1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 11357
024*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(
2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.324.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\sqrt{2x^2-x+3}\cdot\left(\frac{10x^6}{7}+\frac{317x^5}{84}+\frac{97x^4}{240}+\frac{52877x^3}{8960}+\frac{50701x^2}{15360}+\frac{3685583x}{860160}+\frac{2077039}{1146880}\right)$$
$$+\frac{1183005\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(10*x**6/7 + 317*x**5/84 + 97*x**4/240 + 52877*x**3/8
960 + 50701*x**2/15360 + 3685583*x/860160 + 2077039/1146880) + 1183005*sqr
t(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/131072
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$= \frac{5}{7}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4 + \frac{377}{168}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3 + \frac{283}{1120}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2 - \frac{5179}{17920}\left(2x^2-x+3\right)^{\frac{3}{2}}x + \frac{242329}{215040}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{51435}{8192}\sqrt{2x^2-x+3}x + \frac{1183005}{131072}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{51435}{32768}\sqrt{2x^2-x+3}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="ma
xima")
```

output
```
5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/11
20*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329
/215040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005
/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2
- x + 3)
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$= \frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2x^2-} - \frac{1183005}{131072}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gi
ac")
```

output
```
1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*
x + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log
(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx\\
&=\frac{283\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{1120}+\frac{377\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{168}\\
&\quad+\frac{5\,x^4\,{\left(2\,x^2-x+3\right)}^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{573440}\\
&\quad+\frac{194737\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{17920}+\frac{242329\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{3440640}\\
&\quad-\frac{5179\,x\,{\left(2\,x^2-x+3\right)}^{3/2}}{17920}+\frac{5573567\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{4587520}
\end{aligned}
$$

input
```
int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
(283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
 + (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^
(1/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x
+ 3)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/344
0640 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^
2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

3.325 $\int \sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

3.325.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71x(3-x+2x^2)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}-\frac{106007\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

output
```
287/5120*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3
)^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-106007/65536*arcsinh(1/23*(1-4*x)*23^(1
/2))*2^(1/2)-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.325.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{4\sqrt{3-x+2x^2}(-27807+221868x+105696x^2+258432x^3-59392x^4+204800x^5)-1590105\sqrt{2}\log\left(1\right.}{983040}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 5939
2*x^4 + 204800*x^5) - 1590105*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/983040
```

3.325.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(7x^3-6x^2+8x+16\right)dx+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(7x^3-6x^2+8x+16\right)dx+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{10}\int \frac{1}{2}\left(-71x^2+76x+320\right)\sqrt{2x^2-x+3}dx+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{20}\int \left(-71x^2+76x+320\right)\sqrt{2x^2-x+3}dx+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int \frac{1}{2}(861x+5546)\sqrt{2x^2-x+3}dx-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\int (861x+5546)\sqrt{2x^2-x+3}dx-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \textbf{1160}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\int \sqrt{2x^2-x+3}dx+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \textbf{1087}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-x\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \textbf{1090}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{3/\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \textbf{222}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-x+\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((7*x^2*(3 - x + 2*x^2)^(3/2))/10 + ((-
71*x*(3 - x + 2*x^2)^(3/2))/8 + ((287*(3 - x + 2*x^2)^(3/2))/2 + (23045*(-
1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(1
6*Sqrt[2])))/4)/16)/20)/8
```

3.325. $\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\, dx$

3.325.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.325. $\int \sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

3.325.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807)\sqrt{2x^2-x+3}}{245760}+\frac{106007\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{5}{6}x^5-\frac{29}{120}x^4+\frac{673}{640}x^3+\frac{1101}{2560}x^2+\frac{18489}{20480}x-\frac{9269}{81920}\right)\sqrt{2x^2-x+3}+\frac{106007\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_\right.\right.}{}$
default	$\frac{287(2x^2-x+3)^{\frac{3}{2}}}{5120}+\frac{4609(-1+4x)\sqrt{2x^2-x+3}}{16384}+\frac{106007\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}+\frac{5x^3(2x^2-x+3)^{\frac{3}{2}}}{12}+\frac{7x^2(2x^2-x+3)^{\frac{3}{2}}}{80}$

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/245760*(204800*x^5-59392*x^4+258432*x^3+105696*x^2+221868*x-27807)*(2*x^
2-x+3)^(1/2)+106007/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{245760}\left(204800\,x^5-59392\,x^4+258432\,x^3+105696\,x^2+221868\,x-27807\right)\sqrt{2\,x^2-x+3}+\frac{106007}{131072}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 27
807)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.325.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^5}{6}-\frac{29x^4}{120}+\frac{673x^3}{640}+\frac{1101x^2}{2560}+\frac{18489x}{20480}-\frac{9269}{81920}\right)+\frac{106007\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**5/6 - 29*x**4/120 + 673*x**3/640 + 1101*x**2/25
60 + 18489*x/20480 - 9269/81920) + 106007*sqrt(2)*asinh(4*sqrt(23)*(x - 1/
4)/23)/65536
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= \frac{5}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2x^2-x+3\right)^{\frac{3}{2}}x\\&\quad+\frac{287}{5120}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2x^2-x+3}x\\&\quad+\frac{106007}{65536}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4609}{16384}\sqrt{2x^2-x+3}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*
(2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(
2*x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 4609/16384*sqrt(2*x^2 - x + 3)
```

3.325.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right) dx$$
$$= \frac{1}{245760}\left(4\left(8\left(4\left(16\left(100x-29\right)x+2019\right)x+3303\right)x+55467\right)x-27807\right)\sqrt{2x^2-x+3} - \frac{106007}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 2780
7)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3)) + 1)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 13.96 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right) dx$$
$$= \frac{7x^2\left(2x^2-x+3\right)^{3/2}}{80} + \frac{5x^3\left(2x^2-x+3\right)^{3/2}}{12} + \frac{63779\sqrt{2}\ln\left(\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(2x-\frac{1}{2}\right)}{2}\right)}{40960} + \frac{2773\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2x^2-x+3}}{1280} + \frac{287\sqrt{2x^2-x+3}\left(32x^2-4x+45\right)}{81920} - \frac{71x\left(2x^2-x+3\right)^{3/2}}{1280} + \frac{19803\sqrt{2}\ln\left(2\sqrt{2x^2-x+3}+\frac{\sqrt{2}\left(4x-1\right)}{2}\right)}{327680}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
(7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (637
79*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (
2773*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*
(32*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^
(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

3.326 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

3.326.1 Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096} + \frac{4535}{768}\left(3-x+2x^2\right)^{3/2} - \frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2} + \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2} + \frac{5627989\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}} - \frac{11001\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{16\sqrt{2}}$$

output
```
4535/768*(2*x^2-x+3)^(3/2)-127/128*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^
2*(2*x^2-x+3)^(3/2)+5627989/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1
1001/32*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/4096*(
489587-80844*x)*(2*x^2-x+3)^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$= \frac{4\sqrt{3-x+2x^2}(1561161-300404x+79840x^2-21120x^3+6144x^4)+33795072\sqrt{2}\text{arctanh}\left(\frac{1}{6}(5+2x-\right.}{49152}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*
x^4) + 33795072*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 168
83967*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/49152
```

3.326.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2184, 27, 2184, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{160}\int -\frac{5\sqrt{2x^2-x+3}(1016x^3+1860x^2+1298x+161)}{2x+5}dx+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}(2x+5)^2\left(2x^2-x+3\right)^{3/2}-\frac{1}{32}\int \frac{\sqrt{2x^2-x+3}(1016x^3+1860x^2+1298x+161)}{2x+5}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{32}\left(-\frac{1}{64}\int \frac{8(-18140x^2-20604x+3193)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(-\frac{1}{8}\int \frac{\left(-18140x^2-20604x+3193\right)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}-\frac{1}{24}\int -\frac{12(16289-40422x)\sqrt{2x^2-x+3}}{2x+5}dx\right)-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{(16289-40422x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}\right)-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{8}(489587-80844x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(5655127-11255978x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}\right)\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\int \frac{5655127-11255978x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(489587-80844x)\right)+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}\right)-\frac{12}{4}\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-5627989\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x+3}(489587\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \text{1090}$$

3.326. $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{5627989\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(\right.\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(489587-\right.\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 1154

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(-67590144\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(48\right.\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 219

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-2816256\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2-x+3}(489\right.\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

output
```
((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))/16 + ((-127*(5 + 2*x)*(3 - x + 2*x^2)^
(3/2))/4 + ((4535*(3 - x + 2*x^2)^(3/2))/3 + (((489587 - 80844*x)*Sqrt[3 -
 x + 2*x^2])/8 + ((-5627989*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 281625
6*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16)/2)/8)
/32
```

3.326. $\int\frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

3.326.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.326. $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.326.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$-\frac{20211(4x-1)\sqrt{2x^2-x+3}}{4096}-\frac{5627989\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{16384}+\frac{1925(2x^2-x+3)^{\frac{3}{2}}}{768}+\frac{x^2(2x^2-x+3)^{\frac{3}{2}}}{4}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

output
```
-20211/4096*(4*x-1)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/23*2
3^(1/2)*(x-1/4))+1925/768*(2*x^2-x+3)^(3/2)+1/4*x^2*(2*x^2-x+3)^(3/2)-47/6
4*x*(2*x^2-x+3)^(3/2)+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/
2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx \\
&= \frac{1}{12288}\left(6144\,x^4-21120\,x^3+79840\,x^2-300404\,x+1561161\right)\sqrt{2\,x^2-x+3} \\
&\quad+\frac{5627989}{32768}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right) \\
&\quad+\frac{11001}{64}\,\sqrt{2}\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)
\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fr
icas")
```

output
```
1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
 - x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

3.326.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5),
x)
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{1}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{47}{64}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x \\ &+\frac{1925}{768}\left(2\,x^2-x+3\right)^{\frac{3}{2}} \\ &-\frac{20211}{1024}\sqrt{2\,x^2-x+3}x \\ &-\frac{5627989}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) \\ &+\frac{11001}{32}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}\right. \\ &\left.-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{489587}{4096}\sqrt{2\,x^2-x+3}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="ma
xima")
```

output
```
1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(
2*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sq
rt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(
22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096
*sqrt(2*x^2 - x + 3)
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{1}{12288}\left(4\left(8\left(12\left(16\,x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2\,x^2-x+3} \\ &\quad+\frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) \\ &\quad-\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) \\ &\quad+\frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gi
ac")
```

output
```
1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
 - x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^
2 - x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3)))
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{2\,x+5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.327 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx$

3.327.1 Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2} dx = -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} - \frac{2551847\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}} + \frac{239201\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{384\sqrt{2}}$$

output
```
-541/384*(2*x^2-x+3)^(3/2)-3667/576*(2*x^2-x+3)^(3/2)/(5+2*x)+5/64*(5+2*x)
*(2*x^2-x+3)^(3/2)-2551847/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+239
201/768*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/18432*
(1996953-333380*x)*(2*x^2-x+3)^(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$=\frac{\frac{4\sqrt{3-x+2x^2}(-3539439-728410x+94936x^2-17344x^3+3840x^4)}{5+2x}-15308864\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-7}{24576}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,
x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 384
0*x^4))/(5 + 2*x) - 15308864*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x
^2])/6] - 7655541*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/24576
```

3.327.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {2181, 27, 2184, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^2}\,dx$$
$$\Big\downarrow \text{2181}$$
$$-\frac{1}{72}\int \frac{\sqrt{2x^2-x+3}\left(-2880x^3+7776x^2-50504x+19341\right)}{16(2x+5)}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}\left(-2880x^3+7776x^2-50504x+19341\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$
$$\Big\downarrow \text{2184}$$
$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int \frac{128\sqrt{2x^2-x+3}\left(9738x^2-19762x+9333\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 27$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\int \frac{\sqrt{2x^2-x+3}\left(9738x^2-19762x+9333\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 2184$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{24}\int \frac{6(61677-166690x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1623}{2}\left(2x^2-x+3\right)^{3/2}\right)}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 27$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\int \frac{(61677-166690x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1623}{2}\left(2x^2-x+3\right)^{3/2}\right)}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 1231$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{1}{8}(1996953-333380x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{18(2549629-5103694x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1623}{2}\left(\right.\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 27$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\int \frac{2549629-5103694x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(1996953-333380x)\right)+\frac{1623}{2}\left(2x^2-\right.\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 1269$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2551847\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x+\right.\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \ 1090$$

3.327. $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2551847\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 222

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 1154

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(-30617728\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 219

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{3827216}{3}\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (90*(5 + 2*x)*(3 - x + 2*x
^2)^(3/2) - 2*((1623*(3 - x + 2*x^2)^(3/2))/2 + (((1996953 - 333380*x)*Sqr
t[3 - x + 2*x^2])/8 + (9*((-2551847*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2]
- (3827216*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/
3))/16)/4))/1152
```

3.327. $\int\frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

3.327.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.327. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.327.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$

$$= \frac{7655541\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+7654432\sqrt{2}(2x+5)\log(}{49}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="
fricas")
```

output
```
1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x +
25)) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^
2 - x + 3))/(2*x + 5)
```

3.327.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
2, x)
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx &= \frac{5}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}} \\ &\quad + \frac{6001}{512}\sqrt{2x^2-x+3}x \\ &\quad + \frac{2551847}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &\quad - \frac{239201}{768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) - \frac{182769}{2048}\sqrt{2x^2-x+3} \\ &\quad - \frac{3667\sqrt{2x^2-x+3}}{32\,(2x+5)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="
maxima")
```

output
```
5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sq
rt(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt
(2*x^2 - x + 3)/(2*x + 5)
```

3.327.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{3 - x + 2x^2}(2 + x + 3x^2 - x^3 + 5x^4)}{(5 + 2x)^2}\,dx$$

$$= \frac{1}{24576}\sqrt{2}\left(7654432 \log\left(12\sqrt{-\frac{11}{2x+5} + \frac{36}{(2x+5)^2} + 1} + \frac{72}{2x+5} - 11\right)\mathrm{sgn}\left(\frac{1}{2x+5}\right) + 7655541 \log\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="
giac")
```

output
```
1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(a
bs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x
 + 5)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5))
 + 2*(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*
sgn(1/(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/
(2*x + 5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 319219
20*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x +
 5)) - 2445813*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sg
n(1/(2*x + 5)) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x
+ 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^4)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.328 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

3.328.1 Optimal result

Integrand size = 40, antiderivative size = 151

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944} + \frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2} + \frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{117315\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} - \frac{12670805\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{55296\sqrt{2}}$$

output
```
5/48*(2*x^2-x+3)^(3/2)-3667/1152*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391/82944*
(2*x^2-x+3)^(3/2)/(5+2*x)+117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)-12670805/110592*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)+5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(4880551+2959330x+272520x^2-25632x^3+3840x^4)}{(5+2x)^2}+12670805\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+}{55296}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 3
840*x^4))/(5 + 2*x)^2 + 12670805*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x +
 4*x^2])/6] + 6335010*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/5529
6
```

3.328.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{1}{144}\int \frac{\sqrt{2x^2-x+3}(-5760x^3+15552x^2-57004x+27681)}{16(2x+5)^2}dx-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-5760x^3+15552x^2-57004x+27681)}{(2x+5)^2}dx}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{72}\int \frac{5\sqrt{2x^2-x+3}(41472x^2-787480x+306261)}{2x+5}dx+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{5}{72}\int \frac{\sqrt{2x^2-x+3}(41472x^2-787480x+306261)}{2x+5}dx+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 2184

$$\frac{\frac{5}{72}\left(\frac{1}{24}\int \frac{24(332181-880792x)\sqrt{2x^2-x+3}}{2x+5}dx+3456(2x^2-x+3)^{3/2}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{5}{72}\left(\int \frac{(332181-880792x)\sqrt{2x^2-x+3}}{2x+5}dx+3456(2x^2-x+3)^{3/2}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1231

$$\frac{\frac{5}{72}\left(-\frac{1}{32}\int -\frac{576(422491-844668x)}{(2x+5)\sqrt{2x^2-x+3}}dx+3456(2x^2-x+3)^{3/2}+2(661065-110099x)\sqrt{2x^2-x+3}\right)+\frac{357391(2x^2-x+3}{36(2x+5)}}{2304}$$

$$\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{5}{72}\left(18\int \frac{422491-844668x}{(2x+5)\sqrt{2x^2-x+3}}dx+3456(2x^2-x+3)^{3/2}+2(661065-110099x)\sqrt{2x^2-x+3}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-$$

$$\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1269

$$\frac{\frac{5}{72}\left(18\left(2534161\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-422334\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+3456(2x^2-x+3)^{3/2}+2(661065-110099x)\sqrt{2}\right.}{2304}$$

$$\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

3.328. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

$\Big\downarrow$ 1090

$$\frac{\frac{5}{72}\left(18\left(2534161\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-211167\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(66106\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 222

$$\frac{\frac{5}{72}\left(18\left(2534161\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-110099x\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{5}{72}\left(18\left(-5068322\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{5}{72}\left(18\left(-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{2534161\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-110\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + ((357391*(3 - x + 2*x^2
)^(3/2))/(36*(5 + 2*x)) + (5*(2*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2] +
3456*(3 - x + 2*x^2)^(3/2) + 18*(-211167*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[2
3]] - (2534161*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*S
qrt[2]))))/72)/2304
```

3.328. $\int\frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

3.328.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.328. $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.328.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$

$$= \frac{12670020\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+12670805\sqrt{2}(4x}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="
fricas")
```

output
```
1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x +
25)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x +
 1153)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959
330*x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

3.328.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
3, x)
```

3.328.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{5}{48}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{149}{64}\sqrt{2\,x^2-x+3}x - \frac{117315}{1024}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{12670805}{110592}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{3877}{144}\sqrt{2\,x^2-x+3} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{1152\left(4\,x^2+20\,x+25\right)} + \frac{357391\sqrt{2\,x^2-x+3}}{4608\left(2\,x+5\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="
maxima")
```

output
```
5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sq
rt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*a
rcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877
/144*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x +
 25) + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

3.328.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{1}{768}\,(4\,(40\,x-467)x+19695)\sqrt{2\,x^2-x+3} + \frac{117315}{1024}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) - \frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{\sqrt{2}\left(10693526\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3+79895946\,\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2-124044603\,\sqrt{2}\left(\sqrt{2}x\right.\right.}{9216\left(2\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="
giac")
```

output
```
1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)
*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*s
qrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805
/110592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 +
 79895946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.329 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

3.329.1 Optimal result

Integrand size = 40, antiderivative size = 158

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968}-\frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3}+\frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2}-\frac{6467659(3-x+2x^2)^{3/2}}{5971968(5+2x)}-\frac{10939\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}+\frac{170114729\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3981312\sqrt{2}}$$

output
```
-3667/1728*(2*x^2-x+3)^(3/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2
*x)^2-6467659/5971968*(2*x^2-x+3)^(3/2)/(5+2*x)-10939/512*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2
*x^2-x+3)^(1/2))*2^(1/2)-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)
```

3.329.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$=\frac{\frac{12\sqrt{3-x+2x^2}(-327735797-329667508x-97682900x^2-5453568x^3+414720x^4)}{(5+2x)^3}-170114729\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x}\right.\right.}{3981312}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 545356
8*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170114729*Sqrt[2]*ArcTanh[(5 + 2*x - Sq
rt[6 - 2*x + 4*x^2])/6] - 85061664*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/3981312
```

3.329.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2181, 2181, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\,(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$
$$\Big\downarrow \text{ 2181}$$
$$-\frac{1}{216}\int \frac{3\sqrt{2x^2-x+3}\,(-2880x^3+7776x^2-21168x+12007)}{16(2x+5)^3}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-2880x^3+7776x^2-21168x+12007)}{(2x+5)^3}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$
$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{144}\int \frac{\sqrt{2x^2-x+3}(207360x^2-1712380x+890709)}{(2x+5)^2}dx+\frac{158527(2x^2-x+3)^{3/2}}{72(2x+5)^2}}{1152}-\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 2181

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{(22099149-59206232x)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{158527(2x^2-x+3)^{3/2}}{72(2x+5)^2}}{1152}-\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 1231

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(\frac{1}{32}\int -\frac{576(28345289-56707776x)}{(2x+5)\sqrt{2x^2-x+3}}dx-2(44378877-7400779x)\sqrt{2x^2-x+3}\right)-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{15852}{}}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 27

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\int \frac{28345289-56707776x}{(2x+5)\sqrt{2x^2-x+3}}dx-2\sqrt{2x^2-x+3}(44378877-7400779x)\right)-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{158527(2x}{72(2x}}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 1269

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-28353888\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)-2\sqrt{2x^2-x+3}(44378877-7400779x\right.\right.}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 1090

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-14176944\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)-2\sqrt{2x^2-x+3}(4437\right.\right.}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 222

3.329. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-2\sqrt{2x^2-x+3}(44378877-74007\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 1154

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(-340229458\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-2\sqrt{2x^2-x+3}(4437887\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

$\downarrow$ 219

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{170114729\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-2\sqrt{2x^2-x+3}(44378877-7\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(1728*(5 + 2*x)^3) + ((158527*(3 - x + 2*x^2
)^(3/2))/(72*(5 + 2*x)^2) + ((-6467659*(3 - x + 2*x^2)^(3/2))/(36*(5 + 2*x
)) + (-2*(44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2] - 18*(-14176944*Sqrt[2
]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (170114729*ArcTanh[(17 - 22*x)/(12*Sqrt[2
]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2])))/72)/144)/1152
```

3.329.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.329. $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

3.329.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{170123328\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+170}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="
fricas")
```

output
```
1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*
(8*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x -
 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 5
453568*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/
(8*x^3 + 60*x^2 + 150*x + 125)
```

3.329.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
4, x)
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{5}{32}\sqrt{2\,x^2-x+3}x + \frac{10939}{512}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{170114729}{7962624}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{693775}{165888}\sqrt{2\,x^2-x+3} - \frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{1728\,(8\,x^3+60\,x^2+150\,x+125)} + \frac{158527\,(2\,x^2-x+3)^{\frac{3}{2}}}{82944\,(4\,x^2+20\,x+25)} - \frac{6467659\sqrt{2\,x^2-x+3}}{331776\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="
maxima")
```

output
```
5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1
/23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x
 + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) -
 3667/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/8
2944*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2
 - x + 3)/(2*x + 5)
```

3.329.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{1}{128}\sqrt{2\,x^2-x+3}(20\,x-413) - \frac{10939}{512}\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$
$$+ \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$
$$- \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$
$$- \frac{\sqrt{2}\Big(575810908\,\sqrt{2}\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^5+9206213116\,\big(\sqrt{2}x-\sqrt{2\,x^2-x+3}\big)^4+9688786604\,\sqrt{2}(}{663552\,\Big(2\,\big(\sqrt{2}x-\sqrt{2\,x^2}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="
giac")
```

output
```
1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs
(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt
(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552
*sqrt(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 92062131
16*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
49481952947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^3
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.330 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx$

3.330.1 Optimal result

Integrand size = 40, antiderivative size = 165

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5} dx = \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} + \frac{259\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} - \frac{4640586097\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1146617856\sqrt{2}}$$

output
```
-3667/2304*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+
2*x)^3-9363383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(
1-4*x)*23^(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1
/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+7/95551488*(52836655+9616196*x)*(2*x^2-x+3)
^(1/2)/(5+2*x)
```

3.330.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(44676885233+62847867486x+31323229164x^2+6105343976x^3+238878720x^4)}{(5+2x)^4} + 4640586097\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}(5+2x\right.}{1146617856}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 +
6105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640586097*Sqrt[2]*ArcTanh[
(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 2320109568*Sqrt[2]*Log[1 - 4*x + 2*
Sqrt[6 - 2*x + 4*x^2]])/1146617856
```

3.330.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$
$$\Big\downarrow \; 2181$$
$$-\frac{1}{288}\int \frac{\sqrt{2x^2-x+3}(-11520x^3+31104x^2-70004x+44361)}{16(2x+5)^4}dx - \frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-11520x^3+31104x^2-70004x+44361)}{(2x+5)^4}dx}{4608} - \frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$
$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{216}\int \frac{3\sqrt{2x^2-x+3}(414720x^2-2156544x+1380023)}{(2x+5)^3}dx+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\int \frac{\sqrt{2x^2-x+3}(414720x^2-2156544x+1380023)}{(2x+5)^3}dx+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 2181

$$\frac{\frac{1}{72}\left(-\frac{1}{144}\int \frac{7(4755675-9616196x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\int \frac{(4755675-9616196x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1230

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(-\frac{1}{8}\int \frac{2(110533831-220962816x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(9616196x+52836655)}{2(2x+5)}\right)-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(-\frac{1}{4}\int \frac{110533831-220962816x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(9616196x+52836655)}{2(2x+5)}\right)-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1269

3.330. $\displaystyle\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(110481408\int\frac{1}{\sqrt{2x^2-x+3}}dx-662940871\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{936338}{}\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 1090

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-662940871\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{2x}}{2(2x+5)}\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 222

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-662940871\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{936}{}\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 1154

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(1325881742\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+}}{2(2x+5)}\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 219

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{662940871\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

3.330. $\int\frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + ((593771*(3 - x + 2*x^2
)^(3/2))/(108*(5 + 2*x)^3) + ((-9363383*(3 - x + 2*x^2)^(3/2))/(72*(5 + 2*
x)^2) - (7*(-1/2*((52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) +
(55240704*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (662940871*ArcTanh[(17 -
22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4))/144)/72)/4608
```

3.330.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.330. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.330.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \frac{4640219136\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="
fricas")
```

output
```
1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 464
0586097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(
2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20
*x + 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 6284786
7486*x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1
000*x + 625)
```

3.330.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
5, x)
```

3.330.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx &= -\frac{259}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\&\quad+\frac{4640586097}{2293235712}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{16828343}{47775744}\sqrt{2\,x^2-x+3}\\&\quad-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}+\frac{593771\,(2\,x^2-x+3)^{\frac{3}{2}}}{497664\,(8\,x^3+60\,x^2+150\,x+125)}\\&\quad-\frac{9363383\,(2\,x^2-x+3)^{\frac{3}{2}}}{23887872\,(4\,x^2+20\,x+25)}+\frac{201573155\sqrt{2\,x^2-x+3}}{95551488\,(2\,x+5)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="
maxima")
```

output
```
-259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/229
3235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs
(2*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x
+ 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*
x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^
2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x +
 3)/(2*x + 5)
```

3.330.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx =$$

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="
giac")
```

output
```
-1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)
^2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4
640219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) -
 1))*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 8
35793*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) -
 464569597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 17
9159040*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1
/(2*x + 5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 6/(2*x + 5))^2 - 1))
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.331 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

3.331.1 Optimal result

Integrand size = 40, antiderivative size = 165

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx &= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5}\\ &\quad+\frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4}-\frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3}\\ &\quad-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}+\frac{12895597463\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{82556485632\sqrt{2}}\end{aligned}$$

output
```
-3667/2880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+
2*x)^4-38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1
-4*x)*23^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^
(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/6879707136*(4583087983+3174439702*x)*(2
*x^2-x+3)^(1/2)/(5+2*x)^2
```

3.331.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{-\frac{12\sqrt{3-x+2x^2}(3110673952831+5608297138216x+3919478861832x^2+1285267446304x^3+186470433136x^4)}{(5+2x)^5} - 64477987315\sqrt{2}\text{arcta}}{412782428160}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,
x]
```

output
```
((-12*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832
*x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64477987315*Sq
rt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 3224862720O*Sqrt[2]*L
og[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/412782428160
```

3.331.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2181, 27, 2181, 2181, 27, 1229, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{360}\int \frac{\sqrt{2x^2-x+3}(-14400x^3+38880x^2-76504x+52701)}{16(2x+5)^5}dx - \frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-14400x^3+38880x^2-76504x+52701)}{(2x+5)^5}dx}{5760} - \frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{288}\int \frac{\sqrt{2x^2-x+3}(2073600x^2-7934876x+5935131)}{(2x+5)^4}dx+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 2181

$$\frac{\frac{1}{288}\left(-\frac{1}{216}\int \frac{15(9244801-14929920x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\int \frac{(9244801-14929920x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 1229

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{(3174439702x+4583087983)\sqrt{2x^2-x+3}}{288(2x+5)^2}-\frac{\int -\frac{2(2146055063-4299816960x)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}\right)-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+}{144(2x+5)}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\int \frac{2146055063-4299816960x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(3174439702x+4583087983)}{288(2x+5)^2}\right)-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-}{144(2x-}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 1269

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2149908480\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+4583087983)}{288(2x+5)^2}\right.\right.}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\Big\downarrow$ 1090

3.331. $\displaystyle\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1074954240\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-x+3}(31744}{288(}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 222

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+4583087}{288(2x+5)^2}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 1154

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(-25791194926\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(3174439702}{288(2x+5}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 219

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{12895597463\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+45}{288(2x+5)^2}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + ((711961*(3 - x + 2*x^2
)^(3/2))/(144*(5 + 2*x)^4) + ((-38732321*(3 - x + 2*x^2)^(3/2))/(108*(5 +
2*x)^3) - (5*(((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(288*(5 +
2*x)^2) + (-1074954240*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/576))
/72)/288)/5760
```

3.331. $\int\frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

3.331.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

3.331. $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.331.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{64497254400\sqrt{2}(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="
fricas")
```

output
```
1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x
^2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 12852674
46304*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^
2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

3.331.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$=\int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
6, x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{5}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{12895597463}{165112971264}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{46569601}{3439853568}\sqrt{2\,x^2-x+3} - \frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2880\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)} + \frac{711961\,(2\,x^2-x+3)^{\frac{3}{2}}}{829440\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} - \frac{38732321\,(2\,x^2-x+3)^{\frac{3}{2}}}{179159040\,(8\,x^3+60\,x^2+150\,x+125)} + \frac{46569601\,(2\,x^2-x+3)^{\frac{3}{2}}}{1719926784\,(4\,x^2+20\,x+25)} - \frac{562688629\sqrt{2\,x^2-x+3}}{6879707136\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="
maxima")
```

output
```
5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/165112
971264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x
 + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711
961/829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5) - 38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 12
5) + 46569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 56268
8629/6879707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

3.331.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(134) = 268$.

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= -\frac{5}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(4368922304720\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9+124570969998480\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8+63780\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="
giac")
```

output
```
-5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1289
5597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
 - x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*s
qrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3))^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 3763189
300187016*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3))^2 + 10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 25
07056315485767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$=\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.332 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

3.332.1 Optimal result

Integrand size = 40, antiderivative size = 169

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx &= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6}\\ &+\frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5}-\frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4}\\ &+\frac{87677717(3-x+2x^2)^{3/2}}{8599633920(5+2x)^3}-\frac{26972675\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3962711310336\sqrt{2}}\end{aligned}$$

output
```
-3667/3456*(2*x^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2
*x)^5-5703277/39813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*
x^2-x+3)^(3/2)/(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1172725/330225942528*(17-22*x)*(2*x^2-x+3)
^(1/2)/(5+2*x)^2
```

3.332.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-219337079305+27245373694x+158340720344x^2+397498825328x^3+12256250416x^4+271409942624x^5)}{(5+2x)^6}+134863375\sqrt{2}}{19813556551680}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2
 + 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5))/(5 + 2*x)^6 + 1
34863375*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/19813556551
680
```

3.332.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2181, 27, 2181, 27, 2181, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{432}\int \frac{3\sqrt{2x^2-x+3}(-5760x^3+15552x^2-27668x+20347)}{16(2x+5)^6}dx-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-5760x^3+15552x^2-27668x+20347)}{(2x+5)^6}dx}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{360}\int \frac{3\sqrt{2x^2-x+3}(345600x^2-1059208x+895257)}{(2x+5)^5}dx+\frac{92239(2x^2-x+3)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{120}\int \frac{\sqrt{2x^2-x+3}(345600x^2-1059208x+895257)}{(2x+5)^5}dx+\frac{92239(2x^2-x+3)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 2181

$$\frac{\frac{1}{120}\left(-\frac{1}{288}\int \frac{(20294487-26953292x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx-\frac{5703277(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)+\frac{92239(2x^2-x+3)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 1228

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\int \frac{\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{87677717(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)+\frac{92239(2x^2-x+3)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 1152

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(\frac{23}{576}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{(17-22x)\sqrt{2x^2-x+3}}{288(2x+5)^2}\right)+\frac{87677717(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)+}{2304}\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 1154

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(-\frac{23}{288}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{87677717(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277(2x^2-x+}{144(2x+5)^4}}{2304}\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$\big\downarrow$ 219

3.332. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(-\frac{23\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3456\sqrt{2}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{87677717(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + ((92239*(3 - x + 2*x^2)
^(3/2))/(60*(5 + 2*x)^5) + ((-5703277*(3 - x + 2*x^2)^(3/2))/(144*(5 + 2*x
)^4) + ((87677717*(3 - x + 2*x^2)^(3/2))/(108*(5 + 2*x)^3) + (5863625*(-1/
288*((17 - 22*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 - (23*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3456*Sqrt[2])))/72)/288)/120)/2304
```

3.332.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.332. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.332.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

method	result
risch	$\frac{542819885248x^7-246897441792x^6+1596971228112x^5-44048633392x^4+1088646503028x^3+9102628728x^2+301073200387x-6580}{1651129712640(5+2x)^6\sqrt{2x^2-x+3}}$
trager	$\frac{(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-219337079305)\sqrt{2x^2-x+3}}{1651129712640(5+2x)^6}+\frac{26972675}{}$
default	$-\frac{5703277\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{637009920\left(x+\frac{5}{2}\right)^4}+\frac{87677717\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{68797071360\left(x+\frac{5}{2}\right)^3}-\frac{1172725\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{330225942528\left(x+\frac{5}{2}\right)^2}-\frac{12899975\left(2(x+\right.}{11888133931}$

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x,method=_RETURNVERB
OSE)
```

output
```
1/1651129712640*(542819885248*x^7-246897441792*x^6+1596971228112*x^5-44048
633392*x^4+1088646503028*x^3+9102628728*x^2+301073200387*x-658011237915)/(
5+2*x)^6/(2*x^2-x+3)^(1/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(17
/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx = \frac{134863375\sqrt{2}(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(2}{4}\right.}{79254226206720}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="
fricas")
```

output
```
1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x
^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22
*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(2714099426
24*x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 272453736
94*x - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 2
0000*x^3 + 37500*x^2 + 37500*x + 15625)
```

3.332.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^7} dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
7, x)
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= \frac{26972675}{7925422620672}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}$$
$$-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{3456\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}$$
$$+\frac{92239\,(2\,x^2-x+3)^{\frac{3}{2}}}{138240\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}$$
$$-\frac{5703277\,(2\,x^2-x+3)^{\frac{3}{2}}}{39813120\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}$$
$$+\frac{87677717\,(2\,x^2-x+3)^{\frac{3}{2}}}{8599633920\,(8\,x^3+60\,x^2+150\,x+125)}$$
$$-\frac{1172725\,(2\,x^2-x+3)^{\frac{3}{2}}}{82556485632\,(4\,x^2+20\,x+25)}-\frac{12899975\sqrt{2\,x^2-x+3}}{330225942528\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="
maxima")
```

output
```
26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 366
7/3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37
500*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 +
400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 -
 x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/85996
33920*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556
485632*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*s
qrt(2*x^2 - x + 3)/(2*x + 5)
```

3.332.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(139) = 278$.

Time = 0.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= -\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\left(16506981498400\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11}+389429252643040\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10}+22\cdots\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="
giac")
```

output
```
-26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*
sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(1650698149840
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*
x - sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 +
 902212326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519
861840*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x
^2 - x + 3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 + 356306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 823483531
28195465*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2
*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
 - x + 3)) - 11)^6
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^7}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.333 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

3.333.1 Optimal result

Integrand size = 40, antiderivative size = 194

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx &= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2}-\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7}\\&\quad+\frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6}-\frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5}+\frac{19414831(3-x+2x^2)^{3/2}}{4013162496(5+2x)^4}\\&\quad+\frac{246159769(3-x+2x^2)^{3/2}}{866843099136(5+2x)^3}-\frac{289071245\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{285315214344192\sqrt{2}}\end{aligned}$$

output
```
-3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^7+948341/1741824*(2*x^2-x+3)^(3/2)/(5
+2*x)^6-1464037/13934592*(2*x^2-x+3)^(3/2)/(5+2*x)^5+19414831/4013162496*(
2*x^2-x+3)^(3/2)/(5+2*x)^4+246159769/866843099136*(2*x^2-x+3)^(3/2)/(5+2*x
)^3-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(
1/2))*2^(1/2)-12568315/23776267862016*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^
2
```

3.333.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-20465234808721+590492177460x+14716683780036x^2+41058010262368x^3+4982916071952x^4+27976951397184x^5+15743422\ldots}{(5+2x)^7}}{1997206500409344}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 1471668378003
6*x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574
342277056*x^6))/(5 + 2*x)^7 + 2023498715*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6
 - 2*x + 4*x^2])/6])/1997206500409344
```

3.333.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1237, 25, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{504}\int \frac{\sqrt{2x^2-x+3}\left(-20160x^3+54432x^2-89504x+69381\right)}{16(2x+5)^7}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}\left(-20160x^3+54432x^2-89504x+69381\right)}{(2x+5)^7}dx}{8064}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{432}\int \frac{15\sqrt{2x^2-x+3}(290304x^2-750908x+700441)}{(2x+5)^6}dx+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5}{144}\int \frac{\sqrt{2x^2-x+3}(290304x^2-750908x+700441)}{(2x+5)^6}dx+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{5}{144}\left(-\frac{1}{360}\int \frac{3(5149971-5705944x)\sqrt{2x^2-x+3}}{(2x+5)^5}dx-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5}{144}\left(-\frac{1}{120}\int \frac{(5149971-5705944x)\sqrt{2x^2-x+3}}{(2x+5)^5}dx-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1237$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\int -\frac{(52011459-77659324x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx+\frac{19414831(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)}{216(2x+5)^6}}{8064}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{19414831(2x^2-x+3)^{3/2}}{144(2x+5)^4}-\frac{1}{288}\int \frac{(52011459-77659324x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx\right)-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/}}{216(2x+5)^6}}{8064}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1228$$

3.333. $\displaystyle\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\int\frac{\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{246159769(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$\Big\downarrow$ 1152

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(\frac{23}{576}\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{(17-22x)\sqrt{2x^2-x+3}}{288(2x+5)^2}\right)+\frac{246159769(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831(2x^2-x+3)}{144(2x+5)^4}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$\Big\downarrow$ 1154

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(-\frac{23}{288}\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{246159769(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831}{14}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$\Big\downarrow$ 219

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(-\frac{23\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3456\sqrt{2}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{246159769(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831(2x}{144(2x}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + ((948341*(3 - x + 2*x^2
)^(3/2))/(216*(5 + 2*x)^6) + (5*((-1464037*(3 - x + 2*x^2)^(3/2))/(60*(5 +
 2*x)^5) + ((19414831*(3 - x + 2*x^2)^(3/2))/(144*(5 + 2*x)^4) + ((2461597
69*(3 - x + 2*x^2)^(3/2))/(108*(5 + 2*x)^3) + (87978205*(-1/288*((17 - 22*
x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 - (23*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*
Sqrt[3 - x + 2*x^2])])/(3456*Sqrt[2])))/72)/288)/120))/144)/8064
```

3.333. $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

3.333.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.333. $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.333.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

method	result
risch	$\frac{3148684554112x^8+54379560517312x^7-13288092422112x^6+161063958644336x^5+3324105513560x^4+109638331361988x^3+262908}{166433875034112(5+2x)^7\sqrt{2x^2-x+3}}$
trager	$\frac{(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+14716683780036x^2+590492177460x-2046523480}{166433875034112(5+2x)^7}$
default	$\frac{19414831\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{64210599936\left(x+\frac{5}{2}\right)^4}+\frac{246159769\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{6934744793088\left(x+\frac{5}{2}\right)^3}-\frac{12568315\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{23776267862016\left(x+\frac{5}{2}\right)^2}-\frac{138251465\left(2(x-\right.}{8559456430}$

input
```
int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x,method=_RETURNVERB
OSE)
```

output
```
1/166433875034112*(3148684554112*x^8+54379560517312*x^7-13288092422112*x^6
+161063958644336*x^5+3324105513560*x^4+109638331361988*x^3+2629089545206*x
^2+22236711341101*x-61395704426163)/(5+2*x)^7/(2*x^2-x+3)^(1/2)-289071245/
570630428688384*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x
-19/2)^(1/2))
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{2023498715\sqrt{2}(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\,\mathrm{lo}}{7}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="
fricas")
```

output
```
1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 7
0000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
25)) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 41
058010262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*s
qrt(2*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x
^3 + 262500*x^2 + 218750*x + 78125)
```

3.333.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
8, x)
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{289071245}{570630428688384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{12568315}{11888133931008}\sqrt{2\,x^2-x+3} - \frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{4032\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)} + \frac{948341\,(2\,x^2-x+3)^{\frac{3}{2}}}{1741824\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)} - \frac{1464037\,(2\,x^2-x+3)^{\frac{3}{2}}}{13934592\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)} + \frac{19414831\,(2\,x^2-x+3)^{\frac{3}{2}}}{4013162496\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} + \frac{246159769\,(2\,x^2-x+3)^{\frac{3}{2}}}{866843099136\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{12568315\,(2\,x^2-x+3)^{\frac{3}{2}}}{5944066965504\,(4\,x^2+20\,x+25)} - \frac{138251465\,\sqrt{2\,x^2-x+3}}{23776267862016\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="
maxima")
```

output
```
289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3)
 - 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000
*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2
 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 +
 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/8668430991
36*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066
965504*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/2377626786201
6*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

3.333.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= -\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(129503917760\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{13} - 3320259746027840\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{12} - 239\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="
giac")
```

output
```
-289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x
 - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950
3917760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3))^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9
 + 796135370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 25315231391710
05408*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^4 - 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^3 + 16204706960604668100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 319625459
3191113265*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) - 11)^7
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$=\int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^8}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.334 $\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right) dx$

3.334.1 Optimal result

Integrand size = 38, antiderivative size = 166

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right) dx = -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304}+\frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2}-\frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360}-\frac{147157749\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

output
```
-92727/131072*(1-4*x)*(2*x^2-x+3)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^
(5/2)-1121/2304*(5+2*x)^3*(2*x^2-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5
/2)-3/143360*(661397+215900*x)*(2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh
(1/23*(1-4*x)*23^(1/2))*2^(1/2)-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.334.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{4\sqrt{3-x+2x^2}(1592737263+12357760788x+4870637856x^2+12669290112x^3+379086848x$$

input
```
Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 1266
9290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 292683776
0*x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x
 + 4*x^2]])/2642411520
```

3.334.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {2184, 25, 2184, 27, 2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\int -\left((2x+5)\left(2x^2-x+3\right)^{3/2}\left(8968x^3+15996x^2+11262x+2299\right)\right)dx+\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{144}(2x+5)^4\left(2x^2-x+3\right)^{5/2}-\frac{1}{288}\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(8968x^3+15996x^2+11262x+2299\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\left(-\frac{1}{128}\int -8(2x+5)\left(2x^2-x+3\right)^{3/2}\left(277660x^2+281660x+24871\right)dx-\frac{1121}{8}\left(2x^2-x+3\right)^{5/2}(2x+5)^3\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 27

$$\frac{1}{288}\left(\frac{1}{16}\int(2x+5)\left(2x^2-x+3\right)^{3/2}\left(277660x^2+281660x+24871\right)dx-\frac{1121}{8}(2x+5)^3\left(2x^2-x+3\right)^{5/2}\right)+$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 2184

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{1}{56}\int 108(15467-64770x)(2x+5)\left(2x^2-x+3\right)^{3/2}dx+\frac{69415}{7}(2x+5)^2\left(2x^2-x+3\right)^{5/2}\right)-\frac{1121}{8}(2\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 27

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\int(15467-64770x)(2x+5)\left(2x^2-x+3\right)^{3/2}dx+\frac{69415}{7}(2x+5)^2\left(2x^2-x+3\right)^{5/2}\right)-\frac{1121}{8}(2x+\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 1225

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\int\left(2x^2-x+3\right)^{3/2}dx-\frac{1}{20}(215900x+661397)\left(2x^2-x+3\right)^{5/2}\right)+\frac{69415}{7}(2x+5)^2\left(2x^2\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 1087

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\int\sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{20}(215900x+661397)\left(2x^2-\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 1087

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\downarrow$ 1090

3.334. $\int(5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\right.\right.\right.\right.\right.$$

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow 222$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\right.\right.\right.\right.$$

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

input
```
Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*(5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))/144 + ((-1121*(5 + 2*x)^3*(3 - x + 2
*x^2)^(5/2))/8 + ((69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/7 + (27*(-1/20
*((661397 + 215900*x)*(3 - x + 2*x^2)^(5/2)) + (216363*(-1/16*((1 - 4*x)*(
3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/8))/14)/16)/288
```

3.334.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.334. $\int(5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.334.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x+1}{660602880}$
trager	$\left(\frac{20}{9}x^8+\frac{319}{72}x^7+\frac{1051}{672}x^6+\frac{295847}{16128}x^5+\frac{26443}{46080}x^4+\frac{32992943}{1720320}x^3+\frac{2415991}{327680}x^2+\frac{343271133}{18350080}x+\frac{176970807}{73400320}\right)\sqrt{2}$
default	$\frac{92727(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{131072}+\frac{6398163\sqrt{2x^2-x+3}\,(4x-1)}{2097152}+\frac{147157749\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{8388608}+\frac{2005x^2(2x^2-x+3)^{\frac{5}{2}}}{8064}+$

input
```
int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
1/660602880*(1468006400*x^8+2926837760*x^7+1033175040*x^6+12117893120*x^5+
379086848*x^4+12669290112*x^3+4870637856*x^2+12357760788*x+1592737263)*(2*
x^2-x+3)^(1/2)+147157749/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{660602880}\left(1468006400\,x^8+2926837760\,x^7+1033175040\,x^6+12117893120\,x^5+379086848\,x\right.$$
$$+\frac{147157749}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 1211789312
0*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x +
 1592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(
2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.334.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{20x^8}{9}+\frac{319x^7}{72}+\frac{1051x^6}{672}+\frac{295847x^5}{16128}+\frac{26443x^4}{46080}+\frac{32992943x^3}{1720320}+\frac{2415991x^2}{327680}+\frac{343271133x}{18350080}+\frac{176970807}{73400320}\right)+\frac{147157749\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}$$

input
```
integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(20*x**8/9 + 319*x**7/72 + 1051*x**6/672 + 295847*x**
5/16128 + 26443*x**4/46080 + 32992943*x**3/1720320 + 2415991*x**2/327680 +
 343271133*x/18350080 + 176970807/73400320) + 147157749*sqrt(2)*asinh(4*sq
rt(23)*(x - 1/4)/23)/8388608
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{5}{9}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{479}{288}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{2005}{8064}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{5645}{21504}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{120809}{143360}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{92727}{32768}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{92727}{131072}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{6398163}{524288}\sqrt{2\,x^2-x+3}x+\frac{147157749}{8388608}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{6398163}{2097152}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/8
064*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 12080
9/143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 927
27/131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 1
47157749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/209715
2*sqrt(2*x^2 - x + 3)
```

3.334.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{660602880}\left(4\left(8\left(4\left(16\left(20\left(8\left(28\left(160\,x+319\right)x+3153\right)x+295847\right)x+185101\right)x+98978829\right)\right.\right.\right.\right.\right.\right.\right.\cdots-\frac{147157749}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x
+ 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqr
t(2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\int (2\,x+5)\,\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)\,dx$$

input
```
int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.335 $\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx$

3.335.1 Optimal result

Integrand size = 33, antiderivative size = 147

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536} + \frac{1167(3-x+2x^2)^{5/2}}{14336} + \frac{125x(3-x+2x^2)^{5/2}}{3584} + \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2} - \frac{13643439\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

output
```
-8597/65536*(1-4*x)*(2*x^2-x+3)^(3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/358
4*x*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5
/2)-13643439/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-593193/1048576
*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = \frac{4\sqrt{3-x+2x^2}(-1663407+27845612x+3845856x^2+27023744x^3-7497728x^4+29335552x}{29360128}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
 - 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt
[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/29360128
```

3.335.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2} \left(5x^4-x^3+3x^2+x+2\right) \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16} \int \frac{1}{2} \left(2x^2-x+3\right)^{3/2} \left(23x^3+6x^2+32x+64\right)dx+\frac{5}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \int \left(2x^2-x+3\right)^{3/2} \left(23x^3+6x^2+32x+64\right)dx+\frac{5}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32} \left(\frac{1}{14} \int \frac{1}{2} \left(2x^2-x+3\right)^{3/2} \left(375x^2+620x+1792\right)dx+\frac{23}{14} x^2 \left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \left(\frac{1}{28} \int \left(2x^2-x+3\right)^{3/2} \left(375x^2+620x+1792\right)dx+\frac{23}{14} x^2 \left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32} \left(\frac{1}{28} \left(\frac{1}{12} \int \frac{3}{2} (5835x+13586) \left(2x^2-x+3\right)^{3/2}dx+\frac{125}{4} x \left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14} x^2 \left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\int (5835x+13586)\left(2x^2-x+3\right)^{3/2}dx+\frac{125}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\int \left(2x^2-x+3\right)^{3/2}dx+\frac{1167}{2}\left(2x^2-x+3\right)^{5/2}\right)+\frac{125}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14}x^2\left(2x^2-x\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1167}{2}\left(2x^2-x+3\right)^{5/2}\right)+\frac{125}{4}x(2\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x\right.\right.\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \; 222$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{116}{2}\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

3.335. $\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

output
```
(5*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((23*x^2*(3 - x + 2*x^2)^(5/2))/14 + ((
125*x*(3 - x + 2*x^2)^(5/2))/4 + ((1167*(3 - x + 2*x^2)^(5/2))/2 + (60179*
(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x
 + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4)/8)/2
8)/32
```

3.335.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.335. $\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4) \, dx$

3.335.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-1663407)\sqrt{2x^2-x+3}}{7340032}+\frac{13643439\sqrt{2}\,\mathrm{a}}{}$
trager	$\left(\frac{5}{4}x^7-\frac{117}{112}x^6+\frac{3581}{896}x^5-\frac{523}{512}x^4+\frac{211123}{57344}x^3+\frac{17169}{32768}x^2+\frac{6961403}{1835008}x-\frac{1663407}{7340032}\right)\sqrt{2x^2-x+3}-\frac{1364343}{}$
default	$\frac{1167(2x^2-x+3)^{\frac{5}{2}}}{14336}+\frac{8597(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{65536}+\frac{593193\sqrt{2x^2-x+3}\,(4x-1)}{1048576}+\frac{13643439\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{4194304}+\frac{5x^3(2}{}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/7340032*(9175040*x^7-7667712*x^6+29335552*x^5-7497728*x^4+27023744*x^3+3
845856*x^2+27845612*x-1663407)*(2*x^2-x+3)^(1/2)+13643439/4194304*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3 +5x^4)\,dx = \frac{1}{7340032}\left(9175040\,x^7-7667712\,x^6+29335552\,x^5-7497728\,x^4+27023744\,x^3+3845856\,x^2+\right.$$
$$+\frac{13643439}{8388608}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 270237
44*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 1364343
9/8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25)
```

3.335.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int (3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^7}{4}-\frac{117x^6}{112}+\frac{3581x^5}{896}-\frac{523x^4}{512}+\frac{211123x^3}{57344}+\frac{17169x^2}{32768}+\frac{6961403x}{1835008}-\frac{1663407}{7340032}\right)+\frac{13643439\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**7/4 - 117*x**6/112 + 3581*x**5/896 - 523*x**4/5
12 + 211123*x**3/57344 + 17169*x**2/32768 + 6961403*x/1835008 - 1663407/73
40032) + 13643439*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4194304
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{5}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{8597}{65536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{593193}{262144}\sqrt{2x^2-x+3}x+\frac{13643439}{4194304}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{593193}{1048576}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/35
84*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384
*(2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/26214
4*sqrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

3.335.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = \frac{1}{7340032}(4(8(4(16(4(8(140x-117)x+3581)x-3661)x+211123)x+120183)x+6961403 - \frac{13643439}{4194304}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)
*x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/4194
304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) \, dx = \int \left(2x^2-x+3\right)^{3/2} \left(5x^4-x^3+3x^2+x+2\right) \, dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

3.336.1 Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} + \frac{3505}{896}(3-x+2x^2)^{5/2} - \frac{311}{448}(5+2x)(3-x+2x^2)^{5/2} + \frac{5}{112}(5+2x)^2(3-x+2x^2)^{5/2} + \frac{1622009981\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}} - \frac{99009\text{arctanh}\left(\frac{1}{}\right.}{8\sqrt{2}}$$

output
```
1/12288*(500141-123060*x)*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311
/448*(5+2*x)*(2*x^2-x+3)^(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+162200998
1/262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-
22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/65536*(141051019-23482924*x)*(2
*x^2-x+3)^(1/2)
```

3.336.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{4\sqrt{3-x+2x^2}(3149403255-609499532x+159973408$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),
x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 4647667
2*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 68130865152*Sqrt[2]*Arc
Tanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 34062209601*Sqrt[2]*Log[1 - 4*
x + 2*Sqrt[6 - 2*x + 4*x^2]])/5505024
```

3.336.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2184, 2184, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\int \frac{(2x^2-x+3)^{3/2}(-7464x^3-14508x^2-9926x+573)}{2x+5}dx+\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\left(\frac{1}{96}\int -\frac{24(-70100x^2-85940x+17923)(2x^2-x+3)^{3/2}}{2x+5}dx-\frac{311}{2}(2x+5)(2x^2-x+3)^{5/2}\right)+\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(-\frac{1}{4}\int \frac{\left(-70100x^2-85940x+17923\right)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx-\frac{311}{2}(2x+5)\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 2184$$

$$\frac{1}{224}\left(\frac{1}{4}\left(3505\left(2x^2-x+3\right)^{5/2}-\frac{1}{40}\int -\frac{140(7397-20510x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx\right)-\frac{311}{2}(2x+5)\left(2x^2-x+3\right)\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\int \frac{(7397-20510x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+3505\left(2x^2-x+3\right)^{5/2}\right)-\frac{311}{2}(2x+5)\left(2x^2-x+3\right)^{5/2}\right)+$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{48}(500141-123060x)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int -\frac{2(4441417-11741462x)\sqrt{2x^2-x+3}}{2x+5}dx\right)+350\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\int \frac{(4441417-11741462x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1}{48}(500141-123060x)\left(2x^2-x+3\right)^{3/2}\right)+3505(\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{8}(141051019-23482924x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(1622930831-3244019962x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1}{48}(50\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

3.336. $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\int\frac{1622930831-3244019962x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(141051019-23482924x)\right)+\frac{1}{48}(500141\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1269

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1622009981\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1090

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{1622009981\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 222

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1154

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(-19465961472\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 219

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-811081728\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

3.336. $\int\frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

output
```
(5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/112 + ((-311*(5 + 2*x)*(3 - x + 2*x^
2)^(5/2))/2 + (3505*(3 - x + 2*x^2)^(5/2) + (7*(((500141 - 123060*x)*(3 -
x + 2*x^2)^(3/2))/48 + (((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/8 +
 ((-1622009981*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 811081728*Sqrt[2]*A
rcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16)/32))/2)/4)/224
```

3.336.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.336. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.336.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

3.336. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

3.336.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{1376256}\left(983040\,x^6-3710976\,x^5+14493696\,x^4-4647\right.$$
$$+\frac{1622009981}{524288}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$
$$+\frac{99009}{32}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fr
icas")
```

output
```
1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 159973
408*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/52428
8*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
) + 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 10
60*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

3.336.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{5}{28}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2 - \frac{111}{224}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x + \frac{1395}{896}\left(2\,x^2-x+3\right)^{\frac{5}{2}} - \frac{10255}{1024}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{500141}{12288}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{5870731}{16384}\sqrt{2\,x^2-x+3}x - \frac{1622009981}{262144}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{99009}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{141051019}{65536}\sqrt{2\,x^2-x+3}$$

input

```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="ma
xima")
```

output

```
5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/89
6*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/1228
8*(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981
/262144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2
)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 1
41051019/65536*sqrt(2*x^2 - x + 3)
```

3.336.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40\,x-151\right)x+2359\right)x-12103\right.\right.\right.\right.\right. + \frac{1622009981}{262144}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) - \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

input

```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gi
ac")
```

output
```
1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)
*x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sq
rt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(
2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqr
t(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx=\int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{2\,x+5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

3.337 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

3.337.1 Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} - \frac{839}{960}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)(3-x+2x^2)^{5/2} - \frac{982669459\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}} + \frac{959625\,\mathrm{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{64\sqrt{2}}$$

output

```
-1/18432*(909513-226052*x)*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-366
7/576*(2*x^2-x+3)^(5/2)/(5+2*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/1
31072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/32768*(85448933-14243732*x)*(2*x
^2-x+3)^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}(-6814208295-1404323114x+182033816x^2-35369408x^3+8283}{5+2x}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
2,x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 3536
9408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 2947968000
0*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 14740041885*Sqrt[
2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/1966080
```

3.337.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 2184, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-79840x+26675)}{16(2x+5)}dx-\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-79840x+26675)}{2x+5}dx}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{60(2x+5)(2x^2-x+3)^{5/2}-\frac{1}{96}\int \frac{96(2x^2-x+3)^{3/2}(20136x^2-67720x+24725)}{2x+5}dx}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{60(2x+5)\left(2x^2-x+3\right)^{5/2}-\int \frac{\left(2x^2-x+3\right)^{3/2}\left(20136x^2-67720x+24725\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 2184

$$\frac{-\frac{1}{40}\int \frac{80(18655-56513x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+60(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{5034}{5}\left(2x^2-x+3\right)^{5/2}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 27

$$\frac{-2\int \frac{(18655-56513x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+60(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{5034}{5}\left(2x^2-x+3\right)^{5/2}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 1231

$$\frac{-2\left(\frac{1}{32}(909513-226052x)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int -\frac{9(2667335-7121866x)\sqrt{2x^2-x+3}}{2x+5}dx\right)+60(2x+5)\left(2x^2-x+3\right)^{5}}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 27

$$\frac{-2\left(\frac{9}{64}\int \frac{(2667335-7121866x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1}{32}(909513-226052x)\left(2x^2-x+3\right)^{3/2}\right)+60(2x+5)\left(2x^2-x+3\right)^{5/2}}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 1231

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{8}(85448933-14243732x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(982588705-1965338918x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1}{32}(909513-226052x)\left(2x\right.\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\Big\downarrow$ 27

3.337. $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\int \frac{982588705-1965338918x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-14243732x)\right)+\frac{1}{32}(909513-226052x)\left(2x^2-\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1269

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-982669459\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1424373\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1090

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{982669459\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-142\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 222

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{982669459\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1424373\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1154

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(-11791872000\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{982669459\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1\right.\right.}{115}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 219

3.337. $\displaystyle\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

$$-2\left(\frac{9}{64}\left(\frac{1}{16}\left(-\frac{982669459\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-491328000\sqrt{2}\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1\ldots\right.\right.$$

$$\overline{\qquad 115\ldots}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + ((-5034*(3 - x + 2*x^2)^(5
/2))/5 + 60*(5 + 2*x)*(3 - x + 2*x^2)^(5/2) - 2*(((909513 - 226052*x)*(3 -
 x + 2*x^2)^(3/2))/32 + (9*(((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/
8 + ((-982669459*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 491328000*Sqrt[2]
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16))/64))/1152
```

3.337.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.337. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

3.337.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{14740041885\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
fricas")
```

output
```
1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*
sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2
 + 20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3
+ 182033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5
)
```

3.337.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**2, x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{5}{48}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x - \frac{589}{960}\left(2\,x^2-x+3\right)^{\frac{5}{2}} + \frac{9059}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x - \frac{185827}{6144}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{3560933}{8192}\sqrt{2\,x^2-x+3}x + \frac{982669459}{131072}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{959625}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{85448933}{32768}\sqrt{2\,x^2-x+3} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{32\,(2\,x+5)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
maxima")
```

output
```
5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(
2*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*
sqrt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x -
 1/23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5)
 - 17/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 366
7/32*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.337.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(137) = 274.

Time = 0.36 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.11

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
giac")
```

output
```
1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147
40041885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) -
1))*sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
*sgn(1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) +
33178982400*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn
(1/(2*x + 5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 17132461
0560*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x
 + 5)) + 60059281615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^3*sgn(1/(2*x + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(
2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 ...
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx=\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^2}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

3.338 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

3.338.1 Optimal result

Integrand size = 40, antiderivative size = 174

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \frac{129342063\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}} - \frac{8083915\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1024\sqrt{2}}$$

output
```
1/82944*(2154633-534617*x)*(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/1
152*(2*x^2-x+3)^(5/2)/(5+2*x)^2+438065/82944*(2*x^2-x+3)^(5/2)/(5+2*x)+129
342063/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1
/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24576*(33741483-5623292
*x)*(2*x^2-x+3)^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}(298966737+181223072x+16667188x^2-1620944x^3+253312x^4-}{(5+2x)^2}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
3,x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*
x^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 258685280*Sqrt[2]*
ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 129342063*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/32768
```

3.338.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{144}\int \frac{(2x^2-x+3)^{3/2}(-5760x^3+15552x^2-86340x+35015)}{16(2x+5)^2}dx-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-5760x^3+15552x^2-86340x+35015)}{(2x+5)^2}dx}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(207360x^2-8087312x+2737465)}{2x+5}dx+\frac{438065(2x^2-x+3)^{5/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{1}{72}\left(\frac{1}{40}\int \frac{40(2867065-8553872x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+10368\left(2x^2-x+3\right)^{5/2}\right)+\frac{438065\left(2x^2-x+3\right)^{5/2}}{36(2x+5)}}{2304}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(\int \frac{(2867065-8553872x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+10368\left(2x^2-x+3\right)^{5/2}\right)+\frac{438065\left(2x^2-x+3\right)^{5/2}}{36(2x+5)}}{2304}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{72}\left(-\frac{1}{64}\int -\frac{6912(527400-1405823x)\sqrt{2x^2-x+3}}{2x+5}dx+10368\left(2x^2-x+3\right)^{5/2}+2(2154633-534617x)\left(2x^2-x+3\right)^{3/2}\right)}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(108\int \frac{(527400-1405823x)\sqrt{2x^2-x+3}}{2x+5}dx+10368\left(2x^2-x+3\right)^{5/2}+2(2154633-534617x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{438}{}}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{72}\left(108\left(\frac{1}{16}(33741483-5623292x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{9(43115175-86228042x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+10368\left(2x^2-x+3\right)^{5/2}+2(2\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\int \frac{43115175-86228042x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292x)\right)+10368\left(2x^2-x+3\right)^{5/2}+2(2154\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

3.338. $\displaystyle\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

$\Big\downarrow$ 1269

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-43114021\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292x}{2304}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1090

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{43114021\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-562}{2304}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 222

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{43114021\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292x)}{2304}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(-517370560\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{43114021\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-562}{2304}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(-\frac{43114021\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{64671320}{3}\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-562}{2304}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

3.338. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(1152*(5 + 2*x)^2) + ((438065*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)) + (2*(2154633 - 534617*x)*(3 - x + 2*x^2)^(3/2) +
10368*(3 - x + 2*x^2)^(5/2) + 108*(((33741483 - 5623292*x)*Sqrt[3 - x + 2*
x^2])/16 + (9*((-43114021*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - (6467132
0*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/3))/32))/
72)/2304
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.338. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3} \, dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.338. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

3.338.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{129342063\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
fricas")
```

output
```
1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x +
 25)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620
944*x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*
x^2 + 20*x + 25)
```

3.338.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**3, x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{1}{16}\left(2x^2-x+3\right)^{\frac{5}{2}} - \frac{149}{128}\left(2x^2-x+3\right)^{\frac{3}{2}}x + \frac{46691}{4608}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1152\left(4x^2+20x+25\right)} - \frac{1405823}{6144}\sqrt{2x^2-x+3}x - \frac{129342063}{32768}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{8083915}{2048}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{11247161}{8192}\sqrt{2x^2-x+3} + \frac{438065\left(2x^2-x+3\right)^{\frac{3}{2}}}{4608\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
maxima")
```

output
```
1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*
(2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25
) - 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4
/23*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(
23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x
^2 - x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.54

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+8460\right. + \frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{\sqrt{2}\left(14243182\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+109906674\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-170996871\sqrt{2}\left(\sqrt{2}x-\right.\right.}{512\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
giac")
```

output
```
1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
 - x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
 - x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*
sqrt(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqr
t(2) + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*
x - sqrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 - 170996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 11)^2
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

3.339 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

3.339.1 Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} - \frac{(138006843-34265045x)(3-x+2x^2)^{3/2}}{17915904} - \frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} + \frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2} - \frac{32865365(3-x+2x^2)^{5/2}}{17915904(5+2x)} - \frac{19176431\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}} + \frac{517762327\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{221184\sqrt{2}}$$

output
```
-1/17915904*(138006843-34265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)
^(5/2)/(5+2*x)^3+556255/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365/179159
04*(2*x^2-x+3)^(5/2)/(5+2*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))
*2^(1/2)+517762327/442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2)
)*2^(1/2)-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-1994650739-2006873194x-594798908x^2-33595416x^3+262}{(5+2x)^3}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
4,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 335
95416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 103552465
4*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 517763637*Sqrt[2]
*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/442368
```

3.339.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 2181, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{216}\int \frac{(2x^2-x+3)^{3/2}(-8640x^3+23328x^2-92840x+43355)}{16(2x+5)^3}dx-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-8640x^3+23328x^2-92840x+43355)}{(2x+5)^3}dx}{3456}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{(2x^2-x+3)^{3/2}(622080x^2-9909876x+4202675)}{(2x+5)^2}dx+\frac{556255(2x^2-x+3)^{5/2}}{72(2x+5)^2}}{3456}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{(182685181-548240720x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{36(2x+5)}\right)+\frac{556255\left(2x^2-x+3\right)^{5/2}}{72(2x+5)^2}}{3456}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(\frac{1}{64}\int -\frac{3456(67520547-180096712x)\sqrt{2x^2-x+3}}{2x+5}dx-2(138006843-34265045x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{32865365\left(2x^2\right.}{36(2x+}\right.}{3456}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\int \frac{(67520547-180096712x)\sqrt{2x^2-x+3}}{2x+5}dx-2(138006843-34265045x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{32865365\left(2x^2-x+\right.}{36(2x+5)}\right.}{3456}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(2(135068604-22512089x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{288(172585259-345175758x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-2(138006843-34\right.\right.}{3456}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\int \frac{172585259-345175758x}{(2x+5)\sqrt{2x^2-x+3}}dx+2\sqrt{2x^2-x+3}(135068604-22512089x)\right)-2(138006843-34265045\right.\right.}{3456}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1269}$$

3.339. $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-172587879\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+2\sqrt{2x^2-x+3}(135068604-225\right.\right.\right.}{3456}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 1090

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{172587879\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+2\sqrt{2x^2-x+3}(135068604\right.\right.\right.}{345}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 222

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{172587879\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(135068604-225\right.\right.\right.}{3456}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(-2071049308\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{172587879\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(13506860\right.\right.\right.}{345}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(-\frac{172587879\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{517762327\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(135068604-2\right.\right.\right.}{3456}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

3.339. $\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(1728*(5 + 2*x)^3) + ((556255*(3 - x + 2*x^2
)^(5/2))/(72*(5 + 2*x)^2) + ((-32865365*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*
x)) + (-2*(138006843 - 34265045*x)*(3 - x + 2*x^2)^(3/2) - 54*(2*(13506860
4 - 22512089*x)*Sqrt[3 - x + 2*x^2] + 9*((-172587879*ArcSinh[(-1 + 4*x)/Sq
rt[23]])/Sqrt[2] - (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x +
 2*x^2])])/(3*Sqrt[2]))))/72)/144)/3456
```

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.339. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.339.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{517763637\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{\right.}}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
fricas")
```

output
```
1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 1
7) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 3156
48*x^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 19946
50739)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

3.339.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**4, x)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{5}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{1094743}{497664}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1728\left(8x^3+60x^2+150x+125\right)} + \frac{556255\left(2x^2-x+3\right)^{\frac{5}{2}}}{248832\left(4x^2+20x+25\right)} + \frac{22512089}{331776}\sqrt{2x^2-x+3}x + \frac{19176431}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{517762327}{442368}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{11255717}{27648}\sqrt{2x^2-x+3} - \frac{32865365\left(2x^2-x+3\right)^{\frac{3}{2}}}{995328\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
maxima")
```

output
```
5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667
/1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832
*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 -
x + 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23))
 - 517762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995
328*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.339.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.73

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{1}{4096}(4(8(20x-287)x+23341)x-1004633)\sqrt{2x^2-\ldots}$$
$$-\frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(1092794276\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5+18284336132\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+20314214356\sqrt{2}\ldots\right.}{36864\left(2\left(\sqrt{2}x-\sqrt{2x^2-\ldots\right.\right.}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
giac")
```

output
```
1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 19
176431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
 + 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
+ 20314214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

3.339. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

3.340 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

3.340.1 Optimal result

Integrand size = 40, antiderivative size = 188

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx &= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496}\\ &+\frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)}-\frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4}\\ &+\frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3}-\frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2}\\ &+\frac{432565\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}-\frac{8969688643\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{21233664\sqrt{2}}\end{aligned}$$

output
```
1/95551488*(762984903+67865260*x)*(2*x^2-x+3)^(3/2)/(5+2*x)-3667/2304*(2*x
^2-x+3)^(5/2)/(5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995
/23887872*(2*x^2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)-8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x
+3)^(1/2))*2^(1/2)+1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(86386856771+121473790266x+60528581892x^2+11761910072}{(5+2x)^4}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
5,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
 11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^
4 + 8969688643*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 4484
833920*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/21233664
```

3.340.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 27, 2181, 1230, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}\,(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{288}\int \frac{(2x^2-x+3)^{3/2}\,(-11520x^3+31104x^2-99340x+51695)}{16(2x+5)^4}dx-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-11520x^3+31104x^2-99340x+51695)}{(2x+5)^4}dx}{4608}-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{216}\int \frac{3(2x^2-x+3)^{3/2}(414720x^2-3955064x+1998335)}{(2x+5)^3}dx+\frac{224815(2x^2-x+3)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\int \frac{\left(2x^2-x+3\right)^{3/2}\left(414720x^2-3955064x+1998335\right)}{(2x+5)^3}dx+\frac{224815\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\left(-\frac{1}{144}\int \frac{(84332303-203595780x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^2}dx-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{72(2x+5)^2}\right)+\frac{224815\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{1230}$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{1}{8}\int \frac{6(1170176463-3119804872x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{(67865260x+762984903)\left(2x^2-x+3\right)^{3/2}}{2(2x+5)}\right)-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{72(2x+5)^2}\right)+22}{4608}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\int \frac{(1170176463-3119804872x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{(67865260x+762984903)\left(2x^2-x+3\right)^{3/2}}{2(2x+5)}\right)-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{72(2x+5)^2}\right)+224}{4608}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(2(2339916063-389975609x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{576(1494965443-2989889280x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{(67865260x+7629849}{2(2x}\right.\right.}{4608}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\int \frac{1494965443-2989889280x}{(2x+5)\sqrt{2x^2-x+3}}dx+2\sqrt{2x^2-x+3}(2339916063-389975609x)\right)+\frac{(67865260x+762984903)(2x}{2(2x+5)}\right.\right.}{4608}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{1269}$$

3.340. $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}dx$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1494944640\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+2\sqrt{2x^2-x+3}(2339916063-38\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\downarrow$ 1090

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-747472320\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+2\sqrt{2x^2-x+3}(2\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\downarrow$ 222

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-747472320\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+2\sqrt{2x^2-x+3}(2339916063-\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\downarrow$ 1154

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(-17939377286\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-747472320\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+2\sqrt{2x^2-x+3}(2339\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\downarrow$ 219

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(-747472320\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{8969688643\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(23399160\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

3.340. $\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}dx$

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(2304*(5 + 2*x)^4) + ((224815*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)^3) + ((-14477995*(3 - x + 2*x^2)^(5/2))/(72*(5 + 2*
x)^2) + (((762984903 + 67865260*x)*(3 - x + 2*x^2)^(3/2))/(2*(5 + 2*x)) +
(3*(2*(2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2] + 18*(-747472320*Sqrt
[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (8969688643*ArcTanh[(17 - 22*x)/(12*Sqr
t[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))))/4)/144)/72)/4608
```

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.340. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.340. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

3.340.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{8969667840\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
fricas")
```

output
```
1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 89696
88643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)
*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x
 + 25)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3
 + 60528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(1
6*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

3.340.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**5, x)
```

3.340.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{16966315}{47775744}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{2304\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)} + \frac{224815\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{165888\left(8\,x^3+60\,x^2+150\,x+125\right)} - \frac{14477995\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{23887872\left(4\,x^2+20\,x+25\right)} - \frac{389975609}{31850496}\sqrt{2\,x^2-x+3}x - \frac{432565}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{8969688643}{42467328}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{779972021}{10616832}\sqrt{2\,x^2-x+3} + \frac{593321753\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{95551488\left(2\,x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
maxima")
```

output
```
16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/
(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3
)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)
^(5/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 43
2565/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/42
467328*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.340.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(153) = 306.

Time = 0.36 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.68

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx =$$

$$-\frac{1}{42467328}\sqrt{2}\left(8969688643\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+8969\right.$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
giac")
```

output
```
-1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896
9667840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1
))*sgn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 429
07*sgn(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54
1312039*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824
*(806241*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/
(2*x + 5)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^4*sgn(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
+ 6/(2*x + 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x
+ 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5)
 + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x
 + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3
)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

3.340. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

3.341 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx$

3.341.1 Optimal result

Integrand size = 40, antiderivative size = 195

$$\begin{aligned}
\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx &= \\
&-\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} \\
&+\frac{(246012435+44773976x)(3-x+2x^2)^{3/2}}{95551488(5+2x)^2}-\frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} \\
&+\frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4}-\frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} \\
&-\frac{23775\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}+\frac{70991525167\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1528823808\sqrt{2}}
\end{aligned}$$

output
```
1/95551488*(246012435+44773976*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2
*x^2-x+3)^(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-373050
7/11943936*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/127401984*(5658774871+1028823716*x)*(2*x^2-x+3)^(1
/2)/(5+2*x)
```

3.341.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-17093312738327-30872393829992x-21590439797064x^2-71}{(5+2x)^5}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
6,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 21590439797
064*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 159252
4800*x^6))/(5 + 2*x)^5 - 354957625835*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 -
2*x + 4*x^2])/6] - 177479424000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x
^2]])/7644119040
```

3.341.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2181, 27, 2181, 2181, 27, 1230, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{360}\int \frac{5(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-21168x+12007)}{16(2x+5)^5}dx-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-21168x+12007)}{(2x+5)^5}dx}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{288}\int \frac{(2x^2-x+3)^{3/2}(414720x^2-2790652x+1622891)}{(2x+5)^4}dx+\frac{158527(2x^2-x+3)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{2181}$$

$$\frac{\frac{1}{288}\left(-\frac{1}{216}\int \frac{3(22142555-44773976x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^3}dx-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+\frac{158527\left(2x^2-x+3\right)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{288}\left(-\frac{1}{72}\int \frac{(22142555-44773976x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^3}dx-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+\frac{158527\left(2x^2-x+3\right)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{1230}$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{32}\int \frac{4(514656291-1028823716x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx+\frac{(44773976x+246012435)\left(2x^2-x+3\right)^{3/2}}{4(2x+5)^2}\right)-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+158}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\int \frac{(514656291-1028823716x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx+\frac{(44773976x+246012435)\left(2x^2-x+3\right)^{3/2}}{4(2x+5)^2}\right)-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+15852}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{1230}$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(-\frac{1}{8}\int \frac{2(11831717167-23663923200x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(1028823716x+5658774871)}{2(2x+5)}\right)+\frac{(44773976x+246012435)\left(2x^2-x+3\right)}{4(2x+5)^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\downarrow \text{27}$$

3.341. $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(-\frac{1}{4}\int\frac{11831717167-23663923200x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(1028823716x+5658774871)}{2(2x+5)}\right)+\frac{(44773976x+246012435)\left(2x^2-x+3\right)^{3/2}}{4(2x+5)^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$\downarrow$ 1269

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(11831961600\int\frac{1}{\sqrt{2x^2-x+3}}dx-70991525167\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+5658774871)\sqrt{2x^2-x+}}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$\downarrow$ 1090

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-70991525167\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+565}{2(2}\right.\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$\downarrow$ 222

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-70991525167\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+5658774871)\sqrt{2x^2-}}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$\downarrow$ 1154

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(141983050334\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{(1028823716x+56587748}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$\downarrow$ 219

3.341. $\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}dx$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{70991525167\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{(1028823716x+5658774871)\sqrt{2x}}{2(2x+5)}\right.\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + ((158527*(3 - x + 2*x^2
)^(5/2))/(144*(5 + 2*x)^4) + ((-3730507*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*
x)^3) + (((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(4*(5 + 2*x)^2)
+ (3*(-1/2*((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) + (
5915980800*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (70991525167*ArcTanh[(17
 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4))/8)/72)/288)/1
152
```

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.341. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.341.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

3.341. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$

3.341.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{354958848000\,\sqrt{2}(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
fricas")
```

output
```
1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 3049684992
0*x^5 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872
393829992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 200
0*x^3 + 5000*x^2 + 6250*x + 3125)
```

3.341.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**6, x)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.29

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = -\frac{134077495}{3439853568}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{2880\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)} + \frac{158527\left(2x^2-x+3\right)^{\frac{5}{2}}}{165888\left(16x^4+160x^3+600x^2+1000x+625\right)} - \frac{3730507\left(2x^2-x+3\right)^{\frac{5}{2}}}{11943936\left(8x^3+60x^2+150x+125\right)} + \frac{134077495\left(2x^2-x+3\right)^{\frac{5}{2}}}{1719926784\left(4x^2+20x+25\right)} + \frac{3086715581}{2293235712}\sqrt{2x^2-x+3}x + \frac{23775}{1024}\sqrt{2}\,\text{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{70991525167}{3057647616}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{6173186729}{764411904}\sqrt{2x^2-x+3} - \frac{4698578717\left(2x^2-x+3\right)^{\frac{3}{2}}}{6879707136\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
maxima")
```

output
```
-134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5
/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/1658
88*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373
0507/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 13407
7495/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/229
3235712*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x
 - 1/23*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(
2*x^2 - x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.341.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.08

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{1}{256}\sqrt{2x^2-x+3}(20x-633) - \frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(8281387393360\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9+275661428628240\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8+1560382\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
giac")
```

output
```
1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*l
og(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057
647616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 - 1/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3))^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270
3345760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^4 + 70060241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 9773
0658088823880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11
)^5
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^6}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

3.342 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

3.342.1 Optimal result

Integrand size = 40, antiderivative size = 195

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} + \frac{369\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} - \frac{1903976002333\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{660451885056\sqrt{2}}$$

output
```
-1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-3667/
3456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^5
-14087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4*x
)*23^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/55037657088*(151764102421+27596573612*x)
*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.342.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{12\sqrt{3-x+2x^2}(458411625354581+1011372787716826x+910256842473992x^2\cdots}{(\cdots}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
7,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 910256842
473992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^
5 + 275188285440*x^6))/(5 + 2*x)^6 + 1903976002333*Sqrt[2]*ArcTanh[(5 + 2*
x - Sqrt[6 - 2*x + 4*x^2])/6] + 951979474944*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[
6 - 2*x + 4*x^2]])/660451885056
```

3.342.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2181, 27, 2181, 27, 2181, 1229, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{432}\int \frac{(2x^2-x+3)^{3/2}(-17280x^3+46656x^2-112340x+68375)}{16(2x+5)^6}dx-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-17280x^3+46656x^2-112340x+68375)}{(2x+5)^6}dx}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{360}\int \frac{5(2x^2-x+3)^{3/2}(622080x^2-3234816x+2112205)}{(2x+5)^5}dx+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(622080x^2-3234816x+2112205)}{(2x+5)^5}dx+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 2181

$$\frac{\frac{1}{72}\left(-\frac{1}{288}\int \frac{(84010769-145928500x)(2x^2-x+3)^{3/2}}{(2x+5)^4}dx-\frac{14087245(2x^2-x+3)^{5/2}}{144(2x+5)^4}\right)+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 1229

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{\int -\frac{6(13781234361-27596573612x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx}{1152}-\frac{(6793718806x+9802984711)(2x^2-x+3)^{3/2}}{96(2x+5)^3}\right)-\frac{14087245(2x^2-x+3)^{5/2}}{144(2x+5)^4}\right)+18}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(-\frac{1}{192}\int \frac{(13781234361-27596573612x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{(6793718806x+9802984711)(2x^2-x+3)^{3/2}}{96(2x+5)^3}\right)-\frac{14087245(2x^2-x+3)^{5/2}}{144(2x+5)^4}\right.}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 1230

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{8}\int \frac{2(317343544093-634652983296x)}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(27596573612x+151764102421)}{2(2x+5)}\right)-\frac{(6793718806x+9802984711)(2}{96(2x+5)^3}\right.\right.}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\int \frac{317343544093-634652983296x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(27596573612x+151764102421)}{2(2x+5)}\right)-\frac{(6793718806x+9802984711)(2x^2-}{96(2x+5)^3}\right.\right.}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

3.342. $\displaystyle\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

$\downarrow$ 1269

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-317326491648\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(27596573612x+151}{2(2x+5)}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$\downarrow$ 1090

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-158663245824\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-x+}}{}\right.\right.\right.}{691}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$\downarrow$ 222

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(27596573612x+}{2(2x+5)}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$\downarrow$ 1154

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(-3807952004666\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(275}{}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$\downarrow$ 219

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{1903976002333\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(27596573}{2(2x}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

3.342. $\int\frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}dx$

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + ((182165*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)^5) + ((-14087245*(3 - x + 2*x^2)^(5/2))/(144*(5 + 2
*x)^4) + (-1/96*((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(5 + 2
*x)^3 + (((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(2*(5 + 2*x)
) + (-158663245824*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (1903976002333*A
rcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4)/192)
/288)/72)/6912
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.342. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.342. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} \, dx$

3.342.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{1903958949888\,\sqrt{2}(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
fricas")
```

output
```
1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 2000
0*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6
000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
 + 48*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255
4114856528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 45841162535458
1)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x
^2 + 37500*x + 15625)
```

3.342.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**7, x)
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.52

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{3607708597}{1486016741376}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)} + \frac{182165\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{248832\left(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125\right)} - \frac{14087245\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{71663616\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)} + \frac{149610673\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5159780352\left(8\,x^3+60\,x^2+150\,x+125\right)} - \frac{3607708597\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{743008370688\left(4\,x^2+20\,x+25\right)} - \frac{82772668391}{990677827584}\sqrt{2\,x^2-x+3}x - \frac{369}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{1903976002333}{1320903770112}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{165562389227}{330225942528}\sqrt{2\,x^2-x+3} + \frac{125860542215\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{2972033482752\left(2\,x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
maxima")
```

output
```
3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)
^(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15
625) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 +
5000*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^
4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x +
3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 -
 x + 3)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 -
x + 3)*x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1903
976002333/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17
/23*sqrt(23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3)
 + 125860542215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.342.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.32

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{5}{64}\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(159278433934432\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11} + 6347903280912544\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10} + 485\ldots\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
giac")
```

output
```
369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 19
03976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*
x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/11
0075314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^11 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268
40833424*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 88313821135911024*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 -
x + 3))^6 - 397211131697032056*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5
 + 11708897232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 128034848607
28491138*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 1259303319786757723
4*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 589526263249780195)/(2*(sqrt(2)*x - sqrt(2*x
^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^7}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

3.343 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

3.343.1 Optimal result

Integrand size = 40, antiderivative size = 195

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx =\\ &-\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2}\\ &-\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4}-\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7}\\ &+\frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6}-\frac{1930441(3-x+2x^2)^{5/2}}{13934592(5+2x)^5}\\ &-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}+\frac{412760561351\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5283615080448\sqrt{2}}\end{aligned}$$

output
```
-1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/403
2*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-19
30441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(
1/2))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(
2*x^2-x+3)^(1/2))*2^(1/2)-1/440301256704*(146583836191+101679102454*x)*(2*
x^2-x+3)^(1/2)/(5+2*x)^2
```

3.343.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{12\sqrt{3-x+2x^2}(3479517268702637+9065154700300572x+997606536749818}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
8,x]
```

output
```
((-12*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 9976065
367498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 402255822731
712*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889323929457*Sqrt[2]*ArcTanh
[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1444738498560*Sqrt[2]*Log[1 - 4*x
+ 2*Sqrt[6 - 2*x + 4*x^2]])/36985305563136
```

3.343.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1229, 27, 1229, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{504}\int \frac{(2x^2-x+3)^{3/2}(-20160x^3+54432x^2-118840x+76715)}{16(2x+5)^7}dx-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-20160x^3+54432x^2-118840x+76715)}{(2x+5)^7}dx}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{432}\int \frac{9(2x^2-x+3)^{3/2}(483840x^2-2058628x+1481635)}{(2x+5)^6}dx+\frac{114335(2x^2-x+3)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{48}\int \frac{\left(2x^2-x+3\right)^{3/2}\left(483840x^2-2058628x+1481635\right)}{(2x+5)^6}dx+\frac{114335\left(2x^2-x+3\right)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{48}\left(-\frac{1}{360}\int \frac{35(1640279-2488320x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^5}dx-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^5}\right)+\frac{114335\left(2x^2-x+3\right)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\int \frac{(1640279-2488320x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^5}dx-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^5}\right)+\frac{114335\left(2x^2-x+3\right)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1229$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{(411822458x+463558457)\left(2x^2-x+3\right)^{3/2}}{576(2x+5)^4}-\frac{\int -\frac{6(300244177-477757440x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx}{2304}\right)-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^5}\right)+\frac{114335(\cdots}{24(\cdots}}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\int \frac{(300244177-477757440x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{(411822458x+463558457)\left(2x^2-x+3\right)^{3/2}}{576(2x+5)^4}\right)-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^5}\right)+11\cdots}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1229$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{(101679102454x+146583836191)\sqrt{2x^2-x+3}}{288(2x+5)^2}-\frac{\int -\frac{2(68775204551-137594142720x)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}\right)+\frac{(411822458x+463558457)\left(2x^2-x\cdots\right.}{576(2x+5)^4}\cdots\right.\right.}{8064}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 27$$

3.343. $\displaystyle\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\int\frac{68775204551-137594142720x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(101679102454x+146583836191)}{288(2x+5)^2}\right)+\frac{(411822458x+463558457)(2x^2}{576(2x+5)^4}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\big\downarrow$ 1269

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-68797071360\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(101679102454x+14}{288(2x+5)^2}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\big\downarrow$ 1090

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-34398535680\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-x+}}{}\right.\right.\right.}{806}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\big\downarrow$ 222

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(101679102454x}{288(2x+5)}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\big\downarrow$ 1154

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(-825521122702\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(101}{}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\big\downarrow$ 219

3.343. $\displaystyle\int\frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{412760561351\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(10167910}{288(2}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + ((114335*(3 - x + 2*x^2
)^(5/2))/(24*(5 + 2*x)^6) + ((-1930441*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*x
)^5) - (7*(((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x
)^4) + (((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(288*(5 + 2*
x)^2) + (-34398535680*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (412760561351
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/576)/
384))/72)/48)/8064
```

3.343.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.343. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.343. $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

3.343.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \frac{2889476997120\sqrt{2}(128\,x^7+2240\,x^6+16800\,x^5+70000$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
fricas")
```

output
```
1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*
(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 21
8750*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2
 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582
2731712*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 99760653674981
88*x^2 + 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*
x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*
x + 78125)
```

3.343.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x***4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**8, x)
```

3.343.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.78

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{769352975}{11888133931008}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{4032\left(128x^7+2240x^6+16800x^5+70000x^4+175000x^3+262500x^2+218750x+78125\right)} + \frac{114335\left(2x^2-x+3\right)^{\frac{5}{2}}}{193536\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)} - \frac{1930441\left(2x^2-x+3\right)^{\frac{5}{2}}}{13934592\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)} + \frac{7861079\left(2x^2-x+3\right)^{\frac{5}{2}}}{573308928\left(16x^4+160x^3+600x^2+1000x+625\right)} - \frac{32967491\left(2x^2-x+3\right)^{\frac{5}{2}}}{41278242816\left(8x^3+60x^2+150x+125\right)} + \frac{769352975\left(2x^2-x+3\right)^{\frac{5}{2}}}{5944066965504\left(4x^2+20x+25\right)} + \frac{17957520133}{7925422620672}\sqrt{2x^2-x+3}\,x + \frac{5}{128}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{412760561351}{10567230160896}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right) - \frac{35893173457}{2641807540224}\sqrt{2x^2-x+3} - \frac{27452157541\left(2x^2-x+3\right)^{\frac{3}{2}}}{23776267862016\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
maxima")
```

output
```
-769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3
)^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*
x^2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 96
0*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/1393
4592*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*
x + 3125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 60
0*x^2 + 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3
+ 60*x^2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4
*x^2 + 20*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/12
8*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230
160896*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) - 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23
776267862016*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

3.343.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.51

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx =$$

$$-\frac{5}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(1121897398412224\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{13}+48260296303776704\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{12}+\cdots\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
giac")
```

output
```
-5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 412
760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x
 - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(1121897
398412224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704
*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^9 - 17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130
12094737246112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 1065158801360
64432096*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(
2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3
))^2 + 92292080735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15
161716093827501349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^7
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^8}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

3.344 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

3.344.1 Optimal result

Integrand size = 38, antiderivative size = 120

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} - \frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048} - \frac{85429\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

output

```
-85429/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+761/256*(5+2*x)^2*(2*x^
2-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/16*(5+2*x)^4*(2*x^2-x+3
)^(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)
```

3.344.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(2973-6916x+352x^2+7040x^3+2048x^4)-85429\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 8
5429*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8192
```

3.344.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {2184, 27, 2184, 27, 2184, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{160}\int -\frac{5(2x+5)\left(840x^3+1116x^2+878x+1011\right)}{\sqrt{2x^2-x+3}}dx+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}(2x+5)^4\sqrt{2x^2-x+3}-\frac{1}{32}\int \frac{(2x+5)\left(840x^3+1116x^2+878x+1011\right)}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{32}\left(-\frac{1}{64}\int -\frac{8(2x+5)\left(9132x^2+2636x+8187\right)}{\sqrt{2x^2-x+3}}dx-\frac{105}{4}\sqrt{2x^2-x+3}(2x+5)^3\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{8}\int \frac{(2x+5)\left(9132x^2+2636x+8187\right)}{\sqrt{2x^2-x+3}}dx-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{24}\int \frac{12(1915-2338x)(2x+5)}{\sqrt{2x^2-x+3}}dx+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{(1915-2338x)(2x+5)}{\sqrt{2x^2-x+3}}dx+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429}{8}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{8\sqrt{46}}-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right)\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2x\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

3.344. $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx$

output
```
((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 + ((-105*(5 + 2*x)^3*Sqrt[3 - x + 2*x
^2])/4 + (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2] + (-1/4*((19227 + 4676*x)*Sq
rt[3 - x + 2*x^2]) + (85429*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2]))/2)/
8)/32
```

3.344.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.344. $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

3.344.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(2048x^4+7040x^3+352x^2-6916x+2973)\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\left(x^4+\frac{55}{16}x^3+\frac{11}{64}x^2-\frac{1729}{512}x+\frac{2973}{2048}\right)\sqrt{2x^2-x+3}-\frac{85429\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootC}\right.}{8192}$
default	$\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}+\frac{11x^2\sqrt{2x^2-x+3}}{64}-\frac{1729x\sqrt{2x^2-x+3}}{512}+\frac{2973\sqrt{2x^2-x+3}}{2048}+x^4\sqrt{2x^2-x+3}+\frac{55}{}$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2048*(2048*x^4+7040*x^3+352*x^2-6916*x+2973)*(2*x^2-x+3)^(1/2)+85429/819
2*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3} + \frac{85429}{16384}\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
 + 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\left(x^4+\frac{55x^3}{16}+\frac{11x^2}{64}-\frac{1729x}{512}+\frac{2973}{2048}\right)+\frac{85429\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(x**4 + 55*x**3/16 + 11*x**2/64 - 1729*x/512 + 2973/2
048) + 85429*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/8192
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2\,x^2-x+3}x^4+\frac{55}{16}\,\sqrt{2\,x^2-x+3}x^3+\frac{11}{64}\,\sqrt{2\,x^2-x+3}x^2-\frac{1729}{512}\,\sqrt{2\,x^2-x+3}x+\frac{85429}{8192}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{2973}{2048}\,\sqrt{2\,x^2-x+3}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="ma
xima")
```

output
```
sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
 - x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsin
h(1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{2048}(4(8(4(16x+55)x+11)x-1729)x+2973)\sqrt{2x^2-x+3} - \frac{85429}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gi
ac")
```

output
```
1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3
) - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) +
1)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{\sqrt{2x^2-x+3}}\,dx$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

3.345.1 Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}-\frac{6863\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

output
```
-6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-505/1024*(2*x^2-x+3)^(1/
2)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)+5/8*x^3*(2*x^2-
x+3)^(1/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-1515-1636x+608x^2+1920x^3)-20589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[
2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

3.345.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{19x^3-42x^2+16x+32}{2\sqrt{2x^2-x+3}}dx+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{19x^3-42x^2+16x+32}{\sqrt{2x^2-x+3}}dx+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{6}\int \frac{-409x^2-36x+384}{2\sqrt{2x^2-x+3}}dx+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\int \frac{-409x^2-36x+384}{\sqrt{2x^2-x+3}}dx+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{1}{4}\int \frac{3(1842-505x)}{2\sqrt{2x^2-x+3}}dx-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\int \frac{1842-505x}{\sqrt{2x^2-x+3}}dx-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(5*x^3*Sqrt[3 - x + 2*x^2])/8 + ((19*x^2*Sqrt[3 - x + 2*x^2])/6 + ((-409*x
*Sqrt[3 - x + 2*x^2])/4 + (3*((-505*Sqrt[3 - x + 2*x^2])/2 + (6863*ArcSinh
[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/12)/16
```

3.345.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.345. $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.345.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$\frac{(1920x^3+608x^2-1636x-1515)\sqrt{2x^2-x+3}}{3072}+\frac{6863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$
trager	$\left(\frac{5}{8}x^3+\frac{19}{96}x^2-\frac{409}{768}x-\frac{505}{1024}\right)\sqrt{2x^2-x+3}+\frac{6863\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\mathrm{Roo}\right.}{4096}$
default	$-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{6863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}+\frac{5x^3\sqrt{2x^2-x+3}}{8}+\frac{19x^2\sqrt{2x^2-x+3}}{96}-\frac{409x\sqrt{2x^2-x+3}}{768}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(1920*x^3+608*x^2-1636*x-1515)*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3} + \frac{6863}{8192}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/819
2*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^3}{8}+\frac{19x^2}{96}-\frac{409x}{768}-\frac{505}{1024}\right) + \frac{6863\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**3/8 + 19*x**2/96 - 409*x/768 - 505/1024) + 6863
*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4096
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}\sqrt{2\,x^2-x+3}x^3+\frac{19}{96}\sqrt{2\,x^2-x+3}x^2 - \frac{409}{768}\sqrt{2\,x^2-x+3}x + \frac{6863}{4096}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{505}{1024}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt
(2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 5
05/1024*sqrt(2*x^2 - x + 3)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\,(4\,(8\,(60\,x+19)x-409)x-1515)\sqrt{2\,x^2-x+3} - \frac{6863}{4096}\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/409
6*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

3.346.1 Optimal result

Integrand size = 40, antiderivative size = 126

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{1669}{128}\sqrt{3-x+2x^2}-\frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2}+\frac{9657\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}} - \frac{3667\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{96\sqrt{2}}$$

output

```
9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/24*(17-
22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1669/128*(2*x^2-x+3)^(1/2)-337/19
2*(5+2*x)*(2*x^2-x+3)^(1/2)+5/48*(5+2*x)^2*(2*x^2-x+3)^(1/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(2637-548x+160x^2)+58672\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+28971\sqrt{2}\log\left(\right.}{1536}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 58672*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 28971*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6
- 2*x + 4*x^2]])/1536
```

3.346.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2184, 25, 2184, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\int -\frac{2696x^3+4092x^2+3054x+2183}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{48}(2x+5)^2\sqrt{2x^2-x+3}-\frac{1}{96}\int \frac{2696x^3+4092x^2+3054x+2183}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\left(-\frac{1}{32}\int -\frac{24(6676x^2+5364x+1021)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{337}{2}\sqrt{2x^2-x+3}(2x+5)\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{96}\left(\frac{3}{4}\int \frac{6676x^2+5364x+1021}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{4(10387-19314x)}{(2x+5)\sqrt{2x^2-x+3}}dx+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{10387-19314x}{(2x+5)\sqrt{2x^2-x+3}}dx+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-9657\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{9657\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\sqrt{\ldots}\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(-117344\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{14668}{3}\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

3.346. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + ((-337*(5 + 2*x)*Sqrt[3 - x + 2*x
^2])/2 + (3*(1669*Sqrt[3 - x + 2*x^2] + ((-9657*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/Sqrt[2] - (14668*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x +
2*x^2])])/3)/2))/4)/96
```

3.346.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.346.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{1}{384}\left(160\,x^2-548\,x+2637\right)\sqrt{2\,x^2-x+3} + \frac{9657}{1024}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right) + \frac{3667}{384}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fr
icas")
```

output
```
1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log
(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*
sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036
*x + 1153)/(4*x^2 + 20*x + 25))
```

3.346.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3))
, x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx &= \frac{5}{12}\sqrt{2x^2-x+3}x^2 - \frac{137}{96}\sqrt{2x^2-x+3}x \\ &\quad - \frac{9657}{512}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &\quad + \frac{3667}{192}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) \\ &\quad + \frac{879}{128}\sqrt{2x^2-x+3}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="ma
xima")
```

output
```
5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(2
2/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt
(2*x^2 - x + 3)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{1}{384}\left(4\left(40\,x-137\right)x+2637\right)\sqrt{2\,x^2-x+3} + \frac{9657}{512}\,\sqrt{2}\log\left(-4\,\sqrt{2}x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right) - \frac{3667}{192}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{3667}{192}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gi
ac")
```

output
```
1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log
(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(ab
s(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(
abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

3.347.1 Optimal result

Integrand size = 40, antiderivative size = 126

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx=-\frac{243}{64}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)}+\frac{5}{32}(5+2x)\sqrt{3-x+2x^2}-\frac{2943\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}+\frac{158527\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{6912\sqrt{2}}$$

output
```
-2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-243/64*(2*x^2-x+3)^(1/2)-366
7/576*(2*x^2-x+3)^(1/2)/(5+2*x)+5/32*(5+2*x)*(2*x^2-x+3)^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx=\frac{\frac{24\sqrt{3-x+2x^2}(-6176-1287x+180x^2)}{5+2x}-158527\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-79461\sqrt{2}\log\left(1-4x+\right.}{6912}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((24*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158527*Sq
rt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 79461*Sqrt[2]*Log[1 -
 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6912
```

3.347.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2184, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{72}\int \frac{-2880x^3+7776x^2-21168x+12007}{16(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2880x^3+7776x^2-21168x+12007}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{180(2x+5)\sqrt{2x^2-x+3}-\frac{1}{32}\int \frac{32(17496x^2-13608x+15157)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{180(2x+5)\sqrt{2x^2-x+3}-\int \frac{17496x^2-13608x+15157}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{-\frac{1}{8}\int \frac{16(13046-26487x)}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-2\int \frac{13046-26487x}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \; 1269$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487}{2}\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \; 1090$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \; 222$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \; 1154$$

$$\frac{-2\left(-158527\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \; 219$$

$$\frac{-2\left(-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{158527\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{12\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

3.347. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x)) + (-4374*Sqrt[3 - x + 2*x^2] +
 180*(5 + 2*x)*Sqrt[3 - x + 2*x^2] - 2*((-26487*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/(2*Sqrt[2]) - (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x
^2])])/(12*Sqrt[2])))/1152
```

3.347.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.347. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.347.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{158922\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+158527\sqrt{2}(2x+5)\log\left(24\right.}{27648\,(2x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25
)) + 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

3.347.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x +
3)), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \frac{5}{16}\sqrt{2x^2-x+3}x + \frac{2943}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{158527}{13824}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{193}{64}\sqrt{2x^2-x+3} - \frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5)
- 17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqr
t(2*x^2 - x + 3)/(2*x + 5)
```

3.347.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(99) = 198.

Time = 0.34 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.69

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{158922\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}\right.\right.}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) +
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x +
5)) - 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 10
8*(3393*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*
(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x
+ 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{(2\,x+5)}^2\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)
```

3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

3.348.1 Optimal result

Integrand size = 40, antiderivative size = 128

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{331776\sqrt{2}}$$

output
```
149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1546507/663552*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+5/16*(2*x^2-x+3)^(1/2)-3667/1
152*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239/27648*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.348.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(589187+357278x+34560x^2)}{(5+2x)^2}+1546507\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+772416\sqrt{2}\log\left(1\right.}{331776}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 15
46507*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 772416*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/331776
```

3.348.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2181, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{144}\int \frac{-5760x^3+15552x^2-27668x+20347}{16(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-5760x^3+15552x^2-27668x+20347}{(2x+5)^2\sqrt{2x^2-x+3}}dx}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int \frac{3(69120x^2-359424x+215947)}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{24}\int \frac{69120x^2-359424x+215947}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{1}{24}\left(\frac{1}{8}\int \frac{8(259147-514944x)}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{24}\left(\int \frac{259147-514944x}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\big\downarrow$ 1269

$$\frac{\frac{\frac{1}{24}\left(-257472\int \frac{1}{\sqrt{2x^2-x+3}}dx+1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}}{}$$

$\big\downarrow$ 1090

$$\frac{\frac{1}{24}\left(1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-128736\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\big\downarrow$ 222

$$\frac{\frac{1}{24}\left(1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\big\downarrow$ 1154

$$\frac{\frac{1}{24}\left(-3093014\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{24}\left(-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{1546507\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

3.348. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2) + ((92239*Sqrt[3 - x + 2*x^
2])/(12*(5 + 2*x)) + (17280*Sqrt[3 - x + 2*x^2] - 128736*Sqrt[2]*ArcSinh[(
-1 + 4*x)/Sqrt[23]] - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
+ 2*x^2])])/(6*Sqrt[2]))/24)/2304
```

3.348.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.348. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.348.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1544832\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+1546507\sqrt{2}(4x^2 \cdots}{1327104(4 \cdots}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x +
1153)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2
 - x + 3))/(4*x^2 + 20*x + 25)
```

3.348.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x +
3)), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx &= -\frac{149}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\ &\quad+\frac{1546507}{663552}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\ &\quad+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3667\sqrt{2\,x^2-x+3}}{1152\,(4\,x^2+20\,x+25)}\\ &\quad+\frac{92239\sqrt{2\,x^2-x+3}}{27648\,(2\,x+5)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
-149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5
)) + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*
x + 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

3.348.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.94

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) - \frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{5}{16}\sqrt{2\,x^2-x+3} + \frac{\sqrt{2}\left(2381290\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3+16628406\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2-25697445\sqrt{2}\left(\sqrt{2}x-\right.\right.}{55296\left(2\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 154
6507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)
)) + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645
)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3)) - 11)^2
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,\sqrt{2\,x^2-x+3}} \,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

3.349.1 Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{5\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} + \frac{22389491\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{71663616\sqrt{2}}$$

output
```
-5/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/
24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/1728*(2*x^2-x+3)^(1/2
)/(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3163415/5971968*(2*x
^2-x+3)^(1/2)/(5+2*x)
```

3.349.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{-\frac{12\sqrt{3-x+2x^2}(44369687+44312764x+12653660x^2)}{(5+2x)^3}-22389491\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-1119744}{71663616}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((-12*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x
)^3 - 22389491*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1119
7440*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/71663616
```

3.349.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2181, 27, 2181, 2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{216}\int \frac{-8640x^3+23328x^2-34168x+28687}{16(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-8640x^3+23328x^2-34168x+28687}{(2x+5)^3\sqrt{2x^2-x+3}}dx}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{622080x^2-1655188x+1464275}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{3(3727091-7464960x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{24}\int \frac{3727091-7464960x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(3732480\int\frac{1}{\sqrt{2x^2-x+3}}dx-22389491\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 1090$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-22389491\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2}}{72(2x}}{3456}\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-22389491\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(44778982\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+}}{72(2x+5)^2}}{3456}\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{22389491\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

3.349. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + ((394907*Sqrt[3 - x + 2*x
^2])/(72*(5 + 2*x)^2) + ((-3163415*Sqrt[3 - x + 2*x^2])/(12*(5 + 2*x)) + (
1866240*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (22389491*ArcTanh[(17 - 22*
x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/24)/144)/3456
```

3.349.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.349. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.349.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{22394880\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+2238}{2}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(
8*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x -
17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 +
44312764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

3.349.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x +
3)), x)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx &= \frac{5}{32}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\ &\quad-\frac{22389491}{143327232}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\ &\quad-\frac{3667\sqrt{2\,x^2-x+3}}{1728\,(8\,x^3+60\,x^2+150\,x+125)}\\ &\quad+\frac{394907\sqrt{2\,x^2-x+3}}{248832\,(4\,x^2+20\,x+25)}-\frac{3163415\sqrt{2\,x^2-x+3}}{5971968\,(2\,x+5)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232
*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 39490
7/248832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*
x^2 - x + 3)/(2*x + 5)
```

3.349.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(108) = 216.

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(215012404\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+3010410772\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+2740802468\sqrt{2}(\right.}{11943936\left(2\left(\sqrt{2}x-\sqrt{2\,x^2\right.\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
-5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 2238
9491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*s
qrt(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 +
2740802468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt
(2)*x - sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqr
t(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

3.350.1 Optimal result

Integrand size = 40, antiderivative size = 139

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)} + \frac{2053207\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{20639121408\sqrt{2}}$$

output
```
2053207/41278242816*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1
/2)-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/
(5+2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085/171992678
4*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.350.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-298655447-255525906x+43592076x^2+214400680x^3)}{(5+2x)^4}-2053207\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20639121408}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 214400
680*x^3))/(5 + 2*x)^4 - 2053207*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x +
4*x^2])/6])/20639121408
```

3.350.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2181, 27, 2181, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{288}\int \frac{-11520x^3+31104x^2-40668x+37027}{16(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-11520x^3+31104x^2-40668x+37027}{(2x+5)^4\sqrt{2x^2-x+3}}dx}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{216}\int \frac{1244160x^2-2364856x+2607829}{(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{216}\left(-\frac{1}{144}\int \frac{13(1493165-1876588x)}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\int \frac{1493165-1876588x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(\frac{157939}{24}\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(-\frac{157939}{12}\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(-\frac{157939\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{144\sqrt{2}}-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + ((513097*Sqrt[3 - x + 2*x
^2])/(108*(5 + 2*x)^3) + ((-16295969*Sqrt[3 - x + 2*x^2])/(72*(5 + 2*x)^2)
 - (13*((-2061545*Sqrt[3 - x + 2*x^2])/(12*(5 + 2*x)) - (157939*ArcTanh[(1
7 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(144*Sqrt[2])))/144)/216)/460
8
```

3.350.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.350. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.350.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{2053207\sqrt{2}(16x^4+160x^3+600x^2+1000x+625)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48(2}{82556485632(16x^4+160x^3+600x^2+100}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 115
3)/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x -
 298655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5)
```

3.350.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x +
3)), x)
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{2053207}{41278242816}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{3667\,\sqrt{2\,x^2-x+3}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} + \frac{513097\,\sqrt{2\,x^2-x+3}}{497664\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{16295969\,\sqrt{2\,x^2-x+3}}{71663616\,(4\,x^2+20\,x+25)} + \frac{26800085\,\sqrt{2\,x^2-x+3}}{1719926784\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
-2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3
+ 600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*
x^2 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{1}{41278242816}\,\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{792072}{(2\,x+5)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}-\frac{16295969}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}+\frac{26800085}{\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)}\right)\sqrt{-\frac{11}{2\,x+5}+ \cdots}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
 + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5)
+ 26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20
53207*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)
/sgn(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^5\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

3.351 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

3.351.1 Optimal result

Integrand size = 40, antiderivative size = 124

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}}-\frac{13153}{512}\sqrt{3-x+2x^2}+\frac{2645}{128}x\sqrt{3-x+2x^2}+\frac{153}{16}x^2\sqrt{3-x+2x^2}+\frac{5}{4}x^3\sqrt{3-x+2x^2}+\frac{144217\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

output
```
144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4/23*(346-533*x)/(2*x^2
-x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128*x*(2*x^2-x+3)^(1/2)+153/1
6*x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{4(-1616165+2124123x-510554x^2+418232x^3+210496x^4+29440x^5)}{\sqrt{3-x+2x^2}}+3316991\sqrt{}}{47104}$$

input
```
Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2
),x]
```

output
```
((4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x
^5))/Sqrt[3 - x + 2*x^2] + 3316991*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/47104
```

3.351.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2191, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23\left(-10x^4-53x^3-70x^2+25x+66\right)}{2\sqrt{2x^2-x+3}}dx-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{-10x^4-53x^3-70x^2+25x+66}{\sqrt{2x^2-x+3}}dx-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$-\frac{1}{8}\int \frac{-459x^3-470x^2+200x+528}{\sqrt{2x^2-x+3}}dx-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{153}{2}x^2\sqrt{2x^2-x+3}-\frac{1}{6}\int \frac{3\left(-2645x^2+2636x+2112\right)}{2\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{153}{2}x^2\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{-2645x^2+2636x+2112}{\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{2645}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int\frac{13153x+32766}{2\sqrt{2x^2-x+3}}dx\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{2645}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int\frac{13153x+32766}{\sqrt{2x^2-x+3}}dx\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$\Big\downarrow$ 1160

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right.$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$\Big\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-x\right.$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$\Big\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)-$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

input
```
Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*x^3*Sqrt[3 - x + 2*x^2])/
4 + ((153*x^2*Sqrt[3 - x + 2*x^2])/2 + ((2645*x*Sqrt[3 - x + 2*x^2])/4 + (
(-13153*Sqrt[3 - x + 2*x^2])/2 - (144217*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*
Sqrt[2]))/8)/4)/8
```

3.351. $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

3.351.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.351. $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

3.351.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{3316991\sqrt{2}(2x^2-x+3)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32\right.}{}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3
- 510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

3.351.6 Sympy [F]

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(2x+5)^2\cdot(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(
3/2), x)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{5\,x^5}{2\sqrt{2\,x^2-x+3}} + \frac{143\,x^4}{8\sqrt{2\,x^2-x+3}} + \frac{2273\,x^3}{64\sqrt{2\,x^2-x+3}} - \frac{11099\,x^2}{256\sqrt{2\,x^2-x+3}} - \frac{144217}{2048}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{2124123\,x}{11776\sqrt{2\,x^2-x+3}} - \frac{1616165}{11776\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/
sqrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt
(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3)
 - 1616165/11776/sqrt(2*x^2 - x + 3)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(46\,(4\,(8\,(20\,x+143)x+2273)x-11099)x+2124123)x-1616165}{11776\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 16
16165)/sqrt(2*x^2 - x + 3)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{{\left(2\,x+5\right)}^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

input
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.352 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

3.352.1 Optimal result

Integrand size = 38, antiderivative size = 103

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx=\frac{-53+373x}{23\sqrt{3-x+2x^2}}+\frac{33}{64}\sqrt{3-x+2x^2}+\frac{193}{48}x\sqrt{3-x+2x^2}+\frac{5}{6}x^2\sqrt{3-x+2x^2}+\frac{3111\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

output
```
3111/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/23*(-53+373*x)/(2*x^2-x+
3)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2-x+3)^(1/2)+5/6*x^2*(2*x^2
-x+3)^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx=\frac{-3345+122607x-2162x^2+31832x^3+7360x^4}{4416\sqrt{3-x+2x^2}}+\frac{3111\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),
x]
```

output
```
(-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*
x^2]) + (3111*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(128*Sqrt[2])
```

3.352.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2191, 27, 2192, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23\left(-10x^3-28x^2+25\right)}{4\sqrt{2x^2-x+3}}dx-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{-10x^3-28x^2+25}{\sqrt{2x^2-x+3}}dx-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2}\left(\frac{5}{3}x^2\sqrt{2x^2-x+3}-\frac{1}{6}\int \frac{-193x^2+60x+150}{\sqrt{2x^2-x+3}}dx\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{9(262-11x)}{2\sqrt{2x^2-x+3}}dx\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\int \frac{262-11x}{\sqrt{2x^2-x+3}}dx\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

output
```
-1/23*(53 - 373*x)/Sqrt[3 - x + 2*x^2] + ((5*x^2*Sqrt[3 - x + 2*x^2])/3 +
((193*x*Sqrt[3 - x + 2*x^2])/4 - (9*((-11*Sqrt[3 - x + 2*x^2])/2 + (1037*A
rcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/6)/2
```

3.352.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.352. $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.352.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}+\frac{3111\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x}\right.}{256}$
default	$\frac{\frac{10185x}{2944}-\frac{10185}{11776}}{\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}+\frac{55}{512\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{5x^4}{3\sqrt{2x^2-x+3}}+$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4416*(7360*x^4+31832*x^3-2162*x^2+122607*x-3345)/(2*x^2-x+3)^(1/2)-3111/
256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx = \frac{214659\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x\right.}{}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fr
icas")
```

output
```
1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 122
607*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

3.352.6 Sympy [F]

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2
), x)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{(3-x+2x^2)^{3/2}}\,dx = \frac{5\,x^4}{3\,\sqrt{2\,x^2-x+3}} + \frac{173\,x^3}{24\,\sqrt{2\,x^2-x+3}} - \frac{47\,x^2}{96\,\sqrt{2\,x^2-x+3}} - \frac{3111}{256}\,\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{40869\,x}{1472\,\sqrt{2\,x^2-x+3}} - \frac{1115}{1472\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="ma
xima")
```

output
```
5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/s
qrt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 4
0869/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(40\,x+173)x-47)x+122607)x-3345}{4416\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gi
ac")
```

output
```
3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x +
3)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx$

3.353.1 Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} + \frac{213\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

output
```
213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/92*(89+219*x)/(2*x^2-x+3)
^(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3)^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{2575+2511x+782x^2+920x^3}{736\sqrt{3-x+2x^2}} + \frac{213\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (213*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

3.353.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23(-20x^2-6x+15)}{16\sqrt{2x^2-x+3}}dx+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{219x+89}{92\sqrt{2x^2-x+3}}-\frac{1}{8}\int \frac{-20x^2-6x+15}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{6(20-9x)}{\sqrt{2x^2-x+3}}dx\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\int \frac{20-9x}{\sqrt{2x^2-x+3}}dx\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

3.353. $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}dx$

output
```
(89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (5*x*Sqrt[3 - x + 2*x^2] - (3*((-9
*Sqrt[3 - x + 2*x^2])/2 + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/
2)/8
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.353. $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx$

3.353.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	size
risch	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$	45
trager	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\mathrm{RootOf}\left(\_Z^2-2\right)\right)}{128}$	72
default	$\frac{901}{256\sqrt{2x^2-x+3}}+\frac{\frac{123x}{1472}-\frac{123}{5888}}{\sqrt{2x^2-x+3}}+\frac{5x^3}{4\sqrt{2x^2-x+3}}+\frac{17x^2}{16\sqrt{2x^2-x+3}}+\frac{213x}{64\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$	98

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/736*(920*x^3+782*x^2+2511*x+2575)/(2*x^2-x+3)^(1/2)-213/128*2^(1/2)*arcs
inh(4/23*23^(1/2)*(x-1/4))
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{4899\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right.}{5888\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(
2*x^2 - x + 3))/(2*x^2 - x + 3)
```

3.353.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}} + \frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}} - \frac{213}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}} + \frac{2575}{736\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt
(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 25
75/736/sqrt(2*x^2 - x + 3)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{213}{128}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(46\,(20\,x+17)x+2511)x+2575}{736\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

3.354.1 Optimal result

Integrand size = 40, antiderivative size = 101

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} - \frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}$$

output
```
39/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22
*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/3312*(1191+917*x)/(2*x^2-x+3)^(1/
2)+5/8*(2*x^2-x+3)^(1/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{\frac{12(7401-1153x+4140x^2)}{\sqrt{3-x+2x^2}} + 84341\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) + 4}{39744}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),
x]
```

output
```
((12*(7401 - 1153*x + 4140*x^2))/Sqrt[3 - x + 2*x^2] + 84341*Sqrt[2]*ArcTa
nh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 48438*Sqrt[2]*Log[1 - 4*x + 2*Sq
rt[6 - 2*x + 4*x^2]])/39744
```

3.354.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int -\frac{23\left(-720x^2-216x+293\right)}{576(2x+5)\sqrt{2x^2-x+3}}dx+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}-\frac{1}{288}\int \frac{-720x^2-216x+293}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\left(180\sqrt{2x^2-x+3}-\frac{1}{8}\int -\frac{8(157-1404x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{288}\left(\int \frac{157-1404x}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{288}\left(-702\int \frac{1}{\sqrt{2x^2-x+3}}dx+3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{288}\left(3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-351\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{288}\left(3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{288}\left(-7334\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{288}\left(-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (180*Sqrt[3 - x + 2*x^2] - 351
*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt
[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/288
```

3.354.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.354. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx$

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.354. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx$

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

3.354.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{96876\sqrt{2}(2x^2-x+3)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fr
icas")
```

output
```
1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*s
qrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
+ 20*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2
- x + 3)
```

3.354.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/
2)), x)
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{5\,x^2}{4\,\sqrt{2\,x^2-x+3}} - \frac{39}{32}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x-\frac{1}{23}\,\sqrt{23}\right) + \frac{3667}{3456}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{1153\,x}{3312\,\sqrt{2\,x^2-x+3}} + \frac{2467}{1104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="ma
xima")
```

output
```
5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23
*sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sq
rt(2*x^2 - x + 3)
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{39}{32}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right) - \frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{(4140x-1153)x+7401}{3312\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gi
ac")
```

output
```
39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/3
456*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 366
7/3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\,(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)
```

3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx$

3.355.1 Optimal result

Integrand size = 40, antiderivative size = 108

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)} - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}} + \frac{25951\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}$$

output
```
-5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24*(17-
22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/119232*(9897+2203*x)/(2*x^2-x+3
)^(1/2)-3667/10368*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.355.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = -\frac{\sqrt{3-x+2x^2}(51351-48653x+53290x^2)}{79488(15+x+8x^2+4x^3)} - \frac{25951\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20736\sqrt{2}} - \frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
-1/79488*(Sqrt[3 - x + 2*x^2]*(51351 - 48653*x + 53290*x^2))/(15 + x + 8*x
^2 + 4*x^3) - (25951*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(20736*
Sqrt[2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

3.355.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{23}\int -\frac{23\left(-25920x^2-11410x+1463\right)}{20736(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{\int \frac{-25920x^2-11410x+1463}{(2x+5)^2\sqrt{2x^2-x+3}}dx}{10368}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{72}\int -\frac{108(4351-8640x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{3}{2}\int \frac{4351-8640x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-4320\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2160\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\frac{3}{2}\left(-51902\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{3}{2}\left(-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{25951\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) + ((-3667*Sqrt[3 - x + 2*x^2]
)/(5 + 2*x) - (3*(-2160*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (25951*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2])))/2)/10368
```

3.355.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.355. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\, dx$

3.355.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.45

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{596160\,\sqrt{2}(4\,x^3+8\,x^2+x+15)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x
^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^
2 + x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1
036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt
(2*x^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

3.355.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^2\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**
(3/2)), x)
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{5}{16}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{25951}{82944}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{26645\,x}{79488\sqrt{2\,x^2-x+3}} + \frac{30313}{26496\sqrt{2\,x^2-x+3}} - \frac{3667}{576\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2
)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 2
6645/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/
576/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(85) = 170.

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{1}{1907712}\sqrt{2}\left(\frac{12\left(\frac{\frac{315103}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{1012092}{(2\,x+5)\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}-\frac{26645}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}} + \frac{596873\log\left(\right.}{}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1
/(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
+ 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(ab
s(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x
+ 5)))
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^2\,(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx$

3.356.1 Optimal result

Integrand size = 40, antiderivative size = 112

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5971968\sqrt{2}}$$

output
```
-52631/11943936*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+
1/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5
+2*x)^2+115369/1492992*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.356.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{12(11594283+5842933x+3263288x^2+3444340x^3)}{(5+2x)^2\sqrt{3-x+2x^2}} + 1210513\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\right.\right.}{137355264}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/((5 + 2*x)^2*Sqrt
[3 - x + 2*x^2]) + 1210513*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2
])/6])/137355264
```

3.356.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int \frac{23\left(977500x^2+632660x+224707\right)}{746496(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{977500x^2+632660x+224707}{(2x+5)^3\sqrt{2x^2-x+3}}dx}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{-\frac{1}{144}\int \frac{288(73238-178369x)}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-2\int \frac{73238-178369x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{-2\left(-\frac{157893}{16}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-2\left(\frac{157893}{8}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-2\left(\frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{32\sqrt{2}}-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) + ((-66006*Sqrt[3 - x + 2*x
^2])/(5 + 2*x)^2 - 2*((-115369*Sqrt[3 - x + 2*x^2])/(8*(5 + 2*x)) + (52631
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(32*Sqrt[2])))/373
248
```

3.356.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

3.356. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx$

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.356.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1210513\sqrt{2}(8x^4+36x^3+42x^2+35x+75)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-1}{4x^2+2}\right.}{549421056\,(8}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(2
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x
^2 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*s
qrt(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

3.356.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^3\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**
(3/2)), x)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx &= \frac{52631}{11943936}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\ &+\frac{861085\,x}{11446272\sqrt{2\,x^2-x+3}}-\frac{1163201}{3815424\sqrt{2\,x^2-x+3}}\\ &-\frac{3667}{1152\left(4\sqrt{2\,x^2-x+3}x^2+20\sqrt{2\,x^2-x+3}x+25\sqrt{2\,x^2-x+3}\right)}\\ &+\frac{196043}{82944\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/381542
4/sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x
^2 - x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x +
 3)*x + 5*sqrt(2*x^2 - x + 3))
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = -\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{8779x-65991}{4292352\sqrt{2x^2-x+3}} + \frac{\sqrt{2}\left(3594214\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+19874490\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-30140067\sqrt{2}\left(\sqrt{2}x-\sqrt{\right.\right.}}{2985984\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
-52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298
5984*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 198744
90*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*
sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx$

3.357.1 Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx = \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1289945088\sqrt{2}}$$

output
```
-3505819/2579890176*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1
/2)+1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)
^(1/2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799/1074954
24*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.357.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{12(1873786587+1257975811x+441046842x^2+572739684x^3+56754760x^4)}{(5+2x)^3\sqrt{3-x+2x^2}}+80633837\sqrt{2}\text{ar}}{29668737024}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
((12*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760
*x^4))/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]) + 80633837*Sqrt[2]*ArcTanh[(5 + 2
*x - Sqrt[6 - 2*x + 4*x^2])/6])/29668737024
```

3.357.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int \frac{23\left(453064x^3+38587980x^2+31270710x+15168577\right)}{26873856(2x+5)^4\sqrt{2x^2-x+3}}dx+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{453064x^3+38587980x^2+31270710x+15168577}{(2x+5)^4\sqrt{2x^2-x+3}}dx}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{-\frac{1}{216}\int \frac{216\left(-226532x^2-12391084x+3461275\right)}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\int \frac{-226532x^2-12391084x+3461275}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{72(2061152x+1275689)}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{2061152x+1275689}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{1}{2}\left(\frac{3505819}{8}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{2}\left(\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}-\frac{3505819}{4}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{2}\left(\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}-\frac{3505819\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{48\sqrt{2}}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) + ((-1584144*Sqrt[3 -
x + 2*x^2])/(5 + 2*x)^3 + (458655*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 + ((430
799*Sqrt[3 - x + 2*x^2])/(4*(5 + 2*x)) - (3505819*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(48*Sqrt[2]))/2)/13436928
```

3.357.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.357. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.357. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx$

3.357.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx = \frac{80633837\,\sqrt{2}(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375)\log\left(-\frac{24\sqrt{\ldots}}{\ldots}\right.}{1\ldots}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 3
25*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 -
1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4
41046842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 1
12*x^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

3.357.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\,(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**
(3/2)), x)
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 (3-x+2x^2)^{3/2}}\,dx = \frac{3505819}{2579890176}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}} + \frac{6128291}{824131584\sqrt{2\,x^2-x+3}} - \frac{3667}{1728\left(8\sqrt{2\,x^2-x+3}x^3+60\sqrt{2\,x^2-x+3}x^2+150\sqrt{2\,x^2-x+3}x+125\sqrt{2\,x^2-x+3}\right)} + \frac{314233}{248832\left(4\sqrt{2\,x^2-x+3}x^2+20\sqrt{2\,x^2-x+3}x+25\sqrt{2\,x^2-x+3}\right)} - \frac{3127169}{17915904\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*s
qrt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291
/824131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60
*sqrt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x
+ 3)) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*
x + 25*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x +
5*sqrt(2*x^2 - x + 3))
```

3.357.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(111) = 222.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4 (3-x+2x^2)^{3/2}}\,dx = -\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{175877\,x-369609}{154524672\sqrt{2\,x^2-x+3}} - \frac{\sqrt{2}\Big(10398764\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5 - 303070900\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4 - 529738052\sqrt{2}\big(\sqrt{2}x -}{214990848\Big(2\left(\sqrt{2}x-\sqrt{2\,x^2-x-\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
-3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
+ 2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x
 + 3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3))^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052
284471)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) - 11)^3
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

3.358 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

3.358.1 Optimal result

Integrand size = 40, antiderivative size = 105

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{1471\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

output
```
-4/69*(346-533*x)/(2*x^2-x+3)^(3/2)-1471/64*arcsinh(1/23*(1-4*x)*23^(1/2))
*2^(1/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(1/2)+247/16*(2*x^2-x+3)^(1/2)
+5/4*x*(2*x^2-x+3)^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{6663133-6410082x+8639625x^2-3764360x^3+1440996x^4+}{25392(3-x+2x^2)^{3/2}} - \frac{1471\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{32\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2
),x]
```

output
```
(6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^
5)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]])/(32*Sqrt[2])
```

3.358.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)^2 \left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69} \int -\frac{-690x^4-3657x^3-4830x^2+1725x+290}{2 \left(2x^2-x+3\right)^{3/2}}dx-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{69} \int \frac{-690x^4-3657x^3-4830x^2+1725x+290}{\left(2x^2-x+3\right)^{3/2}}dx-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{69} \left(\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}-\frac{2}{23} \int -\frac{1587 \left(5x^2+29x+42\right)}{2\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{69} \left(69 \int \frac{5x^2+29x+42}{\sqrt{2x^2-x+3}}dx+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{69} \left(69 \left(\frac{1}{4} \int \frac{247x+306}{2\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{69} \left(69 \left(\frac{1}{8} \int \frac{247x+306}{\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69 \left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + ((4*(18982 - 20383*x))/(23
*Sqrt[3 - x + 2*x^2]) + 69*((5*x*Sqrt[3 - x + 2*x^2])/4 + ((247*Sqrt[3 - x
 + 2*x^2])/2 + (1471*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8))/69
```

3.358.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.358. $\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.358.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{2334477\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x}\right.}{}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 14
40996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 -
x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.358.6 Sympy [F]

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(2x+5)^2\cdot(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(
5/2), x)
```

3.358.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(84) = 168.

Time = 0.29 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.09

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{5\,x^5}{(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{227\,x^4}{4\,(2\,x^2-x+3)^{\frac{3}{2}}}$$
$$+ \frac{1471}{50784}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)$$
$$+ \frac{1471}{64}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{104441}{25392}\,\sqrt{2\,x^2-x+3} - \frac{383581\,x}{12696\,\sqrt{2\,x^2-x+3}}$$
$$+ \frac{321\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{15965}{4232\,\sqrt{2\,x^2-x+3}} - \frac{4147\,x}{46\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{42883}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784
*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2
*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2))
+ 1471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x
^2 - x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)
^(3/2) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2)
+ 42883/138/(2*x^2 - x + 3)^(3/2)
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{((4\,(1587\,(20\,x+227)x-941090)x+8639625)x-6410082)x+6663133}{25392\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
-1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/25392*((((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 6
663133)/(2*x^2 - x + 3)^(3/2)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(2\,x+5)^2\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.358. $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

3.359 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

3.359.1 Optimal result

Integrand size = 38, antiderivative size = 86

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{-53+373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} - \frac{71\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

output
```
1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^
(1/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)+5/4*(2*x^2-x+3)^(1/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{102869-199290x+185337x^2-147664x^3+31740x^4}{6348(3-x+2x^2)^{3/2}} - \frac{71\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),
x]
```

output
```
(102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2
*x^2)^(3/2)) - (71*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

3.359.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2191, 27, 2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int -\frac{-690x^3-1932x^2+233}{4\left(2x^2-x+3\right)^{3/2}}dx-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{138}\int \frac{-690x^3-1932x^2+233}{\left(2x^2-x+3\right)^{3/2}}dx-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{138}\left(\frac{6055-28981x}{23\sqrt{2x^2-x+3}}-\frac{2}{23}\int -\frac{1587(10x+33)}{4\sqrt{2x^2-x+3}}dx\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{138}\left(\frac{69}{2}\int \frac{10x+33}{\sqrt{2x^2-x+3}}dx+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71}{2}\int \frac{1}{\sqrt{2x^2-x+3}}dx+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow 222$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

output
```
-1/69*(53 - 373*x)/(3 - x + 2*x^2)^(3/2) + ((6055 - 28981*x)/(23*Sqrt[3 -
x + 2*x^2]) + (69*(5*Sqrt[3 - x + 2*x^2] + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/(2*Sqrt[2])))/2)/138
```

3.359.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.359.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{112677\sqrt{2}(4x^4-4x^3+13x^2-6x+9)\log\left(-4\sqrt{2}\sqrt{2x^2-x-\right.}{}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fr
icas")
```

output
```
1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 14766
4*x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

3.359.6 Sympy [F]

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(2x+5)(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2
), x)
```

3.359.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\begin{aligned}\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx &= \frac{5\,x^4}{(2\,x^2-x+3)^{\frac{3}{2}}} \\ &+ \frac{71}{12696}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) \\ &+ \frac{71}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{5041}{6348}\sqrt{2\,x^2-x+3} - \frac{10007\,x}{3174\sqrt{2\,x^2-x+3}} \\ &+ \frac{59\,x^2}{2\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{2959}{2116\sqrt{2\,x^2-x+3}} - \frac{807\,x}{92\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{7603}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="ma
xima")
```

output
```
5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174
*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3
)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23
)*(4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x
 + 3) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 8
07/92*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{\left(\left(4\,(7935\,x-36916)x+185337\right)x-199290\right)x+102869}{6348\,\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gi
ac")
```

output
```
-71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/6
348*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x +
 3)^(3/2)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx$

3.360.1 Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx=\frac{89+219x}{276\,(3-x+2x^2)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

output
```
1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)+1/2116*(-1465-2604*x)/(2*x^2-x+3)^(1/2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx=\frac{-5569-7002x-489x^2-7812x^3}{3174\,(3-x+2x^2)^{3/2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-5569 - 7002*x - 489*x^2 - 7812*x^3)/(3174*(3 - x + 2*x^2)^(3/2)) - (5*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

3.360.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{69}\int -\frac{3(-460x^2-138x+53)}{16\,(2x^2-x+3)^{3/2}}dx+\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}-\frac{1}{184}\int \frac{-460x^2-138x+53}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{184}\left(-\frac{2}{23}\int -\frac{2645}{\sqrt{2x^2-x+3}}dx-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{184}\left(230\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{184}\left(5\sqrt{46}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{184}\left(115\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276\,(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) + ((-2*(1465 + 2604*x))/(23*Sqrt[
3 - x + 2*x^2]) + 115*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/184
```

3.360. $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx$

3.360.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.360.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{7935\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)\right.}{25392\,(4\,x^4-4\,x^3}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2
+ 7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.360.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

3.360.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{5}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x+}\right.$$
$$+\frac{5}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{355}{3174}\sqrt{2\,x^2-x+3}-\frac{58\,x}{1587\,\sqrt{2\,x^2-x+3}}$$
$$+\frac{x^2}{2\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{1897}{6348\,\sqrt{2\,x^2-x+3}}-\frac{95\,x}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{41}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(
3/2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2
 - x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2)
- 1897/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/
(2*x^2 - x + 3)^(3/2)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx =$$
$$-\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{3\left((2604\,x+163)x+2334\right)x+5569}{3174\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/317
4*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx$

3.361.1 Optimal result

Integrand size = 40, antiderivative size = 85

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx=\frac{1191+917x}{9936(3-x+2x^2)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}$$

output
```
1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-3667/62208*arctanh(1/24*(17-22*x)*2^
(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/1371168*(-335337-146729*x)/(2*x^2-x+3)^
(1/2)
```

3.361.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx=\frac{-841653+21696x-523945x^2-293458x^3}{1371168(3-x+2x^2)^{3/2}}+\frac{3667\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{15552\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),
x]
```

output
```
(-841653 + 21696*x - 523945*x^2 - 293458*x^3)/(1371168*(3 - x + 2*x^2)^(3/
2)) + (3667*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(15552*Sqrt[2])
```

3.361.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2177, 27, 2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int -\frac{-49680x^2-22240x+1877}{576(2x+5)\left(2x^2-x+3\right)^{3/2}}dx+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{\int \frac{-49680x^2-22240x+1877}{(2x+5)\left(2x^2-x+3\right)^{3/2}}dx}{19872}$$

$$\Big\downarrow \text{2177}$$

$$\frac{-\frac{2}{23}\int -\frac{1939843}{12(2x+5)\sqrt{2x^2-x+3}}dx-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{84341}{6}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\frac{84341}{3}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{84341\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{36\sqrt{2}}-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) + (-1/69*(335337 + 146729*x)/S
qrt[3 - x + 2*x^2] - (84341*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(36*Sqrt[2]))/19872
```

3.361.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.361. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx$

3.361.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1939843\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+10}{4\,x^2+20\,x+2}\right.}{65816064\,(4\,x^4-}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fr
icas")
```

output
```
1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 +
 20*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2
- x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.361.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/
2)), x)
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx = \frac{3667}{62208}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}} - \frac{5\,x^2}{4\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{173881}{457056\sqrt{2\,x^2-x+3}} + \frac{7127\,x}{9936\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{5813}{3312\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="ma
xima")
```

output
```
3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/
abs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x
+ 3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x +
3)^(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{5/2}}\,dx = -\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{((293458\,x+523945)x-21696)x+841653}{1371168\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gi
ac")
```

output
```
-3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)
)) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2
- x + 3)^(3/2)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx$

3.362.1 Optimal result

Integrand size = 40, antiderivative size = 110

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \frac{9897+2203x}{357696(3-x+2x^2)^{3/2}} - \frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)} - \frac{2821\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{2239488\sqrt{2}}$$

output
```
1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-2821/4478976*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24681024*(-1255878+62021*x)/(2*x^2
-x+3)^(1/2)-3667/186624*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.362.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \frac{-\frac{12(79153407-18840090x+63941915x^2+10350004x^3+6767036x^4)}{(5+2x)(3-x+2x^2)^{3/2}} + 1492309\sqrt{2}\text{arctanh}}{1184689152}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((-12*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 6767036*x^4))
/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 1492309*Sqrt[2]*ArcTanh[(5 + 2*x - Sq
rt[6 - 2*x + 4*x^2])/6])/1184689152
```

3.362.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2177, 27, 2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int \frac{1823728x^2+963530x+119353}{20736(2x+5)^2\left(2x^2-x+3\right)^{3/2}}dx+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1823728x^2+963530x+119353}{(2x+5)^2\left(2x^2-x+3\right)^{3/2}}dx}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2177}$$

$$\frac{\frac{2}{23}\int \frac{529(19111-18758x)}{6(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{23}{3}\int \frac{19111-18758x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{23}{3}\left(\frac{2821}{4}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{23}{3}\left(-\frac{2821}{2}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{23}{3}\left(-\frac{2821\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{24\sqrt{2}}-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) + ((-2*(1255878 - 62021*x))
/(69*Sqrt[3 - x + 2*x^2]) + (23*((-3667*Sqrt[3 - x + 2*x^2])/(2*(5 + 2*x))
 - (2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(24*Sqrt[2
])))/3)/715392
```

3.362.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.362. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.362.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1492309\sqrt{2}(8\,x^5+12\,x^4+6\,x^3+53\,x^2-12\,x+45)\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+}}{}\right.}{4738}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2
- 18840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53
*x^2 - 12*x + 45)
```

3.362.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**
(5/2)), x)
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \frac{2821}{4478976}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{1691759\,x}{98724096\sqrt{2\,x^2-x+3}} + \frac{265339}{32908032\sqrt{2\,x^2-x+3}} - \frac{248617\,x}{715392\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3667}{576\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)} + \frac{259621}{238464\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23
)/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032
/sqrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2
*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2
 - x + 3)^(3/2)
```

3.362.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(88) = 176$.

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx =$$

$$-\frac{1}{2369378304}\sqrt{2}\left(\frac{1492309\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{12\left(\frac{\frac{\frac{48\left(\frac{23642785}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{52375761}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right.}{2x+5}}{2x+5}}{2x+}\right.}{\left(\frac{11}{2x+5}-\frac{36}{(2x+5)^2}-\right.}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
-1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x
 + 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(
1/(2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/s
gn(1/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^2\,{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx$

3.363.1 Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx = \frac{65991-8779x}{12877056(3-x+2x^2)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2} - \frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)} + \frac{774079\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{322486272\sqrt{2}}$$

output
```
1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)+774079/644972544*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/592344576*(-4679797+2148263
*x)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979/268738
56*(2*x^2-x+3)^(1/2)/(5+2*x)
```

3.363.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-8953831359+2280511668x-5919924791x^2-1503926130x^3+107028732x^4+217883}{(15+x+8x^2+4x^3)^2}}{17059523788}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 15
03926130*x^3 + 107028732*x^4 + 217883368*x^5))/(15 + x + 8*x^2 + 4*x^3)^2
- 409487791*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/17059523
7888
```

3.363.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{2177}$$

$$\frac{2}{69}\int \frac{-280928x^3+65340540x^2+38386140x+11115283}{746496(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{-280928x^3+65340540x^2+38386140x+11115283}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textbf{2177}$$

$$\frac{\frac{2}{23}\int -\frac{529\left(125300x^2+1076692x+324461\right)}{4(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{23}{2}\int \frac{125300x^2+1076692x+324461}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textbf{2181}$$

$$\frac{-\frac{23}{2}\left(\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1}{144}\int -\frac{288(53327x+64349)}{(2x+5)^2\sqrt{2x^2-x+3}}dx\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{23}{2}\left(2\int \frac{53327x+64349}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1228$$

$$\frac{-\frac{23}{2}\left(2\left(\frac{774079}{48}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{-\frac{23}{2}\left(2\left(\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}-\frac{774079}{24}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{23}{2}\left(2\left(\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}-\frac{774079\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{288\sqrt{2}}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) + (-1/23*(4679797 - 2148
263*x)/Sqrt[3 - x + 2*x^2] - (23*((22002*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2
+ 2*((45979*Sqrt[3 - x + 2*x^2])/(24*(5 + 2*x)) - (774079*ArcTanh[(17 - 22
*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(288*Sqrt[2]))))/2)/25754112
```

3.363. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx$

3.363.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

3.363. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx$

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.363.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{409487791\,\sqrt{2}(16\,x^6+64\,x^5+72\,x^4+136\,x^3+241\,x^2+30\,x+225)\log$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 24
1*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060
*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732
*x^4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2
*x^2 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

3.363.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**
(5/2)), x)
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.32

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx=-\frac{774079}{644972544}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}-\frac{36393601}{4738756608\sqrt{2\,x^2-x+3}}+\frac{2323723\,x}{103016448\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{1152\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}+\frac{115369}{82944\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{5254255}{34338816\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
-774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 363936
01/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3
/2) - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x
+ 25*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*
(2*x^2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

3.363.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$
$$-\frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\left(44558\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3-10136238\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+16812201\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x\right.\right.}{53747712\left(2\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right)^2}$$
$$+\frac{((4296526\,x-11507857)x+10720752)x-11003805}{592344576\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*s
qrt(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 168122
01*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)
^2 + 1/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x
^2 - x + 3)^(3/2)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)^3\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx$

3.364.1 Optimal result

Integrand size = 40, antiderivative size = 160

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = \frac{369609-175877x}{463574016(3-x+2x^2)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2} + \frac{475357\sqrt{3-x+2x^2}}{1934917632(5+2x)} + \frac{4778789\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{7739670528\sqrt{2}}$$

output
```
1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)+4778789/15479341056*arctan
h(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/31986607104*(-277545
39+31190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-8
9137/80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357/1934917632*(2*x^2-x+3)^(1
/2)/(5+2*x)
```

3.364.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{\frac{12(-95241881529+73621973154x-6702882569x^2+27484986184x^3+46210466520x^4+34872810880}{(5+2x)^3(3-x+2x^2)^{3/2}}}{4094285}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((12*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 46
210466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2
*x^2)^(3/2)) - 2527979381*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2]
)/6])/4094285709312
```

3.364.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2177, 27, 2177, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int \frac{-11256128x^4-81299864x^3+2240465820x^2+1454170990x+606939313}{26873856(2x+5)^4\left(2x^2-x+3\right)^{3/2}}dx+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-11256128x^4-81299864x^3+2240465820x^2+1454170990x+606939313}{(2x+5)^4(2x^2-x+3)^{3/2}}dx}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{\frac{2}{23}\int -\frac{1058(301804x^3+3955080x^2+20194167x+9095911)}{3(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{92}{3}\int \frac{301804x^3+3955080x^2+20194167x+9095911}{(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{92}{3}\left(\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}-\frac{1}{216}\int -\frac{216(150902x^2+2392357x+2631056)}{(2x+5)^3\sqrt{2x^2-x+3}}dx\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{92}{3}\left(\int \frac{150902x^2+2392357x+2631056}{(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{92}{3}\left(-\frac{1}{144}\int -\frac{9(2276860x+9970363)}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\int \frac{2276860x+9970363}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(\frac{14336367}{8}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-311909}{69\sqrt{2x^2-x+3}}}{927148032}$$

$$\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1154

3.364. $\displaystyle\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx$

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(-\frac{14336367}{4}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(277545}{69\sqrt{}}}{927148032}$$

$$\frac{369609-175877x}{463574016\,(2x^2-x+3)^{3/2}}$$

$\downarrow$ 219

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(-\frac{4778789\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{16\sqrt{2}}-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-}{69\sqrt{2x^2}}}{927148032}$$

$$\frac{369609-175877x}{463574016\,(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) + ((-2*(27754539 - 3
1190998*x))/(69*Sqrt[3 - x + 2*x^2]) - (92*((198018*Sqrt[3 - x + 2*x^2])/(
5 + 2*x)^3 + (267411*Sqrt[3 - x + 2*x^2])/(8*(5 + 2*x)^2) + ((-475357*Sqrt
[3 - x + 2*x^2])/(4*(5 + 2*x)) - (4778789*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*
Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2]))/16))/3)/927148032
```

3.364.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.364. $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx$

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

3.364.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{2527979381\sqrt{2}(32\,x^7+208\,x^6+464\,x^5+632\,x^4+1162\,x^3+1265\,x^2+$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
 + 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)
*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440
4208*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 670288256
9*x^2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^
6 + 464*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

3.364.6 Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**
(5/2)), x)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.54

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = -\frac{4778789}{15479341056}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{416525263\,x}{341190475776\,\sqrt{2\,x^2-x+3}} - \frac{245375387}{113730158592\,\sqrt{2\,x^2-x+3}} + \frac{16932905\,x}{2472394752\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3667}{1728\left(8\,(2\,x^2-x+3)^{\frac{3}{2}}x^3+60\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+150\,(2\,x^2-x+3)^{\frac{3}{2}}x+125\,(2\,x^2-x+3)^{\frac{3}{2}}\right)} + \frac{25951}{27648\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)} - \frac{34861}{1990656\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)} - \frac{10570421}{824131584\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
-4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 2
45375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 -
 x + 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3
)^(3/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 2
5951/27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*
(2*x^2 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x
^2 - x + 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

3.364.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{((15595499x-21675019)x+27298005)x-14440149}{7996651776(2x^2-x+3)^{\frac{3}{2}}} + \frac{\sqrt{2}\left(38030012\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5+734231900\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+122834956\sqrt{2}\left(\sqrt{2}x-\right.\right.}{3869835264\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+\right.\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27
298005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(380300
12*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sq
rt(2*x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^3 - 2154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4(2x^2-x+3)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

3.365.1 Optimal result

Integrand size = 35, antiderivative size = 354

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg}{3c^3(b^2-4ac)(a+bx+}$$
$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4f-c^3(8b}{3c^3(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{j\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^
4*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x
)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+j*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(-3*a*i+2*c*
g)-b^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-c*(16*c^4*f-c
^3*(-8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*
h))*x)/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

3.365.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-3b^5jx^2-2b^4jx(3a+2cx^2)+b^3(-3a^2j+18acjx^2+c^2(-f-3gx+}{}$$
$$+\frac{2j\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-3*b^5*j*x^2 - 2*b^4*j*x*(3*a + 2*c*x^2) + b^3*(-3*a^2*j + 18*a*c*j*x^
2 + c^2*(-f - 3*g*x + 3*h*x^2 + i*x^3)) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f
 - 6*g*x + h*x^2) - a*c*(g - 6*h*x + 3*i*x^2 - 14*j*x^3)) - 8*c^2*(-2*c^3*
f*x^3 + a^3*(2*i + 3*j*x) - a*c^2*x*(3*f + h*x^2) + a^2*c*(g + 3*i*x^2 + 4
*j*x^3)) + 4*b*c*(5*a^3*j - 2*c^3*x^2*(-3*f + g*x) + 2*a^2*c*(h - 3*i*x) +
 3*a*c^2*(f - x*(g - h*x + i*x^2)))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))^(3/2)) + (2*j*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])]
)/c^(5/2)
```

3.365.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2191, 27, 2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{2191}$$
$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$
$$\frac{2\int \frac{3\left(4a-\frac{b^2}{c}\right)jx^2-\frac{3(b^2-4ac)(ci-bj)x}{c^2}+\frac{jb^4-c(bi+aj)b^2+8c^4f-c^3(4bg-4ah)+c^2(b^2h-4a^2j)}{c^3}}{2(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$\Big\downarrow$ 27

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\int \frac{\frac{jb^4}{c^3}-\frac{(bi+aj)b^2}{c^2}-4gb+3\left(4a-\frac{b^2}{c}\right)jx^2+8cf+4ah+\frac{b^2h-4a^2j}{c}-\frac{3(b^2-4ac)(ci-bj)x}{c^2}}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$\Big\downarrow$ 2191

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^3i-b^3c(ch-10aj)+b^2(4c^3g-6ac^2i)}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 27

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^3i-b^3c(ch-10aj)+b^2(4c^3g-6ac^2i)}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 1092

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^3i-b^3c(ch-10aj)+b^2(4c^3g-6ac^2i)}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 219

$$\frac{2(-x(c^2(2a^2j+3abi+b^2h)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f)-bc(-3a^2j+ach+c^2f)-ab^3j+ab}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f)-4bc^2(8a^2j+ach+2c^2f)+24a^2c^3i-b^3c(ch-10aj)+b^2(4c^3g-6ac^2i)}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

input
```
Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

3.365. $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

output
```
(2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2
*j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^
2*h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)
) - ((2*(b^4*c*i + 24*a^2*c^3*i + b^2*(4*c^3*g - 6*a*c^2*i) - b^5*j - b^3*
c*(c*h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3
*(8*b*g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i
 - 16*a^2*j))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*
a*c)*j*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2))/(3
*(b^2 - 4*a*c))
```

3.365.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.365. $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

3.365.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

3.365.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(337) = 674.

Time = 22.78 (sec) , antiderivative size = 1373, normalized size of antiderivative = 3.88

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

output
```
[1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*
j)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b
*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 -
 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 +
 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2
 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c -
 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5
)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*
b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a
^2*b^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c
 + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b
^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^
3*c^3*i + (16*c^6*f - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - ...
```

3.365.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.365.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.31

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{(16c^5f-8bc^4g+2b^2c^3h+8ac^4h+b^3c^2i-12abc^3i-4b^4cj+28ab^2c^2j-32a^2c^3j)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3(8bc\cdots}{\cdots}\right.\right.\right.}{\cdots} - \frac{j\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

output
```
2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*
b*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h -
 2*a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g +
 4*a*b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)
/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a
*b^2*c^2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20
*a^3*b*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
- j*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}} \, dx = \int \frac{j\,x^4 + i\,x^3 + h\,x^2 + g\,x + f}{(c\,x^2 + b\,x + a)^{5/2}} \, dx$$

input
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}} \, dx$

3.366.1 Optimal result

Integrand size = 36, antiderivative size = 353

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \frac{2(ab^2ci+2ac^2(cg+ai)+ab^3j-bc(c^2f-ach-3a^2j)+(2c^4f+c^3(bg-}{3c^3(b^2+4ac)(a+bx-}$$
$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg+3ai)+b^5j+b^3c(ch+10aj)+4bc^2(2c^2f-ach+8a^2j)-c(16c^4f+8c^3(b}{3c^3(b^2+4ac)^2\sqrt{a+bx-cx^2}}$$
$$-\frac{j\arctan\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4
*f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)
/c^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-j*arctan(1/2*(-2*c*x+b)/c^(1/2)/(-c*
x^2+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(3*a*i+2*c*g
)+b^5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(16*c^4*f+8*c
^3*(-a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*
x)/c^3/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

3.366.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx =$$
$$-\frac{2(3b^5jx^2+b^4(6ajx-4cjx^3)+b^3(3a^2j+18acjx^2+c^2(f+3gx-x^2(3h+ix)))+8c^2(2c^3fx^3+a^3(2i+3$$

$$+\frac{2j\arctan\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b-cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

output
```
(-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
 + c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*
j*x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a
^3*j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g -
h*x + i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*
(g + x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*
x))^(3/2)) + (2*j*ArcTan[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b - c*x)])])/
c^(5/2)
```

3.366.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2191, 27, 2191, 27, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2191$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$
$$\frac{2\int -\frac{-\frac{3\left(b^2+4ac\right)jx^2}{c}-\frac{3\left(b^2+4ac\right)(ci+bj)x}{c^2}+\frac{jb^4+c(bi+aj)b^2+8c^4f+4c^3(bg-ah)+c^2\left(b^2h-4a^2j\right)}{c^3}}{2(-cx^2+bx+a)^{3/2}}dx}{3\left(4ac+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{jb^4}{c^3}+\frac{(bi+aj)b^2}{c^2}+4gb-\frac{3\left(b^2+4ac\right)jx^2}{c}+8cf-4ah+\frac{b^2h-4a^2j}{c}-\frac{3\left(b^2+4ac\right)(ci+bj)x}{c^2}}{(-cx^2+bx+a)^{3/2}}dx}{3\left(4ac+b^2\right)}+$$
$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 2191

$$\frac{-\frac{2\int -\frac{3\left(b^2+4ac\right)^2j}{2c^2\sqrt{-cx^2+bx+a}}dx}{4ac+b^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3i+b^3}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$
$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{3j\left(4ac+b^2\right)\int \frac{1}{\sqrt{-cx^2+bx+a}}dx}{c^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3i\right.}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$
$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{6j\left(4ac+b^2\right)\int \frac{1}{-\frac{(b-2cx)^2}{-cx^2+bx+a}-4c}d\frac{b-2cx}{\sqrt{-cx^2+bx+a}}}{c^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+\right.\right.}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$
$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

$\Big\downarrow$ 217

3.366. $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$

$$-\frac{\frac{2(-cx(2c^2(-16a^2j-6abi+b^2h)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f)+4bc^2(8a^2j-ach+2c^2f)+24a^2c^3i+b^3c(10aj+ch)+2b^2c^2(3ai+2cg)}{c^3(4ac+b^2)\sqrt{a+bx-cx^2}}}{3(4ac+b^2)}$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

input
```
Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

output
```
(2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2
*j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^
2*h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)
) + ((-2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3
*c*(c*h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*
c^3*(b*g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i
- 16*a^2*j))*x))/(c^3*(b^2 + 4*a*c)*Sqrt[a + b*x - c*x^2]) - (3*(b^2 + 4*a
*c)*j*ArcTan[(b - 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2))/(3*(
b^2 + 4*a*c))
```

3.366.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

3.366. $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.366.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. 2(337) = 674.

Time = 2.46 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	1147

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
f*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*
(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))+j*(1/3*x^3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c
*(x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c
*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+
b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a
/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2
*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b
/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2
*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))))-1/c*(x/c/(-c*x^2+b*x+a)^(1/2)+1/2*b/c*
(1/c/(-c*x^2+b*x+a)^(1/2)+b/c*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2)
)-1/c^(3/2)*arctan(c^(1/2)*(x-1/2*b/c)/(-c*x^2+b*x+a)^(1/2))))+i*(x^2/c/(-
c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-
c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/
2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-
2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b
)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-
2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b
)/(-c*x^2+b*x+a)^(1/2))))+h*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(
-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3
/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/...
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(339) = 678$.

Time = 23.13 (sec) , antiderivative size = 1385, normalized size of antiderivative = 3.92

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fri
cas")
```

output
```
[-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*
c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^
5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)
*j)*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c
*x - b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f +
8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^
2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^
3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*
c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*
c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
 (b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*
a^2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 +
 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^
2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b
^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a
^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x
 + a)*(2*c*x - b)*sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16
*a^3*c^3*i - (16*c^6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3...
```

3.366.6 Sympy [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.366.7 Maxima [F]

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=\int \frac{jx^4+ix^3+hx^2+gx+f}{(-cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="max
ima")
```

output
```
-1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt
(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^(3/2)*(b
^2 + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*
c)^2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - 3*x^2/((-c*
x^2 + b*x + a)^(3/2)*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 +
 4*a*c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*a/
((-c*x^2 + b*x + a)^(3/2)*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*
(b^2 + 4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/
((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(
b^2 + 4*a*c)*c) + 1/((-c*x^2 + b*x + a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(
-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 +
4*a*c)^2) + 2*c*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 +
b*x + a)^(3/2)*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b
*x + a)*(b^2 + 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c))
+ b^2*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt
(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2
 + 4*a*c)*c)) + j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2
*a*c)*x^2 + a^2)*sqrt(-c*x^2 + b*x + a)), x)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.38

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=$$

$$-\frac{2\sqrt{-cx^2+bx+a}\left(\left(\left(\frac{(16\,c^5f+8\,bc^4g+2\,b^2c^3h-8\,ac^4h-b^3c^2i-12\,abc^3i-4\,b^4cj-28\,ab^2c^2j-32\,a^2c^3j)x}{b^4c^2+8\,ab^2c^3+16\,a^2c^4}-\frac{3\,(8\,bc^4f+4\,b^2c^3g+b^3c^2h-4}{b^4c^2+}\right.\right.\right.}{}$$

$$-\frac{j\log\left(\left|2\left(\sqrt{-c}x-\sqrt{-cx^2+bx+a}\right)\sqrt{-c}+b\right|\right)}{\sqrt{-c}c^2}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="gia
c")
```

output
```
-2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c
^4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*
j)*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b
^3*c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j
)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b
^3*c^2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^
2*b^2*c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^
2*f + 12*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*
c^2*i + 3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/
(c*x^2 - b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sq
rt(-c) + b))/(sqrt(-c)*c^2)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}} \, dx = \int \frac{j\,x^4 + i\,x^3 + h\,x^2 + g\,x + f}{(-c\,x^2 + b\,x + a)^{5/2}} \, dx$$

input
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
```

output
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right.$

3.367.1 Optimal result

Integrand size = 38, antiderivative size = 588

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^{11}(1+m)} \\
&\quad - \frac{\left(5d^2-2de+3e^2\right)^2 \left(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5\right)(d+ex)^{2+m}}{e^{11}(2+m)} \\
&\quad + \frac{3\left(5d^2-2de+3e^2\right)\left(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6\right)(d+ex)^{3+m}}{e^{11}(3+m)} \\
&\quad - \frac{2\left(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7\right)(d+ex)^{4+m}}{e^{11}(4+m)} \\
&\quad + \frac{\left(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6\right)(d+ex)^{5+m}}{e^{11}(5+m)} \\
&\quad - \frac{6\left(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5\right)(d+ex)^{6+m}}{e^{11}(6+m)} \\
&\quad + \frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)(d+ex)^{7+m}}{e^{11}(7+m)} \\
&\quad - \frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{8+m}}{e^{11}(8+m)} \\
&\quad + \frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{9+m}}{e^{11}(9+m)} - \frac{25(200d+e)(d+ex)^{10+m}}{e^{11}(10+m)} + \frac{500(d+ex)^{11+m}}{e^{11}(11+m)}
\end{aligned}$$

output
```
(5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/
e^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3
+86*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6
+660*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3
+m)/e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3
*e^4-2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6
+3150*d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*
(e*x+d)^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+9
99*d*e^4-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2
*e^2+686*d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+30
60*d*e^2+49*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d
)^(9+m)/e^11/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11
+m)/e^11/(11+m)
```

3.367.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)^3(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(5d^2-2de+3e^2)^2(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5)(d+e\right.}{2+m}}{}$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^
4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (
3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3
 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7
+ 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5
+ 2218*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e +
 53550*d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d
+ e*x)^4)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3
 + 999*d*e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e +
21420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3
+ 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5
*d*e + 17*e^2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m
) + (500*(d + e*x)^10)/(11 + m)))/e^11
```

3.367.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^3 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^m \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{m+8}}{e^{10}}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{m+7}}{e^{10}}+\frac{\left(5d^2-2de+\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{m+9}}{e^{11}(m+9)}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{m+8}}{e^{11}(m+8)}+\frac{\left(5d^2-2de+3e^2\right)^3\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^{11}(m+1)}+\frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)(d+ex)^{m+7}}{e^{11}(m+7)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5\right)(d+ex)^{m+2}}{e^{11}(m+2)}-\frac{6\left(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5\right)(d+ex)^{m+6}}{e^{11}(m+6)}+\frac{3\left(5d^2-2de+3e^2\right)\left(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6\right)(d+ex)^{m+3}}{e^{11}(m+3)}+\frac{\left(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6\right)(d+ex)^{m+5}}{e^{11}(m+5)}-\frac{2\left(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7\right)(d+ex)^{m+4}}{e^{11}(m+4)}-\frac{25(200d+e)(d+ex)^{m+10}}{e^{11}(m+10)}+\frac{500(d+ex)^{m+11}}{e^{11}(m+11)}$$

input
```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 1
69*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m)
)/(e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*
d^4*e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m)
)/(e^11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e
^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m)
)/(e^11*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^
3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m
)) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 -
85*e^5)*(d + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21
420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*
(30000*d^3 + 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8
+ m)) + (45*(500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) -
 (25*(200*d + e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11
+ m))/(e^11*(11 + m))
```

3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.367.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5923 vs. $2(588) = 1176$.

Time = 0.45 (sec) , antiderivative size = 5924, normalized size of antiderivative = 10.07

method	result	size
gosper	Expression too large to display	5924
risch	Expression too large to display	6934
parallelrisch	Expression too large to display	11277

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
result too large to display
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4795 vs. 2(588) = 1176.

Time = 0.35 (sec) , antiderivative size = 4795, normalized size of antiderivative = 8.15

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
(54*d*e^10*m^10 + 500*(e^11*m^10 + 55*e^11*m^9 + 1320*e^11*m^8 + 18150*e^1
1*m^7 + 157773*e^11*m^6 + 902055*e^11*m^5 + 3416930*e^11*m^4 + 8409500*e^1
1*m^3 + 12753576*e^11*m^2 + 10628640*e^11*m + 3628800*e^11)*x^11 + 1814400
000*d^11 + 99792000*d^10*e + 3392928000*d^9*e^2 + 488980800*d^8*e^3 + 5696
697600*d^7*e^4 - 3392928000*d^6*e^5 + 8853546240*d^5*e^6 - 5728060800*d^4*
e^7 + 6346771200*d^3*e^8 - 2694384000*d^2*e^9 + 2155507200*d*e^10 - 25*(39
91680*e^11 - (20*d*e^10 - e^11)*m^10 - 4*(225*d*e^10 - 14*e^11)*m^9 - 15*(
1160*d*e^10 - 91*e^11)*m^8 - 60*(3150*d*e^10 - 317*e^11)*m^7 - 21*(60260*d
*e^10 - 7963*e^11)*m^6 - 84*(64125*d*e^10 - 11492*e^11)*m^5 - 5*(2894720*d
*e^10 - 737251*e^11)*m^4 - 20*(1172700*d*e^10 - 456659*e^11)*m^3 - 36*(570
320*d*e^10 - 386841*e^11)*m^2 - 144*(50400*d*e^10 - 80939*e^11)*m)*x^10 -
135*(d^2*e^9 - 26*d*e^10)*m^9 + 5*(678585600*e^11 - (5*d*e^10 - 153*e^11)*
m^10 - (1000*d^2*e^9 + 235*d*e^10 - 8721*e^11)*m^9 - 6*(6000*d^2*e^9 + 785
*d*e^10 - 36006*e^11)*m^8 - 6*(91000*d^2*e^9 + 8785*d*e^10 - 509031*e^11)*
m^7 - 105*(43200*d^2*e^9 + 3445*d*e^10 - 259029*e^11)*m^6 - 21*(1069000*d^
2*e^9 + 74815*d*e^10 - 7560189*e^11)*m^5 - 2*(33642000*d^2*e^9 + 2145620*d
*e^10 - 306036567*e^11)*m^4 - 4*(29531000*d^2*e^9 + 1761185*d*e^10 - 38217
2121*e^11)*m^3 - 72*(1522000*d^2*e^9 + 86510*d*e^10 - 32587351*e^11)*m^2 -
 1440*(28000*d^2*e^9 + 1540*d*e^10 - 1370727*e^11)*m)*x^9 + 9*(106*d^3*e^8
 - 945*d^2*e^9 + 11160*d*e^10)*m^8 - (488980800*e^11 - (765*d*e^10 - 98...
```

3.367.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136733 vs. 2(564) = 1128.

Time = 38.08 (sec) , antiderivative size = 136733, normalized size of antiderivative = 232.54

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(500*x**11/11 - 5*x**10/2 + 85*x**9 - 49*x**8/4 + 999*x**7
/7 + 85*x**6 + 1109*x**5/5 + 287*x**4/2 + 159*x**3 + 135*x**2/2 + 54*x), E
q(e, 0)), (1260000*d**10*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12
*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x
**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*
x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 3
690500*d**10/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x*
*2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x
**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d**9*e*x*log(d/e
+ x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302
400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52
9200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 35645000*d**9*e*x/(2520*d**10*e**1
1 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 +
 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6
+ 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2
520*e**21*x**10) + 6300*d**9*e/(2520*d**10*e**11 + 25200*d**9*e**12*x + 11
3400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 6
35040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7...
```

3.367.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2292 vs. 2(588) = 1176.

Time = 0.27 (sec) , antiderivative size = 2292, normalized size of antiderivative = 3.90

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
135*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) +
54*(e*x + d)^(m + 1)/(e*(m + 1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m
)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e
^3) + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^
3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*
m^3 + 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*
d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^
m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m
^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50
*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(
m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x
- 120*d^6)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 176
4*m + 720)*e^6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764
*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*
e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4
+ 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3
+ 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 +
 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
 - 98*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306...
```

3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10965 vs. 2(588) = 1176.

Time = 0.39 (sec) , antiderivative size = 10965, normalized size of antiderivative = 18.65

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
(500*(e*x + d)^m*e^11*m^10*x^11 + 500*(e*x + d)^m*d*e^10*m^10*x^10 - 25*(e
*x + d)^m*e^11*m^10*x^10 + 27500*(e*x + d)^m*e^11*m^9*x^11 - 25*(e*x + d)^
m*d*e^10*m^10*x^9 + 765*(e*x + d)^m*e^11*m^10*x^9 + 22500*(e*x + d)^m*d*e^
10*m^9*x^10 - 1400*(e*x + d)^m*e^11*m^9*x^10 + 660000*(e*x + d)^m*e^11*m^8
*x^11 + 765*(e*x + d)^m*d*e^10*m^10*x^8 - 98*(e*x + d)^m*e^11*m^10*x^8 - 5
000*(e*x + d)^m*d^2*e^9*m^9*x^9 - 1175*(e*x + d)^m*d*e^10*m^9*x^9 + 43605*
(e*x + d)^m*e^11*m^9*x^9 + 435000*(e*x + d)^m*d*e^10*m^8*x^10 - 34125*(e*x
 + d)^m*e^11*m^8*x^10 + 9075000*(e*x + d)^m*e^11*m^7*x^11 - 98*(e*x + d)^m
*d*e^10*m^10*x^7 + 999*(e*x + d)^m*e^11*m^10*x^7 + 225*(e*x + d)^m*d^2*e^9
*m^9*x^8 + 37485*(e*x + d)^m*d*e^10*m^9*x^8 - 5684*(e*x + d)^m*e^11*m^9*x^
8 - 180000*(e*x + d)^m*d^2*e^9*m^8*x^9 - 23550*(e*x + d)^m*d*e^10*m^8*x^9
+ 1080180*(e*x + d)^m*e^11*m^8*x^9 + 4725000*(e*x + d)^m*d*e^10*m^7*x^10 -
 475500*(e*x + d)^m*e^11*m^7*x^10 + 78886500*(e*x + d)^m*e^11*m^6*x^11 + 9
99*(e*x + d)^m*d*e^10*m^10*x^6 + 510*(e*x + d)^m*e^11*m^10*x^6 - 6120*(e*x
 + d)^m*d^2*e^9*m^9*x^7 - 4998*(e*x + d)^m*d*e^10*m^9*x^7 + 58941*(e*x + d
)^m*e^11*m^9*x^7 + 45000*(e*x + d)^m*d^3*e^8*m^8*x^8 + 8775*(e*x + d)^m*d^
2*e^9*m^8*x^8 + 780300*(e*x + d)^m*d*e^10*m^8*x^8 - 143178*(e*x + d)^m*e^1
1*m^8*x^8 - 2730000*(e*x + d)^m*d^2*e^9*m^7*x^9 - 263550*(e*x + d)^m*d*e^1
0*m^7*x^9 + 15270930*(e*x + d)^m*e^11*m^7*x^9 + 31636500*(e*x + d)^m*d*e^1
0*m^6*x^10 - 4180575*(e*x + d)^m*e^11*m^6*x^10 + 451027500*(e*x + d)^m*...
```

3.367.9 Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 4341, normalized size of antiderivative = 7.38

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output

```
(500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m
^4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628
800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395
35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
+ 39916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 18144000
00*d^11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8
853546240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^
8*e^3 + 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m -
 5364581040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432
604960*d^7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*
d*e^10*m^2 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^
10*m^5 + 17637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 35
10*d*e^10*m^9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e
^8*m^2 - 2127097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e
^5*m^2 + 387691920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m
^2 - 1011746160*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m
^3 + 318992760*d^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 +
 493920*d^8*e^3*m^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 5978
7840*d^4*e^7*m^4 + 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7
*e^4*m^4 - 36710415*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^...
```

3.367. $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

3.368.1 Optimal result

Integrand size = 38, antiderivative size = 432

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx \\
&= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{1+m}}{e^9(1+m)} \\
&\quad - \frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^{2+m}}{e^9(2+m)} \\
&\quad + \frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^{3+m}}{e^9(3+m)} \\
&\quad - \frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^{4+m}}{e^9(4+m)} \\
&\quad + \frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^{5+m}}{e^9(5+m)} \\
&\quad - \frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{6+m}}{e^9(6+m)} \\
&\quad + \frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{7+m}}{e^9(7+m)} - \frac{5(160d+9e)(d+ex)^{8+m}}{e^9(8+m)} + \frac{100(d+ex)^{9+m}}{e^9(9+m)}
\end{aligned}$$

output
```
(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/
e^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d
*e^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*
d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1
575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+
m)+(7000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/
(5+m)-(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*
d^2+315*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e
^9/(8+m)+100*(e*x+d)^(9+m)/e^9/(9+m)
```

3.368.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{(d+ex)^{1+m} \left(\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)}{2+m}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*
e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
 107*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 +
370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575
*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600
*d^3 + 945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 +
 315*d*e + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(
8 + m) + (100*(d + e*x)^8)/(9 + m)))/e^9
```

3.368.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+6}}{e^8}+\frac{\left(-5600d^3-945d^2e-666de^2-37e^3\right)(d+ex)^{m+5}}{e^8}+\frac{(5d^2-2de-$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^{m+5}}{e^9(m+5)}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^{m+2}}{e^9(m+2)}-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^{m+4}}{e^9(m+4)}+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^{m+3}}{e^9(m+3)}-\frac{5(160d+9e)(d+ex)^{m+8}}{e^9(m+8)}+\frac{100(d+ex)^{m+9}}{e^9(m+9)}$$

input
```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*
d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^
9*(2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2
*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5
+ 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^
(4 + m))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^
3 + 148*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 6
66*d*e^2 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^
(8 + m))/(e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.368.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3221 vs. $2(432) = 864$.

Time = 0.39 (sec) , antiderivative size = 3222, normalized size of antiderivative = 7.46

method	result	size
gosper	Expression too large to display	3222
risch	Expression too large to display	3895
parallelrisch	Expression too large to display	6428

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
1/e^9*(e*x+d)^(1+m)/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+7236
80*m^3+1172700*m^2+1026576*m+362880)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*
e^8*m^7*x^8-800*d*e^7*m^7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m
^6*x^8+315*d*e^7*m^7*x^6-22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x
^6-25830*e^8*m^6*x^7+453600*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7
*x^5+9450*d*e^7*m^6*x^6-257600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*
x^5+67044*e^8*m^6*x^6-218610*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*
m^6*x^5+117600*d^2*e^6*m^5*x^6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+11466
0*d*e^7*m^5*x^6-1568000*d*e^7*m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-2353
2*e^8*m^6*x^5+579642*e^8*m^5*x^6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-3
3600*d^3*e^5*m^5*x^5+3330*d^2*e^6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2
*e^6*m^4*x^6-592*d*e^7*m^7*x^3+6290*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727
650*d*e^7*m^4*x^6-5415200*d*e^7*m^3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+9
9160*e^8*m^6*x^4-208458*e^8*m^5*x^5+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^
7+11812400*e^8*m^2*x^8+9450*d^3*e^5*m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2
*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-415800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*
m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*m^6*x^3+86210*d*e^7*m^5*x^4-1831500*
d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-10505600*d*e^7*m^2*x^7+33*e^8*m^8*x+44
94*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*e^8*m^5*x^4-1090353*e^8*m^4*x^5+91
34412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+10958400*e^8*m*x^8+168000*d^4*e^4...
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2796 vs. $2(432) = 864$.

Time = 0.31 (sec) , antiderivative size = 2796, normalized size of antiderivative = 6.47

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
(18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 2
2449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*
x^9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10
741248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 65
31840*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^
9)*m^7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7
*(19340*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(12
10*d*e^8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^
7 - 24*d*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*
e^7 + 675*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9
)*m^6 - 14*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^
7 + 17655*d*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 3262
29*e^9)*m^3 - 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(400
0*d^2*e^7 + 2025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7
 + 7434*d*e^8)*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^
7 + 1184*d*e^8 - 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e
^8 - 5883*e^9)*m^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 3474
3*e^9)*m^5 - (476000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^
9)*m^4 - 3*(420000*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*
m^3 - 2*(767200*d^3*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)...
```

3.368.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65193 vs. $2(410) = 820$.

Time = 13.90 (sec) , antiderivative size = 65193, normalized size of antiderivative = 150.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**
5/5 + 65*x**4/4 + 107*x**3/3 + 33*x**2/2 + 18*x), Eq(e, 0)), (84000*d**8*l
og(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4
7040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 235
20*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 228300*d**8/(84
0*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12
*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 672000*d**7*e*x*log(d/e + x)/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**
12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15
*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 1742400*d**7*e*x/(840*d**8*e
**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 67
20*d*e**16*x**7 + 840*e**17*x**8) + 4725*d**7*e/(840*d**8*e**9 + 6720*d**7
*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**1
3*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7
 + 840*e**17*x**8) + 2352000*d**6*e**2*x**2*log(d/e + x)/(840*d**8*e**9 +
6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*
d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e
**16*x**7 + 840*e**17*x**8) + 5762400*d**6*e**2*x**2/(840*d**8*e**9 + 6...
```

3.368.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(432) = 864$.

Time = 0.24 (sec) , antiderivative size = 1414, normalized size of antiderivative = 3.27

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 1
8*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)
*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^
3) + 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3
- 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^
5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2
*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/(
(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 +
 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2
 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3
+ 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 12
0*d^6)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
+ 720)*e^6) + 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m +
 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*
x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*
m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 36
0*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*
m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 4
5*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m ...
```

3.368.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6226 vs. $2(432) = 864$.

Time = 0.32 (sec) , antiderivative size = 6226, normalized size of antiderivative = 14.41

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
(100*(e*x + d)^m*e^9*m^8*x^9 + 100*(e*x + d)^m*d*e^8*m^8*x^8 - 45*(e*x + d
)^m*e^9*m^8*x^8 + 3600*(e*x + d)^m*e^9*m^7*x^9 - 45*(e*x + d)^m*d*e^8*m^8*
x^7 + 111*(e*x + d)^m*e^9*m^8*x^7 + 2800*(e*x + d)^m*d*e^8*m^7*x^8 - 1665*
(e*x + d)^m*e^9*m^7*x^8 + 54600*(e*x + d)^m*e^9*m^6*x^9 + 111*(e*x + d)^m*
d*e^8*m^8*x^6 - 37*(e*x + d)^m*e^9*m^8*x^6 - 800*(e*x + d)^m*d^2*e^7*m^7*x
^7 - 1350*(e*x + d)^m*d*e^8*m^7*x^7 + 4218*(e*x + d)^m*e^9*m^7*x^7 + 32200
*(e*x + d)^m*d*e^8*m^6*x^8 - 25830*(e*x + d)^m*e^9*m^6*x^8 + 453600*(e*x +
 d)^m*e^9*m^5*x^9 - 37*(e*x + d)^m*d*e^8*m^8*x^5 + 148*(e*x + d)^m*e^9*m^8
*x^5 + 315*(e*x + d)^m*d^2*e^7*m^7*x^6 + 3552*(e*x + d)^m*d*e^8*m^7*x^6 -
1443*(e*x + d)^m*e^9*m^7*x^6 - 16800*(e*x + d)^m*d^2*e^7*m^6*x^7 - 16380*(
e*x + d)^m*d*e^8*m^6*x^7 + 67044*(e*x + d)^m*e^9*m^6*x^7 + 196000*(e*x + d
)^m*d*e^8*m^5*x^8 - 218610*(e*x + d)^m*e^9*m^5*x^8 + 2244900*(e*x + d)^m*e
^9*m^4*x^9 + 148*(e*x + d)^m*d*e^8*m^8*x^4 + 65*(e*x + d)^m*e^9*m^8*x^4 -
666*(e*x + d)^m*d^2*e^7*m^7*x^5 - 1258*(e*x + d)^m*d*e^8*m^7*x^5 + 5920*(e
*x + d)^m*e^9*m^7*x^5 + 5600*(e*x + d)^m*d^3*e^6*m^6*x^6 + 7560*(e*x + d)^
m*d^2*e^7*m^6*x^6 + 45732*(e*x + d)^m*d*e^8*m^6*x^6 - 23532*(e*x + d)^m*e^
9*m^6*x^6 - 140000*(e*x + d)^m*d^2*e^7*m^5*x^7 - 103950*(e*x + d)^m*d*e^8*
m^5*x^7 + 579642*(e*x + d)^m*e^9*m^5*x^7 + 676900*(e*x + d)^m*d*e^8*m^4*x^
8 - 1098405*(e*x + d)^m*e^9*m^4*x^8 + 6728400*(e*x + d)^m*e^9*m^3*x^9 + 65
*(e*x + d)^m*d*e^8*m^8*x^3 + 107*(e*x + d)^m*e^9*m^8*x^3 + 185*(e*x + d...
```

3.368.9 Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 2625, normalized size of antiderivative = 6.08

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^
7 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^
3 + 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4
*e^5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 916207
2*d*e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14
868*d*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 52588
36*d^3*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^
3*m^2 + 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 18
9150*d^4*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*
m^4 + 133750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^
2*e^7*m^5 + 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^
3*e^6*m^6 - 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(102
6576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 87
0*m^7 + 45*m^8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/
(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6
+ 870*m^7 + 45*m^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531
840*e^9 + 9162072*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*
m^5 + 14868*e^9*m^6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896
800*d^3*e^6*m - 10741248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^...
```

3.368. $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.369 $\int (d+ex)^m (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

3.369.1 Optimal result

Integrand size = 36, antiderivative size = 292

$$\int (d+ex)^m (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx = \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{1+m}}{e^7(1+m)} - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^{2+m}}{e^7(2+m)} + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{3+m}}{e^7(3+m)} - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{4+m}}{e^7(4+m)} + \frac{(300d^2+85de+17e^2)(d+ex)^{5+m}}{e^7(5+m)} - \frac{(120d+17e)(d+ex)^{6+m}}{e^7(6+m)} + \frac{20(d+ex)^{7+m}}{e^7(7+m)}$$

output
```
(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^
7/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2
+m)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m
)/e^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(3
00*d^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e
^7/(6+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

3.369.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.89

$$\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)}{2+m} + \frac{(300d^4+170d^3e}{}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2
- d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3
 + 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*
e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 3
4*d*e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d +
e*x)^4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/
(7 + m)))/e^7
```

3.369.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{(300d^2+85de+17e^2)(d+ex)^{m+4}}{e^6} - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{m+3}}{e^6} + \frac{(300d^4+170d^3e+10}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{(300d^2+85de+17e^2)(d+ex)^{m+5}}{e^7(m+5)}-\frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{m+4}}{e^7(m+4)}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{m+1}}{e^7(m+1)}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{m+3}}{e^7(m+3)}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^{m+2}}{e^7(m+2)}-\frac{(120d+17e)(d+ex)^{m+6}}{e^7(m+6)}+\frac{20(d+ex)^{m+7}}{e^7(m+7)}$$

input
```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
+ e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2
*e^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 17
0*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)
) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4
+ m)) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((
120*d + 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e
^7*(7 + m))
```

3.369.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.369.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. 2(292) = 584.

Time = 0.50 (sec) , antiderivative size = 1220, normalized size of antiderivative = 4.18

method	result	size
norman	Expression too large to display	1220
gosper	Expression too large to display	1504
risch	Expression too large to display	1908
parallelrisch	Expression too large to display	3215

input
```
int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
d*(6*e^6*m^6-7*d*e^5*m^5+162*e^6*m^5+42*d^2*e^4*m^4-175*d*e^5*m^4+1770*e^6
*m^4+24*d^3*e^3*m^3+924*d^2*e^4*m^3-1715*d*e^5*m^3+9990*e^6*m^3+408*d^4*e^
2*m^2+432*d^3*e^3*m^2+7518*d^2*e^4*m^2-8225*d*e^5*m^2+30624*e^6*m^2+2040*d
^5*e*m+5304*d^4*e^2*m+2568*d^3*e^3*m+26796*d^2*e^4*m-19278*d*e^5*m+48168*e
^6*m+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280*d^2*e^4-17640*
d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068
*m+5040)*exp(m*ln(e*x+d))+(20*d*m-17*e*m-119*e)/e/(m^2+13*m+42)*x^6*exp(m*
ln(e*x+d))+(17*d*e^2*m^3-4*e^3*m^3+85*d^2*e*m^2+221*d*e^2*m^2-72*e^3*m^2+6
00*d^3*m+595*d^2*e*m+714*d*e^2*m-428*e^3*m-840*e^3)/e^3/(m^4+22*m^3+179*m^
2+638*m+840)*x^4*exp(m*ln(e*x+d))+(21*d*e^4*m^5+7*e^5*m^5+12*d^2*e^3*m^4+4
62*d*e^4*m^4+175*e^5*m^4+204*d^3*e^2*m^3+216*d^2*e^3*m^3+3759*d*e^4*m^3+17
15*e^5*m^3+1020*d^4*e*m^2+2652*d^3*e^2*m^2+1284*d^2*e^3*m^2+13398*d*e^4*m^
2+8225*e^5*m^2+7200*d^5*m+7140*d^4*e*m+8568*d^3*e^2*m+2520*d^2*e^3*m+17640
*d*e^4*m+19278*e^5*m+17640*e^5)/e^5/(m^6+27*m^5+295*m^4+1665*m^3+5104*m^2+
8028*m+5040)*x^2*exp(m*ln(e*x+d))+20/(7+m)*x^7*exp(m*ln(e*x+d))-(17*d*e*m^
2-17*e^2*m^2+120*d^2*m+119*d*e*m-221*e^2*m-714*e^2)/e^2/(m^3+18*m^2+107*m+
210)*x^5*exp(m*ln(e*x+d))-(4*d*e^3*m^4-21*e^4*m^4+68*d^2*e^2*m^3+72*d*e^3*
m^3-462*e^4*m^3+340*d^3*e*m^2+884*d^2*e^2*m^2+428*d*e^3*m^2-3759*e^4*m^2+2
400*d^4*m+2380*d^3*e*m+2856*d^2*e^2*m+840*d*e^3*m-13398*e^4*m-17640*e^4)/e
^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+d))-(-7*d...
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(292) = 584$.

Time = 0.29 (sec) , antiderivative size = 1448, normalized size of antiderivative = 4.96

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
(6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624
*e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5
*e^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280
*e^7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e
^6 - 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)
*m^2 - 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (
17136*e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5
 - 3*(400*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^
6 - 3145*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(24
0*d^2*e^5 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 177
0*d*e^6)*m^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^
6 - 96*e^7)*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4
- (3600*d^3*e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*
e^4 + 1411*d^2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2
*e^5 + 714*d*e^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2
*e^5 + 9990*d*e^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5
 + 84*d*e^6 - 525*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 518
7*e^7)*m^4 - (2400*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25
599*e^7)*m^3 - 4*(1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 -
 16338*e^7)*m^2 - 4*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d...
```

3.369.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26165 vs. $2(272) = 544$.

Time = 4.85 (sec) , antiderivative size = 26165, normalized size of antiderivative = 89.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**
2/2 + 6*x), Eq(e, 0)), (1200*d**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e*
*8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 3
60*d*e**12*x**5 + 60*e**13*x**6) + 2940*d**6/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360
*d*e**12*x**5 + 60*e**13*x**6) + 7200*d**5*e*x*log(d/e + x)/(60*d**6*e**7
+ 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e
**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 16440*d**5*e*x/(60*d**6*e*
*7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**
2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 170*d**5*e/(60*d**6*e**
7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2
*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 18000*d**4*e**2*x**2*log
(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3
*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 37
500*d**4*e**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 +
1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x
**6) + 1020*d**4*e**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x*
*2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e*
*13*x**6) - 34*d**4*e**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x
**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 6...
```

3.369.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(292) = 584$.

Time = 0.22 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.70

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{7\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m}{(m^2+3m+2)e^2} + \frac{6\,(ex+d)^{m+1}}{e(m+1)}$$
$$+ \frac{21\left((m^2+3m+2)e^3x^3+(m^2+m)de^2x^2-2d^2emx+2d^3\right)(ex+d)^m}{(m^3+6m^2+11m+6)e^3}$$
$$- \frac{4\left((m^3+6m^2+11m+6)e^4x^4+(m^3+3m^2+2m)de^3x^3-3(m^2+m)d^2e^2x^2+6d^3emx-6d^4\right)(ex-}{(m^4+10m^3+35m^2+50m+24)e^4}$$
$$+ \frac{17\left((m^4+10m^3+35m^2+50m+24)e^5x^5+(m^4+6m^3+11m^2+6m)de^4x^4-4(m^3+3m^2+2m)\right.}{(m^5+15m^4+85m^3+225m^2+274m+120}$$
$$- \frac{17\left((m^5+15m^4+85m^3+225m^2+274m+120)e^6x^6+(m^5+10m^4+35m^3+50m^2+24m)de^5x^5\right.}{(m^6+21m^5+175m}$$
$$+ \frac{20\left((m^6+21m^5+175m^4+735m^3+1624m^2+1764m+720)e^7x^7+(m^6+15m^5+85m^4+225m^3\right.}{}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*
(e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*
e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
- 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 +
35*m^2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*
x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m
^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24
*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m
^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6
)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720
)*e^6) + 20*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*
e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 -
6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 +
11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2
 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

3.369. $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.369.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. 2(292) = 584.

Time = 0.29 (sec) , antiderivative size = 3099, normalized size of antiderivative = 10.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
(20*(e*x + d)^m*e^7*m^6*x^7 + 20*(e*x + d)^m*d*e^6*m^6*x^6 - 17*(e*x + d)^
m*e^7*m^6*x^6 + 420*(e*x + d)^m*e^7*m^5*x^7 - 17*(e*x + d)^m*d*e^6*m^6*x^5
 + 17*(e*x + d)^m*e^7*m^6*x^5 + 300*(e*x + d)^m*d*e^6*m^5*x^6 - 374*(e*x +
 d)^m*e^7*m^5*x^6 + 3500*(e*x + d)^m*e^7*m^4*x^7 + 17*(e*x + d)^m*d*e^6*m^
6*x^4 - 4*(e*x + d)^m*e^7*m^6*x^4 - 120*(e*x + d)^m*d^2*e^5*m^5*x^5 - 289*
(e*x + d)^m*d*e^6*m^5*x^5 + 391*(e*x + d)^m*e^7*m^5*x^5 + 1700*(e*x + d)^m
*d*e^6*m^4*x^6 - 3230*(e*x + d)^m*e^7*m^4*x^6 + 14700*(e*x + d)^m*e^7*m^3*
x^7 - 4*(e*x + d)^m*d*e^6*m^6*x^3 + 21*(e*x + d)^m*e^7*m^6*x^3 + 85*(e*x +
 d)^m*d^2*e^5*m^5*x^4 + 323*(e*x + d)^m*d*e^6*m^5*x^4 - 96*(e*x + d)^m*e^7
*m^5*x^4 - 1200*(e*x + d)^m*d^2*e^5*m^4*x^5 - 1785*(e*x + d)^m*d*e^6*m^4*x
^5 + 3519*(e*x + d)^m*e^7*m^4*x^5 + 4500*(e*x + d)^m*d*e^6*m^3*x^6 - 13940
*(e*x + d)^m*e^7*m^3*x^6 + 32480*(e*x + d)^m*e^7*m^2*x^7 + 21*(e*x + d)^m*
d*e^6*m^6*x^2 + 7*(e*x + d)^m*e^7*m^6*x^2 - 68*(e*x + d)^m*d^2*e^5*m^5*x^3
 - 84*(e*x + d)^m*d*e^6*m^5*x^3 + 525*(e*x + d)^m*e^7*m^5*x^3 + 600*(e*x +
 d)^m*d^3*e^4*m^4*x^4 + 1105*(e*x + d)^m*d^2*e^5*m^4*x^4 + 2227*(e*x + d)^
m*d*e^6*m^4*x^4 - 904*(e*x + d)^m*e^7*m^4*x^4 - 4200*(e*x + d)^m*d^2*e^5*m
^3*x^5 - 5015*(e*x + d)^m*d*e^6*m^3*x^5 + 15725*(e*x + d)^m*e^7*m^3*x^5 +
5480*(e*x + d)^m*d*e^6*m^2*x^6 - 31433*(e*x + d)^m*e^7*m^2*x^6 + 35280*(e*
x + d)^m*e^7*m*x^7 + 7*(e*x + d)^m*d*e^6*m^6*x + 6*(e*x + d)^m*e^7*m^6*x +
 12*(e*x + d)^m*d^2*e^5*m^5*x^2 + 483*(e*x + d)^m*d*e^6*m^5*x^2 + 182*(...
```

3.369.9 Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 1425, normalized size of antiderivative = 4.88

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 3528
0*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4
*m + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 +
1770*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3
*e^4*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*
e^4*m^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^
5 + 48168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 +
1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m +
 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2
+ 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(3
5280*d^2*e^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 -
 162*e^7*m^5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m
+ 14280*d^5*e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 17
5*d*e^6*m^5 - 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^
4*e^3*m^2 + 2040*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^
4*e^3*m^3 + 924*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^
6*m + 14400*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 32
2*m^5 + 28*m^6 + m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4
*m - 13398*e^4*m - 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2
856*d^2*e^2*m + 428*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*...
```

3.369. $\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$

3.370 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

3.370.1 Optimal result

Integrand size = 38, antiderivative size = 255

$$\begin{aligned}
&\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx \\
&= \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(d+ex)^{3+m}}{5e^3(3+m)} \\
&\quad - \frac{(6412i-423\sqrt{14})(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-e+i\sqrt{14}e}\right)}{3500\left(5id-\left(i+\sqrt{14}\right)e\right)(1+m)} \\
&\quad - \frac{(6412i+423\sqrt{14})(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500\left(5id-\left(i-\sqrt{14}\right)e\right)(1+m)}
\end{aligned}$$

output
```
1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e
*x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(e*x+d)^(1+m)*hyp
ergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(6412*I+423*14^(1/
2))/(1+m)/(5*I*d-e*(I-14^(1/2)))-1/3500*(e*x+d)^(1+m)*hypergeom([1, 1+m],[
2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(6412*I-423*14^(1/2))/(1+m)/(5*I*d-e*
(I+14^(1/2)))
```

3.370.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{28(100d^2+165de+81e^2)}{e^3(1+m)} - \frac{140(40d+33e)(d+ex)}{e^3(2+m)} + \frac{2800(d+ex)^2}{e^3(3+m)} - \frac{\left(6412i+423\sqrt{14}\right)\text{Hypergeometric2F1}\left(1,1+m,2+\right.}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)}\right.}{3500}$$

input
```
Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140
*(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m))
 - ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x
))/(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) -
 ((-6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x)
)/(5*d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3
500
```

3.370.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^m}{5x^2+2x+3}\,dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{(100d^2+165de+81e^2)(d+ex)^m}{125e^2} + \frac{(-40d-33e)(d+ex)^{m+1}}{25e^2} + \frac{4(d+ex)^{m+2}}{5e^2} + \frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex)}{10x-2i\sqrt{14}+2}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{5(d+ex)}{5d+i\sqrt{14}e-e}\right)}{3500(m+1)\left(5id-\left(\sqrt{14}+i\right)e\right)}-\frac{(423\sqrt{14}+6412i)(d+ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500(m+1)\left(5id-\left(-\sqrt{14}+i\right)e\right)}$$

input
```
Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*
d + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e
^3*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d
- (I + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e
)])/(3500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

3.370.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.370.4 Maple [F]

$$\int \frac{(ex+d)^m\,(4x^4-5x^3+3x^2+x+2)}{5x^2+2x+3}dx$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

3.370.5 Fricas [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

3.370.7 Maxima [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x
)
```

3.370.8 Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x
)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(d+e\,x)^m\,(4\,x^4-5\,x^3+3\,x^2+x+2)}{5\,x^2+2\,x+3}\,dx$$

input
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

3.371 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

3.371.1 Optimal result

Integrand size = 38, antiderivative size = 377

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem\right.}{19600\left(5d+i\left(i+\sqrt{14}\right)e\right)\left(5d^2-2de+\right.}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2-i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem\right.}{19600\left(5d-\left(1+i\sqrt{14}\right)e\right)\left(5d^2-2de\right.}$$

output
```
4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(
1+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(e*x+d)^(1+m)*hypergeom([1,
 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(80360*d^2-32144*d*e+48216*e
^2-5922*d*e*m+19138*e^2*m-I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))
*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d-e*(1+I*14^(1/2)))+1/19600*(e*x+d
)^(1+m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(80360*d^
2-32144*d*e+48216*e^2-5922*d*e*m+19138*e^2*m+I*(6565*d^2-2*d*e*(1313-3206*
m)+e^2*(3939-98*m))*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d+I*e*(I+14^(1/
2)))
```

3.371.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{3136}{e+em}-\frac{28(d(1367+423x)-e(293+1367x))}{(5d^2-2de+3e^2)(3+2x+5x^2)}+\frac{56\left(287i+31\sqrt{14}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)} - \cdots\right.}{\cdots}$$

input
```
Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367
*x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[1
4])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[1
4])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[1
4])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14
])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 +
d*e*(-846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqr
t[14])*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I
*Sqrt[14])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6
412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hype
rgeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/
((5*I)*d - (I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

3.371.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2179, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^m}{(5x^2+2x+3)^2}dx$$

$\downarrow$ 2179

$$\frac{\int \frac{2(d+ex)^m(1845d^2-e(738-1367m)d+560(5d^2-2ed+3e^2)x^2+e^2(1107-293m)-(4620d^2-3e(141m+616)d+e^2(1367m+2772))x)}{25(5x^2+2x+3)}dx}{56(5d^2-2de+3e^2)}-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$\downarrow$ 27

$$\frac{\int \frac{(d+ex)^m(1845d^2-e(738-1367m)d+560(5d^2-2ed+3e^2)x^2+e^2(1107-293m)-(4620d^2-3e(141m+616)d+e^2(1367m+2772))x)}{5x^2+2x+3}dx}{700(5d^2-2de+3e^2)}-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$\downarrow$ 2159

$$-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}+\frac{\int \left(112(5d^2-2ed+3e^2)(d+ex)^m+\frac{\left(-5740d^2+2296ed+423emd-3444e^2-1367e^2m-\frac{i(6565d^2-2626ed+6412emd+3939e^2-98e^2m)}{\sqrt{14}}\right)(}{10x-2i\sqrt{14}+2}\right.}{700(5d^2-2de+3e}$$

$\downarrow$ 2009

$$-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}+\frac{\left(i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m-\right.}{28(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)}$$

input
```
Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

3.371. $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}dx$

output
```
-1/700*((1367*d - 293*e + (423*d - 1367*e)*x)*(d + e*x)^(1 + m))/((5*d^2 -
 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((112*(5*d^2 - 2*d*e + 3*e^2)*(d + e*
x)^(1 + m))/(e*(1 + m)) + ((80360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]
*(6565*d^2 - 2*d*e*(1313 - 3206*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 191
38*e^2*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x
))/(5*d - e + I*Sqrt[14]*e)])/(28*(5*d + I*(I + Sqrt[14])*e)*(1 + m)) + ((
80360*d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3
206*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e
)])/(28*(5*d - (1 + I*Sqrt[14])*e)*(1 + m)))/(700*(5*d^2 - 2*d*e + 3*e^2))
```

3.371.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2179
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*((R*(b*c*d - b^2*e + 2*a*c*e) - a*S*(2*c*d - b*e) + c*
(R*(2*c*d - b*e) - S*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Qx + R*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - S*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(S*(b*d - 2*a*e) - R
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x
] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& LtQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] |
| ILtQ[p + 1/2, 0]))
```

3.371. $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

3.371.4 Maple [F]

$$\int \frac{(ex+d)^m (4x^4-5x^3+3x^2+x+2)}{(5x^2+2x+3)^2}dx$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

3.371.5 Fricas [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34
*x^2 + 12*x + 9), x)
```

3.371.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

3.371.7 Maxima [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
 x)
```

3.371.8 Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
 x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(d+e\,x)^m\,(4\,x^4-5\,x^3+3\,x^2+x+2)}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3} \, dx$

3.372.1 Optimal result

Integrand size = 38, antiderivative size = 528

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3} \, dx =$$
$$-\frac{ab^3ch+bc^2(c^2d+acf-3a^2h)-ab^4i-ab^2c(cg-4ai)-2ac^2(c^2e-acg+a^2i)+(2c^5d-c^4(be+2af}{2c^4(b^2-4ac)(a+bx+cx^2}$$
$$+\frac{b^5ch+b^3c^2(cf-8ah)+2bc^3(3c^2d+acf+11a^2h)-b^6i-b^4c(cg-11ai)-16a^2c^3(cg-2ai)-b^2c^2(3}{2c^4(b^2}$$
$$-\frac{(12c^5d-c^4(6be-4af)+2c^3(b^2f-3abg+6a^2h)-b^5i+10ab^3ci-30a^2bc^2i)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{5/2}}$$
$$+\frac{i\log(a+bx+cx^2)}{2c^3}$$

output
```
1/2*(-a*b^3*c*h-b*c^2*(-3*a^2*h+a*c*f+c^2*d)+a*b^4*i+a*b^2*c*(-4*a*i+c*g)+
2*a*c^2*(a^2*i-a*c*g+c^2*e)-(2*c^5*d-c^4*(2*a*f+b*e)+c^3*(2*a^2*h+3*a*b*g+
b^2*f)-b^5*i+b^3*c*(5*a*i+b*h)-b*c^2*(5*a^2*i+4*a*b*h+b^2*g))*x)/c^4/(-4*a
*c+b^2)/(c*x^2+b*x+a)^2+1/2*(b^5*c*h+b^3*c^2*(-8*a*h+c*f)+2*b*c^3*(11*a^2*
h+a*c*f+3*c^2*d)-b^6*i-b^4*c*(-11*a*i+c*g)-16*a^2*c^3*(-2*a*i+c*g)-b^2*c^2
*(39*a^2*i-5*a*c*g+3*c^2*e)+2*c*(6*c^5*d-c^4*(-2*a*f+3*b*e)+c^3*(-10*a^2*h
-3*a*b*g+b^2*f)+2*b^5*i-b^3*c*(15*a*i+b*h)+a*b*c^2*(25*a*i+8*b*h))*x)/c^4/
(-4*a*c+b^2)^2/(c*x^2+b*x+a)-(12*c^5*d-c^4*(-4*a*f+6*b*e)+2*c^3*(6*a^2*h-3
*a*b*g+b^2*f)-b^5*i+10*a*b^3*c*i-30*a^2*b*c^2*i)*arctanh((2*c*x+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*i*ln(c*x^2+b*x+a)/c^3
```

3.372.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.92

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{\frac{b^5ix+b^4(ai-chx)+2c^2\left(a^3i-c^3dx+ac^2(e+fx)-a^2c(g+hx)\right)+b^2c\left(-4a^2i-c^2fx+ac(g+4hx)\right)+b^3c(cgx-a(h+5ix))+bc^2\left(c^2(-d+ex)-ac(f+3gx\right.}{(b^2-4ac)(a+x(b+cx))^2}}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

output
```
((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
 a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(
c*g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*
h + 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h +
4*i*x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)
) + 4*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^
2*(-39*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(
d - e*x) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x
*(b + c*x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*
b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*
c^4)
```

3.372.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2191, 2191, 27, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$-\frac{\int \frac{-\frac{ib^5}{c^4}+\frac{(bh+3ai)b^3}{c^3}-3eb-\frac{\left(-ia^2+2bha+b^2g\right)b}{c^2}+2\left(4a-\frac{b^2}{c}\right)ix^3-\frac{2\left(b^2-4ac\right)(ch-bi)x^2}{c^2}+6cd+2af+\frac{-2ha^2+bga+b^2f}{c}-\frac{2\left(b^2-4ac\right)\left(ib^2+c^2g-c(bh+ai)\right.}{c^3}}{\left(cx^2+bx+a\right)^2}}{2\left(b^2-4ac\right)}$$

$$\frac{x\left(c^3\left(2a^2h+3abg+b^2f\right)-bc^2\left(5a^2i+4abh+b^2g\right)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2\left(-3a^2\right.}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 2191

$$-\frac{-\frac{\int \frac{2\left(\left(\frac{aib^3}{c^2}+fb^2-3ceb-3agb-\frac{7a^2ib}{c}+6c^2d+2acf+6a^2h\right)c^2+\left(b^2-4ac\right)^2ix\right)}{c^2\left(cx^2+bx+a\right)}dx}{b^2-4ac}-\frac{-b^2c^2\left(39a^2i-5acg+3c^2e\right)+2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(1\right.}{}}{2\left(b^2\right.}$$

$$\frac{x\left(c^3\left(2a^2h+3abg+b^2f\right)-bc^2\left(5a^2i+4abh+b^2g\right)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2\left(-3a^2\right.}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 27

$$-\frac{-\frac{2\int \frac{\left(\frac{aib^3}{c^2}+fb^2-3ceb-3agb-\frac{7a^2ib}{c}+6c^2d+2acf+6a^2h\right)c^2+\left(b^2-4ac\right)^2ix}{cx^2+bx+a}dx}{c^2\left(b^2-4ac\right)}-\frac{-b^2c^2\left(39a^2i-5acg+3c^2e\right)+2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(15c\right.}{}}{2\left(b^2-\right.}$$

$$\frac{x\left(c^3\left(2a^2h+3abg+b^2f\right)-bc^2\left(5a^2i+4abh+b^2g\right)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2\left(-3a^2\right.}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 1142

$$-\frac{-\frac{2\left(\frac{\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)\int \frac{1}{cx^2+bx+a}dx}{2c}+\frac{i\left(b^2-4ac\right)^2\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}\right)}{c^2\left(b^2-4ac\right)}-\frac{-b^2c^2\left(39a^2i-5acg+\right.}{}}{}$$

$$\frac{x\left(c^3\left(2a^2h+3abg+b^2f\right)-bc^2\left(5a^2i+4abh+b^2g\right)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2\left(-3a^2\right.}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 1083

$$-\frac{-\frac{2\left(\frac{i\left(b^2-4ac\right)^2\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}\right)}{c^2\left(b^2-4ac\right)}-\frac{-b^2c^2\left(\right.}{}}{}$$

$$\frac{x\left(c^3\left(2a^2h+3abg+b^2f\right)-bc^2\left(5a^2i+4abh+b^2g\right)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2\left(-3a^2\right.}{2c^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

$\downarrow$ 219

3.372. $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}dx$

$$-\frac{-\frac{2\left(\frac{i(b^2-4ac)^2\int\frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3(6a^2h-3abg+b^2f)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)}-\frac{-b^2c^2(39a^2i\ldots}{\ldots}}{\ldots}$$

$$\frac{x\left(c^3(2a^2h+3abg+b^2f)-bc^2(5a^2i+4abh+b^2g)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2(-3a^2\ldots}{2c^4(b^2-4ac)(a+bx+cx^2)^2}$$

$\downarrow$ 1103

$$-\frac{-\frac{2\left(\frac{i(b^2-4ac)^2\log(a+bx+cx^2)}{2c}-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3(6a^2h-3abg+b^2f)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)}-\frac{-b^2c^2(39a^2\ldots}{\ldots}}{\ldots}$$

$$\frac{x\left(c^3(2a^2h+3abg+b^2f)-bc^2(5a^2i+4abh+b^2g)+b^3c(5ai+bh)-c^4(2af+be)+b^5(-i)+2c^5d\right)+bc^2(-3a^2\ldots}{2c^4(b^2-4ac)(a+bx+cx^2)^2}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g
 - 4*a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f)
 + c^3*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(
b^2*g + 4*a*b*h + 5*a^2*i))*x)/(c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (
-((b^5*c*h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h)
- b^6*i - b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2
*e - 5*a*c*g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f
 - 3*a*b*g - 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h +
 25*a*i))*x)/(c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2))) - (2*(-(((12*c^5*d - c
^4*(6*b*e - 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*
c*i - 30*a^2*b*c^2*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2
- 4*a*c])) + ((b^2 - 4*a*c)^2*i*Log[a + b*x + c*x^2])/(2*c)))/(c^2*(b^2 -
4*a*c)))/(2*(b^2 - 4*a*c))
```

3.372. $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$

3.372.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.372. $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$

3.372.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.46

method	result
default	$\frac{\left(25a^2b\,c^2i-10a^2c^3h-15a\,b^3ci+8a\,b^2c^2h-3ab\,c^3g+2a\,c^4f+2b^5i-b^4ch+b^2c^3f-3b\,c^4e+6c^5d\right)x^3}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(32a^3c^3i+11a^2b^2c^2i+2a^2b\,c^3h-16a^2c^4g-19a\ldots\right.}{}$
risch	Expression too large to display

input
```
int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOS
E)
```

output
```
((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c
^4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-1
9*a*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*
g+3*b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(
31*a^3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2
*a^2*c^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*
c^5*d-b^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*
a^4*c^2*i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*
h-a^2*b^2*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3
*d)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/c^2/(16*a^2*c^2-8*a*b^2*
c+b^4)*(1/2*(16*a^2*c^2*i-8*a*b^2*c*i+b^4*i)/c*ln(c*x^2+b*x+a)+2*(-7*a^2*b
*i*c+6*a^2*c^2*h+a*b^3*i-3*a*b*c^2*g+2*a*c^3*f+b^2*c^2*f-3*b*c^3*e+6*c^4*d
-1/2*(16*a^2*c^2*i-8*a*b^2*c*i+b^4*i)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2)))
```

3.372.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1730 vs. 2(518) = 1036.

Time = 0.37 (sec) , antiderivative size = 3480, normalized size of antiderivative = 6.59

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fr
icas")
```

output
```
[1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 -
2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12
*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85
*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^
4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*
c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2
 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2
- 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - (12*a^2*c^5*d - 6*a^2*b*c^4*e - 6
*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2
*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)
*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2
*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3
+ (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a
*b^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3
+ 2*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2
+ 2*(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*
i + 2*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2
*(a*b^3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*
x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4
*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^...
```

3.372.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

3.372.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx + cx^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.22

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx + cx^2)^3} dx$$
$$= \frac{(12 c^5 d - 6 bc^4 e + 2 b^2 c^3 f + 4 ac^4 f - 6 abc^3 g + 12 a^2 c^3 h - b^5 i + 10 ab^3 ci - 30 a^2 bc^2 i) \arctan\left(\frac{2 cx+b}{\sqrt{-b^2+4 ac}}\right)}{(b^4 c^3 - 8 ab^2 c^4 + 16 a^2 c^5)\sqrt{-b^2 + 4 ac}}$$
$$+ \frac{i \log(cx^2 + bx + a)}{2 c^3}$$
$$- \frac{b^3 c^3 d - 10 abc^4 d + ab^2 c^3 e + 8 a^2 c^4 e - 6 a^2 bc^3 f + a^2 b^2 c^2 g + 8 a^3 c^3 g + a^2 b^3 ch - 10 a^3 bc^2 h - 3 a^2 b^4 i + 21}{}$$

input
```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gi
ac")
```

output
```
(12*c^5*d - 6*b*c^4*e + 2*b^2*c^3*f + 4*a*c^4*f - 6*a*b*c^3*g + 12*a^2*c^3
*h - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*i
*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d + a*b^2*c^3*e +
8*a^2*c^4*e - 6*a^2*b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h -
10*a^3*b*c^2*h - 3*a^2*b^4*i + 21*a^3*b^2*c*i - 24*a^4*c^2*i - 2*(6*c^6*d
- 3*b*c^5*e + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^
3*h - 10*a^2*c^4*h + 2*b^5*c*i - 15*a*b^3*c^2*i + 25*a^2*b*c^3*i)*x^3 - (1
8*b*c^5*d - 9*b^2*c^4*e + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^
3*g - 16*a^2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h + 3*b^6*i - 1
9*a*b^4*c*i + 11*a^2*b^2*c^2*i + 32*a^3*c^3*i)*x^2 - 2*(2*b^2*c^4*d + 10*a
*c^5*d - b^3*c^3*e - 5*a*b*c^4*e + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2
*g - 5*a^2*b*c^3*g - a*b^4*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h + 3*a*b^5*
i - 22*a^2*b^3*c*i + 31*a^3*b*c^2*i)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^3)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 15.65 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{x\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,(\cdots}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\cdots\right.}\right.}{}$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-1024\,i\,a^5\,c^5+1280\,i\,a^4\,b^2\,c^4-640\,i\,a^3\,b^4\,c^3+160\,i\,a^2\,b^6\,c^2-20\,i\,a\,b^8\,c+i\,b^{10}\right)}{2\,\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)}$$
$$-\frac{\frac{-24\,i\,a^4\,c^2+21\,i\,a^3\,b^2\,c-10\,h\,a^3\,b\,c^2+8\,g\,a^3\,c^3-3\,i\,a^2\,b^4+h\,a^2\,b^3\,c+g\,a^2\,b^2\,c^2-6\,f\,a^2\,b\,c^3+8\,e\,a^2\,c^4+e\,a\,b^2\,c^3-10\,d\,a\,b\,c^4+d\,b^3\,c^3}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,(32\,i\,a^3\cdots}{}}{}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

output
```
(atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) -
 16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(
4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c -
 b^2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^
5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^
2*c^3*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^
2*i))/(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5
*c^5*i + 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a
*b^8*c*i))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 +
640*a^3*b^4*c^6 - 1280*a^4*b^2*c^7)) - ((8*a^2*c^4*e + b^3*c^3*d + 8*a^3*c
^3*g - 3*a^2*b^4*i - 24*a^4*c^2*i + a^2*b^2*c^2*g - 10*a*b*c^4*d + a*b^2*c
^3*e - 6*a^2*b*c^3*f + a^2*b^3*c*h - 10*a^3*b*c^2*h + 21*a^3*b^2*c*i)/(2*c
^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(3*b^6*i - 9*b^2*c^4*e - 16*a^2*
c^4*g + 3*b^3*c^3*f - b^4*c^2*g + 32*a^3*c^3*i + 18*b*c^5*d - b^5*c*h + 11
*a^2*b^2*c^2*i + 6*a*b*c^4*f - 19*a*b^4*c*i - a*b^2*c^3*g + 8*a*b^3*c^2*h
+ 2*a^2*b*c^3*h))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(2*a^2*c^4*f
 - 2*b^2*c^4*d + b^3*c^3*e + 6*a^3*c^3*h - 10*a*c^5*d - 3*a*b^5*i - 10*a^2
*b^2*c^2*h + 5*a*b*c^4*e + a*b^4*c*h - 5*a*b^2*c^3*f + a*b^3*c^2*g + 5*a^2
*b*c^3*g + 22*a^2*b^3*c*i - 31*a^3*b*c^2*i))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) - (x^3*(6*c^5*d + 2*b^5*i + b^2*c^3*f - 10*a^2*c^3*h + 2*a*c^4*...
```

3.372. $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx+cx^2)^3}\,dx$

3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

3.373.1 Optimal result

Integrand size = 53, antiderivative size = 765

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$
$$= \frac{(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3(b^3j+}{c^7}$$
$$+\frac{(c^5g-c^4(bh+aj)+c^3(b^2j+2abk+a^2l)-b^5m+b^3c(bl+4am)-bc^2(b^2k+3abl+3a^2m))\,x^2}{2c^6}$$
$$+\frac{(c^4h-c^3(bj+ak)+b^4m-b^2c(bl+3am)+c^2(b^2k+2abl+a^2m))\,x^3}{3c^5}$$
$$+\frac{(c^3j-c^2(bk+al)-b^3m+bc(bl+2am))\,x^4}{4c^4}$$
$$+\frac{(c^2k+b^2m-c(bl+am))\,x^5}{5c^3}+\frac{(cl-bm)x^6}{6c^2}+\frac{mx^7}{7c}$$
$$-\frac{(2c^8d-c^7(be+2af)+c^6(b^2f+3abg+2a^2h)-c^5(b^3g+4ab^2h+5a^2bj+2a^3k)+b^8m-b^6c(bl+8an}{}$$
$$+\frac{(c^7e-c^6(bf+ag)+c^5(b^2g+2abh+a^2j)-c^4(b^3h+3ab^2j+3a^2bk+a^3l)-b^7m+b^5c(bl+6am)-b}{2c^8}$$

output
```
(c^6*f-c^5*(a*h+b*g)+c^4*(a^2*k+2*a*b*j+b^2*h)+b^6*m-b^4*c*(5*a*m+b*l)+b^2
*c^2*(6*a^2*m+4*a*b*l+b^2*k)-c^3*(a^3*m+3*a^2*b*l+3*a*b^2*k+b^3*j))*x/c^7+
1/2*(c^5*g-c^4*(a*j+b*h)+c^3*(a^2*l+2*a*b*k+b^2*j)-b^5*m+b^3*c*(4*a*m+b*l)
-b*c^2*(3*a^2*m+3*a*b*l+b^2*k))*x^2/c^6+1/3*(c^4*h-c^3*(a*k+b*j)+b^4*m-b^2
*c*(3*a*m+b*l)+c^2*(a^2*m+2*a*b*l+b^2*k))*x^3/c^5+1/4*(c^3*j-c^2*(a*l+b*k)
-b^3*m+b*c*(2*a*m+b*l))*x^4/c^4+1/5*(c^2*k+b^2*m-c*(a*m+b*l))*x^5/c^3+1/6*
(-b*m+c*l)*x^6/c^2+1/7*m*x^7/c+1/2*(c^7*e-c^6*(a*g+b*f)+c^5*(a^2*j+2*a*b*h
+b^2*g)-c^4*(a^3*l+3*a^2*b*k+3*a*b^2*j+b^3*h)-b^7*m+b^5*c*(6*a*m+b*l)-b^3*
c^2*(10*a^2*m+5*a*b*l+b^2*k)+b*c^3*(4*a^3*m+6*a^2*b*l+4*a*b^2*k+b^3*j))*ln
(c*x^2+b*x+a)/c^8-(2*c^8*d-c^7*(2*a*f+b*e)+c^6*(2*a^2*h+3*a*b*g+b^2*f)-c^5
*(2*a^3*k+5*a^2*b*j+4*a*b^2*h+b^3*g)+b^8*m-b^6*c*(8*a*m+b*l)+b^4*c^2*(20*a
^2*m+7*a*b*l+b^2*k)-b^2*c^3*(16*a^3*m+14*a^2*b*l+6*a*b^2*k+b^3*j)+c^4*(2*a
^4*m+7*a^3*b*l+9*a^2*b^2*k+5*a*b^3*j+b^4*h))*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^8/(-4*a*c+b^2)^(1/2)
```

3.373.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 754, normalized size of antiderivative = 0.99

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$

$$= \frac{420c(c^6f-c^5(bg+ah)+c^4(b^2h+2abj+a^2k)+b^6m-b^4c(bl+5am)+b^2c^2(b^2k+4abl+6a^2m)-c^3(b}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8
)/(a + b*x + c*x^2),x]
```

output
```
(420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m -
b^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3
*a*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*
(b^2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3
*a*b*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*
(b*l + 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*
(b*k + a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(
b*l + a*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2
*h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k +
 7*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m)
 + c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b
+ 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f +
 a*g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k
 + a^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^
2*m) + b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x
)])/(420*c^8)
```

3.373.3 Rubi [A] (verified)

Time = 4.05 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(\frac{x^2\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)}{c^5}+\frac{x\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\left(3a^2n\right.\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)}{3c^5}+$$
$$\frac{x^2\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\left(3a^2m+3abl+b^2k\right)+b^3c(4am+bl)-c^4(aj+bh)+b^5(-m)+c^5g\right)}{2c^6}+$$
$$\frac{\log\left(a+bx+cx^2\right)\left(c^5\left(a^2j+2abh+b^2g\right)-b^3c^2\left(10a^2m+5abl+b^2k\right)-c^4\left(a^3l+3a^2bk+3ab^2j+b^3h\right)+bc^3\left(4a^3\right.\right.}{2c^8}$$
$$\frac{x\left(c^4\left(a^2k+2abj+b^2h\right)+b^2c^2\left(6a^2m+4abl+b^2k\right)-c^3\left(a^3m+3a^2bl+3ab^2k+b^3j\right)-b^4c(5am+bl)-c^5(ah+\right.}{c^7}$$
$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)+b^4c^2\left(20a^2m+7abl+b^2k\right)-c^5\left(2a^3k+5a^2bj+4ab^2h+b^3g\right)-b^2c^3\right.}{}$$
$$\frac{x^4\left(-c^2(al+bk)+bc(2am+bl)+b^3(-m)+c^3j\right)}{4c^4}+\frac{x^5\left(-c(am+bl)+b^2m+c^2k\right)}{5c^3}+$$
$$\frac{x^6(cl-bm)}{6c^2}+\frac{mx^7}{7c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x + c*x^2),x]
```

output
```
((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*
(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2
*k + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j +
 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l +
 3*a^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l +
 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k
 + a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b
*l + a*m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*
c^8*d - c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g +
 4*a*b^2*h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*
(b^2*k + 7*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 1
6*a^3*m) + c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e -
c^6*(b*f + a*g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j +
 3*a^2*b*k + a^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b
*l + 10*a^2*m) + b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a +
b*x + c*x^2])/(2*c^8)
```

3.373. $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.373.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 1086, normalized size of antiderivative = 1.42

method	result	size
default	Expression too large to display	1086
risch	Expression too large to display	49911

input
```
int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x,meth
od=_RETURNVERBOSE)
```

output
```
-1/c^7*(a*b^2*c^3*m*x^3-1/2*c^6*g*x^2-b^6*m*x-c^6*f*x-1/5*c^6*k*x^5-1/4*c^
6*j*x^4-1/3*c^6*h*x^3-1/6*c^6*l*x^6-1/7*m*x^7*c^6+1/2*b*c^5*h*x^2+m*c^3*a^
3*x-a^2*c^4*k*x+h*c^5*a*x+b^5*c*l*x-b^4*c^2*k*x+b^3*c^3*j*x-b^2*c^4*h*x+b*
c^5*g*x-1/2*a^2*c^4*l*x^2+1/2*a*c^5*j*x^2+1/2*b^5*c*m*x^2-1/2*b^4*c^2*l*x^
2+1/2*b^3*c^3*k*x^2-1/2*b^2*c^4*j*x^2-1/3*a^2*c^4*m*x^3+1/3*a*c^5*k*x^3-1/
3*b^4*c^2*m*x^3+1/3*b^3*c^3*l*x^3-1/3*b^2*c^4*k*x^3+1/3*b*c^5*j*x^3+1/5*a*
c^5*m*x^5-1/5*b^2*c^4*m*x^5+1/5*b*c^5*l*x^5+1/4*a*c^5*l*x^4+1/4*b^3*c^3*m*
x^4-1/4*b^2*c^4*l*x^4+1/4*b*c^5*k*x^4+1/6*b*c^5*m*x^6-1/2*a*b*c^4*m*x^4-2/
3*a*b*c^4*l*x^3+3/2*a^2*b*c^3*m*x^2-2*a*b^3*c^2*m*x^2+3/2*a*b^2*c^3*l*x^2-
a*b*c^4*k*x^2-6*a^2*b^2*c^2*m*x+3*a^2*b*c^3*l*x+5*a*b^4*c*m*x-4*a*b^3*c^2*
l*x+3*a*b^2*c^3*k*x-2*a*b*c^4*j*x)+1/c^7*(1/2*(4*a^3*b*c^3*m-a^3*c^4*l-10*
a^2*b^3*c^2*m+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*b^4*
c^2*l+4*a*b^3*c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^5*c^
2*k+b^4*c^3*j-b^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)/c*ln(c*x^2+b*x+a)+2*(a^4*
c^3*m-6*a^3*b^2*c^2*m+3*a^3*b*c^3*l-a^3*c^4*k+5*a^2*b^4*c*m-4*a^2*b^3*c^2*
l+3*a^2*b^2*c^3*k-2*a^2*b*c^4*j+a^2*c^5*h-a*b^6*m+a*b^5*c*l-a*b^4*c^2*k+a*
b^3*c^3*j-a*b^2*c^4*h+a*b*c^5*g-a*c^6*f+c^7*d-1/2*(4*a^3*b*c^3*m-a^3*c^4*l
-10*a^2*b^3*c^2*m+6*a^2*b^2*c^3*l-3*a^2*b*c^4*k+a^2*c^5*j+6*a*b^5*c*m-5*a*
b^4*c^2*l+4*a*b^3*c^3*k-3*a*b^2*c^4*j+2*a*b*c^5*h-a*c^6*g-b^7*m+b^6*c*l-b^
5*c^2*k+b^4*c^3*j-b^3*c^4*h+b^2*c^5*g-b*c^6*f+c^7*e)*b/c)/(4*a*c-b^2)^(...
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 2643, normalized size of antiderivative = 3.45

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),
x, algorithm="fricas")
```

output
```
[1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^
6 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*
l + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*
j - (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5
*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b
^3*c^6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 -
4*a^3*c^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h +
 (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*
c^6)*k + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2
 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 + 210*(2*c^8*d - b*
c^7*e + (b^2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b
^2*c^5 + 2*a^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2
 - 6*a*b^4*c^3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*
a^2*b^3*c^3 - 7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 2*a^4*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7
 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c
^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 +
 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^...
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Timed out}$$

input
```
integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2
+b*x+a),x)
```

output
```
Timed out
```

3.373.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),
x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.28

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$$
$$= \frac{60\,c^6mx^7+70\,c^6lx^6-70\,bc^5mx^6+84\,c^6kx^5-84\,bc^5lx^5+84\,b^2c^4mx^5-84\,ac^5mx^5+105\,c^6jx^4-105\,bc^5}{}$$
$$+\frac{(c^7e-bc^6f+b^2c^5g-ac^6g-b^3c^4h+2\,abc^5h+b^4c^3j-3\,ab^2c^4j+a^2c^5j-b^5c^2k+4\,ab^3c^3k-3\,a^2bc^4k}{2\,c^8}$$
$$+\frac{(2\,c^8d-bc^7e+b^2c^6f-2\,ac^7f-b^3c^5g+3\,abc^6g+b^4c^4h-4\,ab^2c^5h+2\,a^2c^6h-b^5c^3j+5\,ab^3c^4j-5\,a^2}{}$$

input
```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),
x, algorithm="giac")
```

output
```
1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*
c^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*
k*x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*
c^4*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^
5*k*x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*
a*b^2*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 21
0*b^2*c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2
+ 210*b^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*
m*x^2 + 840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^
5*g*x + 420*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*
x + 420*b^4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x
 + 1680*a*b^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*
x + 2520*a^2*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 + 1/2*(c^7*e - b*c^6*f + b
^2*c^5*g - a*c^6*g - b^3*c^4*h + 2*a*b*c^5*h + b^4*c^3*j - 3*a*b^2*c^4*j +
 a^2*c^5*j - b^5*c^2*k + 4*a*b^3*c^3*k - 3*a^2*b*c^4*k + b^6*c*l - 5*a*b^4
*c^2*l + 6*a^2*b^2*c^3*l - a^3*c^4*l - b^7*m + 6*a*b^5*c*m - 10*a^2*b^3*c^
2*m + 4*a^3*b*c^3*m)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d - b*c^7*e + b^2*c
^6*f - 2*a*c^7*f - b^3*c^5*g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2
*a^2*c^6*h - b^5*c^3*j + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b
^4*c^3*k + 9*a^2*b^2*c^4*k - 2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14...
```

3.373.9 Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 2779, normalized size of antiderivative = 3.63

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{a + bx + cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
 b*x + c*x^2),x)
```

output
```
x^6*(l/(6*c) - (b*m)/(6*c^2)) + x*(f/c + (b*((a*(j/c - (a*(l/c - (b*m)/c^2
))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c - g/c + (b*(h
/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c
 + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))
/c))/c - (a*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)
/c^2))/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (
a*m)/c^2))/c))/c) + x^4*(j/(4*c) - (a*(l/c - (b*m)/c^2))/(4*c) + (b*((b*(l
/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/(4*c)) - x^2*((a*(j/c - (a*(l/c - (
b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(2*c) -
 g/(2*c) + (b*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*
m)/c^2))/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c +
 (a*m)/c^2))/c))/(2*c)) + x^3*(h/(3*c) - (b*(j/c - (a*(l/c - (b*m)/c^2))/c
 + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(3*c) + (a*((b*(l/c
 - (b*m)/c^2))/c - k/c + (a*m)/c^2))/(3*c)) - x^5*((b*(l/c - (b*m)/c^2))/(
5*c) - k/(5*c) + (a*m)/(5*c^2)) + (log((2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c
^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6
*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 1
4*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*
b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k +
7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - b^8*m - ...
```

3.373. $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

3.374 $\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx$

3.374.1 Optimal result

Integrand size = 35, antiderivative size = 208

$$\begin{aligned}\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx &= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} \\ &+ \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\ &- \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\ &+ \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\ &- \frac{343}{50}x^7(3+2x+5x^2)^{3/2} - \frac{540119881\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{15625000\sqrt{5}}\end{aligned}$$

output
```
-1968340667/131250000*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x
+3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5
*x^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5
*x^2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x
+3)^(3/2)-540119881/78125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-771599
83/31250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

3.374.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-93436408944+57768004650x+78839046795x^2-17642392275x^3-56757413000x^4-}{1968750000}$$
$$+\frac{540119881 \log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{15625000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-93436408944 + 57768004650*x + 78839046795*x^2 - 1
7642392275*x^3 - 56757413000*x^4 - 225922362500*x^5 + 34674656250*x^6 + 49
7593468750*x^7 - 248031875000*x^8 - 67528125000*x^9))/1968750000 + (540119
881*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625000*Sqrt[5])
```

3.374.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.20, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^3 \left(x^2+5x+2\right) \sqrt{5x^2+2x+3}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\int \sqrt{5x^2+2x+3}\left(-50519x^7+110453x^6+6350x^5-43550x^4-3050x^3+5750x^2+1450x+100\right)dx- \frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\left(\frac{1}{45}\int 6\sqrt{5x^2+2x+3}\left(954695x^6+199182x^5-326625x^4-22875x^3+43125x^2+10875x+750\right)dx-\frac{50519}{45}\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{50}\left(\frac{2}{15}\int \sqrt{5x^2+2x+3}\left(954695x^6+199182x^5-326625x^4-22875x^3+43125x^2+10875x+750\right)dx-\frac{50519}{45}\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 2192

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{40}\int 5\sqrt{5x^2+2x+3}\left(-888751x^5-5477085x^4-183000x^3+345000x^2+87000x+6000\right)dx+\frac{190939}{8}\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\int \sqrt{5x^2+2x+3}\left(-888751x^5-5477085x^4-183000x^3+345000x^2+87000x+6000\right)dx+\frac{190939}{8}\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 2192

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{1}{35}\int 2\sqrt{5x^2+2x+3}\left(-90960857x^4+2130006x^3+6037500x^2+1522500x+105000\right)dx-\frac{888751}{35}\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\int \sqrt{5x^2+2x+3}\left(-90960857x^4+2130006x^3+6037500x^2+1522500x+105000\right)dx-\frac{888751}{35}\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 2192

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{1}{30}\int 9\sqrt{5x^2+2x+3}\left(98060877x^3+111085857x^2+5075000x+350000\right)dx-\frac{90960857}{30}x^3\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 27

3.374. $\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\int\sqrt{5x^2+2x+3}\left(98060877x^3+111085857x^2+5075000x+350000\right)dx-\frac{90960857}{30}x^3\left(5x^2\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}\left(1045360143x^2-230745131x+4375000\right)dx+\frac{98060877}{25}\left(5x^2+2x\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\int\sqrt{5x^2+2x+3}\left(1045360143x^2-230745131x+4375000\right)dx+\frac{98060877}{25}\left(5x^2+2x+\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int-\left((9841703335x+3048580429)\sqrt{5x^2+2x+3}\right)dx+\frac{1045360143}{20}x\left(5x^2+2x+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 25

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1045360143}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int(9841703335x+3048580429)\sqrt{5x^2+2x+3}dx\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1160

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\int\sqrt{5x^2+2x+3}dx-\frac{1968340667}{3}\left(5x^2+2x+3\right)^{3/2}\right)+\frac{104536014}{20}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1087

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)-\frac{196834066}{3}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1090

3.374. $\int\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2+2x+3}(5\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 222

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)-\frac{1968340667}{3}\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 + ((-50519*x^6*(3 + 2*x + 5*x^2)^(3/
2))/45 + (2*((190939*x^5*(3 + 2*x + 5*x^2)^(3/2))/8 + ((-888751*x^4*(3 + 2
*x + 5*x^2)^(3/2))/35 + (2*((-90960857*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + (
3*((98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/25 + (2*((1045360143*x*(3 + 2*x
+ 5*x^2)^(3/2))/20 + ((-1968340667*(3 + 2*x + 5*x^2)^(3/2))/3 - 1080239762
*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14]
)])/(5*Sqrt[5])))/20))/25))/10))/35)/8))/15)/50
```

3.374.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.374.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{(67528125000x^9+248031875000x^8-497593468750x^7-34674656250x^6+225922362500x^5+56757413000x^4+17642392275x^3-7883}{1968750000}$
trager	$\left(-\frac{343}{10}x^9-\frac{56693}{450}x^8+\frac{2274713}{9000}x^7+\frac{369863}{21000}x^6-\frac{18073789}{157500}x^5-\frac{56757413}{1968750}x^4-\frac{235231897}{26250000}x^3+\frac{5255936453}{131250000}x^2+\right.$
default	$-\frac{77159983(10x+2)\sqrt{5x^2+2x+3}}{62500000}-\frac{540119881\sqrt{5}\,\text{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}-\frac{1968340667(5x^2+2x+3)^{\frac{3}{2}}}{131250000}+\frac{98060877x^2(5x^2+2}{4375000}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/1968750000*(67528125000*x^9+248031875000*x^8-497593468750*x^7-346746562
50*x^6+225922362500*x^5+56757413000*x^4+17642392275*x^3-78839046795*x^2-57
768004650*x+93436408944)*(5*x^2+2*x+3)^(1/2)-540119881/78125000*5^(1/2)*ar
csinh(5/14*14^(1/2)*(x+1/5))
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.47

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx =$$
$$-\frac{1}{1968750000}\left(67528125000\,x^9+248031875000\,x^8-497593468750\,x^7-34674656250\,x^6+22592236250\right.$$
$$+\frac{540119881}{156250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 346
74656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 7883
9046795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 5401198
81/156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
- 10*x - 8)
```

3.374.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$=\sqrt{5x^2+2x+3}\left(-\frac{343x^9}{10}-\frac{56693x^8}{450}+\frac{2274713x^7}{9000}+\frac{369863x^6}{21000}-\frac{18073789x^5}{157500}\right.$$
$$\left.-\frac{56757413x^4}{1968750}-\frac{235231897x^3}{26250000}+\frac{5255936453x^2}{131250000}+\frac{385120031x}{13125000}-\frac{648863951}{13671875}\right)$$
$$-\frac{540119881\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-343*x**9/10 - 56693*x**8/450 + 2274713*x**7/9000
+ 369863*x**6/21000 - 18073789*x**5/157500 - 56757413*x**4/1968750 - 23523
1897*x**3/26250000 + 5255936453*x**2/131250000 + 385120031*x/13125000 - 64
8863951/13671875) - 540119881*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/78125
000
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85

$$\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx$$
$$= -\frac{343}{50} (5x^2+2x+3)^{\frac{3}{2}} x^7 - \frac{50519}{2250} (5x^2+2x+3)^{\frac{3}{2}} x^6 + \frac{190939}{3000} (5x^2+2x+3)^{\frac{3}{2}} x^5 - \frac{888751}{105000} (5x^2+2x+3)^{\frac{3}{2}} x^4 - \frac{90960857}{1575000} (5x^2+2x+3)^{\frac{3}{2}} x^3 + \frac{98060877}{4375000} (5x^2+2x+3)^{\frac{3}{2}} x^2 + \frac{1045360143}{43750000} (5x^2+2x+3)^{\frac{3}{2}} x - \frac{1968340667}{131250000} (5x^2+2x+3)^{\frac{3}{2}} - \frac{77159983}{6250000} \sqrt{5x^2+2x+3}\,x - \frac{540119881}{78125000} \sqrt{5}\, \mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{77159983}{31250000} \sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x
^6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x
+ 3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4
375000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)
^(3/2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000
*sqrt(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14
)*(5*x + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

$$\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx =$$
$$-\frac{1}{1968750000} (5 ((5 (10 (25 (5 (49 (140 (315 x+1157)x-324959)x-1109589)x+36147578)x+227029$$
$$+ \frac{540119881}{78125000} \sqrt{5} \log\left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5x^2+2x+3}\right) - 1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
-1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 11
09589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 11
553600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqr
t(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.374.9 Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{98060877\,x^2\,(5\,x^2+2\,x+3)^{3/2}}{4375000} - \frac{90960857\,x^3\,(5\,x^2+2\,x+3)^{3/2}}{1575000} - \frac{888751\,x^4\,(5\,x^2+2\,x+3)^{3/2}}{105000} + \frac{190939\,x^5\,(5\,x^2+2\,x+3)^{3/2}}{3000} - \frac{50519\,x^6\,(5\,x^2+2\,x+3)^{3/2}}{2250} - \frac{343\,x^7\,(5\,x^2+2\,x+3)^{3/2}}{50} - \frac{3048580429\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,(5\,x+1)}{5}\right)}{156250000} - \frac{3048580429\,\left(\frac{x}{2}+\frac{1}{10}\right)\,\sqrt{5\,x^2+2\,x+3}}{43750000} - \frac{1968340667\,\sqrt{5\,x^2+2\,x+3}\,(200\,x^2+20\,x+108)}{5250000000} + \frac{1045360143\,x\,(5\,x^2+2\,x+3)^{3/2}}{43750000} + \frac{1968340667\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,(10\,x+2)}{5}\right)}{156250000}$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

output
```
(98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^
2 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (190
939*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2)
)/2250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2
*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x
/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 +
3)^(1/2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 +
 3)^(3/2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) +
(5^(1/2)*(10*x + 2))/5))/156250000
```

3.375 $\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2} \, dx$

3.375.1 Optimal result

Integrand size = 35, antiderivative size = 166

$$\begin{aligned}
&\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2} \, dx \\
&= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439(3+2x+5x^2)^{3/2}}{750000} \\
&\quad + \frac{1781669x(3+2x+5x^2)^{3/2}}{250000} - \frac{77509x^2(3+2x+5x^2)^{3/2}}{25000} - \frac{25277x^3(3+2x+5x^2)^{3/2}}{3000} \\
&\quad + \frac{989}{200}x^4(3+2x+5x^2)^{3/2} + \frac{49}{40}x^5(3+2x+5x^2)^{3/2} - \frac{17652061\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}
\end{aligned}$$

output
```
198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-775
09/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/20
0*x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*a
rcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2521723/1250000*(1+5*x)*(5*x^2+2*x+3
)^(1/2)
```

3.375.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-4588584+44333650x+23531995x^2+15583725x^3-65693000x^4-107112500x^5+10}{3750000}$$
$$+ \frac{17652061 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x^
3 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7))/3750000
+ (17652061*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[5]
)
```

3.375.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^2 \left(x^2+5x+2\right) \sqrt{5x^2+2x+3}\, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40} \int \sqrt{5x^2+2x+3}\left(6923x^5-7935x^4-3760x^3+1800x^2+840x+80\right) dx + \frac{49}{40}\left(5x^2+2x+3\right)^{3/2} x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{1}{35} \int 14\sqrt{5x^2+2x+3}\left(-25277x^4-15334x^3+4500x^2+2100x+200\right) dx + \frac{989}{5}\left(5x^2+2x+3\right)^{3/2} x^4\right) + \frac{49}{40}\left(5x^2+2x+3\right)^{3/2} x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{2}{5}\int \sqrt{5x^2+2x+3}\left(-25277x^4-15334x^3+4500x^2+2100x+200\right)dx+\frac{989}{5}\left(5x^2+2x+3\right)^{3/2}x^4\right)+\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{30}\int 3\sqrt{5x^2+2x+3}\left(-77509x^3+120831x^2+21000x+2000\right)dx-\frac{25277}{30}x^3\left(5x^2+2x+3\right)^{3/2}\right)+\frac{989}{5}\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\int \sqrt{5x^2+2x+3}\left(-77509x^3+120831x^2+21000x+2000\right)dx-\frac{25277}{30}x^3\left(5x^2+2x+3\right)^{3/2}\right)+\frac{989}{5}\left(\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}\left(1781669x^2+495027x+25000\right)dx-\frac{77509}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{25277}{30}x^3\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\int \sqrt{5x^2+2x+3}\left(1781669x^2+495027x+25000\right)dx-\frac{77509}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{25277}{30}x^3\left(\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int -\left((4845007-992195x)\sqrt{5x^2+2x+3}\right)dx+\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{77509}{25}x^2\left(5\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int (4845007-992195x)\sqrt{5x^2+2x+3}dx\right)-\frac{77509}{25}x^2\left(5x^2+\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

3.375. $\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

$$\Big\downarrow \ 1160$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\int\sqrt{5x^2+2x+3}dx\right)+\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}\right)\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 222$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right)\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

input
```
Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(49*x^5*(3 + 2*x + 5*x^2)^(3/2))/40 + ((989*x^4*(3 + 2*x + 5*x^2)^(3/2))/5
 + (2*((-25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + ((-77509*x^2*(3 + 2*x + 5
*x^2)^(3/2))/25 + (2*((1781669*x*(3 + 2*x + 5*x^2)^(3/2))/20 + ((198439*(3
 + 2*x + 5*x^2)^(3/2))/3 - 5043446*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 +
 (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5])))/20))/25)/10))/5)/40
```

3.375. $\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.375. $\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2} \, dx$

3.375.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44333650x-4588584)\sqrt{5x^2+2x+3}}{3750000} - \frac{17652...}{}$
trager	$\left(\frac{49}{8}x^7+\frac{1087}{40}x^6-\frac{8569}{300}x^5-\frac{65693}{3750}x^4+\frac{207783}{50000}x^3+\frac{4706399}{750000}x^2+\frac{886673}{75000}x-\frac{191191}{156250}\right)\sqrt{5x^2+2x+3}+\frac{17...}{}$
default	$-\frac{2521723(10x+2)\sqrt{5x^2+2x+3}}{2500000}-\frac{17652061\sqrt{5}\,\mathrm{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}+\frac{198439(5x^2+2x+3)^{\frac{3}{2}}}{750000}+\frac{49x^5(5x^2+2x+3)^{\frac{3}{2}}}{40}+\frac{98...}{}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/3750000*(22968750*x^7+101906250*x^6-107112500*x^5-65693000*x^4+15583725*
x^3+23531995*x^2+44333650*x-4588584)*(5*x^2+2*x+3)^(1/2)-17652061/3125000*
5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.52

$$\int (1+4x-7x^2)^2 (2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$$
$$= \frac{1}{3750000}\left(22968750\,x^7+101906250\,x^6-107112500\,x^5-65693000\,x^4+15583725\,x^3+23531995\,x^2+44\ldots\right.$$
$$+\frac{17652061}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 1
5583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) +
 17652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

3.375.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int (1+4x-7x^2)^2 (2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \sqrt{5x^2+2x+3} \cdot \left(\frac{49x^7}{8}+\frac{1087x^6}{40}-\frac{8569x^5}{300}-\frac{65693x^4}{3750}+\frac{207783x^3}{50000}+\frac{4706399x^2}{750000}+\frac{886673x}{75000}-\frac{191191}{156250}\right) - \frac{17652061\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}(x+\frac{1}{5})}{14}\right)}{3125000}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**7/8 + 1087*x**6/40 - 8569*x**5/300 - 65693*x
**4/3750 + 207783*x**3/50000 + 4706399*x**2/750000 + 886673*x/75000 - 1911
91/156250) - 17652061*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125000
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int (1+4x-7x^2)^2 (2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3 - \frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{1781669}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x + \frac{198439}{750000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{2521723}{250000}\sqrt{5x^2+2x+3}x - \frac{17652061}{3125000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{2521723}{1250000}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 -
25277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/
2)*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 +
 2*x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/312500
0*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 +
2*x + 3)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$$
$$=\frac{1}{3750000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1087\right)x-17138\right)x-262772\right)x+623349\right)x+4706399\right)x+8866730\right)x$$
$$+\frac{17652061}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 623
349)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 176520
61/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.375.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$$
$$=\frac{989\,x^4\left(5\,x^2+2\,x+3\right)^{3/2}}{200}-\frac{25277\,x^3\left(5\,x^2+2\,x+3\right)^{3/2}}{3000}-\frac{77509\,x^2\left(5\,x^2+2\,x+3\right)^{3/2}}{25000}$$
$$+\frac{49\,x^5\left(5\,x^2+2\,x+3\right)^{3/2}}{40}-\frac{33915049\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,(5\,x+1)}{5}\right)}{6250000}$$
$$-\frac{4845007\left(\frac{x}{2}+\frac{1}{10}\right)\sqrt{5\,x^2+2\,x+3}}{250000}+\frac{198439\sqrt{5\,x^2+2\,x+3}\,\left(200\,x^2+20\,x+108\right)}{30000000}$$
$$+\frac{1781669\,x\left(5\,x^2+2\,x+3\right)^{3/2}}{250000}-\frac{1389073\sqrt{5}\,\ln\left(2\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,(10\,x+2)}{5}\right)}{6250000}$$

input

```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

output

```
(989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2)
)/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2
+ 3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*
(5*x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/25
0000 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 +
(1781669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

3.375. $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

3.376 $\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$

3.376.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx &= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x^2)^{3/2}}{7500}+\frac{2149x(3+2x+5x^2)^{3/2}}{2500}\\ &\quad -\frac{289}{250}x^2(3+2x+5x^2)^{3/2}-\frac{7}{30}x^3(3+2x+5x^2)^{3/2}-\frac{32431\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{6250\sqrt{5}}\end{aligned}$$

output
```
7819/7500*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*
(5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*
(1+5*x)*14^(1/2))*5^(1/2)-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\begin{aligned}\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx &= \frac{\sqrt{3+2x+5x^2}(103386+105400x+129895x^2+48225x^3-234250x^4-43750x^5)}{37500}\\ &\quad +\frac{32431\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{6250\sqrt{5}}\end{aligned}$$

input
```
Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 23425
0*x^4 - 43750*x^5))/37500 + (32431*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5
*x^2]])/(6250*Sqrt[5])
```

3.376.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)\left(x^2+5x+2\right)\sqrt{5x^2+2x+3}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{30}\int 3\sqrt{5x^2+2x+3}\left(-289x^3+91x^2+130x+20\right)dx-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \sqrt{5x^2+2x+3}\left(-289x^3+91x^2+130x+20\right)dx-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}\left(2149x^2+2492x+250\right)dx-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(\frac{2}{25}\int \sqrt{5x^2+2x+3}\left(2149x^2+2492x+250\right)dx-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int -\left((1447-39095x)\sqrt{5x^2+2x+3}\right)dx+\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 25

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int(1447-39095x)\sqrt{5x^2+2x+3}dx\right)-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1160

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\int\sqrt{5x^2+2x+3}dx\right)+\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{289}{25}x^2\left(5x^2+\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1087

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right)+\frac{2149}{20}x(5\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1090

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2+2x+3}(5x\right.\right.\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 222

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right)+\frac{2149}{20}x\left(5x^2\right.\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + ((-289*x^2*(3 + 2*x + 5*x^2)^(3/2))/
25 + (2*((2149*x*(3 + 2*x + 5*x^2)^(3/2))/20 + ((7819*(3 + 2*x + 5*x^2)^(3
/2))/3 - 9266*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x
)/(2*Sqrt[14])])/(5*Sqrt[5])))/20))/25)/10
```

3.376. $\int\left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

3.376.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.376. $\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$

3.376.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{(43750x^5+234250x^4-48225x^3-129895x^2-105400x-103386)\sqrt{5x^2+2x+3}}{37500}-\frac{32431\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{31250}$
trager	$\left(-\frac{7}{6}x^5-\frac{937}{150}x^4+\frac{643}{500}x^3+\frac{25979}{7500}x^2+\frac{1054}{375}x+\frac{17231}{6250}\right)\sqrt{5x^2+2x+3}-\frac{32431\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(5\operatorname{RootO}\right.}{}$
default	$-\frac{4633(10x+2)\sqrt{5x^2+2x+3}}{25000}-\frac{32431\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{31250}+\frac{7819(5x^2+2x+3)^{\frac{3}{2}}}{7500}-\frac{7x^3(5x^2+2x+3)^{\frac{3}{2}}}{30}-\frac{289x^2(5x^2+}{250}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/37500*(43750*x^5+234250*x^4-48225*x^3-129895*x^2-105400*x-103386)*(5*x^
2+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int (1+4x-7x^2)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3} + \frac{32431}{62500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 103
386)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

3.376.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$$
$$=\sqrt{5x^2+2x+3}\left(-\frac{7x^5}{6}-\frac{937x^4}{150}+\frac{643x^3}{500}+\frac{25979x^2}{7500}+\frac{1054x}{375}+\frac{17231}{6250}\right)$$
$$-\frac{32431\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-7*x**5/6 - 937*x**4/150 + 643*x**3/500 + 25979*x*
*2/7500 + 1054*x/375 + 17231/6250) - 32431*sqrt(5)*asinh(5*sqrt(14)*(x + 1
/5)/14)/31250
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$$
$$=-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2$$
$$+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2+2x+3}x$$
$$-\frac{32431}{31250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{4633}{12500}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 +
2149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) -
4633/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(
14)*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{37500}(5((5(10(175x+937)x-1929)x-25979)x-21080)x-103386)\sqrt{5x^2+2x+3} + \frac{32431}{31250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
-1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \frac{7819\sqrt{5x^2+2x+3}\,(200x^2+20x+108)}{300000} - \frac{7x^3(5x^2+2x+3)^{3/2}}{30} - \frac{10129\sqrt{5}\,\ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(5x+1)}{5}\right)}{62500} - \frac{1447\left(\frac{x}{2}+\frac{1}{10}\right)\sqrt{5x^2+2x+3}}{2500} - \frac{289x^2(5x^2+2x+3)^{3/2}}{250} + \frac{2149x(5x^2+2x+3)^{3/2}}{2500} - \frac{54733\sqrt{5}\,\ln\left(2\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(10x+2)}{5}\right)}{62500}$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

output
```
(7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
 + 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^
(1/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2
500 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3
/2))/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x
+ 2))/5))/62500
```

3.376. $\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$

3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

3.377.1 Optimal result

Integrand size = 35, antiderivative size = 187

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx \\
&= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} \\
&\quad - \frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right) \\
&\quad + \frac{3}{343}\sqrt{\frac{1}{11}\left(497041+146555\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)
\end{aligned}$$

output
```
-8233/8575*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1/490*(397+35*x)*(5*x^2+
2*x+3)^(1/2)-3/3773*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^
(1/2)/(250-34*11^(1/2))^(1/2))*(5467451-1612105*11^(1/2))^(1/2)+3/3773*arc
tanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))
^(1/2))*(5467451+1612105*11^(1/2))^(1/2)
```

3.377.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.25

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$
$$= \frac{1}{490}(-397-35x)\sqrt{3+2x+5x^2}+\frac{8233\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1715\sqrt{5}}$$
$$+\frac{6}{343}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&,\frac{3317\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+676\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-1}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

output
```
((-397 - 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 + (8233*Log[-1 - 5*x + Sqrt[5]*S
qrt[3 + 2*x + 5*x^2]])/(1715*Sqrt[5]) + (6*RootSum[83 - 16*Sqrt[5]*#1 - 70
*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (3317*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
+ 5*x^2] - #1] + 676*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1
]*#1 - 1331*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[
5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/343
```

3.377.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}\,dx$$
$$\Big\downarrow \text{2138}$$
$$-\frac{1}{490}\int -\frac{2(8233x^2+6704x+1721)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)$$

$$\downarrow 27$$

$$\frac{1}{245}\int \frac{8233x^2+6704x+1721}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 2143$$

$$\frac{1}{245}\left(-\frac{8233}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{60(1331x+338)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 27$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{8233}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 1090$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{8233\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 222$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{8233\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 1365$$

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{1}{11}\left(14641-5028\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(14641+5028\sqrt{11}\right)\int \frac{1}{2\left(-7x+\right.}\right.\right.$$
$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 27$$

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{1}{22}\left(14641-5028\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(14641+5028\sqrt{11}\right)\int \frac{}{\left(-7x+\sqrt{\right.}}\right.\right.$$
$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\downarrow 1154$$

3.377. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

$$\frac{1}{245}\left(\frac{60}{7}\left(-\frac{1}{11}\left(14641-5028\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+\right.}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$

$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{\left(14641-5028\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(14641+5028\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right.}{\sqrt{2\left(125+17\sqrt{\right.}}}\right.}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.\right.$$

$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

output
```
-1/490*((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2]) + ((-8233*ArcSinh[(2 + 10*x)/(
2*Sqrt[14])])/(7*Sqrt[5]) + (60*(((14641 - 5028*Sqrt[11])*ArcTanh[(23 - Sq
rt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((14641 + 5028*Sqrt[11])*ArcT
anh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/245
```

3.377.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.377. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.377. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

3.377.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{(397+35x)\sqrt{5x^2+2x+3}}{490}-\frac{8233\sqrt{5}\,\text{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{8575}+\frac{6\left(-5028+1331\sqrt{11}\right)\sqrt{11}\,\text{arctanh}\left(\frac{250-34\sqrt{11}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{}}{7}\right.}}\right.}{3773\sqrt{250-34\sqrt{}}}$
default	$-\frac{(10x+2)\sqrt{5x^2+2x+3}}{140}-\frac{\sqrt{5}\,\text{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{25}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{1}}{7}\right.}}{49}\right.}{}$
trager	$\left(-\frac{397}{490}-\frac{x}{14}\right)\sqrt{5x^2+2x+3}+\frac{8233\,\text{RootOf}\left(\_Z^2-5\right)\ln\left(-5\,\text{RootOf}\left(\_Z^2-5\right)x+5\sqrt{5x^2+2x+3}-\text{RootOf}\left(\_Z^2-5\right)\right)}{8575}$

input

```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE
)
```

output

```
-1/490*(397+35*x)*(5*x^2+2*x+3)^(1/2)-8233/8575*5^(1/2)*arcsinh(5/14*14^(1
/2)*(x+1/5))+6/3773*(-5028+1331*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)
*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+6/3773*(5028+1331*11^
(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*
(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34
*11^(1/2))^(1/2))
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(133) = 266$.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.63

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$
$$= \frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}(87\sqrt{11}-265)+651\right.}{x}\right.$$
$$-\frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(-\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}(87\sqrt{11}-265)-\right.}{x}\right.$$
$$-\frac{1}{15092}\sqrt{11}\sqrt{-5275980\sqrt{11}+17893476}\log\left(-\frac{\sqrt{5x^2+2x+3}(87\sqrt{11}+265)\sqrt{-5275980\sqrt{11}+17}}{x}\right.$$
$$+\frac{1}{15092}\sqrt{11}\sqrt{-5275980\sqrt{11}+17893476}\log\left(\frac{\sqrt{5x^2+2x+3}(87\sqrt{11}+265)\sqrt{-5275980\sqrt{11}+1789}}{x}\right.$$
$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)$$
$$+\frac{8233}{17150}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fri
cas")
```

output
```
3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3
)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x +
3) + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*
log(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11)
- 265) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sq
rt(-5275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11)
+ 265)*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 11730
6*x + 195510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log
((sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893
476) - 39102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 +
 2*x + 3)*(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x +
 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

3.377.6 Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

output
```
-Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 +
 2*x + 3)/(7*x**2 - 4*x - 1), x)
```

3.377.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(133) = 266.

Time = 0.33 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.67

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = \frac{1}{188650}\sqrt{11}\left(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\,\text{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|} + \frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|} + \frac{\sqrt{}}{7\left|14x\right.}\right.\right.$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="max
ima")
```

output
```
1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/
7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x
- 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 122
5*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*s
qrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(
11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*s
qrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
+ 2*sqrt(11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt
(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x
/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sq
rt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqr
t(-34/49*sqrt(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*
x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) +
1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqr
t(2)/abs(14*x + 2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$
\begin{aligned}
\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = & -\frac{1}{490}\sqrt{5\,x^2+2\,x+3}(35\,x+397) \\
& +\frac{8233}{8575}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5\,x^2+2\,x+3}\right) \\
& +2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\
& \left.+4.41924736459000\right) \\
& -0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\
& \left.+1.25295163054000\right) \\
& -2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\
& \left.-1.02258038113000\right) \\
& +0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}\right. \\
& \left.-2.09411235400000\right)
\end{aligned}
$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="gia
c")
```

output
```
-1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(
5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx=\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

3.378.1 Optimal result

Integrand size = 35, antiderivative size = 199

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)}+\frac{1}{49}\sqrt{5}\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)\\ &-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}\\ &+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}\end{aligned}$$

output
```
1/49*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+3/154*(3+61*x)*(5*x^2+2*x+3)^(
1/2)/(-7*x^2+4*x+1)+1/3011932*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x
^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(454056168467-54668425207*11^(1/2
))^(1/2)-1/3011932*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(
1/2)/(250-34*11^(1/2))^(1/2))*(454056168467+54668425207*11^(1/2))^(1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.15

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$$
$$= \frac{-\frac{5145(3+61x)\sqrt{3+2x+5x^2}}{-1-4x+7x^2} - 5390\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) - 55\text{RootSum}\left[83-16\sqrt{5}\#1-70\#\right.}{}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

output
```
((-5145*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 5390*Sqrt[5
]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]] - 55*RootSum[83 - 16*Sqrt[
5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-314239*Log[-(Sqrt[5]*x) +
Sqrt[3 + 2*x + 5*x^2] - #1] + 28462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1 - 11221*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]
*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*Root
Sum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (599633*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 391895*Log[-(Sqrt[5]
*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 21462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#
1^3) & ])/264110
```

3.378.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2132, 27, 2143, 25, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^2}dx$$
$$\Big\downarrow \text{ 2132}$$
$$\frac{3(61x+3)\sqrt{5x^2+2x+3}}{154(-7x^2+4x+1)} - \frac{1}{308}\int -\frac{4(-55x^2+47x+237)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow 27$$

$$\frac{1}{77}\int \frac{-55x^2+47x+237}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 2143$$

$$\frac{1}{77}\left(\frac{55}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{109x+1604}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 25$$

$$\frac{1}{77}\left(\frac{55}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{7}\int \frac{109x+1604}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 1090$$

$$\frac{1}{77}\left(\frac{1}{7}\int \frac{109x+1604}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx+\frac{11}{14}\sqrt{\frac{5}{14}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 222$$

$$\frac{1}{77}\left(\frac{1}{7}\int \frac{109x+1604}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx+\frac{11}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 1365$$

$$\frac{1}{77}\left(\frac{1}{7}\left(\frac{1}{11}\left(1199-11446\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(1199+11446\sqrt{11}\right)\int \frac{}{2\left(-7x+\sqrt{}\right.}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

$$\Big\downarrow 27$$

$$\frac{1}{77}\left(\frac{1}{7}\left(\frac{1}{22}\left(1199-11446\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(1199+11446\sqrt{11}\right)\int \frac{}{\left(-7x+\sqrt{11}\right.}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154(-7x^2+4x+1)}$$

3.378. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{77}\left(\frac{1}{7}\left(-\frac{1}{11}\left(1199-11446\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+2\right.}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{77}\left(\frac{11}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)+\frac{1}{7}\left(\frac{\left(1199-11446\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(1199+11446\sqrt{\right.}}{}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

output
```
(3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + ((11*Sqrt[5
]*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/7 + (((1199 - 11446*Sqrt[11])*ArcTanh[
(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3
+ 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((1199 + 11446*Sqrt[1
1])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/7)/77
```

3.378.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.378. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

3.378. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.378.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{3(3+61x)\sqrt{5x^2+2x+3}}{154(7x^2-4x-1)}+\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{49}+\frac{\left(-11446+109\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}\right.}{}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+4}}\right.}{11858\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBO
SE)
```

output
```
-3/154*(3+61*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/49*5^(1/2)*arcsinh(5/1
4*14^(1/2)*(x+1/5))+1/11858*(-11446+109*11^(1/2))*11^(1/2)/(250-34*11^(1/2
))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1
/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+1/11858*11^(1
/2)*(11446+109*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/4
9*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1
/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
))+250+34*11^(1/2))^(1/2))
```

3.378.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(145) = 290.

Time = 0.26 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.90

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = -\frac{\sqrt{1397}(7x^2-4x-1)\sqrt{39132731\sqrt{11}+325022311}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{39132731\sqrt{11}+325022311}\left(16943\sqrt{\right.}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="f
ricas")
```

output
```
-1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 32502231
1)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3250223
11)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425
*x + 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11
) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11
) + 325022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) +
78359860425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3913
2731*sqrt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sq
rt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(1
1)*(x + 3) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x -
1)*sqrt(-39132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x
+ 3)*(16943*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 2611
9953475*sqrt(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5
)*(7*x^2 - 4*x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
- 10*x - 8) + 117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

3.378.6 Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

output
```
Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

3.378.7 Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

3.378.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}
+%%%{-248
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{(-7\,x^2+4\,x+1)^2}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.379 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

3.379.1 Optimal result

Integrand size = 35, antiderivative size = 213

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308\,(1+4x-7x^2)^2}-\frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104\,(1+4x-7x^2)} \\ &\quad -\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744} \\ &\quad +\frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{491744}\end{aligned}$$

output
```
3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-1/1721104*(272941-8131
13*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-1/686966368*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(906967747
0265753-1659519919 2187*11^(1/2))^(1/2)+1/686966368*arctanh((23+11^(1/2)+x*
(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(90696774702
65753+16595199192187*11^(1/2))^(1/2)
```

3.379.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.83

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$= \frac{\frac{5764801\sqrt{3+2x+5x^2}(-31807+106279x+737577x^2-813113x^3)}{(1+4x-7x^2)^2} - 60545521580434\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\right.}{}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

output
```
((5764801*Sqrt[3 + 2*x + 5*x^2]*(-31807 + 106279*x + 737577*x^2 - 813113*x
^3))/(1 + 4*x - 7*x^2)^2 - 60545521580434*RootSum[83 - 16*Sqrt[5]*#1 - 70*
#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 20661853520*R
ootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-465*Lo
g[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7
*#1^3) & ] + 22*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*
#1^4 & , (3751778663030*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]*#1 + 2597308755559*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^
2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*RootSum[83 - 16*
Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-11648778057271*Log[-(
Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 13372446682211*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 9645047011740*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ])/1417403151472
```

3.379.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2132, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^3}dx$$

$$\Big\downarrow \text{2132}$$

$$\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308(-7x^2+4x+1)^2}-\frac{1}{616}\int -\frac{4(805x^2+391x+753)}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\int \frac{805x^2+391x+753}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{154}\left(-\frac{\int -\frac{56(126542x+212417)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{11176(-7x^2+4x+1)}\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\left(\frac{7\int \frac{126542x+212417}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{(272941-813113x)\sqrt{5x^2+2x+3}}{11176(-7x^2+4x+1)}\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{154}\left(\frac{7\left(\frac{1}{11}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(1391962+1740003\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\left(\frac{7\left(\frac{1}{22}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(1391962+1740003\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{}}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

3.379. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}dx$

$$\Big\downarrow \; 1154$$

$$\frac{1}{154}\left(\frac{7\left(-\frac{1}{11}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{1}{11}\left(13\right.\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{154}\left(\frac{7\left(\frac{\left(1391962-1740003\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(1391962+1740003\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x}}\right.}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) + (-1/11176
*((272941 - 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (7*(((139
1962 - 1740003*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sq
rt[11])]) + ((1391962 + 1740003*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5
*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sq
rt[2*(125 + 17*Sqrt[11])])))/5588)/154
```

3.379. $\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^3}\,dx$

3.379.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

3.379. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.379.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{\left(813113 x^{3}-737577 x^{2}-106279 x+31807\right) \sqrt{5 x^{2}+2 x+3}}{245872 \left(7 x^{2}-4 x-1\right)^{2}}+\frac{\left(1740003+126542 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctanh}\left(\frac{250+34 \sqrt{11}+\frac{49}{\cdots}}{\sqrt{250+34 \sqrt{11}}\, \sqrt{245 \left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^{2}\cdots}}\cdots\right.}{2704592 \sqrt{250+34 \sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBO
SE)
```

3.379. $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$

output
```
-1/245872*(813113*x^3-737577*x^2-106279*x+31807)/(7*x^2-4*x-1)^2*(5*x^2+2*
x+3)^(1/2)+1/2704592*(1740003+126542*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^
(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*
11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+1/2704592*(-1740
003+126542*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2
))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250-34*11^(1/2))^(1/2))
```

3.379.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(160) = 320$.

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.83

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx =$$

$$-\frac{\sqrt{1397}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{11879169071\,\sqrt{11}+6492253020949}\log\left(\frac{\sqrt{1397}\sqrt{5\,x^2+2\,x+3}\sqrt{1187}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="f
ricas")
```

output
```
-1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(1187916
9071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(
11879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690
71698870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) -
 sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11)
+ 6492253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*s
qrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*s
qrt(11)*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(
49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020
949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*
sqrt(-11879169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x
+ 3) - 1707215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56
*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((s
qrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-118791
69071*sqrt(11) + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 17072
15096611365*x - 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106
279*x + 31807)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

3.379.6 Sympy [F]

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx &= -\int \frac{2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\ &\quad -\int \frac{5x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\ &\quad -\int \frac{x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx\end{aligned}$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

output
```
-Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x
**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x
**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(
x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x)
```

3.379.7 Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \int -\frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^3}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="m
axima")
```

output
```
-integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

3.379.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.77

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$= \frac{6200558\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7 - 835775\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6 - 190947036\left(\sqrt{5}x-\sqrt{5\,x^2-\cdots\right.}}{430276\left(7\left(\sqrt{5}x-\sqrt{5\,x\cdots\right.\right.}}$$
$$+ 0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$- 0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$- 0.139051039089329\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+ 0.138209741946100\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="g
iac")
```

output
```
1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81
6321374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) +
765111048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4
- 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5
*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^
2 + 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.419247364
59000) - 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2529
5163054000) - 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1
.02258038113000) + 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx=\int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{{(-7\,x^2+4\,x+1)}^3}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.380 $\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx$

3.380.1 Optimal result

Integrand size = 35, antiderivative size = 231

$$\begin{aligned}\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = & -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x)(3+2x+5x^2)^{3/2}}{62500000} \\ & - \frac{6133820867(3+2x+5x^2)^{5/2}}{1203125000} + \frac{837379699x(3+2x+5x^2)^{5/2}}{72187500} \\ & + \frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} - \frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000} \\ & - \frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} + \frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625} \\ & - \frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} - \frac{343}{60}x^7(3+2x+5x^2)^{5/2} - \frac{3357568053\,\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250000\sqrt{5}}\end{aligned}$$

output
```
-22840599/62500000*(1+5*x)*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^
2+2*x+3)^(5/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/1732500
00*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-79655
9/123750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-611
03/3300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/
781250000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-479652579/312500000*(1+5*
x)*(5*x^2+2*x+3)^(1/2)
```

3.380.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x +5x^2)^{3/2} dx = \frac{\sqrt{3+2x+5x^2}(-10506617068392+6352777129950x+15865844408685x^2+190416882396}{} + \frac{3357568053 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-10506617068392 + 6352777129950*x + 15865844408685
*x^2 + 19041688239675*x^3 + 2573089891000*x^4 - 85130334087500*x^5 - 52106
830406250*x^6 + 72918247281250*x^7 + 30505457500000*x^8 + 148393743750000*
x^9 - 125007421875000*x^10 - 30950390625000*x^11))/216562500000 + (3357568
053*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250000*Sqrt[5])
```

3.380.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^3 \left(x^2+5x+2\right) \left(5x^2+2x+3\right)^{3/2} dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{60}\int \left(5x^2+2x+3\right)^{3/2} \left(-61103x^7+131103x^6+7620x^5-52260x^4-3660x^3+6900x^2+1740x+120\right)dx - \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{60}\left(\frac{1}{55}\int 2\left(5x^2+2x+3\right)^{3/2}\left(4124708x^6+759477x^5-1437150x^4-100650x^3+189750x^2+47850x+3300\right)dx\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{1}{60}\left(\frac{2}{55}\int \left(5x^2+2x+3\right)^{3/2}\left(4124708x^6+759477x^5-1437150x^4-100650x^3+189750x^2+47850x+3300\right)dx\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{1}{50}\int 30\left(5x^2+2x+3\right)^{3/2}\left(-796559x^5-4457604x^4-167750x^3+316250x^2+79750x+5500\right)dx+\frac{20}{}\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\int \left(5x^2+2x+3\right)^{3/2}\left(-796559x^5-4457604x^4-167750x^3+316250x^2+79750x+5500\right)dx+\frac{20623}{25}\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\int \left(5x^2+2x+3\right)^{3/2}\left(-190236913x^4+2009958x^3+14231250x^2+3588750x+247500\right)dx-\frac{796}{4}\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\int \left(5x^2+2x+3\right)^{3/2}\left(2173004363x^3+2281382217x^2+143550000x+9900000\right)dx-\frac{190236}{40}\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{1}{35}\int 6\left(5x^2+2x+3\right)^{3/2}\left(10048556388x^2-1335629363x+57750000\right)dx+\frac{2173004363}{35}x^2\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 27

3.380. $\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\int\left(5x^2+2x+3\right)^{3/2}\left(10048556388x^2-1335629363x+57750000\right)dx+\frac{2173004363}{35}x^2(\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{30}\int-18(6133820867x+1578509398)\left(5x^2+2x+3\right)^{3/2}dx+\frac{1674759398}{5}x\left(5x^2+2x\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 27

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\int(6133820867x+1578509398)\left(5x^2+2x+3\right)^{3/2}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 1160

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\int\left(5x^2+2x+3\right)^{3/2}dx+\frac{613382086}{25}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 1087

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\int\sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x+\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 1087

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\Big\downarrow$ 1090

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+\cdots}}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

3.380. $\int\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

$$\Big\downarrow 222$$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{7\operatorname{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

input
```
Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 + ((-61103*x^6*(3 + 2*x + 5*x^2)^(5/
2))/55 + (2*((2062354*x^5*(3 + 2*x + 5*x^2)^(5/2))/25 + (3*((-796559*x^4*(
3 + 2*x + 5*x^2)^(5/2))/45 + ((-190236913*x^3*(3 + 2*x + 5*x^2)^(5/2))/40
+ ((2173004363*x^2*(3 + 2*x + 5*x^2)^(5/2))/35 + (6*((1674759398*x*(3 + 2*
x + 5*x^2)^(5/2))/5 - (3*((6133820867*(3 + 2*x + 5*x^2)^(5/2))/25 + (17587
26123*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/20 + (21*(((1 + 5*x)*Sqrt[3 + 2
*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5])))/10))/
5))/5))/35)/40)/45))/5))/55)/60
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.380. $\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.380.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

method	result
risch	$-\frac{(30950390625000x^{11}+125007421875000x^{10}-148393743750000x^{9}-30505457500000x^{8}-72918247281250x^{7}+52106830406250x^{6}+}{21656}$
trager	$\left(-\frac{1715}{12}x^{11}-\frac{76195}{132}x^{10}+\frac{376873}{550}x^{9}+\frac{1743169}{12375}x^{8}+\frac{333340559}{990000}x^{7}-\frac{555806191}{2310000}x^{6}-\frac{6810426727}{17325000}x^{5}+\frac{2573089891}{216562500}\right.$
default	$-\frac{22840599(10x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{125000000}-\frac{479652579(10x+2)\sqrt{5x^2+2x+3}}{625000000}-\frac{3357568053\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}-\frac{6133820867}{120}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/216562500000*(30950390625000*x^11+125007421875000*x^10-148393743750000*
x^9-30505457500000*x^8-72918247281250*x^7+52106830406250*x^6+8513033408750
0*x^5-2573089891000*x^4-19041688239675*x^3-15865844408685*x^2-635277712995
0*x+10506617068392)*(5*x^2+2*x+3)^(1/2)-3357568053/781250000*5^(1/2)*arcsi
nh(5/14*14^(1/2)*(x+1/5))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx =$$
$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}\right.$$
$$+\frac{3357568053}{1562500000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 148393743750
000*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 8
5130334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 1586584440868
5*x^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 33575680
53/1562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

3.380.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.48

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{1715x^{11}}{12}-\frac{76195x^{10}}{132}+\frac{376873x^9}{550}+\frac{1743169x^8}{12375}+\frac{333340559x^7}{990000}-\frac{555806191x^6}{2310000}-\frac{6810426727x^5}{17325000}+\frac{2573089891x^4}{216562500}+\frac{253889176529x^3}{2887500000}+\frac{352574320193x^2}{4812500000}+\frac{14117282511x}{481250000}-\frac{145925237061}{3007812500}\right)-\frac{3357568053\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-1715*x**11/12 - 76195*x**10/132 + 376873*x**9/550
 + 1743169*x**8/12375 + 333340559*x**7/990000 - 555806191*x**6/2310000 - 6
810426727*x**5/17325000 + 2573089891*x**4/216562500 + 253889176529*x**3/28
87500000 + 352574320193*x**2/4812500000 + 14117282511*x/481250000 - 145925
237061/3007812500) - 3357568053*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/781
250000
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = &-\frac{343}{60} (5x^2+2x+3)^{\frac{5}{2}} x^7 \\ &- \frac{61103}{3300} (5x^2+2x+3)^{\frac{5}{2}} x^6 + \frac{1031177}{20625} (5x^2+2x+3)^{\frac{5}{2}} x^5 \\ &- \frac{796559}{123750} (5x^2+2x+3)^{\frac{5}{2}} x^4 - \frac{190236913}{4950000} (5x^2+2x+3)^{\frac{5}{2}} x^3 \\ &+ \frac{2173004363}{173250000} (5x^2+2x+3)^{\frac{5}{2}} x^2 + \frac{837379699}{72187500} (5x^2+2x+3)^{\frac{5}{2}} x \\ &- \frac{6133820867}{1203125000} (5x^2+2x+3)^{\frac{5}{2}} - \frac{22840599}{12500000} (5x^2+2x+3)^{\frac{3}{2}} x \\ &- \frac{22840599}{62500000} (5x^2+2x+3)^{\frac{3}{2}} - \frac{479652579}{62500000} \sqrt{5x^2+2x+3}\, x \\ &- \frac{3357568053}{781250000} \sqrt{5} \operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{479652579}{312500000} \sqrt{5x^2+2x+3}\end{aligned}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x
^6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*
x + 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004
363/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*
x + 3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/
12500000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(
3/2) - 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*s
qrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 +
 2*x + 3)
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.44

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx =$$
$$-\frac{1}{216562500000}(5((5(10(25(5(7(20(105(875(77x+311)x-323034)x-6972676)x-333340559)x+1$$
$$+\frac{3357568053}{781250000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034
)*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 1029235
9564)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617
068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = \int (x^2+5x+2) (5x^2+2x+3)^{3/2} (-7x^2+4x+1)^3 dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

3.381 $\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx$

3.381.1 Optimal result

Integrand size = 35, antiderivative size = 189

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x)(3+2x+5x^2)^{3/2}}{1250000} + \frac{505667(3+2x+5x^2)^{5/2}}{2187500} + \frac{86721x(3+2x+5x^2)^{5/2}}{21875} - \frac{219271x^2(3+2x+5x^2)^{5/2}}{105000} - \frac{18379x^3(3+2x+5x^2)^{5/2}}{3000} + \frac{581}{150}x^4(3+2x+5x^2)^{5/2} + \frac{49}{50}x^5(3+2x+5x^2)^{5/2} - \frac{101512467\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125000\sqrt{5}}$$

output
```
-690561/1250000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(
5/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/
2)-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/5
0*x^5*(5*x^2+2*x+3)^(5/2)-101512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2)
)*5^(1/2)-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

3.381.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{\sqrt{3+2x+5x^2}(-249003936+2291675850x+3721040355x^2+5959365525x^3-3227597000}{13125}$$
$$+\frac{101512467 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{3125000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 59593
65525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250
*x^7 + 15281875000*x^8 + 3215625000*x^9))/131250000 + (101512467*Log[-1 -
5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(3125000*Sqrt[5])
```

3.381.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^2 \left(x^2+5x+2\right) \left(5x^2+2x+3\right)^{3/2} \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\int 5\left(5x^2+2x+3\right)^{3/2}\left(1743x^5-1947x^4-940x^3+450x^2+210x+20\right)dx+\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \left(5x^2+2x+3\right)^{3/2}\left(1743x^5-1947x^4-940x^3+450x^2+210x+20\right)dx+\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{10}\left(\frac{1}{45}\int 6\left(5x^2+2x+3\right)^{3/2}\left(-18379x^4-10536x^3+3375x^2+1575x+150\right)dx+\frac{581}{15}\left(5x^2+2x+3\right)^{5/2}x^4\right)+$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{2}{15}\int \left(5x^2+2x+3\right)^{3/2}\left(-18379x^4-10536x^3+3375x^2+1575x+150\right)dx+\frac{581}{15}\left(5x^2+2x+3\right)^{5/2}x^4\right)+$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 2192

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\int \left(5x^2+2x+3\right)^{3/2}\left(-219271x^3+300411x^2+63000x+6000\right)dx-\frac{18379}{40}x^3\left(5x^2+2x+3\right)^{5/2}\right)+$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 2192

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{1}{35}\int 6\left(5x^2+2x+3\right)^{3/2}\left(2081304x^2+586771x+35000\right)dx-\frac{219271}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\frac{183}{4}$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\int \left(5x^2+2x+3\right)^{3/2}\left(2081304x^2+586771x+35000\right)dx-\frac{219271}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1837}{40}$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 2192

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{30}\int -6(865652-505667x)\left(5x^2+2x+3\right)^{3/2}dx+\frac{346884}{5}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{219271}{35}x^2$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 27

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{346884}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{1}{5}\int (865652-505667x)\left(5x^2+2x+3\right)^{3/2}dx\right)-\frac{219271}{35}x^2(5x^2$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$\downarrow$ 1160

3.381. $\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}(5x^2+2x+3)^{5/2}-\frac{4833927}{5}\int(5x^2+2x+3)^{3/2}dx\right)+\frac{346884}{5}x(5x^2+2x+3)^{5/\right.\right.\right.\right.$$
$$\frac{49}{50}(5x^2+2x+3)^{5/2}x^5$$

$\downarrow$ 1087

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}(5x^2+2x+3)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\int\sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x+1)(5x^2+2x+3\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}(5x^2+2x+3)^{5/2}x^5$$

$\downarrow$ 1087

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}(5x^2+2x+3)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}(5x^2+2x+3)^{5/2}x^5$$

$\downarrow$ 1090

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}(5x^2+2x+3)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}(5x^2+2x+3)^{5/2}x^5$$

$\downarrow$ 222

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}(5x^2+2x+3)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}(5x^2+2x+3)^{5/2}x^5$$

input
```
Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 + ((581*x^4*(3 + 2*x + 5*x^2)^(5/2))/1
5 + (2*((-18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 + ((-219271*x^2*(3 + 2*x +
 5*x^2)^(5/2))/35 + (6*((346884*x*(3 + 2*x + 5*x^2)^(5/2))/5 + ((505667*(3
 + 2*x + 5*x^2)^(5/2))/25 - (4833927*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/
20 + (21*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*
Sqrt[14])])/(5*Sqrt[5])))/10))/5)/5))/35)/40))/15)/10
```

3.381. $\int(1+4x-7x^2)^2(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx$

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.381. $\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx$

3.381.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+5959365525x^3+3721040355x^2+}{131250000}$
trager	$\left(\frac{49}{2}x^9+\frac{3493}{30}x^8-\frac{25423}{600}x^7-\frac{43793}{1400}x^6-\frac{1004341}{10500}x^5-\frac{3227597}{131250}x^4+\frac{79458207}{1750000}x^3+\frac{248069357}{8750000}x^2+\frac{15277839}{875000}x\right.$
default	$-\frac{690561(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{2500000}-\frac{14501781(10x+2)\sqrt{5x^2+2x+3}}{12500000}-\frac{101512467\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}+\frac{505667(5x^2+2x}{2187500}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/131250000*(3215625000*x^9+15281875000*x^8-5561281250*x^7-4105593750*x^6-
12554262500*x^5-3227597000*x^4+5959365525*x^3+3721040355*x^2+2291675850*x-
249003936)*(5*x^2+2*x+3)^(1/2)-101512467/15625000*5^(1/2)*arcsinh(5/14*14^
(1/2)*(x+1/5))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51

$$\int (1+4x-7x^2)^2(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \frac{1}{131250000}\left(3215625000\,x^9+15281875000\,x^8-5561281250\,x^7-4105593750\,x^6-12554262\right.$$
$$+\frac{101512467}{31250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 410559375
0*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2
 + 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sq
rt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \sqrt{5x^2+2x+3} \cdot \left(\frac{49x^9}{2}+\frac{3493x^8}{30}-\frac{25423x^7}{600}-\frac{43793x^6}{1400}-\frac{1004341x^5}{10500}-\frac{3227597x^4}{131250}+\frac{79458207x^3}{1750000} + \frac{248069357x^2}{8750000}+\frac{15277839x}{875000}-\frac{5187582}{2734375}\right)-\frac{101512467\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**9/2 + 3493*x**8/30 - 25423*x**7/600 - 43793*
x**6/1400 - 1004341*x**5/10500 - 3227597*x**4/131250 + 79458207*x**3/17500
00 + 248069357*x**2/8750000 + 15277839*x/875000 - 5187582/2734375) - 10151
2467*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/15625000
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{49}{50}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^5 + \frac{581}{150}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^4 - \frac{18379}{3000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3 - \frac{219271}{105000}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2 + \frac{86721}{21875}\left(5x^2+2x+3\right)^{\frac{5}{2}}x + \frac{505667}{2187500}\left(5x^2+2x+3\right)^{\frac{5}{2}} - \frac{690561}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x - \frac{690561}{1250000}\left(5x^2+2x+3\right)^{\frac{3}{2}} - \frac{14501781}{1250000}\sqrt{5x^2+2x+3}x - \frac{101512467}{15625000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{14501781}{6250000}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 -
18379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(
5/2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 +
 2*x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000
*(5*x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 1015
12467/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000
*sqrt(5*x^2 + 2*x + 3)
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \frac{1}{131250000} (5 ((5 (10 (25 (5 (7 (140 (105 x + 499)x - 25423)x - 131379)x - 2008682)x - 129 + \frac{101512467}{15625000} \sqrt{5} \log \left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5 x^2 + 2 x + 3}\right) - 1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)
*x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x
 - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \int (x^2+5x+2) \left(5 x^2+2 x+3\right)^{3/2} \left(-7 x^2+4 x+1\right)^2 \, dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

3.382 $\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx$

3.382.1 Optimal result

Integrand size = 33, antiderivative size = 147

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)(3+2x+5x^2)^{3/2}}{150000}+\frac{149509(3+2x+5x^2)^{5/2}}{262500}+\frac{2809x(3+2x+5x^2)^{5/2}}{5250}-\frac{1163x^2(3+2x+5x^2)^{5/2}}{1400}-\frac{7}{40}x^3(3+2x+5x^2)^{5/2}-\frac{901453\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{125000\sqrt{5}}$$

output
```
-18397/150000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2
)+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x
^3*(5*x^2+2*x+3)^(5/2)-901453/625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2
)-128779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \frac{\sqrt{3+2x+5x^2}(22275576+36695150x+86464445x^2+78608475x^3-28373000x^4-482375}{5250000} + \frac{901453\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{125000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(22275576 + 36695150*x + 86464445*x^2 + 78608475*x^
3 - 28373000*x^4 - 48237500*x^5 - 127406250*x^6 - 22968750*x^7))/5250000 +
 (901453*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(125000*Sqrt[5])
```

3.382.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-7x^2+4x+1)(x^2+5x+2)(5x^2+2x+3)^{3/2}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\int (5x^2+2x+3)^{3/2}(-1163x^3+343x^2+520x+80)\,dx - \frac{7}{40}x^3(5x^2+2x+3)^{5/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{1}{35}\int 2(5x^2+2x+3)^{3/2}(11236x^2+12589x+1400)\,dx - \frac{1163}{35}x^2(5x^2+2x+3)^{5/2}\right) - \frac{7}{40}x^3(5x^2+2x+3)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{2}{35}\int \left(5x^2+2x+3\right)^{3/2}\left(11236x^2+12589x+1400\right)dx-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{30}\int 2(149509x+4146)\left(5x^2+2x+3\right)^{3/2}dx+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\int (149509x+4146)\left(5x^2+2x+3\right)^{3/2}dx+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\int \left(5x^2+2x+3\right)^{3/2}dx\right)+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}x\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\int \sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x+1)\left(5x^2+2x+3\right)^{3/2}\right)\right)+\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)+\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2+}\right.\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

3.382. $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

$$\Big\downarrow \text{222}$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)+\frac{1}{20}\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

input
```
Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 + ((-1163*x^2*(3 + 2*x + 5*x^2)^(5/2))
/35 + (2*((5618*x*(3 + 2*x + 5*x^2)^(5/2))/15 + ((149509*(3 + 2*x + 5*x^2)
^(5/2))/25 - (128779*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/20 + (21*(((1 +
5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*S
qrt[5])))/10))/5)/15))/35)/40
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c))))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.382.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{\left(22968750x^{7}+127406250x^{6}+48237500x^{5}+28373000x^{4}-78608475x^{3}-86464445x^{2}-36695150x-22275576\right)\sqrt{5x^{2}+2x+3}}{5250000}-\frac{901\ldots}{}$
trager	$\left(-\frac{35}{8}x^{7}-\frac{1359}{56}x^{6}-\frac{3859}{420}x^{5}-\frac{28373}{5250}x^{4}+\frac{1048113}{70000}x^{3}+\frac{17292889}{1050000}x^{2}+\frac{733903}{105000}x+\frac{928149}{218750}\right)\sqrt{5x^{2}+2x+3}\ldots$
default	$-\frac{18397(10x+2)\left(5x^{2}+2x+3\right)^{\frac{3}{2}}}{300000}-\frac{128779(10x+2)\sqrt{5x^{2}+2x+3}}{500000}-\frac{901453\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}+\frac{149509\left(5x^{2}+2x+3\right)^{\frac{5}{2}}}{262500}\ldots$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/5250000*(22968750*x^7+127406250*x^6+48237500*x^5+28373000*x^4-78608475*
x^3-86464445*x^2-36695150*x-22275576)*(5*x^2+2*x+3)^(1/2)-901453/625000*5^
(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx =$$

$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-3669\right.$$

$$+\frac{901453}{1250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 7
8608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3)
+ 901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*
x^2 - 10*x - 8)
```

3.382.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{35x^7}{8}-\frac{1359x^6}{56}-\frac{3859x^5}{420}-\frac{28373x^4}{5250}+\frac{1048113x^3}{70000}+\frac{17292889x^2}{1050000}+\frac{733903x}{105000}+\frac{928149}{218750}\right)-\frac{901453\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-35*x**7/8 - 1359*x**6/56 - 3859*x**5/420 - 28373*
x**4/5250 + 1048113*x**3/70000 + 17292889*x**2/1050000 + 733903*x/105000 +
 928149/218750) - 901453*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/625000
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = -\frac{7}{40}\,(5\,x^2+2\,x+3)^{\frac{5}{2}}x^3 - \frac{1163}{1400}\,(5\,x^2+2\,x+3)^{\frac{5}{2}}x^2 + \frac{2809}{5250}\,(5\,x^2+2\,x+3)^{\frac{5}{2}}x + \frac{149509}{262500}\,(5\,x^2+2\,x+3)^{\frac{5}{2}} - \frac{18397}{30000}\,(5\,x^2+2\,x+3)^{\frac{3}{2}}x - \frac{18397}{150000}\,(5\,x^2+2\,x+3)^{\frac{3}{2}} - \frac{128779}{50000}\,\sqrt{5\,x^2+2\,x+3}x - \frac{901453}{625000}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{128779}{250000}\,\sqrt{5\,x^2+2\,x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2
+ 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5
/2) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x +
3)^(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*ar
csinh(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = -\frac{1}{5250000}\,(5\,((5\,(10\,(25\,(15\,(245\,x+1359)x+7718)x+113492)x-3144339)x-17292889)x-7339030)x + \frac{901453}{625000}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
-1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 314
4339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 901
453/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)\,dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

3.383.1 Optimal result

Integrand size = 35, antiderivative size = 210

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{34425687\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}} - \frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807} + \frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807}$$

output
```
-1/980*(267+35*x)*(5*x^2+2*x+3)^(3/2)-34425687/4201750*arcsinh(1/14*(1+5*x
)*14^(1/2))*5^(1/2)-3/240100*(571621+196105*x)*(5*x^2+2*x+3)^(1/2)-6/18487
7*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(
1/2))^(1/2))*(178175857354-53550689170*11^(1/2))^(1/2)+6/184877*arctanh((2
3+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))
*(178175857354+53550689170*11^(1/2))^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.21

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \frac{\sqrt{3+2x+5x^2}(-1911108-744870x-344225x^2-42875x^3)}{240100} + \frac{34425687\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{840350\sqrt{5}} - \frac{12\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-648783\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-533850\log\left(-\sqrt{\right.}\right.}{-4\sqrt{5}-3}\right.}{16807\sqrt{5}}$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-1911108 - 744870*x - 344225*x^2 - 42875*x^3))/240
100 + (34425687*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(840350*Sqr
t[5]) - (12*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4
 & , (-648783*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 533
850*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 251851*Sqrt[5]*Log
[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*
Sqrt[5]*#1^2 + 7*#1^3) & ])/(16807*Sqrt[5])
```

3.383.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2138, 27, 2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)(5x^2+2x+3)^{3/2}}{-7x^2+4x+1}dx$$

$$\Big\downarrow \text{2138}$$

$$-\frac{\int -\frac{18\sqrt{5x^2+2x+3}(5603x^2+4404x+1131)}{-7x^2+4x+1}dx}{2940} - \frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{490}\int \frac{\sqrt{5x^2+2x+3}(5603x^2+4404x+1131)}{-7x^2+4x+1}dx-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 2138

$$\frac{3}{490}\left(-\frac{1}{490}\int -\frac{2(11475229x^2+7834212x+1411253)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}\sqrt{5x^2+2x+3}(196105x+571621)\right)-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{3}{490}\left(\frac{1}{245}\int \frac{11475229x^2+7834212x+1411253}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}(196105x+571621)\sqrt{5x^2+2x+3}\right)-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 2143

$$\frac{3}{490}\left(\frac{1}{245}\left(-\frac{11475229}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{400(251851x+53385)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(196105x+571\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(196105x+5716\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1090

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}\right)-\frac{1}{490}(196105x\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 222

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}\right)-\frac{1}{490}(196105x+571621)\sqrt{\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\Big\downarrow$ 1365

3.383. $\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{1}{11}\left(2770361-877397\sqrt{11}\right)\int\frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(2770361+877397\sqrt{\right.\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{1}{22}\left(2770361-877397\sqrt{11}\right)\int\frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(2770361+877397\sqrt{1\right.\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\big\downarrow$ 1154

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(-\frac{1}{11}\left(2770361-877397\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11\right.\right.}{\sqrt{5x^2}}\right.\right.\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$\big\downarrow$ 219

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{\left(2770361-877397\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(2770361+877397\sqrt{11}\right)\text{arcta}}{22\sqrt{2\left(12\right.}}\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

output
```
-1/980*((267 + 35*x)*(3 + 2*x + 5*x^2)^(3/2)) + (3*(-1/490*((571621 + 1961
05*x)*Sqrt[3 + 2*x + 5*x^2]) + ((-11475229*ArcSinh[(2 + 10*x)/(2*Sqrt[14])
])/(7*Sqrt[5]) + (400*(((2770361 - 877397*Sqrt[11])*ArcTanh[(23 - Sqrt[11]
 + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2]
)])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((2770361 + 877397*Sqrt[11])*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[
3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/245))/490
```

3.383. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

3.383.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.383. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.383.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{(42875x^3+344225x^2+744870x+1911108)\sqrt{5x^2+2x+3}}{240100} - \frac{34425687\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{4201750} + \frac{12\left(-877397+251851\sqrt{11}\right)}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE
)
```

output
```
-1/240100*(42875*x^3+344225*x^2+744870*x+1911108)*(5*x^2+2*x+3)^(1/2)-3442
5687/4201750*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+12/184877*(-877397+251
851*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*
11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2
)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))
+250-34*11^(1/2))^(1/2))+12/184877*11^(1/2)*(877397+251851*11^(1/2))/(250+
34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2)
)*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^
2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

3.383.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(152) = 304$.

Time = 0.29 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.55

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{12\left(\sqrt{2}\sqrt{5x\right.}}{}\right.$$

$$-\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(-\frac{12\left(\sqrt{2}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+809\right.}}{}\right.$$

$$-\frac{1}{739508}\sqrt{11}\sqrt{-701027203680\sqrt{11}+2332483950816}\log\left(-\frac{\sqrt{5x^2+2x+3}(7690\sqrt{11}+24697)\sqrt{-701}}{}\right.$$

$$+\frac{1}{739508}\sqrt{11}\sqrt{-701027203680\sqrt{11}+2332483950816}\log\left(\frac{\sqrt{5x^2+2x+3}(7690\sqrt{11}+24697)\sqrt{-70102}}{}\right.$$

$$-\frac{1}{240100}\left(42875x^3+344225x^2+744870x+1911108\right)\sqrt{5x^2+2x+3}$$

$$+\frac{34425687}{8403500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fri
cas")
```

output
```
3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(s
qrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x
) - 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-
12*(sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(
7690*sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 2027764
55)/x) - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*lo
g(-(sqrt(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(
11) + 2332483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317
460)/x) + 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*l
og((sqrt(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(
11) + 2332483950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317
460)/x) - 1/240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^
2 + 2*x + 3) + 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*
(5*x + 1) - 25*x^2 - 10*x - 8)
```

3.383.6 Sympy [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

output
```
-Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*
sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x*
*2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x
+ 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x*
*2 - 4*x - 1), x)
```

3.383.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(152) = 304$.

Time = 0.39 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.55

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \frac{1}{92438500}\sqrt{11}\left(19500\sqrt{11}\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}}{7\,|14x-}\right.\right.$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="max
ima")
```

output
```
1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcs
inh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs
(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))
 - 300125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sq
rt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2
*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*s
qrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/
abs(14*x + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcs
inh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs
(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))
 + 15692250*(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*
sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2
*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) -
 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^
2 + 2*x + 3)^(3/2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*
sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) -
60020205*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{240100}(35(35(35x+281)x+21282)x+1911108)\sqrt{5x^2+2x+3} + \frac{34425687}{4201750}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right) + 19.3580321168561\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right) - 0.773682164624264\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right) - 19.3580321168561\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right) + 0.773682164625454\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="gia
c")
```

output
```
-1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x +
3) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 +
2*x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41
924736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) +
 1.25295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x +
3) - 1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2
*x + 3) - 2.09411235400000)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \int \frac{(x^2+5x+2)(5x^2+2x+3)^{3/2}}{-7x^2+4x+1}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

3.384.1 Optimal result

Integrand size = 35, antiderivative size = 222

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}} - \frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411} - \frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411}$$

output
```
3/154*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)+16691/12005*arcsinh(1/14
*(1+5*x)*14^(1/2))*5^(1/2)+1/3773*(5826+3395*x)*(5*x^2+2*x+3)^(1/2)-1/5810
42*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^
(1/2))^(1/2))*(1147858806842-289418283682*11^(1/2))^(1/2)-1/581042*arctanh
((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/
2))*(1147858806842+289418283682*11^(1/2))^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.01

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{\frac{1715\sqrt{3+2x+5x^2}(-12975-81181x+34265x^2+2695x^3)}{-1-4x+7x^2} - 17992898\sqrt{5}\log(-1-$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

output
```
((1715*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-
1 - 4*x + 7*x^2) - 17992898*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]] + 44*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3
 + 7*#1^4 & , (25954129*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1] - 19416530*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 271709
9*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
 - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83 - 16*Sqrt[5
]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (225782939*Sqrt[5]*Log[-(Sqrt
[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 137400830*Log[-(Sqrt[5]*x) + Sqrt[3
 + 2*x + 5*x^2] - #1]*#1 + 7775369*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
12941390
```

3.384.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2132, 27, 2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)(5x^2+2x+3)^{3/2}}{(-7x^2+4x+1)^2}dx$$

$$\Big\downarrow \text{ 2132}$$

$$\frac{3(61x+3)(5x^2+2x+3)^{3/2}}{154(-7x^2+4x+1)} - \frac{1}{308}\int -\frac{4(-970x^2-181x+228)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{77}\int \frac{\left(-970x^2-181x+228\right)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}dx+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 2138$$

$$\frac{1}{77}\left(\frac{1}{49}(3395x+5826)\sqrt{5x^2+2x+3}-\frac{1}{490}\int -\frac{10\left(-183601x^2-107622x+17505\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{77}\left(\frac{1}{49}\int \frac{-183601x^2-107622x+17505}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+5826)\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 2143$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int \frac{2(743879x+30533)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{2}{7}\int \frac{743879x+30533}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}-\frac{2}{7}\int \frac{743879x+30533}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(33\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 222$$

3.384. $\int \frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^2}\,dx$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\int\frac{743879x+30533}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+5826$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$\downarrow$ 1365

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{1}{11}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$\downarrow$ 27

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{1}{22}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$\downarrow$ 1154

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(-\frac{1}{11}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+2\right.}{5x^2+2x+3}}\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$\downarrow$ 219

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{\left(8182669-1701489\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{(81826}{}\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

3.384. $\int\frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^2}dx$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

output
```
(3*(3 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (((5826 +
 3395*x)*Sqrt[3 + 2*x + 5*x^2])/49 + ((183601*ArcSinh[(2 + 10*x)/(2*Sqrt[1
4])])/(7*Sqrt[5]) - (2*(((8182669 - 1701489*Sqrt[11])*ArcTanh[(23 - Sqrt[1
1] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^
2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((8182669 + 1701489*Sqrt[11])*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/49)/77
```

3.384.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.384. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} dx$

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

3.384. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.384.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(2695x^3+34265x^2-81181x-12975)\sqrt{5x^2+2x+3}}{52822x^2-30184x-7546}+\frac{16691\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{12005}-\frac{\left(1701489+743879\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\right.}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/7546*(2695*x^3+34265*x^2-81181*x-12975)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2
)+16691/12005*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-1/290521*(1701489+743
879*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*
11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2
)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2))-1/290521*(-1701489+743879*11^(1/2))*11^(1/2)/(250
-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(164) = 328.

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{5\sqrt{11}(7\,x^2-4\,x-1)\sqrt{26310753062\,\sqrt{11}+104350800622}\log\left(\frac{\sqrt{5}}{\cdots}\right.}{\cdots}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="f
ricas")
```

output
```
1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104350
800622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080062
2)*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x -
 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) +
 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104
350800622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 538557
5055*x + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 6844
1)*sqrt(-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3
) - 5385575055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-263
10753062*sqrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(1
1) + 68441)*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(1
1)*(x + 3) + 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x
- 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 7
70*(2695*x^3 + 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2
- 4*x - 1)
```

3.384.6 Sympy [F]

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)\left(5x^2+2x+3\right)^{\frac{3}{2}}}{(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
```

output
```
Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2,
 x)
```

3.384.7 Maxima [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(5\,x^2+2\,x+3)^{\frac{3}{2}}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="m
axima")
```

output
```
integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

3.384.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7
]%%%}+%%%
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}}{(-7\,x^2+4\,x+1)^2}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)
```

3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

3.385.1 Optimal result

Integrand size = 35, antiderivative size = 234

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716(1+4x-7x^2)} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{308(1+4x-7x^2)^2} - \frac{5}{343}\sqrt{5}\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024} + \frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024}$$

output
```
3/308*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2-5/343*arcsinh(1/14*(1+
5*x)*14^(1/2))*5^(1/2)-1/23716*(9495-37088*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+
4*x+1)-1/927675056*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(
1/2)/(250-34*11^(1/2))^(1/2))*(174049987116977774-5826721433301670*11^(1/2
))^(1/2)+1/927675056*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)
^(1/2)/(250+34*11^(1/2))^(1/2))*(174049987116977774+5826721433301670*11^(1
/2))^(1/2)
```

3.385.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.81 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.72

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{\sqrt{3+2x+5x^2}(-7416+42767x+246464x^2-189161x^3)}{23716(-1-4x+7x^2)^2} + \frac{5}{343}\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) - \frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{4506829\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1320270\log\left(-\sqrt{5}x\cdots\right.}{-4\sqrt{5}-35\#\cdots}\right.}{33614\sqrt{5}} + \frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-16323208013227\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+151120773\cdots}{\cdots}\right.}{71748713246\sqrt{5}} - \frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-4192656948824863\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+24518\cdots}{\cdots}\right.}{34726377211064\sqrt{5}\cdots}$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-7416 + 42767*x + 246464*x^2 - 189161*x^3))/(23716
*(-1 - 4*x + 7*x^2)^2) + (5*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]])/343 - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#
1^4 & , (4506829*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] -
1320270*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 64435*Sqrt[5]*
Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 +
 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(33614*Sqrt[5]) + RootSum[83 - 16*Sqrt[5]*#1
 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-16323208013227*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 151120773150070*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 21832390993791*Sqrt[5]*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ]/(71748713246*Sqrt[5]) - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-4192656948824863*Sqrt[5]*Log[-(Sqrt[5]
*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 24518831643829090*Log[-(Sqrt[5]*x) + S
qrt[3 + 2*x + 5*x^2] - #1]*#1 + 3523608887504055*Sqrt[5]*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 +
 7*#1^3) & ])/(34726377211064*Sqrt[5])
```

3.385.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {2132, 27, 2132, 27, 2143, 25, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\left(5x^2+2x+3\right)^{3/2}}{\left(-7x^2+4x+1\right)^3}dx$$

$$\Big\downarrow \ 2132$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{1}{616}\int -\frac{4\left(-110x^2+163x+744\right)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{154}\int \frac{\left(-110x^2+163x+744\right)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2}dx+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \ 2132$$

$$\frac{1}{154}\left(-\frac{1}{308}\int -\frac{2\left(12100x^2+89403x+128019\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{5x^2+2x+3}(9495-37088x)}{154\left(-7x^2+4x+1\right)}\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{154}\left(\frac{1}{154}\int \frac{12100x^2+89403x+128019}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{154\left(-7x^2+4x+1\right)}\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \ 2143$$

$$\frac{1}{154}\left(\frac{1}{154}\left(-\frac{12100}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{674221x+908233}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{(9495-37088x)\sqrt{5x^2 \cdots}}{154\left(-7x^2+4x+\cdots\right.}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{12100}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{(9495-37088x)\sqrt{5x^2+2x}}{154(-7x^2+4x+1)}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\downarrow$ 1090

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{605}{7}\sqrt{\frac{10}{7}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)\right)-\frac{(9495-37}{154(}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\downarrow$ 222

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{2420}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)\right)-\frac{(9495-37088x)\sqrt{5x^2+2x}}{154(-7x^2+4x+1)}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\downarrow$ 1365

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{1}{11}\left(7416431-7706073\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(7416431+7706073\sqrt{\right.\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\downarrow$ 27

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{1}{22}\left(7416431-7706073\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(7416431+7706073\sqrt{1\right.\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\downarrow$ 1154

3.385. $\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{(1+4x-7x^2)^3}\,dx$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(-\frac{1}{11}\left(7416431-7706073\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)\right.}{\sqrt{5x^2+}}\right.\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$\big\downarrow$ 219

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{\left(7416431-7706073\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(7416431+7706073\sqrt{11}\right)\operatorname{arcta}}{22\sqrt{2\left(125\right.}}\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(3*(3 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) + (-1/154
*((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + ((-2420*Sqrt
[5]*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/7 + (((7416431 - 7706073*Sqrt[11])*A
rcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((7416431 + 77
06073*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])
)/7)/154)/154
```

3.385.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.385. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

3.385. $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.385.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\left(189161 x^{3}-246464 x^{2}-42767 x+7416\right) \sqrt{5 x^{2}+2 x+3}}{23716 \left(7 x^{2}-4 x-1\right)^{2}}-\frac{5 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x+\frac{1}{5}\right)}{14}\right)}{343}+\frac{\left(7706073+674221 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctanh}\left(\right.}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/23716*(189161*x^3-246464*x^2-42767*x+7416)/(7*x^2-4*x-1)^2*(5*x^2+2*x+3
)^(1/2)-5/343*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+1/3652264*(7706073+67
4221*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49
*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/
2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)
)+250+34*11^(1/2))^(1/2))+1/3652264*(-7706073+674221*11^(1/2))*11^(1/2)/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1
/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2
))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(176) = 352$.

Time = 0.29 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.91

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx =$$

$$-\frac{\sqrt{2794}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{2085440742055\,\sqrt{11}+62294197250171}\log\left(\frac{\sqrt{2794}\sqrt{5\,x^2+2\,x+3}\sqrt{208}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="f
ricas")
```

output
```
-1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440
742055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sq
rt(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737)
 + 5426671202560069*sqrt(11)*(x + 3) + 16280013607680207*x - 2713335601280
0345)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407420
55*sqrt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(
2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) -
5426671202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 2713335601280034
5)/x) + sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055
*sqrt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(118405
90*sqrt(11) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 5
426671202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*
sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590
*sqrt(11) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 542
6671202560069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/
x) - 13522960*sqrt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt
(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246
464*x^2 - 42767*x + 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2
+ 8*x + 1)
```

3.385.6 Sympy [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = -\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

output
```
-Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x
**3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*
x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral
(23*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3
 - 27*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*
x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral
(5*x**4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3
- 27*x**2 - 12*x - 1), x)
```

3.385.7 Maxima [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \int -\frac{(5\,x^2+2\,x+3)^{\frac{3}{2}}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^3}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="m
axima")
```

output
```
-integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

3.385.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(176) = 352$.

Time = 0.33 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.76

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{5}{343}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

$$+\frac{264327\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7-3224225\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-87069759\left(\sqrt{5}x-\sqrt{5\,x^2+2}\right.}{83006\left(7\left(\sqrt{5}x-\sqrt{5\,x^2}\right.\right.}$$

$$+0.474028359166807\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.424017987131739\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.474028359166807\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.424017987131739\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="g
iac")
```

output
```
5/343*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) + 1/83
006*(264327*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 3224225*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 87069759*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 36535763*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 41681814
9*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 204858869*sqrt(5)*(sqrt(5)*x - s
qrt(5*x^2 + 2*x + 3))^2 - 411908789*sqrt(5)*x - 187277977*sqrt(5) + 411908
789*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.
474028359166807*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000)
 - 0.424017987131739*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2529516305
4000) - 0.474028359166807*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258
038113000) + 0.424017987131739*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.
09411235400000)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \int \frac{(x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}}{(-7\,x^2+4\,x+1)^3}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

3.386.1 Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{16515809\sqrt{3+2x+5x^2}}{156250}+\frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} - \frac{77513689\,\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

output
```
-77513689/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-16515809/156250*(
5*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2
*(5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*
(5*x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2
*x+3)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)
```

3.386.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-396379416+289653850x+203614255x^2-387039975x^3-47807000x^4+326987500x^5}{3750000}$$
$$+\frac{77513689\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-396379416 + 289653850*x + 203614255*x^2 - 3870399
75*x^3 - 47807000*x^4 + 326987500*x^5 - 106968750*x^6 - 32156250*x^7))/375
0000 + (77513689*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sq
rt[5])
```

3.386.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^3(x^2+5x+2)}{\sqrt{5x^2+2x+3}}\,dx$$
$$\Big\downarrow \; 2192$$
$$\frac{1}{40}\int \frac{-39935x^7+89803x^6+5080x^5-34840x^4-2440x^3+4600x^2+1160x+80}{\sqrt{5x^2+2x+3}}dx - \frac{343}{40}x^7\sqrt{5x^2+2x+3}$$
$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(\frac{1}{35}\int \frac{70\left(52318x^6+12809x^5-17420x^4-1220x^3+2300x^2+580x+40\right)}{\sqrt{5x^2+2x+3}}dx-1141x^6\sqrt{5x^2+2x+3}\right)-\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(2\int \frac{52318x^6+12809x^5-17420x^4-1220x^3+2300x^2+580x+40}{\sqrt{5x^2+2x+3}}dx-1141x^6\sqrt{5x^2+2x+3}\right)-\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(2\left(\frac{1}{30}\int \frac{2\left(-95614x^5-653685x^4-18300x^3+34500x^2+8700x+600\right)}{\sqrt{5x^2+2x+3}}dx+\frac{26159}{15}\sqrt{5x^2+2x+3}x^5\right)-11\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\int \frac{-95614x^5-653685x^4-18300x^3+34500x^2+8700x+600}{\sqrt{5x^2+2x+3}}dx+\frac{26159}{15}\sqrt{5x^2+2x+3}x^5\right)-1141x\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{1}{25}\int \frac{3\left(-5160533x^4+229956x^3+287500x^2+72500x+5000\right)}{\sqrt{5x^2+2x+3}}dx-\frac{95614}{25}x^4\sqrt{5x^2+2x+3}\right)+26\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\int \frac{-5160533x^4+229956x^3+287500x^2+72500x+5000}{\sqrt{5x^2+2x+3}}dx-\frac{95614}{25}x^4\sqrt{5x^2+2x+3}\right)+\frac{26159}{15}\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\int \frac{40722851x^3+52194797x^2+1450000x+100000}{\sqrt{5x^2+2x+3}}dx-\frac{5160533}{20}x^3\sqrt{5x^2+2x+3}\right)-\frac{9561}{25}\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

3.386. $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{1}{15}\int \frac{2(289653850x^2-111293553x+750000)}{\sqrt{5x^2+2x+3}}dx+\frac{40722851}{15}\sqrt{5x^2+2x+3}x^2\right)-\frac{51605}{20}\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\int \frac{289653850x^2-111293553x+750000}{\sqrt{5x^2+2x+3}}dx+\frac{40722851}{15}\sqrt{5x^2+2x+3}x^2\right)-\frac{5160533}{20}x\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int -\frac{30(66063236x+28715385)}{\sqrt{5x^2+2x+3}}dx+28965385\sqrt{5x^2+2x+3}x\right)+\frac{40722851}{15}\sqrt{5x^2}\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\int \frac{66063236x+28715385}{\sqrt{5x^2+2x+3}}dx\right)+\frac{40722851}{15}\sqrt{5x^2+2x+}\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{66063236}{5}\sqrt{5x^2+2x}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}+\frac{66063236}{5}\sqrt{5}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \; 222$$

3.386. $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{66063236}{5}\sqrt{5x^2+2x+3}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

input
```
Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-343*x^7*Sqrt[3 + 2*x + 5*x^2])/40 + (-1141*x^6*Sqrt[3 + 2*x + 5*x^2] + 2
*((26159*x^5*Sqrt[3 + 2*x + 5*x^2])/15 + ((-95614*x^4*Sqrt[3 + 2*x + 5*x^2
])/25 + (3*((-5160533*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((40722851*x^2*Sqrt[
3 + 2*x + 5*x^2])/15 + (2*(28965385*x*Sqrt[3 + 2*x + 5*x^2] - 3*((66063236
*Sqrt[3 + 2*x + 5*x^2])/5 + (77513689*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5
*Sqrt[5]))))/15)/20))/25)/15))/40
```

3.386.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.386.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{\left(32156250 x^{7}+106968750 x^{6}-326987500 x^{5}+47807000 x^{4}+387039975 x^{3}-203614255 x^{2}-289653850 x+396379416\right) \sqrt{5 x^{2}+2 x+3}}{3750000}-$
trager	$\left(-\frac{343}{40} x^{7}-\frac{1141}{40} x^{6}+\frac{26159}{300} x^{5}-\frac{47807}{3750} x^{4}-\frac{5160533}{50000} x^{3}+\frac{40722851}{750000} x^{2}+\frac{5793077}{75000} x-\frac{16515809}{156250}\right) \sqrt{5 x^{2}+2 x}$
default	$-\frac{77513689 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x+\frac{1}{5}\right)}{14}\right)}{3125000}-\frac{16515809 \sqrt{5 x^{2}+2 x+3}}{156250}+\frac{40722851 x^{2} \sqrt{5 x^{2}+2 x+3}}{750000}+\frac{5793077 x \sqrt{5 x^{2}+2 x+3}}{75000}-\frac{343}{}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/3750000*(32156250*x^7+106968750*x^6-326987500*x^5+47807000*x^4+38703997
5*x^3-203614255*x^2-289653850*x+396379416)*(5*x^2+2*x+3)^(1/2)-77513689/31
25000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.386.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx =$$
$$-\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x^2\right.$$
$$+\frac{77513689}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 +
387039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x
+ 3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1
) - 25*x^2 - 10*x - 8)
```

3.386.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{343x^7}{40}-\frac{1141x^6}{40}+\frac{26159x^5}{300}-\frac{47807x^4}{3750}-\frac{5160533x^3}{50000}+\frac{40722851x^2}{750000}+\frac{5793077x}{75000}-\frac{16515809}{156250}\right)-\frac{77513689\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-343*x**7/40 - 1141*x**6/40 + 26159*x**5/300 - 478
07*x**4/3750 - 5160533*x**3/50000 + 40722851*x**2/750000 + 5793077*x/75000
 - 16515809/156250) - 77513689*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125
000
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}\sqrt{5x^2+2x+3}x^7 - \frac{1141}{40}\sqrt{5x^2+2x+3}x^6 + \frac{26159}{300}\sqrt{5x^2+2x+3}x^5 - \frac{47807}{3750}\sqrt{5x^2+2x+3}x^4 - \frac{5160533}{50000}\sqrt{5x^2+2x+3}x^3 + \frac{40722851}{750000}\sqrt{5x^2+2x+3}x^2 + \frac{5793077}{75000}\sqrt{5x^2+2x+3}x - \frac{77513689}{3125000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{16515809}{156250}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 26
159/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 -
 5160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*
x + 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt
(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x +
3)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{3750000}(5((5(10(175(15(49x+163)x-7474)x+191228)x+15481599)x-40722851)x-579307 + \frac{77513689}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
-1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 1548
1599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 7
7513689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3 (2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^3}{\sqrt{5\,x^2+2\,x+3}} dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.387 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

3.387.1 Optimal result

Integrand size = 35, antiderivative size = 143

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875} -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500} -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500}+\frac{5131}{750}x^4\sqrt{3+2x+5x^2} +\frac{49}{30}x^5\sqrt{3+2x+5x^2}-\frac{1719097\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{31250\sqrt{5}}$$

output
```
-1719097/156250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-22053/31250*(5*x^2+
2*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+
3)^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/
2)+49/30*x^5*(5*x^2+2*x+3)^(1/2)
```

3.387.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(1+4x-7x^2)^2 (2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx = \frac{\sqrt{3+2x+5x^2}(-132318+3607300x-1037135x^2-2494425x^3+1282750x^4+306250x^5)}{187500} + \frac{1719097 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{31250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 +
1282750*x^4 + 306250*x^5))/187500 + (1719097*Log[-1 - 5*x + Sqrt[5]*Sqrt[3
 + 2*x + 5*x^2]])/(31250*Sqrt[5])
```

3.387.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2192, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^2 (x^2+5x+2)}{\sqrt{5x^2+2x+3}} dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{30} \int \frac{5131x^5-6135x^4-2820x^3+1350x^2+630x+60}{\sqrt{5x^2+2x+3}} dx + \frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{30}\left(\frac{1}{25} \int \frac{6(-33259x^4-22012x^3+5625x^2+2625x+250)}{\sqrt{5x^2+2x+3}} dx + \frac{5131}{25}\sqrt{5x^2+2x+3}x^4\right) + \frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}\left(\frac{6}{25}\int \frac{-33259x^4-22012x^3+5625x^2+2625x+250}{\sqrt{5x^2+2x+3}}dx+\frac{5131}{25}\sqrt{5x^2+2x+3}x^4\right)+\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 2192

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\int \frac{-207427x^3+411831x^2+52500x+5000}{\sqrt{5x^2+2x+3}}dx-\frac{33259}{20}x^3\sqrt{5x^2+2x+3}\right)+\frac{5131}{25}\sqrt{5x^2+2x+3}x^4\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 2192

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{1}{15}\int \frac{2(3607300x^2+1016031x+37500)}{\sqrt{5x^2+2x+3}}dx-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x^3\sqrt{5x^2+2x+\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 27

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\int \frac{3607300x^2+1016031x+37500}{\sqrt{5x^2+2x+3}}dx-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x^3\sqrt{5x^2+2x+3}\right)\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 2192

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int -\frac{30(22053x+348230)}{\sqrt{5x^2+2x+3}}dx+360730\sqrt{5x^2+2x+3}x\right)-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 27

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\int \frac{22053x+348230}{\sqrt{5x^2+2x+3}}dx\right)-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x^3\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 1160

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right)-\right.\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$\downarrow$ 1090

3.387. $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right)\right.\right.\right.\right.$$

$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{207427}{15}\right.\right.\right.\right.$$

$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

input
```
Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(49*x^5*Sqrt[3 + 2*x + 5*x^2])/30 + ((5131*x^4*Sqrt[3 + 2*x + 5*x^2])/25 +
 (6*((-33259*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((-207427*x^2*Sqrt[3 + 2*x +
5*x^2])/15 + (2*(360730*x*Sqrt[3 + 2*x + 5*x^2] - 3*((22053*Sqrt[3 + 2*x +
 5*x^2])/5 + (1719097*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5]))))/15)
/20))/25)/30
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.387. $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.387.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\left(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318\right)\sqrt{5x^2+2x+3}}{187500}-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$
trager	$\left(\frac{49}{30}x^5+\frac{5131}{750}x^4-\frac{33259}{2500}x^3-\frac{207427}{37500}x^2+\frac{36073}{1875}x-\frac{22053}{31250}\right)\sqrt{5x^2+2x+3}+\frac{1719097\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-\right.}{}$
default	$-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{49x^5\sqrt{5x^2+2x+3}}{30}+\frac{5131x^4\sqrt{5x^2+2x+3}}{750}-\frac{33259x^3\sqrt{5x^2+2x}}{2500}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/187500*(306250*x^5+1282750*x^4-2494425*x^3-1037135*x^2+3607300*x-132318)
*(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.387.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3} + \frac{1719097}{312500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
 - 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt
(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

3.387.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\cdot\left(\frac{49x^5}{30}+\frac{5131x^4}{750}-\frac{33259x^3}{2500}-\frac{207427x^2}{37500}+\frac{36073x}{1875}-\frac{22053}{31250}\right) - \frac{1719097\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**5/30 + 5131*x**4/750 - 33259*x**3/2500 - 207
427*x**2/37500 + 36073*x/1875 - 22053/31250) - 1719097*sqrt(5)*asinh(5*sqr
t(14)*(x + 1/5)/14)/156250
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}\sqrt{5\,x^2+2\,x+3}x^5 + \frac{5131}{750}\sqrt{5\,x^2+2\,x+3}x^4 - \frac{33259}{2500}\sqrt{5\,x^2+2\,x+3}x^3 - \frac{207427}{37500}\sqrt{5\,x^2+2\,x+3}x^2 + \frac{36073}{1875}\sqrt{5\,x^2+2\,x+3}x - \frac{1719097}{156250}\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{22053}{31250}\sqrt{5\,x^2+2\,x+3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 332
59/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2
 + 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/1
4*sqrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{1}{187500}(5\,((5\,(70\,(175\,x+733)x-99777)x-207427)x+721460)x-132318)\sqrt{5\,x^2+2\,x+3} + \frac{1719097}{156250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 1
32318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2 (2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx$

3.388.1 Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx = \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2} - \frac{1901\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{250\sqrt{5}}$$

output

```
-1901/1250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+463/125*(5*x^2+2*x+3)^(1
/2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)-7/20*x^3*(
5*x^2+2*x+3)^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}} dx = \frac{\sqrt{3+2x+5x^2}(5556+2950x-2855x^2-525x^3)}{1500} + \frac{1901\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(5556 + 2950*x - 2855*x^2 - 525*x^3))/1500 + (1901*
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(250*Sqrt[5])
```

3.388.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)(x^2+5x+2)}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\int \frac{-571x^3+203x^2+260x+40}{\sqrt{5x^2+2x+3}}dx-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\left(\frac{1}{15}\int \frac{2(2950x^2+3663x+300)}{\sqrt{5x^2+2x+3}}dx-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\frac{2}{15}\int \frac{2950x^2+3663x+300}{\sqrt{5x^2+2x+3}}dx-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int -\frac{30(195-926x)}{\sqrt{5x^2+2x+3}}dx+295\sqrt{5x^2+2x+3}x\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\int \frac{195-926x}{\sqrt{5x^2+2x+3}}dx\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)$$
$$\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$\downarrow$ 1090

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2}\right.$$
$$\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$\downarrow$ 222

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-$$
$$\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

input
```
Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((-571*x^2*Sqrt[3 + 2*x + 5*x^2])/15 +
 (2*(295*x*Sqrt[3 + 2*x + 5*x^2] - 3*((-926*Sqrt[3 + 2*x + 5*x^2])/5 + (19
01*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5]))))/15)/20
```

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.388. $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.388.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{\left(525x^3+2855x^2-2950x-5556\right)\sqrt{5x^2+2x+3}}{1500}-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}$
trager	$\left(-\frac{7}{20}x^3-\frac{571}{300}x^2+\frac{59}{30}x+\frac{463}{125}\right)\sqrt{5x^2+2x+3}-\frac{1901\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(5\,\mathrm{RootOf}\left(\_Z^2-5\right)x+\mathrm{RootOf}\left(\_Z^2\right.\right.}{1250}$
default	$-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}+\frac{463\sqrt{5x^2+2x+3}}{125}-\frac{7x^3\sqrt{5x^2+2x+3}}{20}-\frac{571x^2\sqrt{5x^2+2x+3}}{300}+\frac{59x\sqrt{5x^2+2x+3}}{30}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/1500*(525*x^3+2855*x^2-2950*x-5556)*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/
2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{1500}\left(525\,x^3+2855\,x^2-2950\,x-5556\right)\sqrt{5\,x^2+2\,x+3} + \frac{1901}{2500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/
2500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x -
 8)
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{7x^3}{20}-\frac{571x^2}{300}+\frac{59x}{30}+\frac{463}{125}\right) - \frac{1901\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-7*x**3/20 - 571*x**2/300 + 59*x/30 + 463/125) - 1
901*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/1250
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{7}{20}\sqrt{5x^2+2x+3}x^3 - \frac{571}{300}\sqrt{5x^2+2x+3}x^2 + \frac{59}{30}\sqrt{5x^2+2x+3}x - \frac{1901}{1250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{463}{125}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/3
0*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x +
 1)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{1500}(5((105x+571)x-590)x-5556)\sqrt{5x^2+2x+3} + \frac{1901}{1250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
-1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/
1250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

3.389 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

3.389.1 Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx &= -\frac{\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} \\ &\quad - \frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right) \\ &\quad + \frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)\end{aligned}$$

output
```
-1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(11430254-
2947670*11^(1/2))^(1/2)+3/39116*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5
*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(11430254+2947670*11^(1/2))^(1/
2)
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{7\sqrt{5}} + \frac{3}{14}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{29\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+10\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-13\,\text{lo}}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]]/(7*Sqrt[5]) + (3*RootSum[83
- 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (29*Log[-(Sqrt[5]*
x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 10*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 +
2*x + 5*x^2] - #1]*#1 - 13*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*
#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/14
```

3.389.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx$$
$$\Big\downarrow \text{2143}$$
$$-\frac{1}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{3(13x+5)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{3}{7}\int \frac{13x+5}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{7}\int \frac{13x+5}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}$$

$$\Big\downarrow \text{222}$$

$$\frac{3}{7}\int \frac{13x+5}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$$\Big\downarrow \text{1365}$$

$$\frac{3}{7}\left(\frac{1}{11}\left(143-61\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(143+61\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2}}\right.$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{1}{22}\left(143-61\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(143+61\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2}}\right.$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{3}{7}\left(-\frac{1}{11}\left(143-61\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{1}{11}\right.$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$$\Big\downarrow \text{219}$$

3.389. $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

$$\frac{3}{7}\left(\frac{(143-61\sqrt{11})\operatorname{arctanh}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2(125-17\sqrt{11})}}+\frac{(143+61\sqrt{11})\operatorname{arctanh}\left(\frac{(17+5\sqrt{11})x+\sqrt{11}+23}{\sqrt{2(125+17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2(125+17\sqrt{11})}}\right)$$

$$\frac{\operatorname{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])]/Sqrt[5] + (3*(((143 - 61*Sqrt[11])*A
rcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((143 + 61*Sqr
t[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7
```

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.389. $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.389.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(61+13\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250}}\right)}{154\sqrt{250+34\sqrt{11}}}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+382515364\operatorname{RootOf}\left(24095456\_Z^4-3240072\_Z^2+29241\right)^2-51436143\right)\ln\left(-\frac{9459586880128\operatorname{RootOf}\left(\_Z^2+382\right.}{}\right.}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(61+13*11^(1/2))*11^(1/
2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11
^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1
/2))+3/154*(-61+13*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2
*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34
*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

3.389.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(114) = 228$.

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.81

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)+\right.}{x}\right.$$

$$+\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(-\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)\right.}{x}\right.$$

$$-\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}(172\sqrt{11}+715)\sqrt{-9495\sqrt{11}+368}}{x}\right.$$

$$+\frac{1}{78232}\sqrt{2794}\sqrt{-9495\sqrt{11}+36819}\log\left(\frac{\sqrt{2794}\sqrt{5x^2+2x+3}(172\sqrt{11}+715)\sqrt{-9495\sqrt{11}+3681}}{x}\right.$$

$$+\frac{1}{70}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output
```
-3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^
2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt
(11)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(1
1) + 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4
091)*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x
) - 1/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(
5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 55740
3*sqrt(11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-94
95*sqrt(11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) +
 715)*sqrt(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x -
 2787015)/x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
25*x^2 - 10*x - 8)
```

3.389.6 Sympy [F]

$$\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx \\
&= -\int \frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx \\
&\quad -\int \frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx \\
&\quad -\int \frac{2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx
\end{aligned}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
-Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3)
- sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x +
3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2
/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x)
```

3.389.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.84

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{1}{10780}\sqrt{11}\left(28\sqrt{11}\sqrt{5}\,\mathrm{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-\frac{1365\sqrt{11}\sqrt{2}\,\mathrm{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{1}{7\left|14\right.}\right.}{\sqrt{17\sqrt{}}}\right.$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/
14*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*s
qrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*
sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) +
23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) +
 390*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*s
qrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
+ 2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5
/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sq
rt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt
(17*sqrt(11) + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x
 + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1
/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt
(2)/abs(14*x + 2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \frac{1}{35}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right) + 0.353184817631429\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right) - 0.0986339689905714\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right) - 0.353184817631429\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right) + 0.0986339689905714\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.353
184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.252951630540
00) - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803
8113000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.0
9411235400000)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,(-7x^2+4x+1)}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

3.390 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

3.390.1 Optimal result

Integrand size = 35, antiderivative size = 178

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx &= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} \\ &\quad - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176} \\ &\quad + \frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176}\end{aligned}$$

output
```
-3/5588*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)+1/31225744*arctanh((
23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2)
)*(8459955268270-39215692714*11^(1/2))^(1/2)-1/31225744*arctanh((23+x*(17-
5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(845995
5268270+39215692714*11^(1/2))^(1/2)
```

3.390.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.98

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$
$$= -\frac{3(-40+371x)\sqrt{3+2x+5x^2}}{5588(-1-4x+7x^2)} - \frac{1}{49}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{-397\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+7\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\Big]$$
$$+\frac{3\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-1510889\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+238966\sqrt{5}\log\left(-\right.}{-4\sqrt{5}-3}}{547624}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(-3*(-40 + 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(-1 - 4*x + 7*x^2)) - RootS
um[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-397*Log[-(
Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^
3) & ]/49 + (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#
1^4 & , (-1510889*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 238966*
Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 60319*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[
5]*#1^2 + 7*#1^3) & ])/547624
```

3.390.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \text{2135}$$

$$-\frac{\int -\frac{8(3693x+6517)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3693x+6517}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1365}$$

$$\frac{\frac{1}{11}\left(40623-53005\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(40623+53005\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{22}\left(40623-53005\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(40623+53005\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\frac{1}{11}\left(40623-53005\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{1}{11}\left(40623+53005\sqrt{}\right.}{5588}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(40623-53005\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(40623+53005\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

3.390. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

output
```
(-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) + (((4062
3 - 53005*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*
(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11
])]) + ((40623 + 53005*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11]
)*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125
 + 17*Sqrt[11])]))/5588
```

3.390.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.390. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.390.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{3(-40+371x)\sqrt{5x^2+2x+3}}{5588(7x^2-4x-1)}+\frac{\left(-53005+3693\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}}\right)}{122936\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	$\frac{161\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250+34\sqrt{11}}}\right)}{484\sqrt{250+34\sqrt{11}}}-\frac{161\sqrt{11}\,\operatorname{arctanh}\left(\frac{}{\sqrt{250-34\sqrt{11}}}\right)}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-3/5588*(-40+371*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/122936*(-53005+369
3*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11
^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/
(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50-34*11^(1/2))^(1/2))+1/122936*(53005+3693*11^(1/2))*11^(1/2)/(250+34*11^
(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

3.390.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.85

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = -\frac{\sqrt{2794}(7\,x^2-4\,x-1)\sqrt{14035681\,\sqrt{11}+3027900955}\log\left(-\frac{\sqrt{2794}\sqrt{5\,x^2+2\,x+3}\sqrt{14035681\,\sqrt{11}+3027900955}\left(7179\right.}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 3027900
955)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 30279
00955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431
01259*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*s
qrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*
sqrt(11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(
x + 3) + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)
*sqrt(-14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x +
3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 26538
1033753*sqrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)
*(7*x^2 - 4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*
sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) +
3027900955) - 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 13269051687
65)/x) + 33528*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

3.390.6 Sympy [F]

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2 \sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,\left(7x^2-4x-1\right)^2}\,dx$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2),
x)
```

3.390.7 Maxima [F]

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2 \sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\left(7\,x^2-4\,x-1\right)^2\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

3.390.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(129) = 258.

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.55

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2 \sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3\left(1231\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3+1735\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2-3913\,\sqrt{5}x-3989\,\sqrt{}\right.}{2794\left(7\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2+1\right.}$$

$$+ 0.0924287071106453 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$

$$- 0.0938608034604765 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$

$$- 0.0924287071106453 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$

$$+ 0.0938608034604765 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(
5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sq
rt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)
)^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.09242870711
06453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386
08034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0
.0924287071106453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811300
0) + 0.0938608034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.0941123
5400000)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,{\left(-7\,x^2+4\,x+1\right)}^2}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.391 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

3.391.1 Optimal result

Integrand size = 35, antiderivative size = 227

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx &= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2}-\frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\,(1+4x-7x^2)}\\ &\quad-\frac{7\left(39370231-2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}}\\ &\quad+\frac{7\left(39370231+2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}}\end{aligned}$$

output
```
-3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7/62451488*(40976
9-1189370*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-7/124902976*arctanh((23+x*
(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(39
370231-2538725*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/124902976*arctanh((23
+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*
(39370231+2538725*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.91

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\frac{235298\sqrt{3+2x+5x^2}(-3538943+3071502x+53381041x^2-58279130x^3)}{(1+4x-7x^2)^2} - 1796775175713\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2}{}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((235298*Sqrt[3 + 2*x + 5*x^2]*(-3538943 + 3071502*x + 53381041*x^2 - 5827
9130*x^3))/(1 + 4*x - 7*x^2)^2 - 1796775175713*RootSum[83 - 16*Sqrt[5]*#1
- 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 11176*Ro
otSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (10486671
792*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 6928653865
*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1
+ 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 +
8*Sqrt[5]*#1^3 + 7*#1^4 & , (36376673721218*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqr
t[3 + 2*x + 5*x^2] - #1]*#1 + 26508461599305*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]
)/14694710223424
```

3.391.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^3\sqrt{5x^2+2x+3}}\,dx$$
$$\Big\downarrow \; 2135$$

$$-\frac{\int -\frac{8(11130x^2+10125x+16253)}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx}{89408}-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{11130x^2+10125x+16253}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{-\frac{\int -\frac{8(17771075x+34292781)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{5588(-7x^2+4x+1)}}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{17771075x+34292781}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-1189370x)\sqrt{5x^2+2x+3}}{5588(-7x^2+4x+1)}}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 1365}$$

$$\frac{\frac{\frac{7}{11}\left(27925975-39370231\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(27925975+39370231\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-118\ldots}{5588(-\ldots}}{11176}$$
$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{7}{22}\left(27925975-39370231\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(27925975+39370231\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-11893\ldots}{5588(-7\ldots}}{11176}$$
$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{-\frac{7}{11}\left(27925975-39370231\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(27925975+39370231\sqrt{11}\right)\int \frac{}{8(1\ldots}}{5588}}{11176}$$
$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{ 219}$$

3.391. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

$$\frac{\frac{\frac{7\left(27925975-39370231\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(27925975+39370231\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}}{5588}-\frac{7(4097}{}}{11176}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) + ((-7
*(409769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) + ((
7*(27925975 - 39370231*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11]
)*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125
 - 17*Sqrt[11])]) + (7*(27925975 + 39370231*Sqrt[11])*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^
2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/5588)/11176
```

3.391.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.391. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.391.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(58279130 x^{3}-53381041 x^{2}-3071502 x+3538943\right) \sqrt{5 x^{2}+2 x+3}}{62451488 \left(7 x^{2}-4 x-1\right)^{2}}+\frac{7 \left(-39370231+2538725 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctanh}\left(\frac{2}{\sqrt{250-34 \sqrt{11}}\, \sqrt{245}}\right)}{1373932736 \sqrt{2}}$
trager	Expression too large to display
default	Expression too large to display

3.391. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/62451488*(58279130*x^3-53381041*x^2-3071502*x+3538943)/(7*x^2-4*x-1)^2*
(5*x^2+2*x+3)^(1/2)+7/1373932736*(-39370231+2538725*11^(1/2))*11^(1/2)/(25
0-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2)
)^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7
/1373932736*(39370231+2538725*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(174) = 348$.

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.72

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx =$$
$$-\frac{\sqrt{2794}(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{1283973697005131\,\sqrt{11}+82616280769148425}\log\left(-\frac{\sqrt{2794}\sqrt{5}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(12839
73697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*
x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt
(11) + 2940638404) + 723215097220611079 7*sqrt(11)*(x + 3) - 2169645291661
8332391*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2
794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148
425)*(358684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x +
 3) + 21696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x
^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 826162807
69148425)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940
638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) + 72321509722
06110797*sqrt(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)
/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-128397369700513
1*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(35
8684877*sqrt(11) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 826162807
69148425) - 7232150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x
+ 36160754861030553985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*
x + 3538943)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

3.391.6 Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx =$$
$$-\int \frac{5x}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2}$$
$$-\int \frac{x^2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2}$$
$$-\int \frac{2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

output
```
-Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*
x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3)
 - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x
**2 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588
*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*
sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2
 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x*
*2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2
*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3)
 - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

3.391.7 Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

3.391.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(174) = 348$.

Time = 0.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.67

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{124397525\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7+26796567\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-3595807617\left(\sqrt{5}x-\sqrt{\right.}\right.}{31225744\left(7\left(\sqrt{\right.}\right.}$$
$$+ 0.0423989586659649 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$- 0.0446437606656958 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$- 0.0423989586659649 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+ 0.0446437606656958 \log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqr
t(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(
5*x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
))^4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 780062
3485*sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x
^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
+ 3) + 4.41924736459000) - 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(
5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0446437606656958*log(-sqrt(5)*x +
 sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,{\left(-7\,x^2+4\,x+1\right)}^3}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.392 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

3.392.1 Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\,\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250\sqrt{5}}$$

output
```
50047657/781250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+16/546875*(6122807-
5338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2*x+3)^(1/2)-3192602
/46875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+2*x+3)^(1/2)+393659
/12500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*x+3)^(1/2)-343/150*
x^5*(5*x^2+2*x+3)^(1/2)
```

3.392.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{3155769618-1045703388x+2135143465x^2-1795638985x^3-174}{6562500\sqrt{3+2x}}$$
$$-\frac{50047657\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(3155769618 - 1045703388*x + 2135143465*x^2 - 1795638985*x^3 - 174819575*x
^4 + 897612625*x^5 - 256821250*x^6 - 75031250*x^7)/(6562500*Sqrt[3 + 2*x +
 5*x^2]) - (50047657*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625
0*Sqrt[5])
```

3.392.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^3(x^2+5x+2)}{(5x^2+2x+3)^{3/2}}dx$$
$$\Big\downarrow \text{2191}$$
$$\frac{1}{28}\int \frac{28(-5359375x^6-15465625x^5+41667500x^4-5403250x^3-36448575x^2+16868255x+16918718)}{78125\sqrt{5x^2+2x+3}}dx+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{-5359375x^6-15465625x^5+41667500x^4-5403250x^3-36448575x^2+16868255x+16918718}{\sqrt{5x^2+2x+3}}dx}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\frac{1}{30}\int \frac{5(-81003125x^5+266083125x^4-32419500x^3-218691450x^2+101209530x+101512308)}{\sqrt{5x^2+2x+3}}dx-\frac{1071875}{6}x^5\sqrt{5x^2+2x+3}}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\int \frac{-81003125x^5+266083125x^4-32419500x^3-218691450x^2+101209530x+101512308}{\sqrt{5x^2+2x+3}}dx-\frac{1071875}{6}x^5\sqrt{5x^2+2x+3}}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{6}\left(\frac{1}{25}\int \frac{150(49207375x^4+1077000x^3-36448575x^2+16868255x+16918718)}{\sqrt{5x^2+2x+3}}dx-3240125x^4\sqrt{5x^2+2x+3}\right)-\frac{1071875}{6}x^5\sqrt{5x^2+}}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(6\int \frac{49207375x^4+1077000x^3-36448575x^2+16868255x+16918718}{\sqrt{5x^2+2x+3}}dx-3240125x^4\sqrt{5x^2+2x+3}\right)-\frac{1071875}{6}x^5\sqrt{5x^2+2x+}}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{20}\int \frac{5(-64582325x^3-234367575x^2+67473020x+67674872)}{\sqrt{5x^2+2x+3}}dx+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3240125x^4\sqrt{5x^2+2x+3}}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\int \frac{-64582325x^3-234367575x^2+67473020x+67674872}{\sqrt{5x^2+2x+3}}dx+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3240125x^4\sqrt{5x^2+2x+3}\right)-}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

3.392. $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{1}{15}\int \frac{10(-319260200x^2+139958925x+101512308)}{\sqrt{5x^2+2x+3}}dx-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\int \frac{-319260200x^2+139958925x+101512308}{\sqrt{5x^2+2x+3}}dx-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-32401\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(\frac{1}{10}\int \frac{330(7143545x+5978496)}{\sqrt{5x^2+2x+3}}dx-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2}\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\int \frac{7143545x+5978496}{\sqrt{5x^2+2x+3}}dx-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+2x}\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 1160

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(4549787\int \frac{1}{\sqrt{5x^2+2x+3}}dx+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x}\right.\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 1090

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(\frac{4549787\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{2\sqrt{70}}+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{1291646}{3}\right.\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 222

3.392. $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(\frac{4549787\operatorname{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{\sqrt{5}}+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

input
```
Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + ((-1071875*x^5
*Sqrt[3 + 2*x + 5*x^2])/6 + (-3240125*x^4*Sqrt[3 + 2*x + 5*x^2] + 6*((9841
475*x^3*Sqrt[3 + 2*x + 5*x^2])/4 + ((-12916465*x^2*Sqrt[3 + 2*x + 5*x^2])/
3 + (2*(-31926020*x*Sqrt[3 + 2*x + 5*x^2] + 33*(1428709*Sqrt[3 + 2*x + 5*x
^2] + (4549787*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/Sqrt[5])))/3)/4))/6)/7812
5
```

3.392.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.392. $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.392.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}+\frac{500476}{}$
trager	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}}+\frac{500476}{}$
default	$\frac{\frac{176049701x}{1093750}+\frac{176049701}{5468750}}{\sqrt{5x^2+2x+3}}+\frac{175268451}{390625\sqrt{5x^2+2x+3}}+\frac{61004099x^2}{187500\sqrt{5x^2+2x+3}}-\frac{50047657x}{156250\sqrt{5x^2+2x+3}}+\frac{50047657\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,(x}{14}\right)}{781250}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/6562500*(75031250*x^7+256821250*x^6-897612625*x^5+174819575*x^4+1795638
985*x^3-2135143465*x^2+1045703388*x-3155769618)/(5*x^2+2*x+3)^(1/2)+500476
57/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1051000797\,\sqrt{5}(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-\right.}{}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6
 - 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457
03388*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

3.392.6 Sympy [F]

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx =$$
$$- \int \left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$
$$- \int \left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$
$$- \int \frac{61x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx$$
$$- \int \frac{871x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx$$
$$- \int \left(-\frac{127x^5}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$
$$- \int \left(-\frac{2065x^6}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$
$$- \int \frac{1127x^7}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx$$
$$- \int \frac{343x^8}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx$$
$$- \int \left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2
 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) -
 Integral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x +
 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**
2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x)
- Integral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*
x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(
5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3))
, x) - Integral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2
 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sq
rt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x +
3)), x) - Integral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2
*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x)
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{343\,x^7}{30\,\sqrt{5\,x^2+2\,x+3}} - \frac{29351\,x^6}{750\,\sqrt{5\,x^2+2\,x+3}} + \frac{1025843\,x^5}{7500\,\sqrt{5\,x^2+2\,x+3}} - \frac{998969\,x^4}{37500\,\sqrt{5\,x^2+2\,x+3}} - \frac{51303971\,x^3}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{61004099\,x^2}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{50047657}{781250}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{87141949\,x}{546875\,\sqrt{5\,x^2+2\,x+3}} + \frac{525961603}{1093750\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) +
1025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x
 + 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sq
rt(5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x +
 1)) - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*
x^2 + 2*x + 3)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\frac{50047657}{781250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$
$$-\frac{(35((5(35(70(175x+599)x-146549)x+998969)x+51303971)x-61004099)x+1045703388)x-31}{6562500\sqrt{5x^2+2x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))
- 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x
+ 51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x
 + 3)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5x+2)(-7x^2+4x+1)^3}{(5x^2+2x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

3.393.1 Optimal result

Integrand size = 35, antiderivative size = 124

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{89583\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1250\sqrt{5}}$$

output
```
89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-8/21875*(12983+136602*x)
/(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1/2)-8749/1250*x*(5*x^2+2*x+
3)^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100*x^3*(5*x^2+2*x+3)^(1/2)
```

3.393.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-168536-1298674x-280805x^2-515655x^3+194775x^4+42875x}{17500\sqrt{3+2x+5x^2}} - \frac{89583\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5)/(
17500*Sqrt[3 + 2*x + 5*x^2]) - (89583*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x
+ 5*x^2]])/(1250*Sqrt[5])
```

3.393.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-7x^2+4x+1\right)^2\left(x^2+5x+2\right)}{\left(5x^2+2x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{28}\int \frac{28\left(30625x^4+105875x^3-173225x^2-52985x+153254\right)}{3125\sqrt{5x^2+2x+3}}dx-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{30625x^4+105875x^3-173225x^2-52985x+153254}{\sqrt{5x^2+2x+3}}dx}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{20}\int \frac{5\left(380625x^3-748025x^2-211940x+613016\right)}{\sqrt{5x^2+2x+3}}dx+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\int \frac{380625x^3-748025x^2-211940x+613016}{\sqrt{5x^2+2x+3}}dx+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{4}\left(\frac{1}{15}\int \frac{30\left(-437450x^2-182095x+306508\right)}{\sqrt{5x^2+2x+3}}dx+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(2\int \frac{-437450x^2-182095x+306508}{\sqrt{5x^2+2x+3}}dx+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{4}\left(2\left(\frac{1}{10}\int \frac{10(437743-50860x)}{\sqrt{5x^2+2x+3}}dx-43745x\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(2\left(\int \frac{437743-50860x}{\sqrt{5x^2+2x+3}}dx-43745x\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{4}\left(2\left(447915\int \frac{1}{\sqrt{5x^2+2x+3}}dx-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{}}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{1}{4}\left(2\left(\frac{89583}{2}\sqrt{\frac{5}{14}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x}\right.}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{4}\left(2\left(89583\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{612}{4}}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

3.393. $\displaystyle \int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

input
```
Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) + ((6125*x^3*Sqrt[3
+ 2*x + 5*x^2])/4 + (25375*x^2*Sqrt[3 + 2*x + 5*x^2] + 2*(-10172*Sqrt[3 +
2*x + 5*x^2] - 43745*x*Sqrt[3 + 2*x + 5*x^2] + 89583*Sqrt[5]*ArcSinh[(2 +
10*x)/(2*Sqrt[14])]))/4)/3125
```

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.393. $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.393.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{6250}$
trager	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(5\operatorname{RootOf}\left(\_Z^2-5\right)x+\operatorname{RootOf}\left(\_\right.\right.}{6250}$
default	$-\frac{5564(10x+2)}{21875\sqrt{5x^2+2x+3}}-\frac{28506}{3125\sqrt{5x^2+2x+3}}+\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}-$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/17500*(42875*x^5+194775*x^4-515655*x^3-280805*x^2-1298674*x-168536)/(5*x
^2+2*x+3)^(1/2)+89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int\frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx=\frac{627081\sqrt{5}(5x^2+2x+3)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2\right.}{}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x +
3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3
 - 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x +
3)
```

3.393.6 Sympy [F]

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5x+2)(7x^2-4x-1)^2}{(5x^2+2x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
 x)
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{49\,x^5}{20\,\sqrt{5\,x^2+2\,x+3}} + \frac{1113\,x^4}{100\,\sqrt{5\,x^2+2\,x+3}} - \frac{14733\,x^3}{500\,\sqrt{5\,x^2+2\,x+3}} - \frac{8023\,x^2}{500\,\sqrt{5\,x^2+2\,x+3}} + \frac{89583}{6250}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{649337\,x}{8750\,\sqrt{5\,x^2+2\,x+3}} - \frac{42134}{4375\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 147
33/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 89
583/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x
^2 + 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right) + \frac{(35((35(35x+159)x-14733)x-8023)x-1298674)x-168536}{17500\sqrt{5x^2+2x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
+ 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 168
536)/sqrt(5*x^2 + 2*x + 3)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5x+2)(-7x^2+4x+1)^2}{(5x^2+2x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

3.394.1 Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{5}}$$

output
```
149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2/875*(2321+2449*x)/(5*x^2+
2*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*x*(5*x^2+2*x+3)^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-2953-2837x-1925x^2-245x^3}{350\sqrt{3+2x+5x^2}} - \frac{149\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{25\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-2953 - 2837*x - 1925*x^2 - 245*x^3)/(350*Sqrt[3 + 2*x + 5*x^2]) - (149*L
og[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(25*Sqrt[5])
```

3.394.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)(x^2+5x+2)}{(5x^2+2x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{28}\int \frac{28(-175x^2-705x+562)}{125\sqrt{5x^2+2x+3}}dx-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\int \frac{-175x^2-705x+562}{\sqrt{5x^2+2x+3}}dx-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{125}\left(\frac{1}{10}\int \frac{5(1229-1305x)}{\sqrt{5x^2+2x+3}}dx-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\left(\frac{1}{2}\int \frac{1229-1305x}{\sqrt{5x^2+2x+3}}dx-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{125}\left(\frac{1}{2}\left(1490\int \frac{1}{\sqrt{5x^2+2x+3}}dx-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{125}\left(\frac{1}{2}\left(149\sqrt{\frac{5}{14}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{125}\left(\frac{1}{2}\left(298\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

input
```
Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) + ((-35*x*Sqrt[3 + 2*x +
5*x^2])/2 + (-261*Sqrt[3 + 2*x + 5*x^2] + 298*Sqrt[5]*ArcSinh[(2 + 10*x)/(
2*Sqrt[14])])/2)/125
```

3.394.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.394.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}$
trager	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}-\frac{149\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-5\operatorname{RootOf}\left(\_Z^2-5\right)x+5\sqrt{5x^2+2x+3}-\operatorname{RootOf}\left(\_Z^2-5\right)\right)}{125}$
default	$-\frac{751(10x+2)}{3500\sqrt{5x^2+2x+3}}-\frac{1001}{125\sqrt{5x^2+2x+3}}-\frac{7x^3}{10\sqrt{5x^2+2x+3}}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}-\frac{149x}{25\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/350*(245*x^3+1925*x^2+2837*x+2953)/(5*x^2+2*x+3)^(1/2)+149/125*5^(1/2)*
arcsinh(5/14*14^(1/2)*(x+1/5))
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1043\sqrt{5}(5x^2+2x+3)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-\right.}{1750\,(5x^2+}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*
(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sq
rt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

3.394.6 Sympy [F]

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\int \left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \frac{31x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \frac{7x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - I
ntegral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 +
2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*
sqrt(5*x**2 + 2*x + 3)), x)
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}} - \frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}} + \frac{149}{125}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}} - \frac{2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125
*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x +
3) - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{149}{125}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right) - \frac{(35\,(7\,x+55)x+2837)x+2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
-149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1
/350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

3.395.1 Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\,\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\,\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}$$

output
```
1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*arctanh((23+x*(17-5*11^(
1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(393525121-34
945955*11^(1/2))^(1/2)+3/1419352*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(
5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(393525121+34945955*11^(1/2))^
(1/2)
```

3.395.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \frac{-131+605x}{3556\sqrt{3+2x+5x^2}} + \frac{3}{254}\text{RootSum}\Bigg[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3 +7\#1^4\&, \frac{22\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+41\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-21\log\left(\right.}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(-131 + 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) + (3*RootSum[83 - 16*Sqrt[5]*#
1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (22*Log[-(Sqrt[5]*x) + Sqrt[3 +
2*x + 5*x^2] - #1] + 41*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]*#1 - 21*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt
[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/254
```

3.395.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}\,dx$$

$$\Big\downarrow \; 2135$$

$$\frac{\int \frac{336(42x+41)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{28448} - \frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{254}\int \frac{42x+41}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx - \frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \ 1365$$

$$\frac{3}{254}\left(\frac{7}{11}\left(66-53\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(66+53\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2}\right.$$
$$\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{254}\left(\frac{7}{22}\left(66-53\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(66+53\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2}}\right.$$
$$\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \ 1154$$

$$\frac{3}{254}\left(-\frac{7}{11}\left(66-53\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{}{1}\right.$$
$$\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{254}\left(\frac{7\left(66-53\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(66+53\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.$$
$$\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
-1/3556*(131 - 605*x)/Sqrt[3 + 2*x + 5*x^2] + (3*((7*(66 - 53*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*S
qrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + (7*(66 + 53*Sqr
t[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/254
```

3.395. $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

3.395.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.395. $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.395.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

method	result
risch	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}+\frac{21\left(53+6\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250+34\sqrt{11}}}\right)}{5588\sqrt{250+34\sqrt{11}}}+$
trager	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}+\frac{27\,\operatorname{RootOf}\left(12905329536\_Z^4-4015815408\_Z^2+285305881\right)\ln\left(\frac{88048478470271768064x\,\operatorname{RootOf}\left(12905329536\right.}{}\right.}{}$
default	$-\frac{10x+2}{196\sqrt{5x^2+2x+3}}-\frac{3\left(61+13\sqrt{11}\right)\sqrt{11}\left(\frac{1}{7\left(\frac{250}{49}+\frac{34\sqrt{11}}{49}\right)\sqrt{5\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+\frac{250}{49}+\frac{34\sqrt{11}}{49}}}-\frac{}{7\left(\frac{250}{49}+34\right.}\right.}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)+21/5588*(53+6*11^(1/2))*11^(1/2)/(
250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/
2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
+21/5588*(-53+6*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(5
00/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11
^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1
/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

3.395.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(117) = 234$.

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.01

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{21\sqrt{1397}(5x^2+2x+3)\sqrt{25015\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+2359\right.}{x}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)
*log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(13
35*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/
x) - 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*
(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt
(11) - 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*
sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1
397)*sqrt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) +
2535237) + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqr
t(1397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)
*sqrt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535
237) - 70790181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt
(5*x^2 + 2*x + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

3.395.6 Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}$$
$$-\int \frac{x^2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}$$
$$-\int \frac{2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt
(5*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) -
6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(
5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sq
rt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2
+ 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

3.395.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(117) = 234.

Time = 0.33 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.68

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\frac{1}{4312}\sqrt{11}\left(\frac{20\sqrt{11}x}{\sqrt{5x^2+2x+3}}-\frac{7890\sqrt{11}x}{17\sqrt{11}\sqrt{5x^2+2x+3}+125\sqrt{5x^2+2x+3}}+\frac{7890\sqrt{1}}{17\sqrt{11}\sqrt{5x^2+2x+3}-}\right.$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(1
7*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(
11)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13
377*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*s
qrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqr
t(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/ab
s(14*x - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^
2 + 2*x + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2
 + 2*x + 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2
 + 2*x + 3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*
x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) +
1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqr
t(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*
sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4)
+ 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)
*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sq
rt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5
*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*
sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(
11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)...
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)\,(3+2x+5x^2)^{3/2}}\,dx &= \frac{605\,x-131}{3556\,\sqrt{5\,x^2+2\,x+3}} \\ &+ 0.0477059376663667\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right) \\ &- 0.0352174957838020\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right) \\ &- 0.0477059376663667\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right) \\ &+ 0.0352174957838020\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)\end{aligned}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376663667*log(-sqrt(
5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957838020*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.047705937666366
7*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.035217495
7838020*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\,(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

3.396 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2 (3+2x+5x^2)^{3/2}}\,dx$

3.396.1 Optimal result

Integrand size = 35, antiderivative size = 215

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2 (3+2x+5x^2)^{3/2}}\,dx = -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588(1+4x-7x^2)\sqrt{3+2x+5x^2}} - \frac{7\left(541543-5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(541543+5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}}$$

output
```
1/19870928*(-76567-22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+
4*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*arctanh((23+x*(17-5*11^(1/2))-11^(1/2
))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(541543-5144*11^(1/2))/(27
50-374*11^(1/2))^(1/2)+7/2838704*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(
5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(541543+5144*11^(1/2))/(2750+3
74*11^(1/2))^(1/2)
```

3.396.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.66 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.93

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \frac{503287-3628805x-444949x^2-159285x^3}{19870928\sqrt{3+2x+5x^2}\,(-1-4x+7x^2)}$$
$$+\frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{116685\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+205710\log\left(-\sqrt{5}x+\sqrt{\ldots}\right.}{-4\sqrt{5}-35\#1\ldots}\right.}{258064\sqrt{5}}$$
$$-\frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{746007\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1016580\log\left(-\sqrt{5}x\ldots\right.}{-4\sqrt{5}-35\ldots}\right.}{2838704\sqrt{5}}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(503287 - 3628805*x - 444949*x^2 - 159285*x^3)/(19870928*Sqrt[3 + 2*x + 5*
x^2]*(-1 - 4*x + 7*x^2)) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5
]*#1^3 + 7*#1^4 & , (116685*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^
2] - #1] + 205710*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 8351
*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(258064*Sqrt[5]) - (3*RootSum[83 -
16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (746007*Sqrt[5]*Log[
-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1016580*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 42623*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]
)/(2838704*Sqrt[5])
```

3.396.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2135}$$

$$-\frac{\int -\frac{8(11130x^2+4719x+6277)}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}dx}{44704}-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11130x^2+4719x+6277}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}dx}{5588}-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{112(36008x+531255)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{28448}-\frac{22755x+76567}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{254}\int \frac{36008x+531255}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{22755x+76567}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1365}$$

$$\frac{\frac{1}{254}\left(\frac{7}{11}\left(56584-541543\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(56584+541543\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}d\right.}{5588}$$
$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{254}\left(\frac{7}{22}\left(56584-541543\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(56584+541543\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right.}{5588}$$
$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{254}\left(-\frac{7}{11}\left(56584-541543\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(56584+5\right.\right.}{5588}$$
$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

3.396. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx$

$$\downarrow \text{219}$$

$$\frac{\frac{1}{254}\left(\frac{7\left(56584-541543\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(56584+541543\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right) -}{5588}$$

$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(-3*(40 - 371*x))/(5588*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) + (-1/355
6*(76567 + 22755*x)/Sqrt[3 + 2*x + 5*x^2] + ((7*(56584 - 541543*Sqrt[11])*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + (7*(56584 + 5
41543*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])
)/254)/5588
```

3.396.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.396. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx$

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1])
 && !IGtQ[q, 0]
```

3.396.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{159285x^3+444949x^2+3628805x-503287}{19870928(7x^2-4x-1)\sqrt{5x^2+2x+3}}+\frac{7\left(-541543+5144\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\cdots}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\cdots}{}\right.}}\right.}{31225744\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

3.396. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/19870928*(159285*x^3+444949*x^2+3628805*x-503287)/(7*x^2-4*x-1)/(5*x^2+
2*x+3)^(1/2)+7/31225744*(-541543+5144*11^(1/2))*11^(1/2)/(250-34*11^(1/2))
^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7/31225744*(541
543+5144*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))
^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(
1/2))+250+34*11^(1/2))^(1/2))
```

3.396.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(162) = 324$.

Time = 0.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.82

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{7\sqrt{1397}(35\,x^4-6\,x^3+8\,x^2-14\,x-3)\sqrt{4294093814065\,\sqrt{11}+35653135368317}\log\left(-\frac{\sqrt{1397}\sqrt{5\,x^2+2\,x+3}\sqrt{}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(429
4093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x +
3)*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949
905) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 1432514722
0857935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(429409
3814065*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*s
qrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905)
 - 2865029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857
935)/x) + 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-429409381
4065*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(560
9479*sqrt(11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) +
 2865029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 1432514722085793
5)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-42940938140
65*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609
479*sqrt(11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) -
2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935
)/x) + 5588*(159285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*
x + 3))/(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)
```

3.396.6 Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5x+2}{(5x^2+2x+3)^{\frac{3}{2}}(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
```

output
```
Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2
), x)
```

3.396.7 Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^2(5\,x^2+2\,x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.37

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \frac{25230\,x+13397}{903224\,\sqrt{5\,x^2+2\,x+3}}$$
$$+\frac{3\left(42623\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3+77302\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2-275511\,\sqrt{5}x-219860\right.}{709676\left(7\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2+\right.}$$
$$+\,0.0218058276254033\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-\,0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-\,0.0218058276254033\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+\,0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x +
 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqr
t(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276254033*log(-sq
rt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.021805827625
4033*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287
4364433911*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2 \left(3+2x+5x^2\right)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^2}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

3.397 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

3.397.1 Optimal result

Integrand size = 35, antiderivative size = 250

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx = -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176(1+4x-7x^2)^2\sqrt{3+2x+5x^2}} - \frac{2701733-9148874x}{62451488(1+4x-7x^2)\sqrt{3+2x+5x^2}} - \frac{7\left(2792860024-84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(2792860024+84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}$$

output
```
-5/222077491328*(461370781+1118731375*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-3
71*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)+1/62451488*(-2701733+9148874*x)
/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*arctanh((23+x*(17-5*11^(
1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(2792860024-8
4865895*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*arctanh((23+11^(
1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(2792
860024+84865895*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

3.397.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.92 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.43

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}} \, dx = \frac{-\frac{1715(14298727813+7828199499x-148022158802x^2+109737266678x^3-200208943655}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}}{}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
((-1715*(14298727813 + 7828199499*x - 148022158802*x^2 + 109737266678*x^3
- 200208943655*x^4 + 274089186875*x^5))/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x
+ 5*x^2]) + 2324168*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[
5]*#1^3 + 7*#1^4 & , (-4989740*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5
*x^2] - #1] + 3790865*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 +
400449*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sq
rt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 22*Sqrt[5]*RootSum[83 - 16*
Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-3200991286865*Sqrt[5]
*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 18470877323690*Log[-(Sqr
t[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2296522946389*Sqrt[5]*Log[-(Sqr
t[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5
]*#1^2 + 7*#1^3) & ] - 9*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*
Sqrt[5]*#1^3 + 7*#1^4 & , (-8189062651053*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[
3 + 2*x + 5*x^2] - #1] + 39132066594240*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 5875617407695*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/380
862897627520
```

3.397.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2135, 27, 2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^3 (5x^2+2x+3)^{3/2}} \, dx$$

$$\Big\downarrow \text{2135}$$

$$-\frac{\int -\frac{8(22260x^2+11151x+16013)}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}dx}{89408}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{22260x^2+11151x+16013}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}dx}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{-\frac{\int -\frac{8\left(91488740x^2+13060267x+26911493\right)}{\left(-7x^2+4x+1\right)\left(5x^2+2x+3\right)^{3/2}}dx}{44704}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{91488740x^2+13060267x+26911493}{\left(-7x^2+4x+1\right)\left(5x^2+2x+3\right)^{3/2}}dx}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\frac{\int \frac{112(594061265x+2623128234)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{28448}-\frac{5(1118731375x+461370781)}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{254}\int \frac{594061265x+2623128234}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{5(1118731375x+461370781)}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1365}$$

3.397. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

$$\frac{\frac{\frac{1}{254}\left(\frac{7}{11}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(933524845+2792860024\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{5(1}{}}{5588}}{11176}$$

$$\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{1}{254}\left(\frac{7}{22}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(933524845+2792860024\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{5(111}{}}{5588}}{11176}$$

$$\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$\big\downarrow$ 1154

$$\frac{\frac{\frac{1}{254}\left(-\frac{7}{11}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(933524845+279286002\right.\right.}{5588}}{11}$$

$$\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{1}{254}\left(\frac{7\left(933524845-2792860024\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(933524845+2792860024\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.}{5588}}{11176}$$

$$\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

3.397. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

output
```
(-3*(40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) + (-1/
5588*(2701733 - 9148874*x)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) + ((-
5*(461370781 + 1118731375*x))/(3556*Sqrt[3 + 2*x + 5*x^2]) + ((7*(93352484
5 - 2792860024*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sq
rt[11])]) + (7*(933524845 + 2792860024*Sqrt[11])*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])
/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/254)/5588)/11176
```

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.397. $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.397.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{274089186875x^5-200208943655x^4+109737266678x^3-148022158802x^2+7828199499x+14298727813}{222077491328(7x^2-4x-1)^2\sqrt{5x^2+2x+3}}+\frac{7(2792860024+848658}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/222077491328*(274089186875*x^5-200208943655*x^4+109737266678*x^3-148022
158802*x^2+7828199499*x+14298727813)/(7*x^2-4*x-1)^2/(5*x^2+2*x+3)^(1/2)+7
/348978914944*(2792860024+84865895*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))+7/348978914944*(-2
792860024+84865895*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2
*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34
*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
```

3.397.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(193) = 386$.

Time = 0.37 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.81

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{7\sqrt{1397}(245\,x^6-182\,x^5+45\,x^4-124\,x^3+27\,x^2+26\,x+3)\sqrt{746933147106396414 67\sqrt{11}+896266498}}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 +
27*x^2 + 26*x + 3)*sqrt(7469331471063964l467*sqrt(11) + 896266498377233657
855)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt
(11) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) + 7550
21206868440551444479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 377510
603434220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^
3 + 27*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 89626649837723
3657855)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*s
qrt(11) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) - 7
55021206868440551444479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 377
510603434220275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124
*x^3 + 27*x^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 8962664983
77233657855)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) -
 407780707037)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855
) + 755021206868440551444479*sqrt(11)*(x + 3) + 226506362060532165433437*x
- 377510603434220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4
- 124*x^3 + 27*x^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 89626
6498377233657855)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt
(11) - 407780707037)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233
657855) - 755021206868440551444479*sqrt(11)*(x + 3) - 226506362060532165...
```

3.397.6 Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}}\, dx =$$
$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640x}$$
$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640x}$$
$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640x}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 +
2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x +
3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 1
10*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x*
*2 + 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 225
4*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5
*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(
5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2
*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**
2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2
*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3
) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38
*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

3.397.7 Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx = \int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3(5\,x^2+2\,x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)),
x)
```

3.397.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(193) = 386$.

Time = 0.30 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.59

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = \frac{501205\,x+1702037}{458837792\,\sqrt{5\,x^2+2\,x+3}}$$
$$+\frac{6871871279\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7+4012856750\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-223088535693\left(\sqrt{5}x\right.}{7931338}$$
$$+0.0107382277384513\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.0107382277384513\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(687
1871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*
x + 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 1
255097956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 56129965
4796*sqrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5
*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2
 + 2*x + 3)) + 83)^2 + 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) + 4.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^
2 + 2*x + 3) + 1.25295163054000) - 0.0107382277384513*log(-sqrt(5)*x + sqr
t(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x
 + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx=\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,{\left(-7\,x^2+4\,x+1\right)}^3}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

3.398 $\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx$

3.398.1 Optimal result

Integrand size = 26, antiderivative size = 166

$$\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx = Ax(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} (d+fx^2)^q \left(1+\frac{fx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{1}{3}Cx^3(a+cx^2)^p \left(1+\frac{cx^2}{a}\right)^{-p} (d+fx^2)^q \left(1+\frac{fx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

output
```
A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+
c*x^2/a)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2
,-p,-q,5/2,-c*x^2/a,-f*x^2/d)/((1+c*x^2/a)^p)/((1+f*x^2/d)^q)
```

3.398.2 Mathematica [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \frac{1}{3}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(\frac{9aAd\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\cdots\right.} + Cx^2\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)$$

input
```
Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

output
```
(x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x
^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((
f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f
*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)
])) + (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 +
 (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

3.398.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Cx^2\right) \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx$$

$$\Big\downarrow \text{406}$$

$$A\int \left(cx^2+a\right)^p \left(fx^2+d\right)^q dx + C\int x^2 \left(cx^2+a\right)^p \left(fx^2+d\right)^q dx$$

$$\Big\downarrow \text{334}$$

$$A\left(a+cx^2\right)^p \left(\frac{cx^2}{a}+1\right)^{-p} \int \left(\frac{cx^2}{a}+1\right)^p \left(fx^2+d\right)^q dx + C\int x^2 \left(cx^2+a\right)^p \left(fx^2+d\right)^q dx$$

$$\Big\downarrow \text{334}$$

$$A\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\int\left(\frac{cx^2}{a}+1\right)^p\left(\frac{fx^2}{d}+1\right)^q dx+ C\int x^2\left(cx^2+a\right)^p\left(fx^2+d\right)^q dx$$

$$\Big\downarrow \text{333}$$

$$C\int x^2\left(cx^2+a\right)^p\left(fx^2+d\right)^q dx+ Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{395}$$

$$C\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\int x^2\left(\frac{cx^2}{a}+1\right)^p\left(fx^2+d\right)^q dx+ Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{395}$$

$$C\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\int x^2\left(\frac{cx^2}{a}+1\right)^p\left(\frac{fx^2}{d}+1\right)^q dx+ Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{394}$$

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+ \frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

input
```
Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

output
```
(A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a),
-((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^
p*(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3
*(1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

3.398. $\int \left(a+cx^2\right)^p\left(A+Cx^2\right)\left(d+fx^2\right)^q\,dx$

3.398.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.398.4 Maple [F]

$$\int \left(c\,x^{2}+a\right)^{p} \left(C\,x^{2}+A\right) \left(f\,x^{2}+d\right)^{q}d x$$

input
```
int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

output
```
int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

3.398.5 Fricas [F]

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int (Cx^2+A)\left(cx^2+a\right)^p \left(fx^2+d\right)^q \, dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
```

output
```
Timed out
```

3.398.7 Maxima [F]

$$\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int (Cx^2+A)\left(cx^2+a\right)^p \left(fx^2+d\right)^q \, dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.398.8 Giac [F]

$$\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx = \int (Cx^2+A)(cx^2+a)^p (fx^2+d)^q \, dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int (a+cx^2)^p (A+Cx^2) (d+fx^2)^q \, dx = \int (C\,x^2+A) \, (c\,x^2+a)^p \, (f\,x^2+d)^q \, dx$$

input
```
int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q,x)
```

output
```
int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q, x)
```

3.399 $\int (A+Bx)(a+cx^2)^p(d+fx^2)^q\,dx$

3.399.1 Optimal result

Integrand size = 24, antiderivative size = 167

$$\int (A+Bx)(a+cx^2)^p(d+fx^2)^q\,dx = Ax(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B(a+cx^2)^{1+p}(d+fx^2)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{f(a+cx^2)}{cd-af}\right)}{2c(1+p)}$$

output
```
A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+
c*x^2/a)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)^(p+1)*(f*x^2+d)^q*hypergeom([-
q, p+1],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(p+1)/((c*(f*x^2+d)/(-a*f+c*d))^q
)
```

3.399.2 Mathematica [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.41

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx=\frac{1}{2}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(1,-p,-q,2,-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{6aAd\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afq\,\text{Ap}\right.}\right.$$

input
```
Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

output
```
(x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
-((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1
/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q,
5/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((
c*x^2)/a), -((f*x^2)/d)]))))/2
```

3.399.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1343, 334, 334, 333, 353, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$$

$$\Big\downarrow \ 1343$$

$$A\int \left(cx^2+a\right)^p\left(fx^2+d\right)^q dx+B\int x\left(cx^2+a\right)^p\left(fx^2+d\right)^q dx$$

$$\Big\downarrow \ 334$$

$$A\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\int \left(\frac{cx^2}{a}+1\right)^p\left(fx^2+d\right)^q dx+B\int x\left(cx^2+a\right)^p\left(fx^2+d\right)^q dx$$

$$\Big\downarrow \text{334}$$

$$A(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}\int\left(\frac{cx^2}{a}+1\right)^p\left(\frac{fx^2}{d}+1\right)^q dx+ B\int x(cx^2+a)^p(fx^2+d)^q dx$$

$$\Big\downarrow \text{333}$$

$$B\int x(cx^2+a)^p(fx^2+d)^q dx+ Ax(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}B\int(cx^2+a)^p(fx^2+d)^q dx^2+ Ax(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{80}$$

$$\frac{1}{2}B(d+fx^2)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\int(cx^2+a)^p\left(\frac{cfx^2}{cd-af}+\frac{cd}{cd-af}\right)^q dx^2+ Ax(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)$$

$$\Big\downarrow \text{79}$$

$$Ax(a+cx^2)^p\left(\frac{cx^2}{a}+1\right)^{-p}(d+fx^2)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+ \frac{B(a+cx^2)^{p+1}(d+fx^2)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{f(cx^2+a)}{cd-af}\right)}{2c(p+1)}$$

input
```
Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

output
```
(A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a),
-((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 +
 p)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c
*d - a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

3.399. $\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$

3.399.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.399. $\int (A + Bx) (a + cx^2)^p (d + fx^2)^q \, dx$

3.399.4 Maple [F]

$$\int (Bx+A)\left(c\,x^{2}+a\right)^{p}\left(f\,x^{2}+d\right)^{q}dx$$

input
```
int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

output
```
int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

3.399.5 Fricas [F]

$$\int (A+Bx)\left(a+cx^{2}\right)^{p}\left(d+fx^{2}\right)^{q}\,dx=\int (Bx+A)(cx^{2}+a)^{p}(fx^{2}+d)^{q}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int (A+Bx)\left(a+cx^{2}\right)^{p}\left(d+fx^{2}\right)^{q}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
```

output
```
Timed out
```

3.399.7 Maxima [F]

$$\int (A+Bx) \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx = \int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q \, dx$$

input
```
integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.399.8 Giac [F]

$$\int (A+Bx) \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx = \int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q \, dx$$

input
```
integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int (A+Bx) \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx = \int \left(c\,x^2+a\right)^p\,\left(f\,x^2+d\right)^q\,\left(A+B\,x\right) \, dx$$

input
```
int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x),x)
```

output
```
int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x), x)
```

3.400 $\int (a+cx^2)^p (A+Bx+Cx^2)(d+fx^2)^q\,dx$

3.400.1 Optimal result

Integrand size = 29, antiderivative size = 252

$$\int (a+cx^2)^p (A+Bx+Cx^2)(d+fx^2)^q\,dx = Ax(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3(a+cx^2)^p\left(1+\frac{cx^2}{a}\right)^{-p}(d+fx^2)^q\left(1+\frac{fx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B(a+cx^2)^{1+p}(d+fx^2)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{f(a+cx^2)}{cd-af}\right)}{2c(1+p)}$$

output
```
A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((1+
c*x^2/a)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2
,-p,-q,5/2,-c*x^2/a,-f*x^2/d)/((1+c*x^2/a)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2
+a)^(p+1)*(f*x^2+d)^q*hypergeom([-q, p+1],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c
/(p+1)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

3.400.2 Mathematica [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.20

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \,dx = \frac{1}{6}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(3Bx\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(1,-p,-q,2,-\frac{cx^2}{a},-\frac{fx^2}{d}\right) + \frac{18aAd\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{3ad\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+2x^2\left(cdp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afq\,\text{Ap}\right.} + 2Cx^2\left(1+\frac{cx^2}{a}\right)^{-p}\left(1+\frac{fx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)$$

input
```
Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

output
```
(x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF
1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p,
-q, 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -
q, 5/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2,
-((c*x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x
^2)/a), -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

3.400.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(A\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Cx^2\left(a+cx^2\right)^p \left(d+fx^2\right)^q\right) dx$$

$$\Big\downarrow \text{2009}$$

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(d+fx^2)}{cd-af}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{f(cx^2+a)}{cd-af}\right)}{2c(p+1)}$$

input
```
Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

output
```
(A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a),
-((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^
p*(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3
*(1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)
^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2
*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

3.400.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.400.4 Maple [F]

$$\int \left(c\,x^2+a\right)^p\left(C\,x^2+Bx+A\right)\left(f\,x^2+d\right)^q\,dx$$

input
```
int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

output
```
int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

3.400.5 Fricas [F]

$$\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q dx = \int (Cx^2+Bx+A)(cx^2+a)^p (fx^2+d)^q dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

output
```
Timed out
```

3.400.7 Maxima [F]

$$\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q dx = \int (Cx^2+Bx+A)(cx^2+a)^p (fx^2+d)^q dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.400.8 Giac [F]

$$\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q \, dx = \int (Cx^2+Bx+A)(cx^2+a)^p(fx^2+d)^q \, dx$$

input
```
integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int (a+cx^2)^p (A+Bx+Cx^2) (d+fx^2)^q \, dx = \int (c\,x^2+a)^p \left(f\,x^2+d\right)^q \left(C\,x^2+B\,x+A\right) \, dx$$

input
```
int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2),x)
```

output
```
int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```